High precision tests of perturbative QCD without renormalization scale and scheme ambiguities

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Outline

- The Renormalization Scale Setting in QED/QCD
- State of the art about renormalization in QCD
- The iCF and the PMC_∞ scale setting procedure
- PMC_∞ and the Event Shape Variables
- Comparison with CSS
- QED and IR Conformal limit for Thrust
- *$\alpha s(Q)$: New method to detemine the coupling*
- … other applications
- Work in progress / Future perspectives

Why The Scale Setting in QCD is a key issue?...truncated series (NLO,NNLO,N3LO...)***

Stückelberg and Peterman

Renormalization: subtraction of infinities at μ0.
RGE- R.Group equations

$$Z_{\alpha_s}^{-1} = (\sqrt{Z_3} Z_2/Z_1)^2,$$

$$\frac{1}{4\pi}\frac{d\alpha_s(Q^2)}{d\log Q^2} = \beta(\alpha_s),$$

$$Z_\alpha\left(Q^2\right) = 1 - \frac{\beta_0 \alpha_s\left(Q^2\right)}{4\pi\varepsilon},$$

$$\beta\left(\alpha_s\right) = -\left(\frac{\alpha_s}{4\pi}\right)^2 \sum_{n=0} \left(\frac{\alpha_s}{4\pi}\right)^n \beta_n.$$

Not only confinement mechanism…

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_0^2)}{1 + \beta_0 \frac{\alpha_s(\mu_0^2)}{4\pi} \ln(Q^2/\mu_0^2)}$$

- *To determine $\alpha s(Q^2)$ to the highest precision;*
- *To make precision tests of the QCD;*
- *To eliminate the renormalization scale ambiguity and the scheme dependence in the observables;*
- *To assess and reach the maximum sensitivity to NP.*

The Renormalization Scale Problem in QED

QED is not only perturbative :

- *No ambiguity in the renormalization scale in QED;*
- *The renormalization scale in QED is physical and set by the exchanged photon virtuality;*
- *An infinite series of Vacuum Polarization diagrams is resummed;*
- *The QED coupling is defined from physical observables (Gell Mann-Low scheme);*
- *No scheme dependence is left;*
- *Analyticity (space-like/time-like);*
- *Exact number of active leptons is set;*
- *Recover of a conformal-like series;*

Effective coupling

$$\alpha(Q) = \frac{\alpha_0}{1 - \Pi(Q)}$$

The VPF contains all β-terms

QED: a Theoretical Constraint for QCD

QCD → ***Abelian Gauge Theory***

In the limit : ***Nc*** → ***0,***

at fixed $\alpha = C_F \alpha_s$, $n_l = T n_F / C_F$

The scale setting procedure used in QCD must be consistent with the QED

Huet, S.J.Brodsky

"In the perspective of a theory unifying all the interactions, electromagnetic, weak and strong nuclear, such as a so-called grand unified theory or GUT, we are constrained to apply the same scale-setting procedure in all sectors of the theory."

The road to scale setting is clouded by some misbeliefs

- ***CSS: we cannot avoid them → we use them:***

Scale is set to the «proper» scale of the process and varied in the range of 2;

- ***«The only way to solve the problem is to compute as much as higher order calculations I can»***

is this the solution to the scale setting? The perturbative series is asymptotic, renormalons, large logs occur …

- ***Scales are judged by results «a posteriori»;***
- ***The renormalization scale is unique for each process;***
- ***The renormalization scale is a simple unphysical parameter;***

These assumptions are wrong for QED and thus too for QCD!

The CSS appears to be more a "lucky guess"!

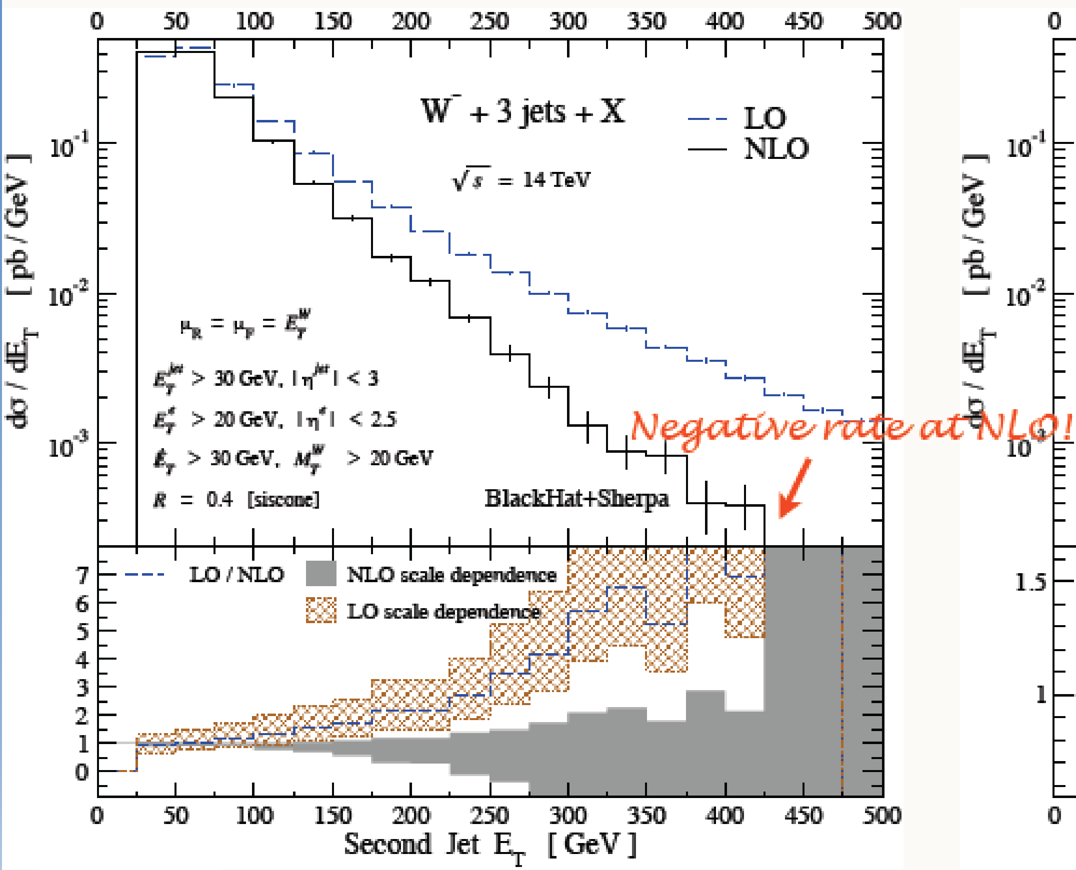
- In general, no one knows the proper renormalization scale value, *Q*;
- CSS does not agree with QED;
- Can the MHO contributions be determined by the scale ambiguities?

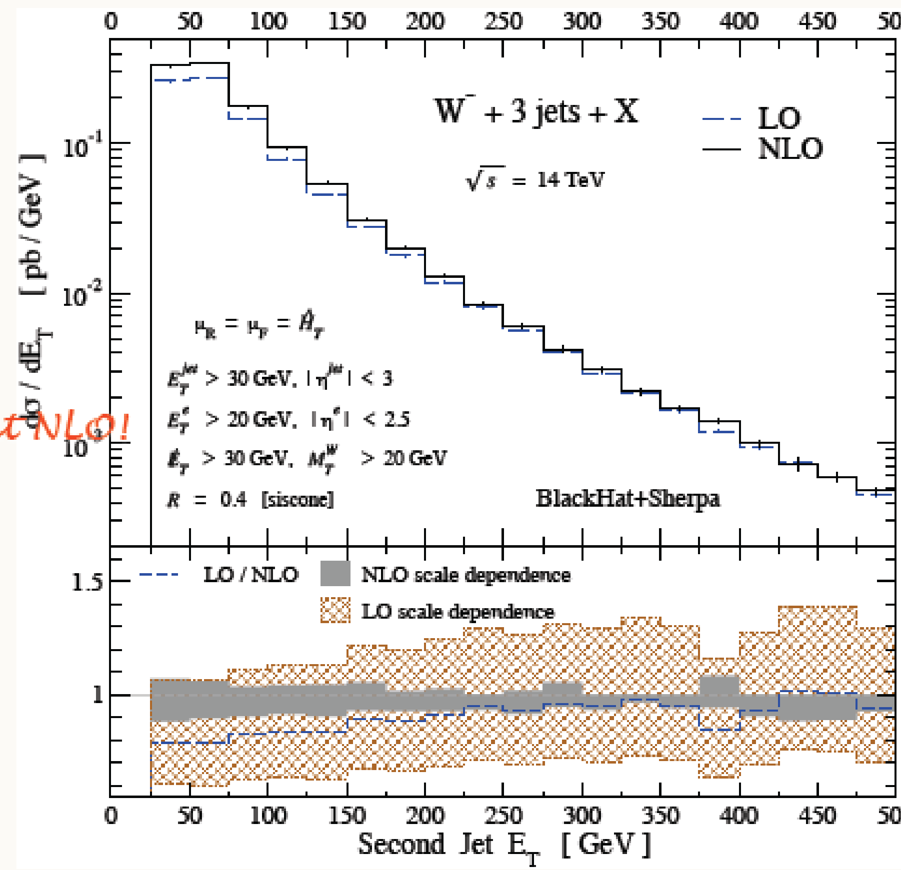
In QCD…

NLO QCD predictions for W+ 3Jet distributions at LHC

Black Hat

$$\mu_R = \mu_F = E_T^W \qquad\qquad \mu_R = \mu_F = \hat{H}_T$$





F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower, and D. Maître

Alternative way to CSS: Optimization Procedures…

- PMS

Scale and scheme parameters are treated as independent variables;

$$\frac{\partial \rho_n}{\partial \tau} = \left(\frac{\partial}{\partial \tau} + \beta\left(\alpha_s\right)\frac{\partial}{\partial \alpha_s}\right)\rho_n \equiv 0$$
$$\frac{\partial \rho_n}{\partial \beta_j} = \left(\frac{\partial}{\partial \beta_j} - \beta\left(\alpha_s\right)\int_0^{\alpha_s} \mathrm{d}\alpha' \frac{\alpha'^{j+2}}{\left[\beta\left(\alpha'\right)\right]^2}\frac{\partial}{\partial \alpha_s}\right)\rho_n \equiv 0$$

- FAC : physical quantities define «effective charges»:

$$\alpha_R \equiv \left(\frac{R}{\mathcal{C}_0}\right)^{1/p} .$$

- ***Optimization procedures such as PMS (xRGE) and FAC (ECH) lead to incorrect and unphysical results violating important RG properties;***

The Reliable Scale-Setting method

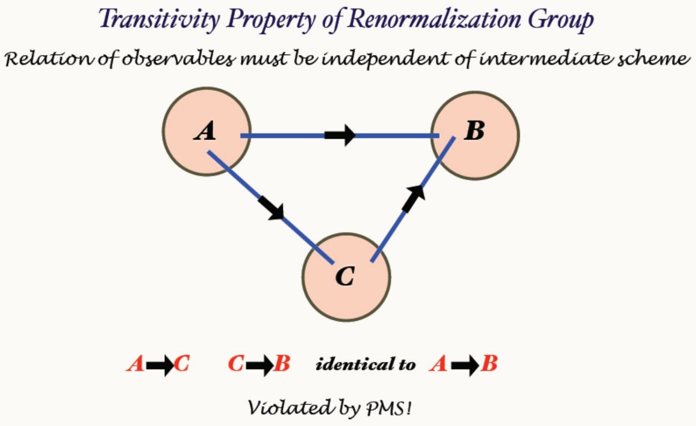
- RG properties: *uniqueness, reflexivity, symmetry,* and *transitivity;*

Other requirements are

- Constraints given by other theories,

(dual theories LFHQCD ,QED, Conformal theories, non-perturbative results)

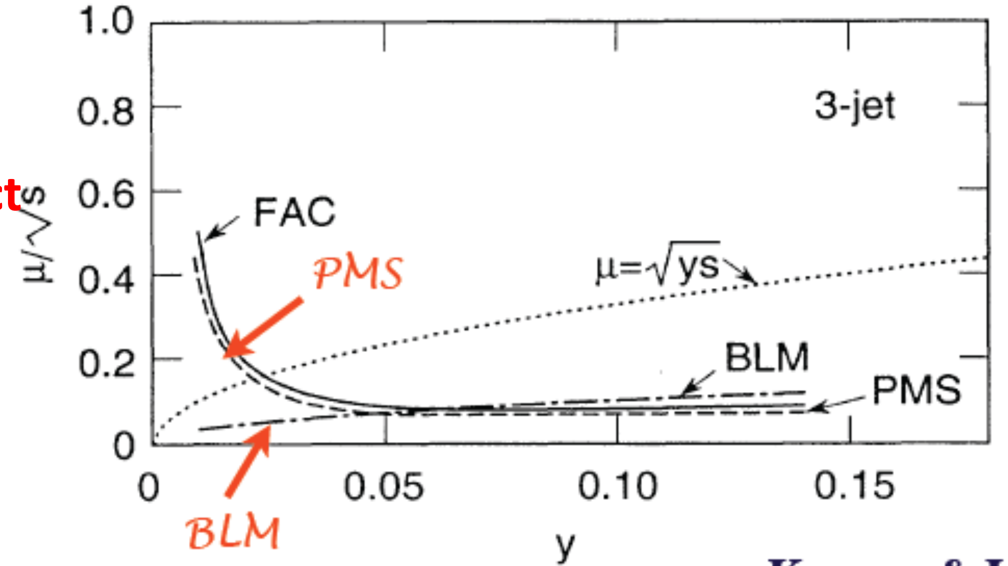
- scheme independence;
- convergence behavior of the series;
- phenomenological results;



e+ e- to 3-jets

PMS and FAC lead to incorrect and unphysical results

PMS violates Transitivity property



Jet Invariant mass squared:

$$\mathcal{M}^2 = ys$$

Kramer & Lampe

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

The Principle of Maximum Conformality is the principle underlying the BLM

S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D 28, **Nov 23rd 1983**

Observable in the initial parametrization

Stanley J. Brodsky, L.D.G.:
Phys. Rev. D 86, 085026 (2011)

Mojaza, Matin and Brodsky, Stanley J. and Wu, Xing-Gang
Phys.Rev.Lett. 110 (2013) 192001

L.D.G., SJ Brodsky, P.G. Ratcliffe, X-G. Wu, S.Q. Wang, **Prog.Part.Nucl.Phys. 135 (2024) 104092.**

$$\begin{aligned}\rho(Q^2) =& r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 \\ &+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ &+ 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5) \qquad (6)\end{aligned}$$

$r_{n,0}$ conformal coefficients

The β-terms are reabsorbed by RGE

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q_1) + r_{2,0}a(Q_2)^2 + r_{3,0}a(Q_3)^3 + r_{4,0}a(Q_4)^4 + \mathcal{O}(a^5) \ ,$$

Conformal-like expansion

$$\ln\frac{Q_k^2}{Q^2} = \frac{R_{k,1} + \Delta_k^{(1)}(a)R_{k,2} + \Delta_k^{(2)}(a)R_{k,3}}{1 + \Delta_k^{(1)}(a)R_{k,1} + \left(\Delta_k^{(1)}(a)\right)^2 (R_{k,2} - R_{k,1}^2) + \Delta_k^{(2)}(a)R_{k,1}^2} \ .$$

PMC scales: reabsorb the RS dependence!

Features of the PMC/PMC$_\infty$

- All terms associated with the beta-function are included into the running coupling;
- PMC agrees with the QED in the Abelian limit;
- PMC is consistent with the IR Conformal limit;
- No scale ambiguities;
- Results are scheme independent ;
- The PMC scale sets the correct number of active flavors;
- Transitivity Property and all RG properties are all preserved;
- No renormalon n! growth in pQCD associated with the beta function;
- Resulting series is identical to conformal series! (CSR - Crewther Relation ;)
- PMC: One procedure from first principles for the whole SM and also for a theory of grand unification.

PMC∞ - Results for Event Shape Variables distributions at NNLO

Work in collaboration with S. J. Brodsky, P.G. Ratcliffe, S.Q.Wang , X.G. Wu and F. Sannino

- arXiv: 2104.12132 [hep]
- *Phys.Rev.D* 102 (2020) 1, 014015

Thrust and C-Par distribution at NNLO: process: e+e- →3jets

$$T = \frac{\max\limits_{\vec{n}} \sum\limits_i |\vec{p}_i \cdot \vec{n}|}{\sum\limits_i |\vec{p}_i|},$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p_i}||\vec{p_j}| \sin^2 \theta_{ij}}{(\sum_i |\vec{p_i}|)^2},$$

Sheng-Quan Wang, S.J. Brodsky, Xing-Gang Wu , Jian-Ming Shen, L.D.G., *Phys.Rev.D* 100 (2019) 9, 094010

Sheng-Quan Wang, S.J. Brodsky, Xing-Gang Wu , L.D.G., Jian-Ming Shen, *Phys.Rev.D* 102 (2020) 1, 014005

Sheng-Quan Wang, S.J. Brodsky, Xing-Gang Wu , L.D.G., Phys. Rev. D 99, no.11, 114020 (2019)

Distributions from EERAD and Event2 codes by:

S. Catani and M. H. Seymour, Phys. Lett. B **378**, 287

A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, Phys. Rev. Lett. **99**, 132002 (2007).
A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, JHEP **0712**, 094 (2007).
S. Weinzierl, JHEP **0906**, 041 (2009).
S. Weinzierl, Phys. Rev. Lett. **101**, 162001 (2008).

Strong Coupling from RunDec program :

K. G. Chetyrkin, J. H. Kuhn and M. Steinhauser, Comput. Phys. Commun. **133**, 43 (2000).

PMC$_\infty$ preserves the iCF: *the intrinsic Conformality*

Observable: Single variable distribution at NNLO calculated at the initial scale μ_0

$$\frac{1}{\sigma_{tot}}\frac{Od\sigma(\mu_0)}{dO} = \frac{\alpha_s(\mu_0)}{2\pi}\frac{Od\overline{A}_O(\mu_0)}{dO} + \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2\frac{Od\overline{B}_O(\mu_0)}{dO} + \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3\frac{Od\overline{C}_O(\mu_0)}{dO} + \mathcal{O}(\alpha_s^4), \quad (3)$$

- No redefinition of the conformal terms at higher orders;
- No initial scale dependence left under a global change of scale;
- The scale dependence is explicit.
- The iCF is the most general and unique RG invariant parametrization;
- Other parametrizations can be reduced to the iCF;

The conformal subsets are the fundamental blocks of the iCF

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)\sigma_N = 0.$$

- Each subset is scale invariant.
- Any combination of conformal subsets is an invariant.
- I can define a scale for each subset preserving the scale invariance.
- If Aconf=0 the whole subset becomes null.

The iCF is an RG invariant parametrization with conformal coefficients and scales.

$$\sigma_I = \left\{\left(\frac{\alpha_s(\mu_0)}{2\pi}\right) + \frac{1}{2}\beta_0 \ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2 + \frac{1}{4}\left[\beta_1 + \beta_0^2\ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\right]\ln\left(\frac{\mu_0^2}{\mu_I^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3\right\}A_{Conf}$$

$$\sigma_{II} = \left\{\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^2 + \beta_0\ln\left(\frac{\mu_0^2}{\mu_{II}^2}\right)\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3\right\}B_{Conf}$$

$$\sigma_{III} = \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^3 C_{Conf} \quad (6)$$

$$\lim_{n\to\infty}\sigma_n \equiv \lim_{n\to\infty}\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^n \mathcal{L}_{nConf} \to \text{Conformal Limit},$$

$$\lim_{n\to\infty}\alpha_s(\mu_0)^n \sim a^n \text{ with } a < 1.$$

The n limit is Conformal

Ordered scale invariance: scale invariance is preserved perturbatively independently from the process, the kinematics and the order.

PMC∞ scales

$$\frac{1}{\sigma_{tot}}\frac{O d\sigma(\mu_I,\mu_{II},\mu_0)}{dO} = \left\{\overline{\sigma}_I + \overline{\sigma}_{II} + \overline{\sigma}_{III} + \mathcal{O}(\alpha_s^4)\right\}, \quad (20)$$

$$\alpha_I \equiv \alpha_s(\mu_I),\ \alpha_{II} \equiv \alpha_s(\tilde{\mu}_{II})$$

Conformal subsets

$$\eta = 3.51$$

$$\begin{aligned}
\overline{\sigma}_I &= A_{Conf}\frac{\alpha_I}{2\pi} \\
\overline{\sigma}_{II} &= (B_{Conf} + \eta A_{tot} A_{Conf})\left(\frac{\alpha_{II}}{2\pi}\right)^2 - \eta A_{tot} A_{Conf}\left(\frac{\alpha_0}{2\pi}\right)^2 \\
&\quad - A_{tot} A_{Conf}\frac{\alpha_0}{2\pi}\frac{\alpha_I}{2\pi} \\
\overline{\sigma}_{III} &= (C_{Conf} - A_{tot} B_{Conf} - (B_{tot} - A_{tot}^2) A_{Conf})\left(\frac{\alpha_0}{2\pi}\right)^3
\end{aligned} \quad (23)$$

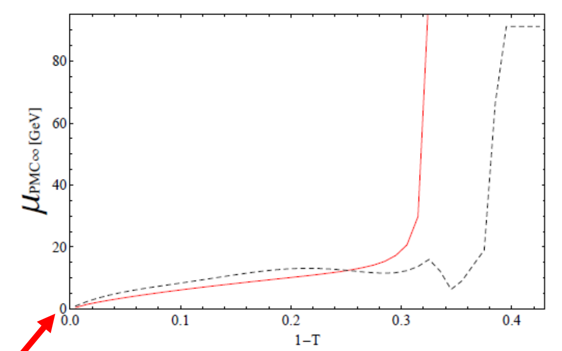
$$\mu_I = \sqrt{s}\cdot e^{-\frac{1}{2}B_{\beta_0}}, \quad (1-T)<0.33$$

Red

$$\tilde{\mu}_{II} = \begin{cases} \sqrt{s}\cdot e^{-\frac{1}{2}C_{\beta_0}\cdot\frac{B_{Conf}}{B_{Conf}+\eta\cdot A_{tot}A_{Conf}}}, & (1-T)<0.33 \\ \sqrt{s}\cdot e^{-\frac{1}{2}\left(\frac{C_1}{11B_1-\frac{2}{3}B_0}\right)}, & (1-T)>0.33 \end{cases}$$

Dashed Black

$$\mu III = \sqrt{s}$$

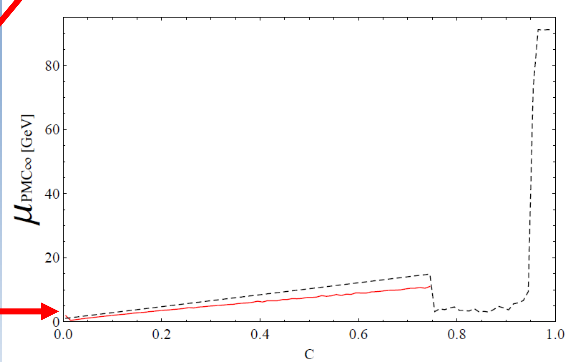


The PMC∞ scales are functions of the Event Shape Variable and of the physical scale of the process: $\sqrt{s}$

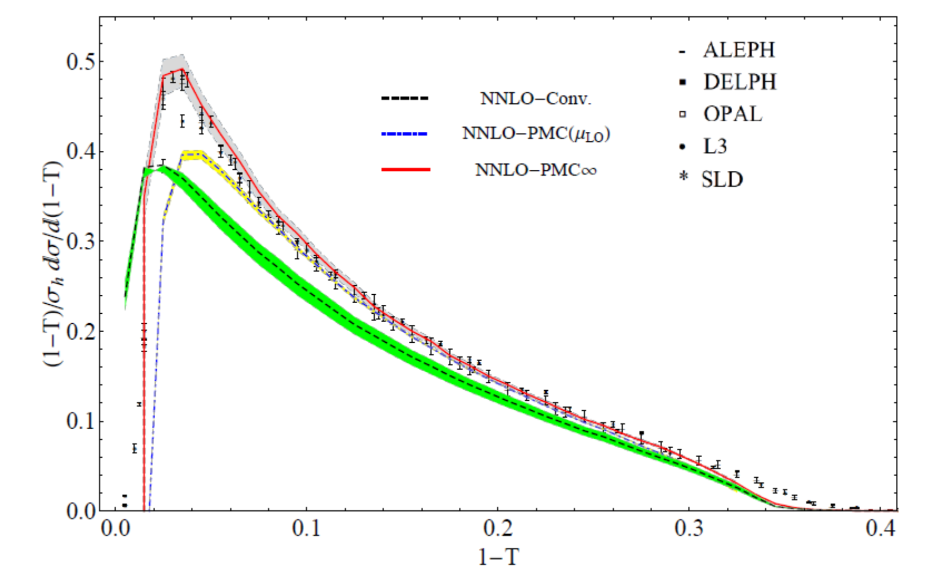
The PMC∞ determines the flow of the running coupling all over the spectrum of the observable reflecting the virtuality of quarks and gluons subprocesses

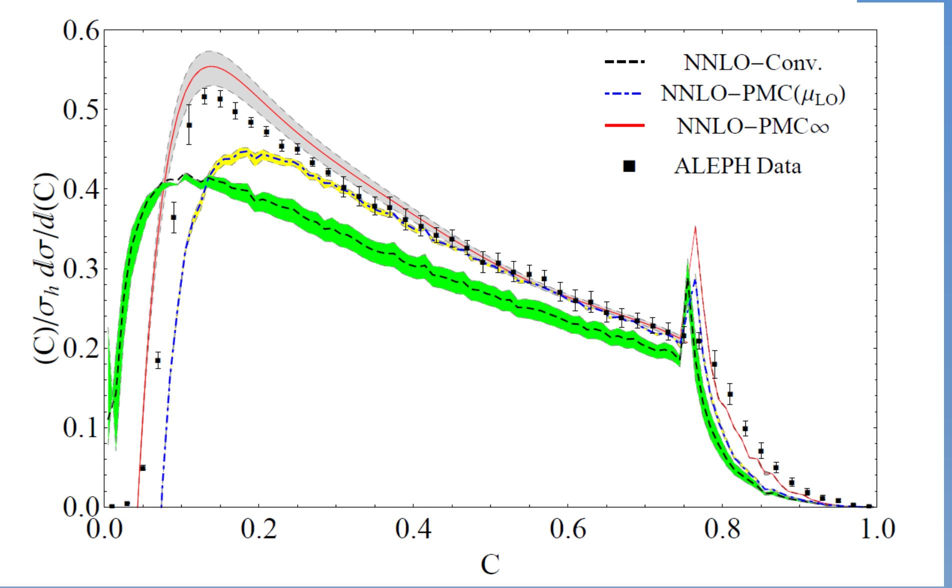
Kinematical constraints cancel the whole LO conformal subset: Aconf=0

Correct physical behavior in the nonperturbative region (unlike other methods e.g. PMS and FAC)



Comparison with Conv. Scale Sett.





$\bar{\delta}[\%]$	Conv.	PMC(μ_{LO})	PMC$_\infty$
$0.10 < (1-T) < 0.33$	6.03	1.41	1.31
$0.21 < (1-T) < 0.33$	6.97	2.19	0.98
$0.33 < (1-T) < 0.42$	8.46		2.61
$0.00 < (1-T) < 0.33$	5.34	1.33	1.77
$0.00 < (1-T) < 0.42$	6.00	-	1.95

$\bar{\delta}[\%]$	Conv.	PMC(μ_{LO})	PMC$_\infty$
$0.00 < (C) < 0.75$	4.77	0.85	2.43
$0.75 < (C) < 1.00$	11.51	3.68	2.42
$0.00 < (C) < 1.00$	6.47	1.55	2.43

PMC∞ improves the precision of the pQCD predictions and the fit with data.

The last unknown scale fixed to the last known leads to stable results.

The error due to the PMC∞ is 1.5% of the whole error $\approx 0.029 - 0.036\ \%$

Errors: 85% depends on not-yet calculated orders.

We can use the standard criteria to evaluate the accuracy and the conformality at NNLO

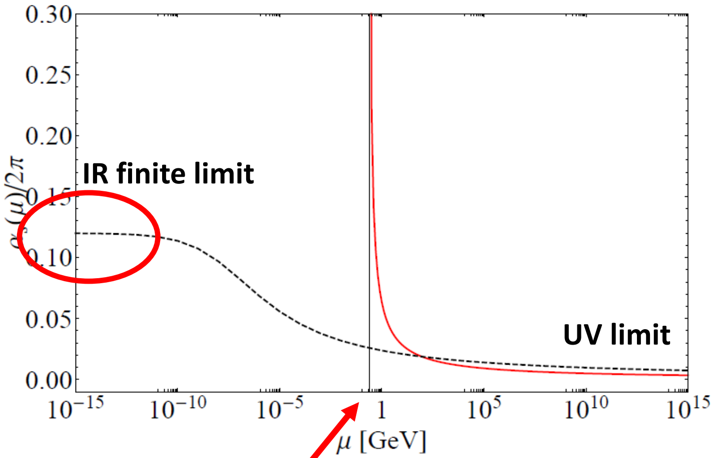
$$\delta = \left| \frac{\sigma(2M)) - \sigma(M/2)}{2\sigma(M)} \right|$$

M = $\sqrt{s}$ = Z0 mass

Thrust in the QCD conformal window

- Banks-Zaks: UV+IR fixed points

L.D.G. , F. Sannino, S.Q. Wang, X.G. Wu,
Phys.Lett.B 823 (2021) 136728



$$\mu^2 \frac{d}{d\mu^2}\left(\frac{\alpha_s}{2\pi}\right) = -\frac{1}{2}\beta_0\left(\frac{\alpha_s}{2\pi}\right)^2 - \frac{1}{4}\beta_1\left(\frac{\alpha_s}{2\pi}\right)^3 + O\left(\alpha_s^4\right)$$

2-loop solution: Lambert function

$$\frac{dx}{dt} = -Bx^2(1+Cx)$$

$$\Lambda = \mu_0\left(1+\frac{|x^*|}{x_0}\right)^{\frac{1}{2B|x^*|}} e^{-\frac{1}{2Bx_0}}$$

$$We^W = z$$

with:

$$W = \left(-\frac{1}{Cx} - 1\right)$$

$$z = e^{-\frac{1}{Cx_0}-1}\left(-\frac{1}{Cx_0} - 1\right)\left(\frac{\mu^2}{\mu_0^2}\right)^{-\frac{B}{C}}.$$

The general solution for the coupling is:

$$x = -\frac{1}{C}\frac{1}{1+W}.$$

Raising the number of flavors Nf , we can compare the two methods CSS and PMC∞ all over the entire energy range from 0 up to ∞.

QCD Conformal Window:

$$\frac{34N_c^3}{13N_c^2-3} < N_f < \bar{N}_f$$

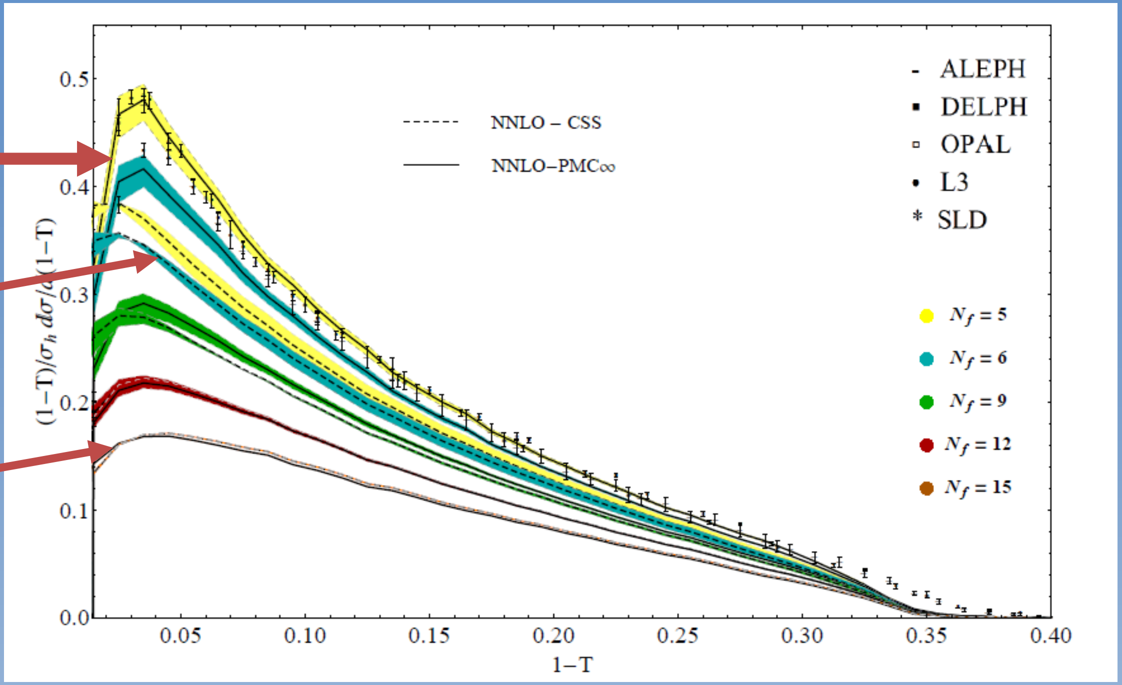
$$\bar{N}_f = x^{*-1}(x_0) \simeq 15.219 \pm 0.012,$$

Thrust in the Conformal Window: C.S.S. and PMC$_\infty$

PMC$_\infty$: the number of active flavors is in agreement with the SM : 5<nf<6

CSS deviates from the conformal distribution

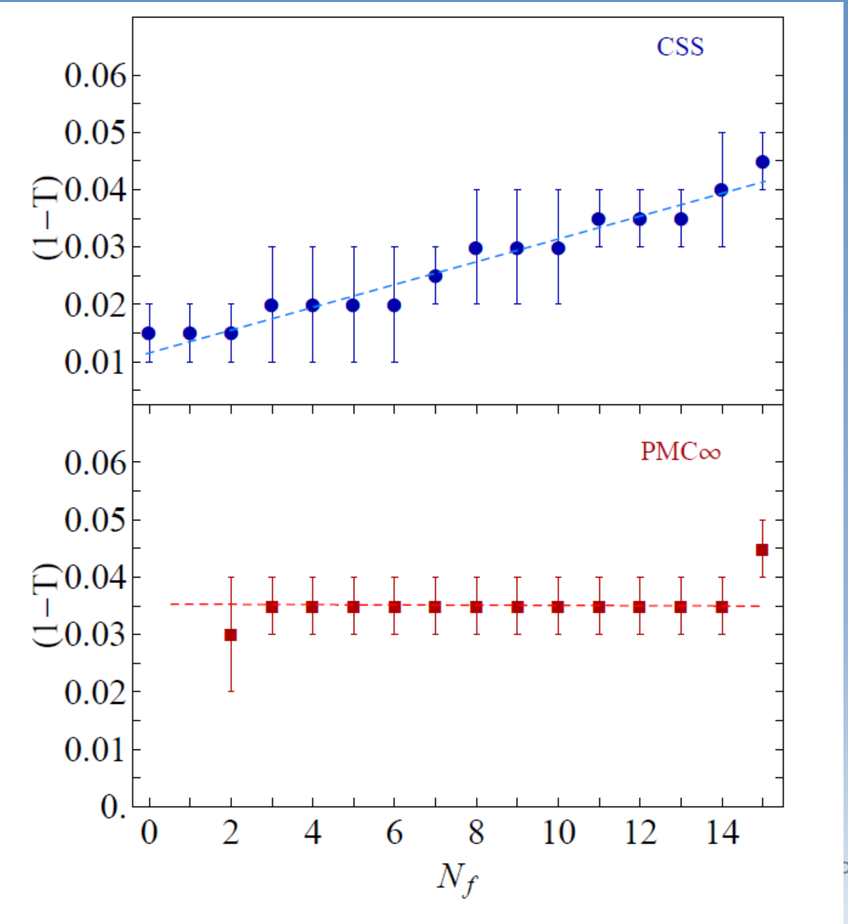
Conformal thrust 9<nf<15

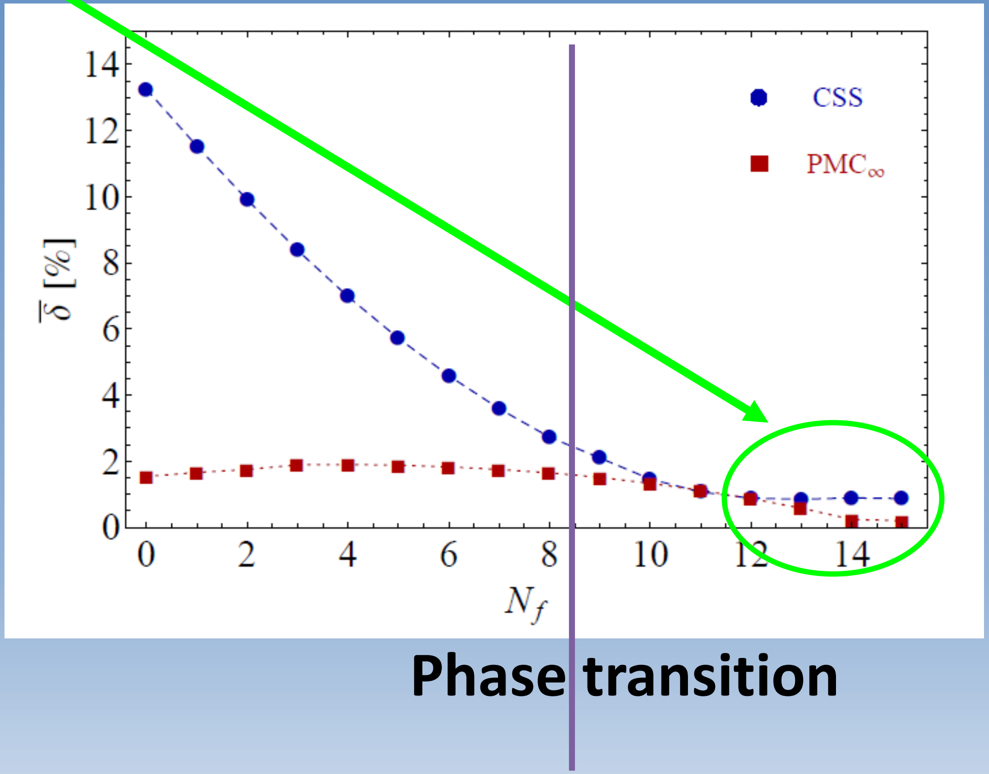


PMC$_\infty$ extends the conformal thrust out of the QCD conformal window

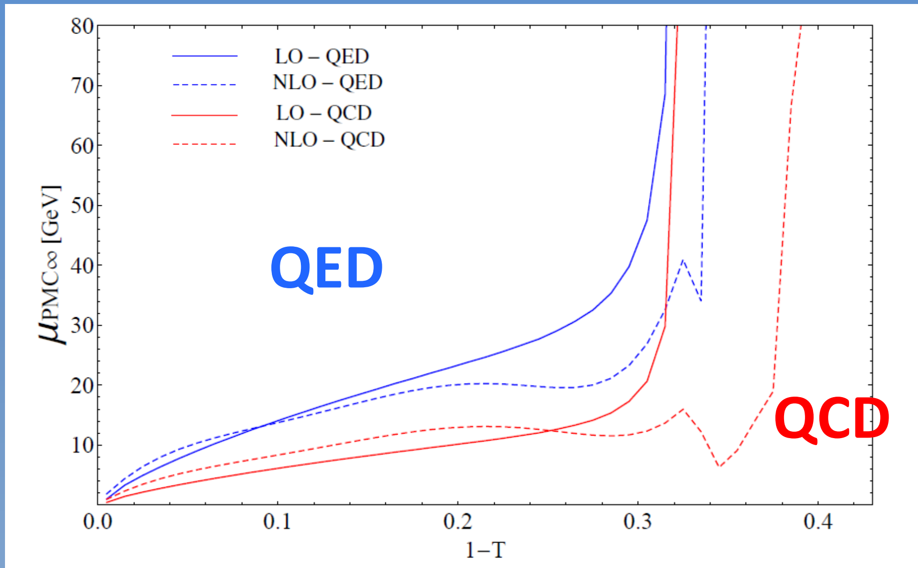
New Features of the PMC_∞

- Shape and the peak position are preserved
- Th. errors calculated with standard criteria show the correct limit in the conformal window





QED Thrust 3-Jet at NNLO, Limit Nc ⟶ 0 is consistent with PMC_∞

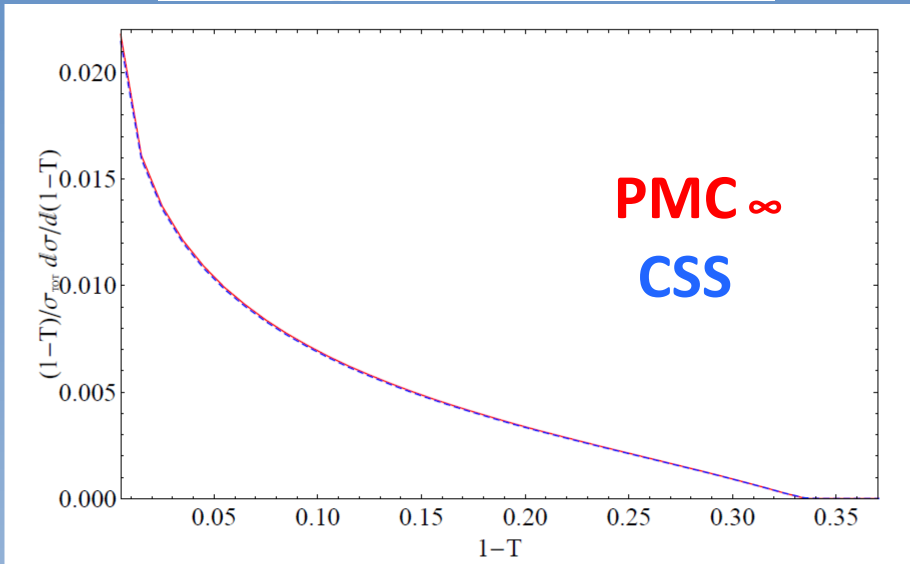


Color factor rescaling for QED:
NA=1, CF=1, TR=1,CA=0,Nc=0,NF=Nl

$$\beta_n / C_F^{n+1} \text{ and } \alpha_s \cdot C_F$$

$$\beta_0 = -\tfrac{4}{3} N_l \text{ and } \beta_1 = -4 N_l$$

QED/QCD PMC_∞ scales differ by the scheme $\overline{\text{MS}}$ factor reabsorption 5/3



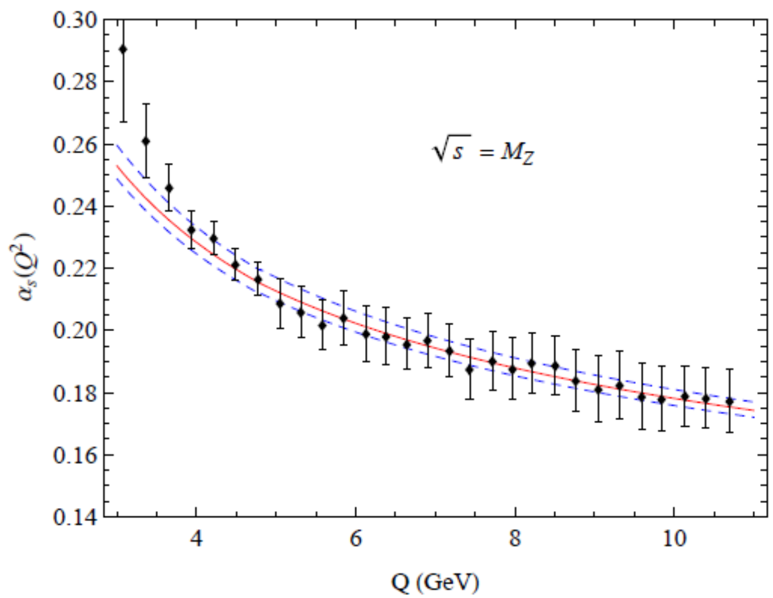
The QED thrust

$$\alpha(Q^2) = \frac{\alpha}{\left(1 - \Re e \Pi^{\overline{\text{MS}}}(Q^2)\right)},$$

Analytic with leptons+quarks+W

«can improve the sensitivity to determine the VPF function for the muon $\frac{g_\mu - 2}{2}$»

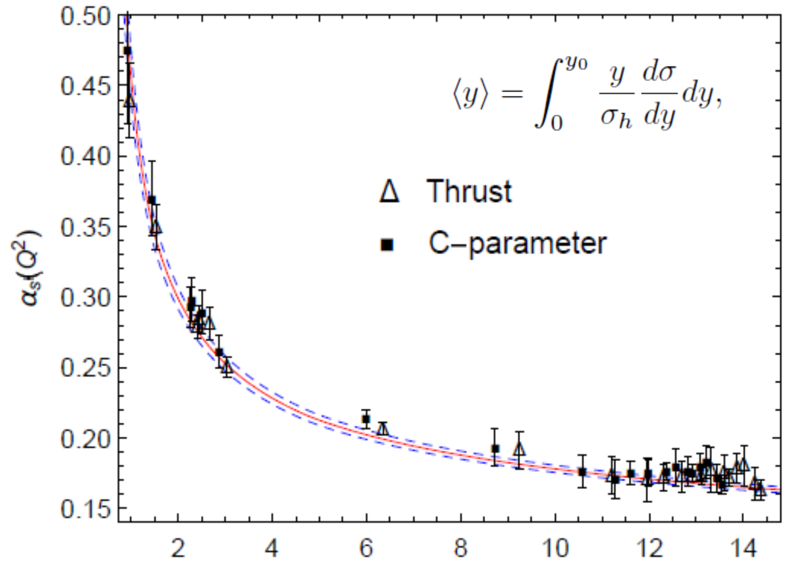
Novel method for the precise determination of $\alpha_s(Q)$



Sheng-Quan Wang, S.J. Brodsky,
Xing-Gang Wu , Jian-Ming Shen, L.D.G.,
Phys.Rev.D 100 (2019) 9, 094010

Asymptotic behavior of $\alpha_s(Q)$ determined from only one experiment

Mean values



T and C-par for e+e- → 3-Jet at a single $\sqrt{s}$

1-T

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0011(\text{exp.}) \pm 0.0005(\text{the.}) = 0.1185 \pm 0.0012,$$

C-par

$$\alpha_s(M_Z^2) = 0.1193^{+0.0009}_{-0.0010}(\text{exp.})^{+0.0019}_{-0.0016}(\text{the.}) = 0.1193^{+0.0021}_{-0.0019},$$

Preliminary results: to appear soon

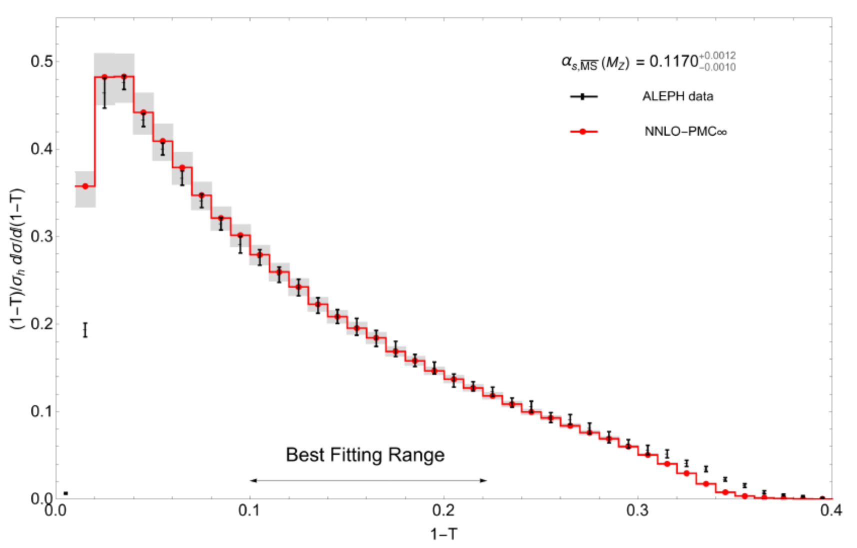
New Method based on the iCF to determine the Entire coupling $\alpha_s(Q)$ at high precision.

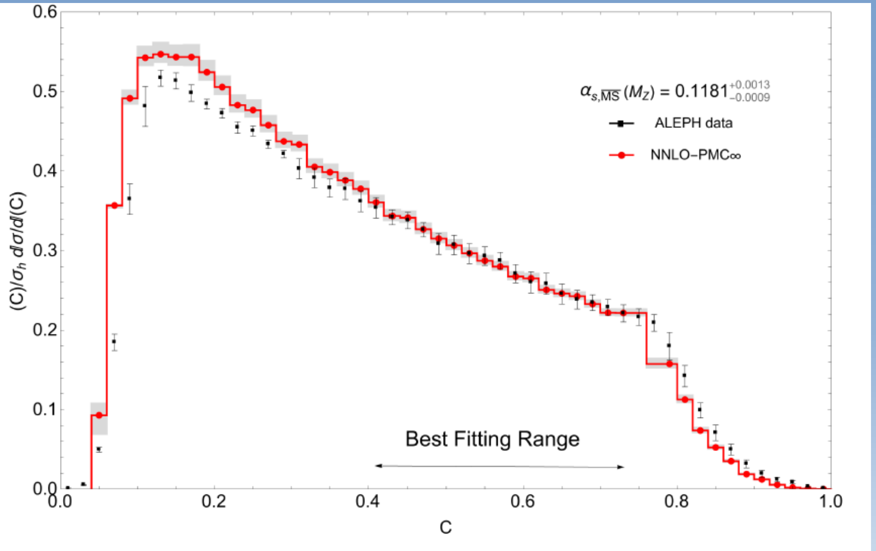
Significant improved precision: `c2 FIT or W.A.` (ALEPH data.)

$$
\begin{aligned}
a_\mu = a_{\mu_0} &+ \beta_0 \ln\left(\frac{\mu_0^2}{\mu^2}\right) a_{\mu_0}^2 + \left[\beta_0^2 \ln^2\left(\frac{\mu_0^2}{\mu^2}\right) + \beta_1 \ln\left(\frac{\mu_0^2}{\mu^2}\right)\right] a_{\mu_0}^3 \\
&+ \left[\beta_0^3 \ln^3\left(\frac{\mu_0^2}{\mu^2}\right) + \frac{5}{2}\beta_0\beta_1 \ln^2\left(\frac{\mu_0^2}{\mu^2}\right) + \beta_2 \ln\left(\frac{\mu_0^2}{\mu^2}\right)\right] a_{\mu_0}^4 \\
&+ \left[\beta_0^4 \ln^4\left(\frac{\mu_0^2}{\mu^2}\right) + \frac{13}{3}\beta_0^2\beta_1 \ln^3\left(\frac{\mu_0^2}{\mu^2}\right) + \frac{3}{2}\beta_1^2 \ln^2\left(\frac{\mu_0^2}{\mu^2}\right) + 3\beta_2\beta_0 \ln^2\left(\frac{\mu_0^2}{\mu^2}\right) + \beta_3 \ln\left(\frac{\mu_0^2}{\mu^2}\right)\right] a_{\mu_0}^5 \\
&+ \left[\beta_0^5 \ln^5\left(\frac{\mu_0^2}{\mu^2}\right) + \frac{77}{12}\beta_1\beta_0^3 \ln^4\left(\frac{\mu_0^2}{\mu^2}\right) + \left(6\beta_2\beta_0^2 + \frac{35}{6}\beta_1^2\beta_0\right) \ln^3\left(\frac{\mu_0^2}{\mu^2}\right) \right. \\
&\left. + \frac{7}{2}(\beta_3\beta_0 + \beta_2\beta_1) \ln^2\left(\frac{\mu_0^2}{\mu^2}\right) + \beta_4 \ln\left(\frac{\mu_0^2}{\mu^2}\right)\right] a_{\mu_0}^6 + \mathcal{O}(a_{\mu_0}^7)\cdots .
\end{aligned}
$$

One experiment Thrust and C-parameter distribution at NNLO for the:
e+e- → 3-Jet process at a single $\sqrt{s}$ energy

5-loop $\alpha_s(Q)$ solution perturbative and RunDec





Preliminary results: to appear soon

Best Fitting Range: results

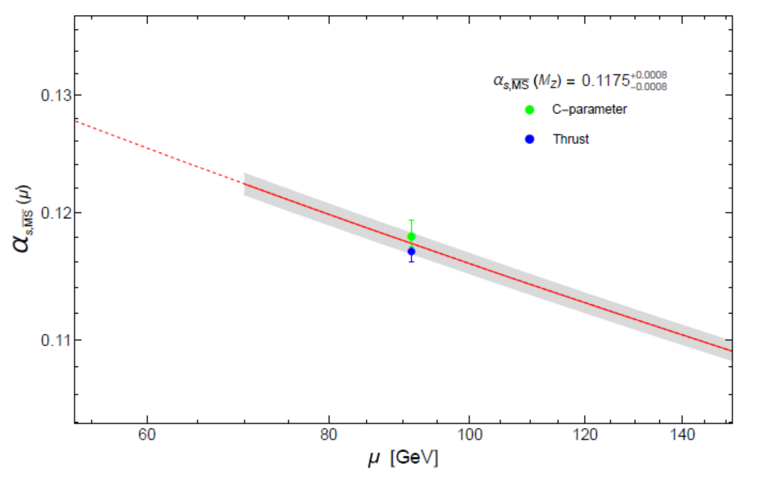
Range	$\alpha_s(M_Z)$	Tot. error	Th. error	Exp. error	$\chi^2_{min}/\text{d.o.f}$	$\text{CDF}(\chi^2_{min}, d.o.f.)$
$0.10 < (1-T) < 0.22$ (BFR)	0.1170	$^{+0.0012}_{-0.0010}$	$^{+0.0010}_{-0.0006}$	$^{+0.0007}_{-0.0007}$	0.80/11	$1.6 \cdot 10^{-5}$

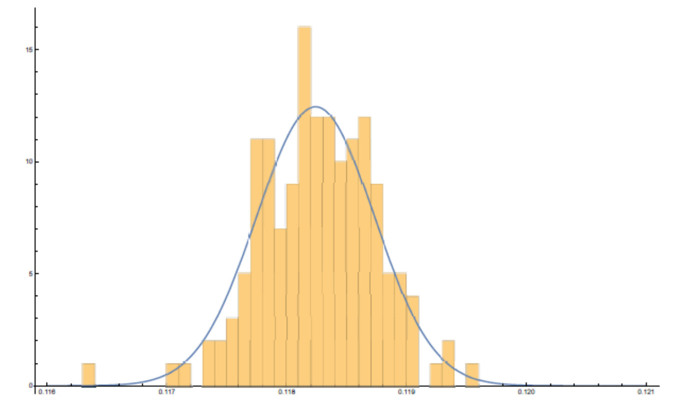
Range	$\alpha_s(M_Z)$	Tot. error	Th. error	Exp. error	$\chi^2_{min}/\text{d.o.f}$	$\text{CDF}(\chi^2_{min}, d.o.f.)$
$0.40 < (C) < 0.74$ (BFR)	0.1181	$^{+0.0012}_{-0.0009}$	$^{+0.0011}_{-0.0007}$	$^{+0.0006}_{-0.0006}$	2.63/16	$6.8 \cdot 10^{-5}$

$$\alpha_s(M_{\text{PL}}) = 0.018837^{+0.000032}_{-0.000024}$$

Lowest CDF

$$CDF(\chi^2_{min}; \text{d.o.f.}) \equiv \int_0^{\chi^2_{min}} d\chi^2 f(\chi^2; \text{d.o.f.}),$$





PMC$_\infty$ fit consistent with the World Average (w/ or w/o latt.)

***Vs*. the previous C.S.S. value:**

$\alpha_s(M_Z) = 0.1274 \pm 0.0021 \pm 0.0042$

Improves statistics and is Consistent with the Maximum Likelihood

LFH low energy and PMC matching

Qing Yu[1,2,*] Xing-Gang Wu[1,3,†] Hua Zhou[1,2,‡] Xu-Dong Huang[1,§] and Jian-Ming Shen[4¶]

$$a_s^{g_1,\mathrm{LFH}}(Q) = e^{-Q^2/4\kappa^2}.$$

Chin.Phys.Lett. 39 (2022) 071201,
ArXiv: 2112.01200 [hep-ph]

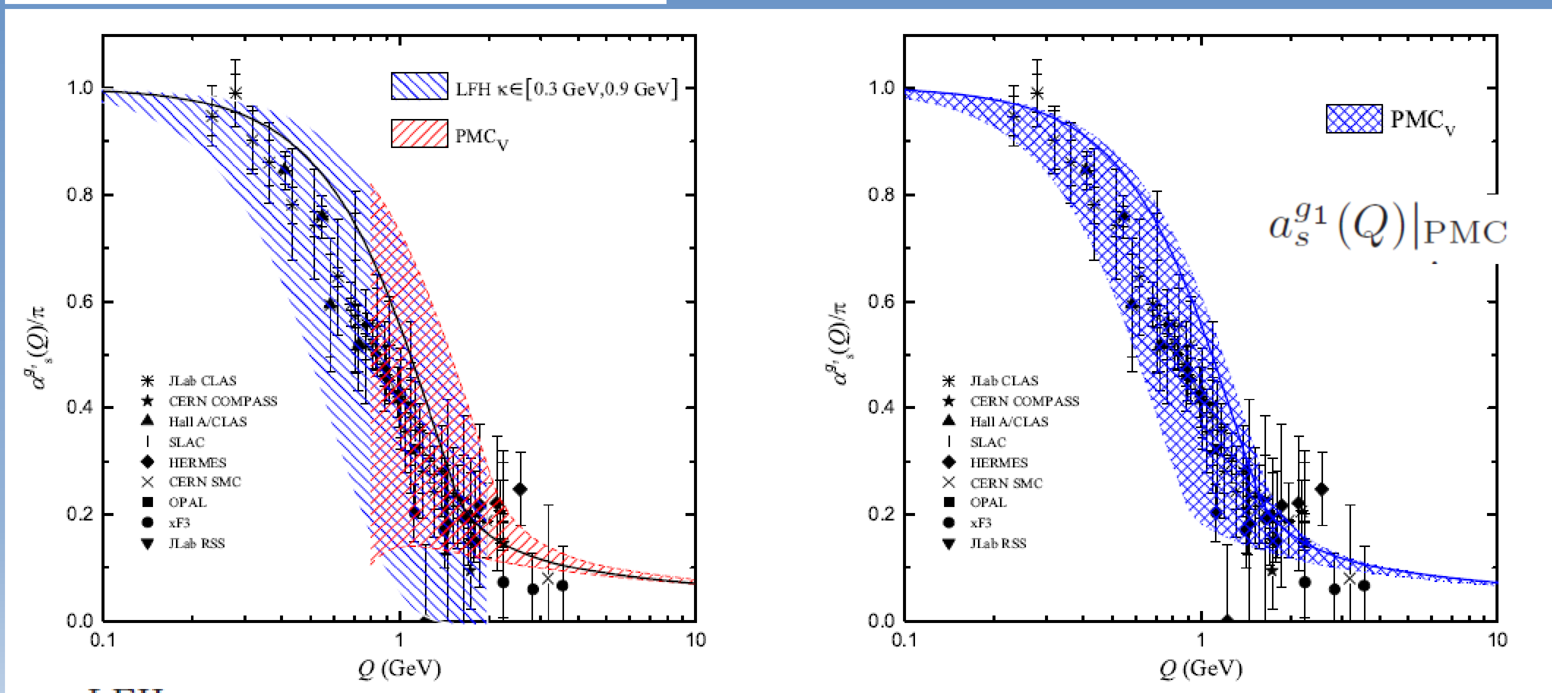
see also : de Tèramond *et al. ArXiv:*2403.16126 [hep-ph]

$$\Gamma_1^{p-n}(Q) = \int_0^1 dx(g_1^p(x) - g_1^n(x)) = \frac{g_A}{6}\left[1 - a_s^{g_1}(Q)\right],$$

Bjorken sum rules (BSR) up to 4 loops

Bjorken sum rules (BSR)

V-scheme



$a_s^{g_1,\mathrm{LFH}}(Q)$

Summary

- The PMC_∞ is based on the PMC and it preserves the iCF;
- The iCF underlies an *ordered* scale invariance;
- We have introduced a new «how to» to easily apply PMC_∞ ;
- Event shape variables results for T and C-par are in very good agreement with data in a wide range of values;
- Thrust in the IR Conformal Window and in the Nc=>0 limit shows consistency with the PMC_∞ ;
- The PMC_∞ eliminates the scale ambiguity and improves the precision of the QCD predictions at any order;
- Measurement of αs agrees with the world average and with the asymptotic behavior;
- PMC_∞ application to $\Gamma(H \to gg)$, $R_{e^+e^-}$, R_τ, and $\Gamma(H \to b\bar{b})$,

  up to 4 loop, improves predictions with respect to CSS.

Future perspectives – wide spread project

- Applications of the PMC_∞ to other fundamental processes: Higgs production,..
- Comparison with other techniques such as Resummation of the IR large logarithms , implementation/interplay of the two methods, analytically and/or numerically… ;
- PMC_∞ for renormalon cancellation (IR renormalons in the mass);
- Application of the PMC_∞ to EW+QCD processes ;
- Large Nc limit ;
- PMC_∞ and Non-perturbative coupling interplay (AdS/CFT ,Holographic QCD);
- PMC_∞ to determine the CSR and Generalized Crewther relation;
- Use of the PMC_∞ to determine the fine structure constant and the hadronic term of the VPF;
- …

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- …

Thank you!

Recent PMC$_\infty$ application to fully integrated quantities (N3LO)

	$n=1$	$n=2$	$n=3$	κ_1	κ_2	κ_3
$R_n\vert_{\mathrm{PMC}\infty}$	0.04750	0.04652	0.03937	7.3%	2.1%	15.4%
$\hat{R}_n\vert_{\mathrm{PMC}\infty}$	0.1805	0.2112	0.2199	102.6%	17.0%	4.1%
$\tilde{R}_n\vert_{\mathrm{PMC}\infty}$	0.2438	0.2448	0.2405	19.9%	0.4%	1.8%

Xu-Dong Huang , Jiang Yan , Hong-Hao Ma , L.D.G., Jian-Ming Shen , Xing-Gang Wu and Stanley J. Brodsky :
Nucl.Phys.B 989 (2023) 116150

$$\kappa_n = \left| \frac{\mathcal{R}_n - \mathcal{R}_{n-1}}{\mathcal{R}_{n-1}} \right|,$$

The ratios in the PMC$_\infty$ are void of any scale dependence

$$R_{e^+e^-}(Q) = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = 3\sum_q e_q^2\left[1 + R(Q)\right],$$

$$R_\tau(M_\tau) = \frac{\sigma\left(\tau \to \nu_\tau + \text{hadrons}\right)}{\sigma\left(\tau \to \nu_\tau + \bar{\nu}_e + e^-\right)} = 3\sum |V_{ff'}|^2 \left[1 + \hat{R}(M_\tau)\right],$$

$$\Gamma(H \to b\bar{b}) = \frac{3G_F M_H m_b^2(M_H)}{4\sqrt{2}\pi}[1 + \tilde{R}(M_H)],$$

CSS

	LO	NLO	N^2LO	N^3LO	Total	Total (CSS)
$R_3\vert_{\mathrm{PMC}\infty}$	0.04383	0.00280	−0.00722	$-0.00004^{+0.00001}_{-0.00004}$	$0.03937^{+0.00001}_{-0.00004}$	$0.04608^{+0.00015}_{-0.00009}$
$\hat{R}_3\vert_{\mathrm{PMC}\infty}$	0.1761	0.0396	0.0035	$0.0007^{+0.0034}_{-0.0005}$	$0.2199^{+0.0034}_{-0.0005}$	$0.1980^{+0.0170}_{-0.0194}$
$\tilde{R}_3\vert_{\mathrm{PMC}\infty}$	0.2265	0.0246	−0.0099	$-0.0007^{+0.0002}_{-0.0004}$	$0.2405^{+0.0002}_{-0.0004}$	$0.2409^{+0.0015}_{-0.0007}$

$\frac{3}{11}R^{\exp}_{e^+e^-} = 1.0527 \pm 0.0050$

$\sqrt{s} = 31.6$ GeV,

$R_\tau^{\exp,\Gamma} = 3.32 \pm 0.12,$

$\Gamma_H = 4.07 \times 10^{-3}$ GeV

Br(H-> bb)= 58.1%
Signal strength : 1.04…