Updating on progress made since the last publication:
M. de Angelis, J. Forshaw, S. Plätzer (2020) [2007.09648]

Developing an amplitude level parton shower – CVolver

Fernando Torre González

With Jeff Forshaw and Simon Plätzer

The University of Manchester

PSR24

Described in detail in Angeles Martinez, de Angelis, Forshaw, Plätzer, Seymour [1802.08531]

The soft evolution algorithm

We dress the hard process density matrix with iterative real and virtual operators:

$$\begin{aligned}
\sigma_0 &= \mathrm{Tr}\left(\mathbf{V}_{\mu,Q}\mathbf{H}(Q)\mathbf{V}^\dagger_{\mu,Q}\right) \equiv \mathrm{Tr}\,\mathbf{A}_0(\mu) \\
\mathrm{d}\sigma_1 &= \mathrm{Tr}\left(\mathbf{V}_{\mu,E_1}\mathbf{D}^\mu_1\mathbf{V}_{E_1,Q}\mathbf{H}(Q)\mathbf{V}^\dagger_{E_1,Q}\mathbf{D}^\dagger_{1\mu}\mathbf{V}^\dagger_{\mu,E_1}\right)\mathrm{d}\Pi_1 \\
&\equiv \mathrm{Tr}\,\mathbf{A}_1(\mu)\mathrm{d}\Pi_1, \\
\mathrm{d}\sigma_2 &= \mathrm{Tr}\left(\mathbf{V}_{\mu,E_2}\mathbf{D}^\nu_2\mathbf{V}_{E_2,E_1}\mathbf{D}^\mu_1\mathbf{V}_{E_1,Q}\mathbf{H}(Q)\mathbf{V}^\dagger_{E_1,Q}\mathbf{D}^\dagger_{1\mu}\mathbf{V}^\dagger_{E_2,E_1}\mathbf{D}^\dagger_{2\nu}\mathbf{V}^\dagger_{\mu,E_2}\right)\mathrm{d}\Pi_1\mathrm{d}\Pi_2 \\
&\equiv \mathrm{Tr}\,\mathbf{A}_2(\mu)\mathrm{d}\Pi_1\mathrm{d}\Pi_2 \\
&\vdots \\
\mathrm{d}\sigma_n &= \mathrm{Tr}\,\mathbf{A}_n(\mu)\prod_{i=1}^{n}\mathrm{d}\Pi_i
\end{aligned}$$

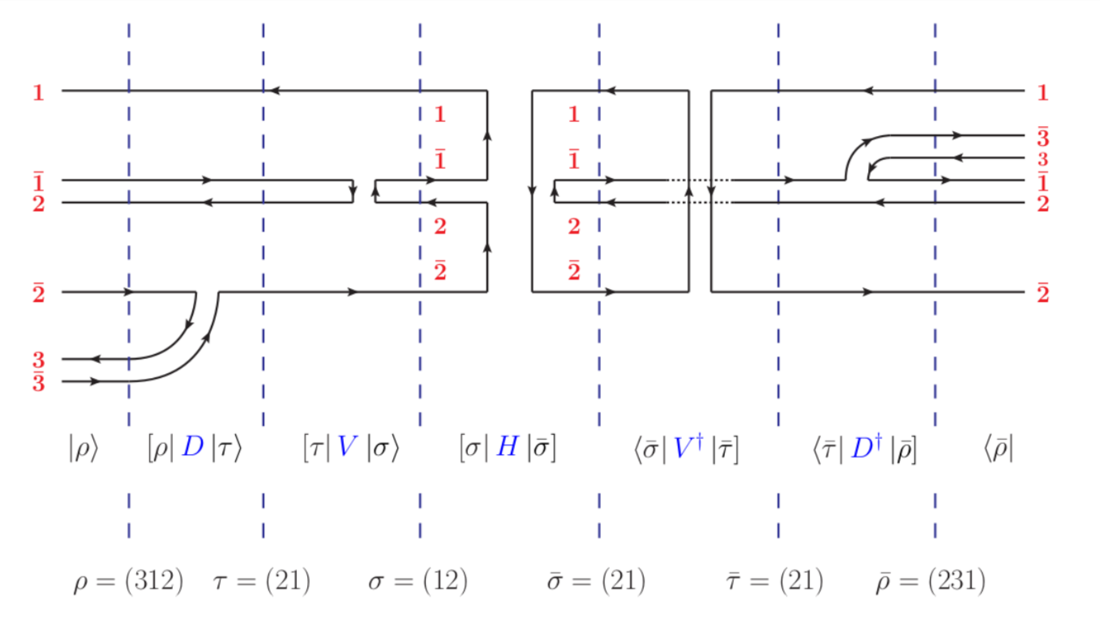
$$\mathbf{D}^\mu_i = \sum_j \mathbf{T}_j \frac{n^\mu_j}{n_j\cdot n_i}$$

$$\mathbf{V}_{a,b} = \exp\left(-\frac{\alpha_s}{\pi}\ln\left(\frac{b}{a}\right)\sum_{i<j}(-\mathbf{T}_i\cdot\mathbf{T}_j)\int\frac{\mathrm{d}\Omega_k}{4\pi}\omega_{ij}(\hat{k})\right)$$

By working in the colour flow basis, we can expand the exponentiated anomalous dimension as a series in $1/N_c$ Plätzer [1312.2448]:

$$[\tau|\,\mathbf{V}_{E,E'}\,|\sigma\rangle \simeq \delta_{\tau\sigma}R(\{\sigma\}) + \sum_{l=1}^{d}\left(-\frac{1}{N_c}\right)^l \sum_{\{\sigma_0,\ldots,\sigma_l\}}\delta_{\tau\sigma_0}\delta_{\sigma_l,\sigma}\left(\prod_{\alpha=0}^{l-1}\Sigma_{\sigma_{\alpha+1},\sigma_\alpha}\right)R(\{\sigma_0,\ldots,\sigma_l\})$$

Keeping track of colour



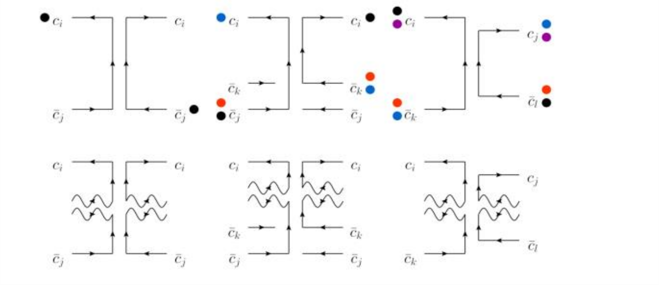
At each step in the evolution, the colour state after the action of the real and virtual operators is sampled. This defines each event as a trajectory in colour space.

We count every factor of $1/N_c$ included at each step. There are four possible sources: the reals, the virtuals, the scalar product of the final colour states, and the hard process.

Thus, we can expand the cross-section in this way:

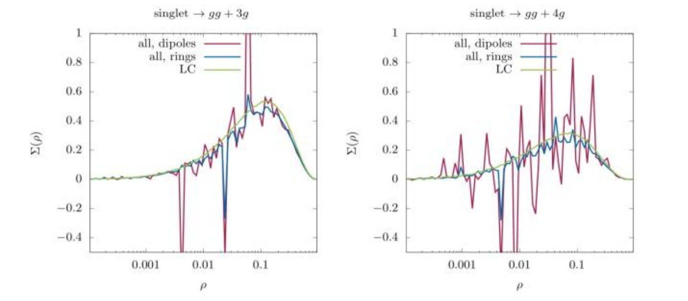
$$\sum_m N_c^m g_m \left(C_{0,m} + \frac{C_{1,m}}{N_c} + \frac{C_{2,m}}{N_c^2} + \ldots \right)$$

Significant developments



- Implemented "rings and strings" in the calculation of real emission matrix elements, which exploit the collinear-finiteness of subleading colour.

  See Forshaw, Holguin, Plätzer [2112.13124]



- Switched from an angular collinear cutoff to a rapidity-type cutoff.

- Added functionality to write event data in HDF5 format, allowing for reanalysis of data and reducing computing time significantly.

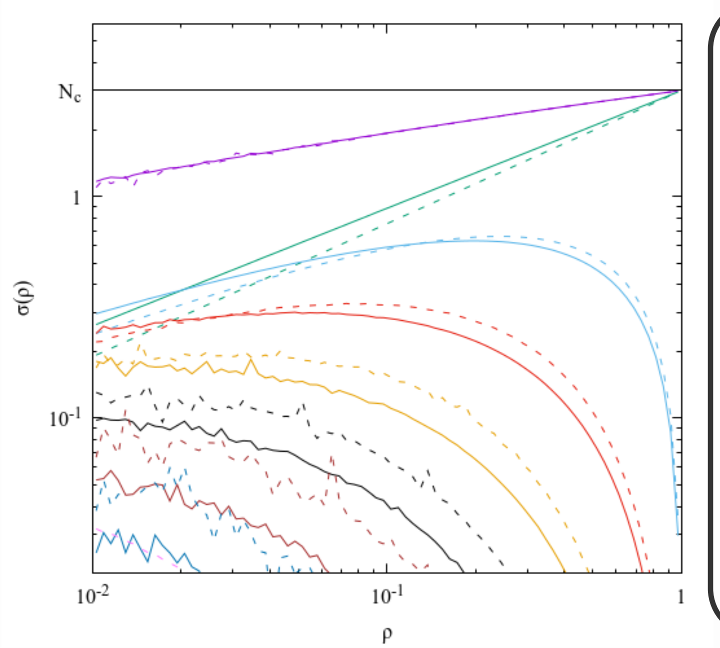
- A lot of different improvements to the sampling – notably the ability to count colour orders and specify the desired colour accuracy of the evolution.

$$\sum_m N_c^m g_m \left(C_{0,m} + \frac{C_{1,m}}{N_c} + \frac{C_{2,m}}{N_c^2} + \dots \right)$$

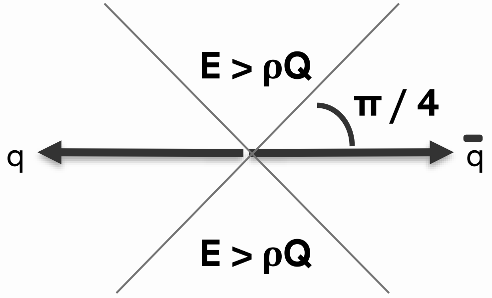
We could force the evolution to stop here, for example

$e^+e^- \rightarrow q\bar{q}$ | gaps between jets

Collinear cutoff agreement



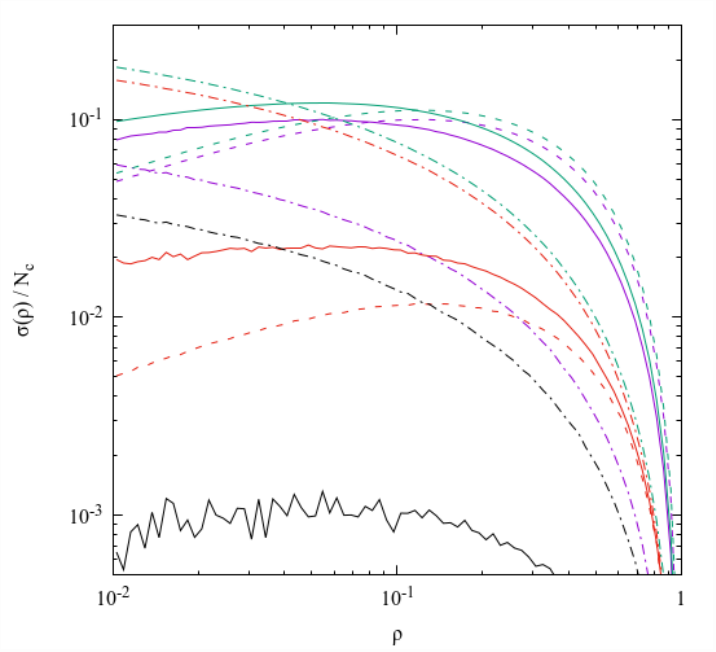
Full cross-section
0 emissions
1 emission
2 emissions
3 emissions
4 emissions
5 emissions
6 emissions
8 emissions



Collinear cutoff: 0.01
Collinear cutoff: 0.005

$e^+e^- \to q\bar{q}$ | gaps between jets

Breakdown in powers of $1/N_c$ for a specific multiplicity



Full colour

LC'

$-(1 / N_c^2)$: NLC'

$(1 / N_c^4)$: NNLC'

$N_c = 3$

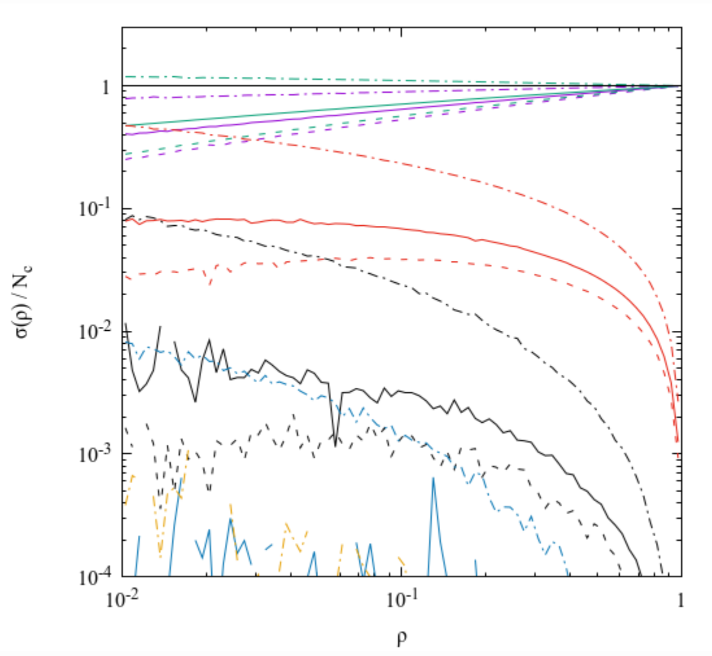
$N_c = 4$

$N_c = \mathrm{sqrt}(2)$

$$N_c^2 g_2 \left(C_{0,2} + \frac{C_{1,2}}{N_c^2} + \frac{C_{2,2}}{N_c^4} \right)$$

$e^+e^- \rightarrow q\bar{q}$ | gaps between jets

Breakdown in powers of $1/N_c$ for the whole cross-section



Full colour

LC'

$-(1/N_c^2)$: NLC'

$(1/N_c^4)$: NNLC'

$-(1/N_c^6)$: N3LC'

$(1/N_c^8)$: N4LC'

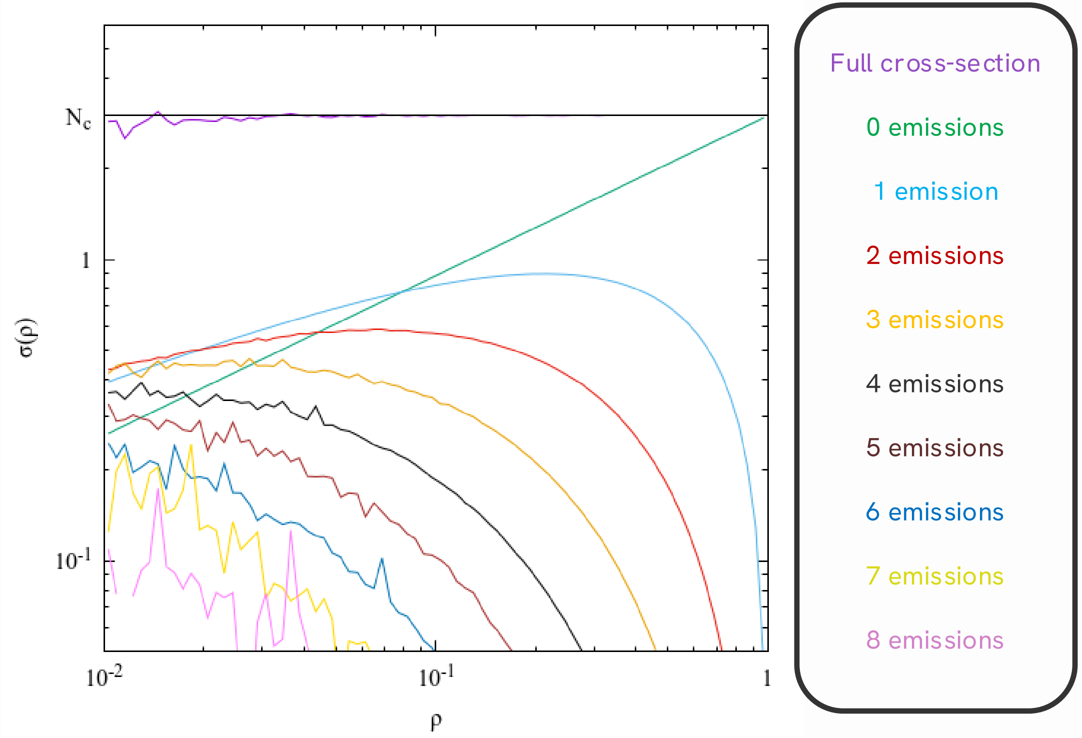
$$\sum_m N_c^m g_m \left(C_{0,m} + \frac{C_{1,m}}{N_c^2} + \frac{C_{2,m}}{N_c^4} + \frac{C_{3,m}}{N_c^6} + \frac{C_{4,m}}{N_c^8} + \ldots \right)$$

$N_c = 3$ (solid line)

$N_c = 4$ (dashed line)

$N_c = \mathrm{sqrt}(2)$ (dash-dotted line)

$e^+e^- \rightarrow q\bar{q}$ | unitarity check



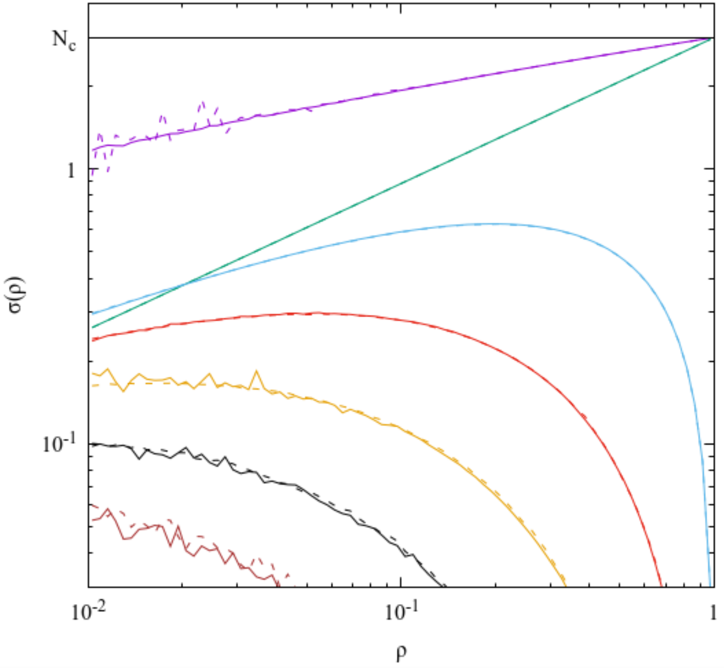
We can run the evolution fully inclusive, accepting every emission.

We expect all the virtual and real emissions to cancel, which serves as a sanity check

No gap...

$e^+e^- \to q\bar{q}$ | gaps between jets

Lorentz invariance



Full cross-section

0 emissions

1 emission

2 emissions

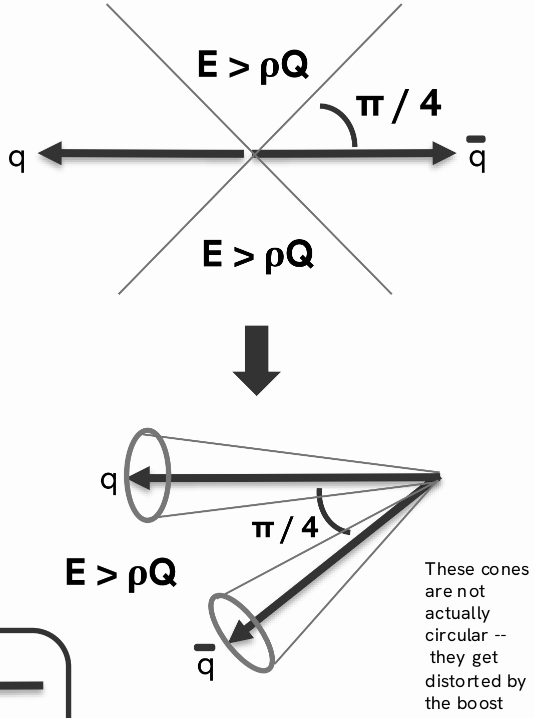
3 emissions

4 emissions

5 emissions

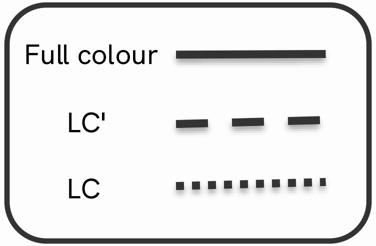
Back-to-back frame
collinear cutoff = 0.01

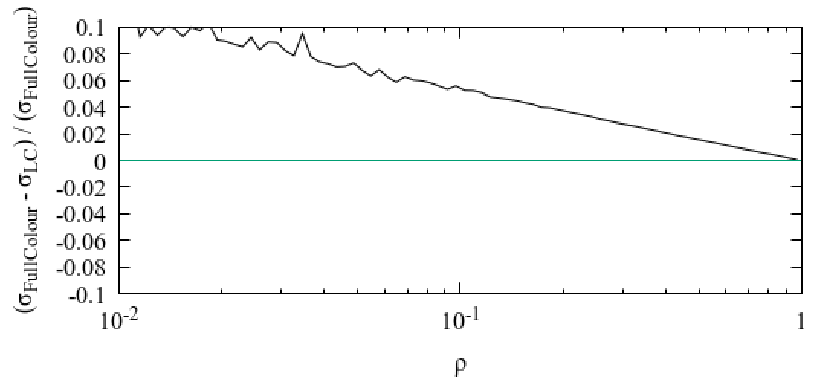
Boosted frame
collinear cutoff = 0.00146

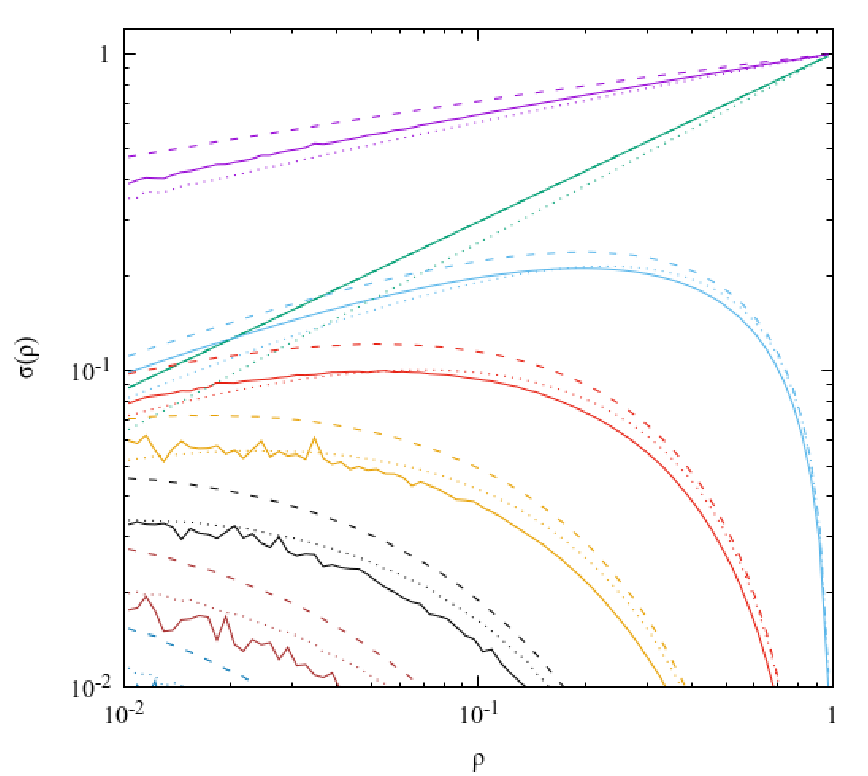


$e^+e^- \to q\bar{q}$ | gaps between jets

Effect of the subleading colour





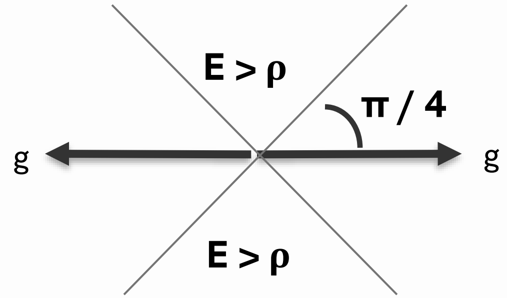


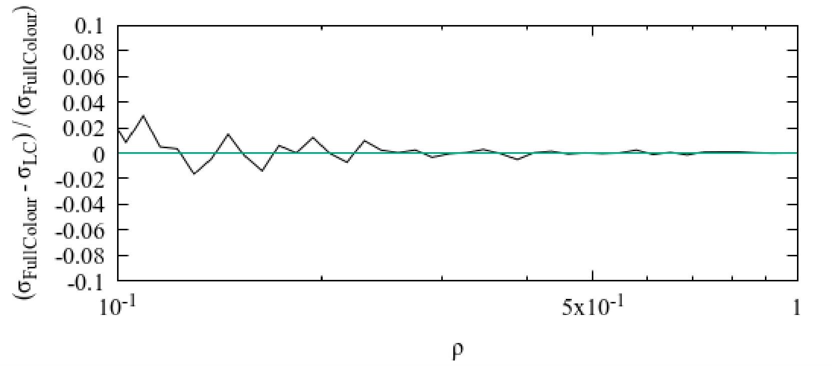
$e^+e^- \to gg$ | gaps between jets

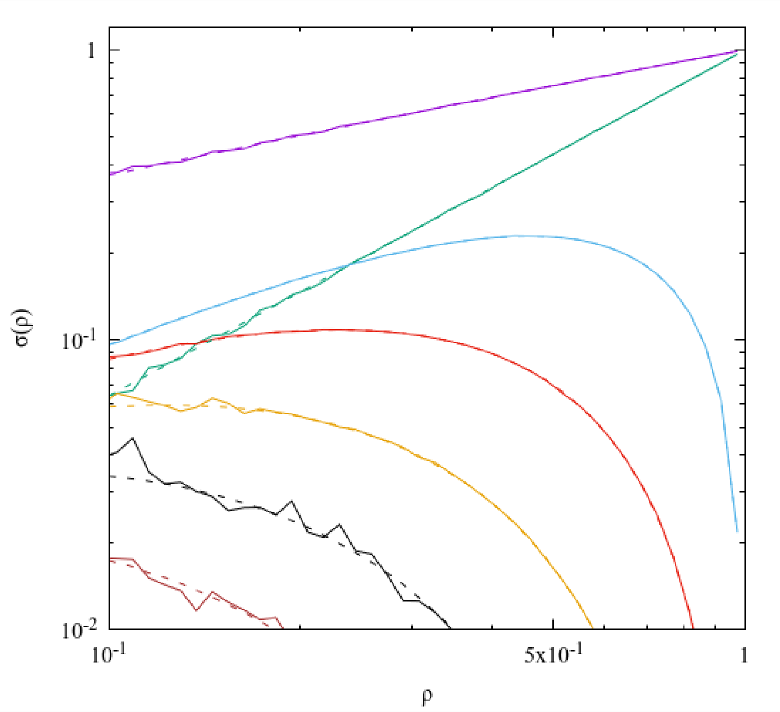
Effect of the subleading colour

Full colour (solid line)
LC (dashed line)

Full cross-section 0 emissions 1 emission
2 emissions 3 emissions 4 emissions 5 emissions

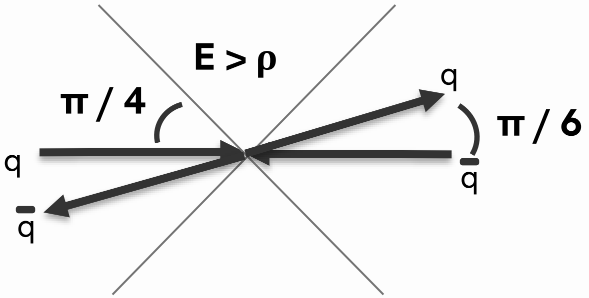


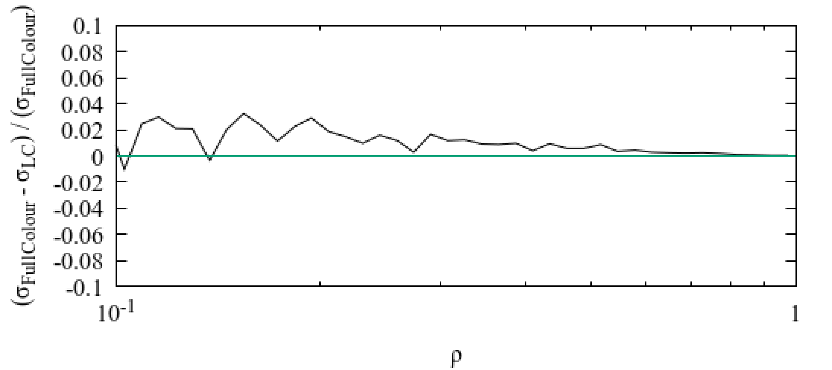


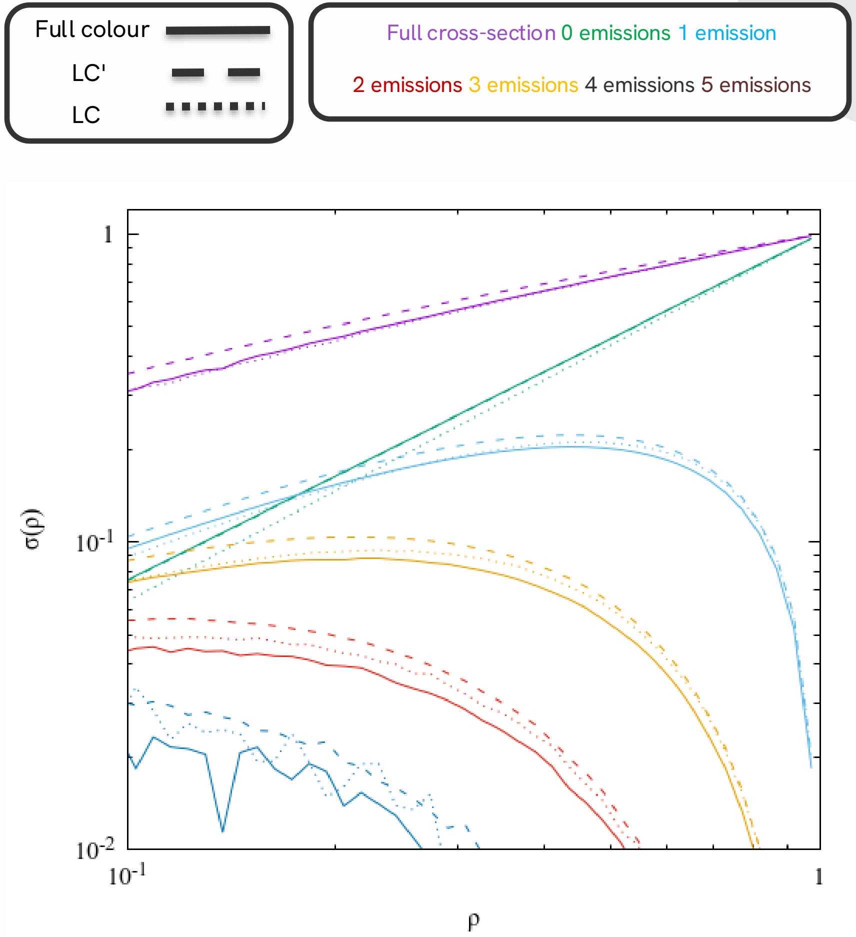


$q\bar{q} \to q\bar{q}$ | gaps between jets

Back-to-back configuration

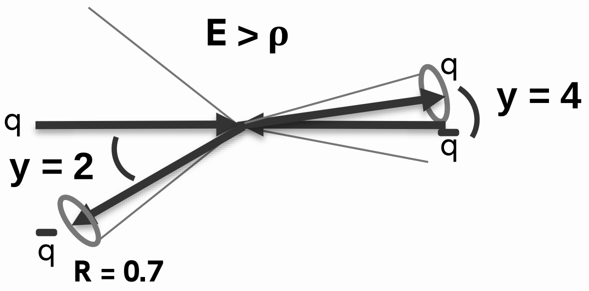


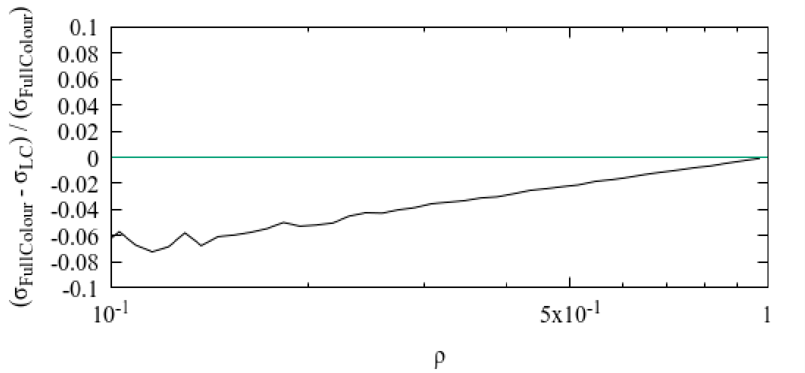


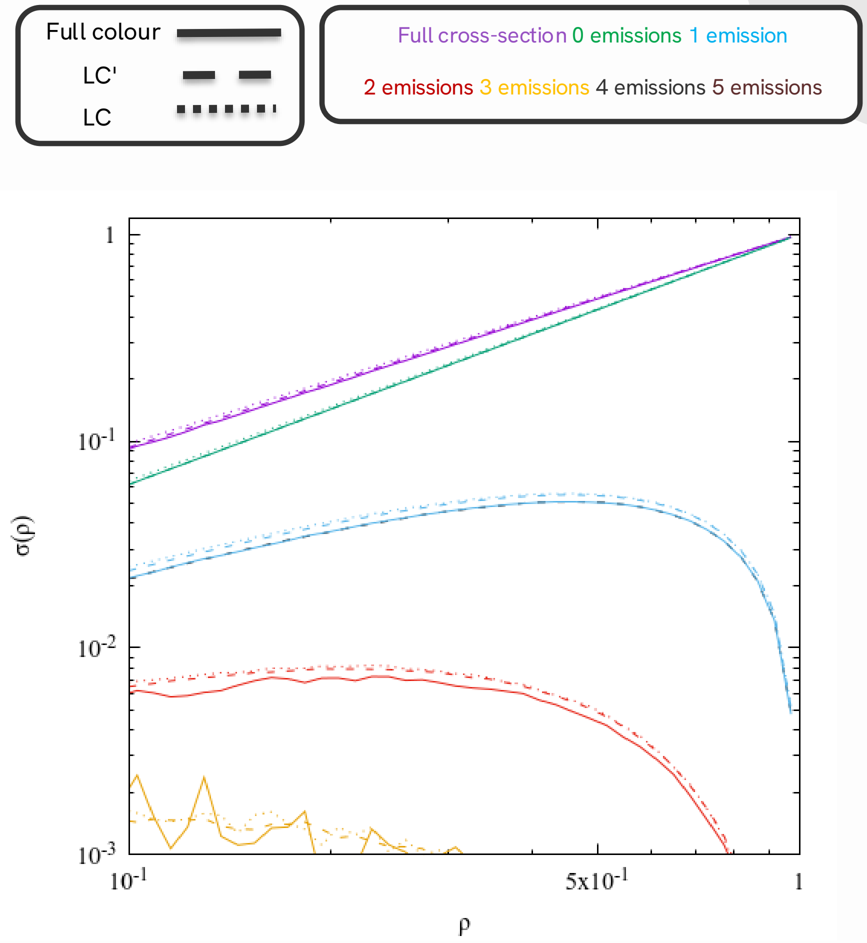


$q\bar{q} \to q\bar{q}$ | gaps between jets

Asymmetic final jet rapidities





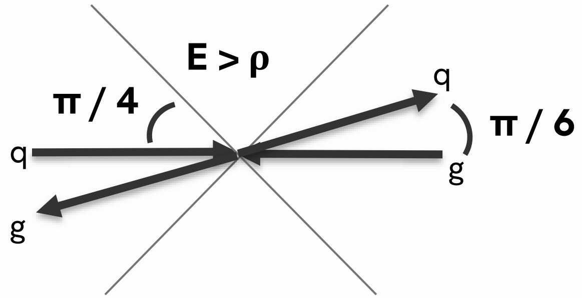


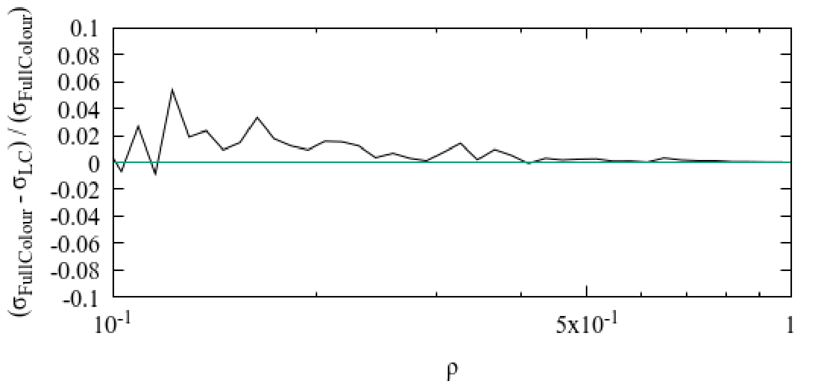
qg → qg | gaps between jets

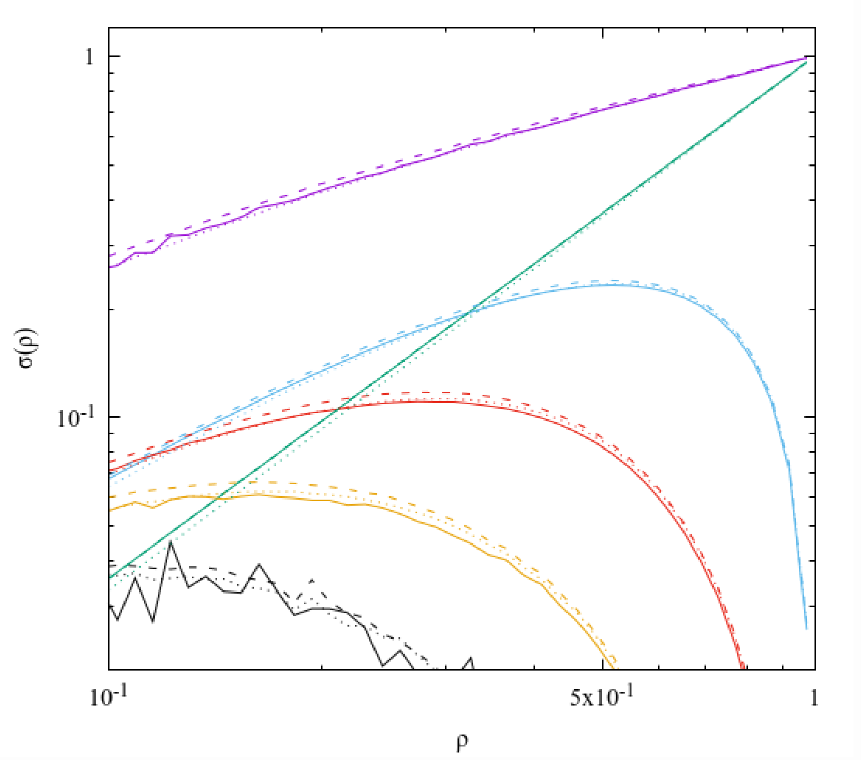
Back-to-back configuration

Full colour
LC'
LC

Full cross-section 0 emissions 1 emission
2 emissions 3 emissions 4 emissions 5 emissions

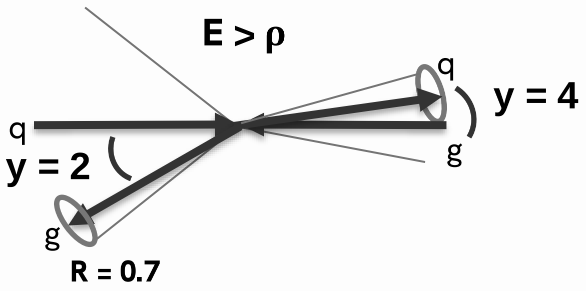


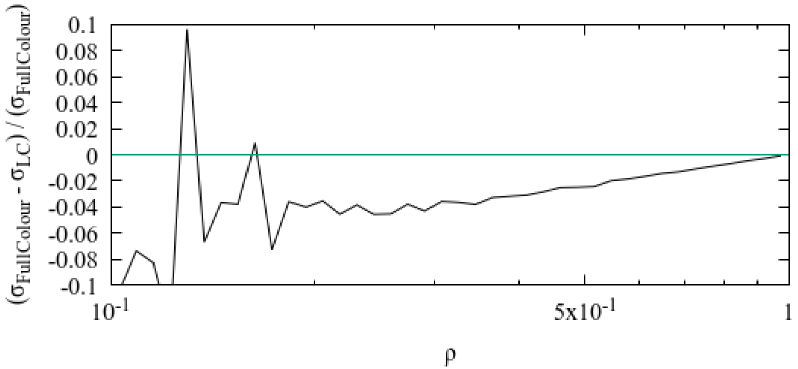


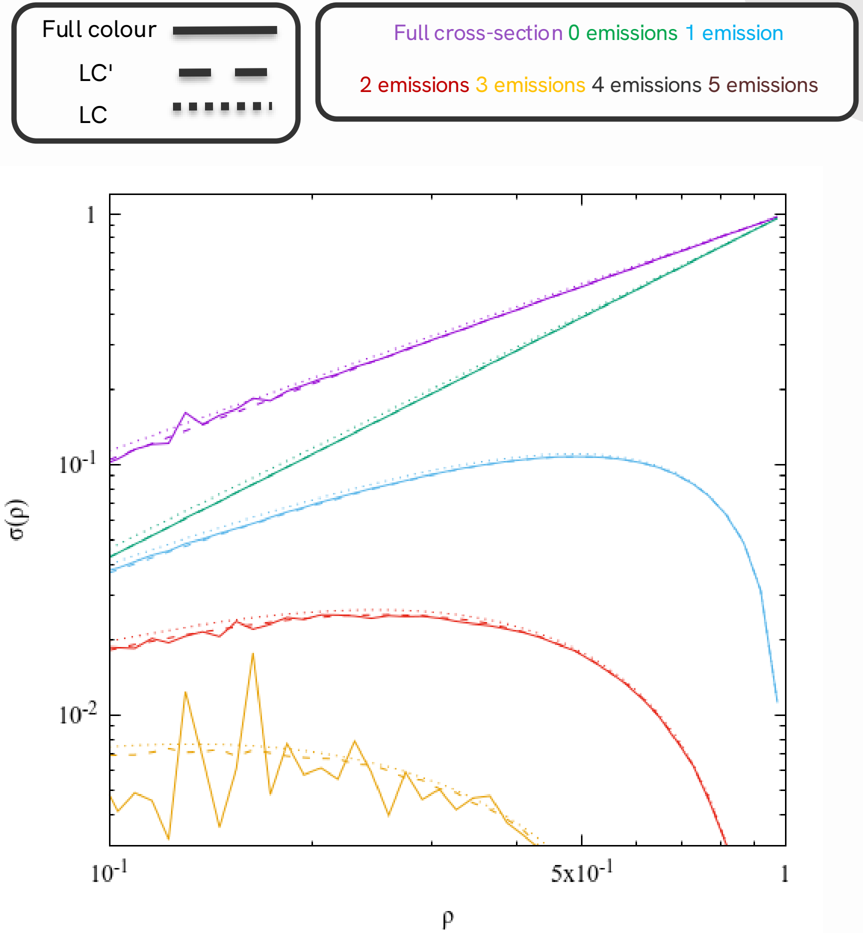


qg → qg | gaps between jets

Asymmetric final jet rapidities





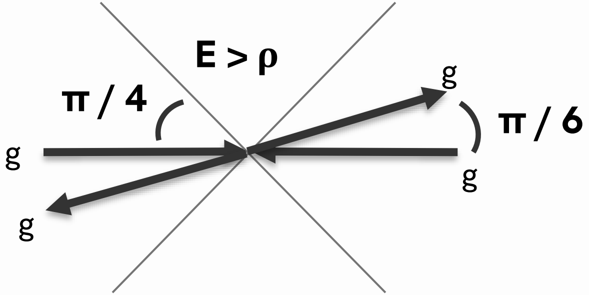


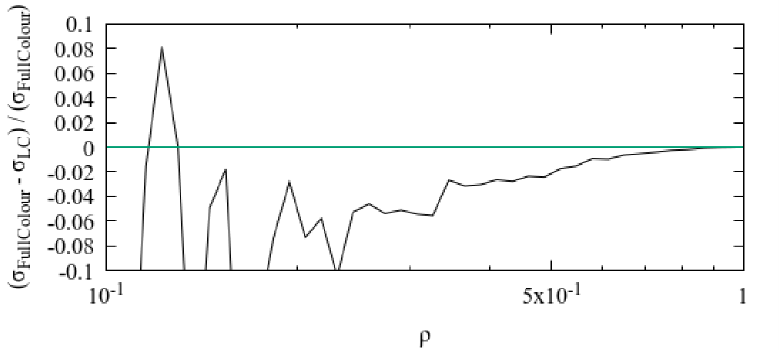
gg → gg | gaps between jets

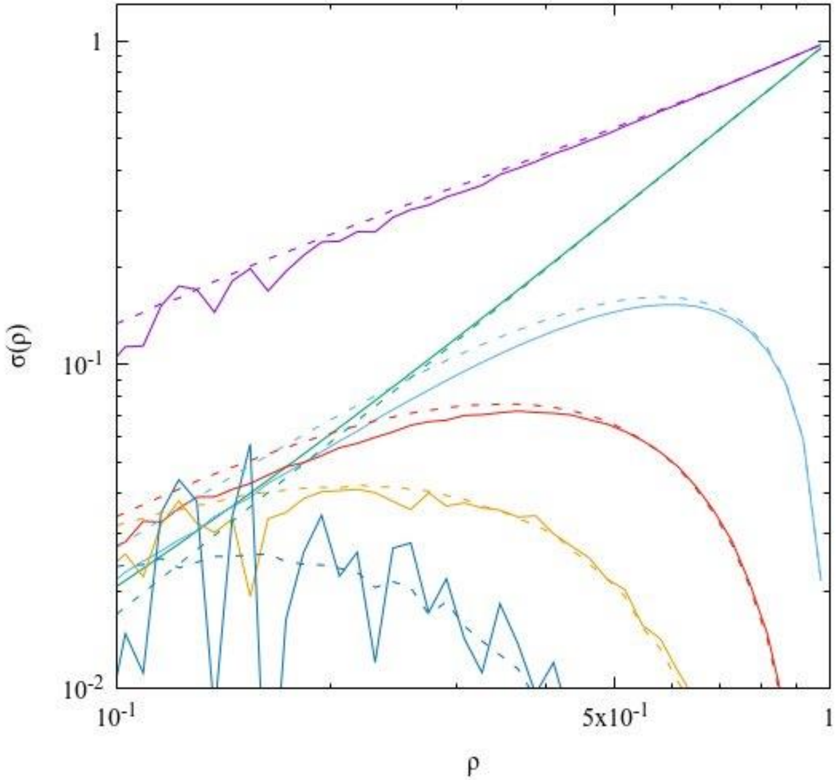
Back-to-back configuration

Full colour (solid line) LC (dashed line)

Full cross-section 0 emissions 1 emission
2 emissions 3 emissions 4 emissions 5 emissions

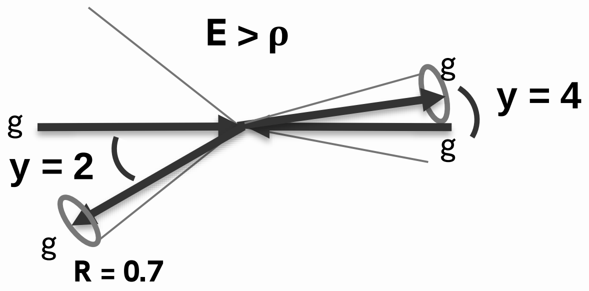


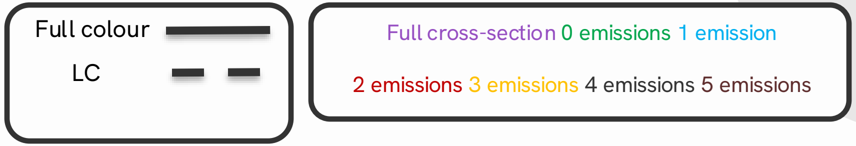


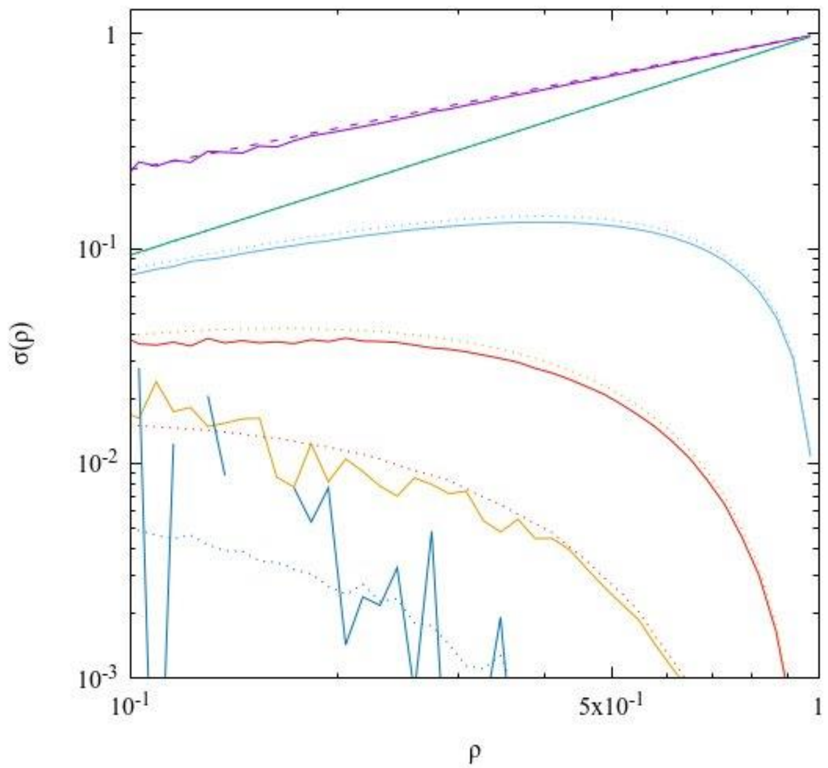


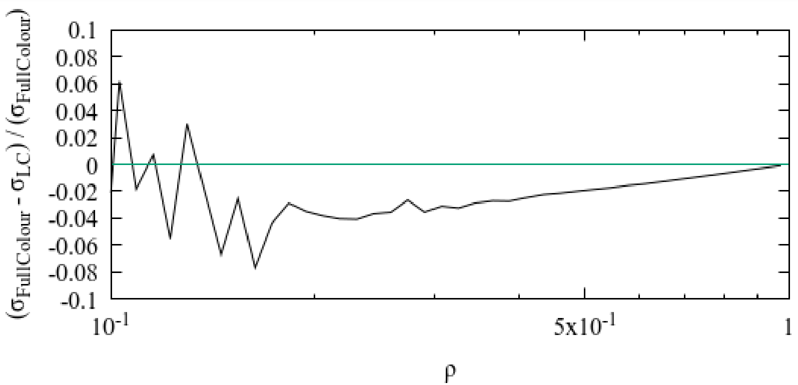
gg → gg | gaps between jets

Asymmetric final jet rapidities



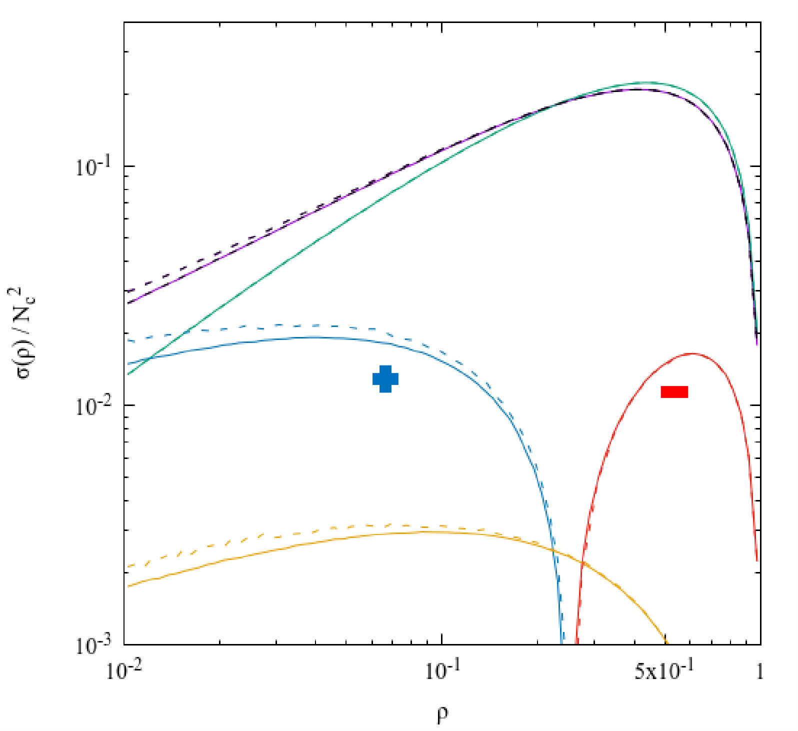






$q\bar{q} \to q\bar{q}$ | Coulomb exchanges

1 emission, broken down in powers of $1/N_c$



Full colour

LC'

$-(1 / N_c^2)$: NLC'

$+(1 / N_c^2)$: NLC'

$+(1 / N_c^4)$: NNLC'

No coulomb gluons

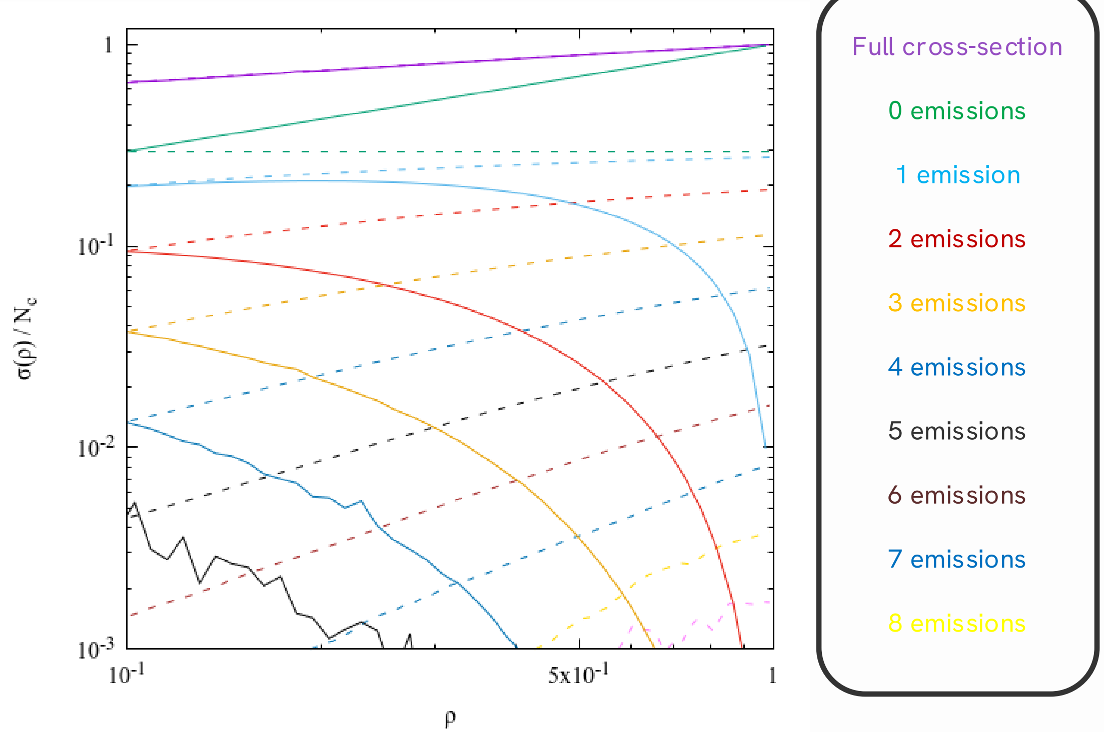
With coulomb gluons

We can simply insert the Coulomb/Glauber phases in the anomalous dimension, and the algorithm is able to resum them.

Perfect agreement with independent calculation.

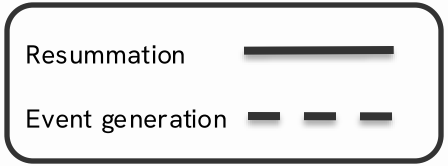
$$\mathbf{V}_{a,b} = \exp\left(-\frac{\alpha_s}{\pi}\ln\left(\frac{b}{a}\right)\sum_{i<j}(-\mathbf{T}_i\cdot\mathbf{T}_j)\left(\int\frac{\mathrm{d}\Omega_k}{4\pi}\omega_{ij}(\hat{k}) - i\pi\tilde{\delta}_{ij}\right)\right)$$

$e^+e^- \rightarrow \bar{q}q$ | event generator mode



Until now we have only discussed using CVolver as a resummation tool for gaps between jets.

But it can also be run as an event generator -- evolving all events down to some infra-red scale μ, and then applying the measurement function to generate a differential cross-section.

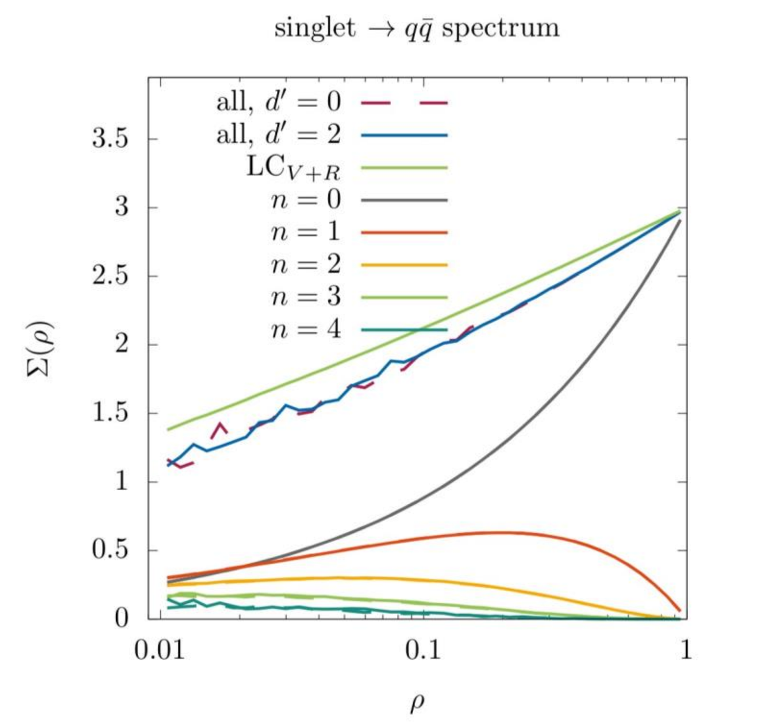


Summary and next steps

- We have implemented a full colour soft evolution algorithm that keeps track of every source of subleading colour in a systematic way.
- We have performed every cross-check we can think of.
- It has provided plenty of evidence that there is significant non-trivial subleading colour coming in at phenomenologically relevant scales.
- It is ready to produce lots of other interesting physics -- for example we can study differential observables in event generator mode (like colour reconnection) and Glauber effects.
- The algorithm design is such that we can "plug in" new features, which are under development:

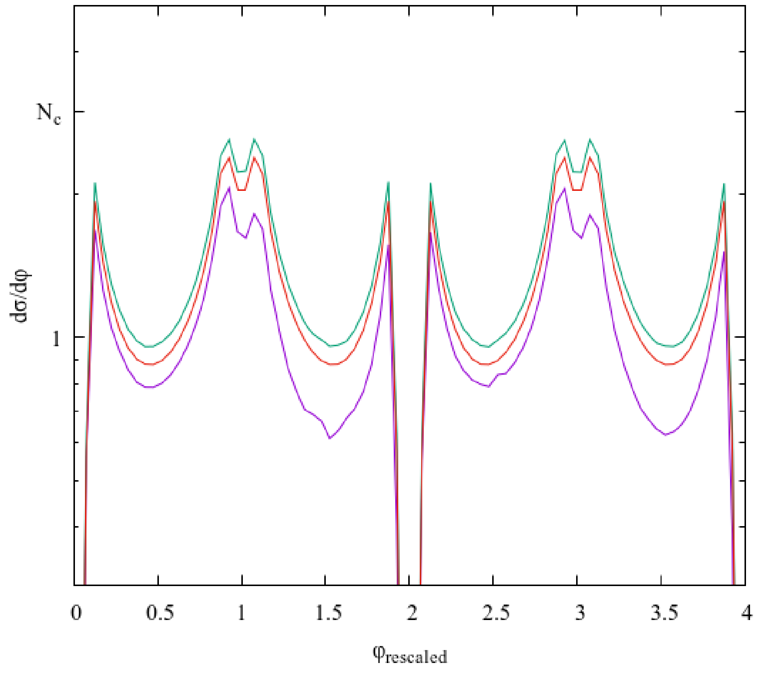
  - Full kinematics parton shower, with hard process elements.
  - k_t ordering.
  - Hard-collinear physics
    See Forshaw, Holguin, Plätzer [1905.08686]

Appendix: PRL plot for qqbar



$e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$

Colour reconnection



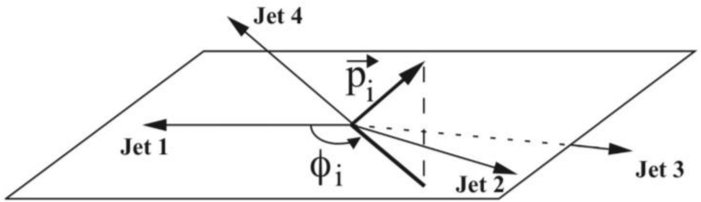


Figure taken from the L3 collaboration [hep-ex/0303042]

Full colour

LC'

LC