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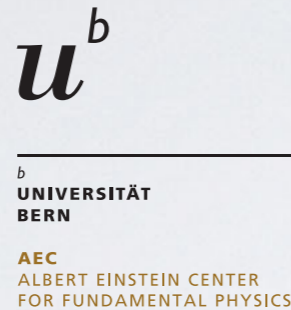
THE NNLO SOFT FUNCTION FOR N-JETTINESS IN HADRONIC COLLISIONS

Rudi Rahn

Based on [\[2312.11626\]](#) with Guido Bell,
Bahman Dehnadi, and Tobias Mohrmann

Parton Showers and Resummation,
Graz, 4th July 2024

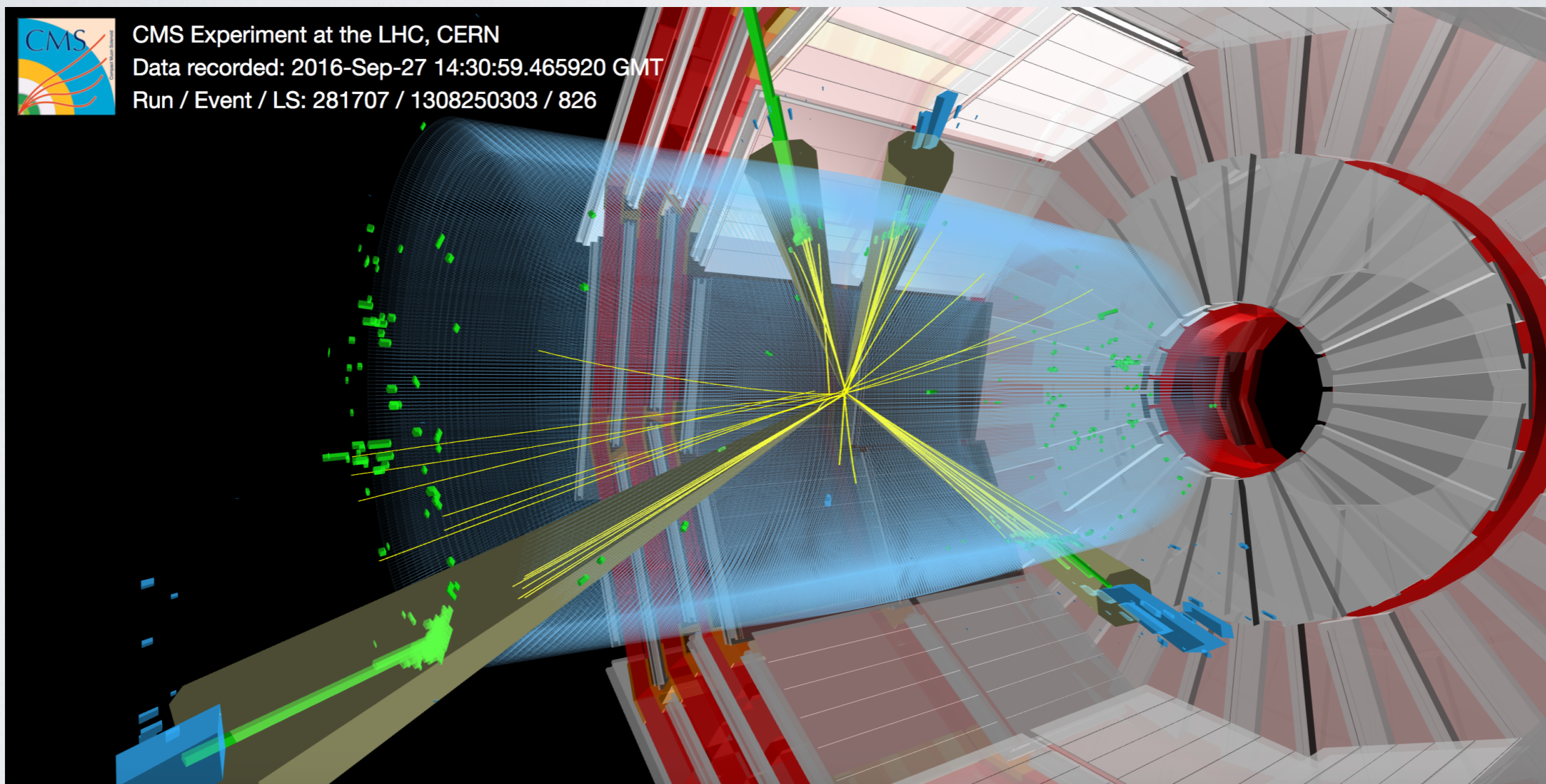
THE CRIME SCENES



OUTLINE

- Background
- Calculation:
SoftSERVE with N jets
- Results
- Endpoints

HOW MANY JETS ARE THERE?



N-JETTINESS

[Stewart, Tackmann, Waalewijn, '10]

- General definition:

$$\mathcal{T}_N = \sum_m \min_{\{q_j\}} \frac{2q_j \cdot k_m}{Q_j}$$

k_i^μ : Momenta of particles in the event

q_j^μ : Momenta of signal jets/beams

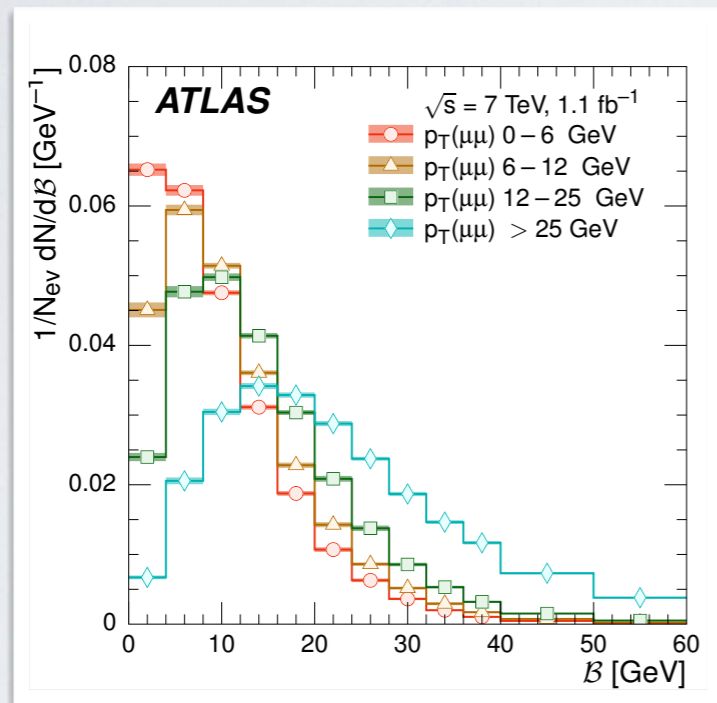
Q_j : Weight factors

- We choose $Q_j = 2E_j$, then $n_j = (1, \vec{n}_j)$, and

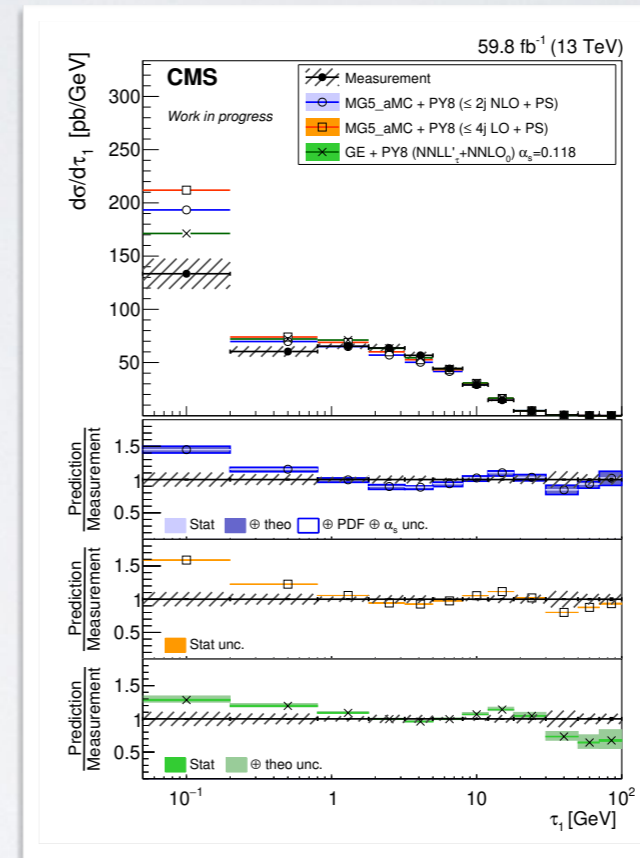
$$\mathcal{T}_N = \sum_m \min_j n_j \cdot k_m$$

UN-OBSERVED OBSERVABLE

- Few measurements



[Aad et al, '16]



[Mijušković, '22]

- Reason: hadronisation, pileup
- Modifications: Leptons (Thrust), DIS, tracks, ...

PHASE SPACE SLICING

[Boughezal, Focke, Liu, Petriello, '15]

[Gaunt, Stahlhofen, Tackmann, Walsh, '15]

- How to get **finite** NNLO cross section predictions?

$$\sigma_{VV} + \sigma_{RV} + \sigma_{RR} = \text{finite}$$

- *Slice* phase space to add unresolved reals to virtuals
- N-jettiness to **resolve** additional emissions

$$\sigma_{\text{NNLO}} = \sigma_{\text{NNLO}}(\mathcal{T}_N < \mathcal{T}_N^{\text{cut}}) + \sigma_{\text{NLO}+1}(\mathcal{T}_N > \mathcal{T}_N^{\text{cut}})$$


approximate using small $\mathcal{T}_N^{\text{cut}}$

STATE OF THE ART

$$\frac{d\sigma}{d\mathcal{T}_N} = \sum_{i,j,\{k_n\}} B_i \otimes B_j \otimes \prod_{n=1}^N J_{k_n} \otimes \text{tr}[H_{ij \rightarrow \{k_n\}} * S_{ij \rightarrow \{k_n\}}] + \mathcal{O}(\mathcal{T}_N)$$

- Hard, beam, and jet functions known to NNLO+
[Broggio et al., '14]
[Stewart et al., '10; Berger et al., '11; Gaunt et al., '14; Ebert et al., '20; Baranowski et al. '22]
[Bauer et al., '04; Bosch et al., '04; Becher et al., '06, '10, '11; Brüser et al., '18; Banerjee et al., '18]
 - NLO soft function, NNLO for 0-, 1-, partial 2-jettiness known
[Jouttenous et al., '11] [Kelley et al., '11; Monni et al., '11; Hornig et al., '17]
[Campbell et al., '18; Boughezal et al., '15]
[Bell et al., '18; Jin et al., '19]
- ⇒ Calculate NNLO Soft function for at least N=2
- Recently: recipe for NNLO N-jettiness (confirming/-ed)
[Agarwal, Melnikov, Pedron, '24]



[Bell, RR, Talbert, '18, '20]

- Soft functions can be calculated from

$$S(\tau, \epsilon) = \int d\Pi_i |\mathcal{A}(\{k_i\}, \epsilon)|^2 \delta(\tau - \tau(\{k_i\}))$$

$$S(\tau, \mu) = \frac{1}{N_c} \sum_X \mathcal{M}(\tau, k_i) \text{Tr} \langle 0 | S_{\bar{n}}^\dagger(0) S_n(0) | X \rangle \langle X | S_n^\dagger(0) S_{\bar{n}}(0) | 0 \rangle$$

- Amplitude is divergent, measurement harmless





Isolate divergences analytically,
do all the rest numerically

QUICK EXAMPLE: NLO

- Matrix element $|\mathcal{A}(k)|^2 \sim \frac{1}{k \cdot n \ k \cdot \bar{n}}$
- Expose divergences $k^\mu = k_T \sqrt{y} n^\mu + \frac{k_T}{\sqrt{y}} \bar{n}^\mu + \dots$
- Divergences exposed: $\int d^d k |\mathcal{A}|^2 \sim \int dk_T dy k_T^{-1-\epsilon} y^{-1}$ (Angles and observable omitted)
- Classify observable behaviour:

$$\tau(k) = k_T y^{\frac{n}{2}} f(y, \{\vartheta\})$$

Mass dimension   Leading behaviour

QUICK EXAMPLE: NLO

- Master formula $S_0(\tau) = 1 + \left(\frac{Z_\alpha \alpha_s}{4\pi} \right) (\mu^2 \bar{\tau}^2) S_R(\epsilon) + \mathcal{O}(\alpha_s^2)$

$$S_R(\epsilon) = \frac{16C_F e^{-\gamma_E \epsilon}}{\sqrt{\pi}} \frac{\Gamma(-2\epsilon)}{\Gamma(1/2 - \epsilon)} \int_0^1 dt \int_0^1 dy y^{-1+n\epsilon} f(y, t)^{2\epsilon} (4t(1-t))^{-1/2-\epsilon}$$

- Leads to a (bare) soft function involving integrals of f

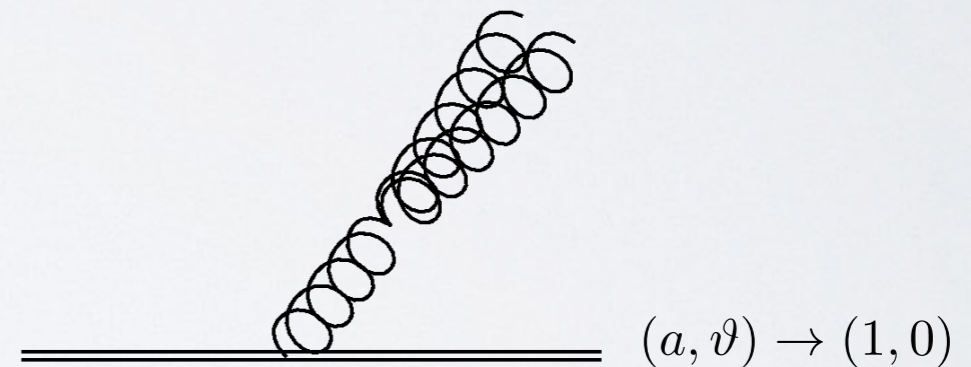
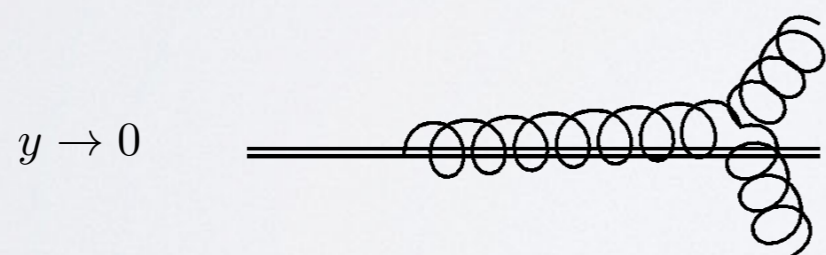
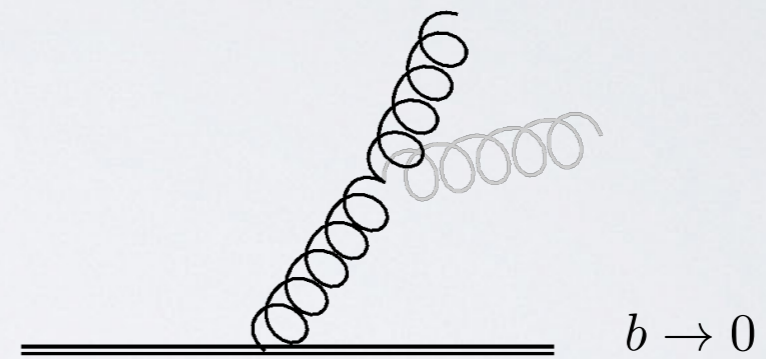
$$S_R(\epsilon) = \frac{-4C_F}{n\epsilon^2} - \frac{8C_F}{n\epsilon} \left[\int_0^1 dt \frac{\ln f(0, t)}{\pi \sqrt{t(1-t)}} + \dots \right] + \dots$$

- SoftSERVE wraps this in C++, and integrates using Cuba

[Hahn, '05]

TWO EMISSIONS

- Four divergence cases:



TWO EMISSIONS

- Four divergence cases:



- Observable:
$$\tau(k, l) = p_T y^{\frac{n}{2}} F(a, b, y, \{\vartheta\})$$



- Extension to N-jet cases:

- ▶ More emitters

- ▶ Nontrivial colour



$$C_1 S_{12} \Rightarrow \sum_{i \neq j=0}^{N+2} \mathbf{T}_i \cdot \mathbf{T}_j S_{ij}$$

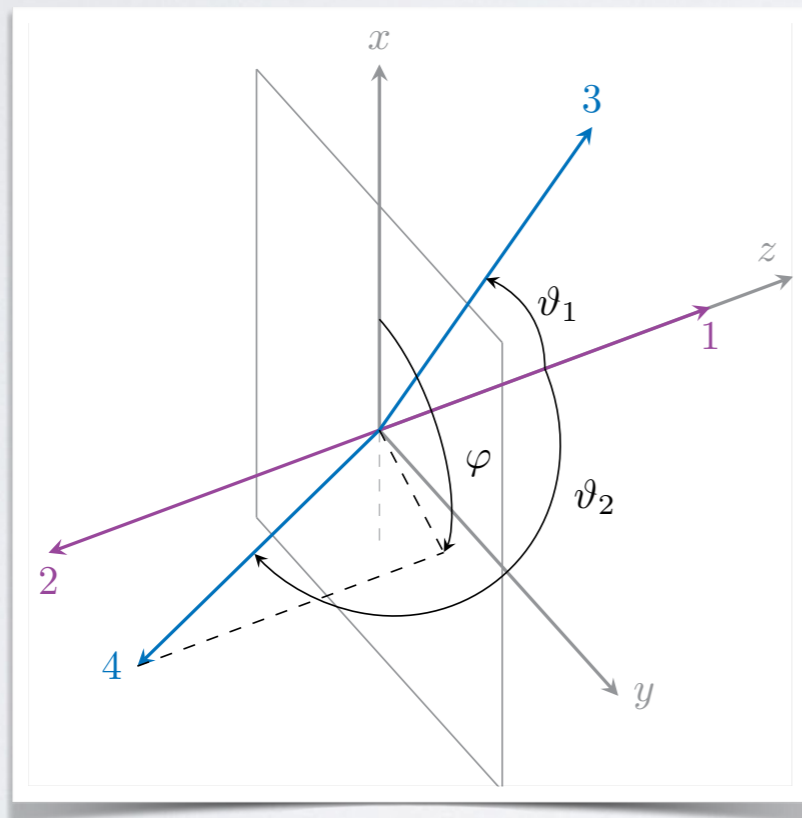
(And tri-/quadrupoles)

- ▶ Complicated geometry

- ▶ Two more integration dimensions



- ▶ Complicated geometry, see e.g. 2-jettiness



1-jettiness: 1 angle
2-jettiness: 3 angles
3-jettiness: 5 angles
N-jettiness: $2N-1$ angles

- ▶ Grids grow very large

THE CALCULATIONS

- Sample in steps of $\frac{\pi}{25}$, yielding 34476 samples (29904 2-jettiness, rest 0- and 1-jettiness)
- Fourfold symmetry reduces this to 8619
- 24 tripole and 18 dipole programs per point (NLO, C_A , and n_f for 6 dipoles)
- 900GB binaries and log files, 30min per point (NNLO programs, particularly C_A , the slowest)

THE COMPETITION

- [Agarwal et al, '24] use simplifications: $f_{N-j_n}(0, t) = 1$

$$S_R(\epsilon) = \frac{-4C_F}{n\epsilon^2} - \frac{8C_F}{n\epsilon} \left[\int_0^1 dt \frac{\ln f(0, t)}{\pi \sqrt{t(1-t)}} + \dots \right] + \dots$$

- Do you need a **recipe** or a **grid**? Do you need $N > 2$?



THE TARGET

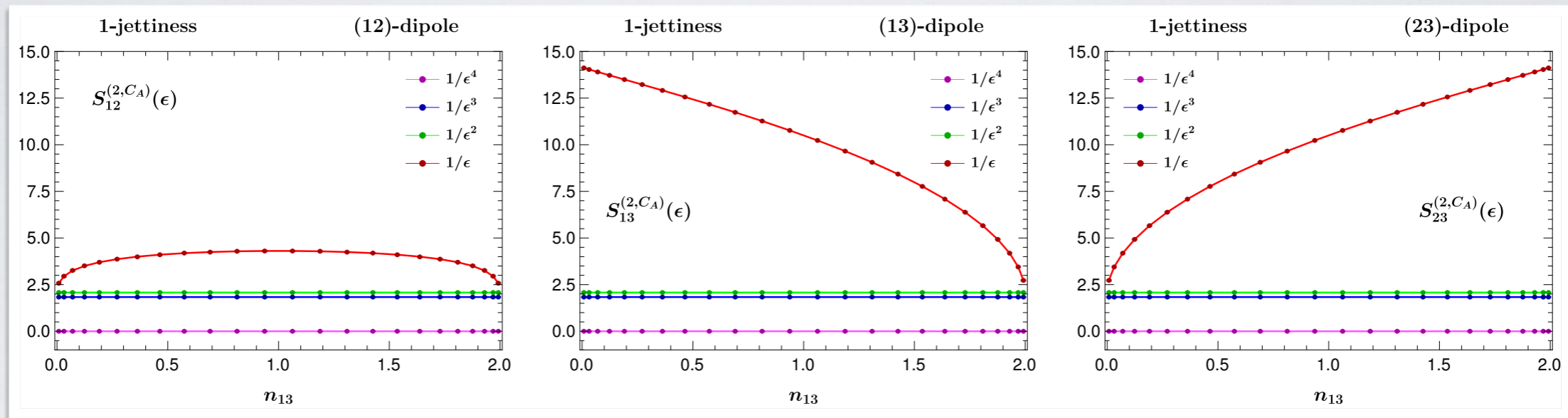
- What I'm about to plot:

$$\begin{aligned}
 S(\tau, \mu) = & 1 + \left(\frac{\alpha_s}{4\pi}\right) \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{\Gamma_0}{n} L_{ij}^2 + \frac{2\gamma_0^S}{n} L_{ij} + c_{ij}^{(1)} \right) + \left(\frac{\alpha_s}{4\pi}\right)^2 \\
 & \times \left\{ \sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{2\beta_0 \Gamma_0}{3n} L_{ij}^3 + \left(\frac{\Gamma_1}{n} + \frac{2\beta_0 \gamma_0^S}{n} \right) L_{ij}^2 + 2 \left(\frac{\gamma_1^S}{n} + \beta_0 c_{ij}^{(1)} \right) L_{ij} + c_{ij}^{(2)} \right) \right. \\
 & + 2\pi \sum_{i \neq j \neq k} f_{ABC} \mathbf{T}_i^A \mathbf{T}_j^B \mathbf{T}_k^C \left(\Gamma_0 \lambda_{ij} \left(\frac{\Gamma_0}{3n} L_{\mu}^3 + \left(\frac{\Gamma_0}{n} \tilde{L}_{jk} + \frac{\gamma_0^S}{n} \right) L_{\mu}^2 + \left(\frac{\Gamma_0}{n} \tilde{L}_{jk}^2 + \frac{2\gamma_0^S}{n} \tilde{L}_{jk} + c_{jk}^{(1)} \right) L_{\mu} \right) + \tilde{c}_{ijk}^{(2)} \right) \\
 & \left. + \frac{1}{4} \sum_{i \neq j} \sum_{k \neq l} \{ \mathbf{T}_i \cdot \mathbf{T}_j, \mathbf{T}_k \cdot \mathbf{T}_l \} \left(\frac{\Gamma_0}{n} L_{ij}^2 + \frac{2\gamma_0^S}{n} L_{ij} + c_{ij}^{(1)} \right) \left(\frac{\Gamma_0}{n} L_{kl}^2 + \frac{2\gamma_0^S}{n} L_{kl} + c_{kl}^{(1)} \right) \right\}
 \end{aligned}$$

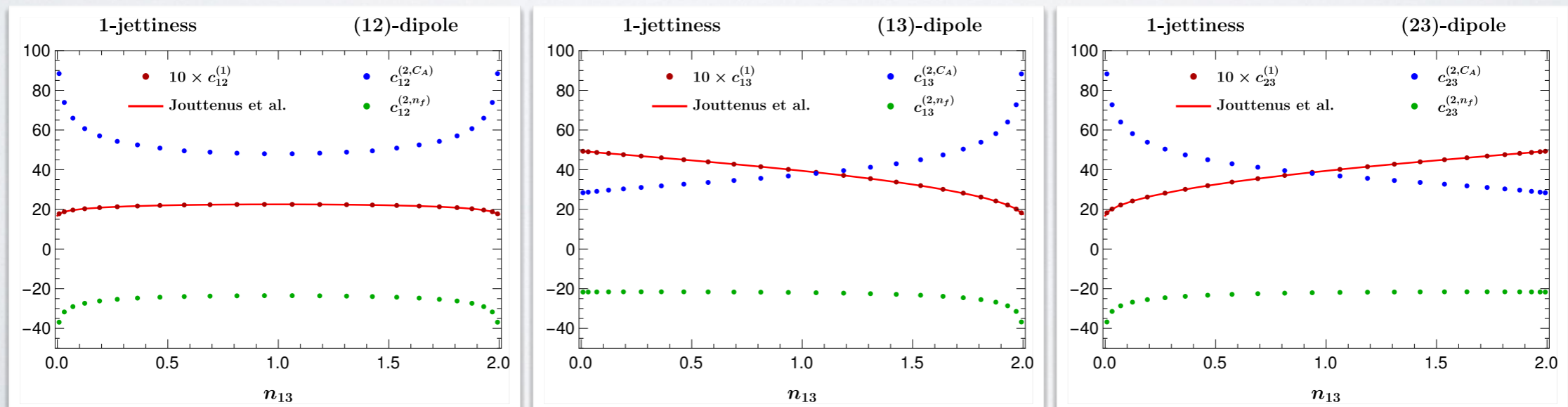
- Full results as ancillary file on the arXiv

1-JETTINESS

- Pole cancellations



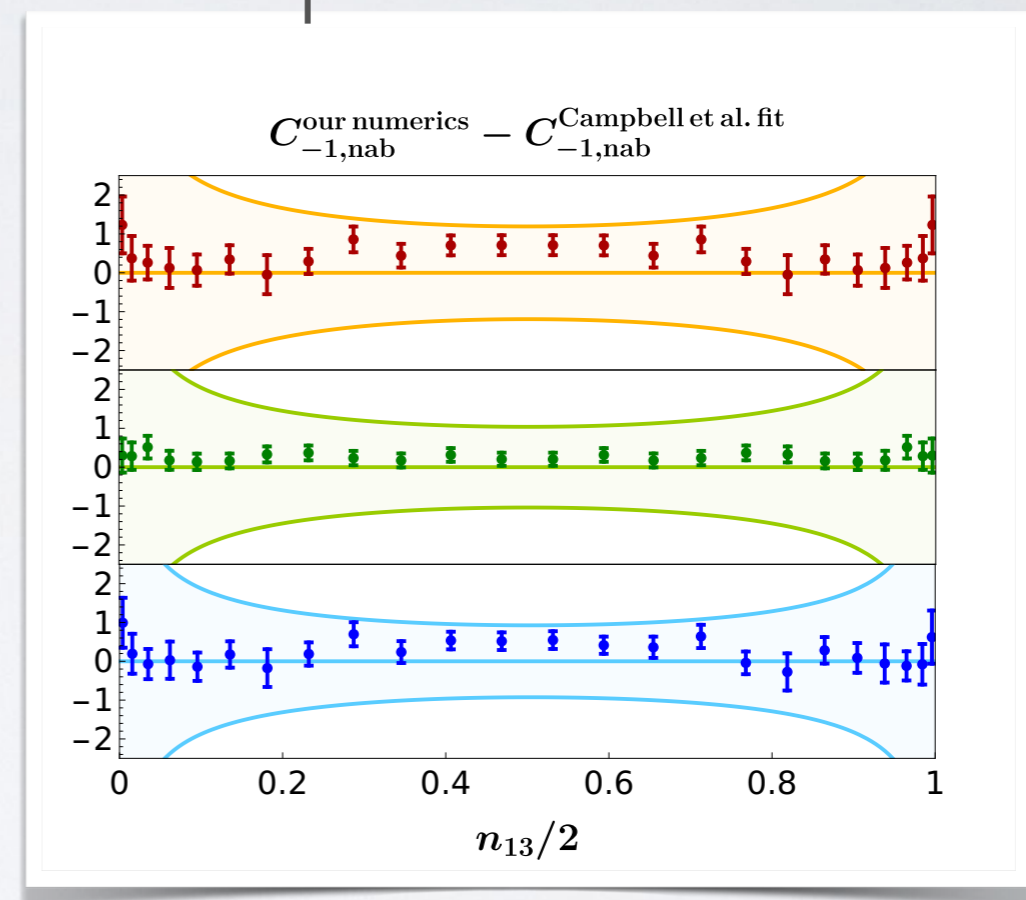
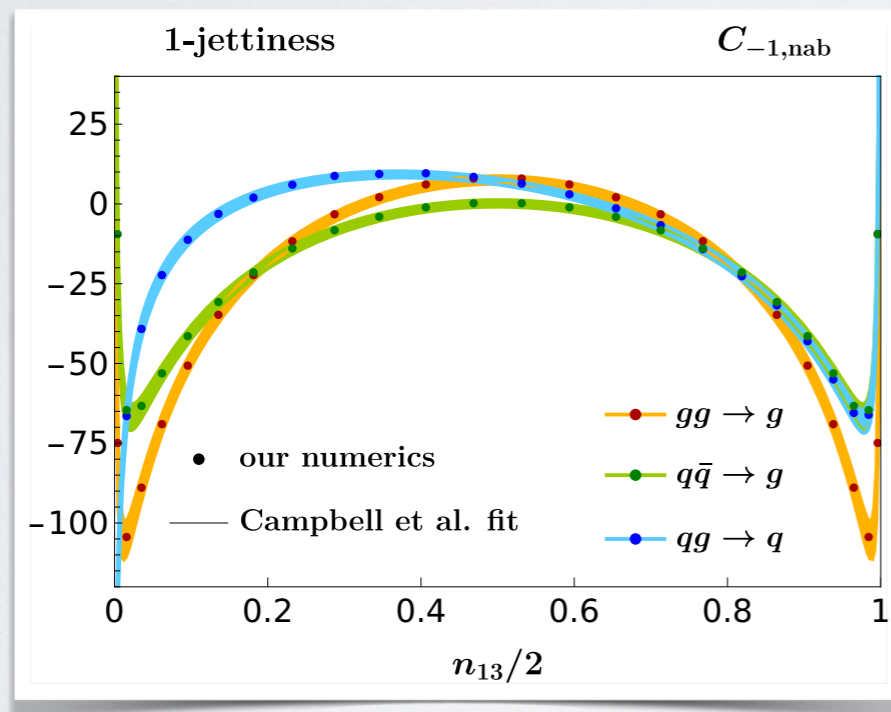
- Finite results



1-JETTINESS COMPARISON

[Campbell, Ellis, Mondini, Williams, '18]

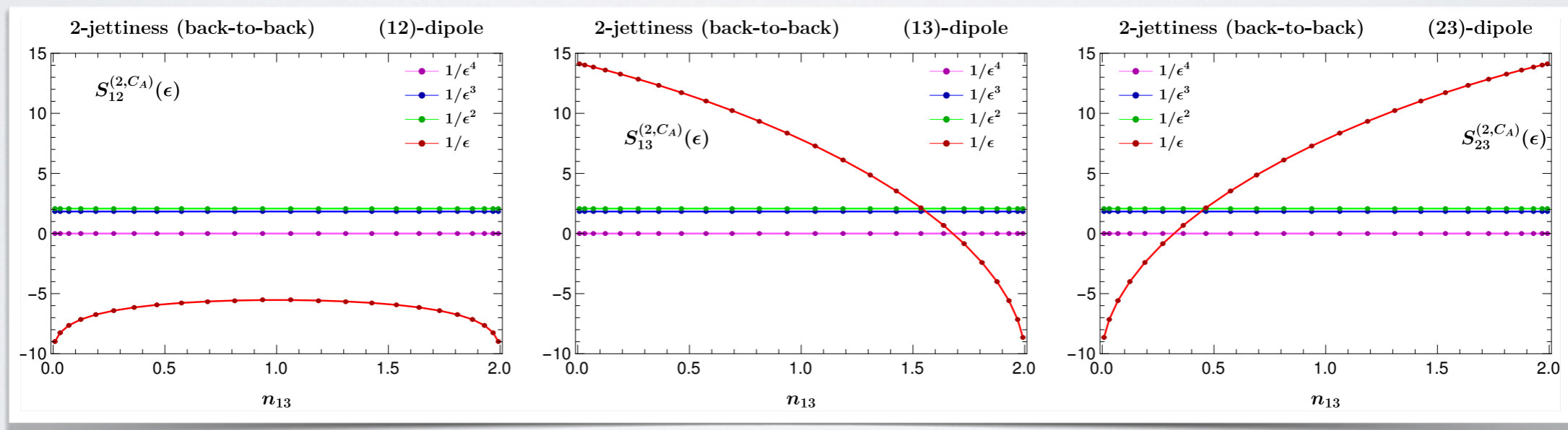
- Compare with result from Campbell et al.



- Only fit functions for physical channels provided

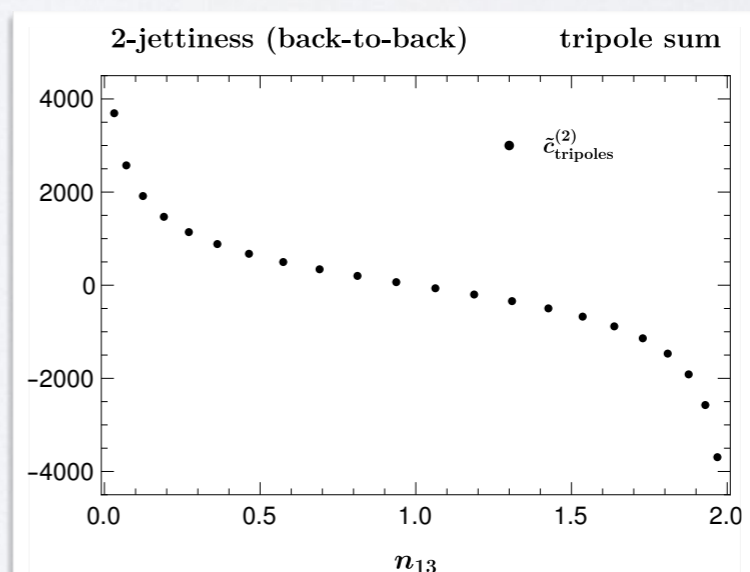
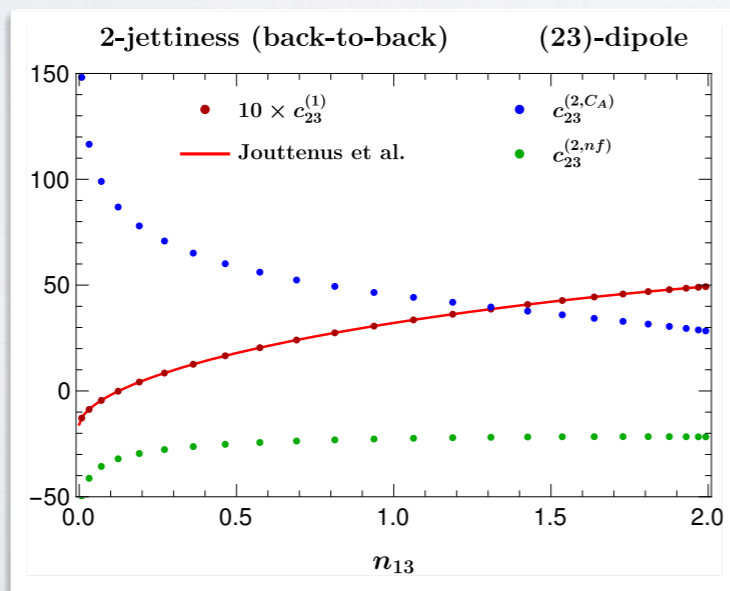
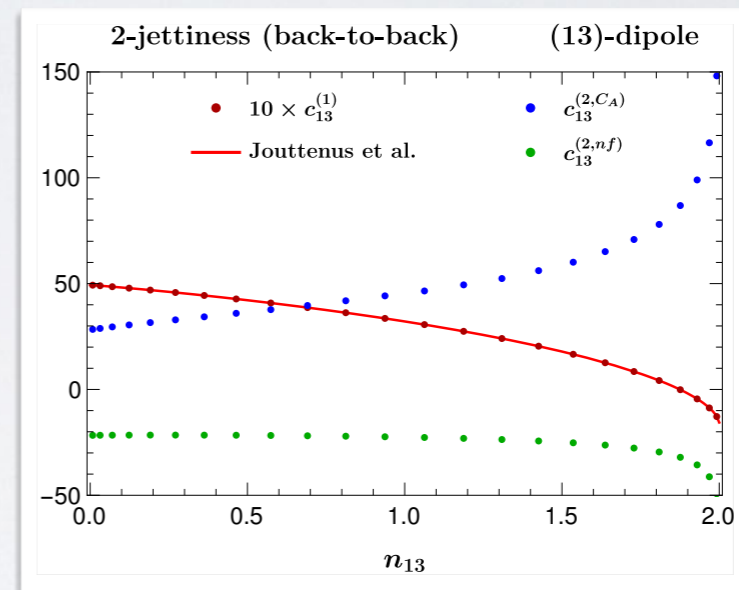
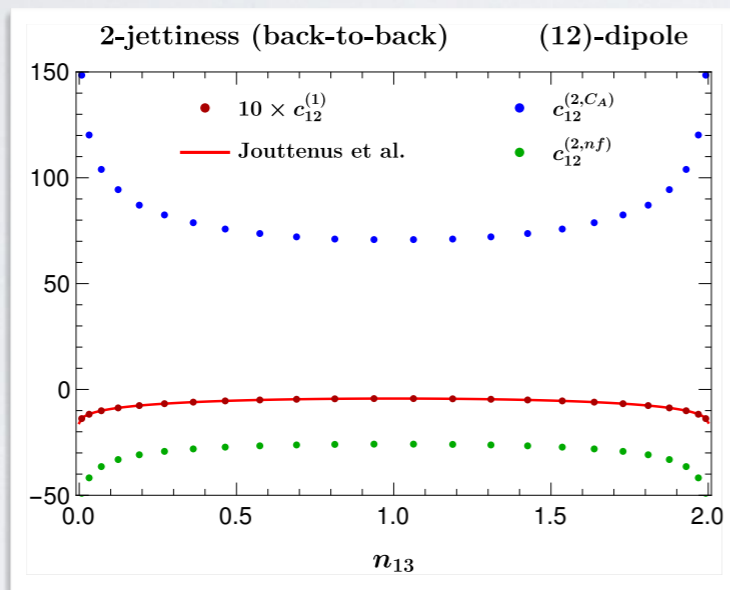
2-JETTINESS - B2B

- Pole cancellations

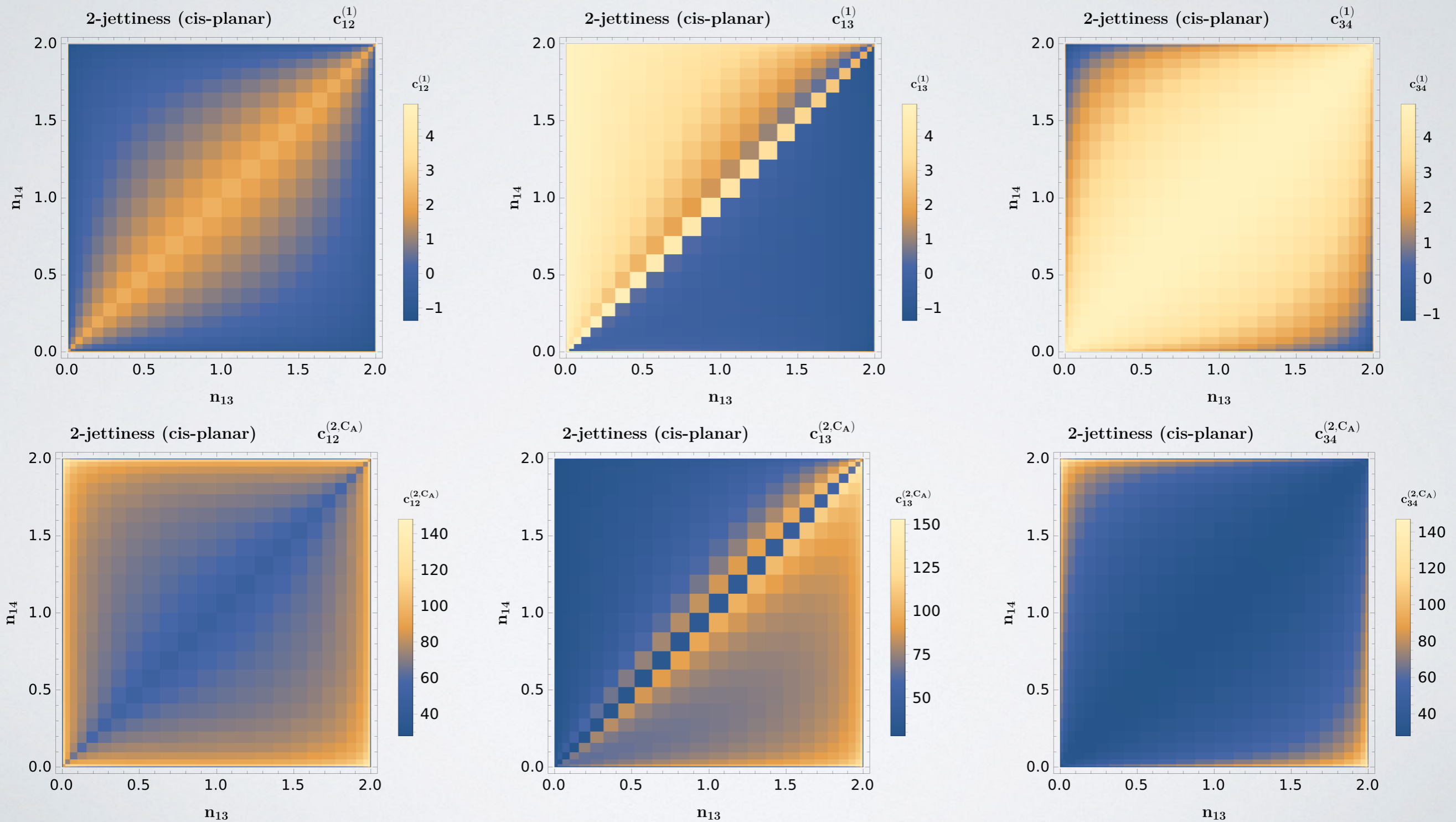


2-JETTINESS - B2B

- Finite results

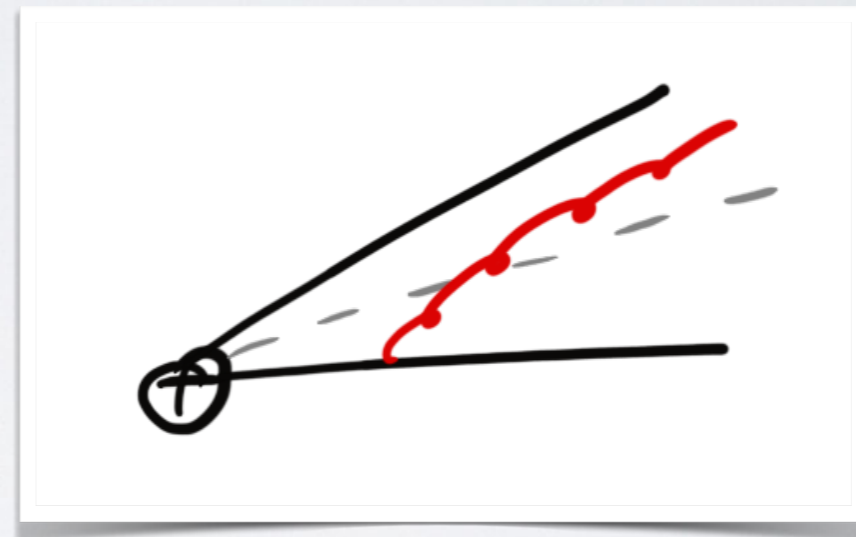
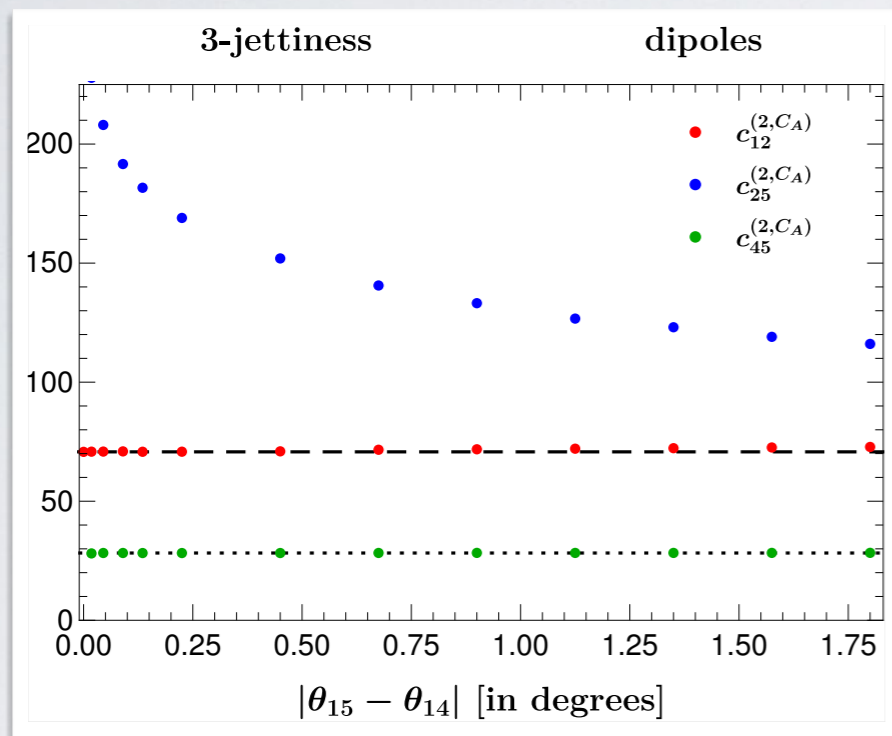


2-JETTINESS - PLANAR



EDGE CASES

- When jets become collinear, some results diverge



- Non-dipole jet collinear to dipole jet: divergence
- Origin: contribution from non-dipole jet enhanced by collinearity to dipole jet

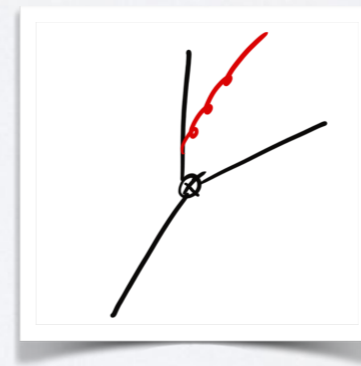
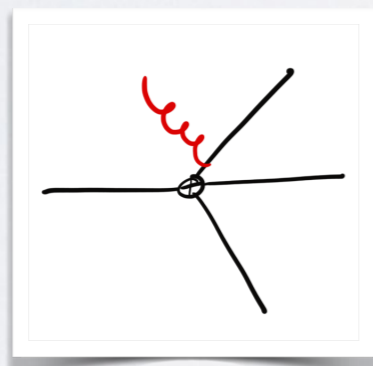
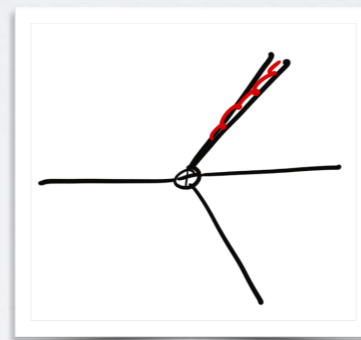
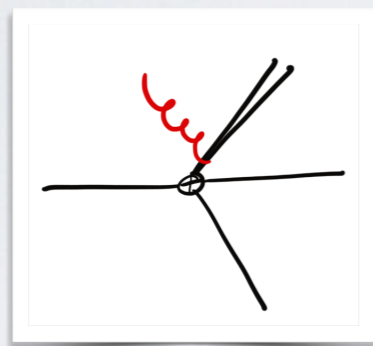
METHOD OF REGIONS!

- Two regions:

bulk

and

collinear



(N-1)-jet base

+

Universal correction

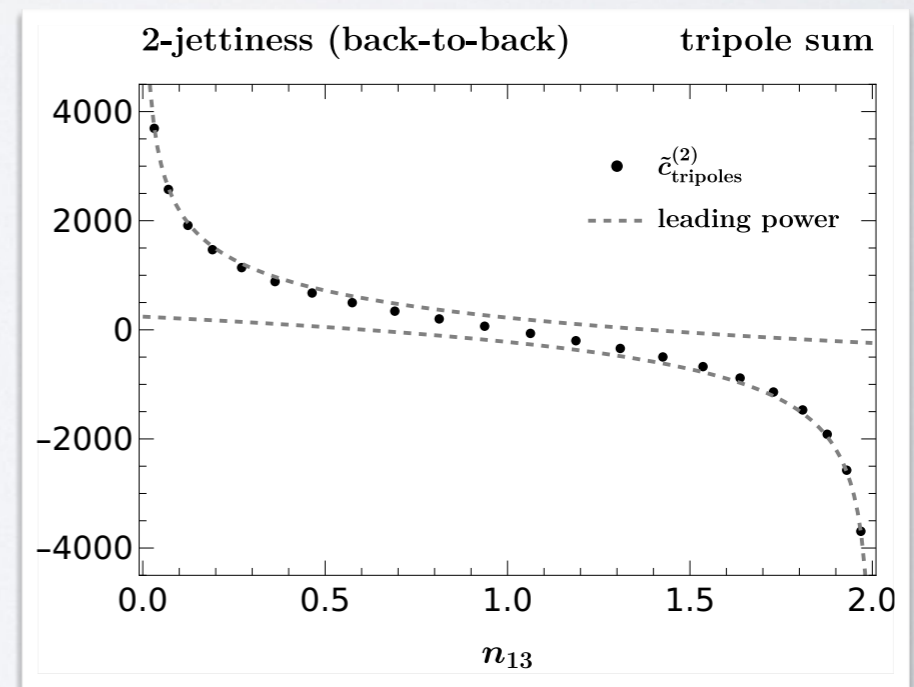
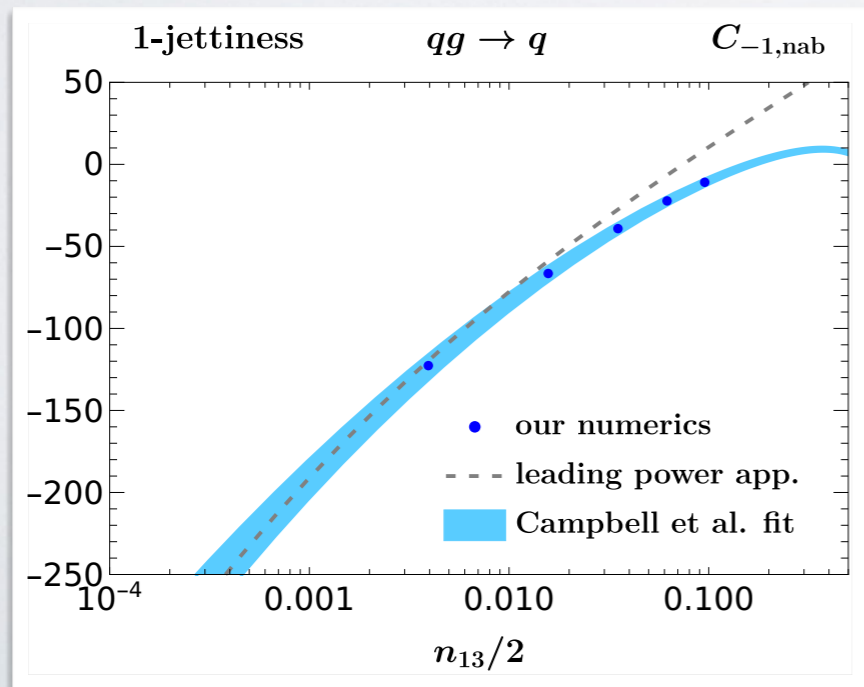
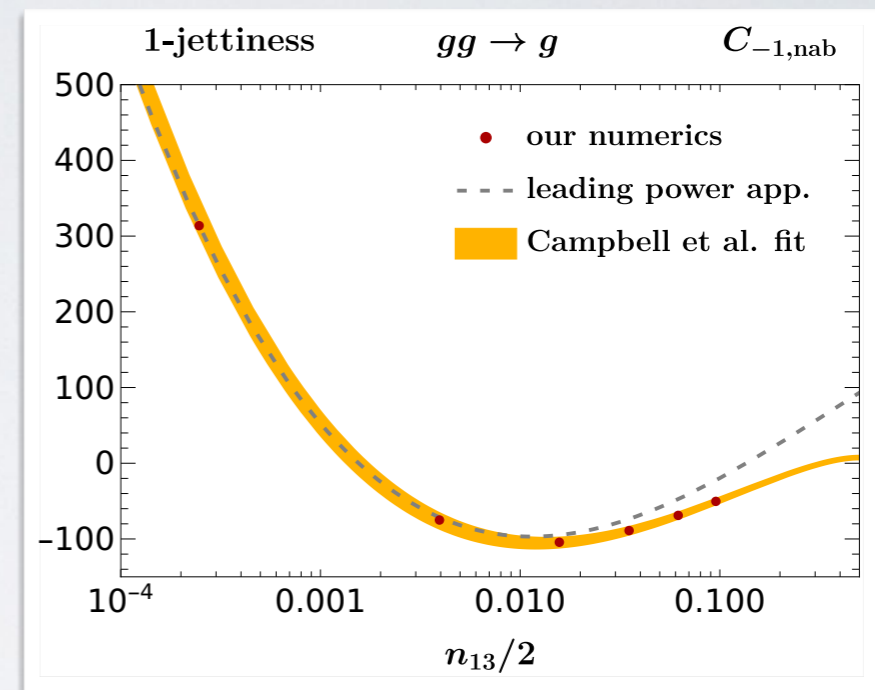
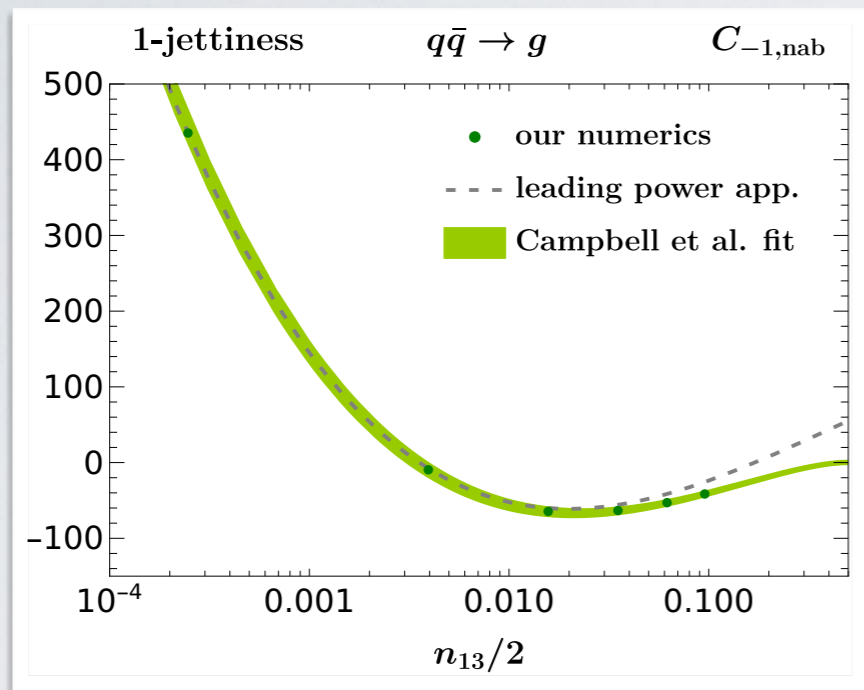
IMPLICATIONS

- A kind of refactorisation
(global soft base and collinear soft correction)
- Pattern should be general
- Correction is observable and dipole dependent

- N-Jettiness: $c_{ij}^{(1,\text{corr})} = -\frac{\pi^2}{3}$

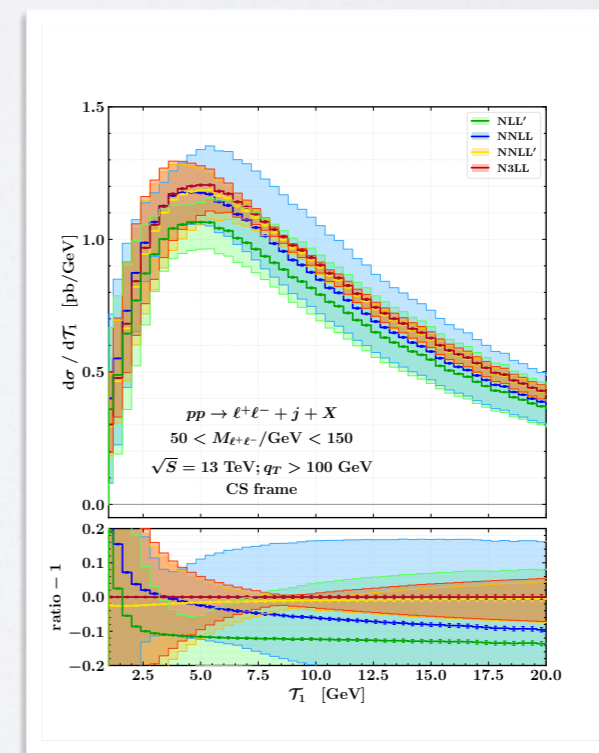
$$c_{ij}^{(2,\text{corr})} = T_F n_f \left(\frac{4\pi^2}{9} \ln \left(\frac{2\delta}{n_{ij}} \right) - 8.023(2) \right) \\ - C_A \left(\frac{11\pi^2}{9} \ln \left(\frac{2\delta}{n_{ij}} \right) - 10.0335(2) \right)$$

CONVERGENCE



IS THIS USEFUL?

- 1-jettiness at N³LL for GENEVA
[Alioli, Bauer, Berggren, Tackmann, Walsh, '15]
- Z+jet production in lab frame vs CS-frame
[Alioli, Bell, Billis, Broggio, Dehnadi, Lim, Marinelli, Nagar, Napoletano, RR '24]
- For some Q_j , small values for n_{13} have to be probed



CONCLUSIONS AND OUTLOOK

- We extended the **SoftSERVE** code to **N-jet** cases and applied it to the N-jettiness at **NNLO**
- Derived a **grid** for the **2-jettiness** soft function
- Investigated **logarithmic divergences** at the **edges of phase space** using the Method of Regions
- Used our results for an **N³LL** resummation
- **Next:** non-global logarithms, N-gularities, ...

THANK YOU!





- ▶ Integration dimensions, dijet:
 - Three physical dimensions
(two beams, one transverse reference)
 - One emission can probe 4th dimension,
Two emissions can probe 4th and 5th dimension:

$$\frac{1}{\Gamma(-\epsilon)} \int_0^1 dt_5 t_5^{-1-\epsilon} \sim \mathcal{O}(\epsilon^0)$$



► Integration dimensions, N-jet:

- Four physical dimensions

(all the beams and jets)

- One emission can probe 5th dimension,

Two emissions can probe 5th and 6th dimension:

$$\frac{1}{\Gamma(-1/2 - \epsilon)} \int_0^1 dt_7 t_7^{-3/2-\epsilon} \sim \mathcal{O}(\epsilon^0)$$

The “awful angle”

- For N-jettiness: analytic integration