Dipole Showers with Global Recoil in Herwig 7

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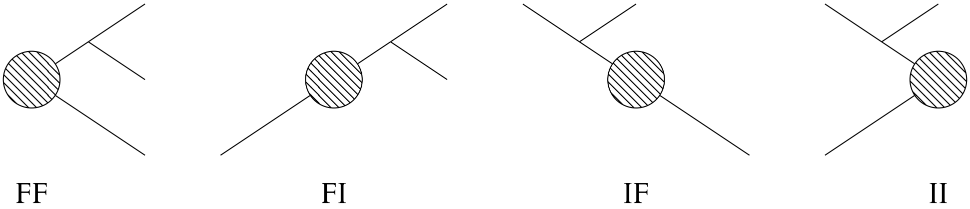
Parton Showers and Resummation, 2 July 2024

Catani-Seymour Dipole Showers

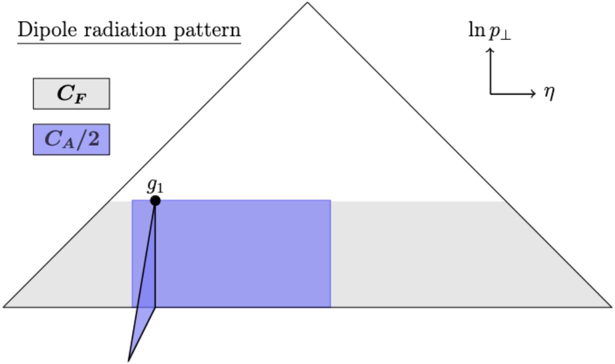
Aim: Formulate the Catani-Seymour dipole kernels and kinematics [hep-ph/9505323] into a parton shower.

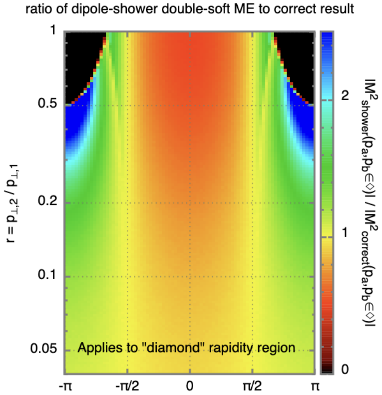
First implementations: Nagy and Soper [hep-ph/0601021], Dinsdale et. al. [0709.1026], Schumann and Krauss [0709.1027] and Plätzer and Gieseke [0909.5593]



Achieving NLC, NLL Accuracy

As seen in Dasgupta et al. [1805.09327]. See also [1904.11866, 2003.06400]





In this Talk

1. Mapping: “Building a Consistent Parton Shower”

2. Tools: “ExSample – A Library for Sampling Sudakov-Type Distributions”

3. Implementation: A Most-General Kinematics Framework

4. Current Progress and Next Steps

Mapping: “Building a Consistent Parton Shower”

Preliminary: The Catani-Seymour Mapping

Looking at Final-Final Case,

$$p_i + p_j = q_i + q + q_j \tag{1}$$

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Looking at Final-Final Case,

$$p_i + p_j = q_i + q + q_j \tag{1}$$

Define Post-Emission Momenta

$$\begin{aligned} q_i &= z\, p_i + y(1-z)\, p_j + k_\perp \\ q &= (1-z)\, p_i + yz\, p_j - k_\perp \\ q_j &= + (1-y)\, p_j \end{aligned} \tag{2}$$

Evolution with $k_\perp$, splitting with z and

$$y = \frac{k_\perp^2}{2p_i.p_j}\frac{1}{z(1-z)} \tag{3}$$

An Introduction to Global Recoil

Rather than resolving the transverse and backward momentum with the particles of the dipole, apply Lorentz boosts and rescaling to conserve them using **all** particles.

As seen in $1 \to 2$ showers, like the Angular-Ordered Shower [0803.0883].

An Introduction to Global Recoil

Credit: M. C. van Beekveld [Indico:1233329]

Possible NLL dipole-shower solutions for e^+e^-

		Ordering	Kinematic map: Dipole-local	Kinematic map: Global	Tests
PanScales showers [2002.11114]	**PanLocal** (Dipole and antenna)	$0 < \beta < 1$	$+, -, \perp$		Fixed- and all-order numerical tests for different observables for e^+e^- and pp (colour singlet) + NNLL e^+e^- (2024)
	PanGlobal	$0 \leq \beta < 1$	$+, -$	$\perp$	
Alaric [2208.06057]		$\beta = 0$	$+$	$-, \perp$	Numerical tests for global event shapes + CEASAR Style analytical tests
Deductor [2011.04777]	**Deductor** k_t	$\beta = 0$	$+$ (Also formulation with $+, -, \perp$)	$-, \perp$	Analytical and to some extent numerical for thrust
	Deductor Λ	$\beta = 1$	$+$	$-, \perp$	
Manchester-Graz [2003.06400]		$\beta = 0$	$+$	$-, \perp$	Analytical for thrust and multiplicity

Showers also differ on the implementation of the splitting functions and how the global imbalance is redistributed

1 **All have different approaches to assess NLL accuracy**

The Forshaw-Holguin-Plätzer Algorithm

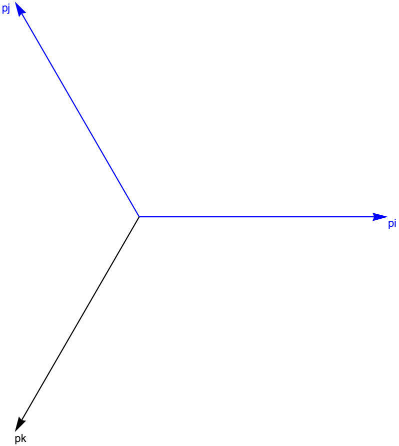
Reference: [2003.06400]

$$\begin{aligned} q_i &= z\, p_i \\ q &= (1-z)\, p_i + yz\, p_j - k_\perp \\ q_j &= p_j \end{aligned} \tag{4}$$

Boost-Rescaling: $p \to \alpha\Lambda p$:

$$\begin{gathered} \Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \frac{2\alpha}{Q''^2} Q''^\mu N''_\nu - \frac{2(Q''^\mu + \alpha N''^\mu)(Q''_\nu + \alpha N''_\nu)}{Q''^2 + \alpha^2 N''^2 + 2\alpha Q'' N''} \, . \\ Q = \sum_l p_l \, , \quad N = Q + yzp_j - k_\perp \, , \quad \alpha = \sqrt{Q^2/N^2} \end{gathered} \tag{5}$$

An Example Emission



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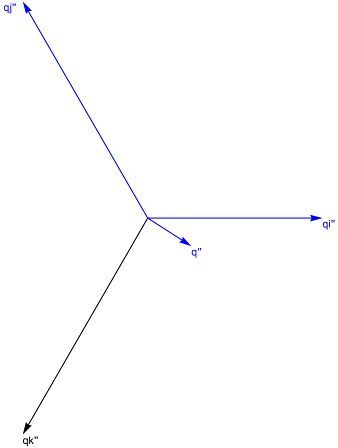

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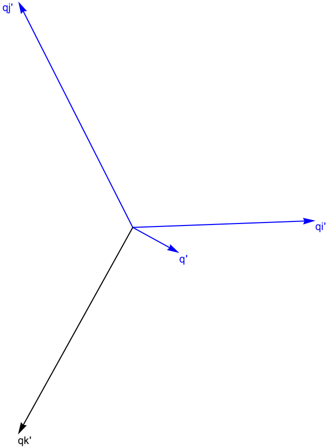

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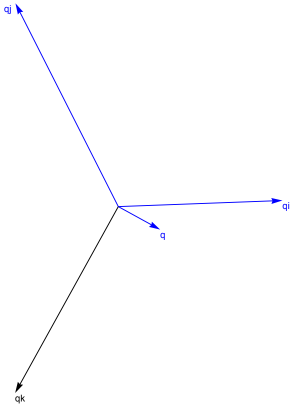
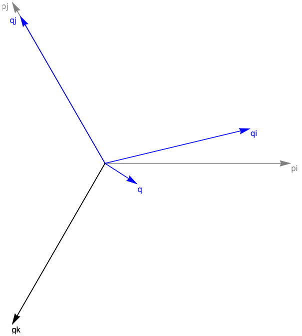
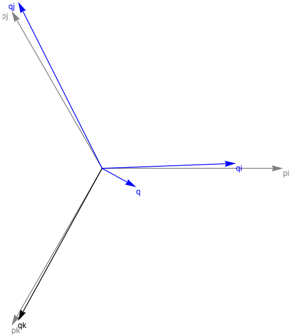


An Example Emission

Catani-Seymour



Forshaw-Holguin-Plätzer



Mathematica Notebook attached to Indico!

Tools: “ExSample – A Library for Sampling Sudakov-Type Distributions”

Motivation - Towards Sophisticated Kernels

Veto Alg: Generate an Overestimate R for a Splitting Kernel P.

- $R \geq P$ everywhere
- R must be of simple form

Can we continue to manually define R for more complicated P? What about numerically defined P, or with PDF ratios?

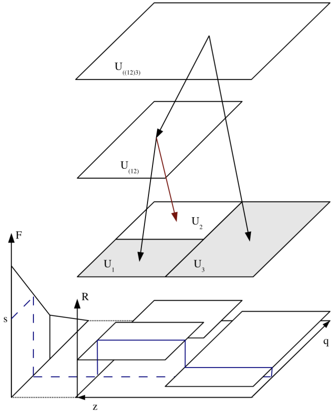
Introducing ExSample

Work by S. Plätzer. Reference: [1108.6182]

Generate Adaptive Overestimate of Splitting Kernel using Phase Space Sampling Methods.

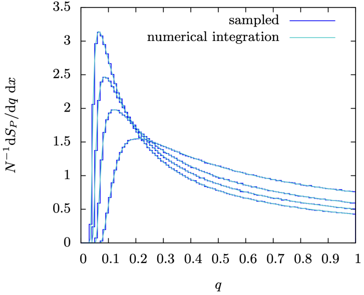
Uses Cell Grid Sampling Methods, as seen in ACDC [ThePEG Reference] and FOAM [physics/0203033].

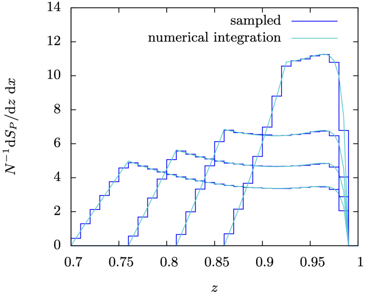
Overestimate distribution R evolves over time, and unexpected overestimates are accounted for by oversampling



Validation and Results

Validated for Toy Kernels, as well as Parton Shower + POWHEG Implementation





Now, completely Integrated with the Herwig Dipole Shower Framework.

Implementation: A Most-General Kinematics Framework

Mappings with Adjustable Parameters

Work by C. B. Duncan and E. Simpson-Dore. Generalise the FHP Mapping using the parameters

$$\begin{aligned} q_i &= a_i \, p_i + b_i \, p_j + g f k_\perp \\ q &= a \, p_i + b \, p_j - k_\perp \\ q_j &= a_j \, p_i + b_j \, p_j + g(1-f) k_\perp \end{aligned} \tag{6}$$

Identical boost, with variables

$$Q = \sum_l p_l, \; N = \left(\sum a - 1\right) p_i + \left(\sum b - 1\right) p_j + (1-g) k_\perp, \; \alpha = \sqrt{\frac{Q^2}{N^2}} \tag{7}$$

in Herwig: Adjust Parameters through options like `KTBalanceFactor` ($= g$) and `SpecatatorBalenceFactor`

Mappings with Adjustable Parameters

Work by C. B. Duncan and E. Simpson-Dore. Generalise the FHP Mapping using the parameters

$$\begin{aligned} q_i &= (1-z) \, p_i + gfy(1-z) \, p_j + gf k_\perp \\ q &= z \, p_i + yz \, p_j - k_\perp \\ q_j &= \frac{(gfk_\perp)^2}{s_ij(1-yz\cdot\texttt{SBF})} \, p_i + (1-yz\cdot\texttt{SBF}) \, p_j + g(1-f) k_\perp \end{aligned} \tag{6}$$

Identical boost, with variables

$$Q = \sum_l p_l, \;\; N = \left(\sum a - 1\right) p_i + \left(\sum b - 1\right) p_j + (1-g) k_\perp, \;\; \alpha = \sqrt{\frac{Q^2}{N^2}} \tag{7}$$

in Herwig: Adjust Parameters through options like `KTBalanceFactor` $(= g)$ and `SpecatatorBalenceFactor`

Replicating Popular Mappings

FHP: `KTBalanceFactor` $= 0$, `SpectatorBalanceFactor` $= 0$

$$
\begin{aligned}
q_i &= z\, p_i \\
q &= (1-z)\, p_i + yz\, p_j - k_\perp \\
q_j &= + p_j
\end{aligned}
\tag{8}
$$

Replicating Popular Mappings

PanGlobal ($\beta = 0$) [2002.11114]: `KTBalanceFactor` $= 0$, `SpectatorBalanceFactor` $= 1$

$$
\begin{aligned}
q_i &= z \, p_i \\
q &= (1 - z) \, p_i + yz \, p_j - k_\perp \\
q_j &= + (1 - yz) \, p_j
\end{aligned}
\tag{9}
$$

Advantages and Next Steps

This allows the user freedom of choice on the recoil scheme, such that they can compare two generators with identical recoil schemes or study the differences between possible recoil schemes.

Progressing towards validating that NLL accuracy is maintained, and generalising phase space, jacobians, …

Current Progress and Next Steps

Where do we go from here?

Framework to implement global recoil is in place

- To do: combine all ingredients and validate $\rightarrow$ Pheno.
- As a cross-check of the workflow, try FHP and PanGlobal $(\beta = 0)$ Mappings, then replace them with General Map

Partitioning and the Splitting Kernel

For NLL Accuracy, Global Recoil needs to be accompanied with a longitudinal momentum conserving partitioning

For PanGlobal ($\beta = 0$), Multiply DGLAP Kernels $P(z)$ by

$$g^{\mathrm{PG}} = \frac{1}{1+e^{-2\eta}} = \frac{1}{1+\frac{p_j \cdot Q}{p_i \cdot Q} / \frac{p_j \cdot q}{p_i \cdot q}}\,, \quad \bar{\eta} = \frac{1}{2}\ln\left(\frac{p_j \cdot q}{p_i \cdot q}\right) - \frac{1}{2}\ln\left(\frac{p_j \cdot Q}{p_i \cdot Q}\right) \tag{10}$$

For FHP, (temporarily) veto emissions if

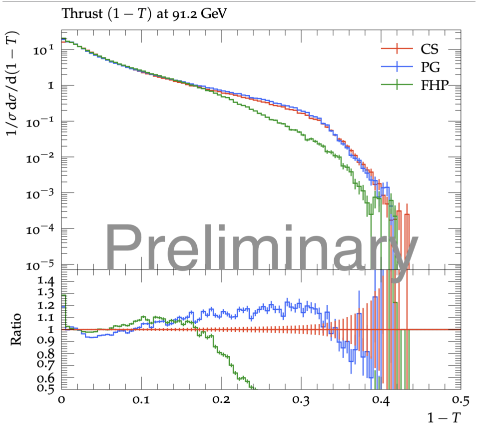
$$\frac{\vec{p_i} \cdot \vec{q}}{|\vec{p_i}||\vec{q}|} < \frac{\vec{p_j} \cdot \vec{q}}{|\vec{p_j}||\vec{q}|} \tag{11}$$

Preliminary Results - Thrust

Here, using consistent parameters

- $\alpha_S(m_Z) = 0.118$
- $t_{\text{cutoff}} = 1\,\text{GeV}^2$

Catani-Seymour and PanGlobal ($\beta = 0$) Agree. Due to the partitioning, FHP lacks high p_T emissions. Also seen in [2012.00622]



A Note on Sampling/Vetoing

Consider the upper limit on the splitting var, $z_+ = 1 - k_\perp/Q$. If $k_\perp^2 = Q^2(1-\varepsilon)$, we will see very small z limits. Furthermore,

$$\int \mathrm{d}\Phi_1 = \int_{k^2_{\perp,\mathrm{cutoff}}}^{Q^2} \mathrm{d}k_\perp^2 \int_0^{1-\frac{k_\perp}{Q}} \frac{\mathrm{d}z}{1-z} \mathcal{J}\left(\propto \frac{1}{z}\right) \dots \tag{12}$$

The divergence is accounted for analytically, BUT in a Monte Carlo, this could potentially give very large/very small numbers $\rightarrow$ potential cause of missing emissions

To avoid this, we may have to:

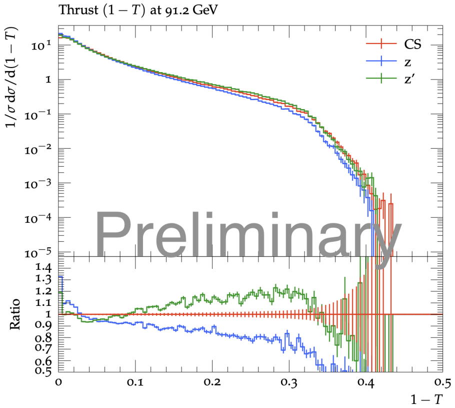
- Do some importance sampling, or sample a different distribution
- Move away from $k_\perp$ and z as shower variables [2312.13275]

A Note on Sampling/Vetoing

We used the substitution:

$$z \to z' = \frac{\left(1 - \frac{k_\perp^2}{Q^2}\right) z}{1 - \frac{k_\perp^2 z}{Q^2}} \qquad (13)$$

Here: PanGlobal $(\beta = 0)$



Summary

NLL Accurate Dipole Showers are an active area of research, and many different approaches are being studied.

In Herwig, we want to study how such a shower behaves and contributes to a general-purpose event generator.

Work has been done to arm users with the framework to choose their own global recoil maps.

With models and technology in place, we are now working towards completing the implementation and beginning validation and phenomenology.

...And That's All!

Thanks for Listening!

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