# Electroweak Sudakov Logarithms 

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NAWI Graz
Natural Sciences
Österreichischer Wissenschaftsfonds


# The issue in gauge theories 



Incoming or outgoing gauged particle

# The issue in gauge theories 

Can emit soft


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Can emit soft


- Resummation yields (double) Sudakov logarithms: $\ln ^{2} \frac{S}{\Lambda}$
- Can dominate the cross sections
- Quick rise can obstruct convergence


# Resolution in gauge theories 



- Involved particle is a gauge singlet


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Incoming or outgoing gauge singlet

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- Cancellations between real and virtual corrections
- Cancel Sudakov logarithms


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- Cancellations between real and virtual corrections
- Cancel Sudakov logarithms
- Initial state: Bloch-Nordsieck theorem/Initial state and final state: Kinoshita-Lee-Naunberg theorem
- Required: Inclusive in the gauge charge


## Invalidation in electroweak physics

[Ciafaloni et al.'00,'22 Bauer et al.'18]


- Brout-Englert-Higgs effect "breaks" gauge symmetry


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- Double Sudakov logarithms suppressed: $\ln ^{2} \frac{s}{m_{W}^{2}}$
- Negligble at small energies


## Invalidation in electroweak physics



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- States like leptons become asymptotic states
- BN and KLN theorems violated
- Double Sudakov logarithms suppressed: $\ln ^{2} \frac{s}{m_{W}^{2}}$
- Negligble at small energies
- At LC@TeV: Same order as strong interactions
- Swamped by jets of vector bosons and Higgs


## Is this inevitable?

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- No!


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- Just a figure of speech
- Actually just ordinary gauge-fixing


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- Actually just ordinary gauge-fixing
- But it is well established?
- Actually, a coincidence of the standard model
- Subleading effects save the day

A toy model

A toy model: Higgs sector of the SM

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- Consider an SU(2) with a fundamental scalar


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- Consider an $\operatorname{SU}(2)$ with a fundamental scalar
- Essentially the standard model Higgs

$$
\begin{gathered}
L=-\frac{1}{4} W_{\mu \nu}^{a} W_{a}^{\mu \nu} \\
W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{\nu} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c}
\end{gathered}
$$

- Ws $W_{\mu}^{a} \mathbb{W}$
- Coupling $g$ and some numbers $f^{a b c}$


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W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{\nu} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c} \\
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- Ws $W_{\mu}^{a}$ W
- Higgs $h_{i}$ h
- Couplings $g, v, \lambda$ and some numbers $f^{a b c}$ and $t_{a}^{i j}$
- Parameters selected for a BEH effect


## A toy model: Symmetries

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- Local $\operatorname{SU}(2)$ gauge symmetry $W_{\mu}^{a} \rightarrow W_{\mu}^{a}+\left(\delta_{b}^{a} \partial_{\mu}-g f_{b c}^{a} W_{\mu}^{c}\right) \phi^{b}$ $h_{i} \rightarrow h_{i}+g t_{a}^{i j} \phi^{a} h_{j}$


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- Local SU(2) gauge symmetry
$W_{\mu}^{a} \rightarrow W_{\mu}^{a}+\left(\delta_{b}^{a} \partial_{\mu}-g f_{b c}^{a} W_{\mu}^{c}\right) \phi^{b}$ $h_{i} \rightarrow h_{i}+g t_{a}^{i j} \phi^{a} h_{j}$
- Global SU(2) custodial (flavor) symmetry
- Acts as (right-)transformation on the scalar field only $W_{\mu}^{a} \rightarrow W_{\mu}^{a}$ $h \rightarrow h \Omega$


## Textbook approach

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- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action
- Choose a suitable gauge and obtain 'spontaneous gauge symmetry breaking': SU(2) $\rightarrow 1$
- Get masses and degeneracies at treelevel
- Perform perturbation theory


## Physical spectrum

Perturbation theory
$0 \quad$ Mass

# Physical spectrum 

Perturbation theory
Scalar
$\backsim \Delta$ fixed charge

Custodial singlet

# Physical spectrum 

Perturbation theory

## Scalar Vector

$\backsim \wedge$ fixed charge gauge triplet

- Both custodial singlets


## Physical states

- No "real" breaking


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- Cannot be the elementary particles
- Non-Abelian nature is relevant


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## Physical states

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- Physical particles are gauge-invariant particles
- Cannot be the elementary particles
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- Need more than one particle: Composite particles
- Higgs-Higgs, W-W, Higgs-Higgs-W etc.

- Has nothing to do with weak coupling
- Think QED (hydrogen atom!)


## Physical spectrum

Perturbation theory

## Scalar Vector

$\backsim$ 』 fixed charge gauge triplet

Both custodial singlets

Remember: Experiment tells that somehow the left is correct!

Physical spectrum
Perturbation theory
Composite (bound) states
n ${ }^{\wedge}$ fixed charge gauge triplet


Experiment tells that somehow the left is correct Theory say the right is correct

Physical spectrum
Perturbation theory
Composite (bound) states
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| Scalar | Vector |
| :---: | :---: |
| $\sim \Delta$ fixed charge | gauge triplet |



Experiment tells that somehow the left is correct Theory say the right is correct There must exist a relation that both are correct

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- JPC and custodial charge only quantum numbers


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- Bound state structure - non-perturbative methods! - Lattice
- Standard lattice spectroscopy problem
- Standard methods
- Smearing, variational analysis, systematic error analysis etc.
- Very large statistics ( $>10^{5}$ configurations)


# Physical spectrum 

Perturbation theory
Scalar Vector
n ${ }^{\wedge}$ fixed charge gauge triplet
Mass

- Both custodial singlets

$$
h(x)^{+} h(x) \quad \text { h }
$$

# Physical spectrum 

Perturbation theory
Scalar Vector
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## Gauge-invariant

 Scalar singlet- Both custodial singlets Custodial singlet

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Perturbation theory
Scalar Vector
n $\sqrt{\text { dixed charge gauge triplet }}$

Gauge-invariant
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singlet

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\operatorname{trt}^{a} \frac{h^{+}}{\sqrt{h^{+} h}} D_{\mu} \frac{h}{\sqrt{h^{+} h}}
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Scalar Vector
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# Physical spectrum 

Perturbation theory
Scalar Vector
n $\sqrt{\text { d }}$ fixed charge gauge triplet

Gauge-invariant
Scalar singlet

Equal!

Custodial singlet Triplet
Vector
singlet

Both custodial singlets

# Physical spectrum 

Perturbation theory
Scalar Vector
$n$
$\sum^{n}$
^ fixed charge gauge triplet

- Equal!

Equal!

- Both custodial singlets Custodial singlet Triplet


## Why?

## How to make predictions

- JPC and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
- Bound state structure - non-perturbative methods?


## How to make predictions

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- JPC and custodial charge only quantum numbers
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- Formulate gauge-invariant, composite operators
- Bound state structure - non-perturbative methods?
- But coupling is still weak and there is a BEH
- Perform double expansion ${ }_{\text {FFroblich etal: } 80, \text { Mas }{ }^{122]}}$
- Vacuum expectation value (FMS mechanism)
- Standard expansion in couplings
- Together: Augmented perturbation theory


## Augmented perturbation theory

1) Formulate gauge-invariant operator

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Higgs field

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## (h) $n$

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[Fröhlich et al.'80,'81
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4) Compare poles on both sides

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\end{gathered}
$$

3) Standard perturbation theory Bound state mass

$$
\frac{\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle}{\left.\frac{\gamma \eta}{}(x) \eta(y)\right\rangle\left\langle\eta^{+}(x) \eta(y)\right\rangle+O(g, \lambda)}
$$

Trivial two-particle state
4) Compare poles on both sides

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Higgs mass
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Perturbation Theory
3) Standard perturbation theory Bound state mass

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Mrohlich et al.'80,'81
Maas \& Sond

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What about this?
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## Consequences: The Higgs

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
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## Consequences: The Higgs




Physical thresholds

## Consequences: The Higgs




Gauge-dependrent

# Consequences: The Higgs 



# Consequences: The Higgs 




Gauge-dependent Unphysical features: Positivity violation Additional thresholds

Not a consequence of instability: Occurs even for an asymptotically stable Higgs in a toy theory

# Consequences: The Higgs 



# Consequences: The Higgs 



## Consequences: The Higgs

Same structure repeats itself in decays and scattering processes

## Consequences: The Higgs



Same structure repeats itself in decays and scattering processes LO: Standard perturbation theory


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Same structure repeats itself in decays and scattering processes LO, NLO: Standard perturbation theory


Augmented perturbation theory only augments Feynman rules

## What about the vector?

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1) Formulate gauge-invariant operator 1- triplet: $\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle$

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\left\langle\left(\tau^{i} h^{+} D_{\sharp} h\right)(x)\left(\tau^{j} h^{+} D_{\sharp} h\right)(y)\right\rangle=v^{2} c_{i j}^{a b}\left\langle W_{\sharp}^{a}(x) W^{b}(y)^{u}\right\rangle+\ldots
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Matrix from group structure

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Matrix from group structure
c projects custodial states to gauge states

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c projects custodial states to gauge states

Exactly one gauge boson for every physical state

## Physical states

- No "real" breaking
- Physical particles are gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant
- Need more than one particle: Composite particles
- Higgs-Higgs, W-W, Higgs-Higgs-W etc.

- Has nothing to do with weak coupling
- Think QED (hydrogen atom!)
- Flavor has two components
- Global SU(3) generation
- Local SU(2) weak gauge (up/down distinction)
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$\left.\left.\left(\begin{array}{cc}h_{2} & -h_{1} \\ h_{1}^{*} & h_{2}^{*}\end{array}\right) \right\rvert\, \begin{array}{l}v_{L} \\ l_{L}\end{array}\right)_{i}(x)$


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$$
\left(\left|\begin{array}{cc}
h_{2} & -h_{1} \\
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\end{array}\right| \begin{array}{l}
v_{L} \\
l_{L}
\end{array} \|_{i}(x)+\left\{\left(\begin{array}{cc}
h_{2} & -h_{1} \\
h_{1}^{*} & h_{2}^{*}
\end{array}| | \begin{array}{l}
v_{L} \\
l_{L}
\end{array} \|\left._{j}\right|_{j}(y) \underset{v^{2}}{\approx}\left(\left.\begin{array}{l}
h=v+\eta \\
l_{L} \\
l_{L}
\end{array}\right|_{i}(x)+\left(\left.\begin{array}{l}
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l_{L}
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$$
\|\left(\begin{array} { c c } 
{ h _ { 2 } } & { - h _ { 1 } } \\
{ h _ { 1 } ^ { * } } & { h _ { 2 } ^ { * } }
\end{array} | | \begin{array} { l } 
{ v _ { L } } \\
{ l _ { L } }
\end{array} \| _ { i } ( x ) + \| \left(\begin{array}{cc}
h_{2} & -h_{1} \\
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- Different masses for doublet members
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- Gauge-invariant state, but custodial doublet
- Yukawa terms break custodial symmetry
- Different masses for doublet members
- Extends non-trivially to hadrons


## Flavor on the lattice

- Only mock-up standard model
- Compressed mass scales
- One generation
- Degenerate leptons and neutrinos
- Dirac fermions: left/righthanded non-degenerate
- Quenched


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Spectrum: Lattice and predictions


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Spectrum: Lattice and predictions


- Supports FMS prediction - grant for unquenching '24-'28


## Invalidation in electroweak physics



- Particles are again (electroweak) gauge-singlets


## Invalidation in electroweak physics



- Particles are again (electroweak) gauge-singlets
- Low energy: FMS expansion
$\langle h e h e \mid h \mu h \mu\rangle=\langle e e \mid \mu \mu\rangle+\langle\eta \eta\rangle\langle e e \mid \mu \mu\rangle+\langle e e\rangle\langle\eta \eta \mid \mu \mu\rangle+\ldots$ Standard Irrelevant at low energies


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$$
\sigma_{\bar{\Psi}_{L}^{2} \Psi_{L}^{2} \rightarrow X}^{L O}=\sigma_{\bar{I}_{L} l_{L} \rightarrow X}^{L O}+\sigma_{\bar{I}_{L} v_{L} \rightarrow X}^{L O}+\sigma_{\bar{v}_{L} l_{L} \rightarrow X}^{L O}+\sigma_{\bar{v}_{L} v_{L} \rightarrow X}^{L O}+\text { higher order }
$$

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- Restores BN theorem and KLN theorem: No Sudakov


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- Restores BN theorem and KLN theorem: No Sudakov
- Interesting consequences for PDFs/FFs


## Summary

- Full gauge invariance also for weak interactions

Review: 1712.04721 Update: 2305.01960

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- Relevant for electroweak resummation at high energies ( $\rightarrow$ FCC/FLC)
- Comparable to strong corrections


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- Full gauge invariance also for weak interactions
- Relevant for electroweak resummation at high energies ( $\rightarrow$ FCC/FLC)
- Comparable to strong corrections
- Effect suppressed at low energies because of standard model structure
- Different in BSM physics

Review: 1712.04721 Update: 2305.01960

