



Running in the Hidden Valley

Exploring near-conformal dark sector theories

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Collaboration with: Suchita Kulkarni & Matthew Strassler

(Paper to appear!)

Confining Hidden Valley models

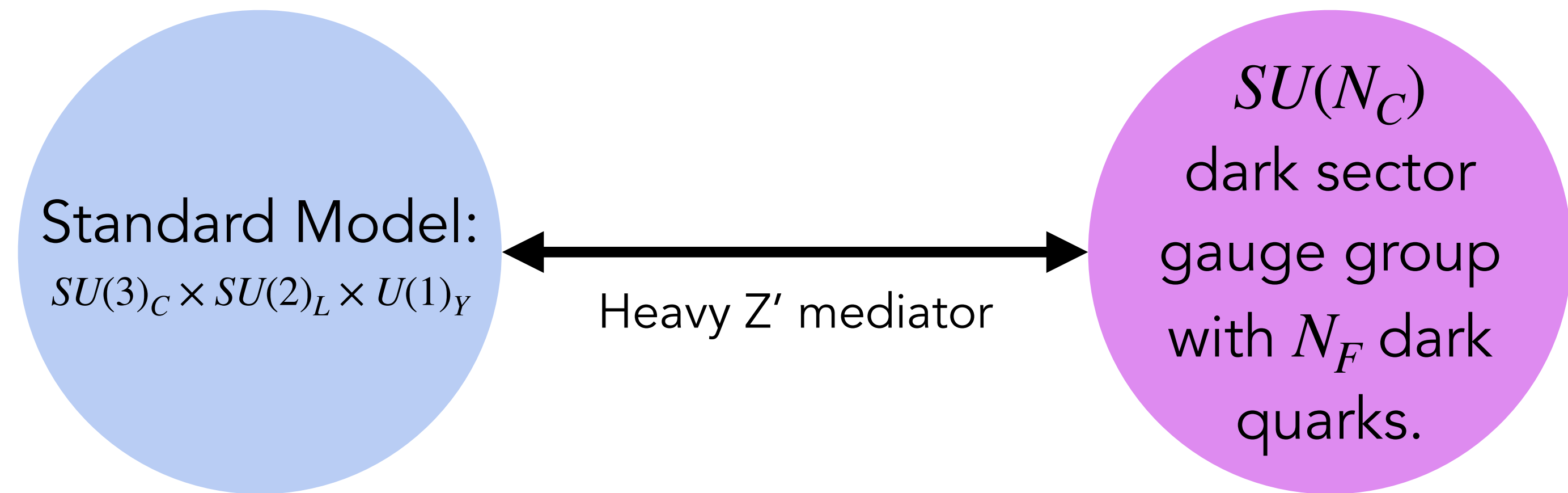
- Hidden Valley models extend the Standard Model (SM) with a new dark sector uncharged under the SM gauge group but instead connected to the SM through a heavy mediator.

arXiv:0604261, M.J. Strassler et al.

arXiv:1502.05409, P. Schwaller et al.

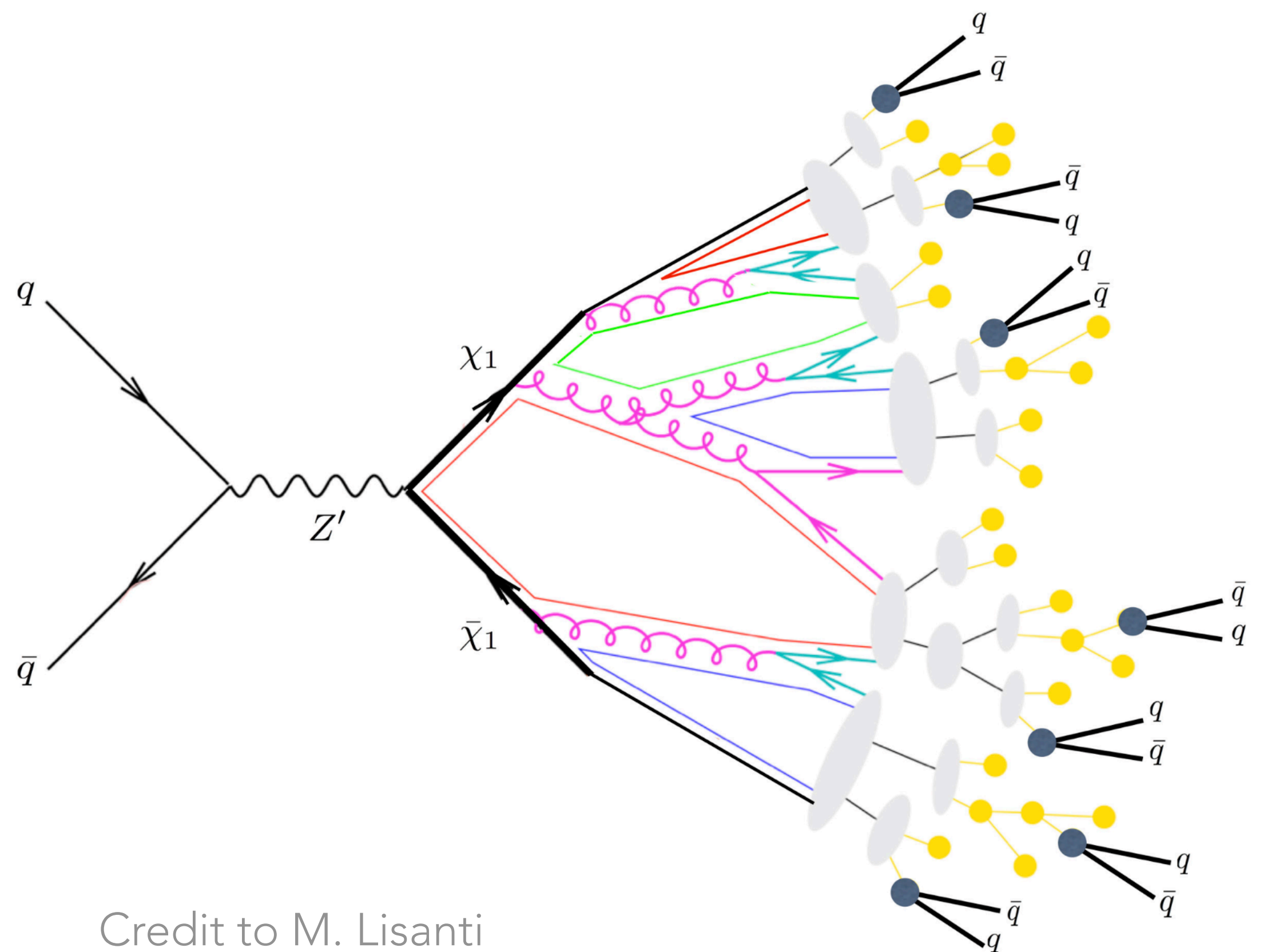
arXiv:1503.00009, T. Cohen et al.

arXiv:0712.2041, T. Han et al.



- Certain classes of these models featuring dark sectors resembling QCD present novel signatures and exciting opportunities for new physics discovery.
- We focus on dark sectors with a non-Abelian $SU(N_C)$ gauge group with N_F flavours of degenerate fundamental Dirac fermions (dark quarks). Such sectors are characterised by four parameters; N_C , N_F , Λ and m_{π_D}/Λ . Confinement ensures the formation of bound states; in our case dark mesons - typically dark pions.

Confining Hidden Valley models



Credit to M. Lisanti

- Searches exist at colliders for the signatures of QCD-like dark sectors; on-going interest in exploring novel signatures at colliders. These signatures are simulated by the event generator Pythia8 for some classes of confining Hidden Valleys.
- While signatures such as “semi-visible” jets are well-known, confining Hidden Valleys are a relatively understudied area. New areas of parameter space could result in exciting and unique phenomenology that have not yet been looked for in colliders.

[arXiv:2112.11125 \(CMS\)](#)

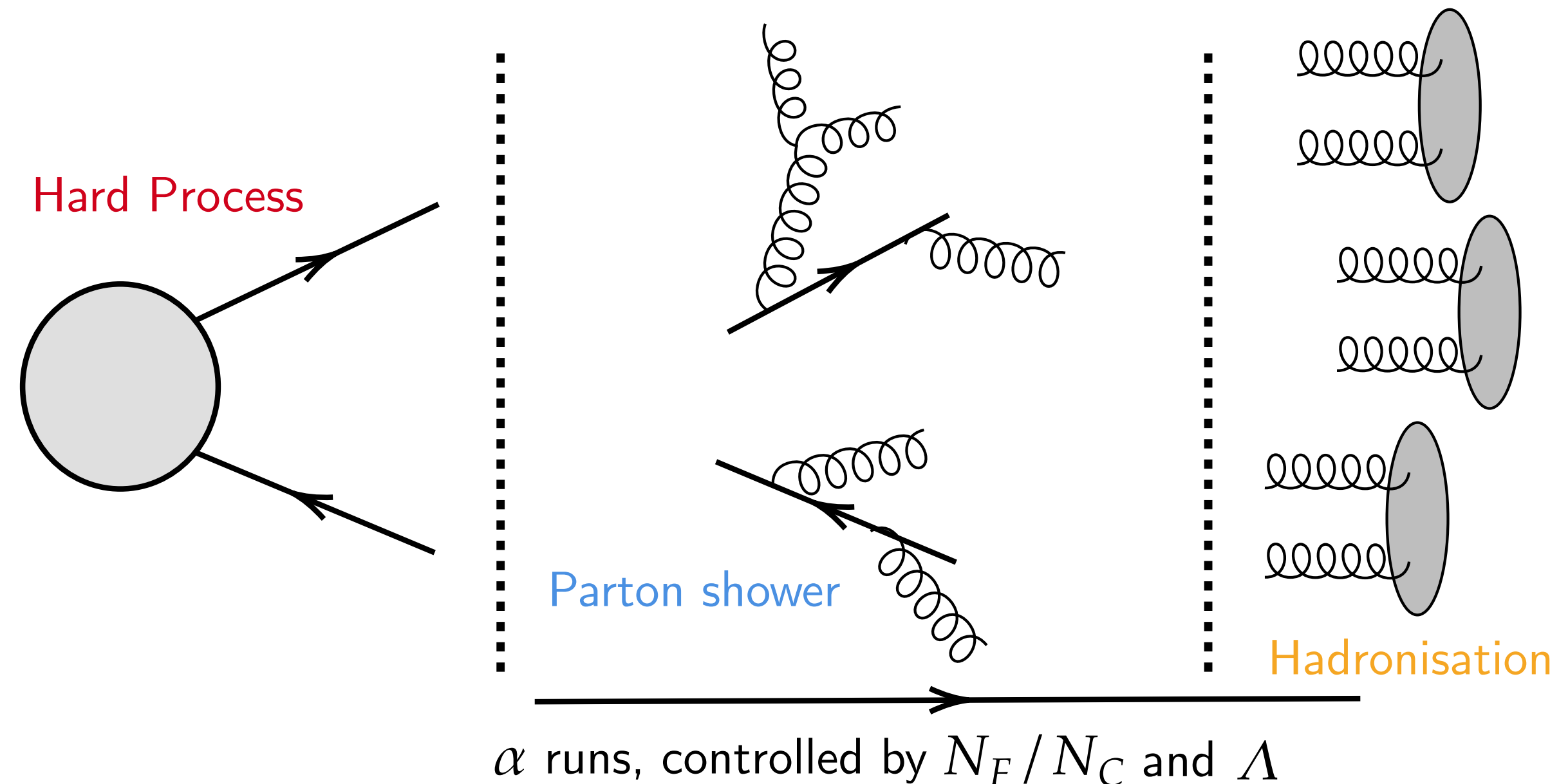
[arXiv:2305.18037 \(ATLAS\)](#)

[arXiv:1910.08447 \(ATLAS\)](#)

[arXiv:2102.10874 \(ATLAS\)](#)

[arXiv:1810.10069 \(CMS\)](#)

Anomalous jet signatures

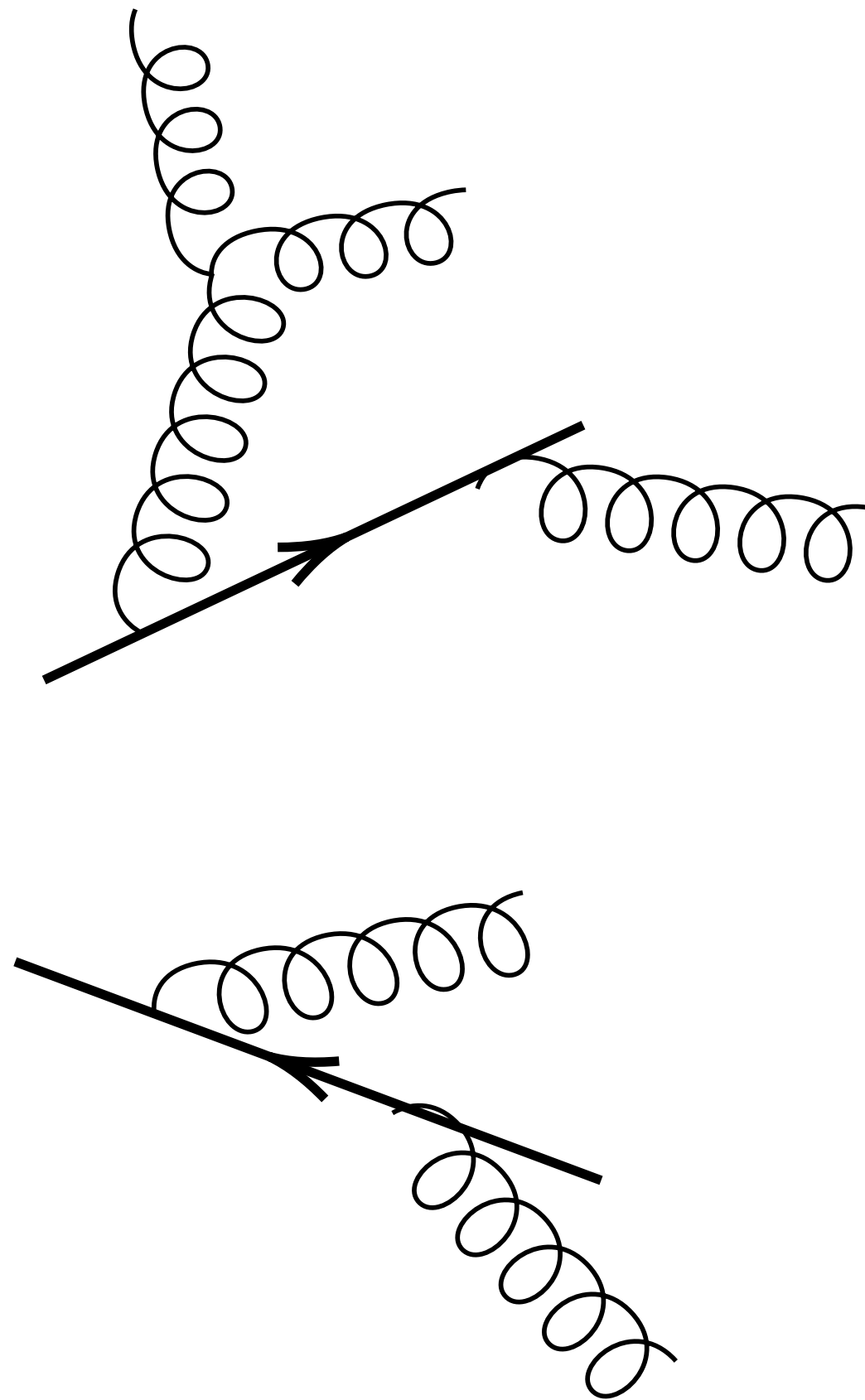


arXiv:1503.00009 - T. Cohen et al.

See talk by Dominic Stafford!

- At colliders, some hard process, such as a p-p collision, produces a Z' which decays into a dark quark-antiquark pair. These dark quarks undergo parton showering and hadronisation within the dark sector.
- A portion of dark mesons will decay to SM particles through the mediator resulting in a jet with a mixture of stable dark hadrons and SM decay products typically known as "dark showers".
- Dark shower signatures e.g. emerging or semi-visible jets are well-known at a small ratio of N_F/N_C . Large N_F/N_C dark sectors are a largely undeveloped area of theory space and could give rise to distinct signatures.

Dark parton showering



- The running coupling, α , in part controls the behaviour of the parton shower, with itself being governed by the Renormalisation Group Equations (RGE).

$$\mu^2 \frac{d\alpha}{d\mu^2} = \beta(\alpha) = -\alpha^2 (\beta_0 + \beta_1 \alpha) \quad (\text{at 2-loop})$$

- Parton shower ends near scale Λ , which characterises breakdown of perturbative expansion of α . To a good approximation, the 't Hooft gauge coupling is governed solely by N_F/N_C and μ/Λ .

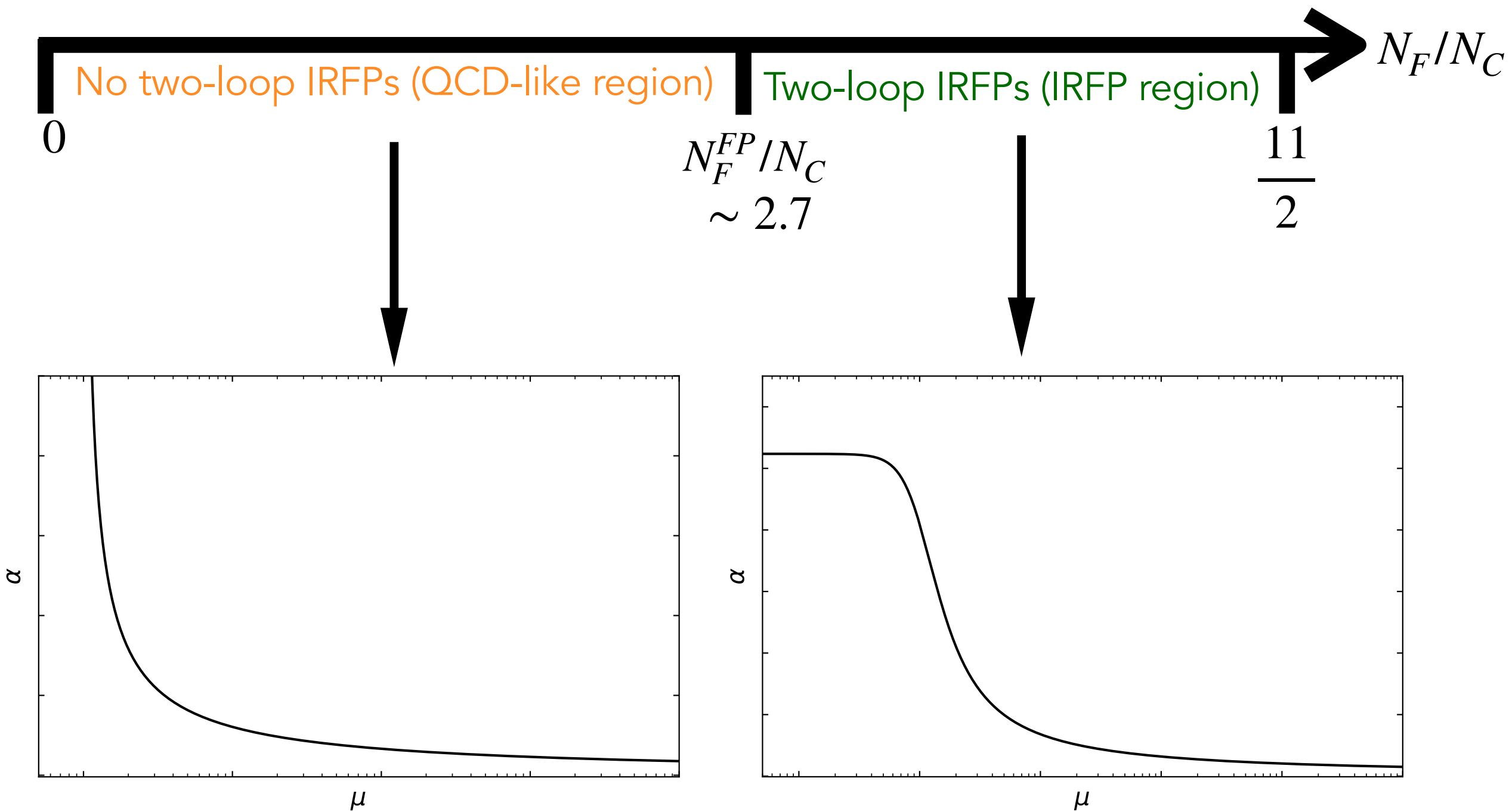
$$N_C \alpha = f(N_F/N_C, \mu/\Lambda) + \mathcal{O}(N_F/N_C^3) \text{ corrections}$$

- At two-loop, for $N_F/N_C \gtrsim 2.7$, α flows to a non-trivial infra-red fixed point (IRFP); as N_F/N_C increases α begins to slow down. New procedures are needed to understand parton showering within this region. T. Banks., A. Zaks, Nucl.Phys.B 196 ('82)

Non-trivial fixed point: $\alpha_* = -\frac{\beta_0}{\beta_1} ; > 0 \text{ for } N_F/N_C \gtrsim 2.7$

Near-conformal dark sector models

Two-loop perturbative description



- Want to capture the effects of running coupling with IRFPs - two-loop is the first order at which this appears.
- There is a plethora of work focused on the non-perturbative structure and the low-energy effective descriptions of large N_F/N_C theories. However the infra-red behaviour and the value of N_F/N_C where IRFPs appear is a topic of hot debate.

arXiv:2306.07236, A. Hasenfratz et al.

arXiv:2312.13761, R. Zwicky

arXiv:2312.08332, A. Pomarol et al.

arXiv:0902.3494, T. Appelquist et al.

arXiv:2008.12223, J.W. Lee

arXiv:0902.3494, F. Sannino

- For now, focus on dark parton showering at high N_F/N_C ; subsequent details such as hadronisation details are not yet fully understood.

Running coupling - current procedure

- The one-loop running coupling is parameterised by a scale Λ , defined to be the divergence of the running coupling; below this scale the perturbative expansion breaks down. For SM QCD, Λ_{QCD} is a useful proxy for the scale of the theory.

$$\alpha = \frac{1}{\beta_0 \ln \left(\frac{\mu^2}{\Lambda^2} \right)}$$

W.-M. Yao et al., Review of Particle Physics (2006), arXiv:0607209 - Prospero et al.

- At two loops, defining Λ in the same way as one-loop, we obtain an implicit equation through RGE integration. "Two-loop exact" solution solvable through special functions, not true at higher-loop order.

$$\beta_0 \ln \left(\frac{\mu^2}{\Lambda^2} \right) = \frac{1}{\alpha} + \frac{1}{\alpha_*} \ln \left(1 - \frac{\alpha_*}{\alpha} \right) \longrightarrow \alpha = \alpha_* [W_{-1}(-z) + 1]^{-1} ; \quad z = \frac{1}{e} \left(\frac{\mu^2}{\Lambda^2} \right)^{\beta_0 \alpha_*}$$

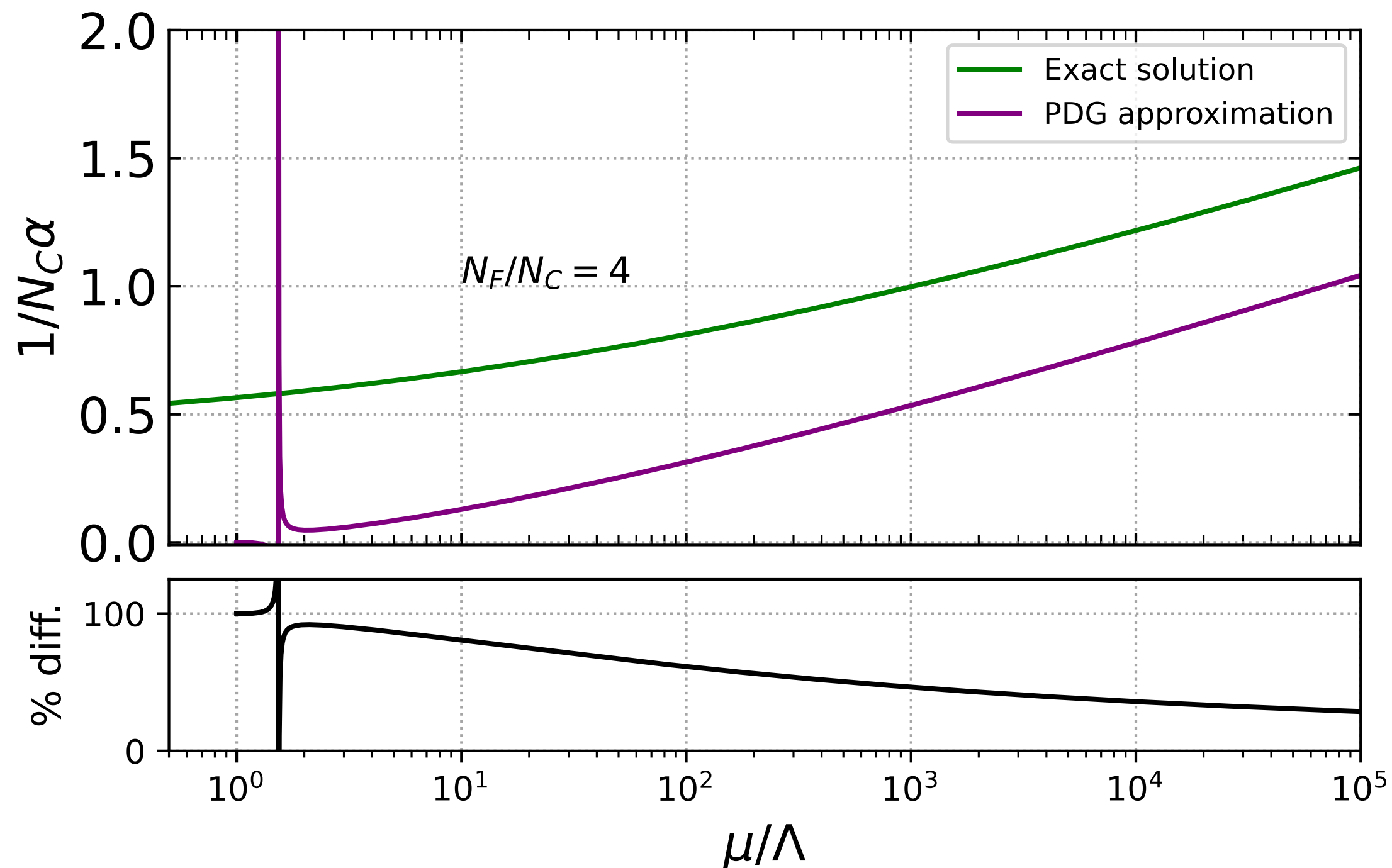
- This solution becomes unphysical when we try to push this solution beyond $N_F/N_C = (N_F/N_C)_* \approx 2.7$ since the presence of the IRFP, α_* , means that the running coupling no longer diverges.

Running coupling - current procedure

- This problem could be avoided through assuming logarithmic terms dominate over the inverse of the magnitude of the IRFP. This gives the two-loop running coupling formula currently used by Pythia and the Particle Data Group (PDG),

$$\frac{1}{\beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)} < |\alpha_*| \quad \longrightarrow \quad \alpha = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)} \left[1 + \frac{1}{\alpha_*} \frac{\ln[\ln(\mu^2/\Lambda^2)]}{\beta_0 \ln(\mu^2/\Lambda^2)} \right]$$

W.-M. Yao et al., Review of Particle Physics (2006), arXiv:0607209, Prospero et al.



- For $N_F/N_C \gg (N_F/N_C)_*$, α_* becomes smaller, thus the PDG approximation begins to break down, inevitably leading to inaccurate showering behaviour.
- Thus the current approximation used within event generators (the PDG formula) is insufficient to describe two-loop α for high N_F/N_C since it neglects important effects of the IRFP.

New procedures for IRFPs

- We want a framework to define both α and Λ in regions with and without IRFPs. In general, perturbatively the scale Λ describes a cross-over between two scaling regions, below which the perturbative expansion breaks down. Our theory is weakly coupled in the UV ($\alpha(\mu_0) < \alpha_*$).
- The traditional definition of Λ remains within the QCD-like region. Unlike the QCD-like region, the low energy behaviour of running coupling in the IRFP region takes on a power-law form,

$$\alpha - \alpha_* \sim \left(\frac{\mu^2}{\mu_0^2} \right)^\gamma \quad ; \quad \gamma = \left. \frac{\partial \beta}{\partial \alpha} \right|_{\alpha=\alpha_*} = \beta_0 \alpha_* \quad (\text{at 2-loop})$$

- We can then define Λ_{FP} as the transition scale between the asymptotic free $\sim \frac{1}{\log}$ and power-law behaviours. The exact scale below which the power-law dominates can be found to be,

$$\beta_0 \ln \left(\frac{\Lambda_{FP}^2}{\mu_0^2} \right) = -\frac{1}{\alpha_0} - \frac{1}{\alpha_*} \ln \left(\frac{\alpha_*}{\alpha_0} - 1 \right)$$

arxiv:9602385,

arxiv:9806409 - T. Appelquist et al.

arxiv:9810192 - E. Gardi et al.

New procedures for IRFPs

- By taking this IRFP into account, we establish a framework of two solutions to the RGE that accurately describe the running coupling in regions with and without IRFPs.
- From this, we can find the explicit forms in both regions in terms of the two real branches of the Lambert W function,

$$\alpha = \alpha_* [W_{-1}(-z) + 1]^{-1} \quad ; \quad \alpha = \alpha_* [W_0(z) + 1]^{-1} \quad ; \quad z = \frac{1}{e} \left(\frac{\mu^2}{\Lambda^2} \right)^{\beta_0 \alpha_*}$$

QCD-like (no IRFP)
IRFP-region

- For large μ/Λ , we can use the expansion in both branches of the Lambert W function of,

$$W(x) = L_1 - L_2 + \frac{L_2}{L_1} + \mathcal{O} \left(\left[\frac{L_2}{L_1} \right]^2 \right)$$

- Where $L_1 = \ln(z)$, $L_2 = \ln(\ln(z))$ for $W_0(z)$ and $L_1 = \ln(z)$, $L_2 = \ln(-\ln(z))$ for $W_{-1}(-z)$,

New procedures for IRFPs

- Gives following approximation in the QCD-like region of,

$$\frac{1}{\alpha} = \beta_0 \ln \left(\frac{\mu^2}{\Lambda^2} \right) - \frac{1}{\alpha_*} \ln \left(1 - \beta_0 \alpha_* \ln \left(\frac{\mu^2}{\Lambda^2} \right) \right) + \frac{1}{\alpha_*} \frac{\ln \left(1 - \beta_0 \alpha_* \ln \left(\frac{\mu^2}{\Lambda^2} \right) \right)}{\beta_0 \alpha_* \ln \left(\frac{\mu^2}{\Lambda^2} \right) - 1}$$

QCD-like approximation

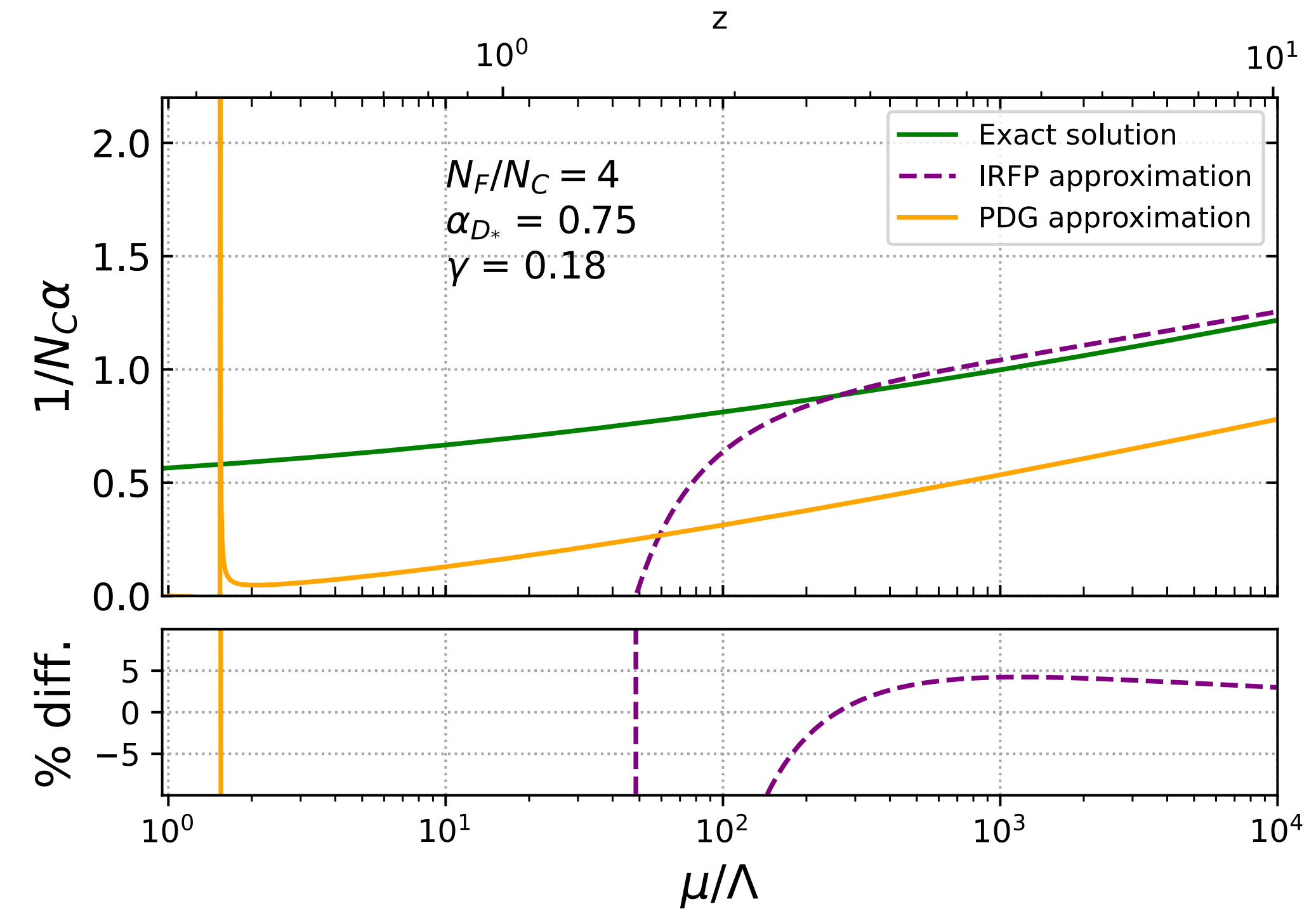
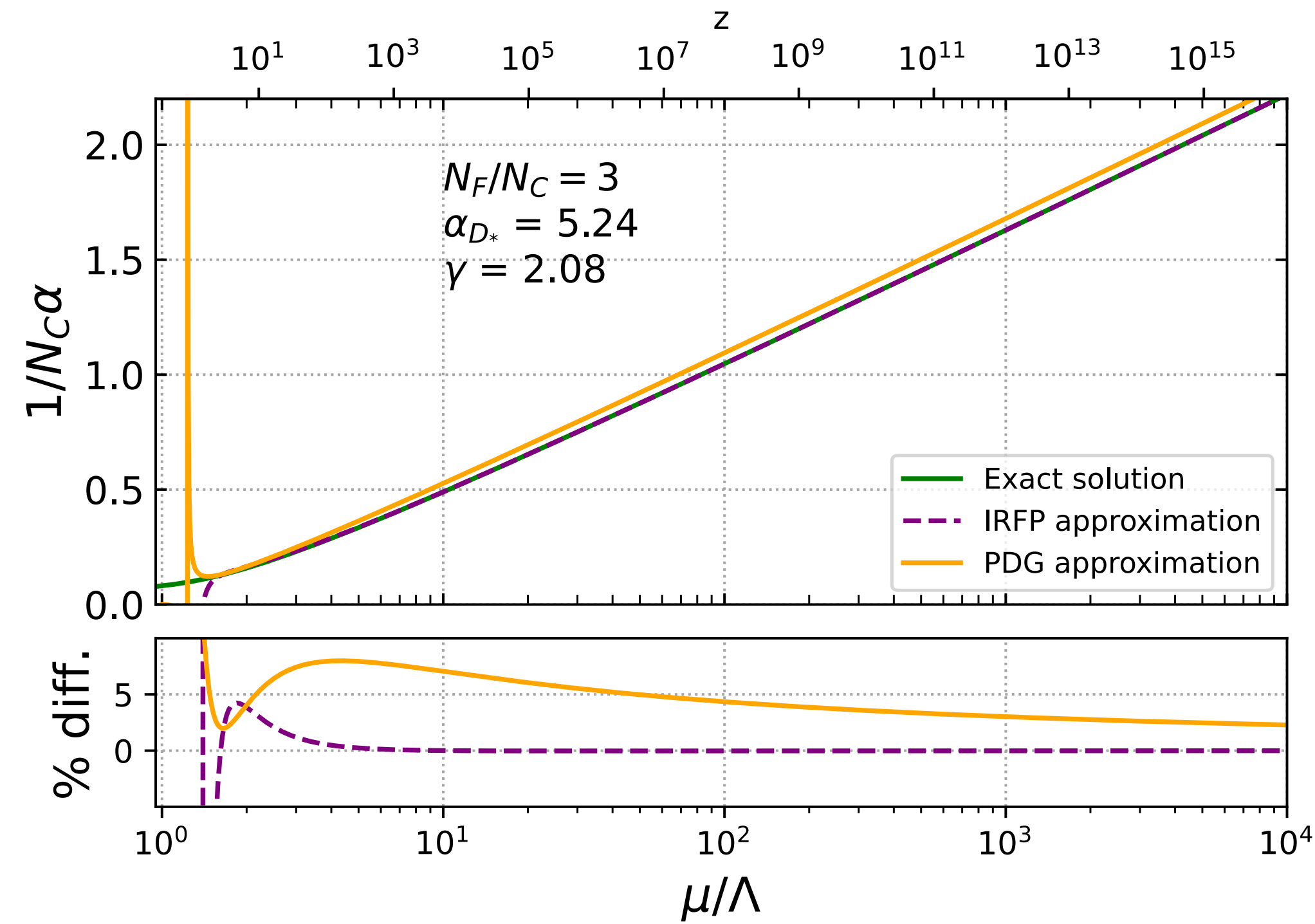
- And in the IRFP region of,

$$\frac{1}{\alpha} = \beta_0 \ln \left(\frac{\mu^2}{\Lambda_{FP}^2} \right) - \frac{1}{\alpha_*} \ln \left(\beta_0 \alpha_* \ln \left(\frac{\mu^2}{\Lambda_{FP}^2} \right) - 1 \right) + \frac{1}{\alpha_*} \frac{\ln \left(\beta_0 \alpha_* \ln \left(\frac{\mu^2}{\Lambda_{FP}^2} \right) - 1 \right)}{\beta_0 \alpha_* \ln \left(\frac{\mu^2}{\Lambda_{FP}^2} \right) - 1}$$

IRFP approximation

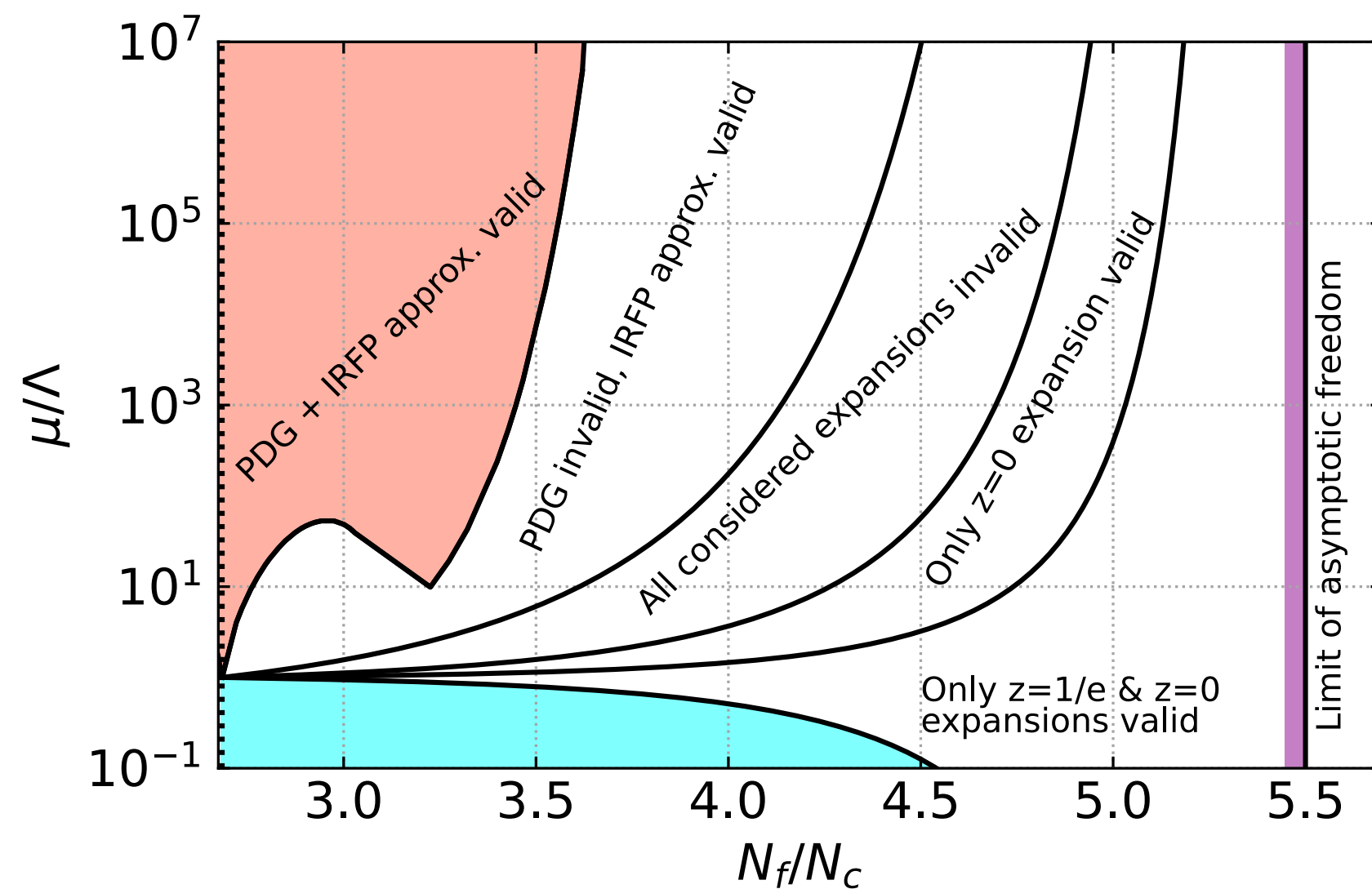
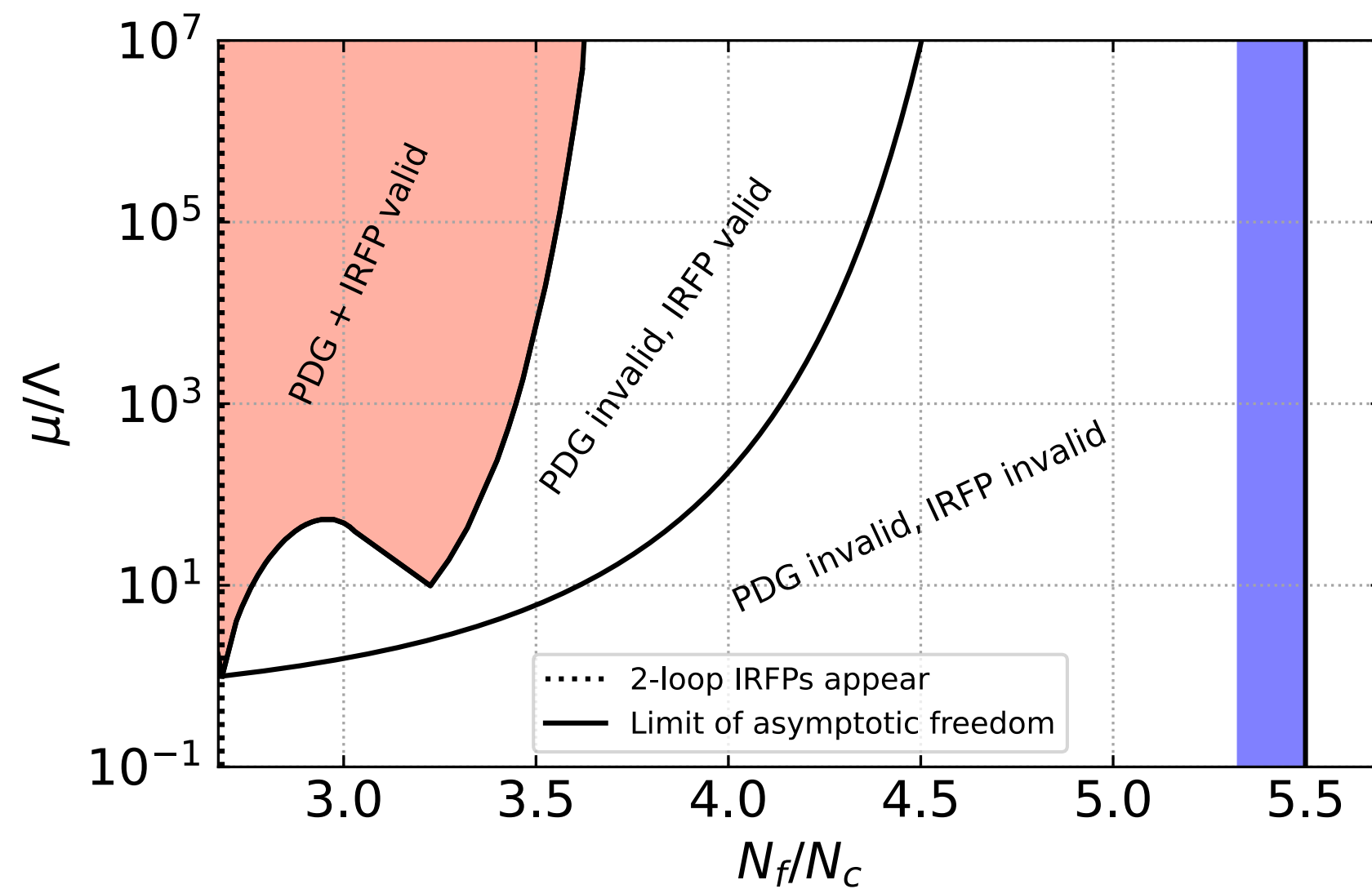
Monte Carlo implementation

- By expanding for large μ/Λ , we obtain closed-form UV expansions of our two solutions.



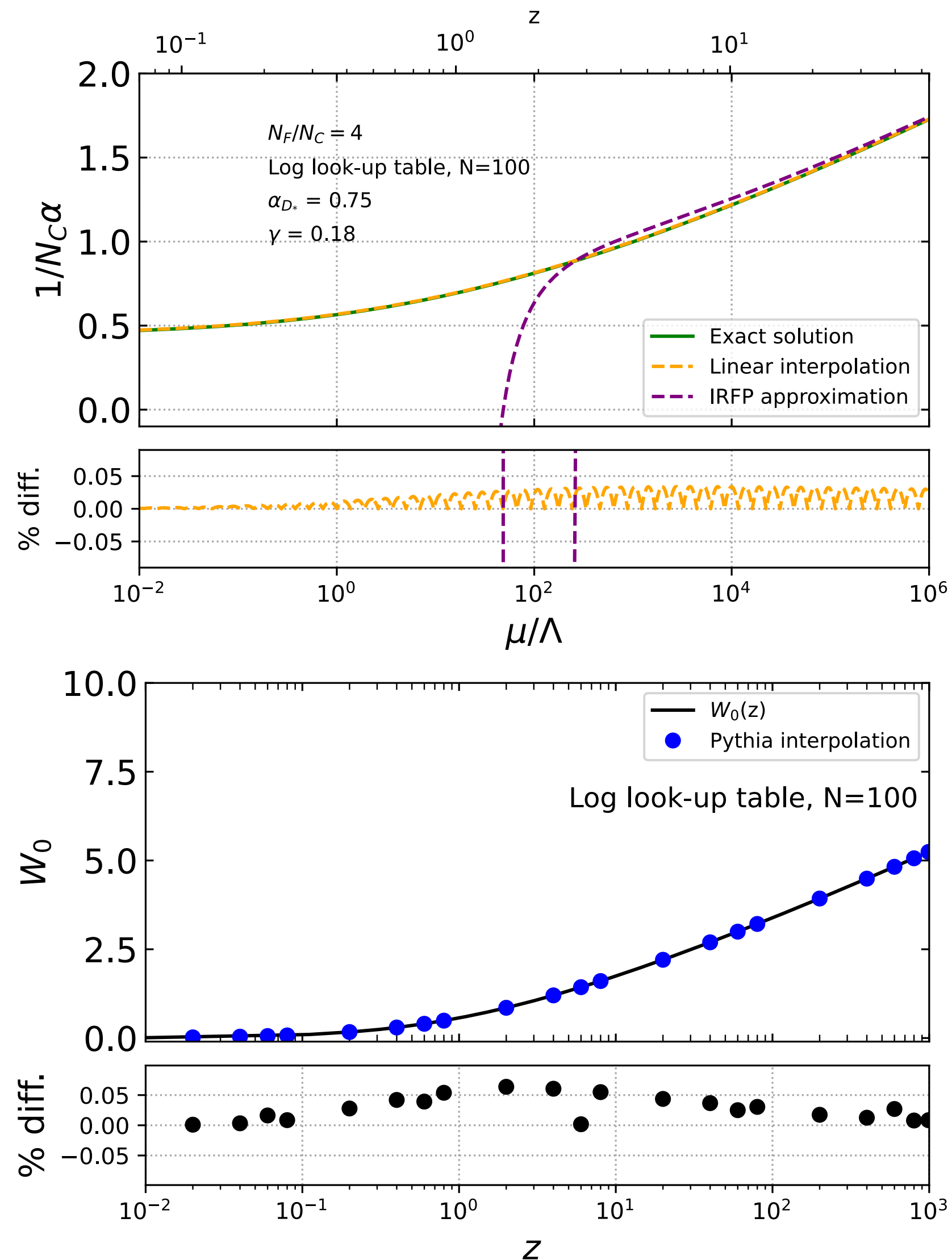
- The IRFP approximation covers more of $N_F/N_C - \mu/\Lambda$ space than the PDG approximation, providing the best approximation when $N_F/N_C \sim (N_F/N_C)_*$. However, increasing N_F/N_C close to 4, the energy range over which the IRFP approximation proves to be reliable begins to decrease, however is still accurate within the UV.

Monte Carlo implementation



- A full comparison of the validity (deviation by over 5%) of the IRFP and PDG approximations in $N_f/N_c - \mu/\Lambda$ space reveals a large area of parameter space covered by neither.
- Various IR expansions were considered, e.g. expanding the exact IRFP solution around $z = 0$ or $z = 1/e$ but none could provide full coverage over all parameter space.
- Since the running of the coupling within the uncovered region is so slow, interpolating the "exact" IRFP solution was found to be the most reliable approximation within this region.

Interpolation procedure



- Entire implementation makes use of the parameter $z = \frac{1}{e} \left(\frac{\mu^2}{\Lambda^2} \right)^{\beta_0 \alpha_s}$ as it allows for scaling between different values of N_F/N_C .
- Within the QCD-like region, the implementation of the running coupling suffices with a large μ/Λ approximation for all $\mu/\Lambda > 1$.
- Within the IRFP region, the implementation also requires a large μ/Λ approximation, however this expansion can not model behaviour near the IRFP.
- Within this region, an interpolation routine is instead used. This was taken over z between a range of 10^{-2} and 10^3 using 100 data points.
- The upper boundary of this interpolation is determined by when the large μ/Λ approximation deviates from the exact solution by 0.1%.

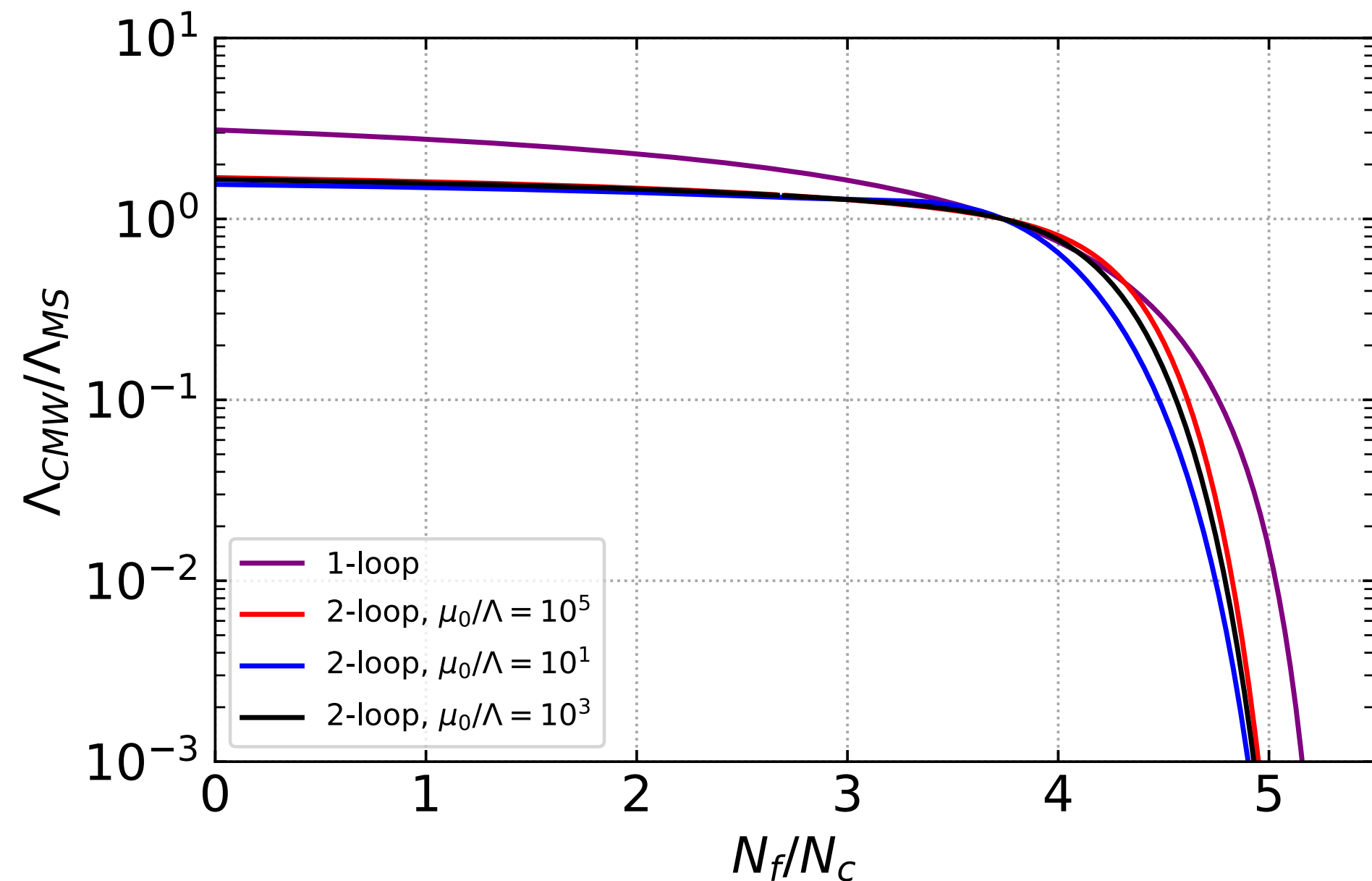
Setup and caveats

Contributing Altarelli-Parisi splitting functions:

$$P_{G_D \rightarrow G_D G_D} = C_A \frac{(1 - \xi(1 - \xi))^2}{\xi(1 - \xi)}$$

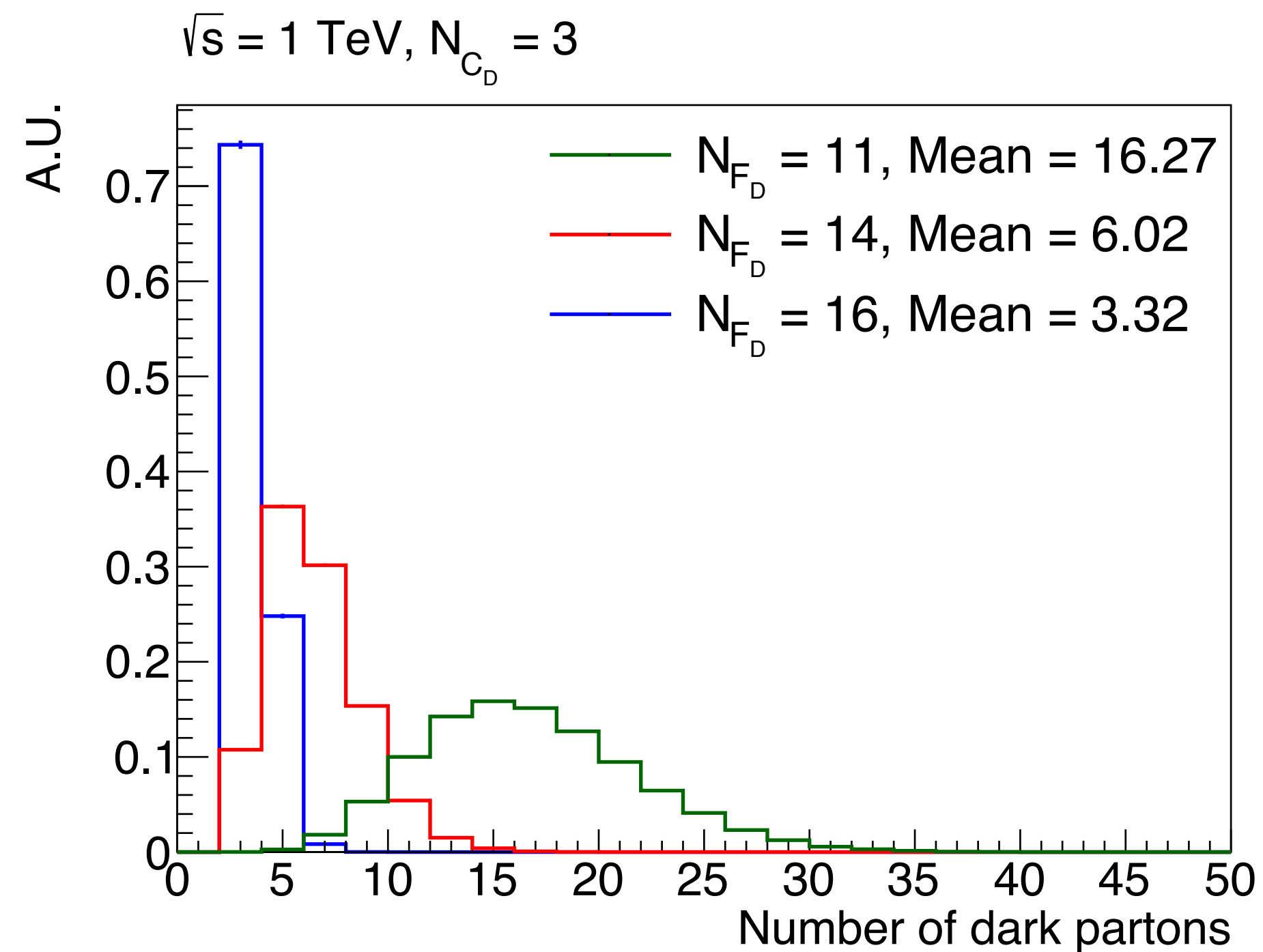
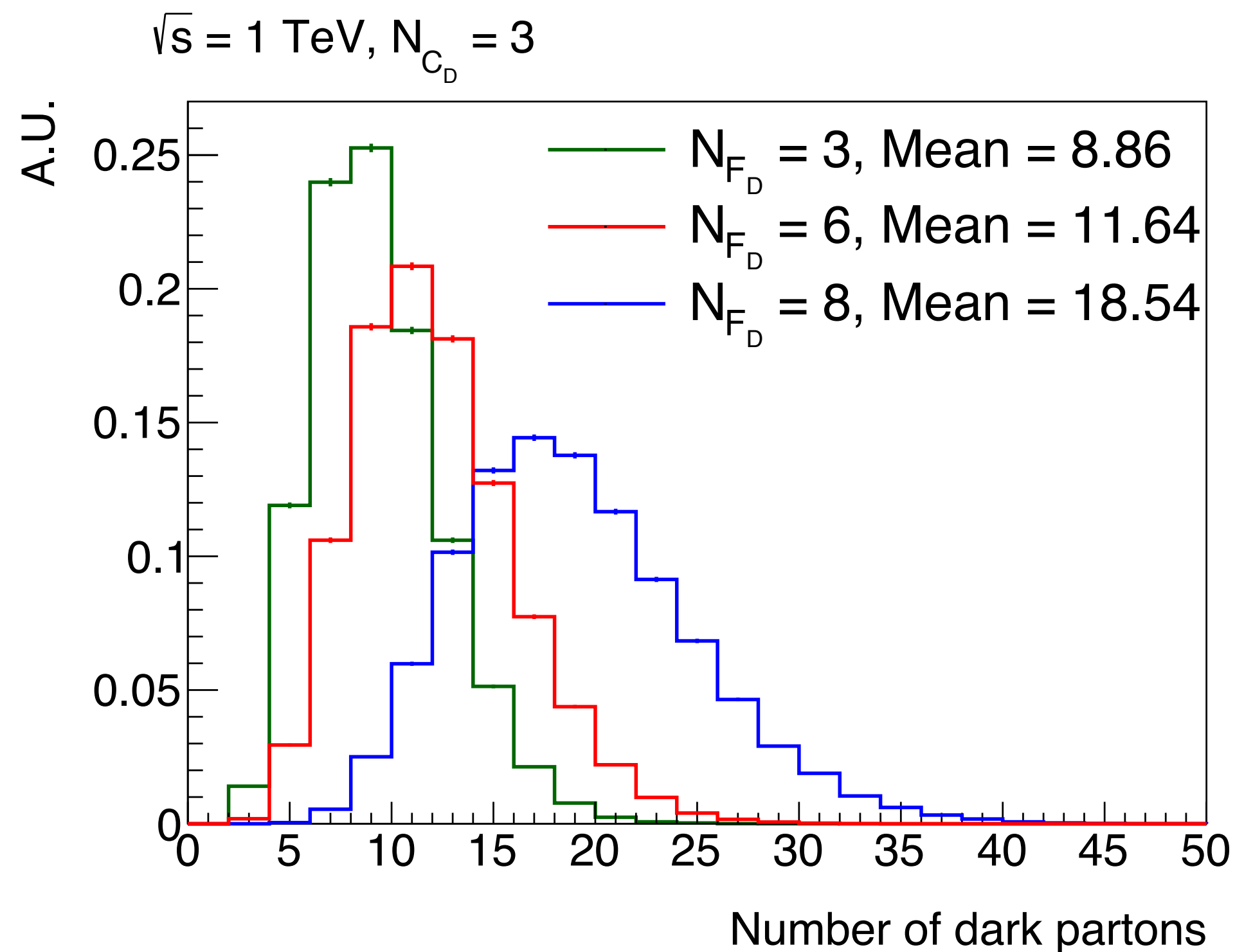
$$P_{q_D \rightarrow q_D G_D} = T_R C_F \frac{1 + \xi^2}{1 - \xi}.$$

G. Altarelli and G. Parisi, Nucl. Phys. B ('77)



- Simulated within a custom version of Pythia 8.307 using a combination of approximation and interpolation. We treat this implementation as a toy-model of near-conformal dark sectors.
- As such, we neglect the $P_{G_D \rightarrow q_D \bar{q}_D}$ branching, as is standard within the Hidden Valley module of Pythia. Additionally, we neglect any implementation of a CMW scheme change. S. Catani, B. R. Webber, G. Marchesini, Nucl. Phys. B 349 ('91)
- Hence the results presented next should be taken as describing qualitative behaviour of near-conformal dark parton showers and not quantitative.

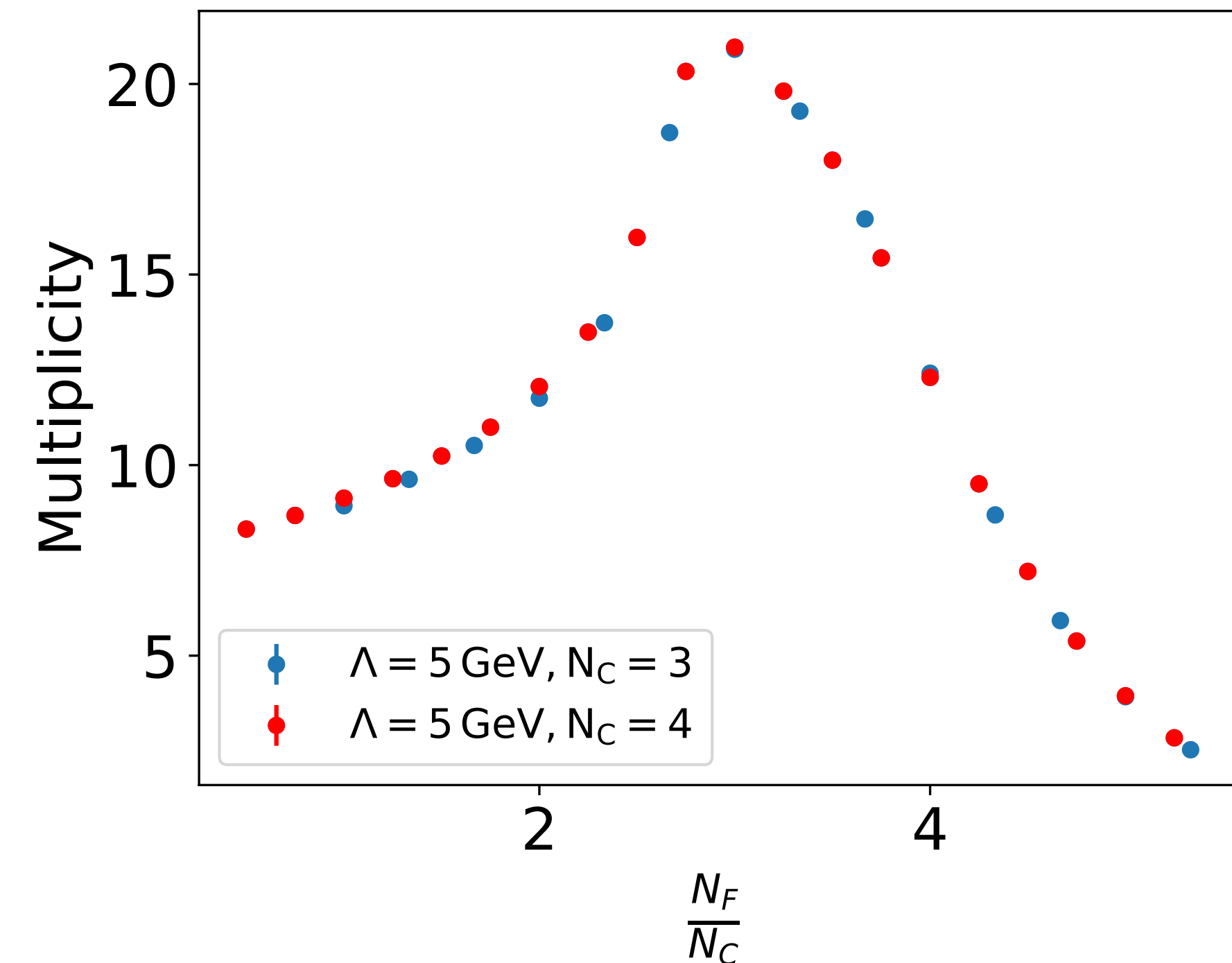
Simulation of dark parton showers



Simulated with a custom Pythia 8.307 with benchmark:
 $e^+e^- \rightarrow Z' \rightarrow q_D \bar{q}_D$,
 $\sqrt{s} = 1.1M_{Z'} = 1.1 \text{ TeV}$,
 hadronisation off,
 $\Lambda = 5 \text{ GeV}, N_C = 3$.
 Cutoff at $Q = 1.1\Lambda$.

- Within the QCD-like region, dark parton multiplicity increases with N_F/N_C . R. K. Ellis, W. J. Stirling and B. R. Webber, QCD and Collider Physics
- Theories with large IRFPs ($\alpha_* \gg 1$) (around $N_F/N_C \sim 3$) have similar dark parton showering behaviour to those without IRFPs, having an overall large multiplicity of soft dark partons. Theories with small IRFPs ($\alpha_* \ll 1$) (around $N_F/N_C \sim 5$) have both hard and soft dark partons; having an overall small multiplicity of hard dark partons.

Average dark parton multiplicity



Simulated with a custom Pythia 8.307 with benchmark:
 $e^+e^- \rightarrow Z' \rightarrow q_D \bar{q}_D$, $\sqrt{s} = 1.1M_{Z'} = 1.1 \text{ TeV}$,
 hadronisation off, $\Lambda = 5 \text{ GeV}$, $N_C = 3$. Cutoff at
 $Q = 1.1\Lambda$.

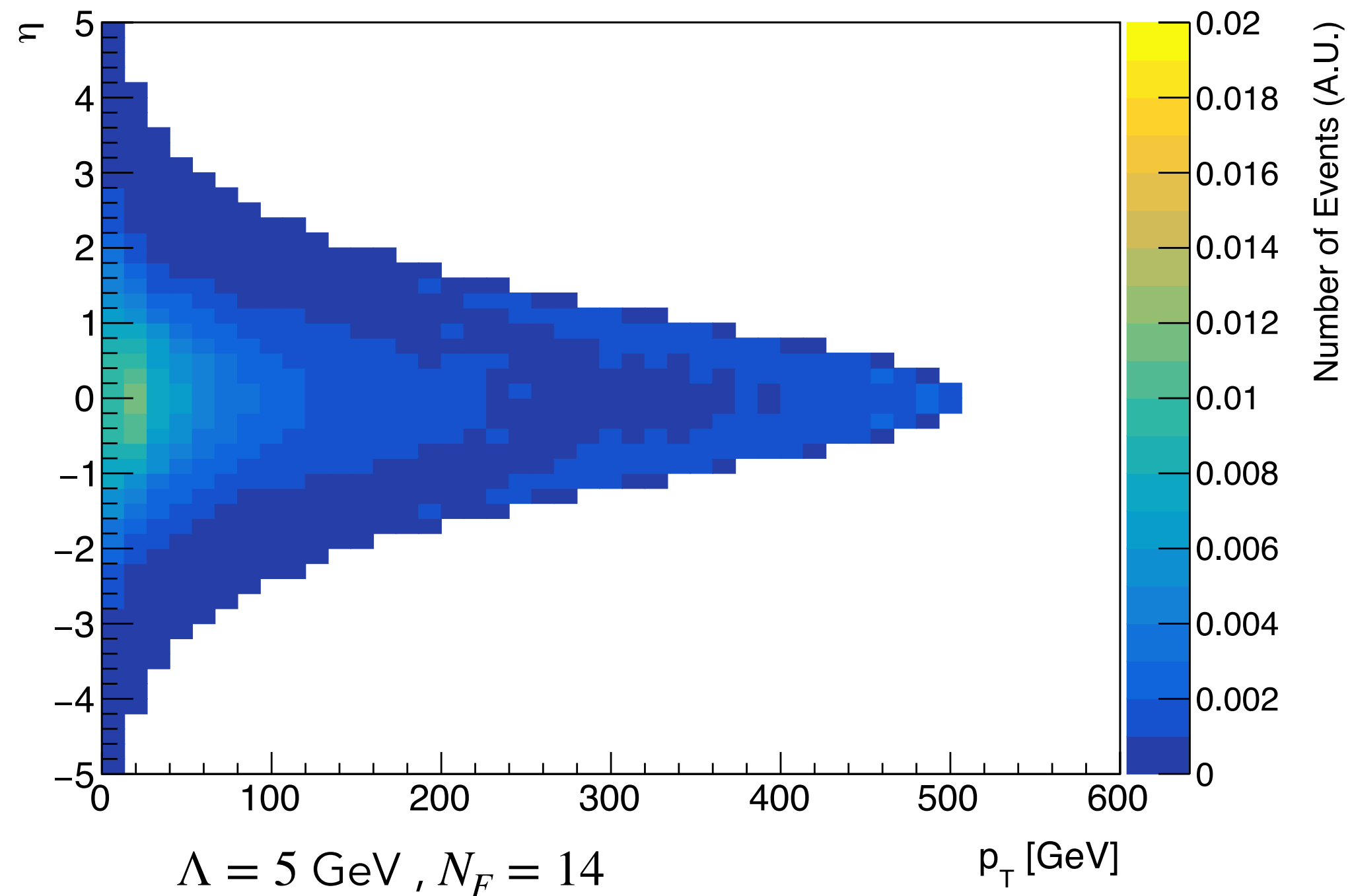
- Since parton splitting probability is proportional to α , it thus vanishes as $N_F/N_C \rightarrow 5.5$. Hence there is very little splitting at $N_F/N_C \sim 5$ and average parton multiplicity tends to 2 - the 2 initial dark quarks.

$$d\mathcal{P}_a(\xi, Q^2) = \frac{dQ^2}{Q^2} \frac{\alpha(Q^2)}{2\pi} \sum_{b,c} P_{a \rightarrow bc}(\xi) d\xi$$

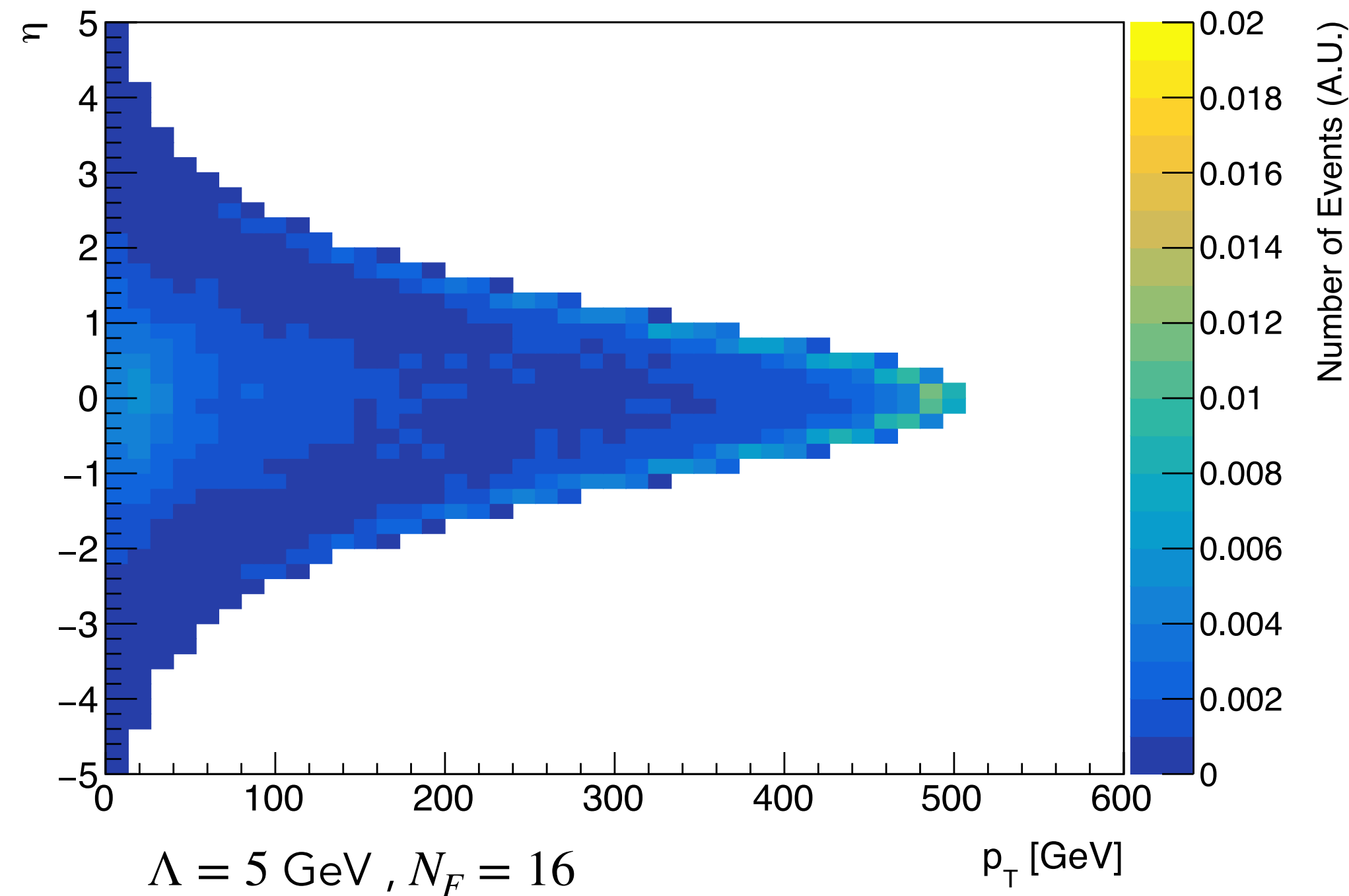
- Maximum of the dark parton multiplicity distribution occurs at $N_F/N_C = 3$ and not $N_F/N_C = (N_F/N_C)^*$.

Simulation of dark parton showers

Showered dark parton p_T and η distribution



Showered dark parton p_T and η distribution



Simulated with a custom Pythia 8.307 with benchmark:
 $e^+e^- \rightarrow Z' \rightarrow q_D \bar{q}_D$,
 $\sqrt{s} = 1.1 M_{Z'} = 1.1 \text{ TeV}$,
 hadronisation off,
 $\Lambda = 5 \text{ GeV}, N_C = 3$.
 Cutoff at $Q = 1.1\Lambda$.

- At around $N_F/N_C \sim 5$, there is a transition in the $p_T - \eta$ plane from the majority of dark partons being soft to a majority being hard, reflecting how branching probability is negligible and the majority of dark partons are initial dark quarks.
- For every dark parton splitting, the two resulting dark partons share the transverse momentum p_T meaning the more splittings, the softer the final state dark partons. In the IRFP region, the average $\eta \rightarrow 0$ as $N_F/N_C \rightarrow 5.5$, more events are back-to-back with respect to the beam line.

Conclusion

- The current approximation used within event generators is insufficient to describe two-loop α for high N_F/N_C since it neglects effects of the IRFP. By taking this IRFP into account, we establish a framework of two solutions to the RGE that allow for dark parton showering to be simulated across a wide range of parameter space.
- The first simulations of near-conformal dark parton showers suggest a wide variety of phenomenology within the IRFP region. On average, theories with large IRFPs ($\alpha_* \gg 1$) (around $N_F/N_C \sim 3$) have an overall large multiplicity of soft dark partons. Whilst theories with small IRFPs ($\alpha_* \ll 1$) (around $N_F/N_C \sim 5$) have an overall small multiplicity of hard dark partons.
- This new procedure allows for the simulation of the anomalous jets signatures of near-conformal Hidden Valley theories. Motivates further investigations into the hadronisation and subsequent decay of near-conformal bound states, as this could have an additionally large influence on the dark shower phenomenology.



Thank you!
Questions?