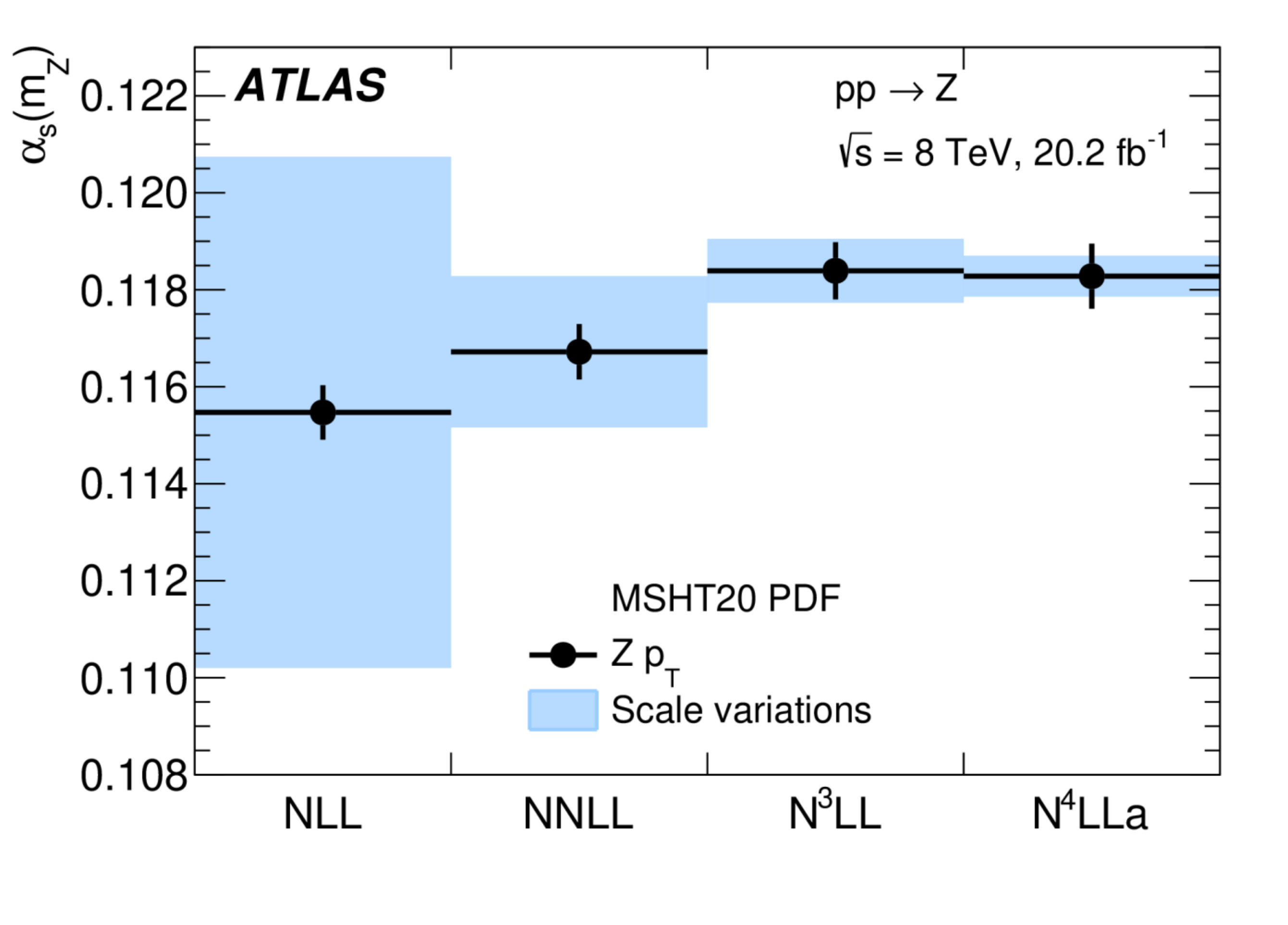
QCD resummation quo vadis?



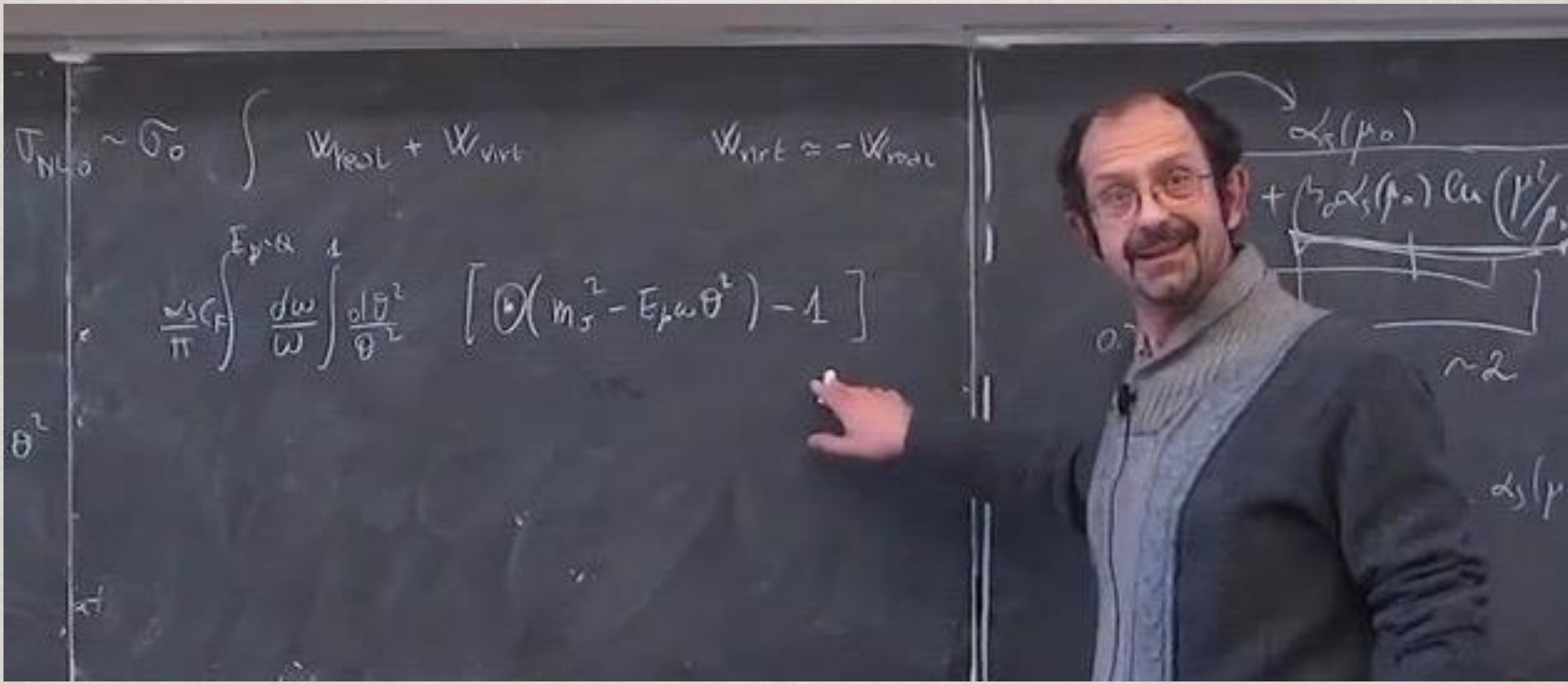
Andrea Banfi

University of Sussex

PSR 2024 – 3 July 2024 – Graz

On the shoulders of giants

On January the 16th, we lost Stefano Catani, one of the most influential scientists in the field of QCD resummations



- His work encompassed all major challenges in the field, from the statement of the problem to QCD factorisation
- As a personal homage to his career, in my talk I will try to review recent developments in the light of his seminal works

Outline

- General considerations
  - Coherence and factorisation
- Accuracy
  - Transverse-momentum distributions
  - Jet-veto resummations
  - N-jettiness
- Impact of QCD resummations
  - IR structure of gauge theories
  - Subtraction schemes for fixed-order calculations
  - Automation of resummations

General considerations

Statement of the problem

Nuclear Physics B407 (1993) 3–42
North-Holland

NUCLEAR
PHYSICS B

Resummation of large logarithms in e^+e^- event shape distributions *

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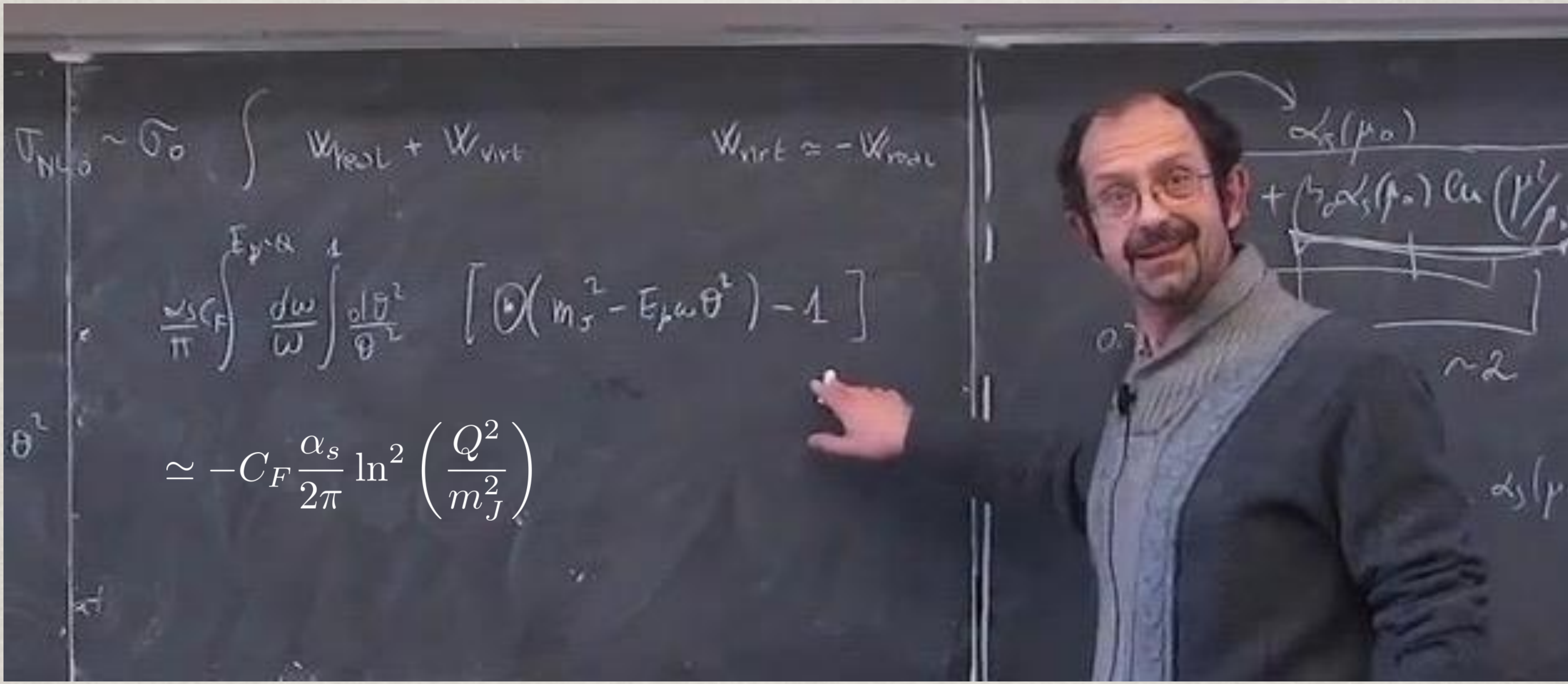
Received 29 January 1993
Accepted for publication 27 May 1993

We describe a method for the resummation of leading and next-to-leading large logarithms to all orders in QCD perturbation theory, applicable to e^+e^- event shape distributions that have the property of exponentiation near the two-jet region. After a general discussion of the conditions for exponentiation and the evaluation of matrix elements and phase space to next-to-leading logarithmic accuracy, we give details of the application of the method to the thrust and heavy jet mass distributions. We show how the resummed expressions can be matched with known second-order results to obtain improved predictions throughout the whole of phase space, and how to suppress spurious higher-order terms generated by resummation outside the physical region. We also give the necessary ingredients for the improvement of third-order predictions by resummation when they become available.

Why large logarithms?

Logarithmically enhanced contributions arise whenever we restrict the phase space of real emissions near the singular soft and collinear regions

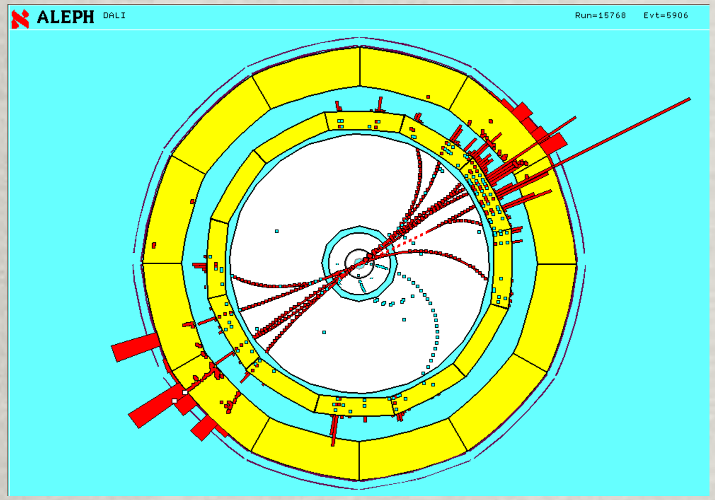


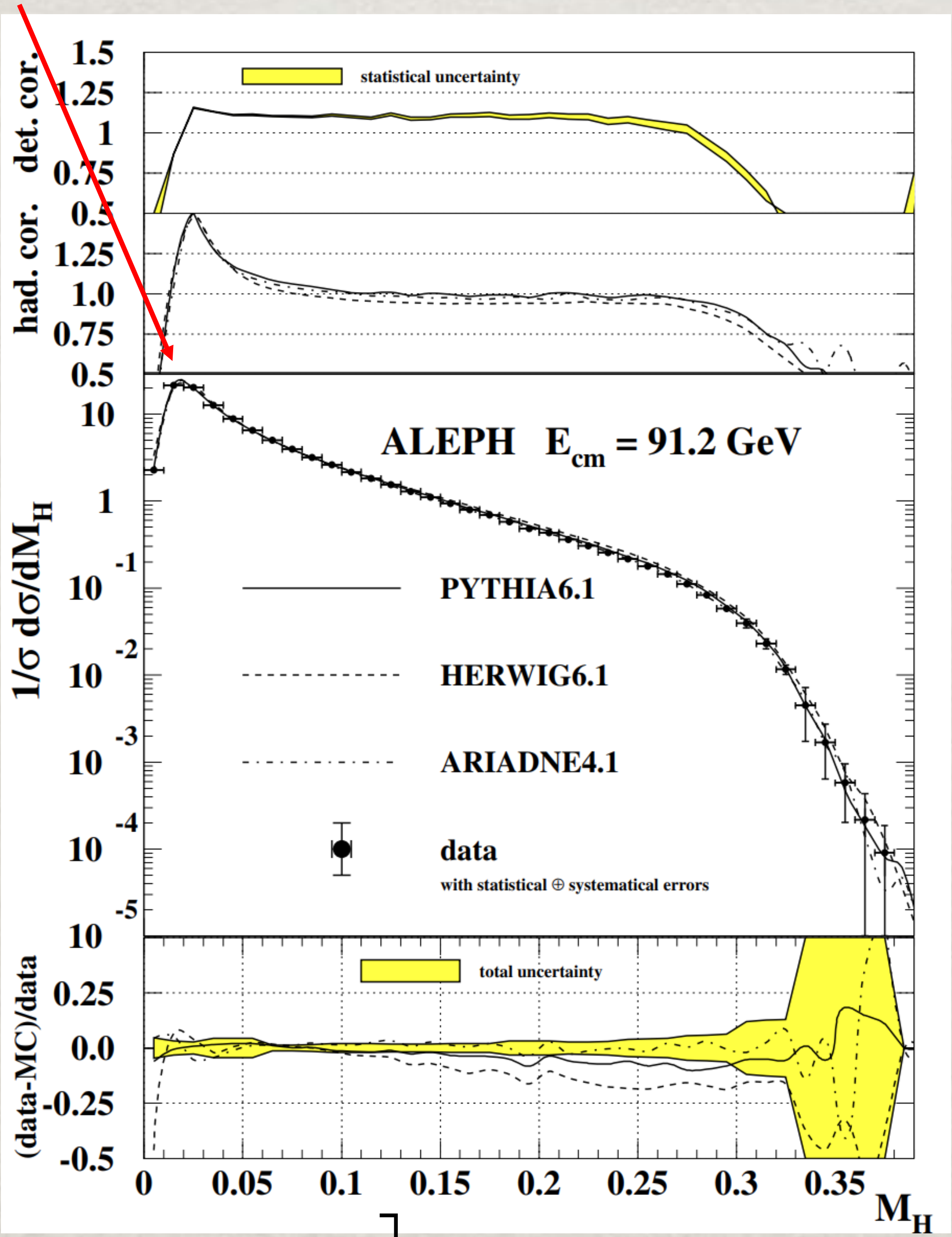
- In the above example, the invariant mass of a jet is constrained to be below the threshold value $m_J^2 \ll Q^2$, thus generating large logarithms $L \equiv \ln(Q^2/m_J^2)$
- Large logarithms are typical of two-scale problems, where one looks inside jets with a resolution Q_0 that is much less than the typical hard scale Q of the process

Where do we resum?

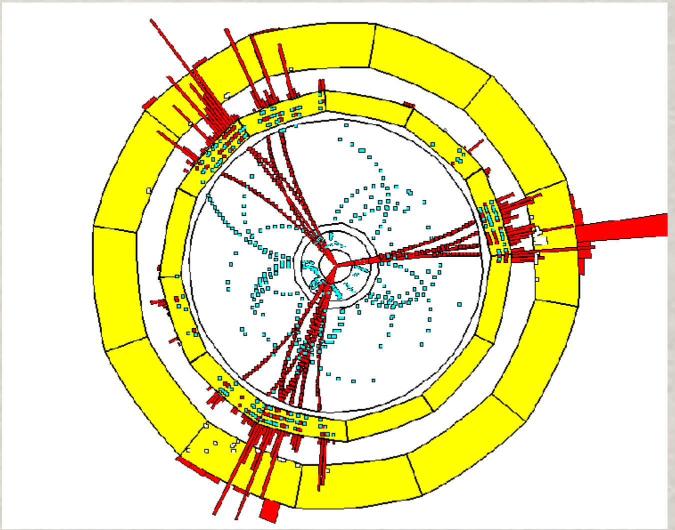
For jet observables that are different from zero with an extra emission, the bulk of events lies in the region $\alpha_s L \sim 1$

resummation





fixed order



$$\sim \exp\left[\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \ldots\right]$$

$$\sim \underbrace{\alpha_s}_{\text{LO}} + \underbrace{\alpha_s^2}_{\text{NLO}} + \underbrace{\alpha_s^3}_{\text{NNLO}} \ldots$$

Ambiguities beyond NLL

Nuclear Physics B 596 (2001) 299–312

NUCLEAR PHYSICS B

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ELSEVIER

Universality of non-leading logarithmic contributions in transverse-momentum distributions ☆

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Received 25 August 2000; accepted 13 October 2000

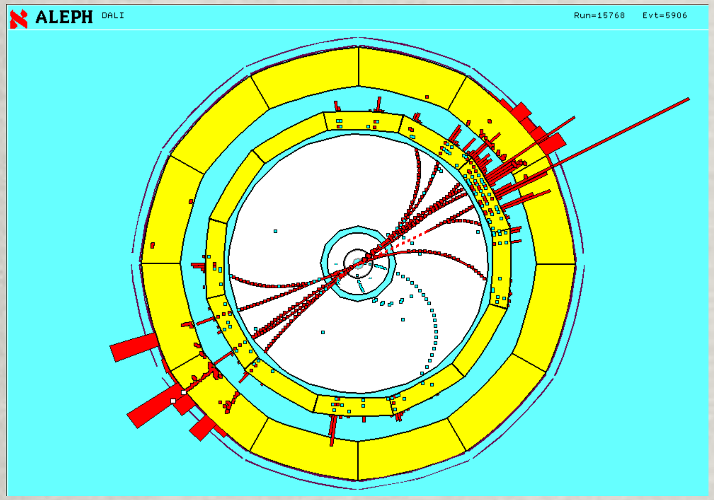
Abstract

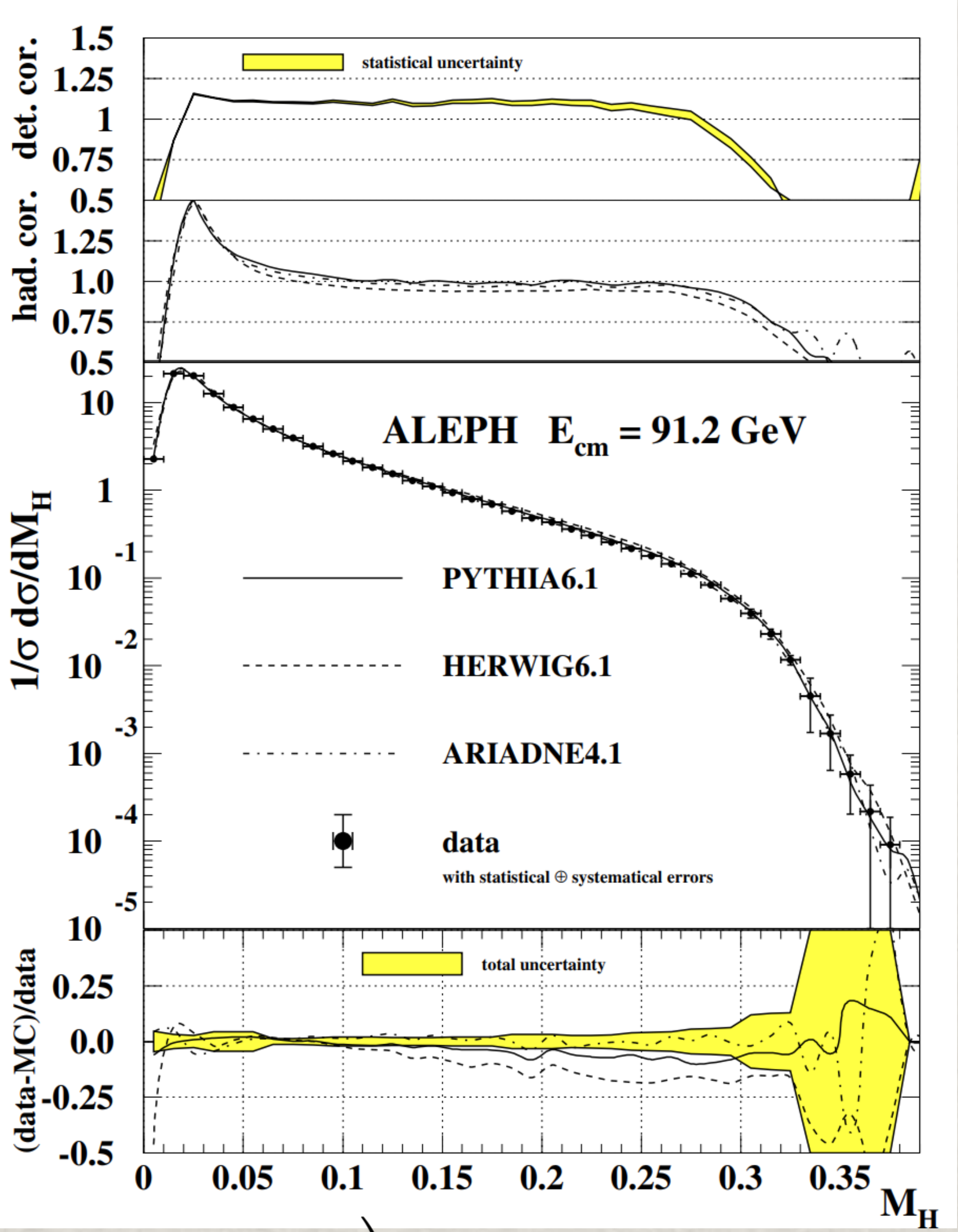
We consider the resummation of the logarithmic contributions to the region of small transverse momenta in the distributions of high-mass systems (lepton pairs, vector bosons, Higgs particles, . . .) produced in hadron collisions. We point out that the resummation formulae that are usually used to compute the distributions in perturbative QCD involve process-dependent form factors and coefficient functions. We present a new universal form of the resummed distribution, in which the dependence on the process is embodied in a single perturbative factor. The new form simplifies the calculation of non-leading logarithms at higher perturbative orders. It can also be useful to systematically implement process-independent non-perturbative effects in transverse-momentum distributions. We also comment on the dependence of these distributions on the factorization and renormalization scales.

How do we resum?

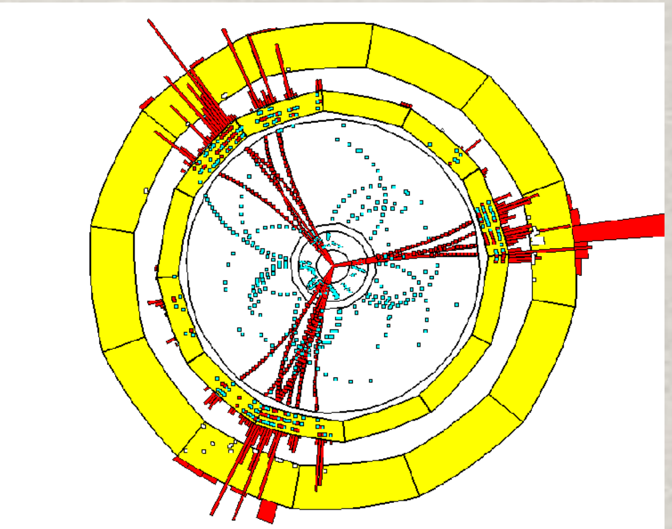
Beyond NLL, there is no unique way to organise subleading contributions, which could be exponentiated or left as coefficient functions

resummation





fixed order



$$\sim e^{\overbrace{Lg_1(\alpha_s L)}^{\mathrm{LL}}} \times \left(\underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}} + \ldots \right)$$

$$\sim \underbrace{\alpha_s}_{\mathrm{LO}} + \underbrace{\alpha_s^2}_{\mathrm{NLO}} + \underbrace{\alpha_s^3}_{\mathrm{NNLO}} \ldots$$

All-order resummation

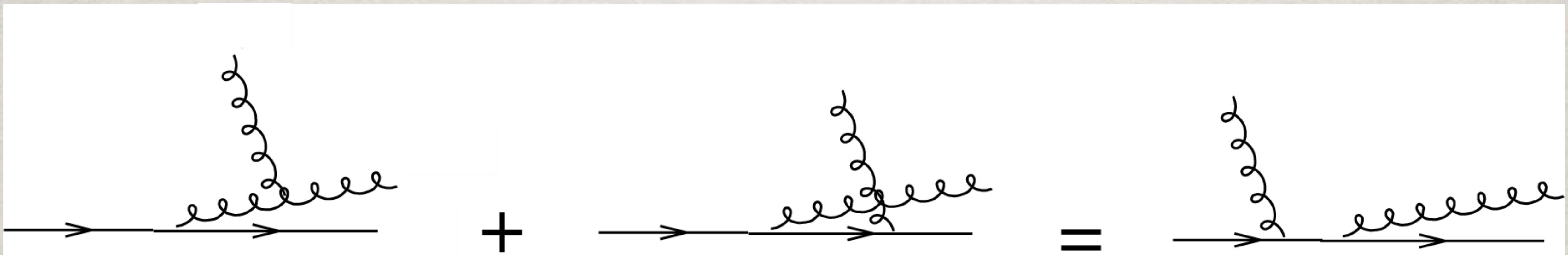
All-order resummation of large logarithms $\Rightarrow$reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with $L \equiv \ln(1/v)$ (e.g. $v = m_J^2/Q^2$)

$$\Sigma(v) \sim e^{\underbrace{Lg_1(\alpha_s L)}_{\text{SLL}}} \times \left(\begin{array}{c} 1 \quad + \quad \alpha_s \quad + \dots \\ \underbrace{G_2(\alpha_s L)}_{\cancel{\text{N}}\text{LL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\cancel{\text{N}}\text{NLL}} + \dots \end{array} \right)$$

- This representation is independent of the formalism used for the resummation, so it can be used to compare different resummed predictions
- It is possible to resum with different logarithmic counting accuracy, e.g. for $\alpha_s L^2 \sim 1$: there we talk about double-logarithmic (DL) accuracy, NDL, etc. The expansion in that case is in powers of $\sqrt{\alpha_s}$
- For some observables (e.g. the transverse momentum of a colour singlet) exponentiation of leading logarithms is not achieved in v-space, but rather in a conjugate variable (e.g. impact parameter)

Coherence is key

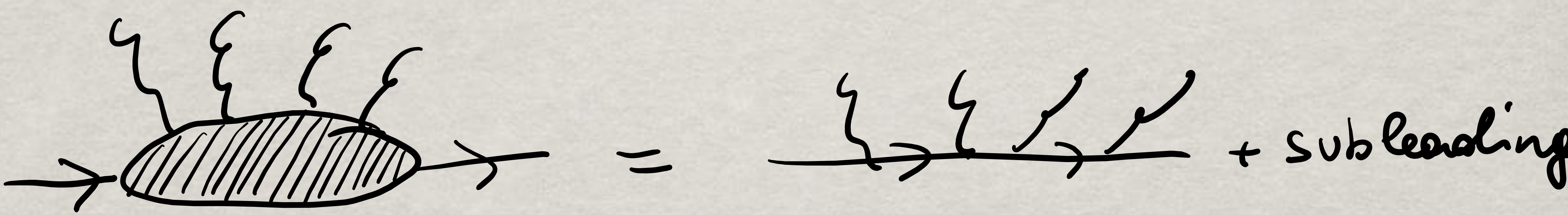
A crucial property that enables all-order resummations is QCD coherence, i.e. an emission sees the total colour charge of all emissions at smaller angles



For recursive IRC collinear safe observables (most global event shapes and jet resolution parameters), and with less than two legs in the initial state, this implies

- Emissions widely separated in angle are emitted independently from the hard legs

[AB Salam Zanderighi hep-ph/0407286]



- It is always possible to separate soft and collinear emissions into a soft and jet function

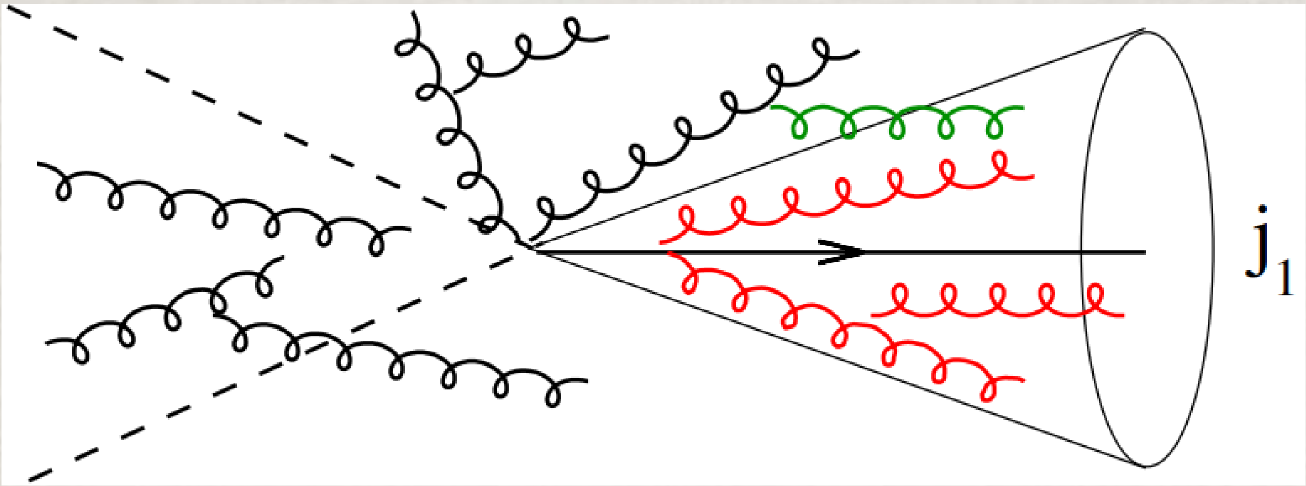
Status of global resummation

The majority of global event shapes and jet resolution parameters can be resummed at very high logarithmic accuracy

- NLL resummation is a solved problem, and can be performed automatically with the CAESAR program [AB Salam Zanderighi hep-ph/0407286]
- Some observables (e.g. thrust, broadening) enjoy factorisation theorems in SCET, which enables NNNLL resummation [Becher Schwartz 0803.0342] [Becher Bell 1210.0580] [Hoang Kolodubrez Mateu Stewart 1501.04111]
- General NNLL resummation of factorisable observables in SCET with the semi-numerical program SoftServe (see R. Rahn's talk) [Bell Rahn Talbert 2004.08396]
- General semi-numerical NNLL resummation of event shapes and jet rates in e^+e^- annihilation with the ARES method [AB Monni McAslan Zanderighi 1412.2126] [AB Monni McAslan Zanderighi 1607.03111] [AB El-Menoufi Monni 1807.11487] [Arpino AB El-Menoufi 1912.09341]

Non-global logarithms

Non-global observables, sensitive to emissions in a restricted portion of the phase space, break coherence by construction [Dasgupta Salam hep-ph/0104277]



Non-Abelian non-global single logarithms ($\alpha_s^n L^n$) arise when the softest emission in a correlated cascade of soft gluons enters the measurement region

- Leading NGLs in the large-N_c limit are resummed via the BMS equation

  [AB Marchesini Smye hep-ph/0206076]

- Numerical solutions exist for N_c=3 based on suitable diffusion equations

  [Hatta Ueda 2011.04154]

Status of non-global resummation

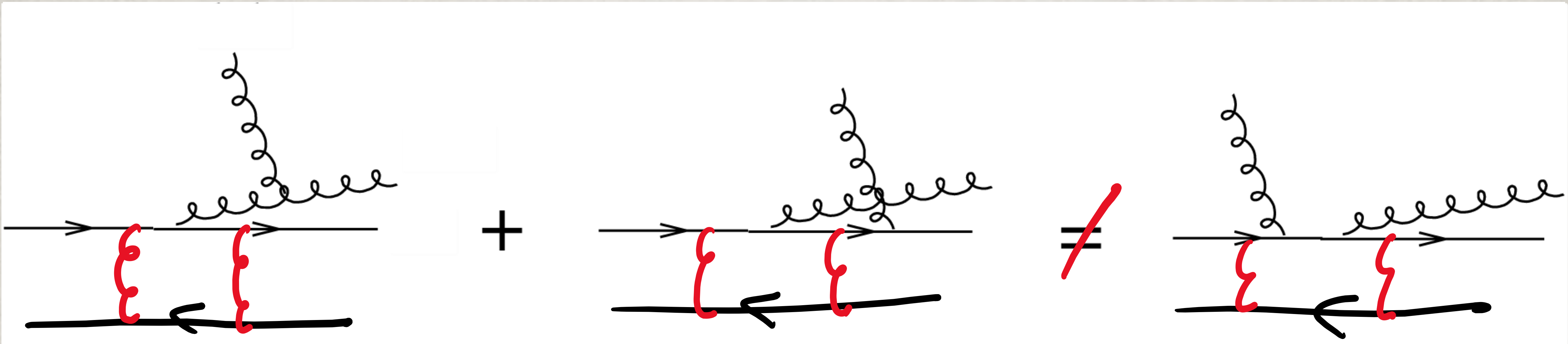
- The general form of NGLs can be obtained from an infinite number of factorisation constraints between soft and hard modes [Becher Neubert 1605.02737]

$$\Sigma(Q, Q_0) = \sum_{n=2}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{W}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

- NL NGLs have been resummed in two equivalent ways
  - By constructing the NL generalisation of BMS equation
    [AB Dreyer Monni 2104.06416 , 2111.02413]
    [GNOLE https://github.com/non-global/gnole]
  - By computing the anomalous dimensions of the hard function at two loops
    [Becher Rau Xu 2112.02108]
- Resummation of NGLs has been extended to hadron collisions (see N. Schalch's talk)
  [Becher Schalch Xu 2307.02283]
- Non-global observables in the presence of jet clustering algorithms give rise to additional Abelian "clustering" logarithms [AB Dasgupta hep-ph/0508159]
  [Becher Haag 2309.17355]

Coherence violating logarithms

Colour can be transferred between two initial-state hard partons



At the amplitude level, splitting functions acquire extra dependence on the momentum of all hard partons [Catani De Florian Rodrigo 1112.4405]

$$|\mathcal{M}^{(1)}(p_1, p_2, \ldots, p_n)\rangle \simeq \boldsymbol{Sp}^{(1)}(p_1, p_2; \widetilde{P}; p_3, \ldots, p_n)\, |\mathcal{M}^{(0)}(\widetilde{P}, \ldots, p_n)\rangle + \boldsymbol{Sp}^{(0)}(p_1, p_2; \widetilde{P})\, |\mathcal{M}^{(1)}(\widetilde{P}, \ldots, p_n)\rangle\,. \tag{4.1}$$

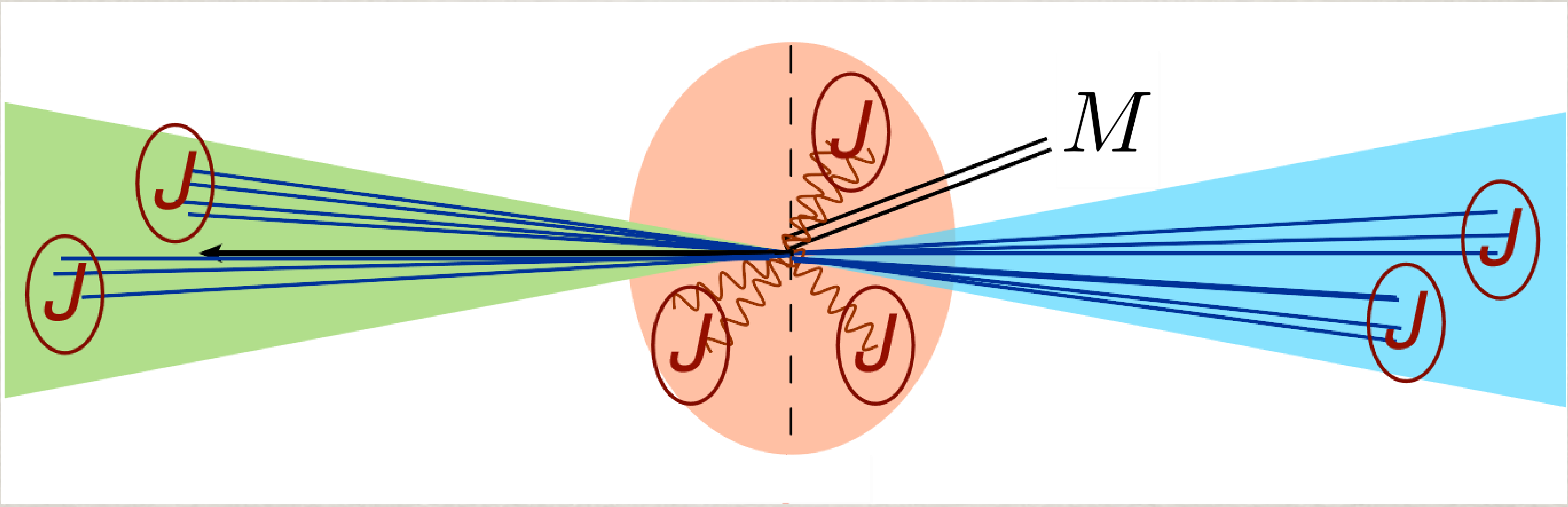
$$\boldsymbol{Sp}^{(1)}(p_1, p_2; \widetilde{P}; p_3, \ldots, p_n) = \boldsymbol{Sp}_H^{(1)}(p_1, p_2; \widetilde{P}) + \boldsymbol{I}_C(p_1, p_2; p_3, \ldots, p_n)\ \boldsymbol{Sp}^{(0)}(p_1, p_2; \widetilde{P})\ , \tag{4.2}$$

- In non-global observables, this leads to super-leading logarithms (see M. Neubert's talk) [Forshaw Kyrieleis Seymour hep-ph/0604094]
- Do coherence-violating logarithms occur in global observables as well? [Forshaw Holguin 2109.03665]

Accuracy

Refactorisation

Both in QCD and Soft Collinear Effective Theory (SCET) it is possible to write "refactorisation" theorems: the resummation consists of the convolution of hard, jet and soft functions, that can be computed independently



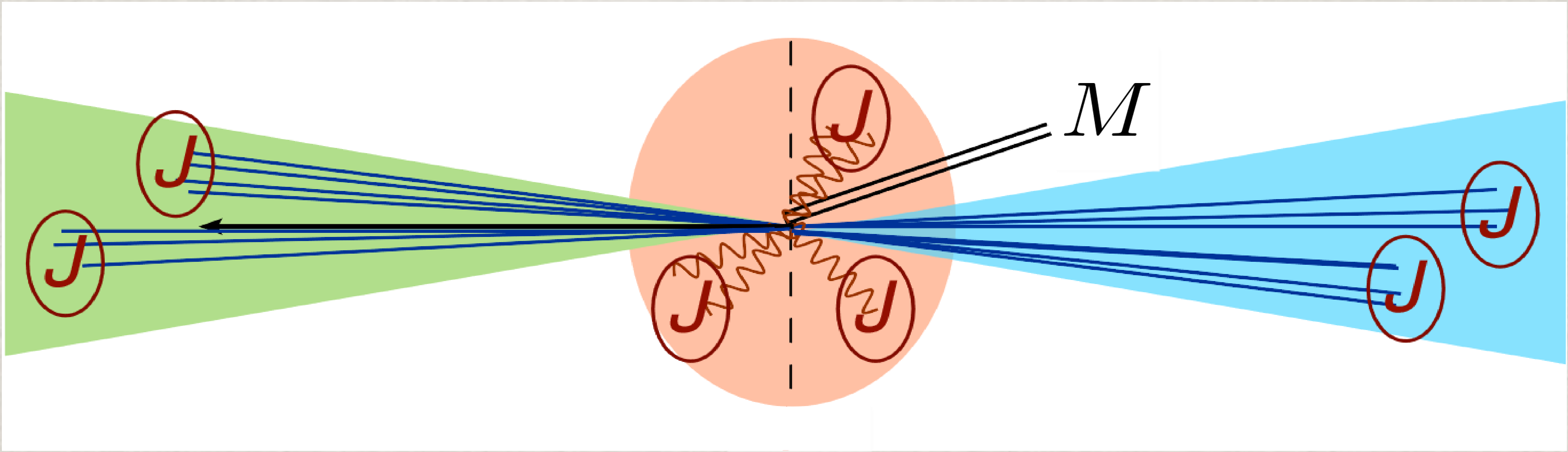
- All resummation functions can be obtained via evolution equations involving anomalous dimensions and initial conditions that can be computed using loop techniques
- Involved observable-dependent coefficients can be computed numerically (see R. Rahn's talk)

Transverse momentum distribution

The distribution in the transverse momentum of a colour singlet (e.g. Higgs, Z boson) is a fully inclusive observable and obeys an all-order refactorisation formula

[Collins Soper Sterman Nucl. Phys. B250 (1985) 199]

$$\frac{d\sigma}{dq_T} \sim \mathcal{B}(\mu_J, \nu) \otimes \mathcal{H}(M, \mu_H) \otimes \mathcal{S}(\mu_S, \nu) \otimes \mathcal{B}(\mu_J, \nu)$$



The independence of the distribution on the scales μ_H, μ_J, μ_S, ν induces renormalisation group equations (RGEs) that can be solved at high accuracy

Transverse momentum distribution

Impressive progress in the calculation of anomalous dimension and coefficient functions allows resummation up to approximate N^{4}LL accuracy

$$\frac{d\sigma}{dq_T} \sim \mathcal{B}(\mu_J,\nu) \otimes \mathcal{H}(M,\mu_H) \otimes \mathcal{S}(\mu_S,\nu) \otimes \mathcal{B}(\mu_J,\nu)$$

Accuracy	$\mathcal{H},\mathcal{B}$	$\Gamma_{\rm cusp}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop	–	–	1-loop ✓
NLL	Tree level	2-loop	1-loop	1-loop	2-loop ✓
NLL$'$	1-loop	2-loop	1-loop	1-loop	2-loop ✓
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop ✓
NNLL$'$	2-loop	3-loop	2-loop	2-loop	3-loop ✓
N^{3}LL	2-loop	4-loop	3-loop	3-loop	4-loop ✓
N^{3}LL$'$	3-loop	4-loop	3-loop	3-loop	4-loop ✓
N^{4}LL	3-loop	5-loop	4-loop	4-loop	5-loop ✓
N^{4}LL$'$	4-loop	5-loop	4-loop	4-loop	5-loop ✓

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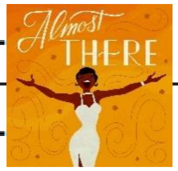
Accuracy	$\mathcal{H},\mathcal{B}$	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop	–	–	1-loop
NLL	Tree level	2-loop	1-loop	1-loop	2-loop
NLL′	1-loop	2-loop	1-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL′	2-loop	3-loop	2-loop	2-loop	3-loop
N^{3}LL	2-loop	4-loop	3-loop	3-loop	4-loop
N^{3}LL′	3-loop	4-loop	3-loop	3-loop	4-loop
N^{4}LL	3-loop	5- 	4-loop	4-loop	5-loop
N^{4}LL′	4-loop	5-	4-loop	4-loop	5-loop

[Herzog Moch Ruijl Ueda, Vermaseren Vogt 1812.11818]

5-loop cusp estimated to have a very small effect

Transverse momentum distribution

Impressive progress in the calculation of anomalous dimension and coefficient functions allows resummation up to approximate N^{4}LL accuracy

$$\frac{d\sigma}{dq_T} \sim \mathcal{B}(\mu_J,\nu) \otimes \mathcal{H}(M,\mu_H) \otimes \mathcal{S}(\mu_S,\nu) \otimes \mathcal{B}(\mu_J,\nu)$$

Accuracy	$\mathcal{H},\mathcal{B}$	$\Gamma_{\rm cusp}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
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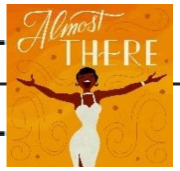
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL$'$	2-loop	3-loop	2-loop	2-loop	3-loop
N^{3}LL	2-loop	4-loop	3-loop	3-loop	4-loop
N^{3}LL$'$	3-loop	4-loop	3-loop	3-loop	4-loop
N^{4}LL	3-loop	5- 	4-loop	4-loop	5-loop
N^{4}LL$'$	4-loop	5-	4-loop	4-loop	5-loop

[Agarwala von Manteuffel Panzerb Schabinger 2102.09725]

Transverse momentum distribution

Impressive progress in the calculation of anomalous dimension and coefficient functions allows resummation up to approximate N^{4}LL accuracy

$$\frac{d\sigma}{dq_T} \sim \mathcal{B}(\mu_J,\nu) \otimes \mathcal{H}(M,\mu_H) \otimes \mathcal{S}(\mu_S,\nu) \otimes \mathcal{B}(\mu_J,\nu)$$

Accuracy	$\mathcal{H},\mathcal{B}$	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
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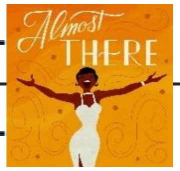
NLL	Tree level	2-loop ✓	1-loop ✓	1-loop ✓	2-loop ✓
NLL$'$	1-loop	2-loop ✓	1-loop ✓	1-loop ✓	2-loop ✓
NNLL	1-loop	3-loop ✓	2-loop ✓	2-loop ✓	3-loop ✓
NNLL$'$	2-loop	3-loop ✓	2-loop ✓	2-loop ✓	3-loop ✓
N^{3}LL	2-loop	4-loop ✓	3-loop ✓	3-loop ✓	4-loop ✓
N^{3}LL$'$	3-loop	4-loop ✓	3-loop ✓	3-loop ✓	4-loop ✓
N^{4}LL	3-loop	5- 	4-loop ✓	4-loop ✓	5-loop ✓
N^{4}LL$'$	4-loop	5-	4-loop ✓	4-loop ✓	5-loop ✓

Vladimirov's conjecture $\Longrightarrow$ [Duhr Mistberger Vita 2205.02242]

[Vladimirov 1610.01404]

[Moult Zhu Zhu 2205.02249]

Transverse momentum distribution

Impressive progress in the calculation of anomalous dimension and coefficient functions allows resummation up to approximate N^{4}LL accuracy

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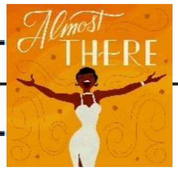
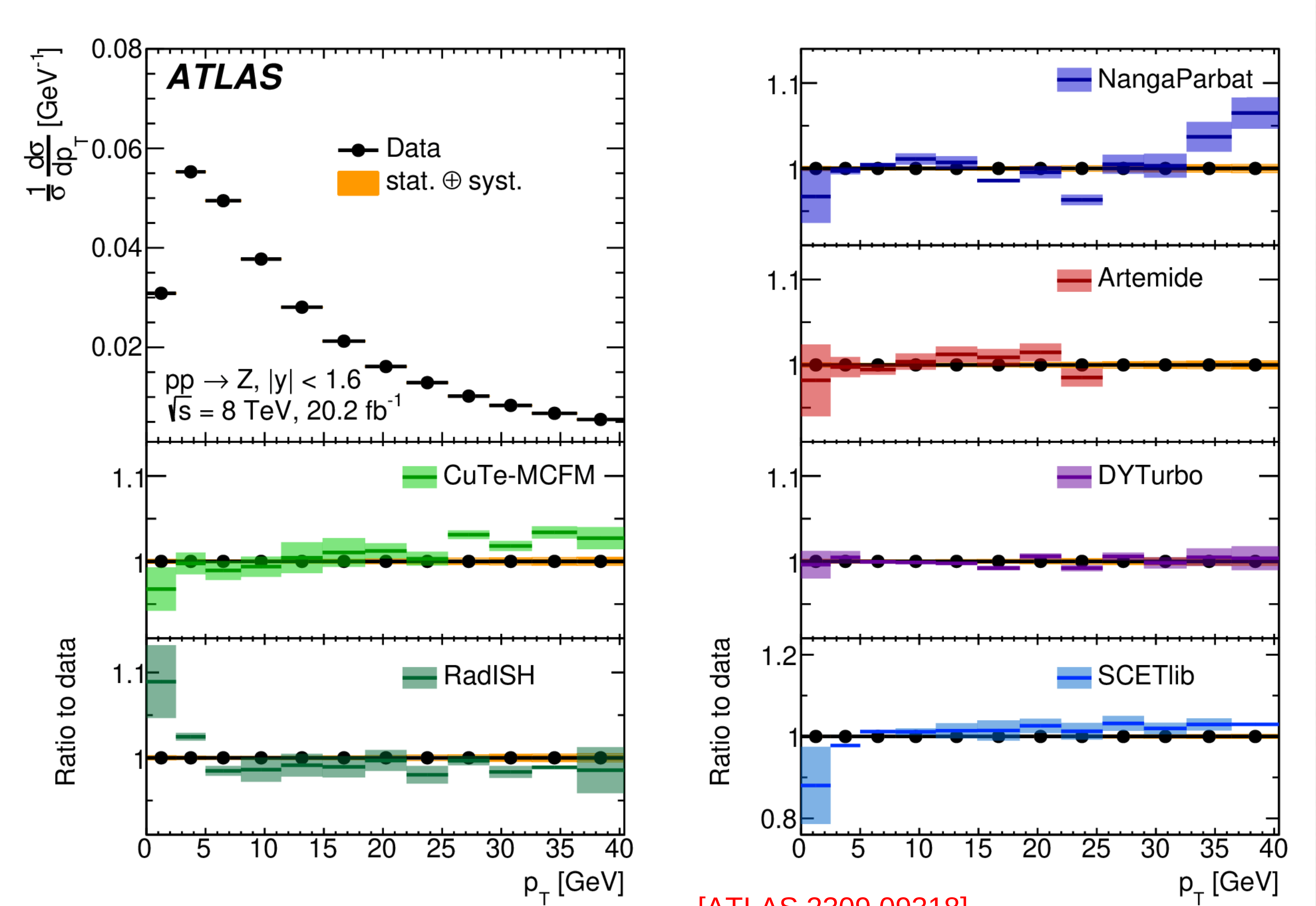
NNLL$'$	2-loop	3-loop	2-loop	2-loop	3-loop
N^{3}LL	2-loop	4-loop	3-loop	3-loop	4-loop
N^{3}LL$'$	3-loop	4-loop	3-loop	3-loop	4-loop
N^{4}LL	3-loop	5- 	4-loop	4-loop	5-loop
N^{4}LL$'$	4-loop	5-	4-loop	4-loop	5-loop

3-loop beam function [Ebert Mistberger Vita 2205.02242]

4-loop hard function [Lee von Manteuffel Schabinger Smirnov Smirnov 2202.04660]

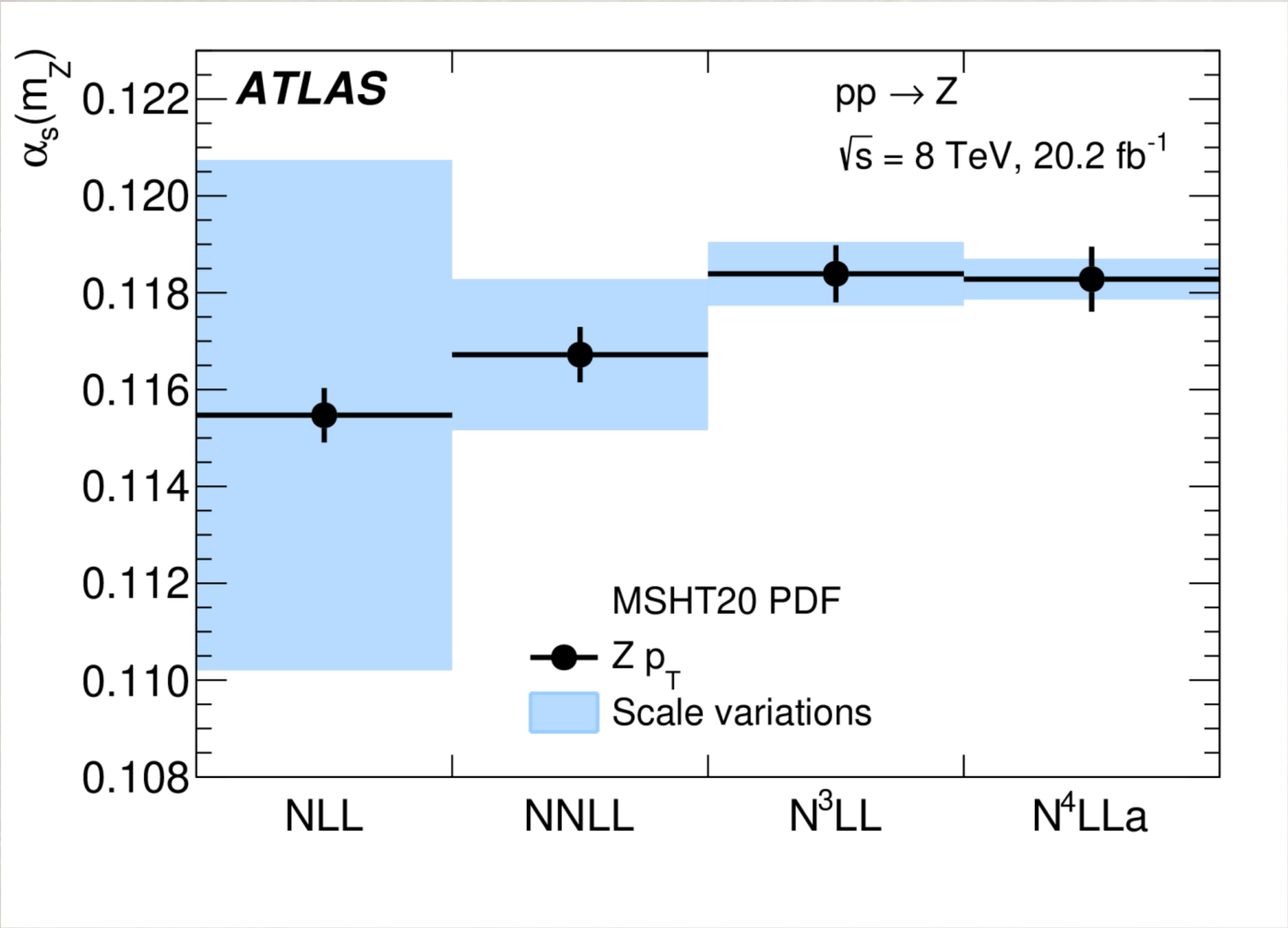
Comparison to LHC data

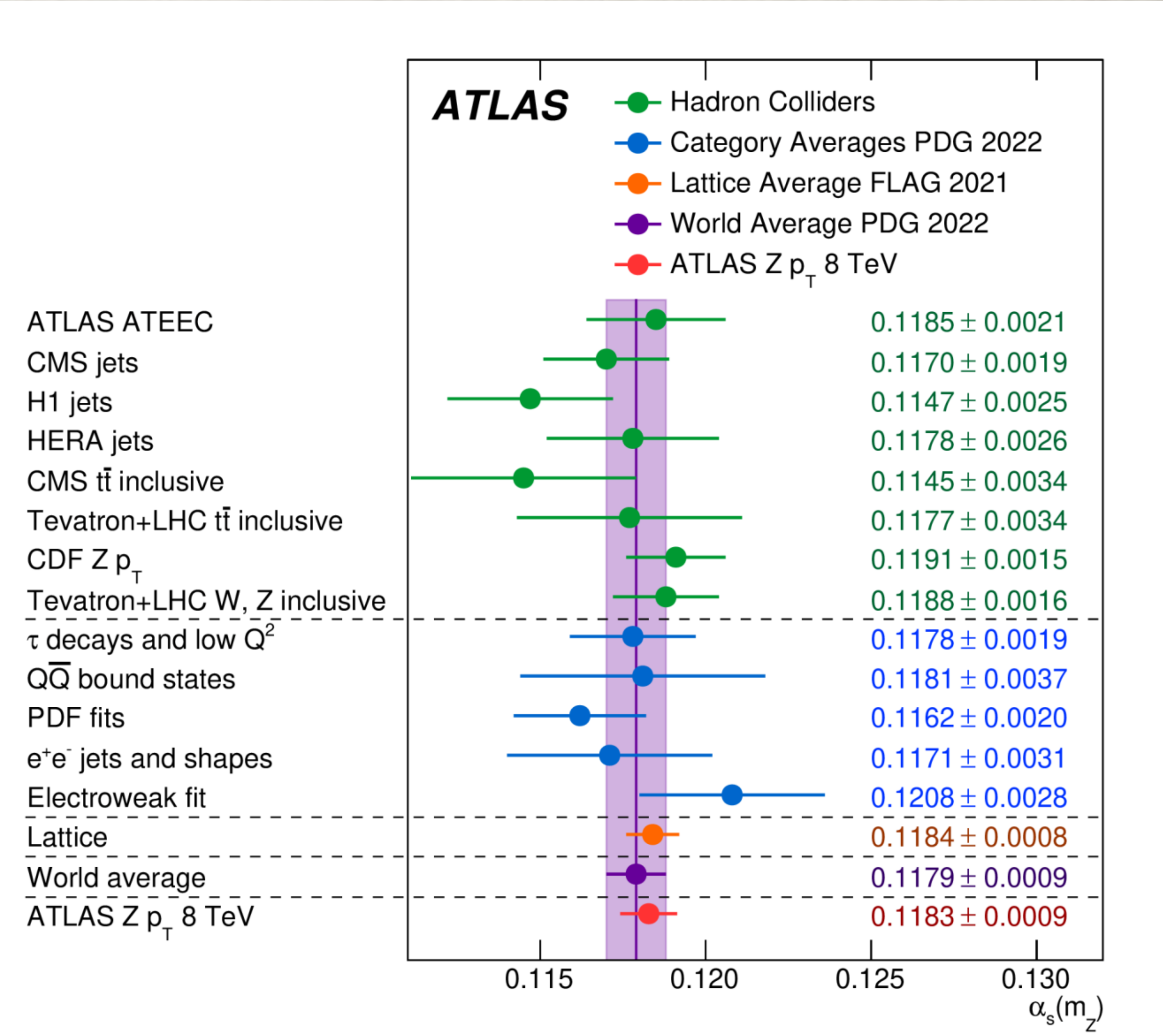


[ATLAS 2309.09318]

All implementations except Artemide include α_s^3 results from MCFM

MEASURING THE STRONG COUPLING





[ATLAS 2309.12986]

- Predictions obtained with DYturbo

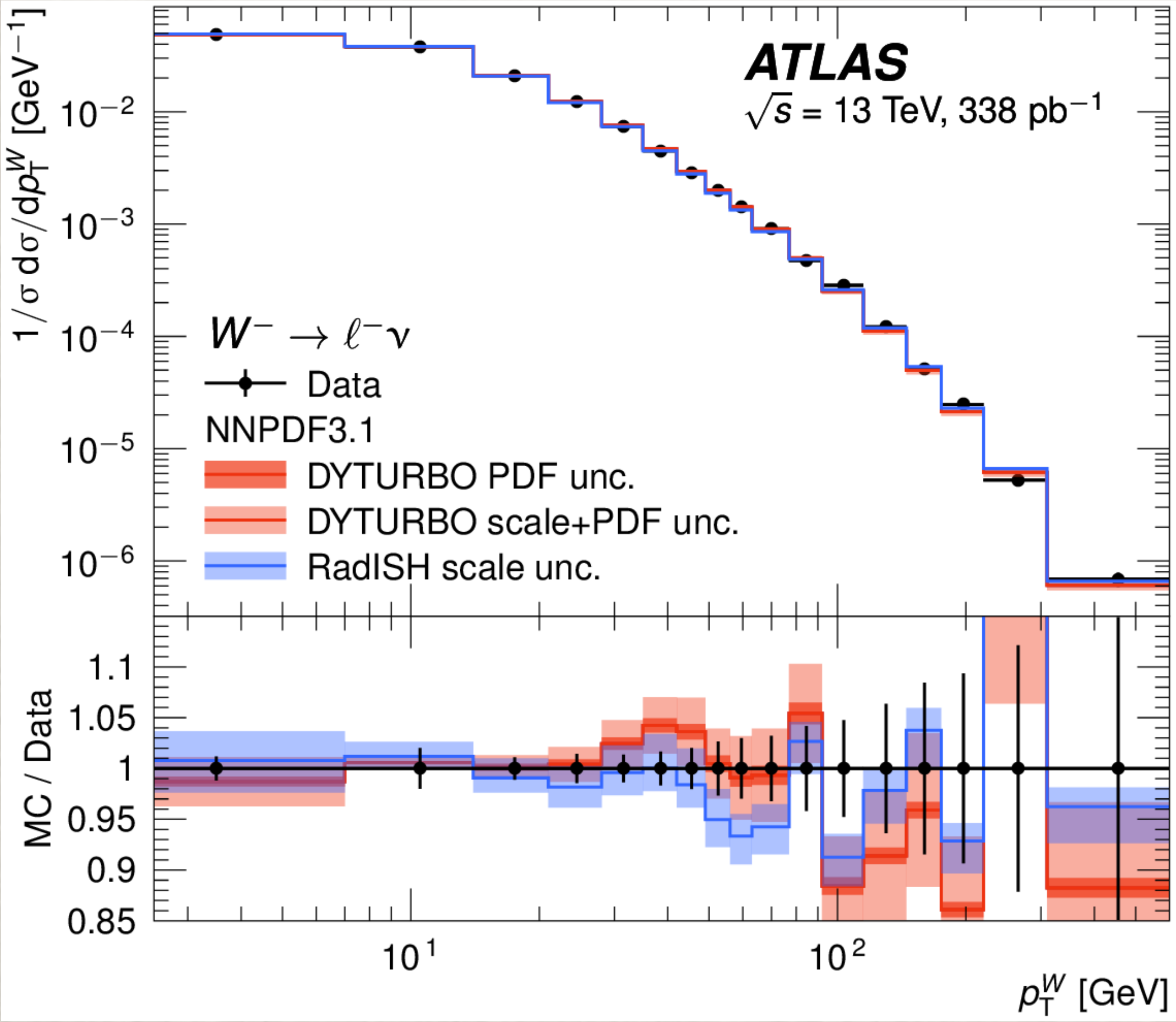
  [Camarda Cieri Ferrera 2303.12781]

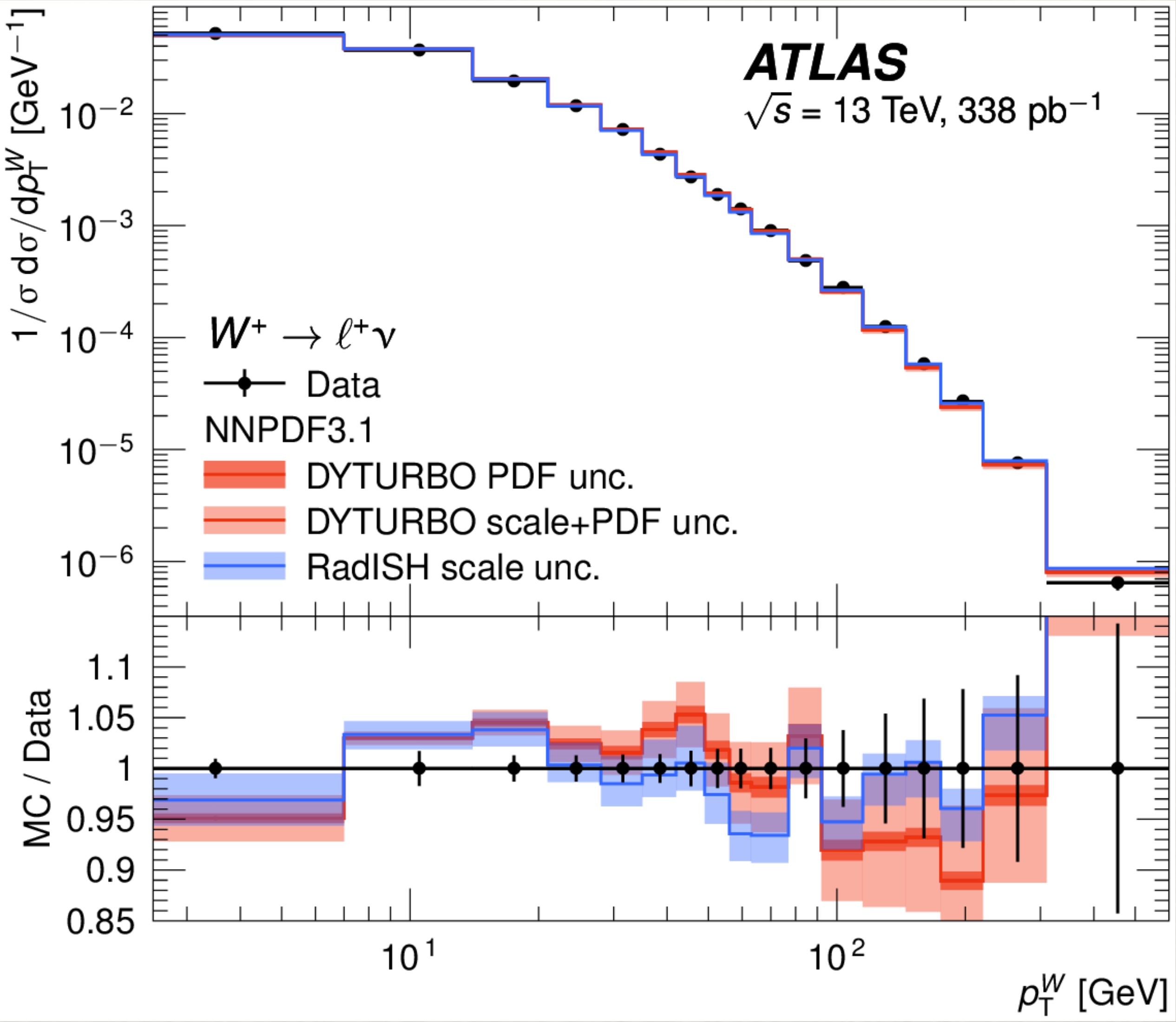
- Two-parameter non-perturbative model

  [Collins Rogers 1412.3820]

Very precise determination of α_s

New: W^+ and W^- p_T distributions





- W transverse momentum reconstructed via hadronic recoil

[ATLAS 2404.06204]

- Very good agreement with QCD resummed predictions (N^{3}LL+NNLO) in all regions of the spectrum

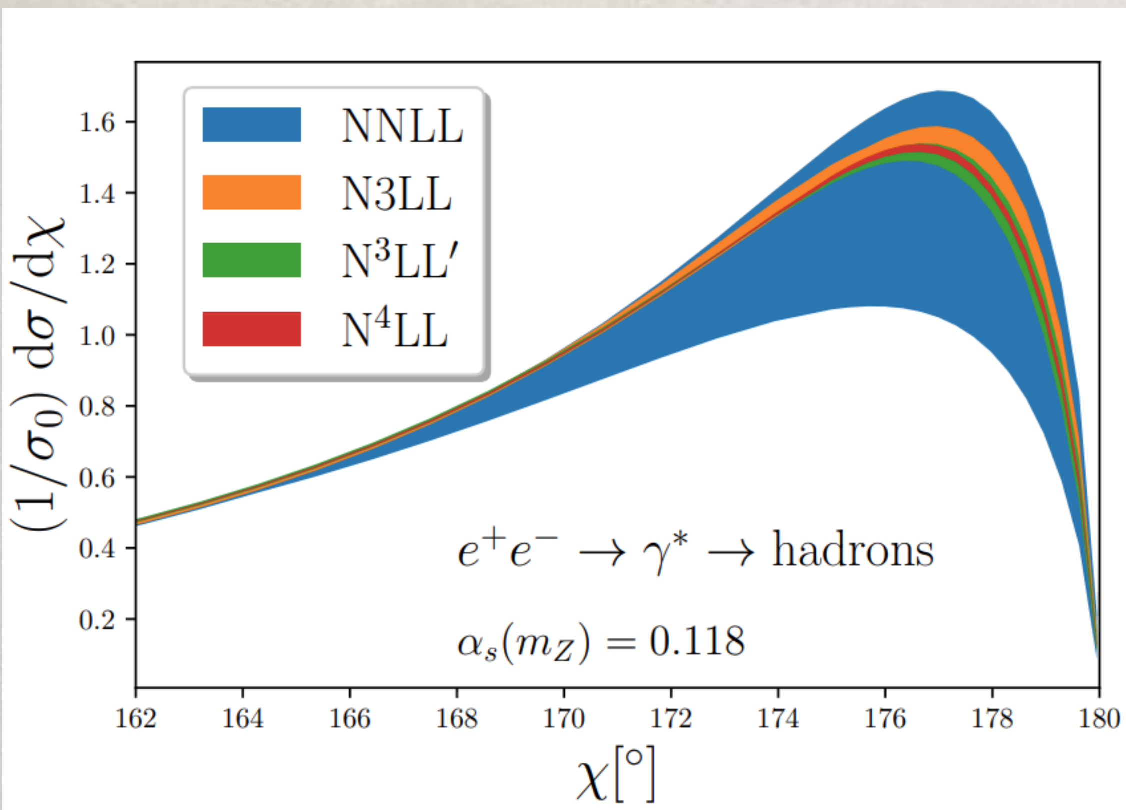
[Camarda Cieri Ferrera 2303.12781]

[Chen Gehrmann Glover Huss Monni Re Rottoli Torrielli 2203.01565]

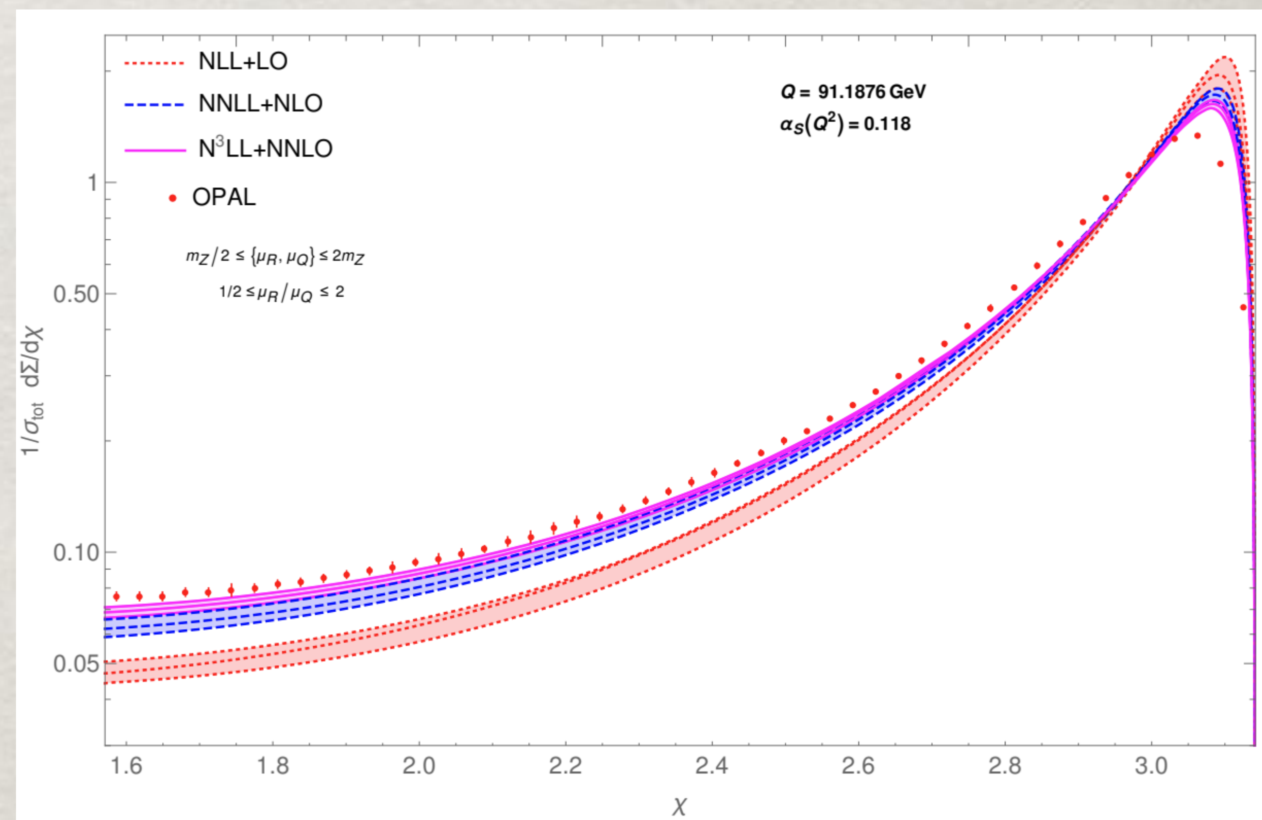
Energy-energy correlation

The new information on the rapidity anomalous dimension makes it possible to resum also the energy-energy correlation in e^+e^-at aN^{4}LL accuracy

$$\mathrm{EEC}(\chi) \equiv \sum_{a,b} \int d\sigma_{ab} \frac{E_a E_b}{Q^2} \delta(\cos\chi_{ab} - \cos\chi)$$



[Duhr Mistberger Vita 2205.02242]



[Aglietti Ferrera 2403.04077]

- The EEC has large 1/Q hadronisation corrections: important at this accuracy (see Friday's mini workshop)

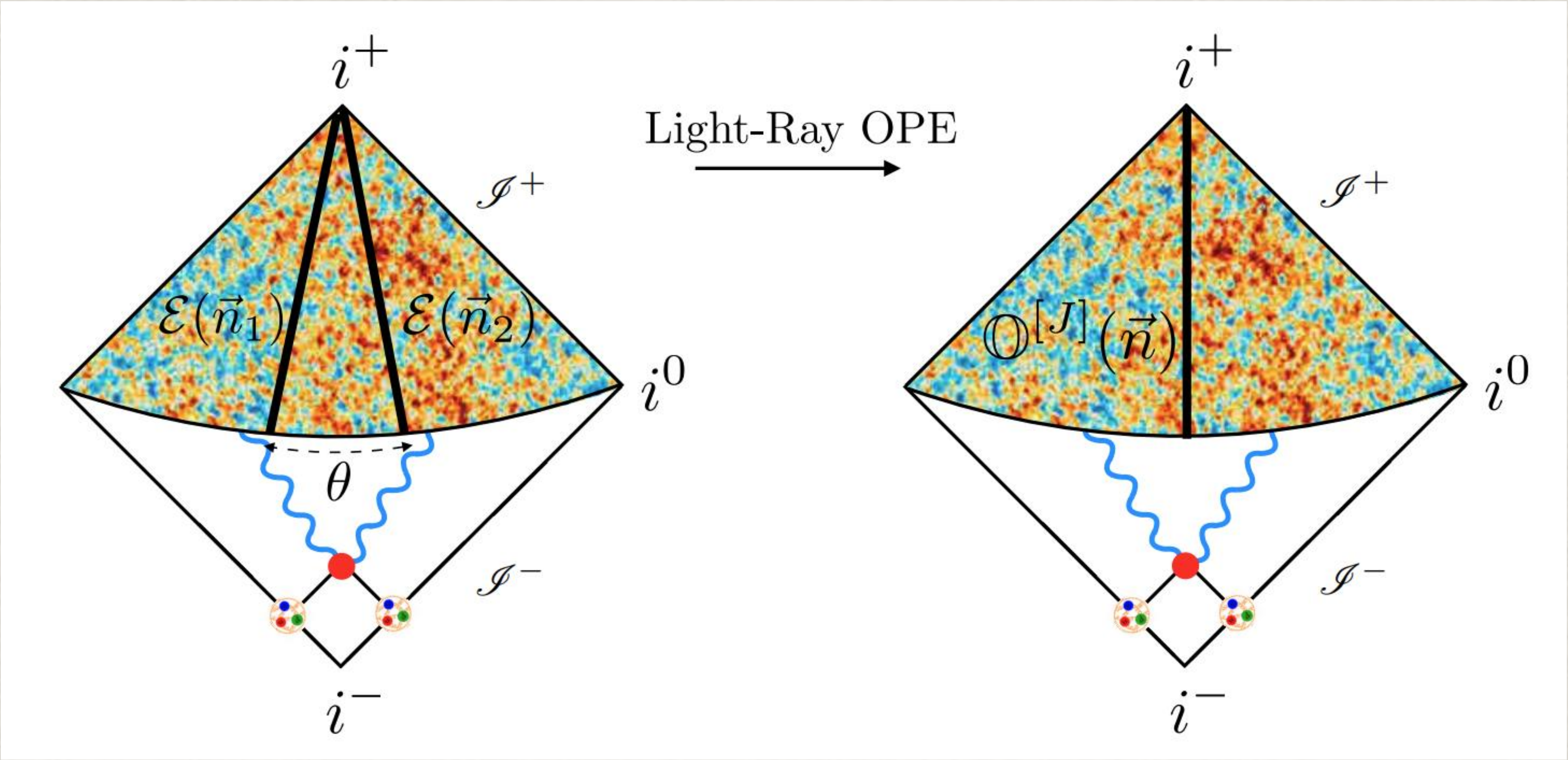
[Dokshitzer Marchesini Webber hep-ph/9905339]

[Schindler Stewart Sun 2305.19311]

Energy-energy correlators

The EEC is just a particular case of energy-energy correlators

[Chen Moult Zhang Zhu 2004.11381]



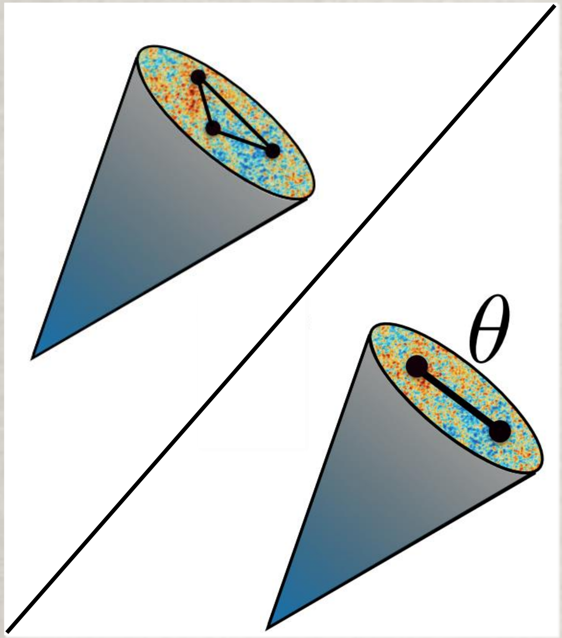
[Ian Moult LHCP 2024]

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, n^i T_{0i}(t, r\vec{n})$$

$$\lim_{\vec{n}_1\to\vec{n}_2} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \frac{1}{\theta^2}\vec{C}\cdot\vec{\mathbb{O}}^{[J=3]}_{\tau=2}(\vec{n}_2) + \frac{\Lambda_{\mathrm{QCD}}}{\theta^3}\vec{D}\cdot\vec{\mathbb{O}}^{[J=2]}_{\tau=2}(\vec{n}_2) + \dots$$

Energy-energy correlators

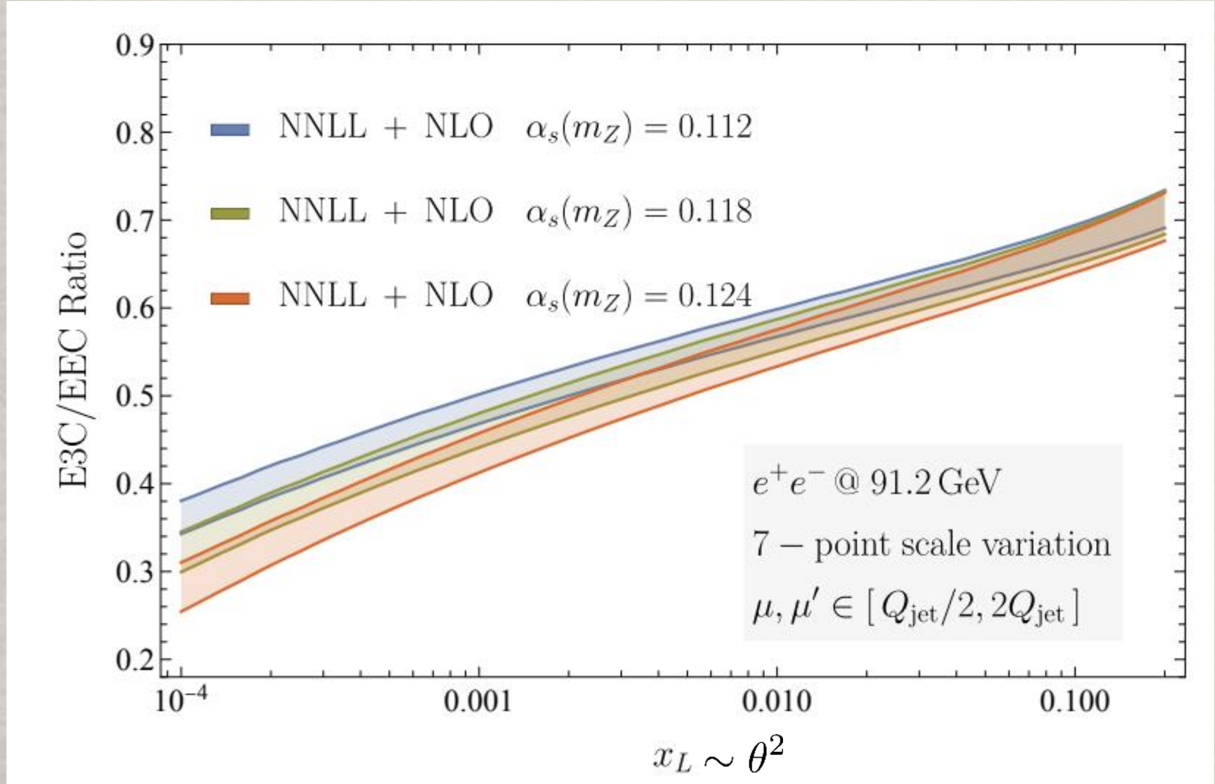
In the small-angle limit, energy-energy correlators exhibit anomalous scalings



$$\lim_{\vec{n}_1 \to \vec{n}_2} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \sim \frac{1}{\theta^2} \sum_i \theta^{\gamma_i^{[J=3]}} \mathbb{O}_i^{[J=3]}(\vec{n}_2) + \frac{\Lambda_{\mathrm{QCD}}}{\theta^3} \sum_i \theta^{\gamma_i^{[J=2]}} \mathbb{O}_i^{[J=2]} + \ldots$$

- Anomalous dimensions can be calculated up to NLL accuracy in hh collisions and NNLL accuracy in e^+e^- annihilation $\Rightarrow$ avenue for precision measurement of α_s

[Chen Gao Li Xu Zhang Zhu 2307.07510]



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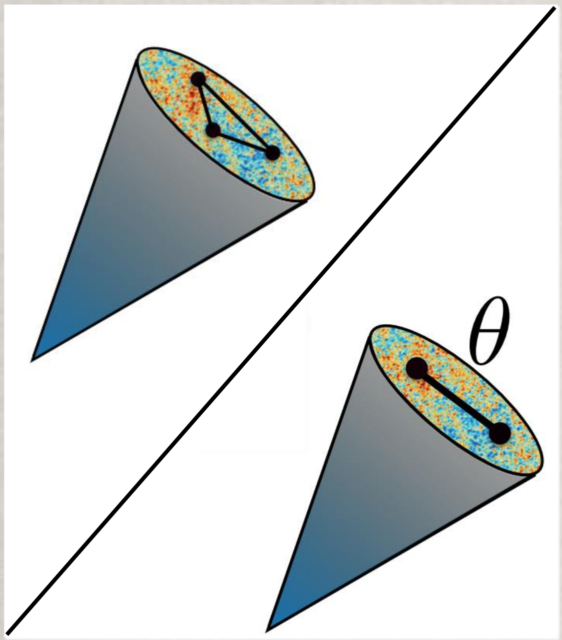
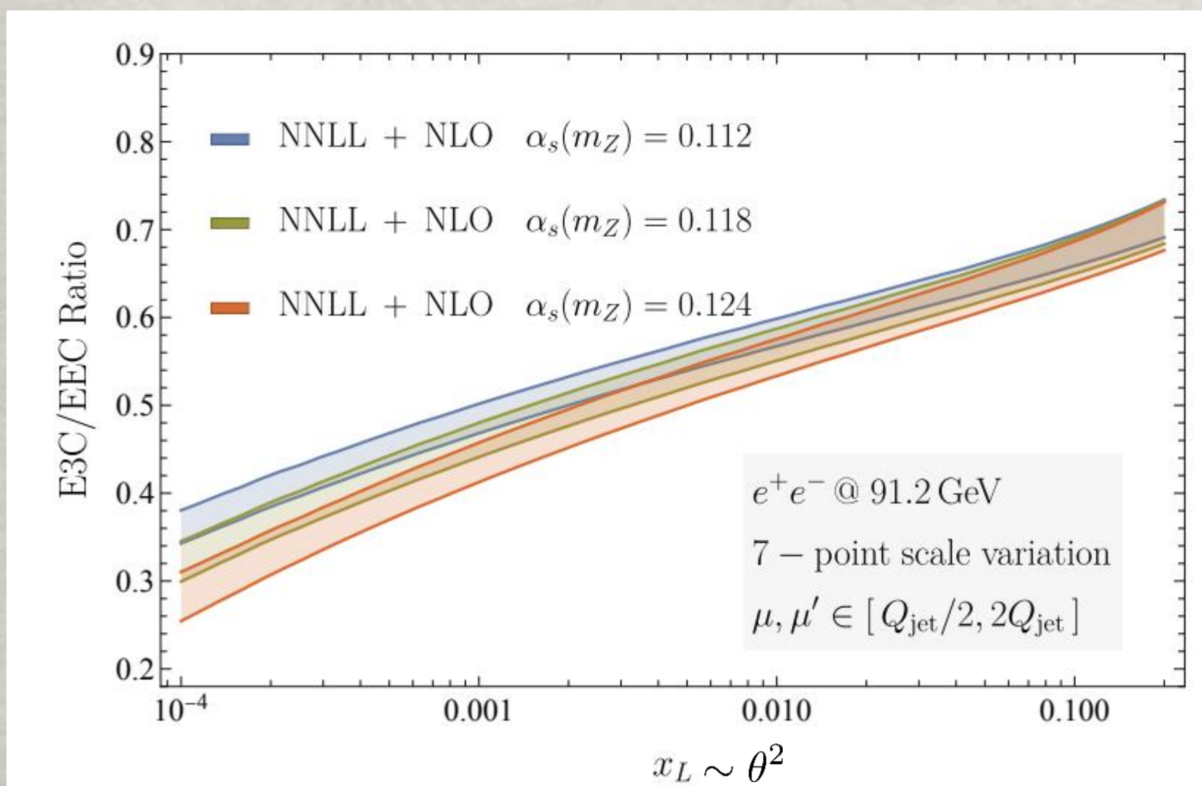


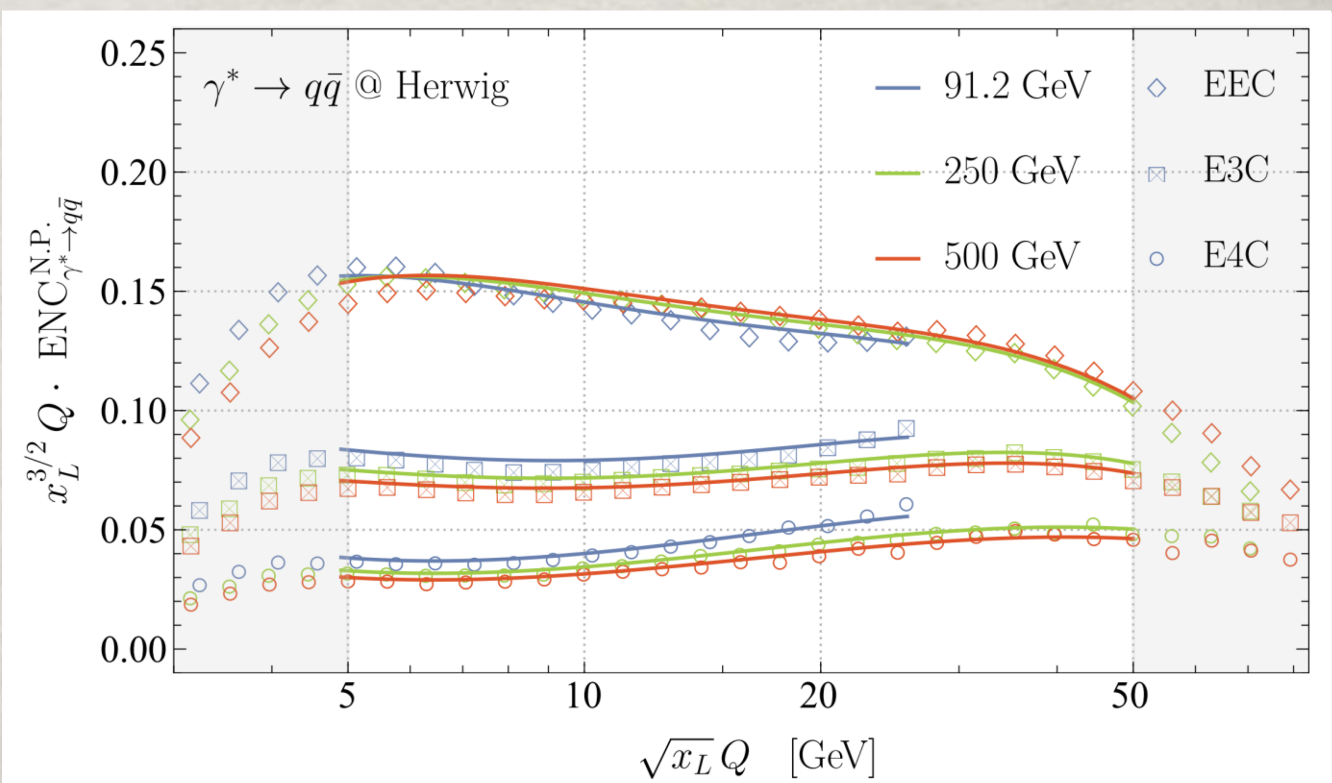
$$\lim_{\vec{n}_1\to\vec{n}_2} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \sim \frac{1}{\theta^2}\sum_i \theta^{\gamma_i^{[J=3]}} \mathbb{O}_i^{[J=3]}(\vec{n}_2) + \frac{\Lambda_{\rm QCD}}{\theta^3}\sum_i \theta^{\gamma_i^{[J=2]}} \mathbb{O}_i^{[J=2]} + \ldots$$

- Anomalous dimensions can be calculated up to NLL accuracy in hh collisions and NNLL accuracy in e^+e^- annihilation $\Rightarrow$ avenue for precision measurement of α_s

  [Chen Gao Li Xu Zhang Zhu 2307.07510]

- Anomalous scaling has been recently calculated for power corrections at LL accuracy [Chen Monni Xu Zhu 2406.06668]

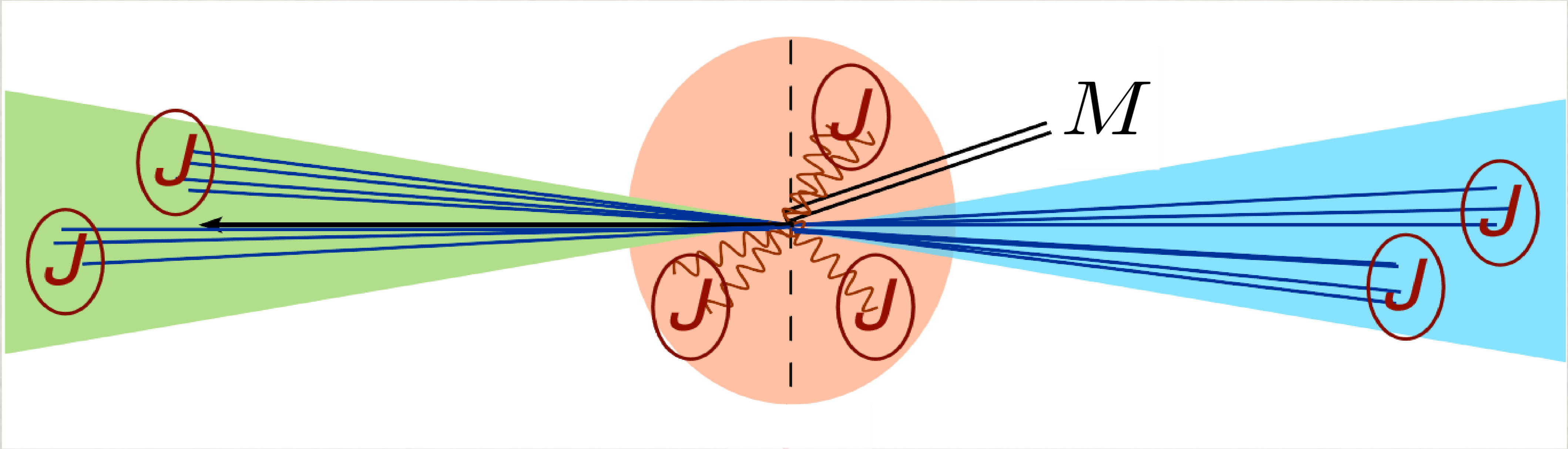




Jet-veto resummation

The cross-section for a colour singlet (e.g. Higgs, Z boson) with the veto-condition that all jets have a transverse momentum below $p_{\mathrm{t,veto}}$ also admits an all-order refactorisation [Becher Neubert 1205.3806]

$$\Sigma(p_{\mathrm{t,veto}}) \sim \mathcal{B}(\mu_J,\nu)\otimes\mathcal{H}(M,\mu_H)\otimes\mathcal{S}(\mu_S,\nu)\otimes\mathcal{B}(\mu_J,\nu)$$



- NNLL corrections have been known for a long time

[AB Monni Salam Zanderighi 1206.4998]

- State of ingredients for N^{3}LL resummation
  - Two-loop soft and beam functions have been recently computed

[Abreu Gaunt Monni Szafron 2204.02987]

[Abreu Gaunt Monni Rottoli Szafron 2207.07037]

  - Calculation of the three-loop rapidity anomalous dimension is in progress

N-jettiness

- N-jettiness is an alternative way to veto jets, with a constrain that dampens the collinear region $\Rightarrow$ no rapidity anomalous dimensions

$$\mathcal{T}_1 \equiv \sum_i \min\left\{\frac{q_i \cdot p_a}{Q_a}, \frac{q_i \cdot p_a}{Q_b}, \frac{q_i \cdot p_{J_1}}{Q_{J_1}}, \ldots, \frac{q_i \cdot p_{J_N}}{Q_{J_N}}\right\}$$

- N-jettiness also admits an all-order refactorisation formula

[Stewart Tackmann Waalewijn 1004.2489]

$$\frac{d\sigma}{d\mathcal{T}_1} \sim \mathcal{B}_a(\mu_J) \otimes \mathcal{H}(M, \mu_H) \otimes \mathcal{S}(\mu_S) \otimes \mathcal{B}_b(\mu_J) \prod_{i=1}^{N} J_i(\mu_J)$$

- NNLL corrections have been known for a long time

[Stewart Tackmann Waalewijn 1005.4060]

- All ingredients exist for N^{3}LL resummation (see A. Broggio's talk)

[Alioli Bell Dehnadi Mohrmann Rahn 2312.11626]

  - Two-loop soft function

[Bell Dehnadi Mohrmann Rahn 2312.11626]

[Agarwal Melnikov Pedron 2403.03078]

  - Three-loop beam function (needed for N^{3}LL' accuracy)

[Ebert Mistlberger Vita 2006.03056]

[Barankowski Behring Melnikov Tancredi Wever 2211.05722]

  - Part of the three-loop soft function (needed for N^{3}LL' accuracy)

[Barankowski Delto Melnikov Wang 2204.09459]

[Barankowski Delto Melnikov Pikelner Wang 2401.05245]

Impact of resummation

IR STRUCTURE OF QCD

- Resummation requires a deep understanding of the infrared structure of gauge theories
- This knowledge is vital information for fixed-order calculations, and provided the foundations of the so-called NNLO revolution

The singular behaviour of QCD amplitudes at two-loop order

Stefano Catani [1]

Theory Division, CERN, CH-1211 Geneva 23, Switzerland
and LPTHE, Université Paris-Sud, Bâtiment 211, F-91405 Orsay Cedex, France

Received 27 February 1998
Editor: R. Gatto

[hep-ph/9802439]

Collinear factorization and splitting functions for next-to-next-to-leading order QCD calculations [1]

Stefano Catani [a,2], Massimiliano Grazzini [a,b]

[a] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
[b] *Institute for Theoretical Physics, ETH-Hönggerberg, CH-8093 Zurich, Switzerland* [3]

Received 20 October 1998
Editor: R. Gatto

[hep-ph/9810389]

Infrared factorization of tree-level QCD amplitudes at the next-to-next-to-leading order and beyond ☆

Stefano Catani [a,2], Massimiliano Grazzini [b,3]

[a] *Theory Division, CERN, CH 1211 Geneva 23, Switzerland*
[b] *Institute for Theoretical Physics, ETH-Hönggerberg, CH 8093 Zurich, Switzerland*

Received 2 September 1999; accepted 10 December 1999

[hep-ph/9908523]

- All-order formulae for IR singularities of gauge theories heavily rely on resummation techniques

[Dixon Magnea Sterman 0805.3515]

[Becher Neubert 0901.0722]

Local subtraction Schemes

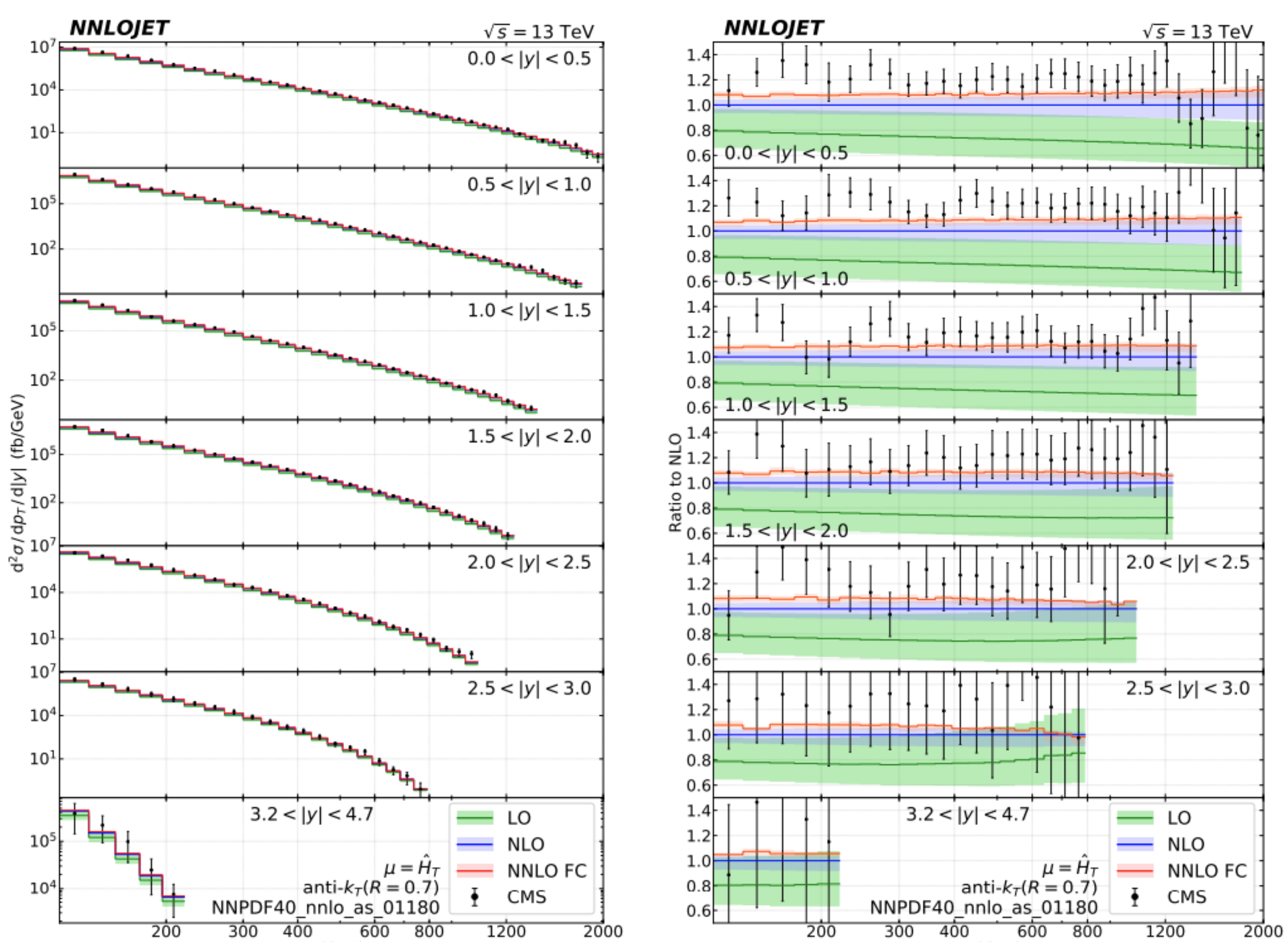
- The knowledge of the singular limits of QCD amplitudes makes it possible to devise local counterterms for fixed-order calculations

A general algorithm for calculating jet cross sections in NLO QCD ⋆

S. Catani [a], M.H. Seymour [b] [hep-ph/9605323]

[a] INFN, Sezione di Firenze, and Dipartimento di Fisica, Università di Firenze, Largo E. Fermi 2, I-50125 Florence, Italy

[b] Theory Division, CERN, CH-1211 Geneva 23, Switzerland



[Chen Gehrmann Glover Huss Mo 2204.10173]

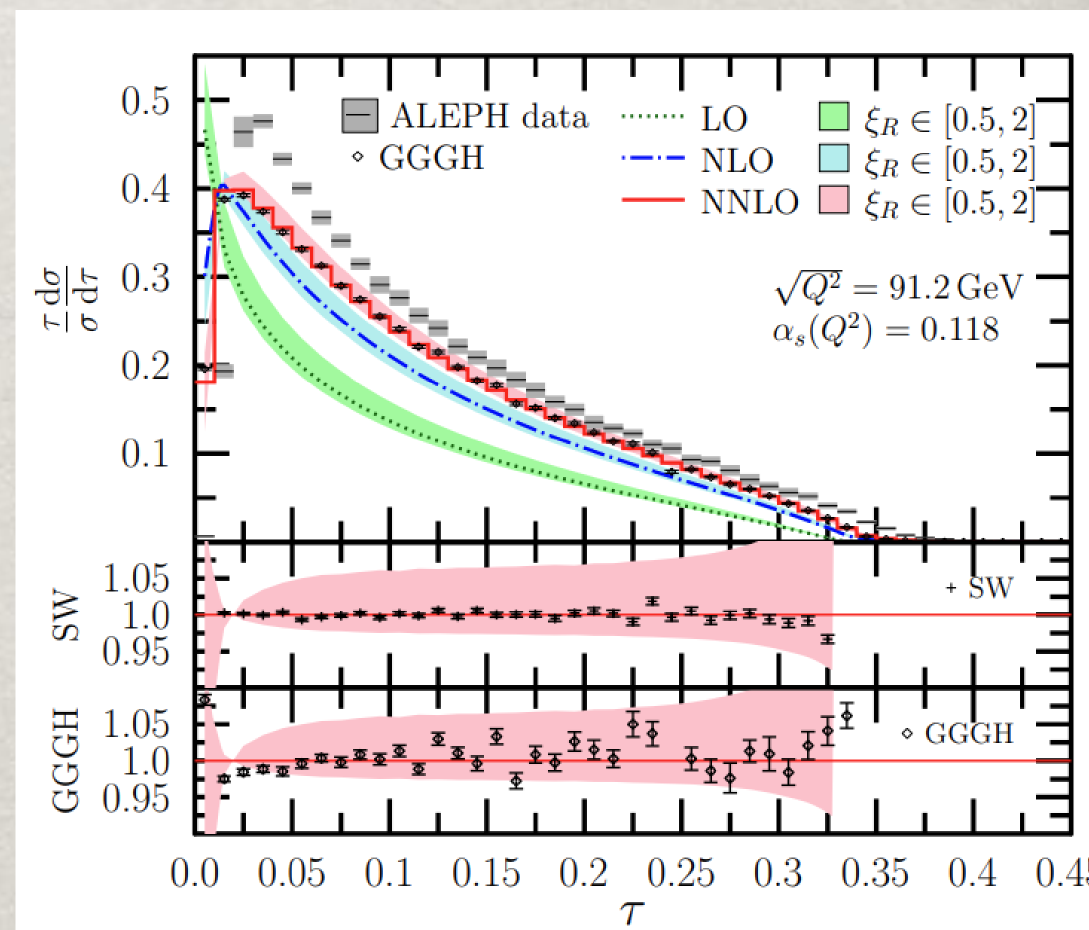
NNLO subtraction schemes

[Gehrmann Gehrmann Glover hep-ph/0505111]

[Del Duca et al 1603.08927]

[Caola Melnikov Röntsch 1902.02081]

[Bertolotti et al 2212.11190]



[Del Duca et al 1606.03453]

Phase-space slicing

Any infrared and collinear safe observable can act as a phase-space slicing parameter

Next-to-Next-to-Leading-Order Subtraction Formalism in Hadron Collisions and its Application to Higgs-Boson Production at the Large Hadron Collider

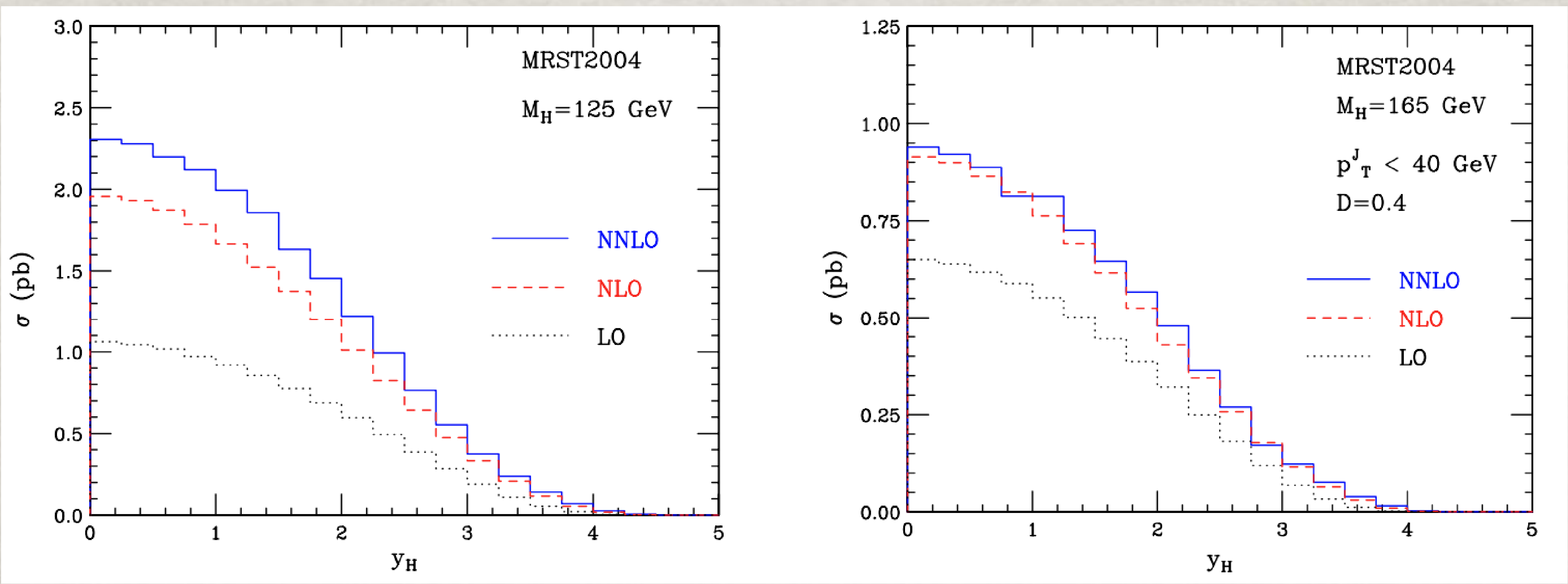
Stefano Catani and Massimiliano Grazzini

INFN, Sezione di Firenze and Dipartimento di Fisica, Università di Firenze, I-50019 Sesto Fiorentino, Florence, Italy

(Received 5 March 2007; published 30 May 2007)

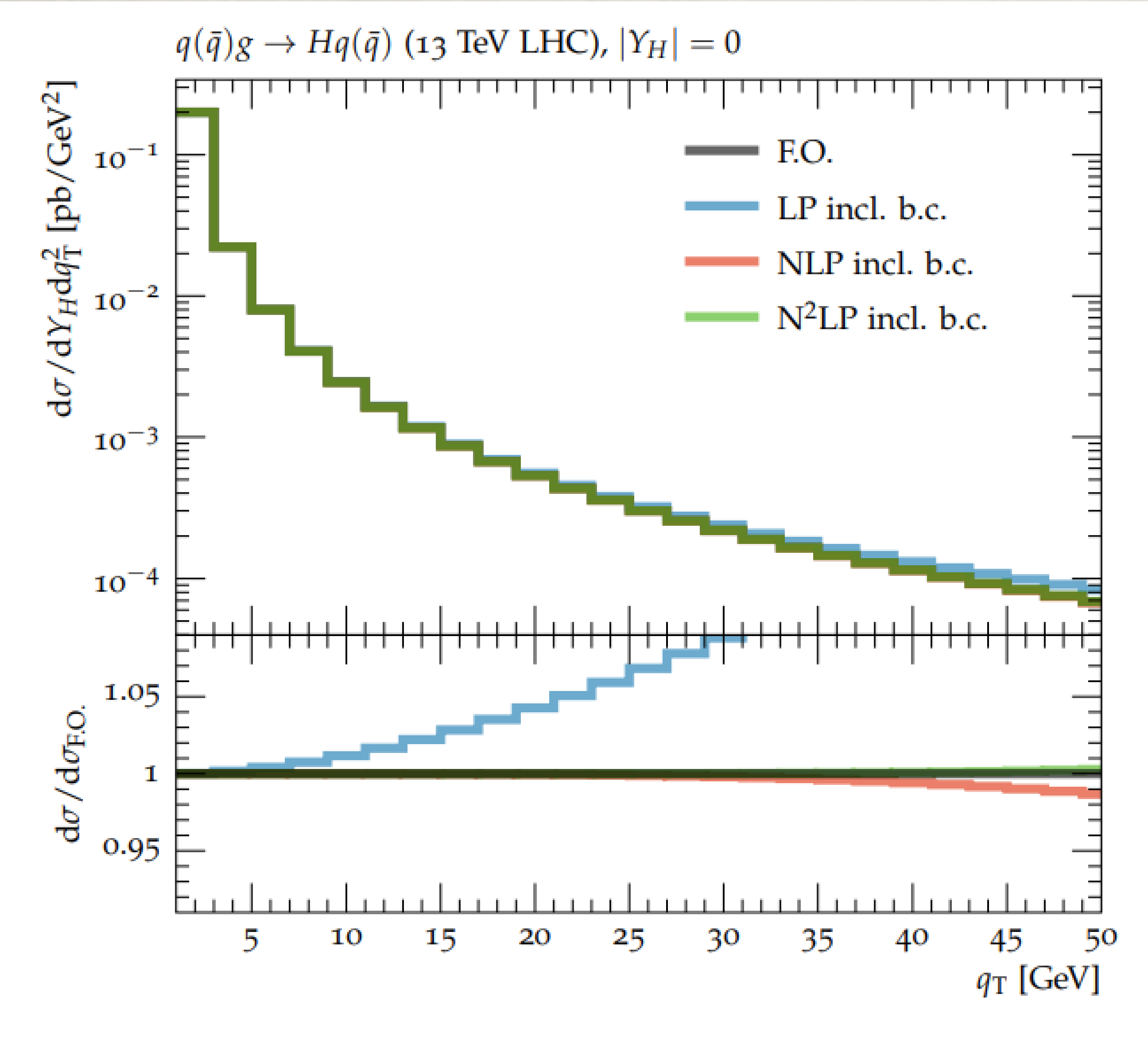
We consider higher-order QCD corrections to the production of colorless high-mass systems (lepton pairs, vector bosons, Higgs bosons, etc.) in hadron collisions. We propose a new formulation of the subtraction method to numerically compute arbitrary infrared-safe observables for this class of processes. To cancel the infrared divergences, we exploit the universal behavior of the associated transverse-momentum (q_T) distributions in the small-q_T region. The method is illustrated in general terms up to the next-to-next-to-leading order in QCD perturbation theory. As a first explicit application, we study Higgs-boson production through gluon fusion. Our calculation is implemented in a parton level Monte Carlo program that includes the decay of the Higgs boson into two photons. We present selected numerical results at the CERN Large Hadron Collider.

[hep-ph/0703012]

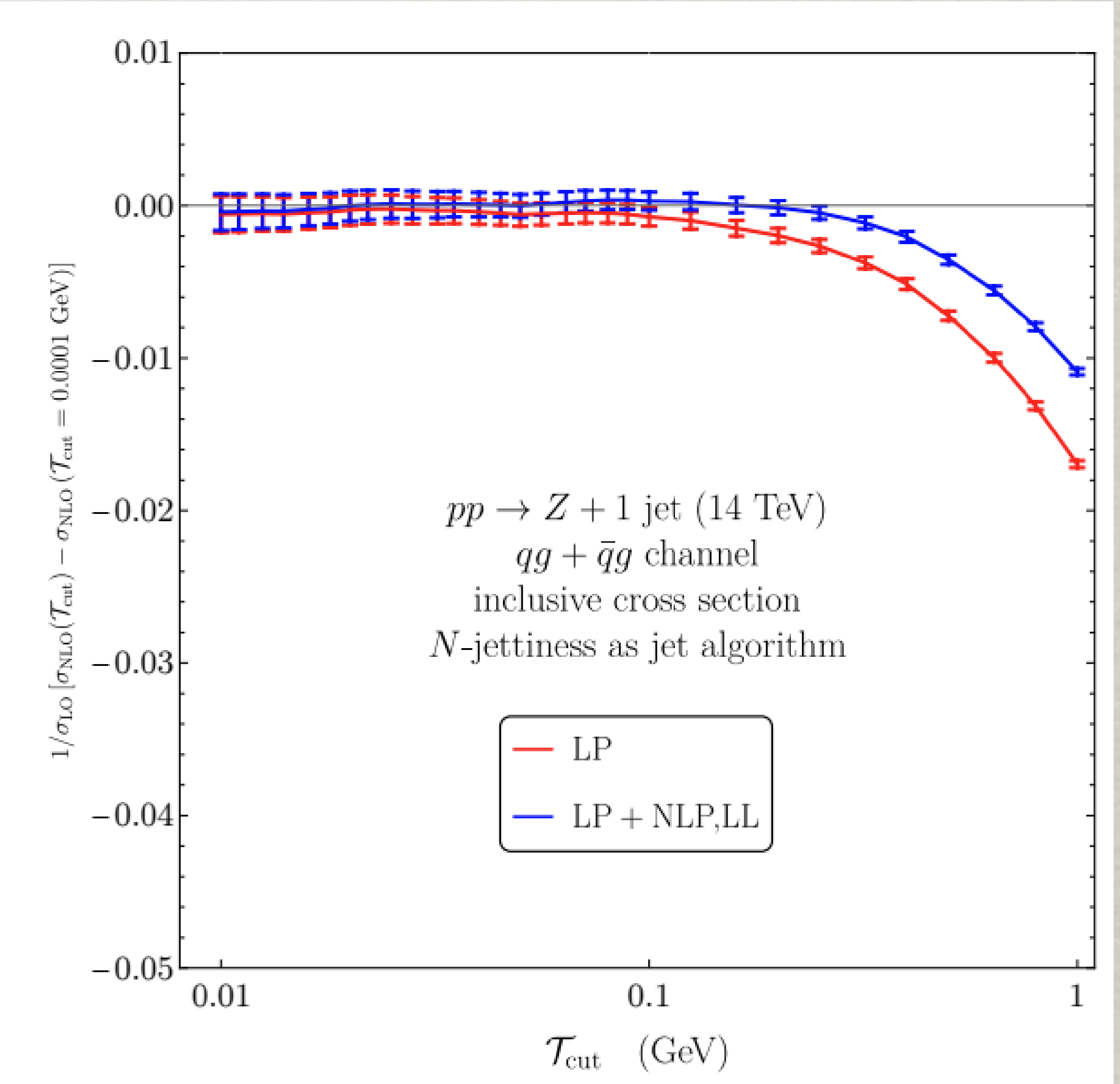


NEXT-TO-LEADING POWER

The subtractions of the logarithmic structure of the slicing variable leaves a power-suppressed leftover $\Rightarrow$ next-to-leading power resummation needed



[Ferrera Ju Schönherr 2312.14911]

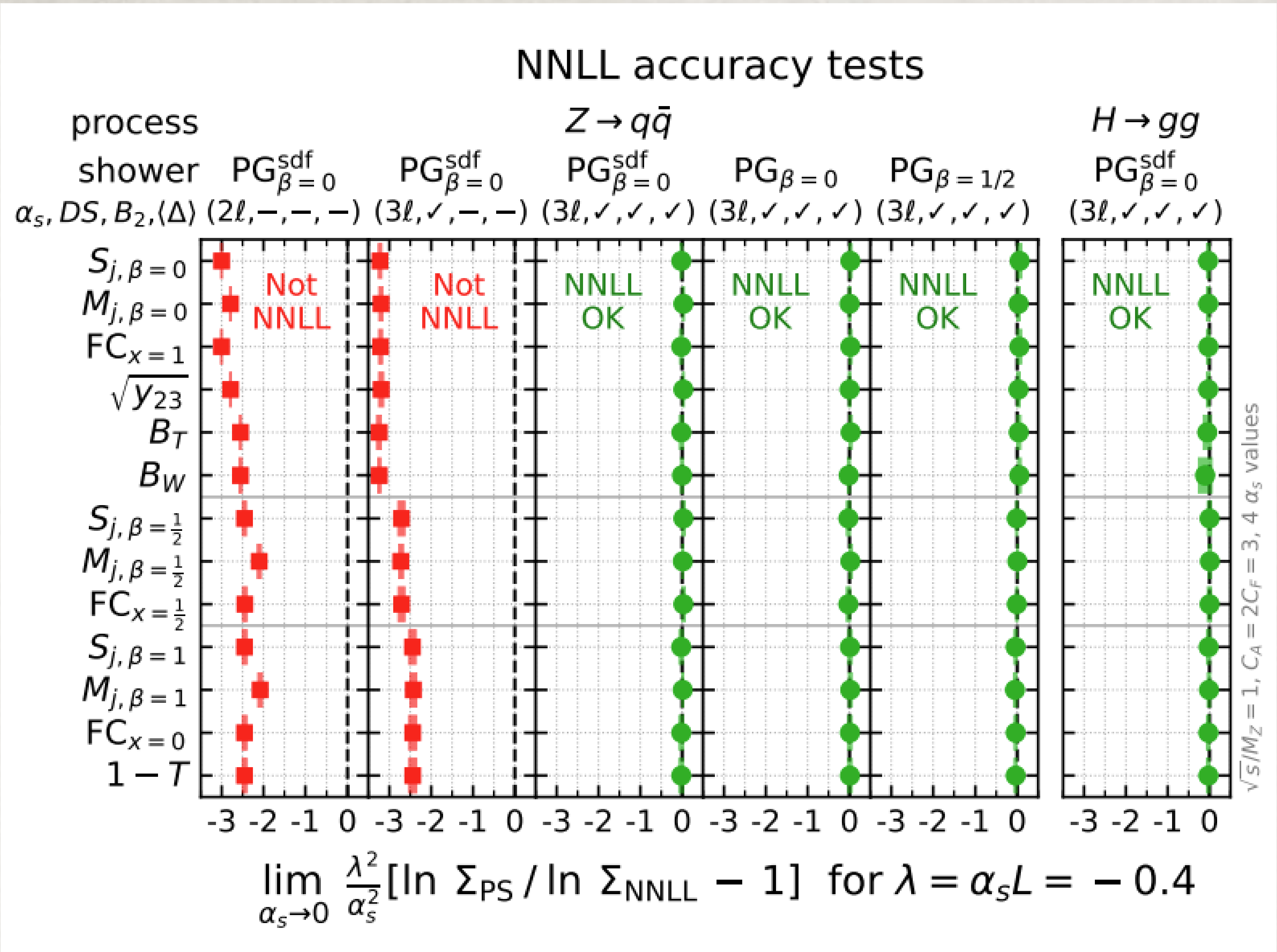


[Boughezal Isgrò Petriello 1907.12213]

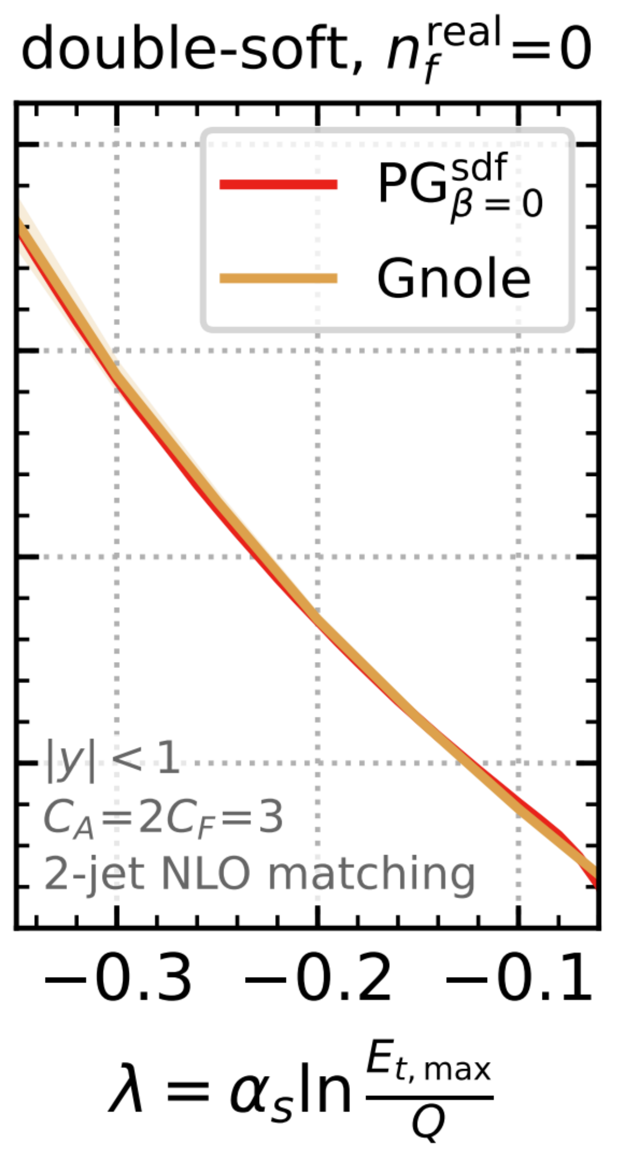
The two variables that are studied at the moment are the transverse momentum of a colour singlet and 0-jettiness

Log accuracy of parton shower

Testing the logarithmic accuracy of parton-shower event generators require the calculation of a number of observables at the required logarithmic accuracy



[van Beekveld Dasgupta El-Menoufi Ferrario-Ravasio et al 2406.02661]



[Ferrario-Ravasio et al 2307.11142]

$$\Sigma(v) = e^{-R(v)} \left[\mathcal{F}_{\rm NLL}(v) + \frac{\alpha_s}{\pi}\delta\mathcal{F}_{\rm NNLL}(v)\right]$$

ARES formula for NNLL resummations

[AB McAslan Monni Zanderighi 1412.2126]

Can we automate?

It is possible to derive general formulae for the resummation of an arbitrary rIRC observable in an arbitrary process

[AB Salam Zanderighi hep-ph/0407286]

- CAESAR performs automatically NLL resummations given only the observable definition in terms of momenta
- Requires careful numerical extrapolations to be extended beyond NLL

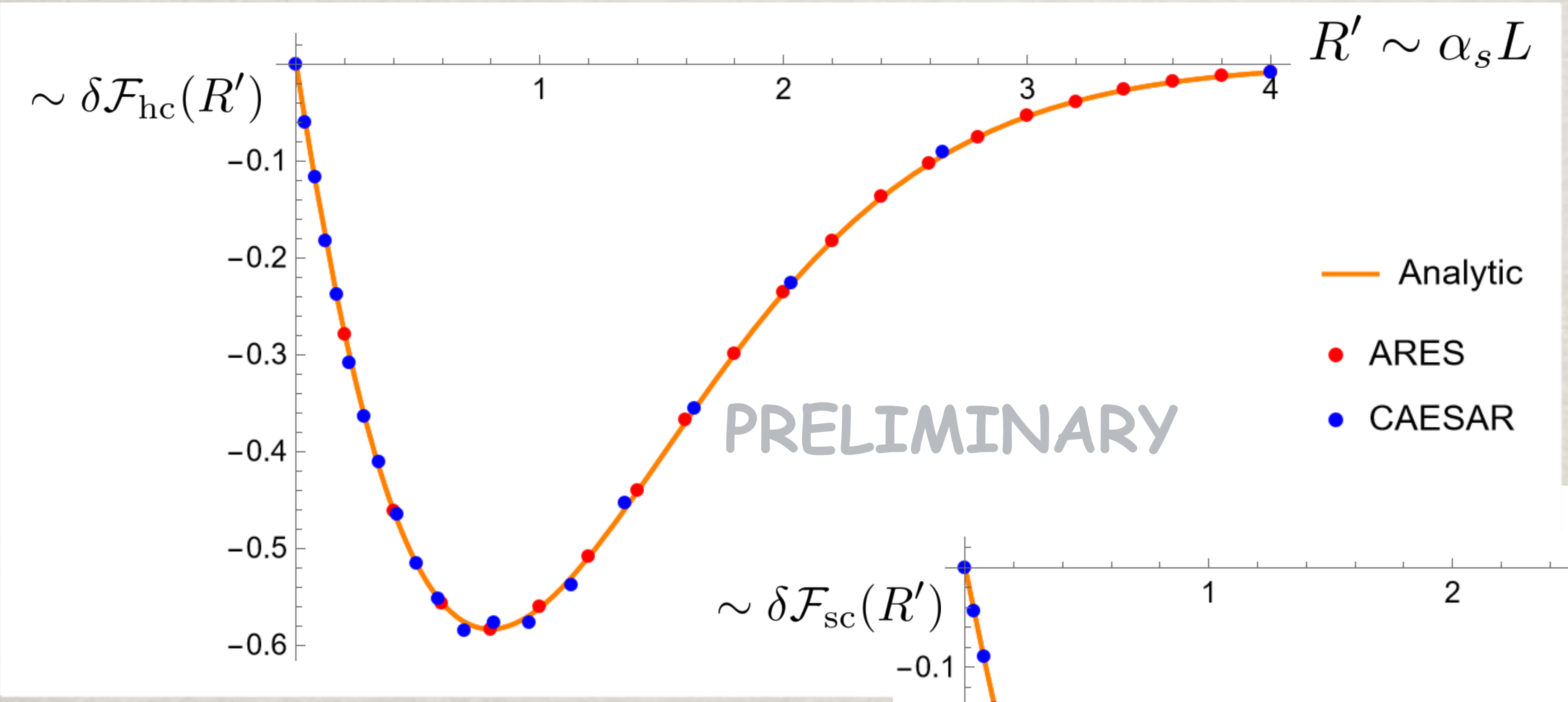
[AB McAslan Monni Zanderighi 1412.2126]

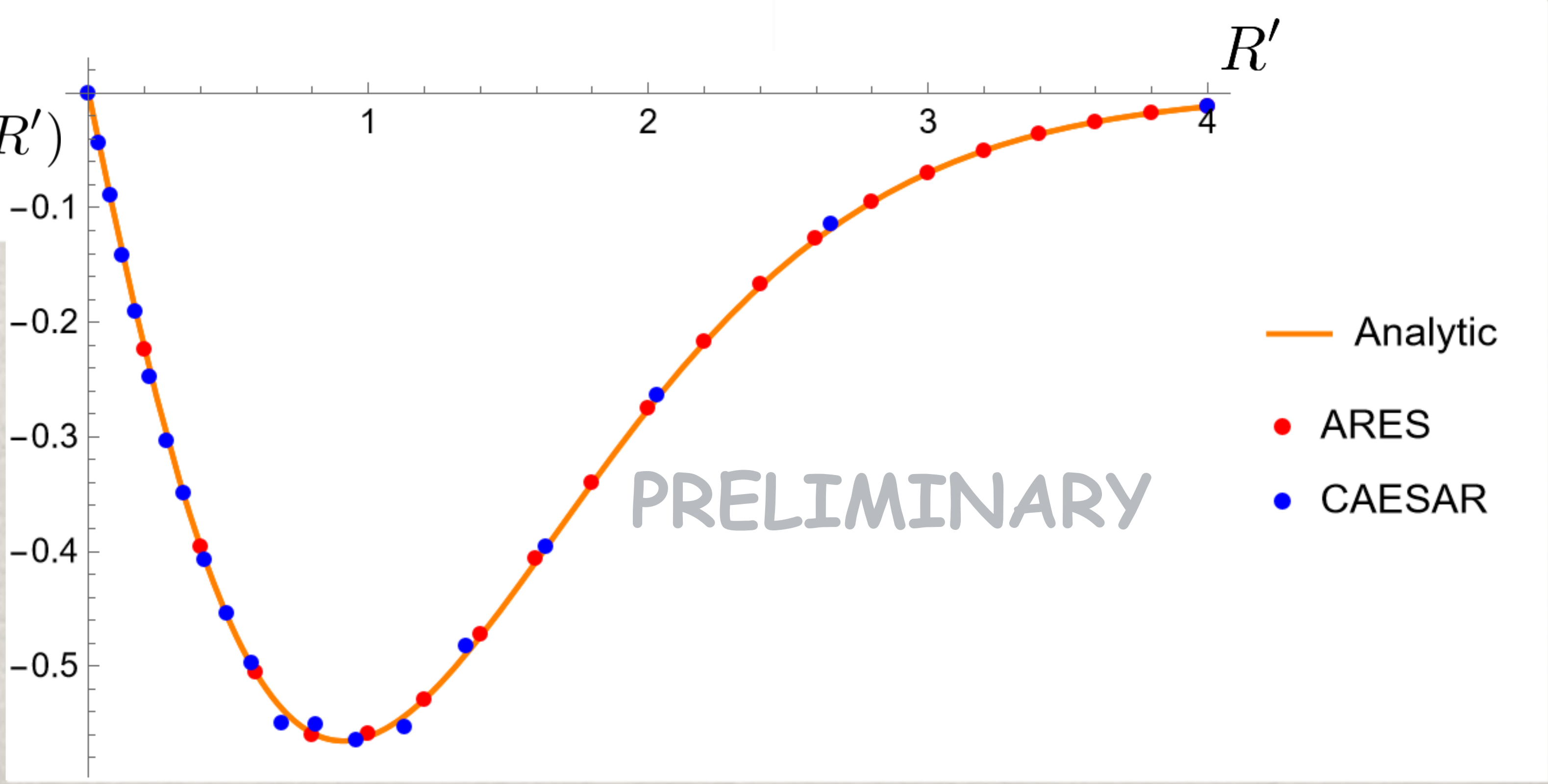
- ARES uses analytically determined soft and collinear limits of each observable
- Currently NNLL, but can be in principle extended to an arbitrary logarithmic accuracy (see e.g. RadISH)

[Monni Re Torrielli 1604.02191]

First-ever NNLL with CAESAR

Soft and collinear emissions are correctly implemented in CAESAR $\Rightarrow$ one can compute the NNLL corrections $\delta\mathcal{F}_{\rm hc}$ and $\delta\mathcal{F}_{\rm sc}$ "out of the box", as all emissions are soft and collinear





Example: additive observables, e.g. thrust

$$V(\{\tilde{p}\}, k_1, \ldots, k_n) = \sum_{i=1}^{n} V(\{\tilde{p}\}, k_i)$$

Resummation vs fixed-order

- For resummations to have an impact on phenomenology, it is vital that predictions are available in the form of publicly available codes
- The great majority of resummed predictions are in the form of in-house programs, typically observable specific
- This is in contrast with fixed-order calculations, where a plethora of general tools is publicly available
  - NLO calculations are fully automated and implemented in general frameworks

    [MCFM https://mcfm.fnal.gov]

    [MADGRAPH_aMC@NLO http://madgraph.phys.ucl.ac.be/]

    [SHERPA https://sherpa-team.gitlab.io/]

    [HELAC https://helac-phegas.web.cern.ch/]

  - NNLO calculations are publicly available for a variety of 2->2 processes

    [MATRIX https://matrix.hepforge.org/]

    [NNLOJET https://nnlojet.hepforge.org/]

Is it possible to establish a common framework to have readily available resummed predictions for a variety of processes?

NLL RESUMMATIONS

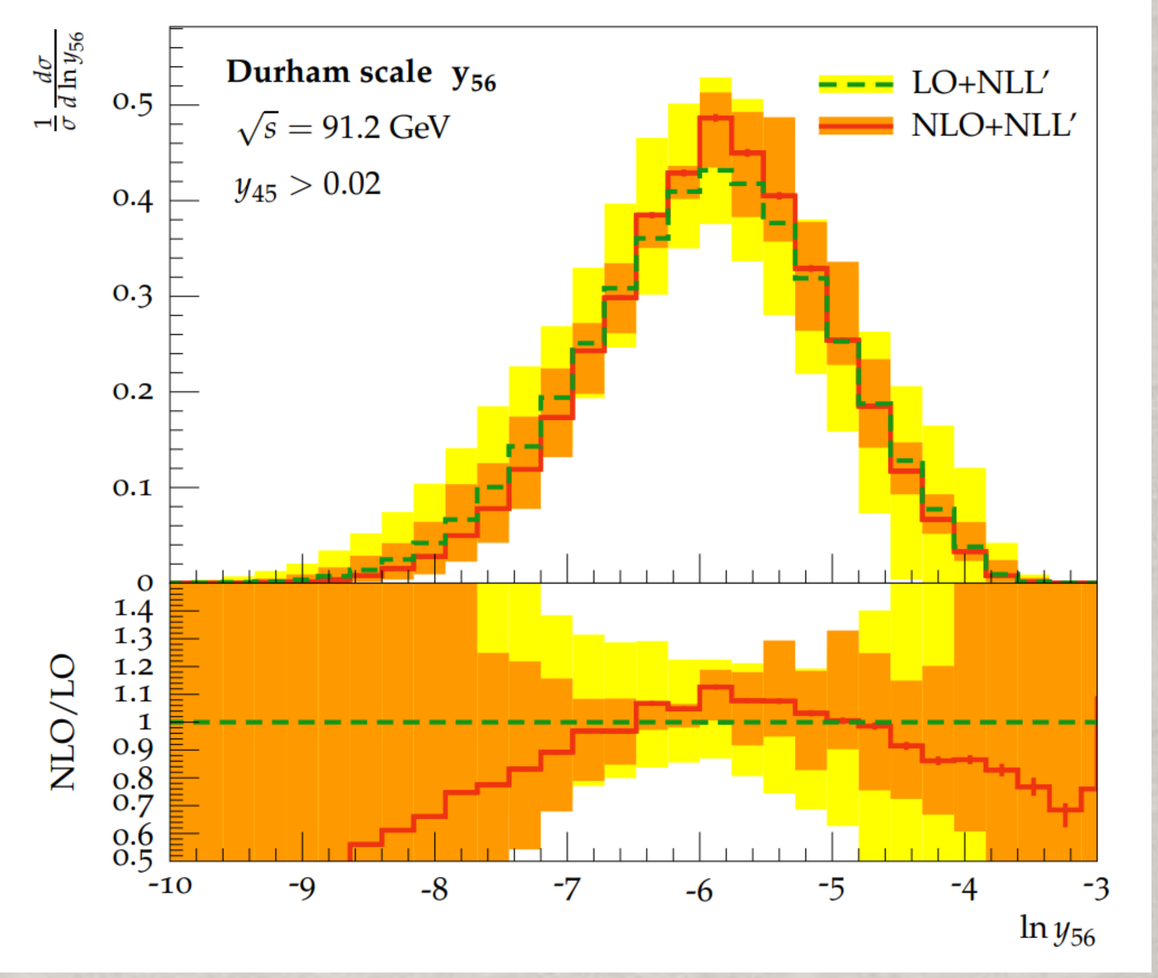
NLL resummations for N hard emitting legs in any framework can be reduced to the CAESAR general formula [AB Salam Zanderighi hep-ph/0407286]

$$\Sigma(v) = e^{-R(v)} \prod_{i=1}^{N} J_i(v) \frac{\langle \mathcal{B} | e^{-\mathbf{\Gamma}^\dagger t(v)} e^{-\mathbf{\Gamma} t(v)} | \mathcal{B} \rangle}{\langle \mathcal{B} | \mathcal{B} \rangle} \qquad \mathbf{\Gamma} = \sum_{i<j} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \Gamma_{ij}$$

- When the colour correlators $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$ are numbers, NLL resummations can be implemented by just reweighting any tree-level event generator
- For more than three-legs, $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$ is a matrix and has to be decomposed in a colour basis

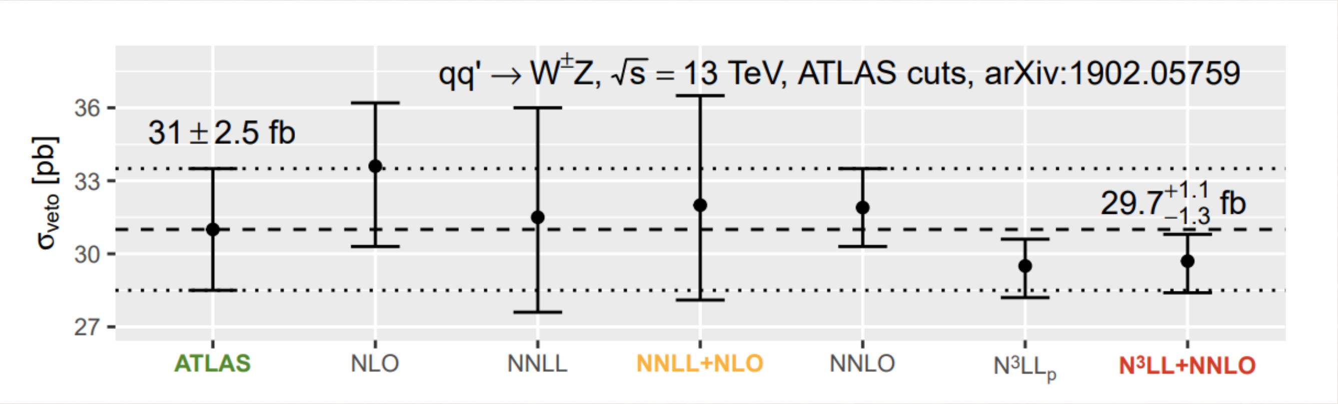
[Baberuxki et al 1912.09396]

[SHERPA https://sherpa-team.gitlab.io/]

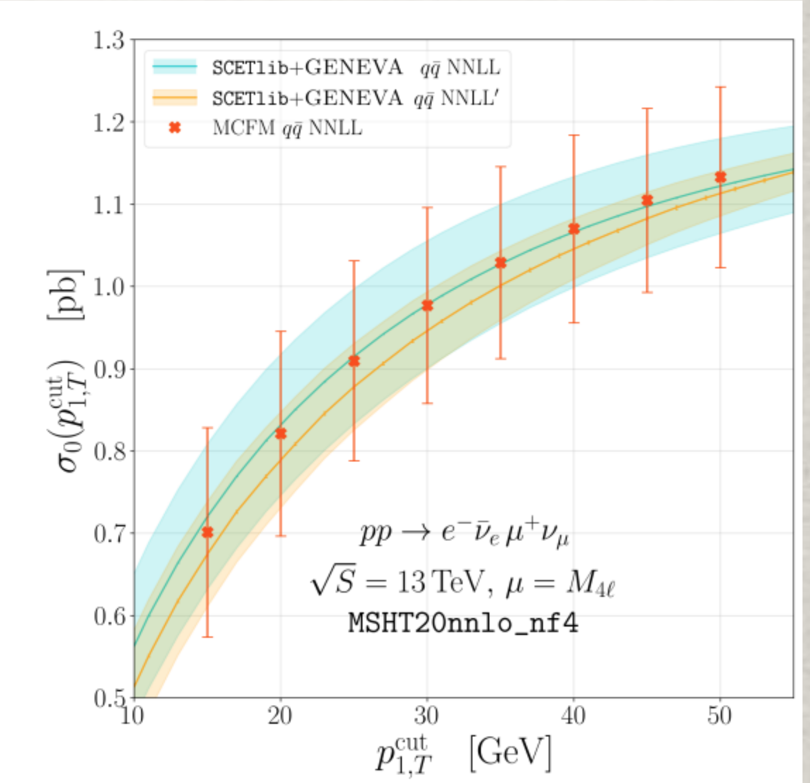


NNLL RESUMMATIONS

NNLL’ resummations have been interfaced to fixed-order programs in the case of the production of a colour singlet



[Campbell Ellis Neumann Seth 2301.11768]



[Gavardi Lim Alioli Tackmann 2308.11577]

Reweighting of tree-level events, but with different strategies

- aMC@NLO-SCET and CuTe-MCFM: beam functions as new LHAPDF tabs [CuTe-MCFM https://mcfm.fnal.gov/cute-mcfm.html]
- MCFM-RE: NNLL corrections of collinear origin as new NLO integrated counterterms [MCFM-RE https://github.com/lcarpino/MCFM-RE]
- MATRIX+RadISH: tree-level matrix elements used as initial condition for the RadISH MC resummation [MATRIX+RadISH https://matrix.hepforge.org/matrix+radish.html]

RESUMMATION MODE

- As a basis for discussion, let's start from the CAESAR NLL formula and see how it can be upgraded to NNLL (and beyond)

$$\Sigma(v) = e^{-R(v)} \prod_{i=1}^{n} J_i(v) \frac{\langle \mathcal{B} | e^{-\mathbf{\Gamma}^\dagger t(v)} e^{-\mathbf{\Gamma} t(v)} | \mathcal{B} \rangle}{\langle \mathcal{B} | \mathcal{B} \rangle}$$

- The Sudakov form factor $R(v)$ is a function of Casimirs only $\Longrightarrow$ it can be accounted for by a simple reweighting factor

- Jet functions $J_i(v)$ involve convolutions with tree-level matrix element squared $\Longrightarrow$ modified integrated counterterms

- For gluons, additional spin-dependent jet-function in near-to-planar kinematics $\Longrightarrow$ need spin density matrices instead of squared amplitudes [HELAC https://helac-phegas.web.cern.ch/]

- For more than three emitters, soft functions and their anomalous dimensions involve colour matrices $\boldsymbol{T}_i \cdot \boldsymbol{T}_j \Longrightarrow$ need colour decomposition of Born matrix elements

[SHERPA https://sherpa-team.gitlab.io/]

[DEDUCTOR https://pages.uoregon.edu/soper/deductor/]

Hadronisation

Strong coupling with jets

- Jet observables constitute an important means of determination of the strong coupling
- Increase in precision of PT QCD calculations resulted in massive decrease in theory uncertainties

Two-jet rate (NNLL+NNLO)

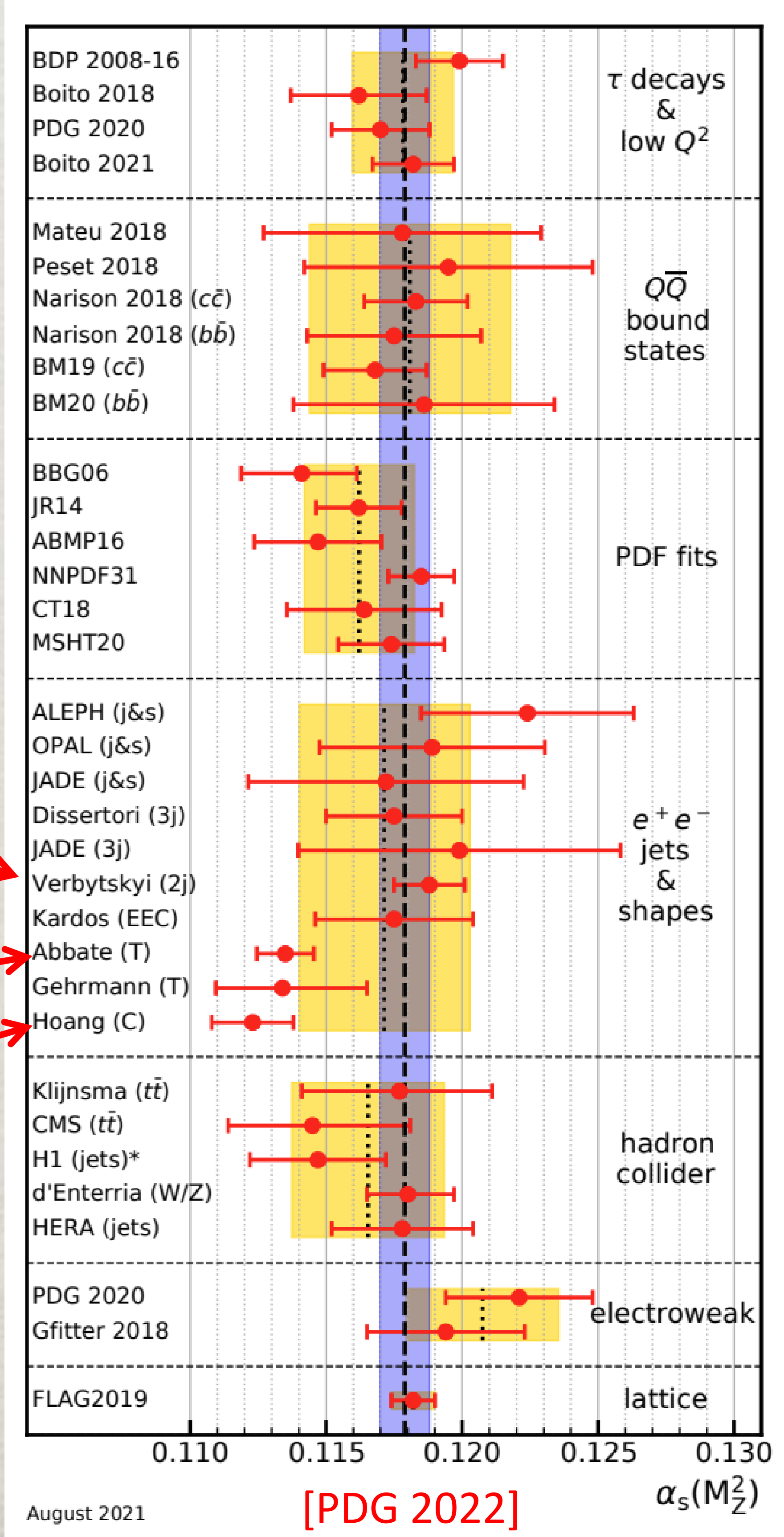
$$\alpha_s(M_Z) = 0.1188 \pm 0.0013$$

Thrust and C-parameter ((N)NNLL+NNLO)

$$\alpha_s(M_Z) = 0.1137^{+0.0034}_{-0.0027}$$

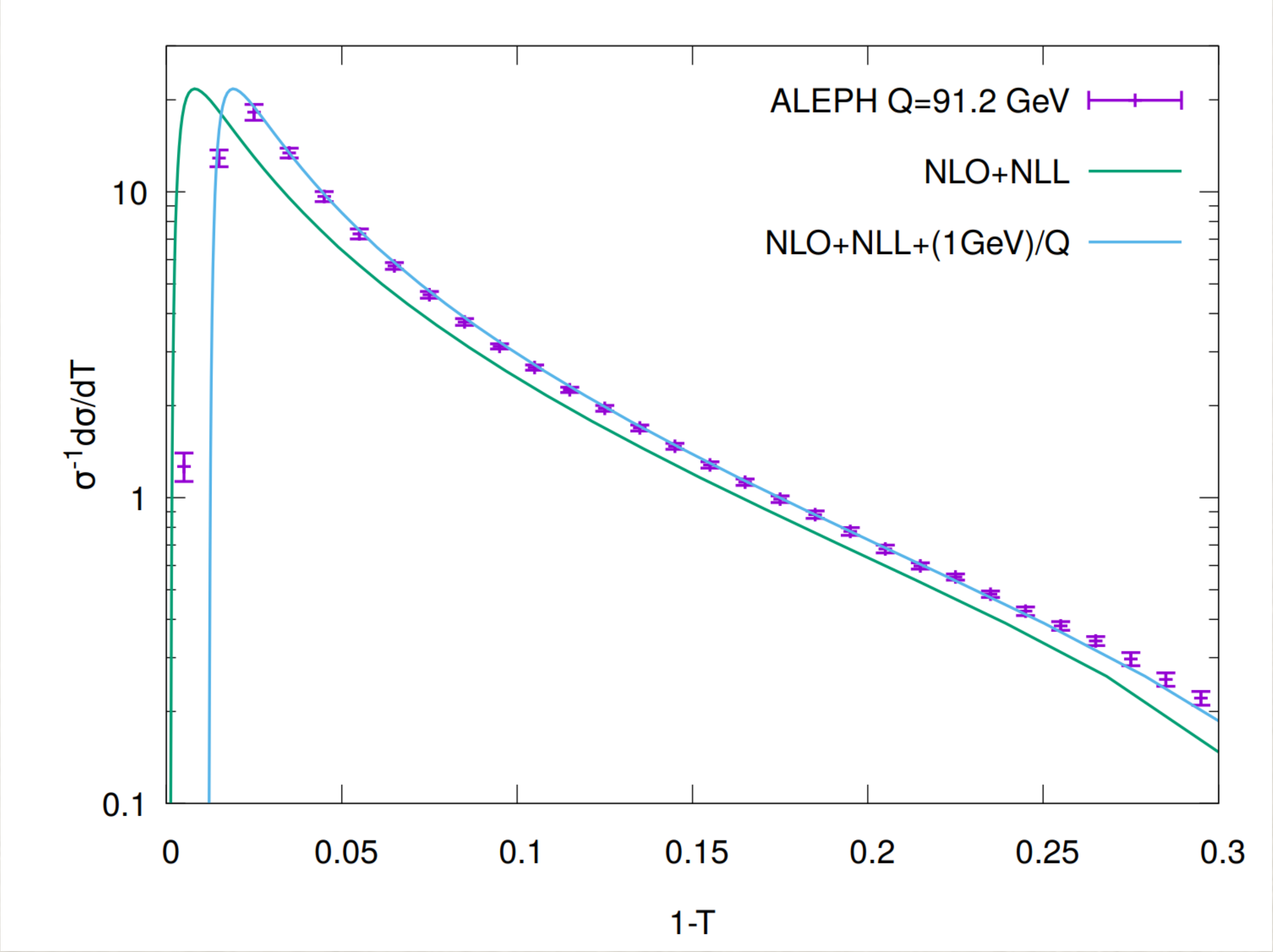
$$\alpha_s(M_Z) = 0.1123 \pm 0.0015$$

- The main difference between these predictions is the modelling of hadronisation corrections



HADRONISATION EFFECTS

At the energies probed so far, perturbative prediction do not agree straightaway with data



- Central hadrons with momenta ~1GeV give rise to a 1/Q suppressed shift of perturbative distributions of jet observables (~10% at LEP energies)

[Dokshitzer Webber hep-ph/9704298]

HADRONISATION EFFECTS

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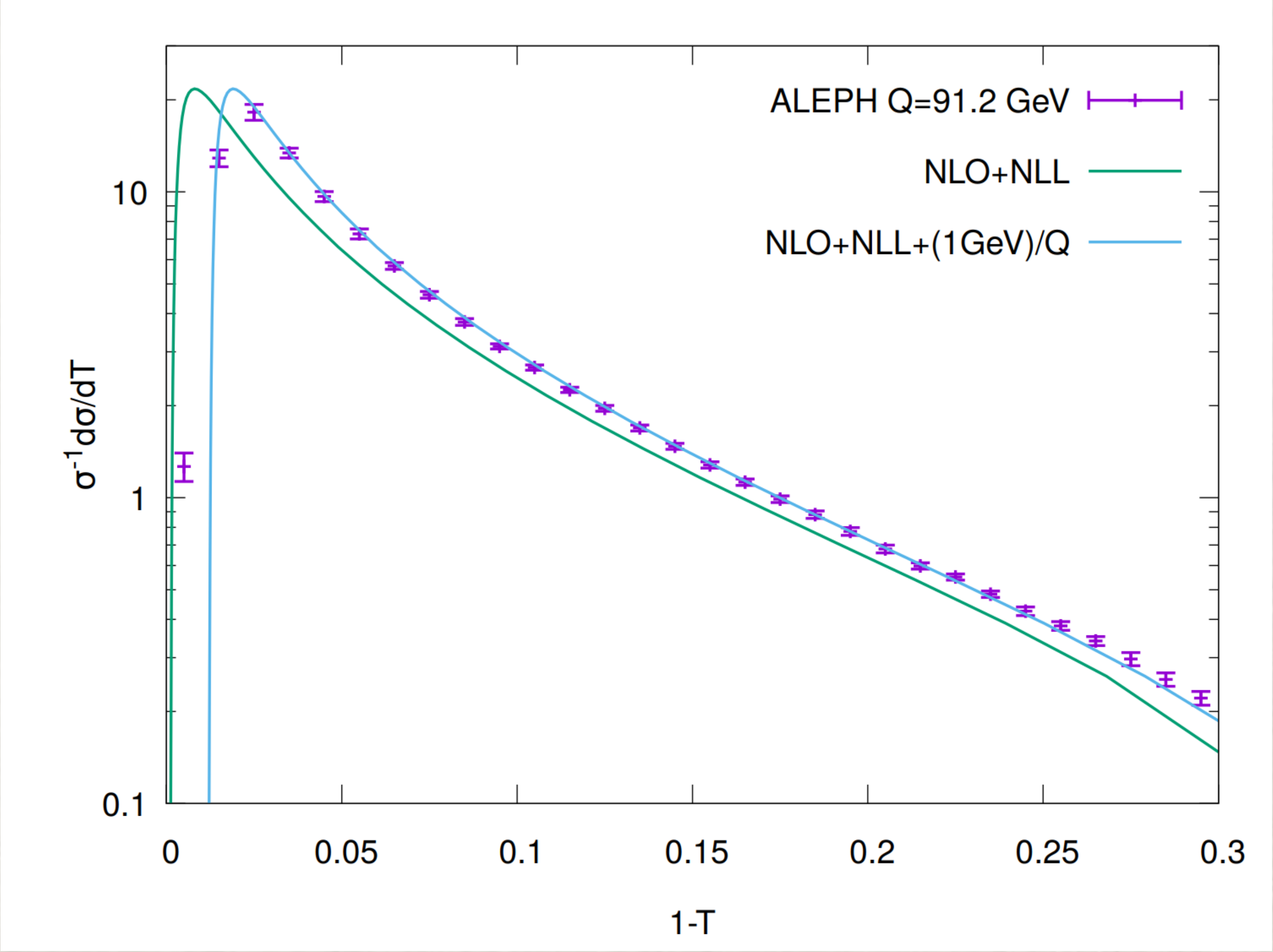
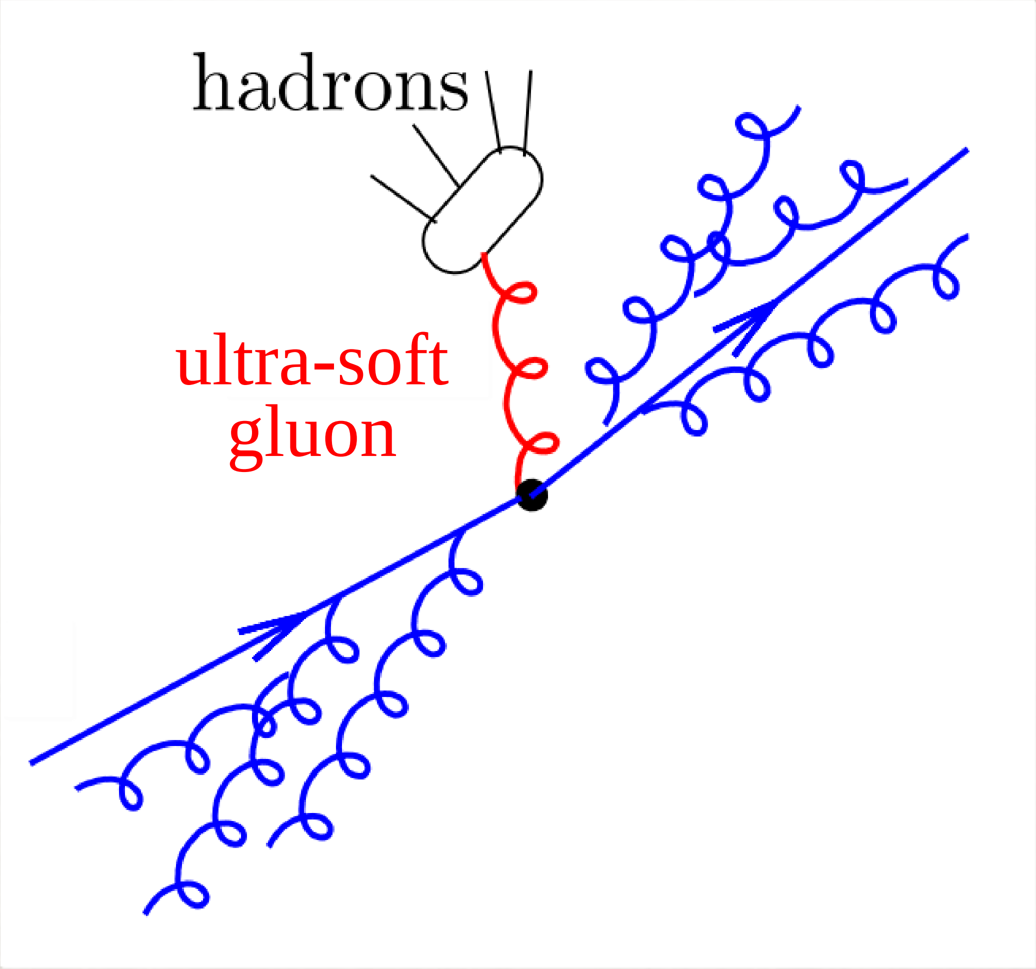




- Central hadrons with momenta ~1GeV give rise to a 1/Q suppressed shift of perturbative distributions of jet observables (~10% at LEP energies)

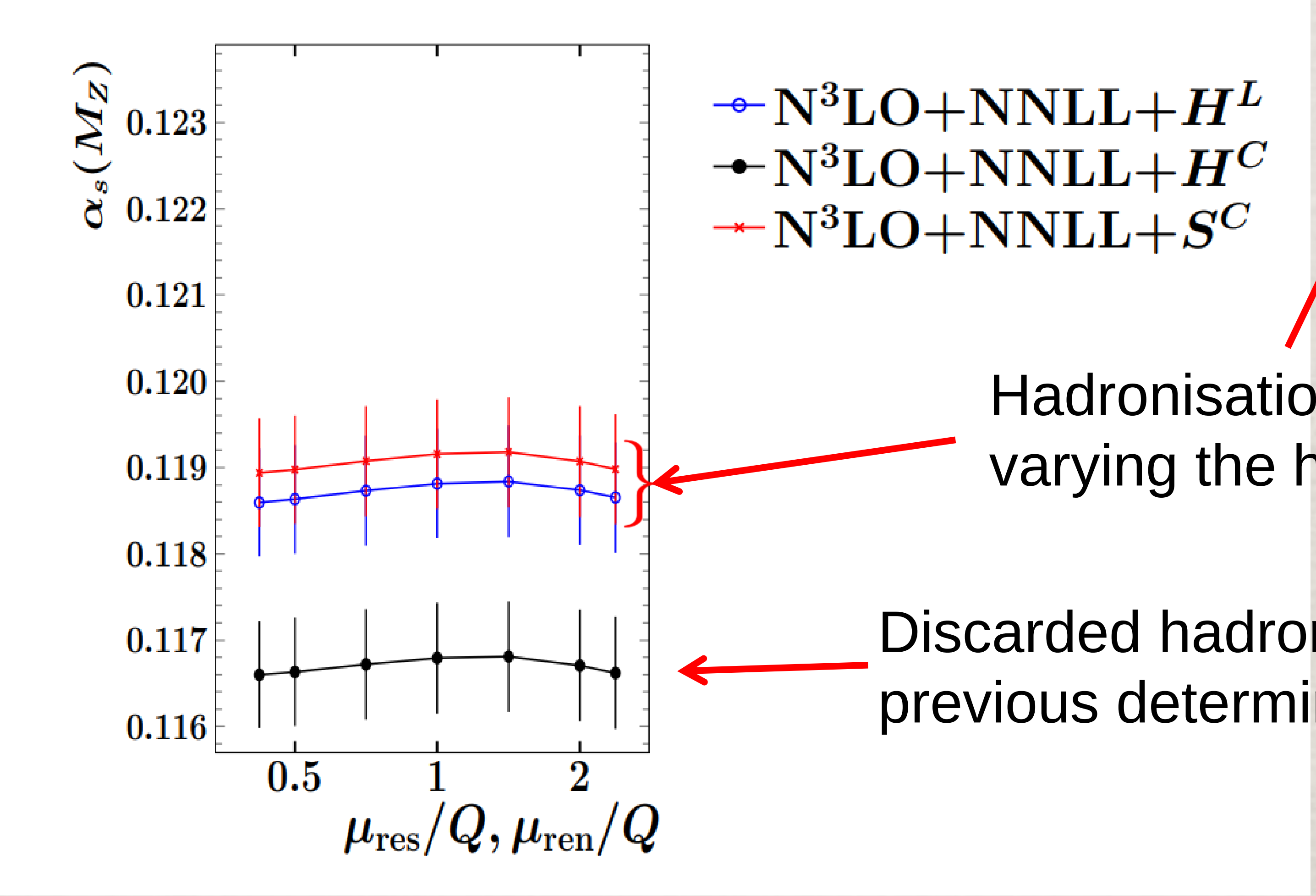
[Dokshitzer Webber hep-ph/9704298]

- Leading 1/Q hadronisation are modelled in terms of the emission of a single ultra-soft gluon $\Rightarrow$ simultaneous fit of α_s and NP parameter α_0

The two-jet rate

- For the two-jet rate, hadronisation corrections are estimated using Monte-Carlo event generators [Verbytskyi et al 1902.08158]

$$\alpha_s(M_Z) = 0.1188 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0010(\text{had}) \pm 0.0006(\text{theo})$$



Perturbative QCD uncertainty

Hadronisation uncertainty estimated by varying the hadronisation model (H^L, H^C, S^C)

Discarded hadronisation model, not an issue in previous determinations due to larger PT uncertainty

- With NNLL resummations available, hadronisation is the main source of uncertainty $\Rightarrow$ try to gain analytical understanding of hadronization corrections

Simultaneous PT-NP fits

Accurate determinations of α_s with event shapes arise from simultaneous fits of 1/Q hadronisation corrections

Thrust (NNLL+NNLO)

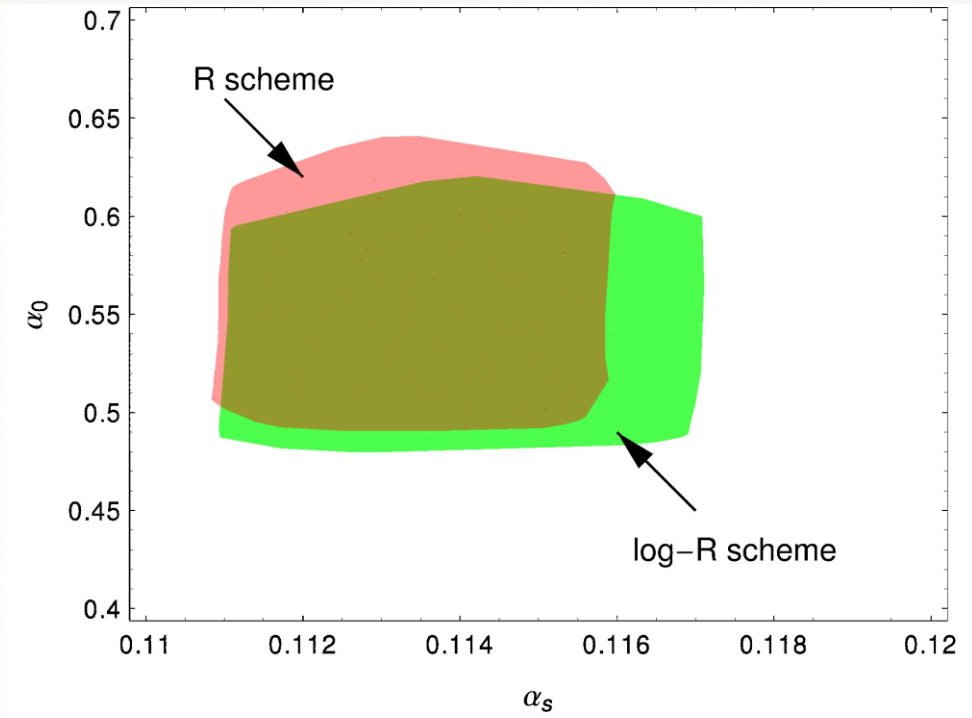
[Gehrmann Luisoni Monni 1210.6945]

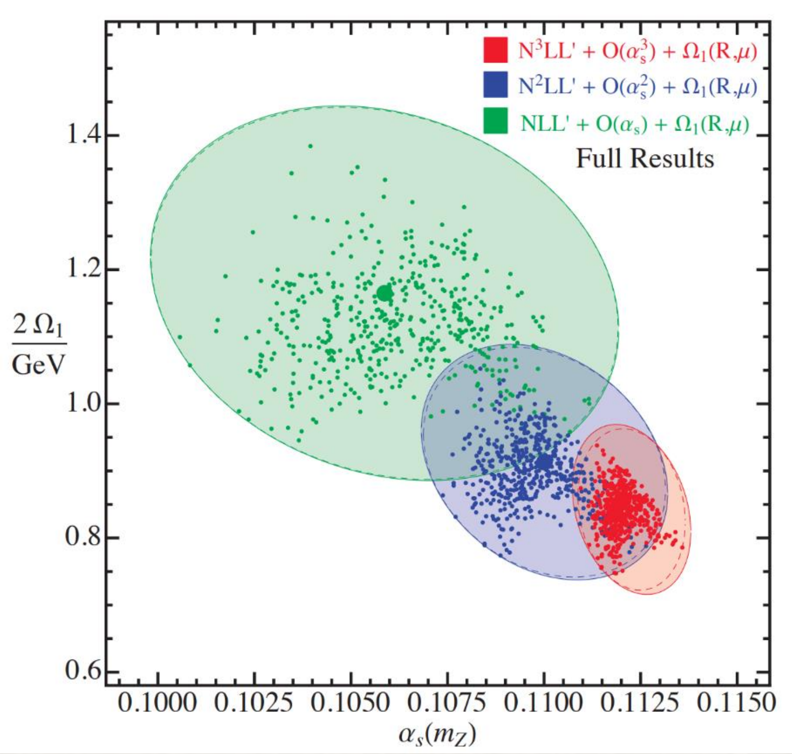
C-parameter (NNNLL+NNLO)

[Hoang Kolodubrez Mateu Stewart 1501.04111]

$$\alpha_s(M_Z) = 0.1137^{+0.0034}_{-0.0027} \qquad \alpha_s(M_Z) = 0.1123 \pm 0.0015$$

$$\alpha_0(2\,\mathrm{GeV}) = 0.524^{+0.096}_{-0.044} \quad \sim \ \text{shift} \ \sim \quad \Omega_1 = 0.421 \pm 0.063\,\mathrm{GeV}$$





Both fits assume that the shift in the fit range is the same as in the two-jet region, where 1-T an C are very small $\Rightarrow$ is this justified?

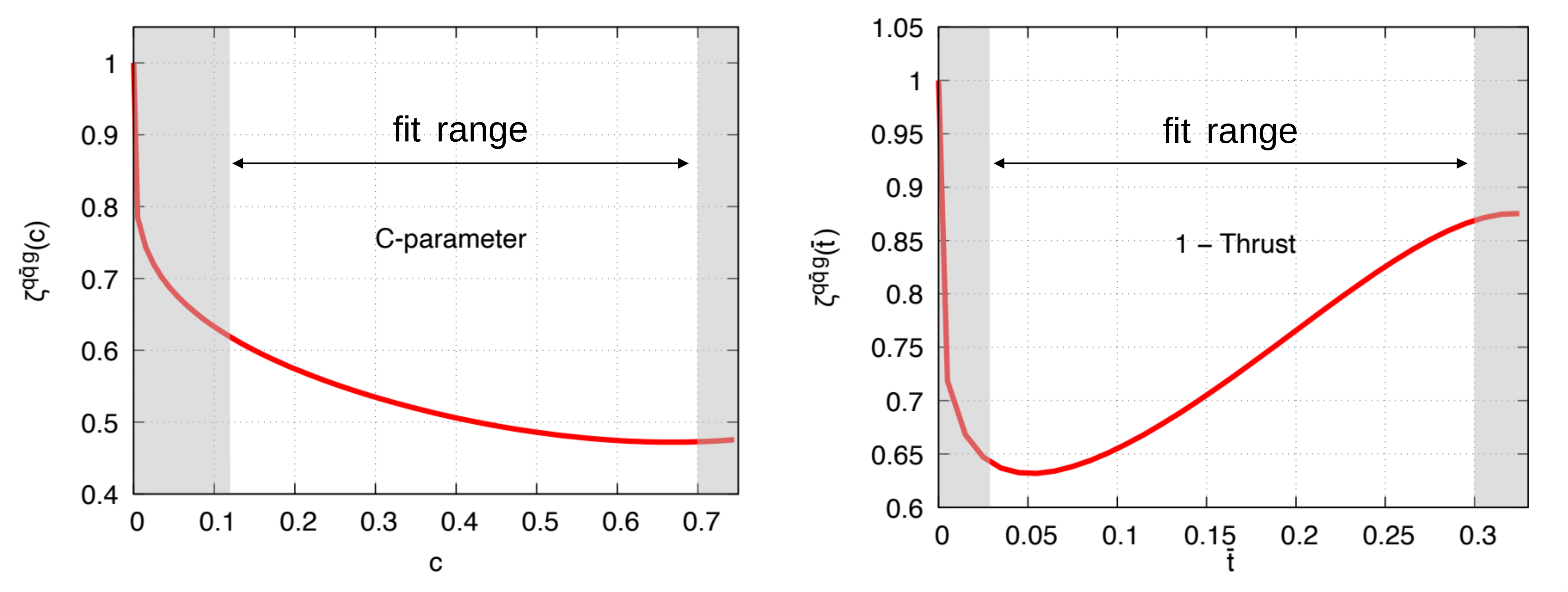
PT-NP INTERPLAY

- The 1/Q shift depends on the observable's value in the fit range $\Rightarrow$ extra 3-4% uncertainty in the determination of α_s

[Luisoni Monni Salam 2012.00622]

- It is possible to calculate analytically the deviation $\zeta(v)$ of the shift from the two-jet limit $\zeta(0) = 1$

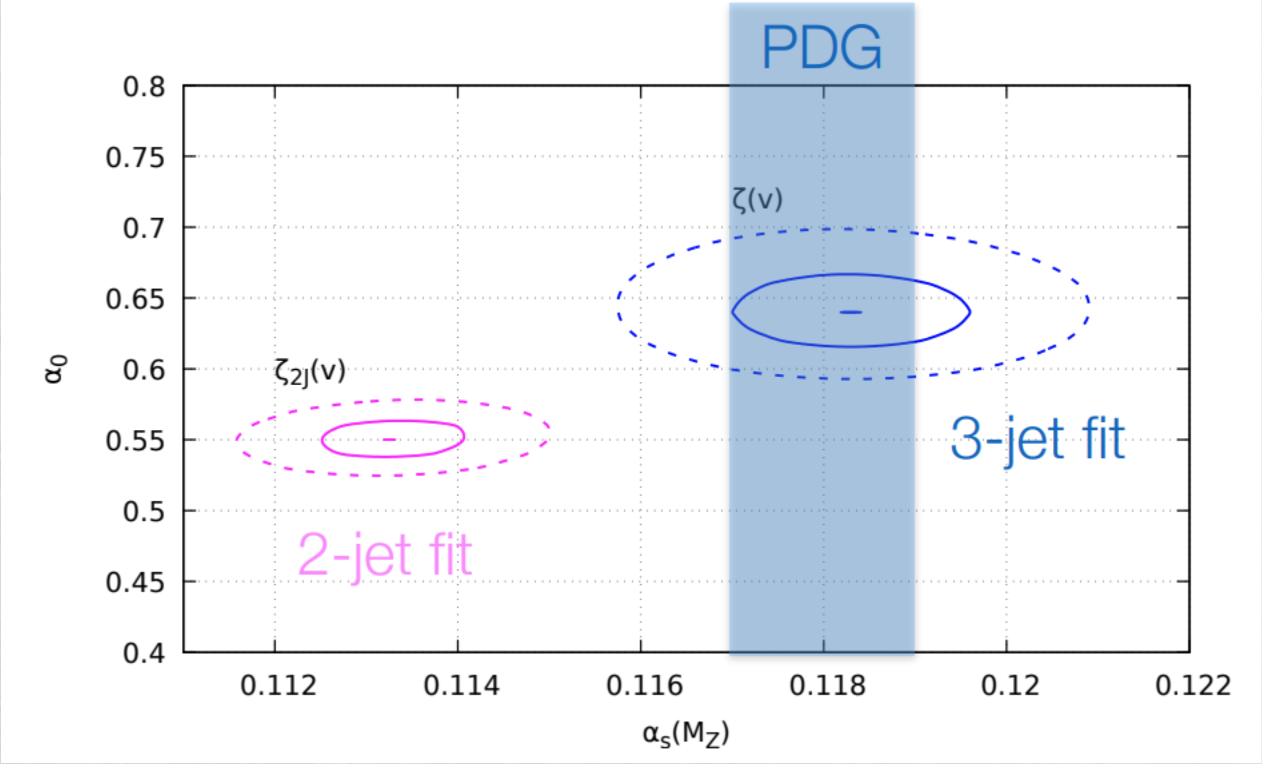
[Caola Ferrario-Ravasio Limatola Melnikov Nason Ozcelik 2204.02247]



- New frontier for precision: calculation of the 1/Q shift in the three-jet region for all event shapes and jet resolution parameters

PT-NP INTERPLAY

- The 1/Q shift can be computed in the three-jet region for most event shapes and for the three-jet resolution parameter [Nason Zanderighi 2301.03607]
- Using NP shifts in the three-jet region greatly improves the universality pattern of hadronisation corrections



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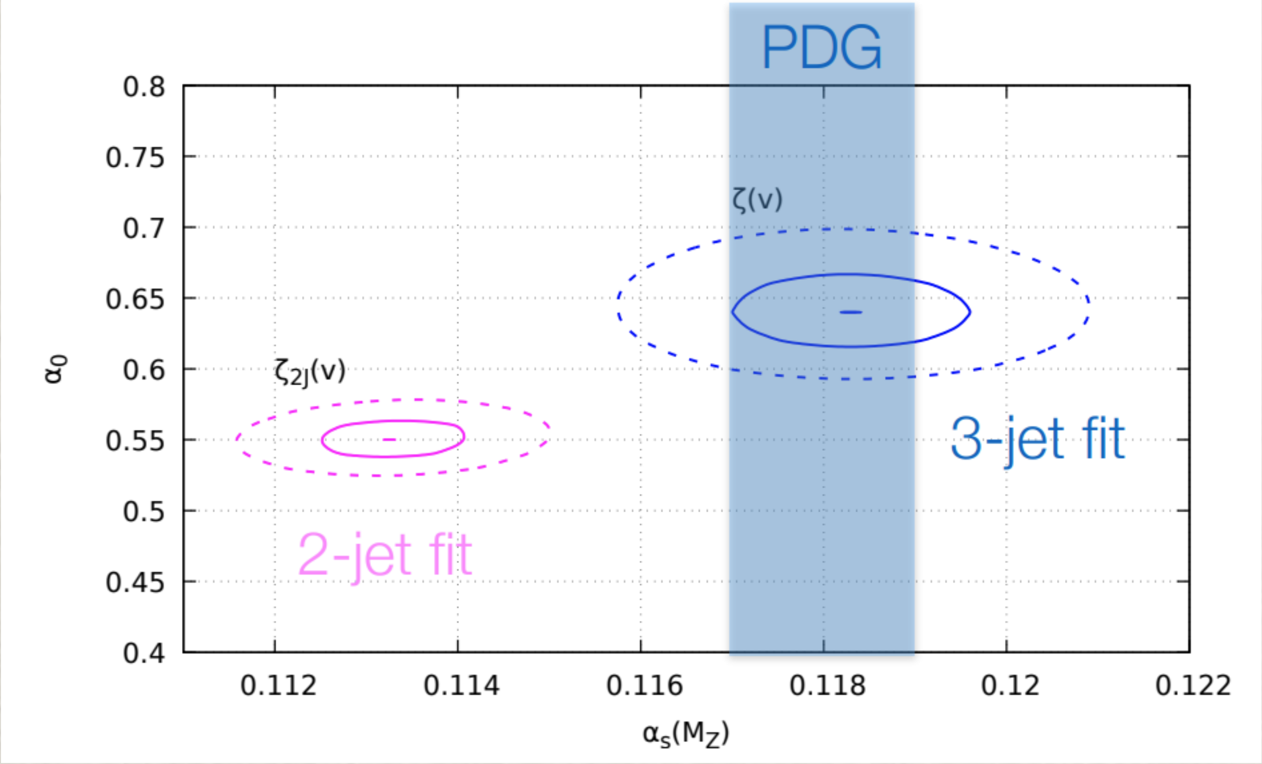
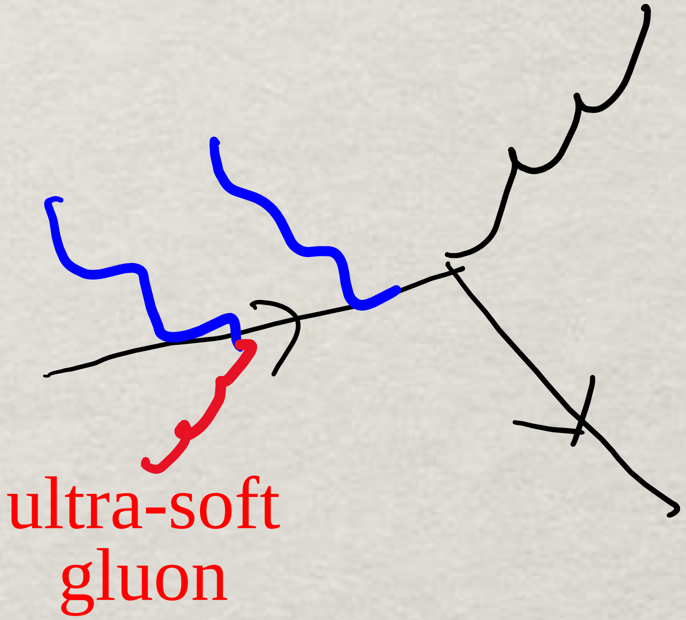


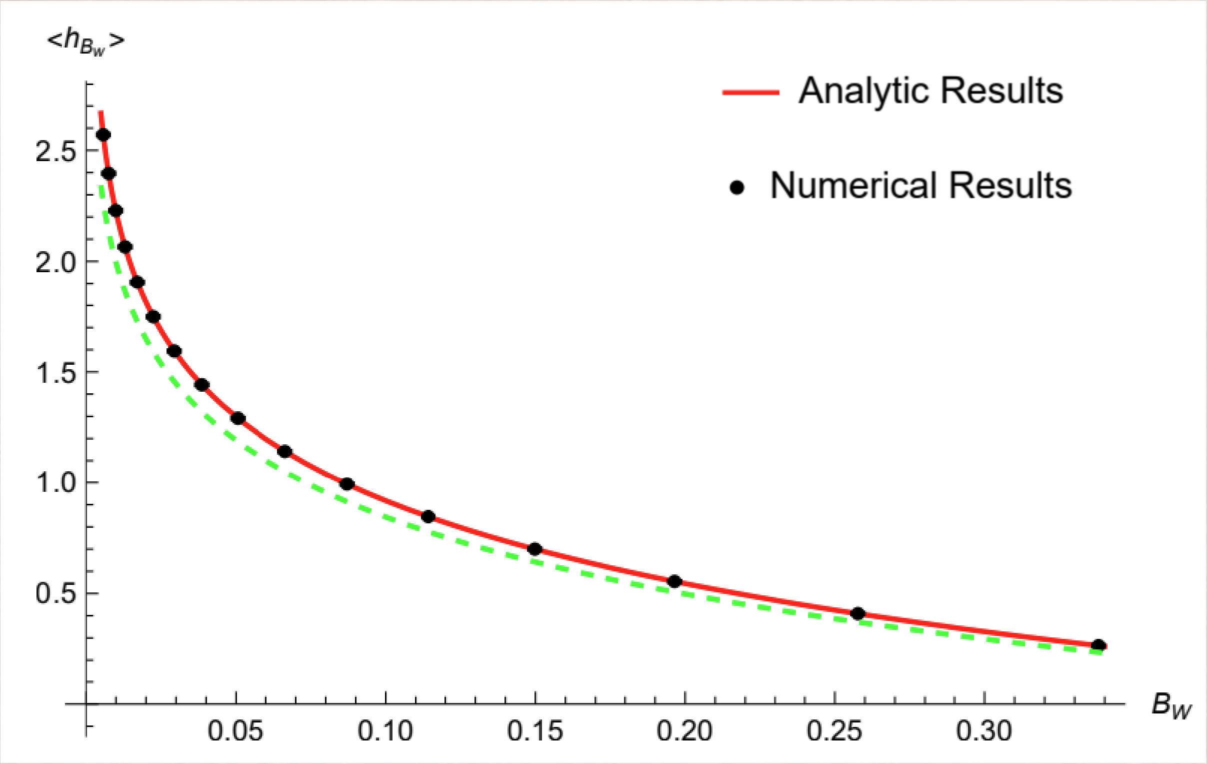


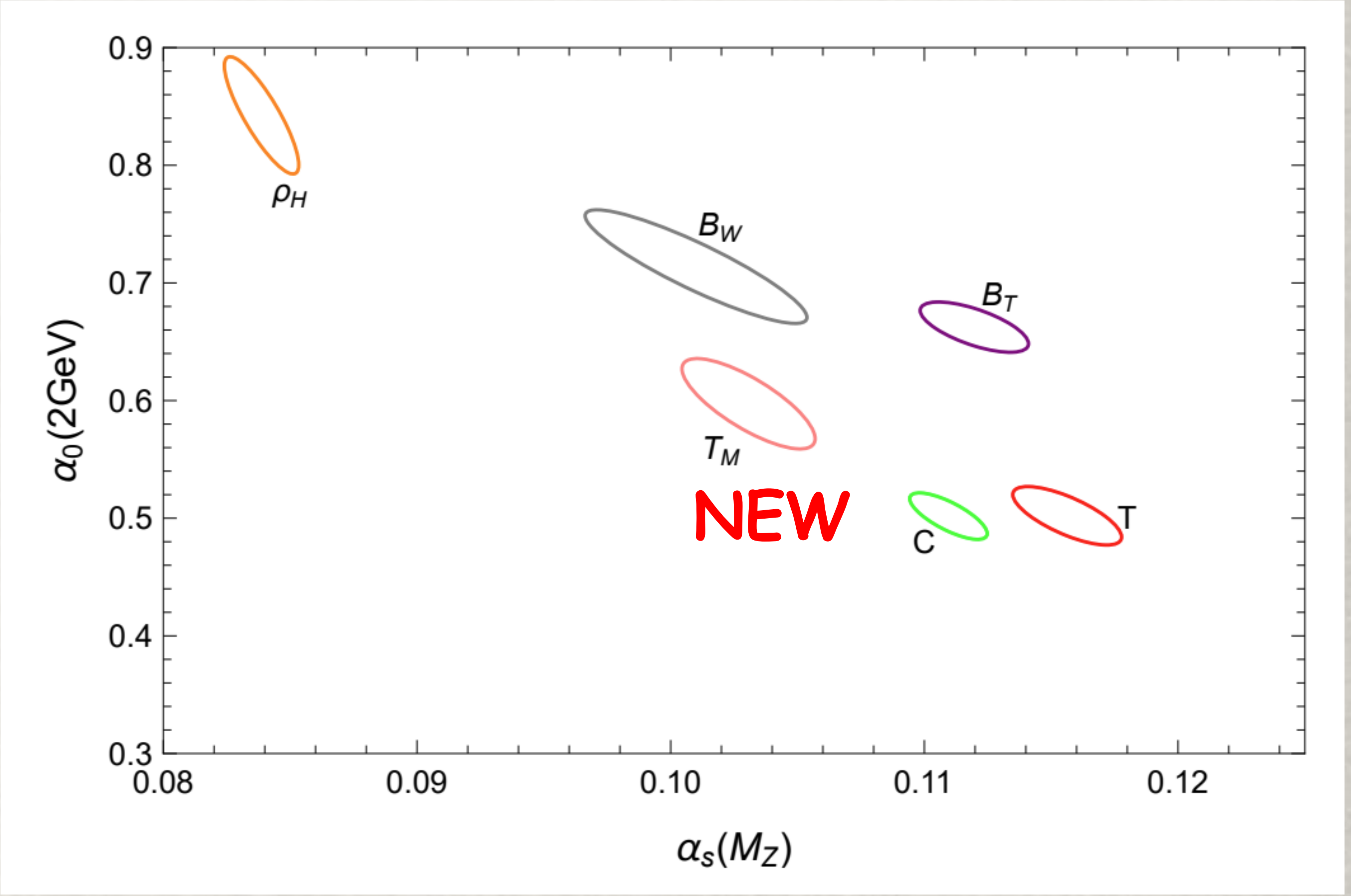
- This approach does not include resummation in the two-jet region, which is essential to describe event shapes (e.g. total jet broadening) where the shift is well defined only in the presence of multiple soft-collinear emissions [Dokshitzer Marchesini Salam hep-ph/9812487]

PT-NP INTERPLAY

With the ARES formalism, it is possible to compute the shift to an arbitrary event-shape variable in the two-jet limit in the presence of multiple soft and collinear emissions

[AB El-Menoufi Wood 2303.01534]





- New results for the thrust major, which does not admit an analytic resummation
- Frontier: merging of hadronisation corrections in the two- and three-jet regions

Concluding remarks

Resummed calculations have seen an impressive progress both in precision and in scope

- Observables for which a factorisation theorem exist are taking advantage of the impressive progress in loop calculations $\Rightarrow$ N^4LL!
- Resummation of non-global logarithms at NL accuracy and first resummation of super-leading logarithms $\Rightarrow$ resummation of an arbitrary observable at hadron colliders is now in sight
- Resummations have an impact beyond their realm of applicability, especially providing input to fixed-order calculations and parton-showers
- Automation and handy interface with event generators would be desirable

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Thank you for your attention