# Electroweak Sudakov Logarithms

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Graz Austria





#### The issue in gauge theories



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- Resummation yields (double) Sudakov logarithms:  $\ln^2 \frac{s}{\Lambda}$
- Can dominate the cross sections
- Quick rise can obstruct convergence



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  - Cancellations between real and virtual corrections
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- Initial state: Bloch-Nordsieck theorem/Initial state and final state: Kinoshita-Lee-Naunberg theorem
  - Required: Inclusive in the gauge charge

[Ciafaloni et al.'00,'22 Bauer et al.'18]



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- At LC@TeV: Same order as strong interactions
  - Swamped by jets of vector bosons and Higgs

• No!

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  - Actually, a coincidence of the standard model
  - Subleading effects save the day

#### A toy model

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- Parameters selected for a BEH effect

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- Global SU(2) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only  $W^a_{\mu} \rightarrow W^a_{\mu}$   $h \rightarrow h \Omega$

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- Perform perturbation theory
Perturbation theory



 $\bigcirc$ 

Perturbation theory Scalar fixed charge

• Custodial singlet

Mass



Both custodial singlets

 $\bigcirc$ 

[Fröhlich et al.'80, Banks et al.'79]

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- Has nothing to do with weak coupling
  - Think QED (hydrogen atom!)



Remember: Experiment tells that somehow the left is correct!



Experiment tells that somehow the left is correct Theory say the right is correct



Experiment tells that somehow the left is correct Theory say the right is correct There must exist a relation that both are correct

[Fröhlich et al.'80,'81, Maas & Törek'16,'18, Maas, Sondenheimer & Törek'17]

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    - Standard lattice spectroscopy problem
    - Standard methods
      - Smearing, variational analysis, systematic error analysis etc.
    - Very large statistics (>10<sup>5</sup> configurations)



Gauge-invariant

Scalar singlet

Both custodial singlets

$$h(x) + h(x)$$





Both custodial singlets

 $\square$ 



**Custodial singlet** 



#### • Both custodial singlets Custodial singlet



Both custodial singlets

Custodial singlet

$$tr t^{a} \frac{h^{+}}{\sqrt{h^{+} h}} D_{\mu} \frac{h}{\sqrt{h^{+} h}}$$





Both custodial singlets

#### Custodial singlet

Triplet

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Why?

# How to make predictions

[Fröhlich et al.'80,'81, Maas & Törek'16,'18, Maas, Sondenheimer & Törek'17 Maas & Sondenheimer '20]

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## How to make predictions

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  - But coupling is still weak and there is a BEH
  - Perform double expansion [Fröhlich et al.'80, Maas'12]
    - Vacuum expectation value (FMS mechanism)
    - Standard expansion in couplings
    - Together: Augmented perturbation theory

[Fröhlich et al.'80,'81 Maas'12,'17]

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Higgs field

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Trivial two-particle state

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[Fröhlich et al.'80,'81
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[Maas'12,'17 Maas & Sondenheimer'20 Dudal et al.'20]



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Additional thresholds

[Maas'12,'17 Maas & Sondenheimer'20 Dudal et al.'20]



Gauge-dependent Unphysical features: Positivity violation Additional thresholds

Not a consequence of instability: Occurs even for an asymptotically stable Higgs in a toy theory





[Maas'12,'17 Maas & Sondenheimer'20 Dudal et al.'20 Maas et al. unpublished]

Same structure repeats itself in decays and scattering processes

[Maas'12,'17 Maas & Sondenheimer'20 Dudal et al.'20 Maas et al. unpublished]

Same structure repeats itself in decays and scattering processes LO: Standard perturbation theory



[Maas'12,'17 Maas & Sondenheimer'20 Dudal et al.'20 Maas et al. unpublished]

Same structure repeats itself in decays and scattering processes LO, NLO: Standard perturbation theory



 $\begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} W^{\pm}, Z \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ h \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{0} \\ \hline \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \hline \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm}, \varphi^{0} \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm} \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{\pm} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{0} \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{0} \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{0} \\ \end{array} \xrightarrow{} \begin{array}{c} h, \varphi^{0} \end{array} \xrightarrow{} \begin{array}{c$ 

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Augmented perturbation theory only augments Feynman rules

[Fröhlich et al.'80,'81 Maas'12]

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*c* projects custodial states to gauge states

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*c* projects custodial states to gauge states

Exactly one gauge boson for every physical state

# **Physical states**

- No "real" breaking
- Physical particles are gauge-invariant particles
  - Cannot be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



- Has nothing to do with weak coupling
  - Think QED (hydrogen atom!)

[Fröhlich et al.'80, Egger, Maas, Sondenheimer'17]

- Flavor has two components
  - Global SU(3) generation
  - Local SU(2) weak gauge (up/down distinction)

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  - Different masses for doublet members

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- Yukawa terms break custodial symmetry
  - Different masses for doublet members
- Extends non-trivially to hadrons

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  - Compressed mass scales
  - One generation
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  - Dirac fermions: left/righthanded non-degenerate
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- Supports FMS prediction grant for unquenching '24-'28



[Maas et al.'22]



• Particles are again (electroweak) gauge-singlets

[Maas et al.'22]



- Particles are again (electroweak) gauge-singlets
- Low energy: FMS expansion

 $\langle hehe | h\mu h\mu \rangle = \langle ee | \mu\mu \rangle + \langle \eta\eta \rangle \langle ee | \mu\mu \rangle + \langle ee \rangle \langle \eta\eta | \mu\mu \rangle + \dots$ Standard Irrelevant at low energies result



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  - Restores BN theorem and KLN theorem: No Sudakov
- Interesting consequences for PDFs/FFs

## Summary

• Full gauge invariance also for weak interactions

Review: 1712.04721 Update: 2305.01960

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Full gauge invariance also for weak interactions

- Relevant for electroweak resummation at high energies (→ FCC/FLC)
  - Comparable to strong corrections

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# Summary

Full gauge invariance also for weak interactions

- Relevant for electroweak resummation at high energies (→ FCC/FLC)
  - Comparable to strong corrections
- Effect suppressed at low energies because of standard model structure
  - Different in BSM physics Review: 1712.04721 Update: 2305.01960