Colour Evolution and Infrared Physics

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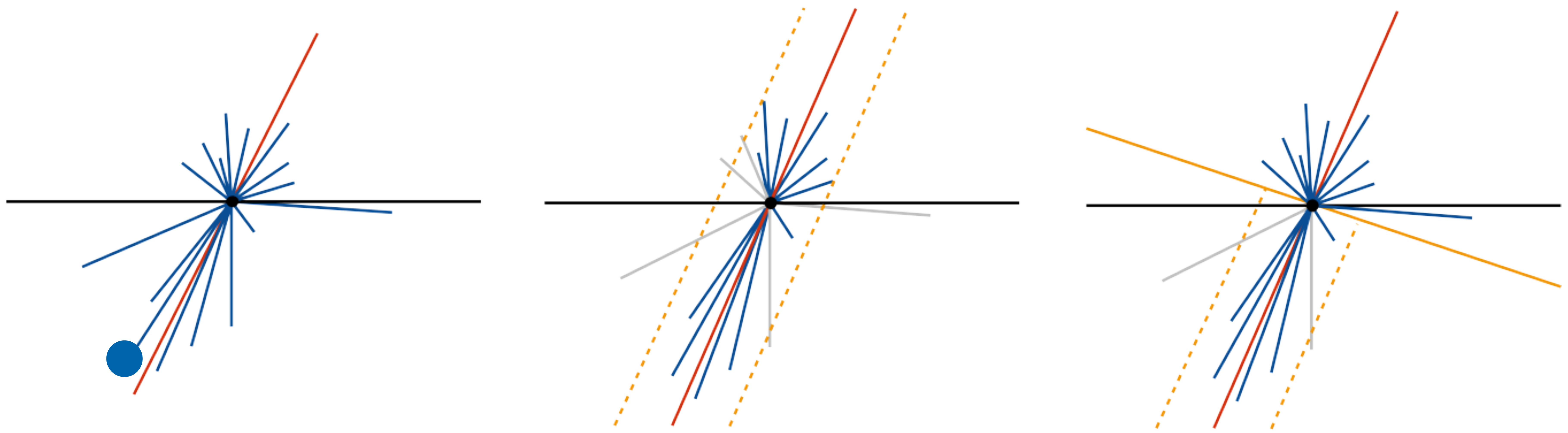
At the
Parton Showers and Resummation Workshop
Graz | 3 July 2024

https://particle.uni-graz.at/en/event-generators-and-resummation/

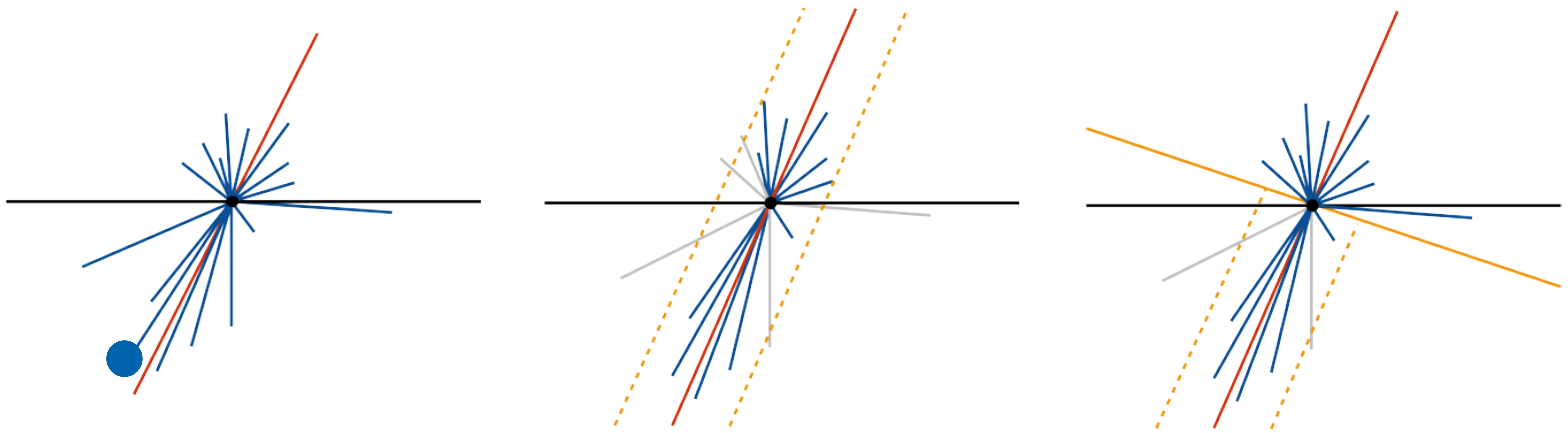
Accuracy of parton showers



Fragmentation is fine if we get collinear physics right.

Accuracy of Parton Showers

[Catani, Trentadue, Webber, Marchesini ….]



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Global event shapes from coherent branching — for two jets.

$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

LL — qualitative NLL — quantitative NNLL — precision

$$\alpha_s L \sim 1$$

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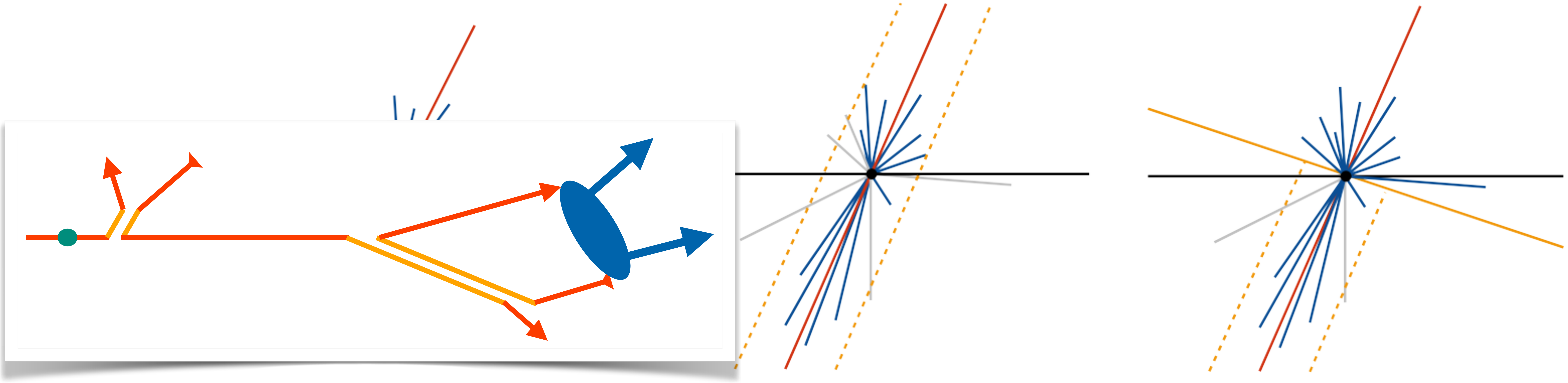

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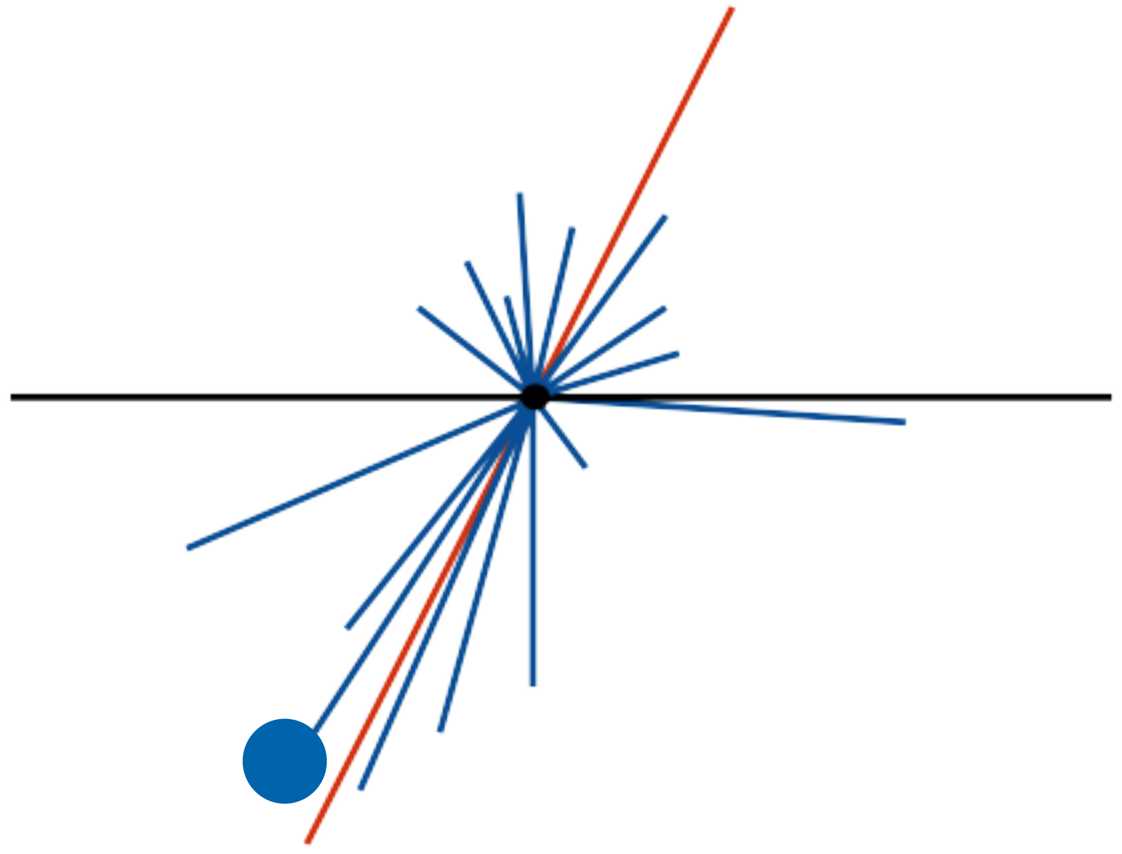

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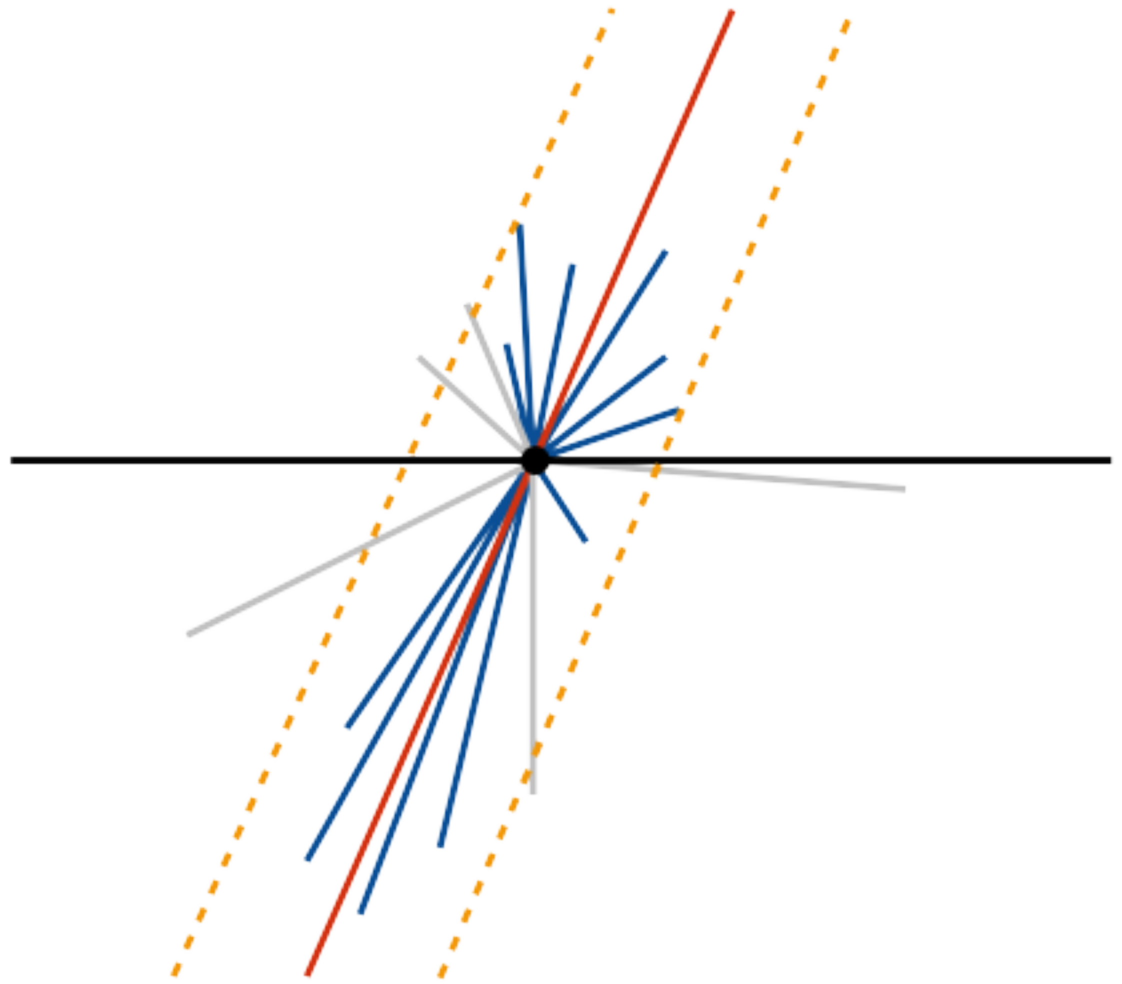

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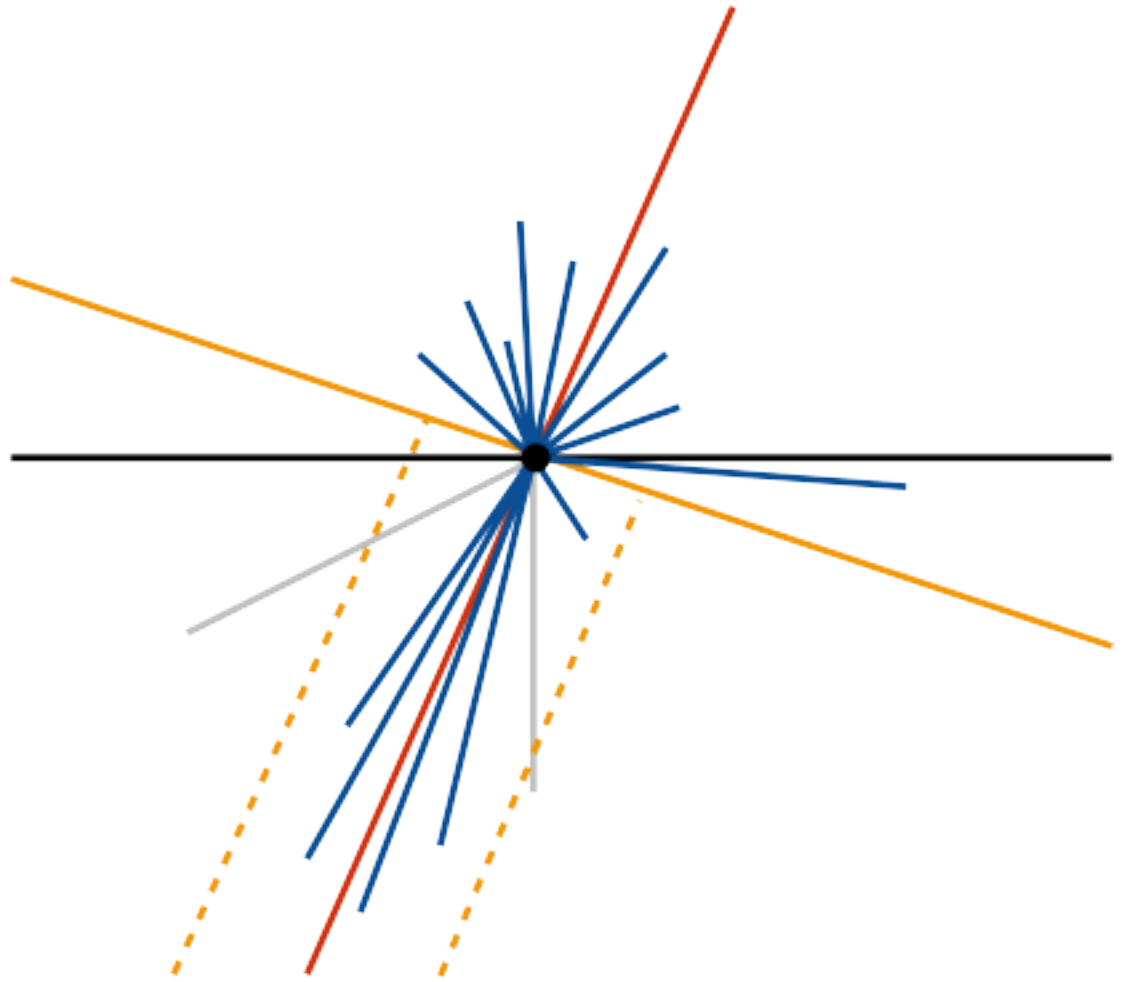

Coherence breaks down for non-global observables.

$$T_k T_l T_i \cdot T_j T_m T_n$$

large-N limit ↓

$$\frac{\partial G_{ab}(t)}{\partial t} = -\int_{\text{in}} \frac{\mathrm{d}\Omega_k}{4\pi} \omega_{ab}(k) G_{ab}(t) + \int_{\text{out}} \frac{\mathrm{d}\Omega_k}{4\pi} \omega_{ab}(k) \left[G_{ak}(t) G_{kb}(t) - G_{ab}(t) \right]$$

[Banfi, Marchesini, Smye ’02]

Full colour and interferences are central to go beyond

Colour reconnection and hadronization is about subleading-N.
So are shower accuracy and interference terms.

Colour factor algorithms

Coherent, NLL-accurate dipole showers

[Gustafson] [PanScales '21]
[Forshaw, Holguin, Plätzer '21]

Colour ME corrections

Colour-exact real emissions as far as possible

[Plätzer, Sjödahl '12, '18]
[Höche, Reichelt '20]

Full amplitude evolution

Colour-exact real and virtual corrections

[Forshaw, Plätzer + … '06, '13 …]
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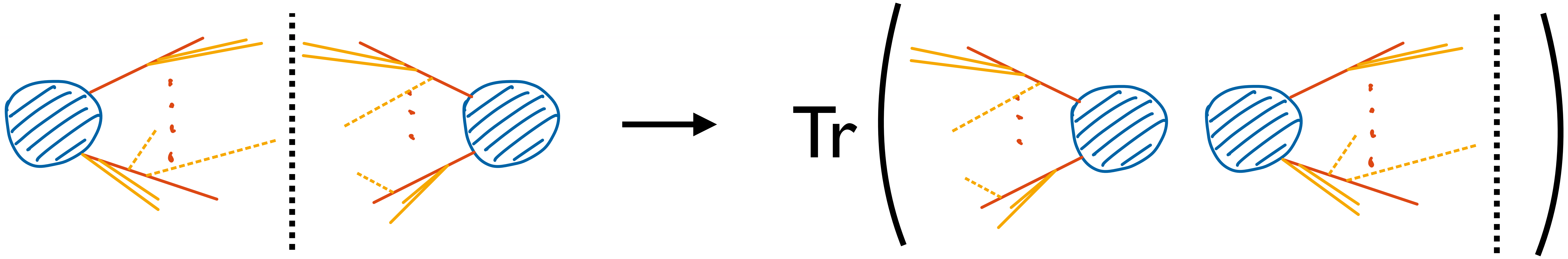
Amplitude evolution



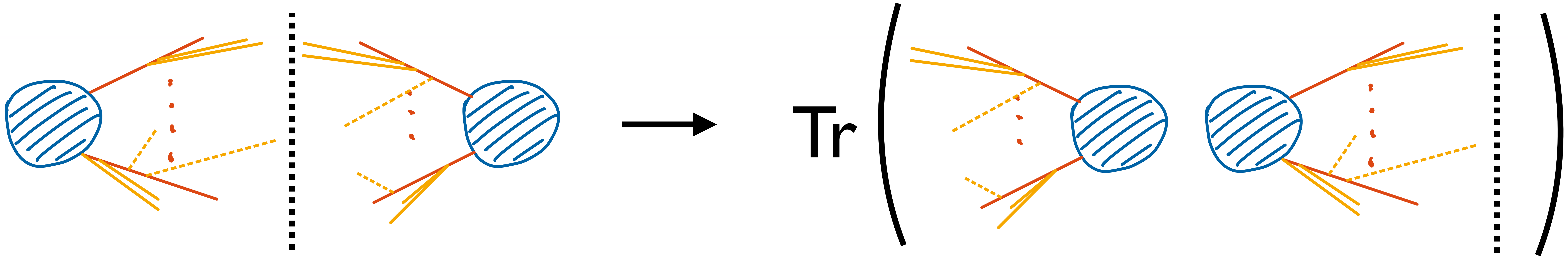
$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}(k')}\ \mathbf{D}_n(k)\ \mathbf{A}_{n-1}(k)\ \mathbf{D}_n^\dagger(k)\ \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level: Iterate gluon exchanges and emission.
Different histories in amplitude and conjugate amplitude needed to include interference.

CVolver solves evolution equations in colour flow space. Flexible for dedicated resummation and new parton showers.

One-loop structures … [Plätzer '13]
Soft evolution … [Angeles, De Angelis, Forshaw, Plätzer, Seymour – '18]
Soft + collinear evolution … [Forshaw, Holguin, Plätzer – '19]
Two-loop structures … [Plätzer, Ruffa — '21]
First Monte Carlo implementation … [De Angelis, Forshaw, Plätzer — '21]
Emissions beyond leading order … [Löschner, Plätzer, Simpson-Dore — '20]

Amplitude evolution



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Simplest case: Eikonal current

$$\mathbf{D}^{(1,0)} \circ \mathbf{D}^{(1,0)\dagger} = \frac{\alpha_S}{2\pi}\sum_{i,j}\omega_{ij}\; \mathbf{T}_i^a \circ \mathbf{T}_j^{\dagger b}$$

Simplest case: soft exchanges

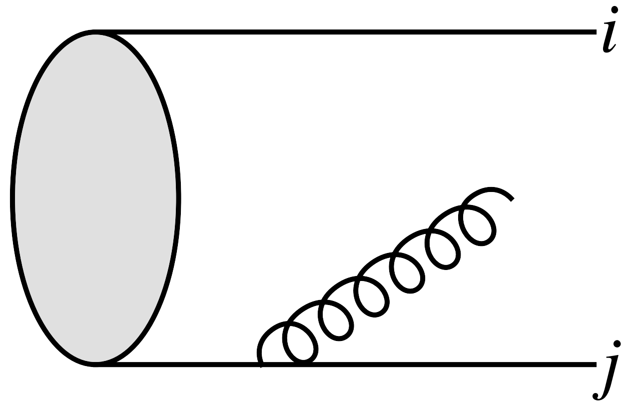
$$\boldsymbol{\Gamma}^{(1)} = \frac{\alpha_s}{2\pi}\sum_{i<j}\int \mathrm{d}\Omega\; \omega_{ij}\mathbf{T}_i\cdot\mathbf{T}_j$$

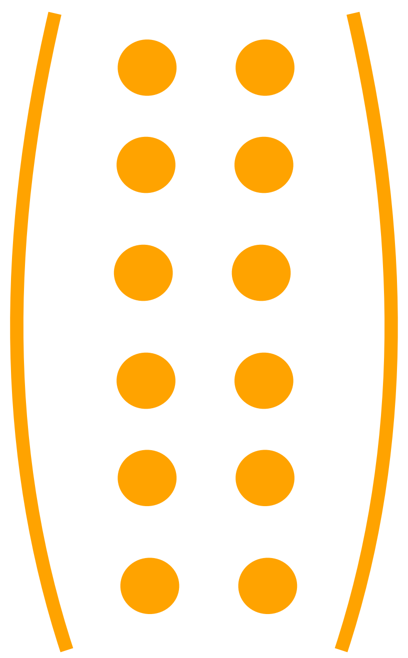
Colour flows

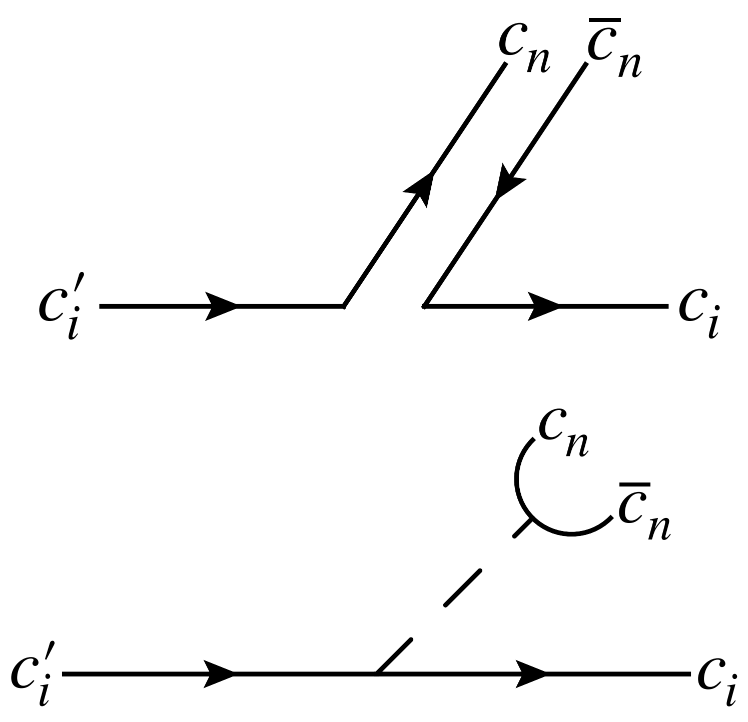
→ Torre's talk

Gluon emission

$\mathbf{D}_n(k)$



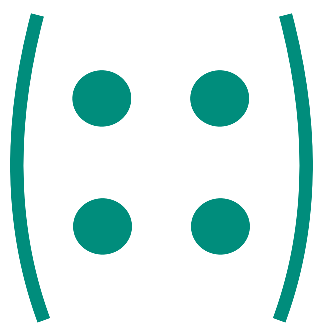


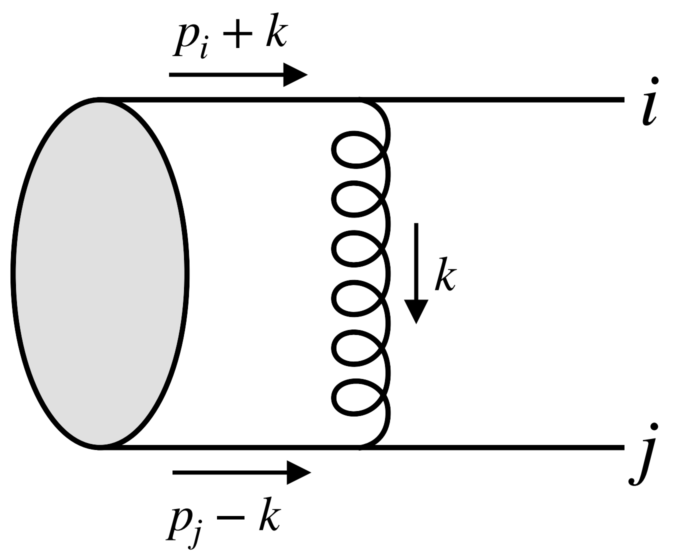


Explicit suppression in 1/N

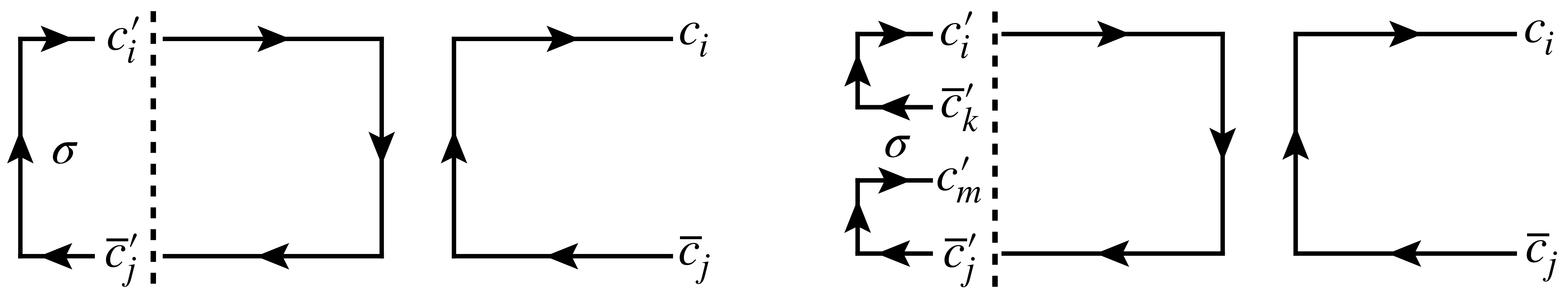
Gluon exchange

$$\mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$





$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$



$$[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma_\sigma^{(1)} + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma_{\sigma\tau}^{(1)}$$

dipole flips — implicit suppression in 1/N

Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

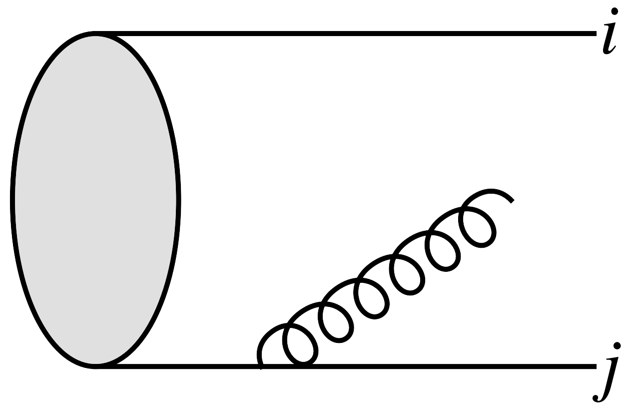
[Plätzer – '13] — diagrams from [Ruffa, MSc thesis 2020]

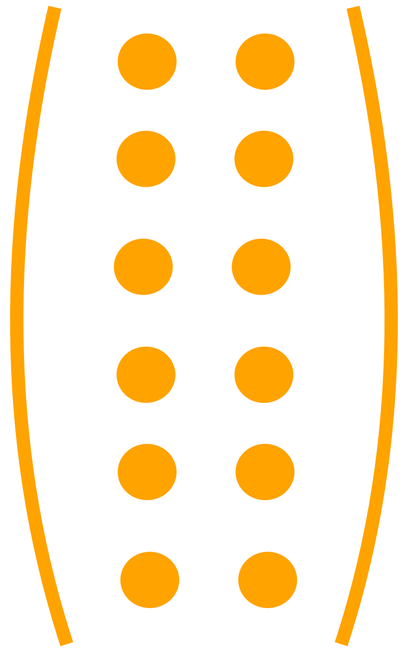
Colour flows

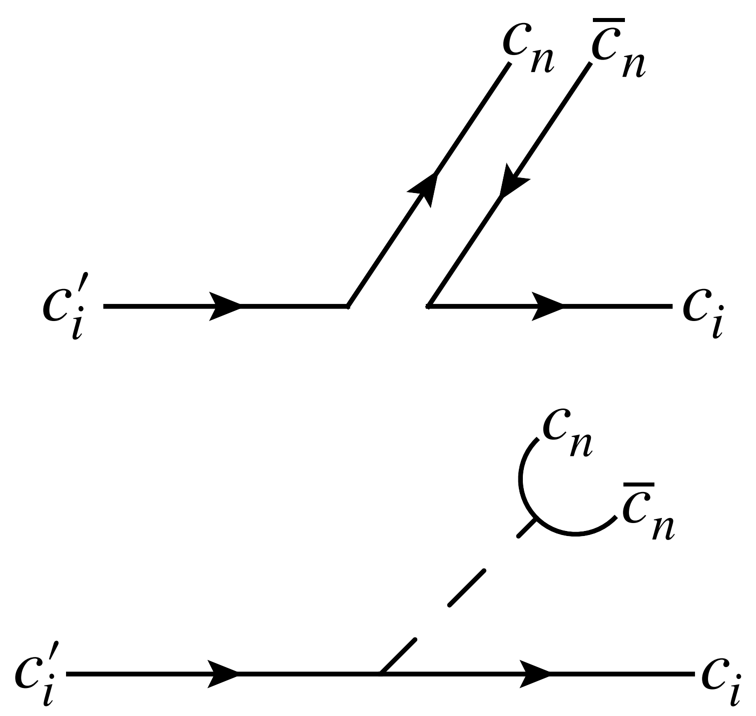
→ Torre's talk

Gluon emission

$$\mathbf{D}_n(k)$$







Explicit suppression in 1/N

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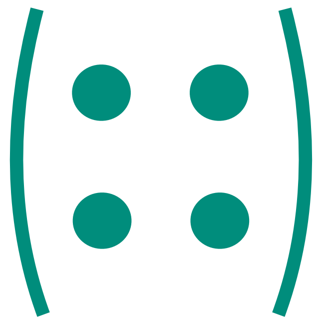
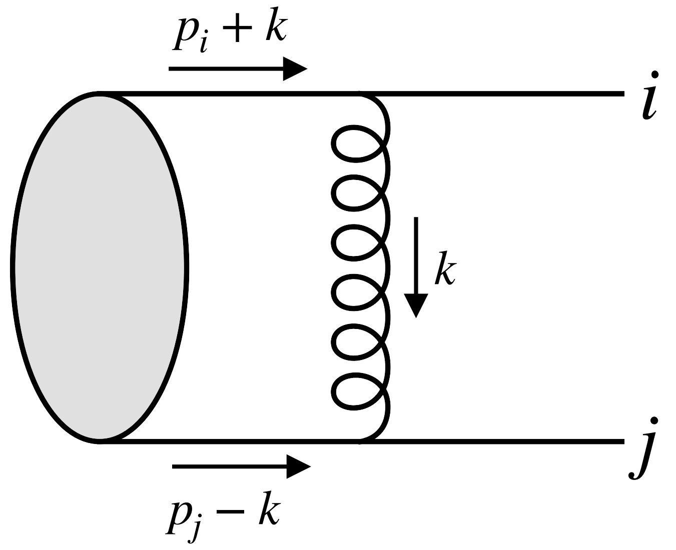
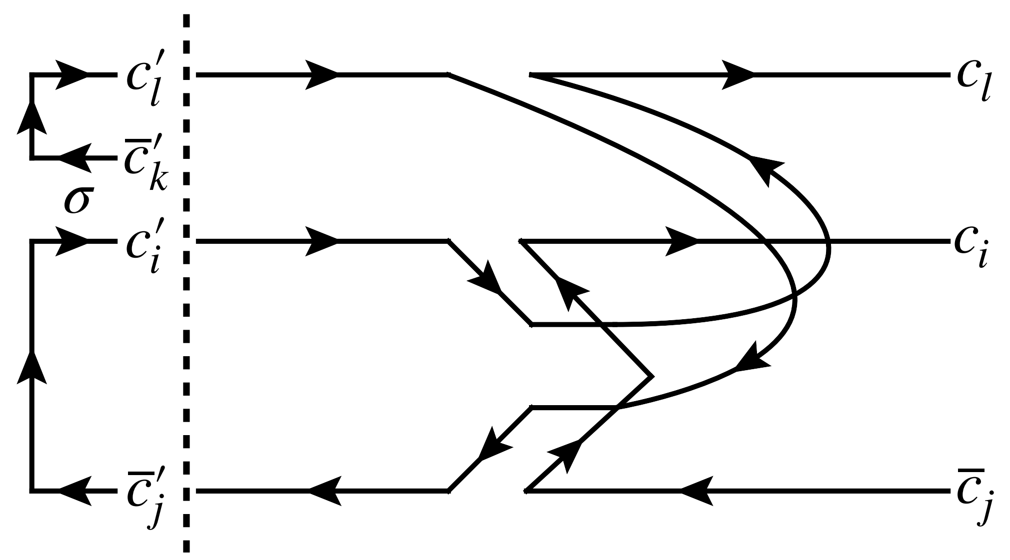
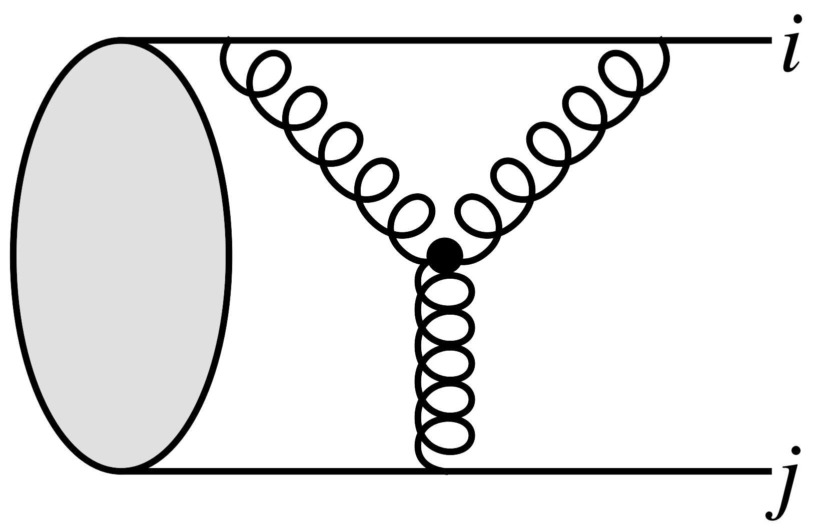
$$P e^{-\int_q^k \frac{dk'}{k'} \mathbf{\Gamma}(k')}$$





$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$





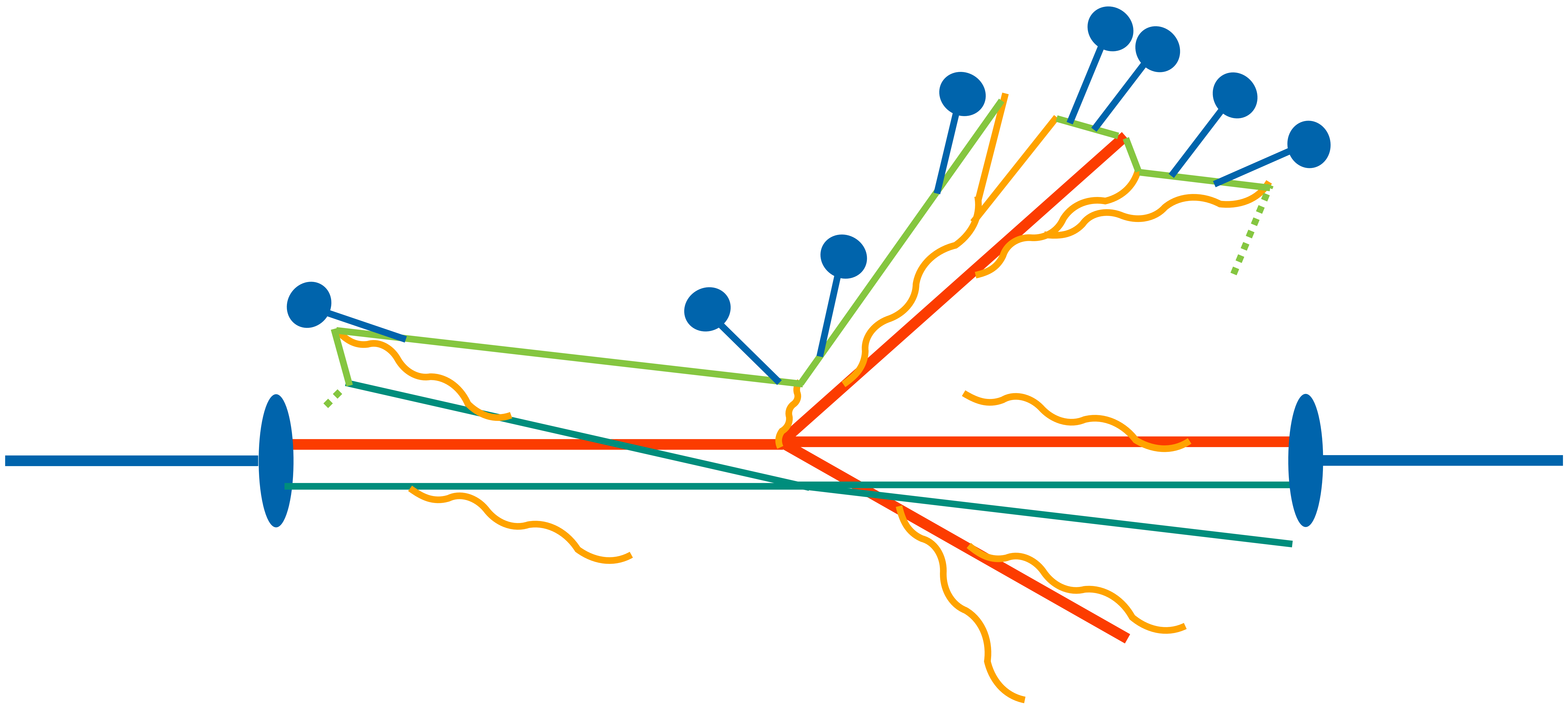
[Plätzer, Ruffa — '21]

dipole flips — implicit suppression in 1/N

Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

[Plätzer – '13] — diagrams from [Ruffa, MSc thesis 2020]

Systematic construction? Hadronization? Higher orders?



$$\mathrm{d}\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q\to\mu)\,\mathrm{d}\mathbf{H}(Q)\,\mathbf{PS}^\dagger(Q\to\mu)\,\mathbf{Had}(\mu\to\Lambda)\right]$$

Factorisation and evolution

[Plätzer – JHEP 07 (2023) 126]

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Factorisation and evolution

[Plätzer – JHEP 07 (2023) 126]

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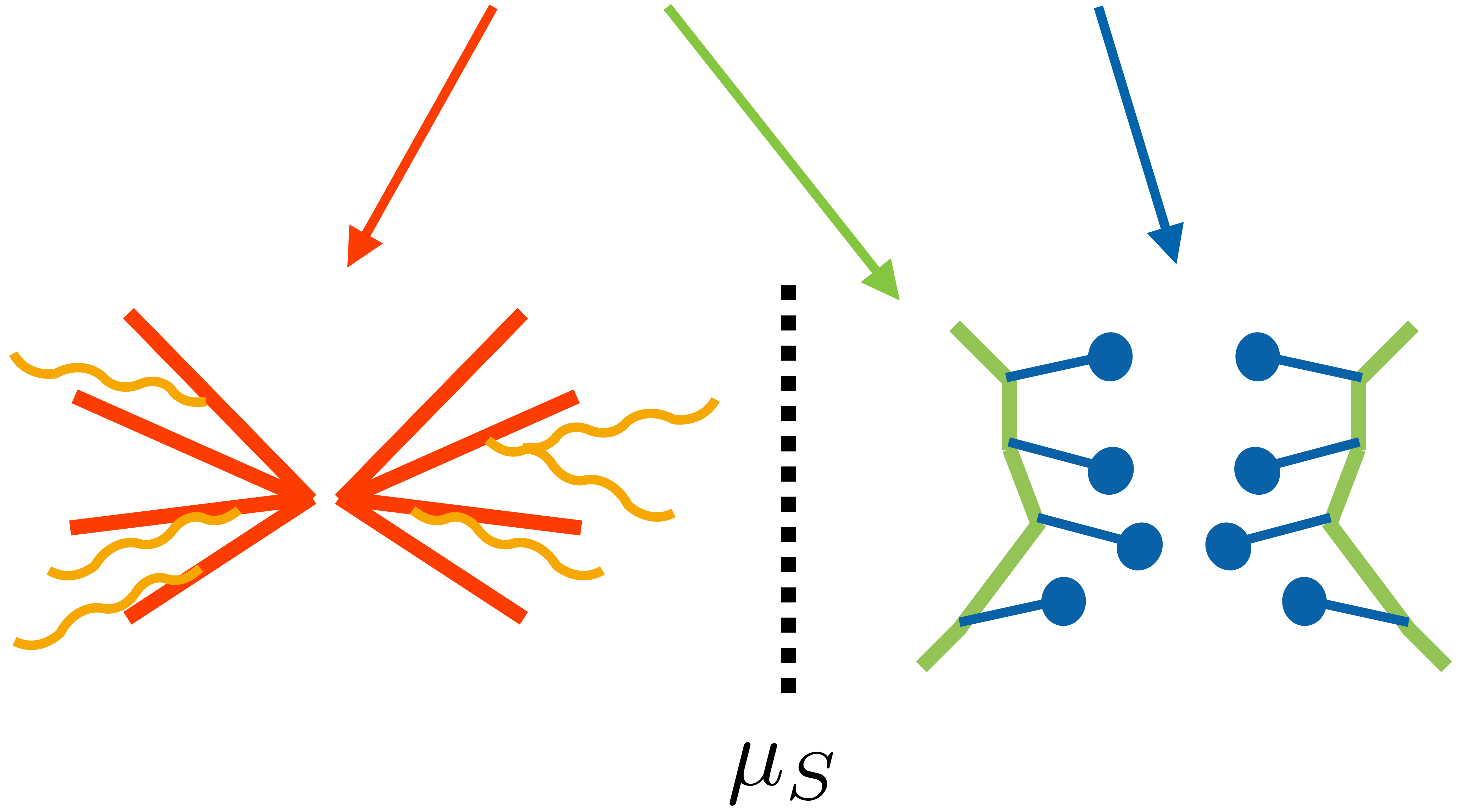

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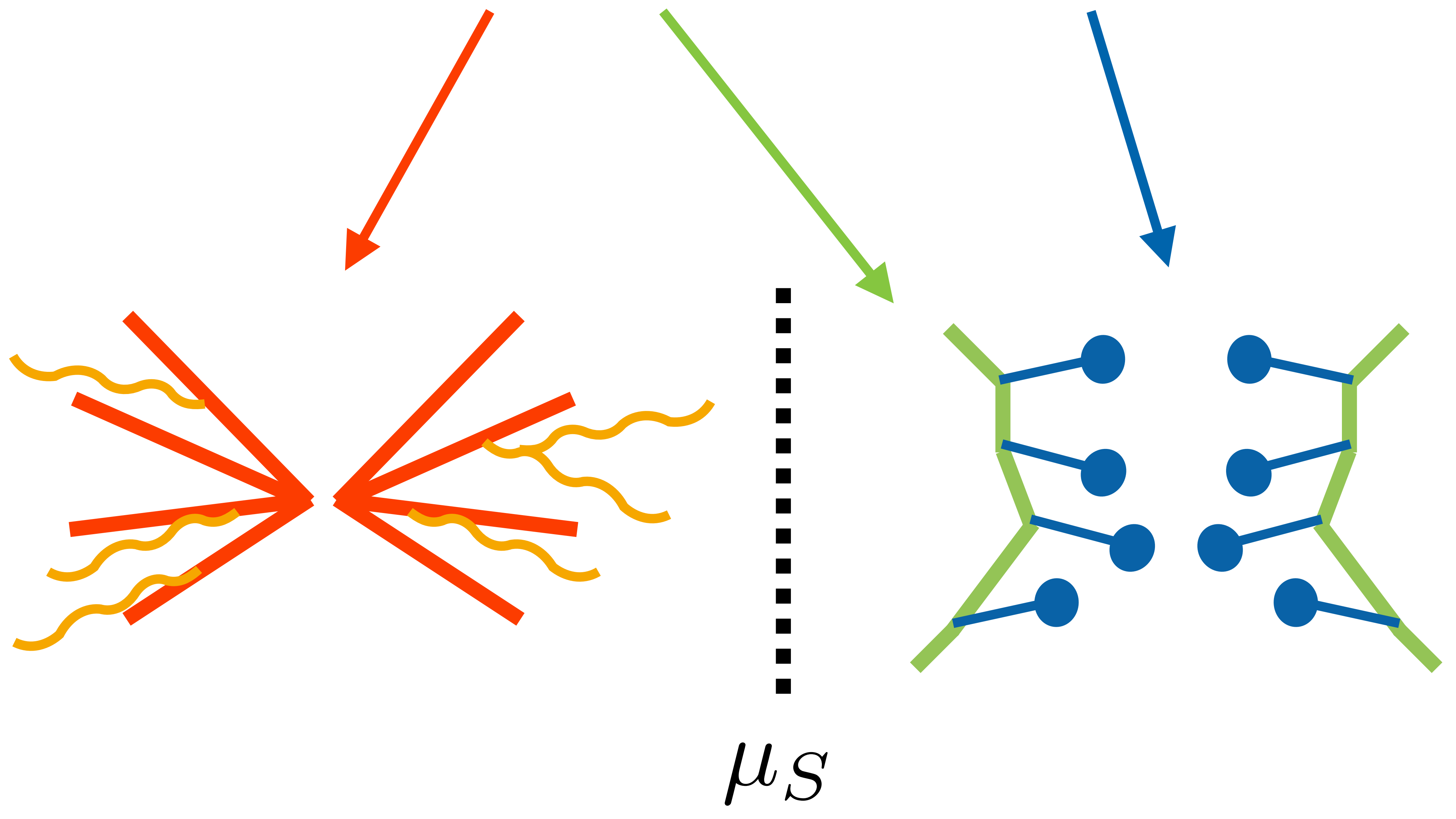

derive evolution

calculate building blocks

construct model response

constrain by data

Factorisation and evolution

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$$\mathbf{U}_n = \mathbf{1}_n u(p_1, \ldots, p_n)$$

Jet cross sections

↓

CVolver algorithm

Factorisation and evolution

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CVolver algorithm

Different resolution variables

↓

Power corrections and link to SCET resummation of NGL

[see Neubert's talk]

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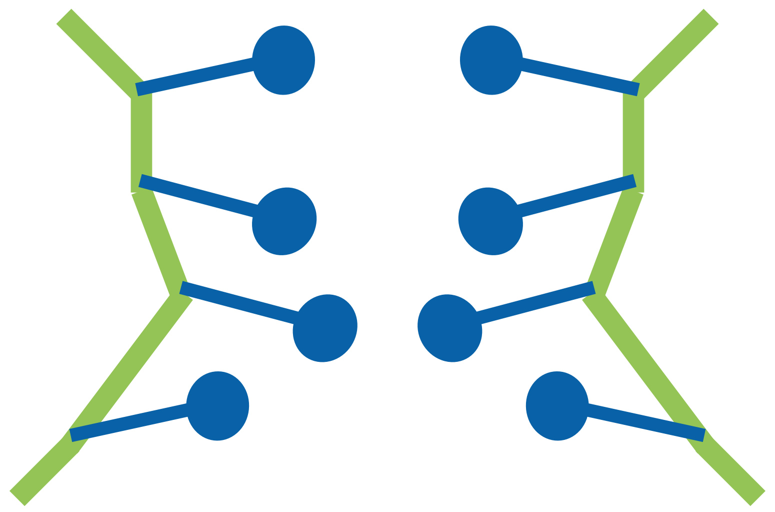

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Jet cross sections → CVolver algorithm

Different resolution variables → Power corrections and link to SCET resummation of NGL

[see Neubert's talk]

Exclusive processes → Hadronization models

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[Plätzer – JHEP 07 (2023) 126]

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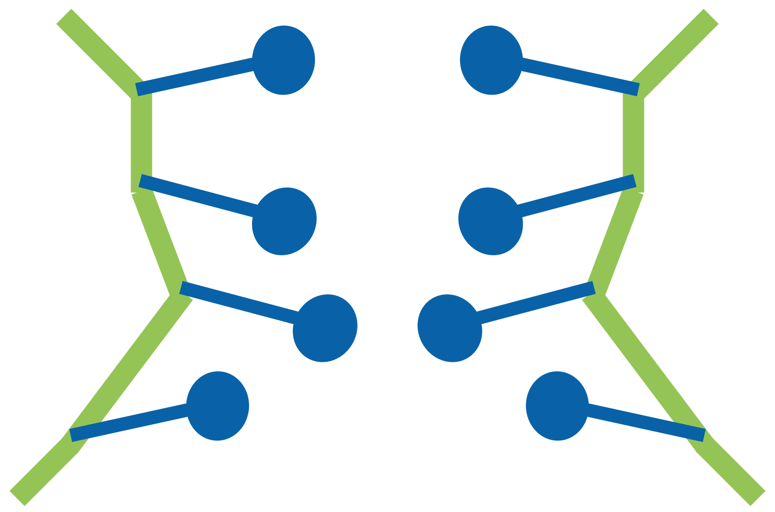

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Jet cross sections → CVolver algorithm

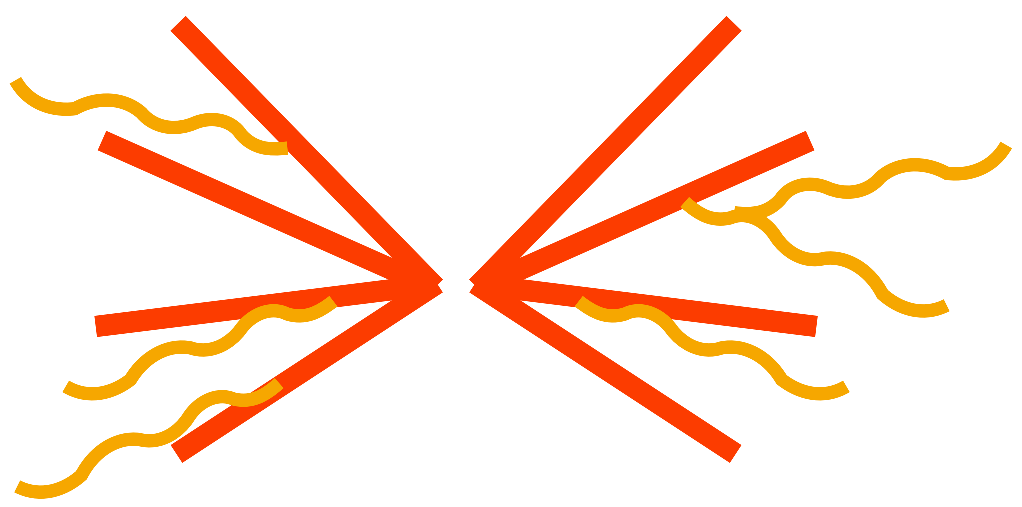
Different resolution variables → Power corrections and link to SCET resummation of NGL

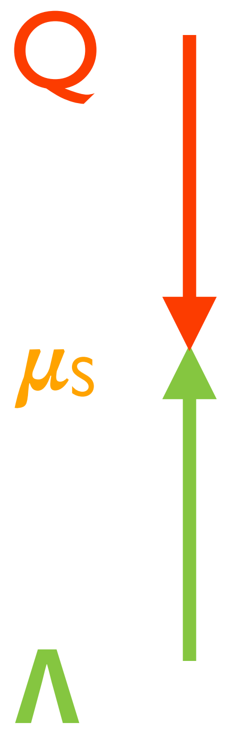
[see Neubert's talk]

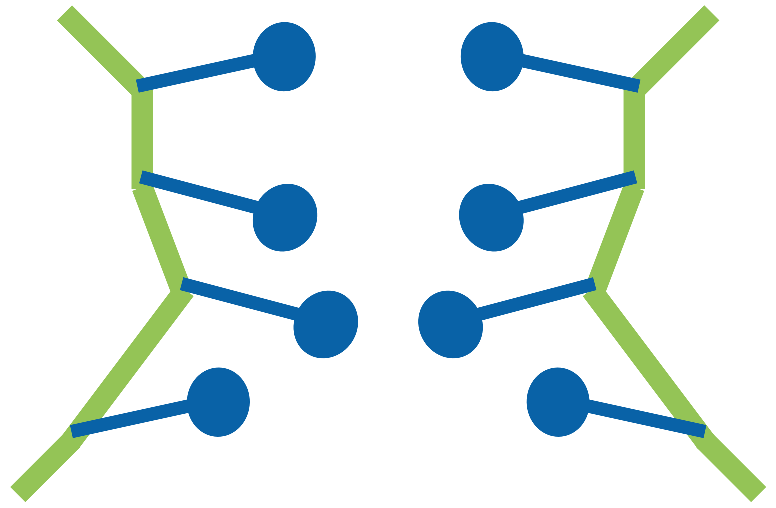
Exclusive processes → Hadronization models

Electroweak processes → Final state definitions?

The role of the IR cutoff







Just a technical parameter?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

- If a shower exploits unitarity, then implicit virtual corrections essentially use a cutoff-dependent renormalisation scheme. [Hoang, Plätzer, Samitz – JHEP 10 (2018) 200]

My approach: "renormalise" bare colour operators. [Plätzer – JHEP 07 (2023) 126]

Subtract IR divergencies
in unresolved regions

Re-arrange to resum
IR enhancements

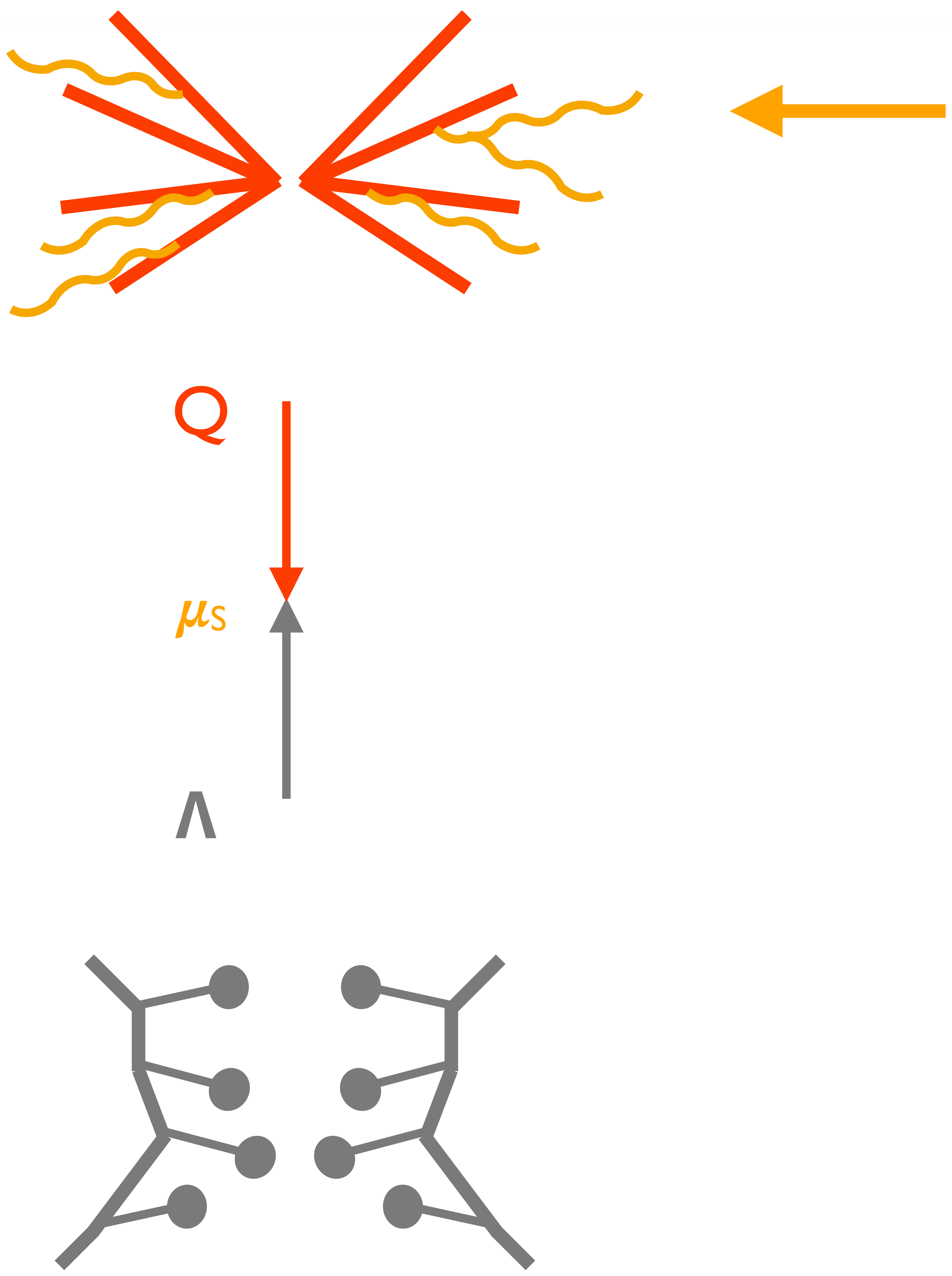
$$\mathbf{U}_n = \mathcal{X}_n \left[\mathbf{S}(\mu_S), \mu_S\right]$$

$$\sigma = \sum_n \alpha_S^n \int \mathrm{Tr}\left[\mathbf{A}_n(\mu_S)\mathbf{S}_n(\mu_S)\right] \mathrm{d}\phi_n$$

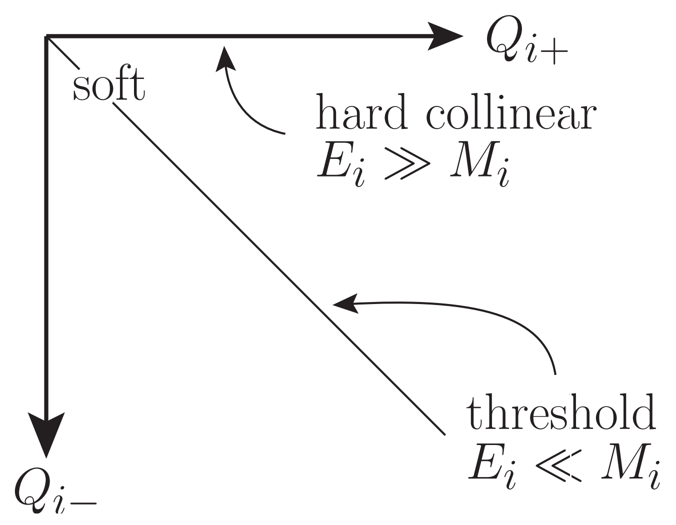
$$\mathbf{M}_n Z_g^n = \mathcal{Z}_n \left[\mathbf{A}(\mu_S), \mu_S\right]$$

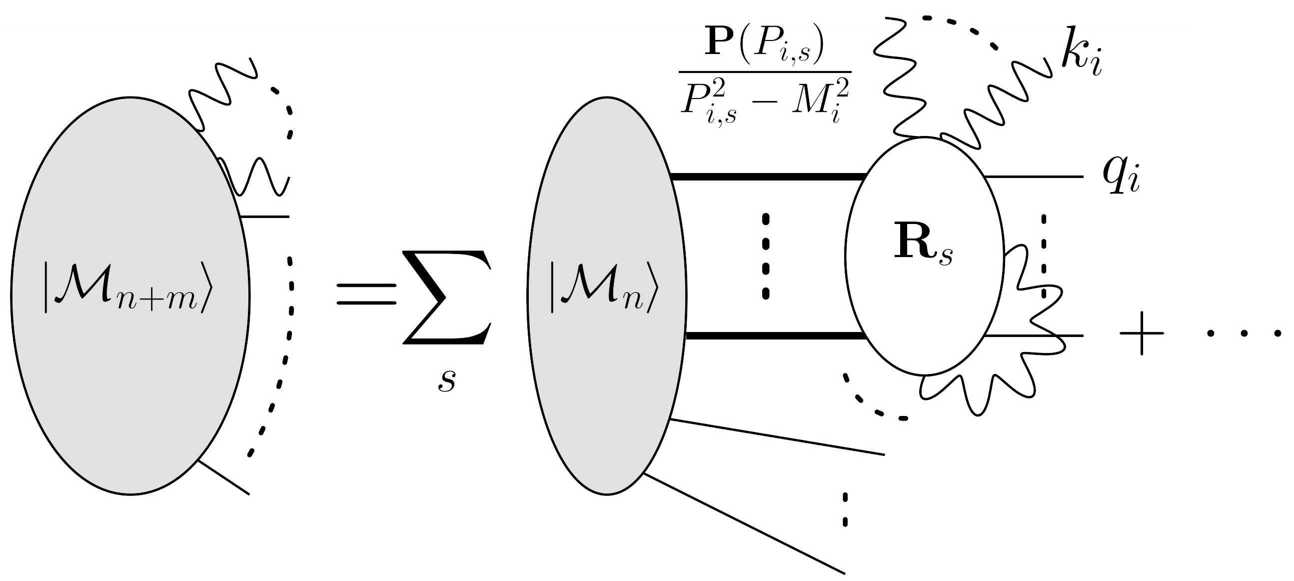
- Even if we include explicit virtual contributions, a cutoff is present. This is independent of the UV renormalisation scheme, and impacts the structure of the resummation. [Plätzer – (slow) progress]

Building shower and resummation algorithms



Factorisation of amplitudes and power expansions.





[Löschner, Plätzer, Ruffa, Sjödahl — '20+]

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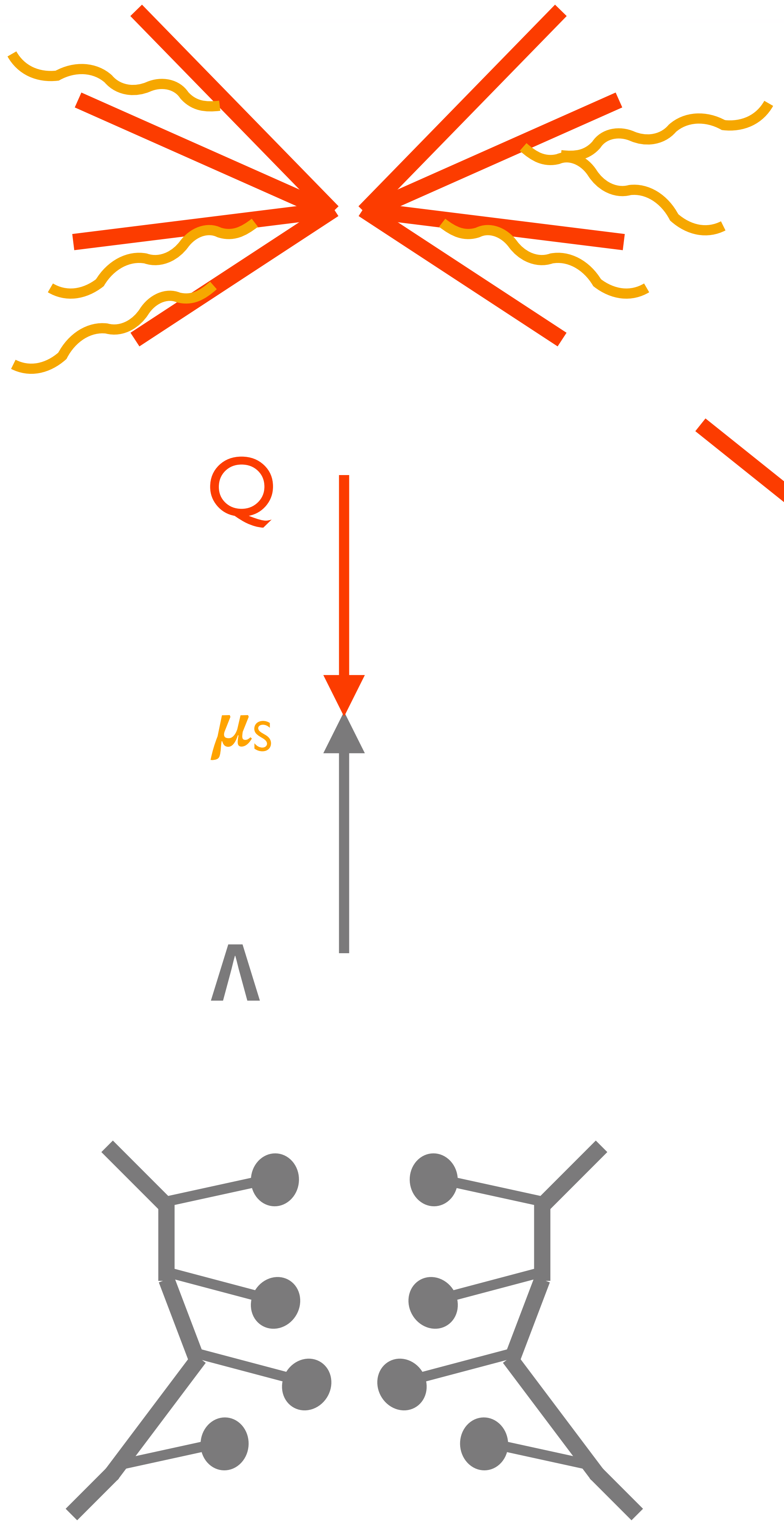

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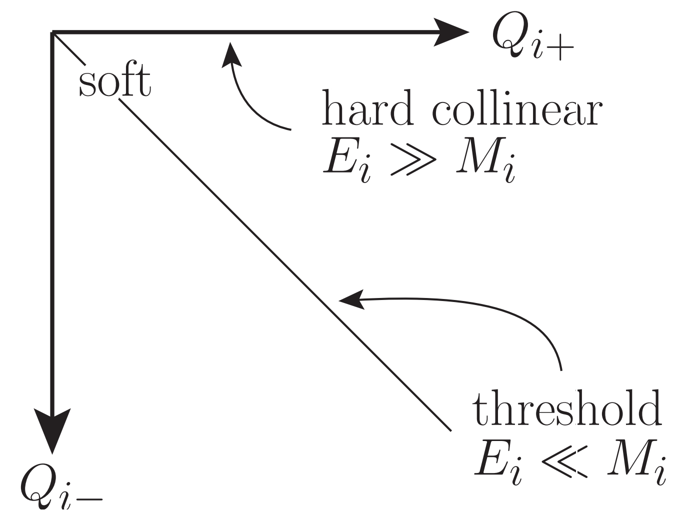
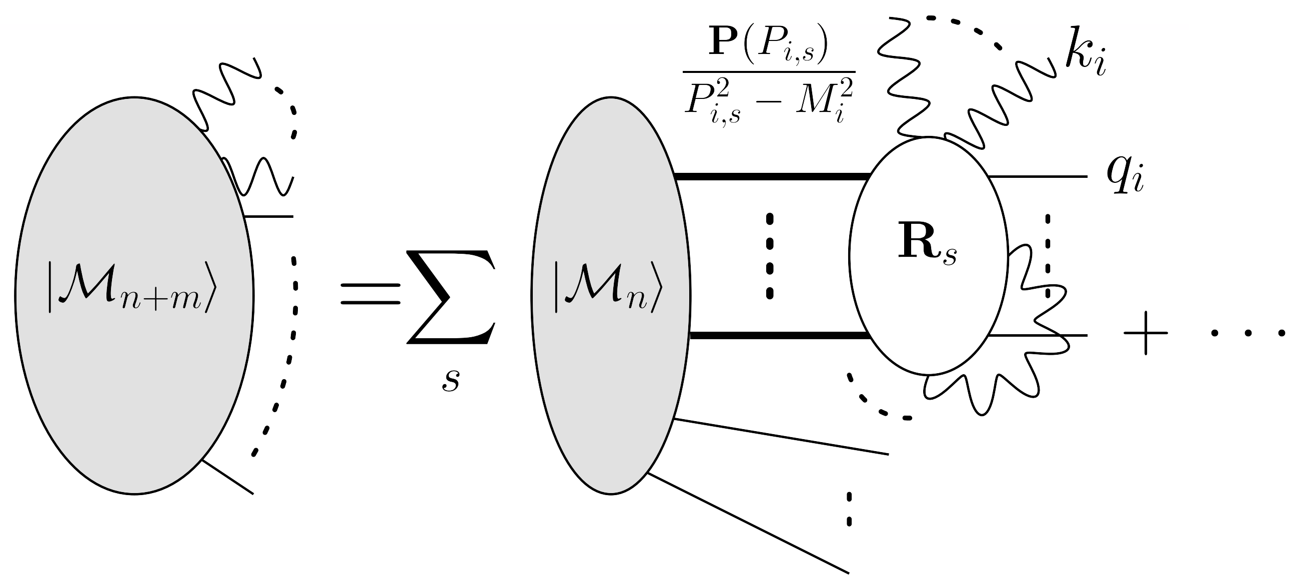




[Löschner, Plätzer, Ruffa, Sjödahl — ’20+]

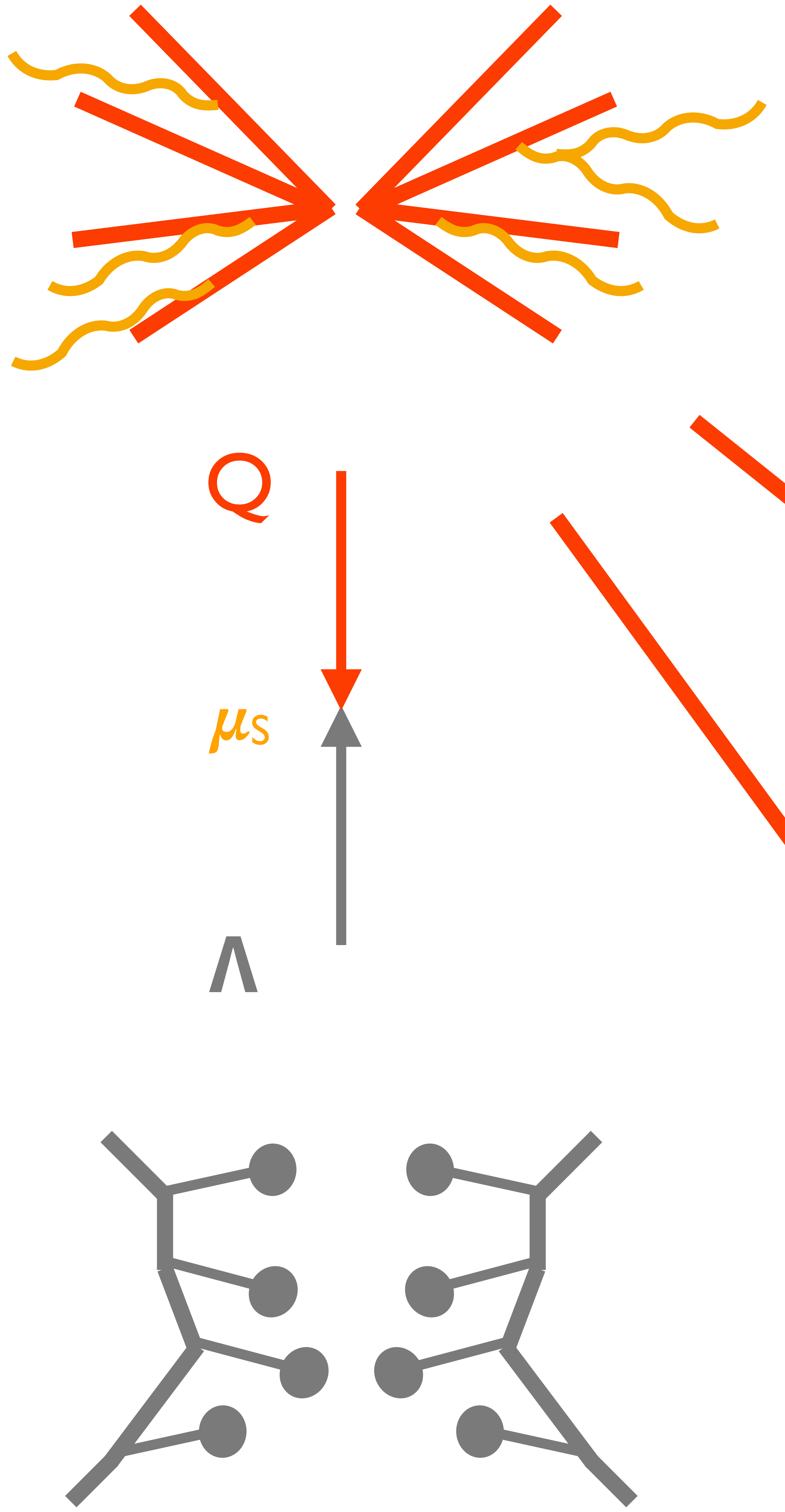
NLL parton showers — Herwig 7 dipole shower

[Forshaw, Holguin, Plätzer — ‘20+] [Holguin, Plätzer, Seymour, Sule – wip]

$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

LL — qualitative   NLL — quantitative   NNLL — precision

Building shower and resummation algorithms



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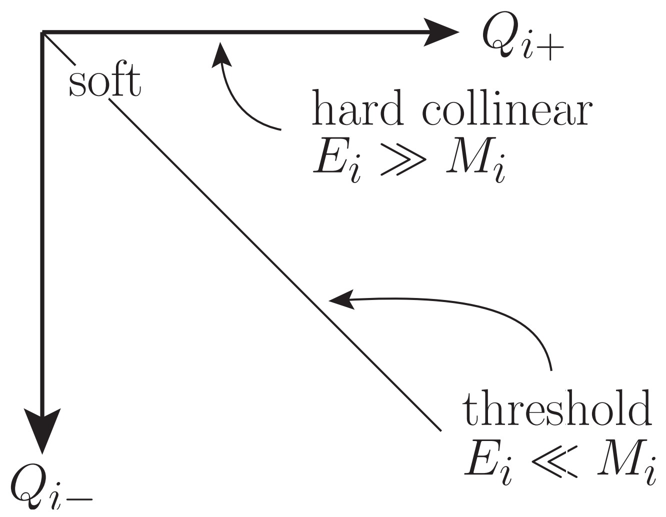

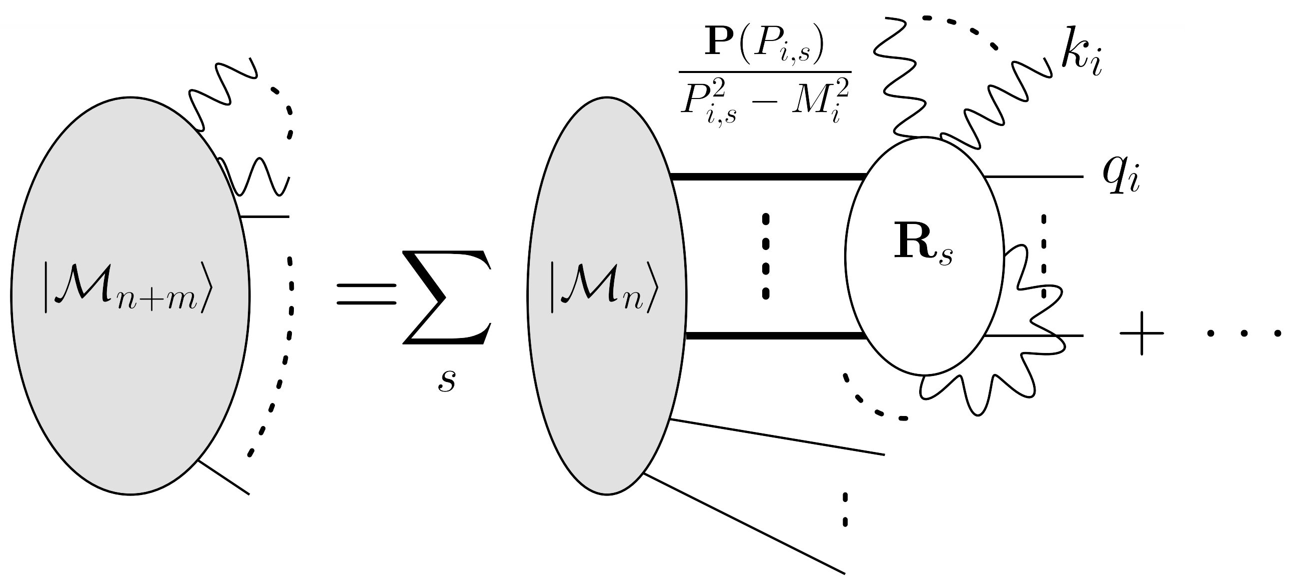

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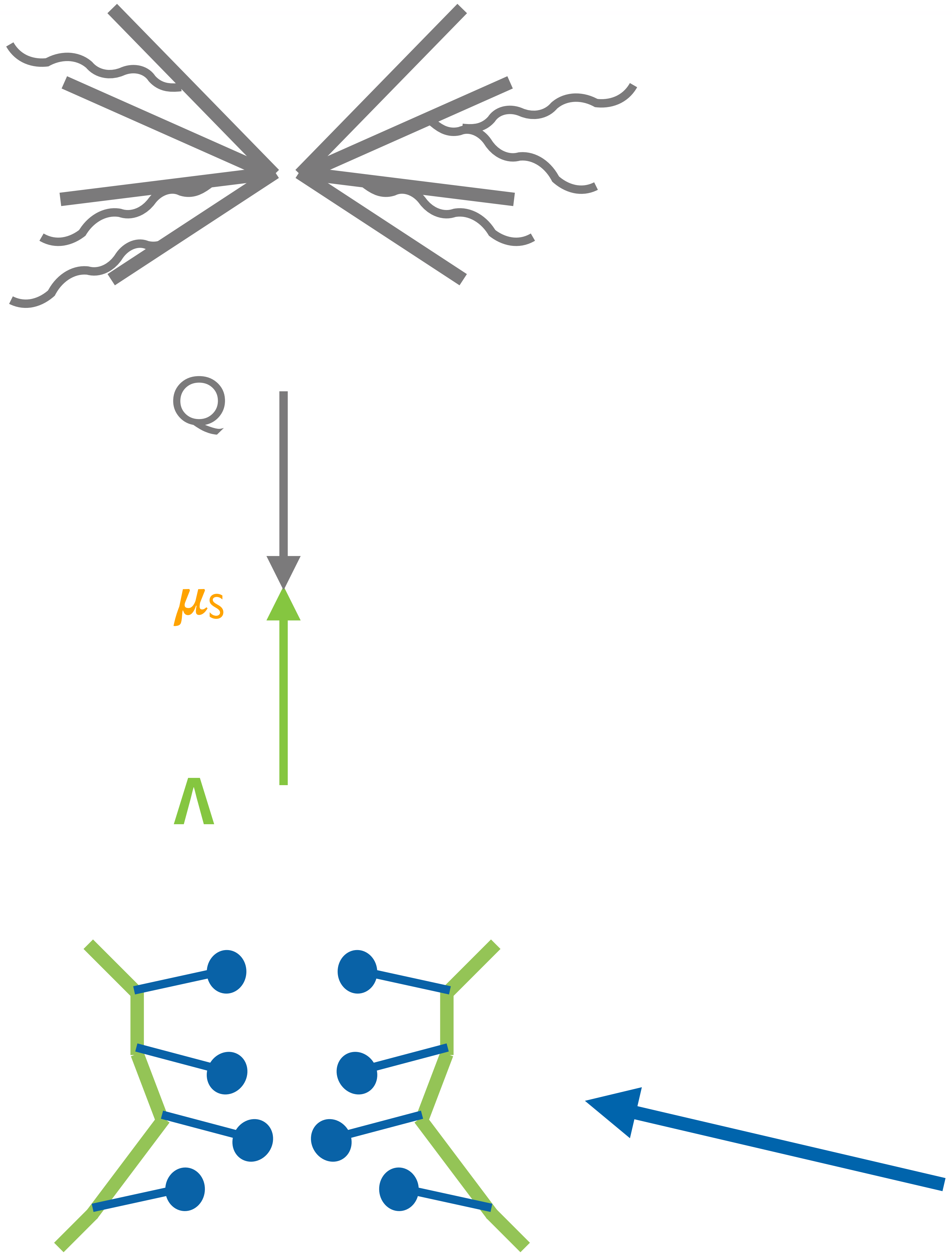
Amplitude evolution and resummation algorithms.

- Started with non-global logarithms. [Forshaw, Plätzer et al. — '18+]
- Establishing links to JIMWLK, EFT, direct QCD resummation.

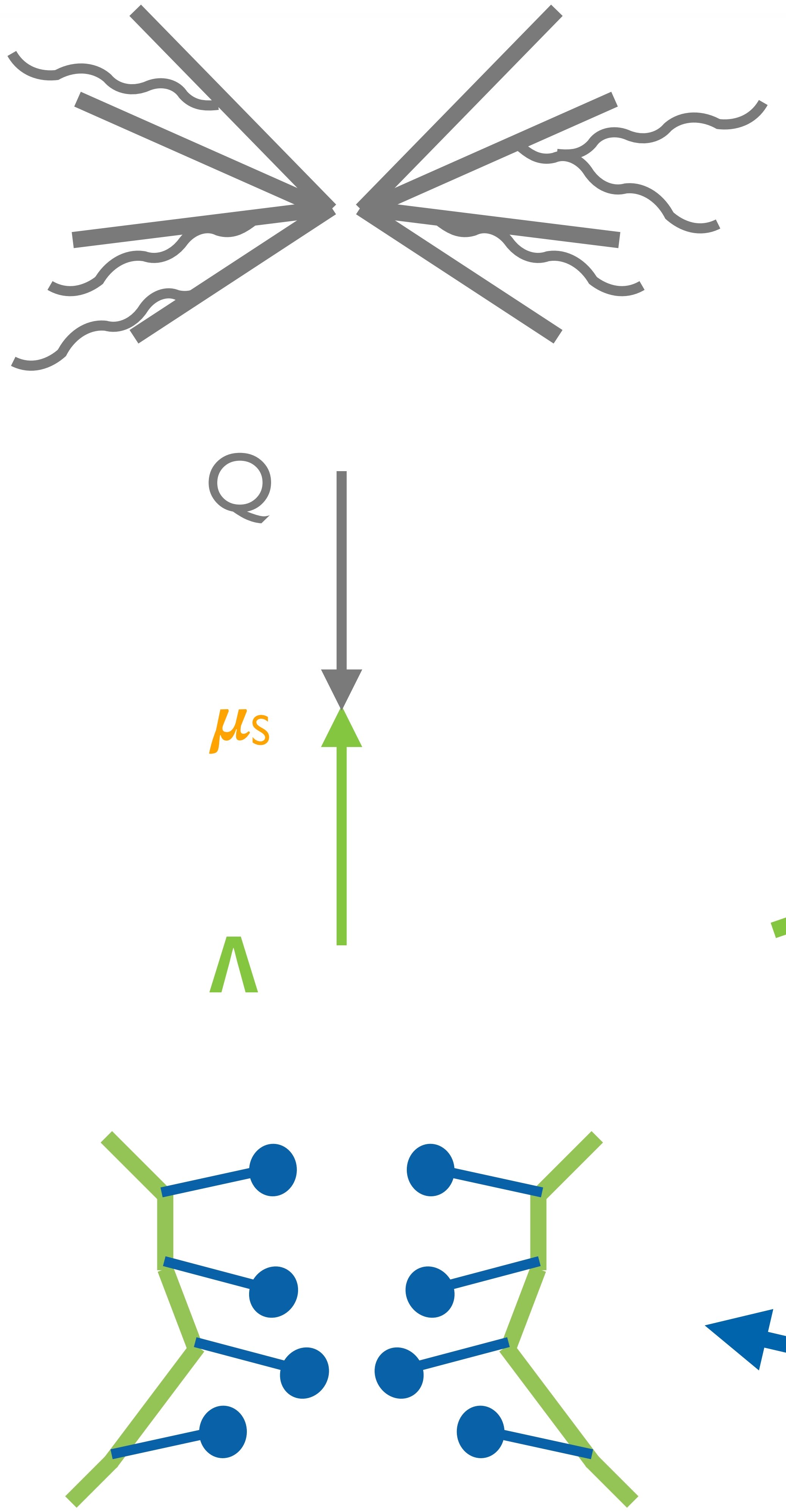
[Plätzer & Weigert – wip]

Building and constraining hadronization models



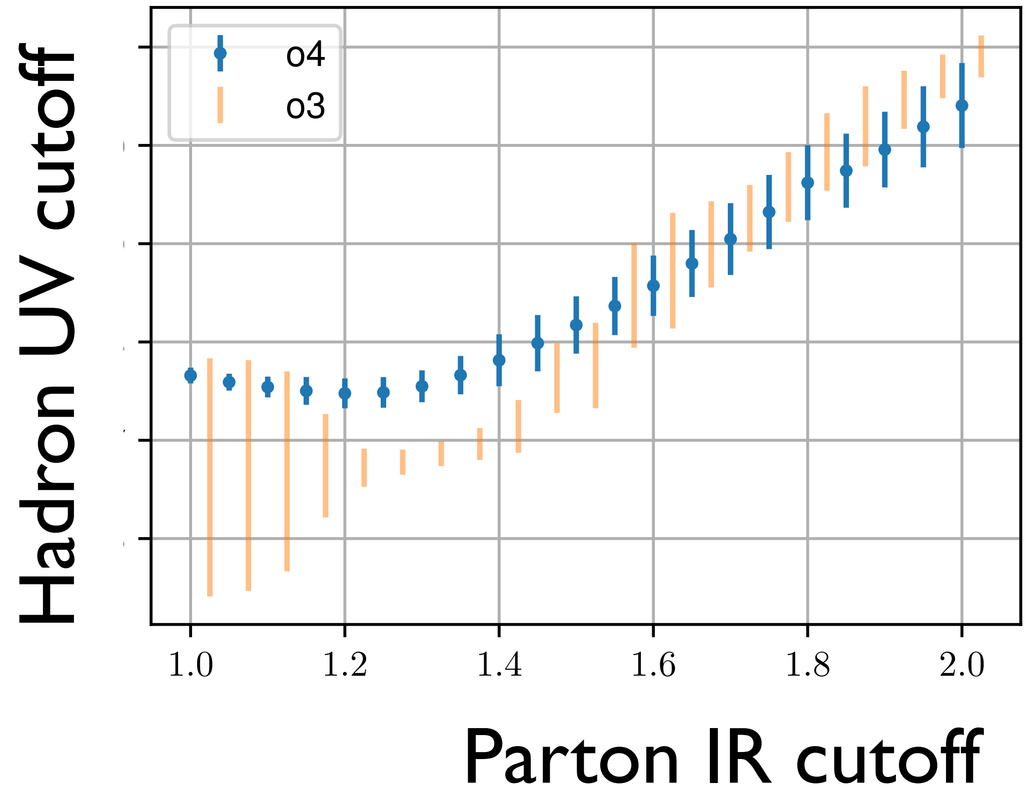
Hadron correlations not predicted or simplistic, baryon rates not right, cutoff dependence uncontrolled (fitted together with strong coupling), uncertainties not quantified … impact to ML more than worrying.

Building and constraining hadronization models



Towards a smooth matching of shower and hadronization at the infrared cutoff — inspired by coherent branching.

[Hoang, Jin, Plätzer, Samitz — 2404.09856]



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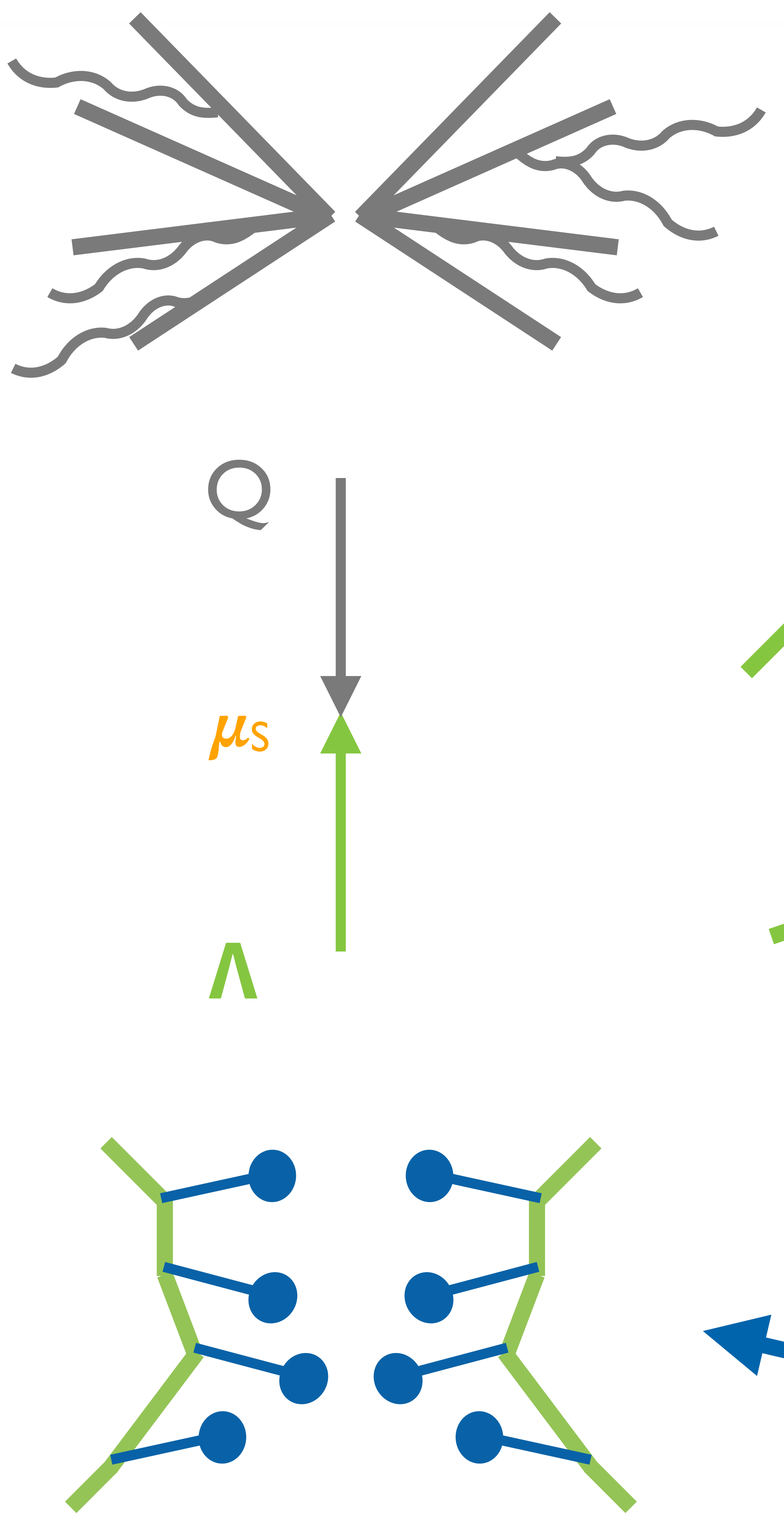
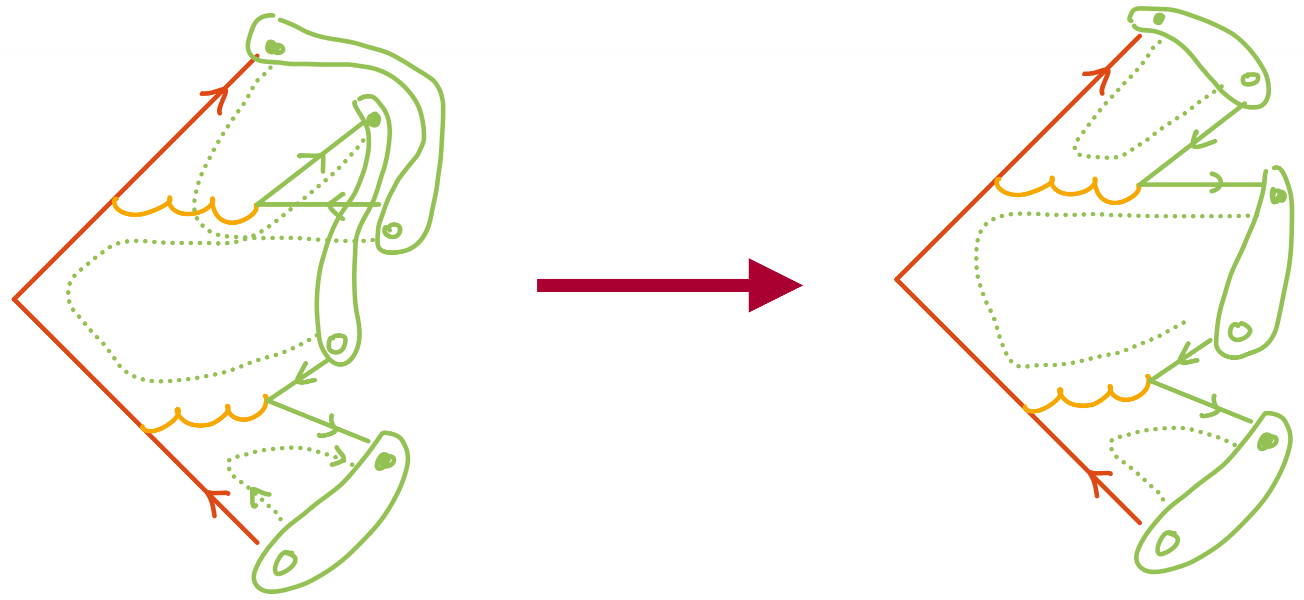


Towards a full model of cluster evolution with fission and colour reconnection informed by perturbative evolution.



[Gieseke, Kiebacher, Plätzer, Priedigkeit — in progress]

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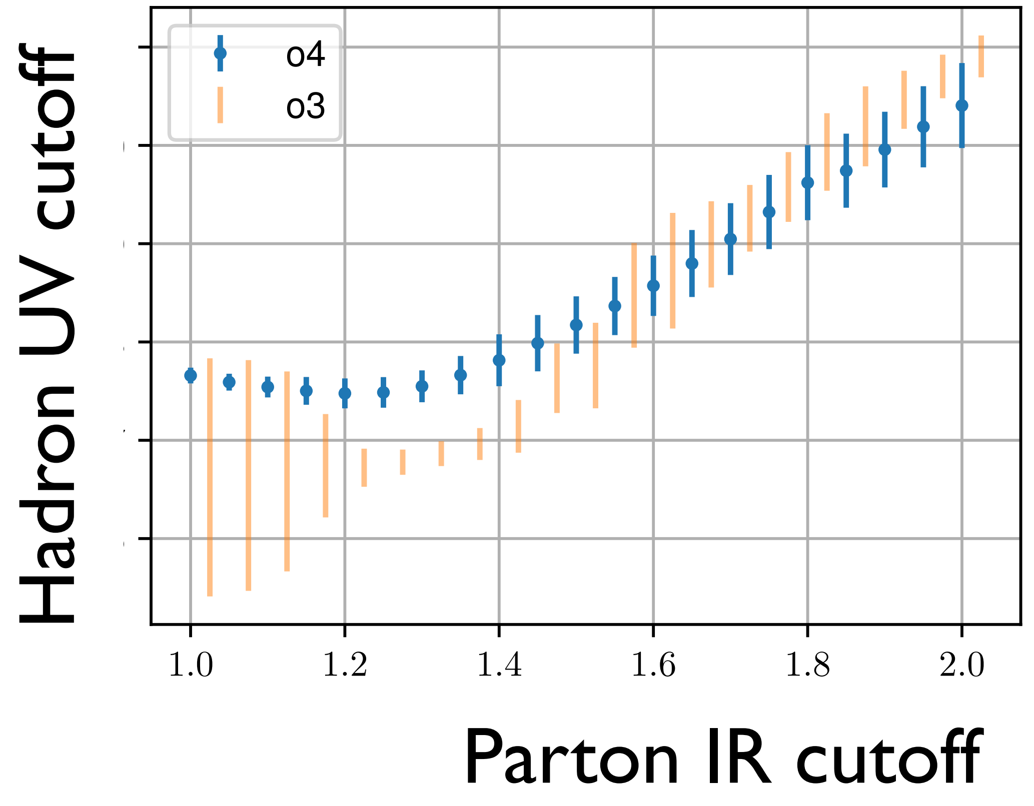

[Hoang, Jin, Plätzer, Samitz — 2404.09856]

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Redefinitions of "bare" operators

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

$$\mathbf{U}_n = \mathcal{X}_n\left[\mathbf{S}(\mu_S), \mu_S\right] = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

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unresolved emission
at leading power

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unresolved emission at leading power

loop divergence at leading power, no unresolved emission

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unresolved emission at leading power ($\mathbf{M}_n^{(0)}$)

loop divergence at leading power, no unresolved emission ($\mathbf{M}_n^{(1)}$)

subtraction for loop divergence ($\mathbf{X}_n^{(1)}$, $\mathbf{X}_n^{(1)\dagger}$)

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subtraction for loop divergence

subtraction for unresolved emission

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unresolved emission at leading power

loop divergence at leading power, no unresolved emission

subtraction for loop divergence

subtraction for unresolved emission

Infer subtractions from singular behaviour.
Complete by re-defining hard process.

$$\mathbf{M}_n Z_g^n = \mathcal{Z}_n \left[\mathbf{A}(\mu_S), \mu_S\right] = \mathbf{Z}_n \mathbf{A}_n \mathbf{Z}_n^\dagger + \sum_{s=1}^{n} \alpha_S^s \mathbf{E}_n^{(s)} \mathbf{A}_{n-s} \mathbf{E}_n^{(s)\dagger}$$

Redefinitions of "bare" operators

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

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inverse to

$$\mathbf{M}_n Z_g^n = \mathcal{Z}_n\left[\mathbf{A}(\mu_S), \mu_S\right] = \mathbf{Z}_n \mathbf{A}_n \mathbf{Z}_n^\dagger + \sum_{s=1}^{n} \alpha_S^s \mathbf{E}_n^{(s)} \mathbf{A}_{n-s} \mathbf{E}_n^{(s)\dagger}$$

$$\sum_n \alpha_0^n \int \mathrm{Tr}\left[\mathbf{M}_n \mathbf{U}_n\right] \mathrm{d}\phi_n = \sum_n \alpha_S^n \int \mathrm{Tr}\left[\mathcal{Z}_n\left[\mathbf{A}(\mu_S), \mu_S\right] \mathcal{X}_n\left[\mathbf{S}(\mu_S), \mu_S\right]\right] \mathrm{d}\phi_n =$$

$$\sum_n \alpha_S^n \int \mathrm{Tr}\left[\mathbf{A}_n(\mu_S) \mathbf{S}_n(\mu_S)\right] \mathrm{d}\phi_n$$

Redefinitions of “bare” operators

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Subtractions necessitate a resolution: what is it we call ‘unresolved’? Encompass all singular regions!

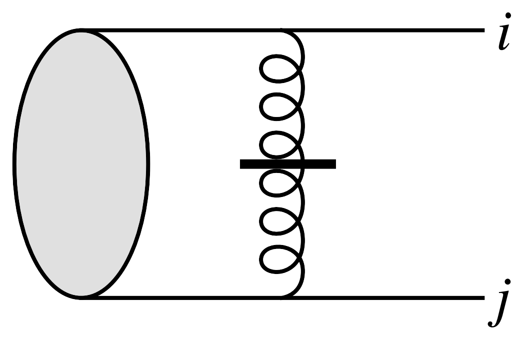
resolution function for (cut) loop momenta

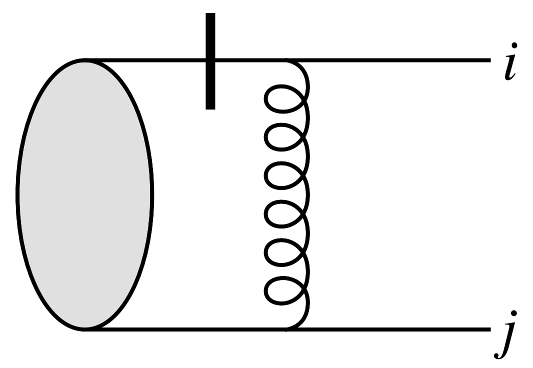
$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}]$$

$$\hat{\mathbf{V}}_n^{(l)}[\Xi_{n,l}] = \sum_\alpha \int \mathcal{I}_{n,\alpha}^{(l)}(p_1, \ldots, p_n; k_1, \ldots, k_l)\ \Xi_{n,l}^{(\alpha)} \prod_{i=1}^{l} \mu_R^{2\epsilon} [\mathrm{d}k_i]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

resolution function for real emission





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resolution function for (cut) loop

$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)} [\Xi_{n,1}]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger}$$

resolution function for real emission

Continues to higher orders …

$$\mathbf{X}_n^{(2)} = \hat{\mathbf{V}}_n^{(2)} [\Xi_{n,2}] - \hat{\mathbf{V}}_n^{(1)} [\Xi_{n,1}] \hat{\mathbf{V}}_n^{(1)}$$

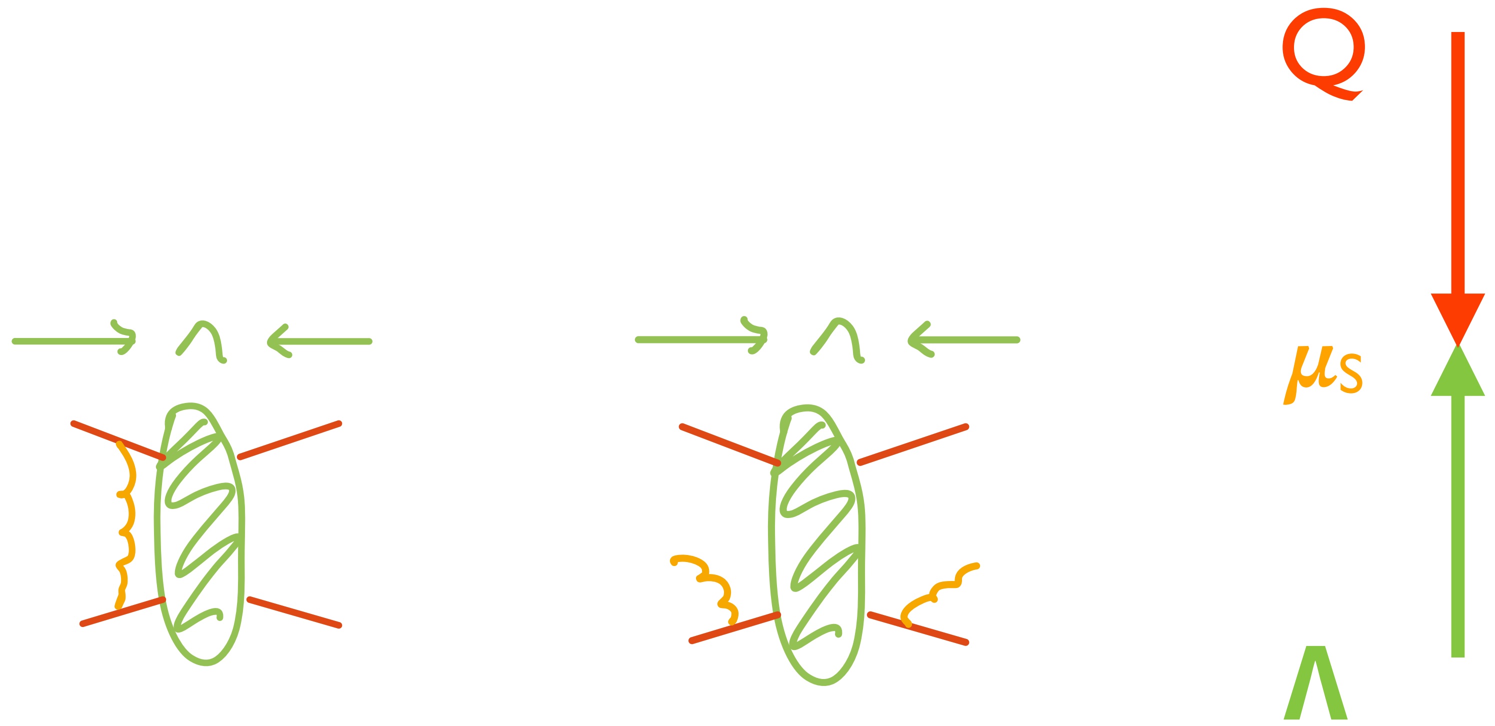
$$\begin{aligned} \mathbf{F}_n^{(1,1)} \circ \mathbf{F}_n^{(1,0)\dagger} = \; & \mathbf{D}_n^{(1,1)} \left[\Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} \\ & + \mathbf{D}_n^{(1,1)} \left[1 - \Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} + \mathbf{D}_n^{(1,1)} \left[\Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger} (1 - \Theta_{n,1}) \\ & - \hat{\mathbf{V}}_n^{(1)} \left[\Xi_{n-1,1}\right] \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)} \hat{\mathbf{V}}_{n-1}^{(1)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} \end{aligned}$$

$$\mathbf{F}_n^{(2,0)} \circ \mathbf{F}_n^{(2,0)\dagger} = \mathbf{D}_n^{(2,0)} \circ \mathbf{D}_n^{(2,0)\dagger} \Theta_{n,2} - \mathbf{D}_n^{(1,0)} \mathbf{D}_{n-1}^{(1,0)} \circ \mathbf{D}_{n-1}^{(1,0)\dagger} \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

Evolution equations

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.



$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S}\mathbf{A}_n + \mathbf{A}_n\mathbf{\Gamma}^\dagger_{n,S} - \sum_{s\geq 1}\alpha_S^s \mathbf{R}^{(s)}_{S,n}\mathbf{A}_{n-s}\mathbf{R}^{(s)\dagger}_{S,n}$$

Evolution equations

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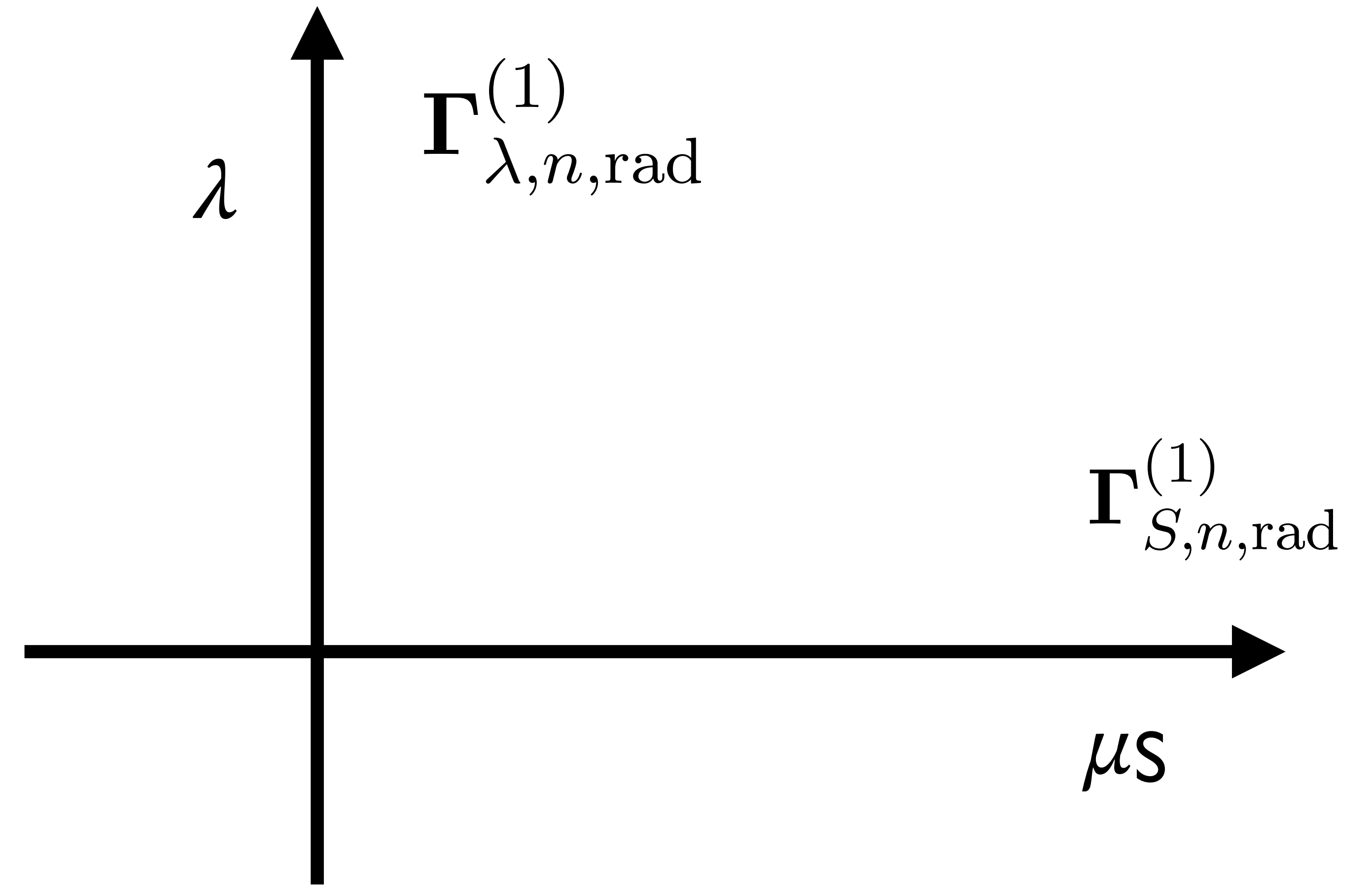
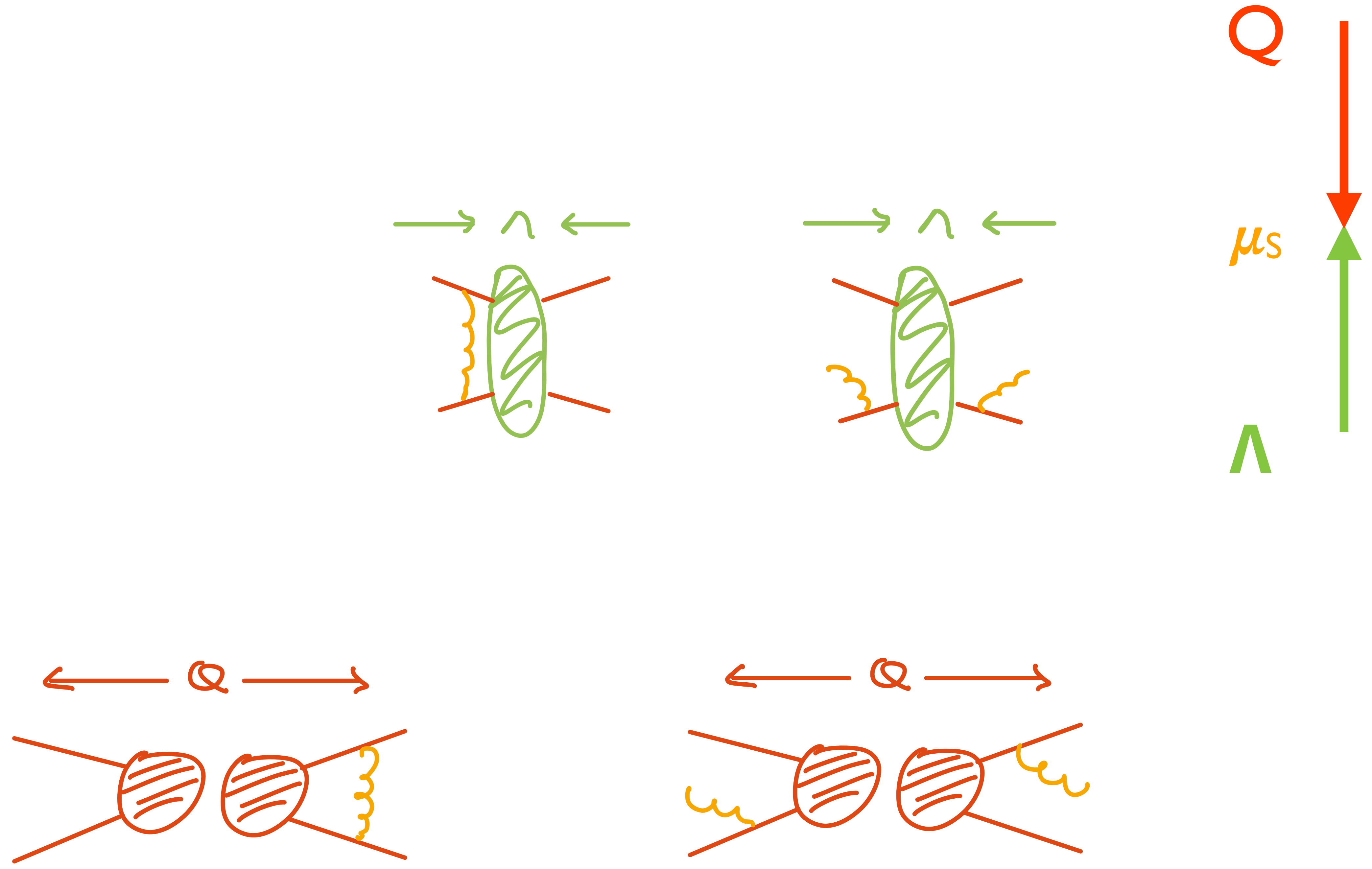


$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1}\theta(E_n - \mu_S)$$

$[\mathrm{d}p_i]\tilde{\delta}(p_i)$

1

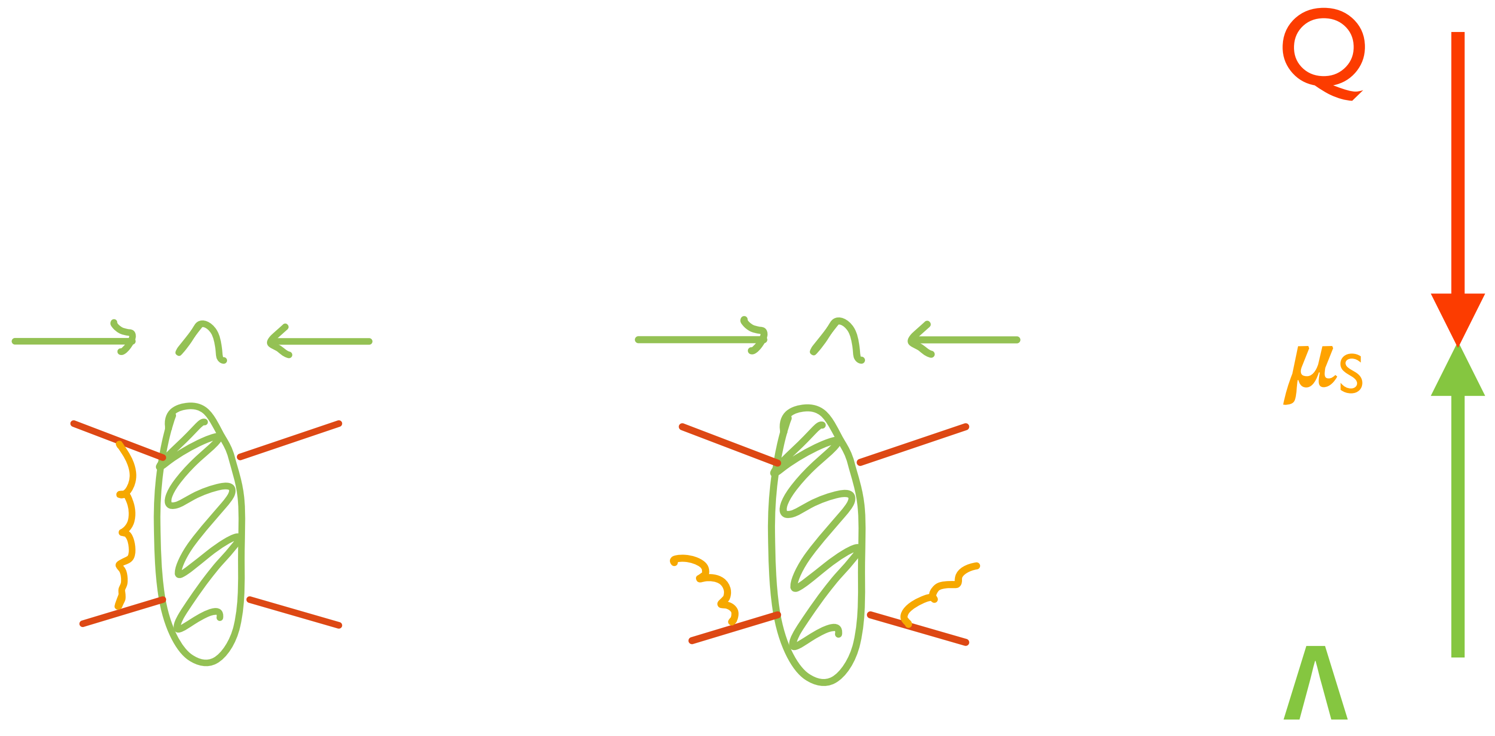


Constructing evolution algorithms

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.

Subtract iterated contribution in ordered phase space.

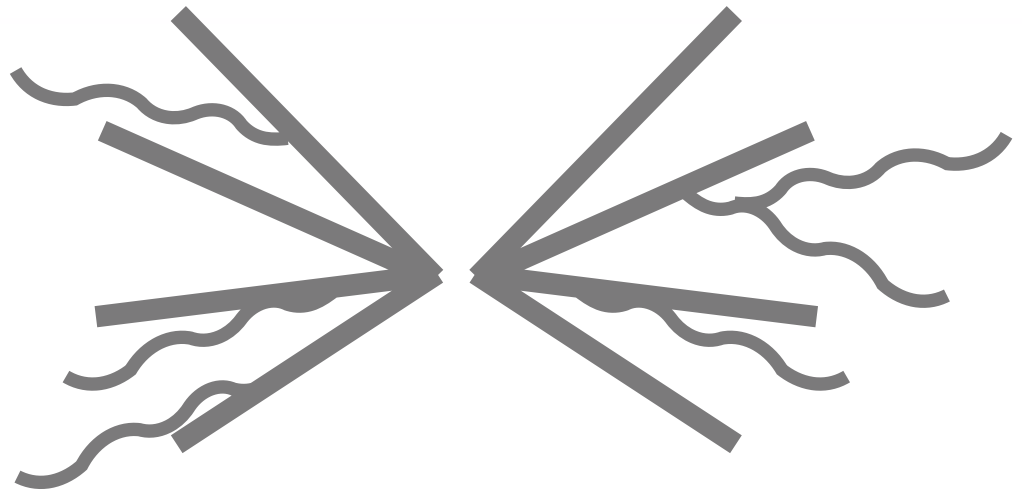


$$\mathbf{R}_n^{(2,0)} \circ \mathbf{R}_n^{(2,0)\dagger} = \left(\hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger}\hat{\Theta}_{n,2} - \hat{\mathbf{D}}_n^{(0,1)}\hat{\mathbf{D}}_{n-1}^{(0,1)} \circ \hat{\mathbf{D}}_{n-1}^{(0,1)\dagger}\hat{\mathbf{D}}_n^{(0,1)\dagger}\hat{\Theta}_{n-1,1}\hat{\Theta}_{n,1}\right)$$
$$\times\theta(E_{n-1} - \mu_S)\delta(E_n - \mu_s)$$
$$+ \hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger}\hat{\Theta}_{n,2}\theta(E_n - \mu_S)\delta(E_{n-1} - \mu_S)$$

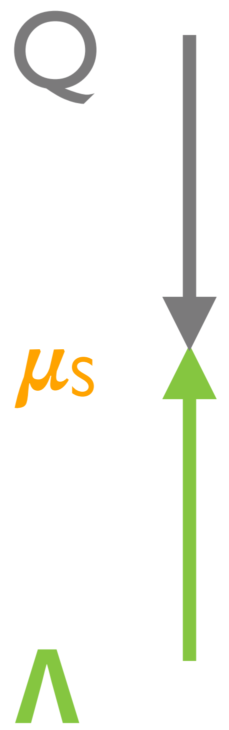
Use full double gluon matrix element outside.

Similar consequences for virtual corrections.

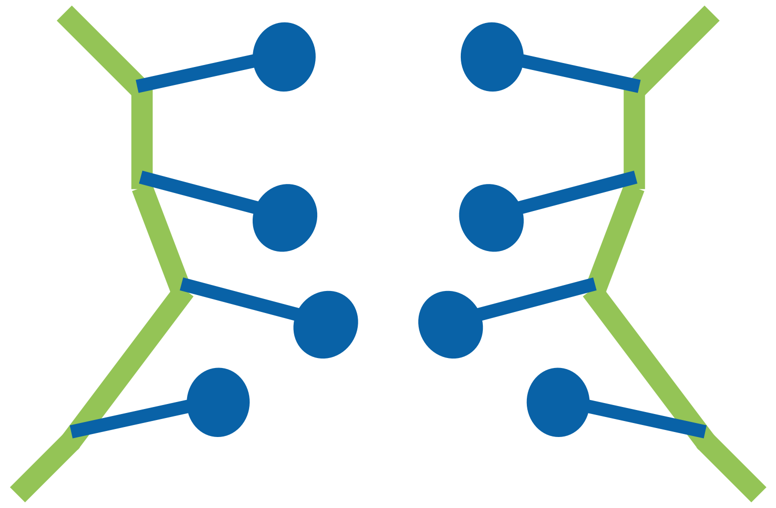
What structures are admissible?



Subtracted ("renormalised") observable defines a very general criterion of infrared safety: finiteness means the bare observable must admit **cancellations local in momentum and colour space.**



$$\mathbf{S}_n = \mathbf{Z}_n^\dagger \mathbf{U}_n \mathbf{Z}_n + \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{E}_{n+s}^{(s)\dagger} \mathbf{U}_{n+s} \mathbf{E}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$



This structure is ubiquitous if we talk about **electroweak final states** (in isospin space) and if we want to predict fully detailed and exclusive final states as needed for an event generator.

Observables singular at his level are genuine non-perturbative.

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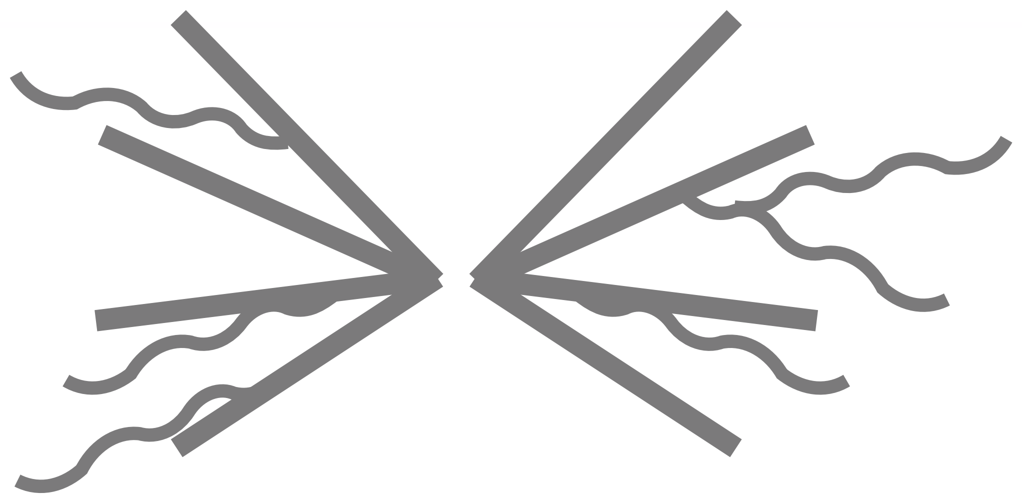

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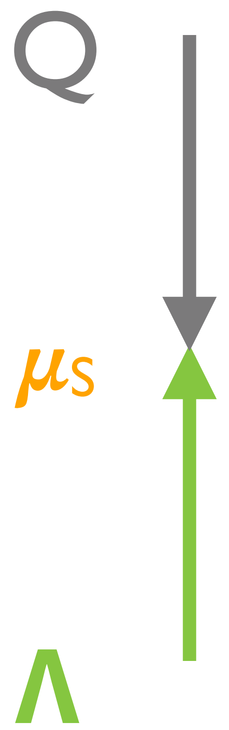
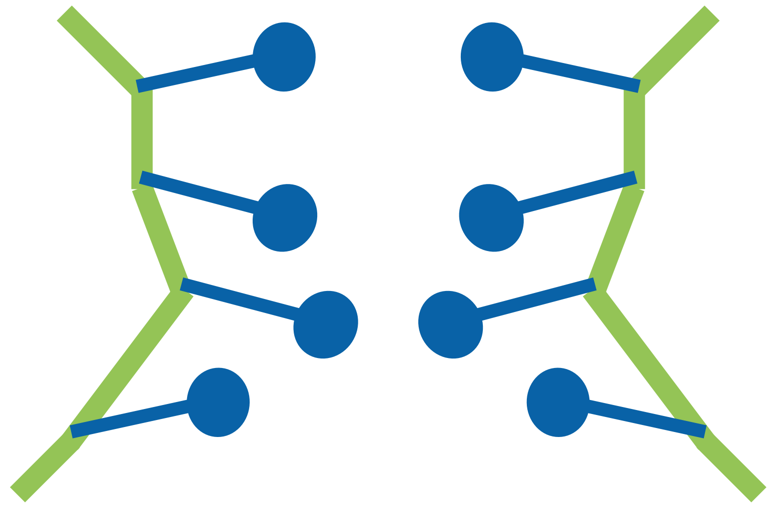


infrared resolution vs observable

$$\mathbf{S}_n = \mathbf{1}_n u(p_1, ..., p_n)$$
$$- \alpha_s \int \mu_R^{2\epsilon} [\mathrm{d}p_{n+1}] \tilde{\delta}(p_{n+1}) \hat{\mathbf{D}}_{n+1}^{(1,0)\dagger} \hat{\mathbf{D}}_{n+1}^{(1,0)} \Theta_{n,1} \left[u(p_1, ..., p_n, p_{n+1}) - u(p_1, ..., p_n) \right] + \mathcal{O}(\alpha_s^2)$$



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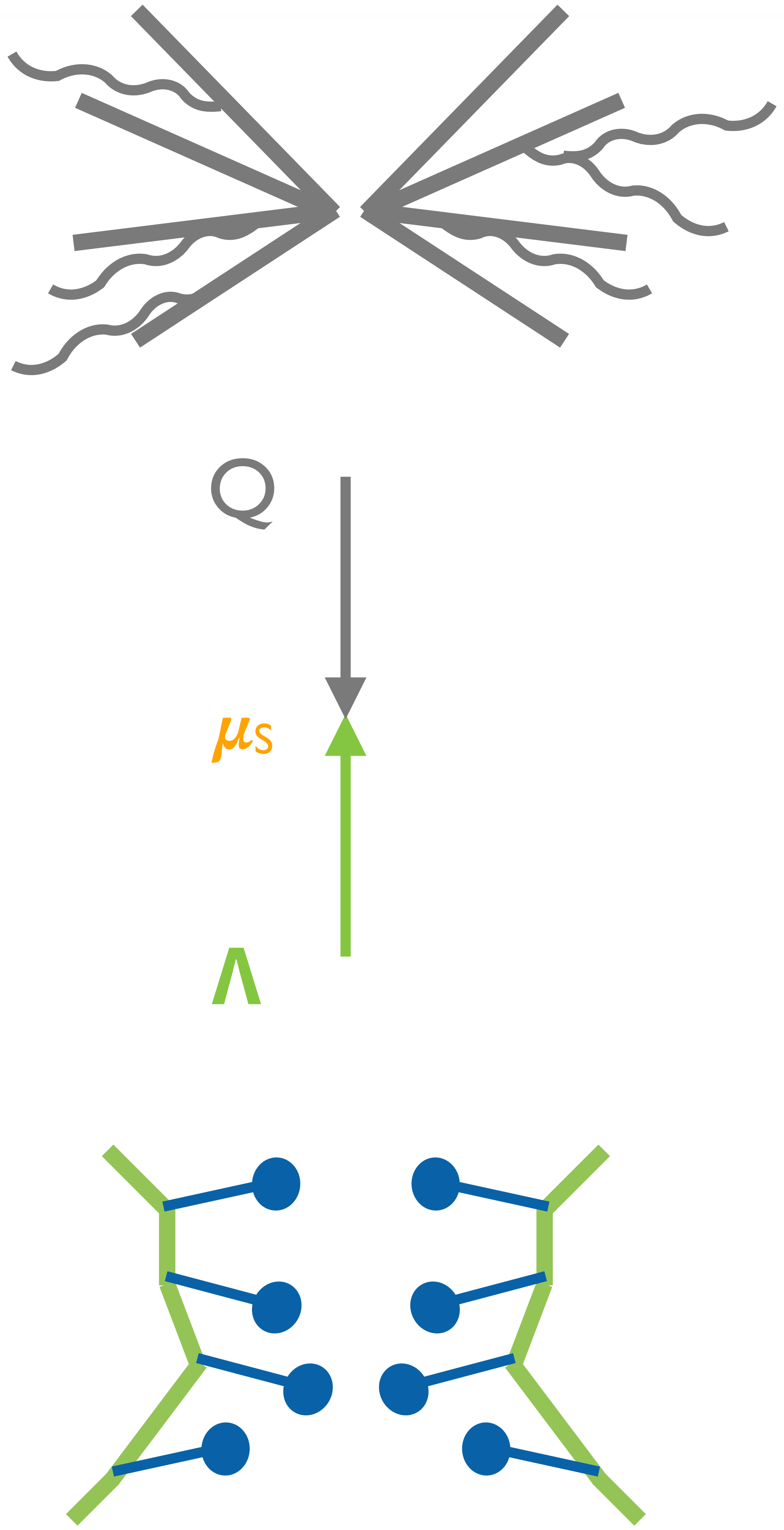
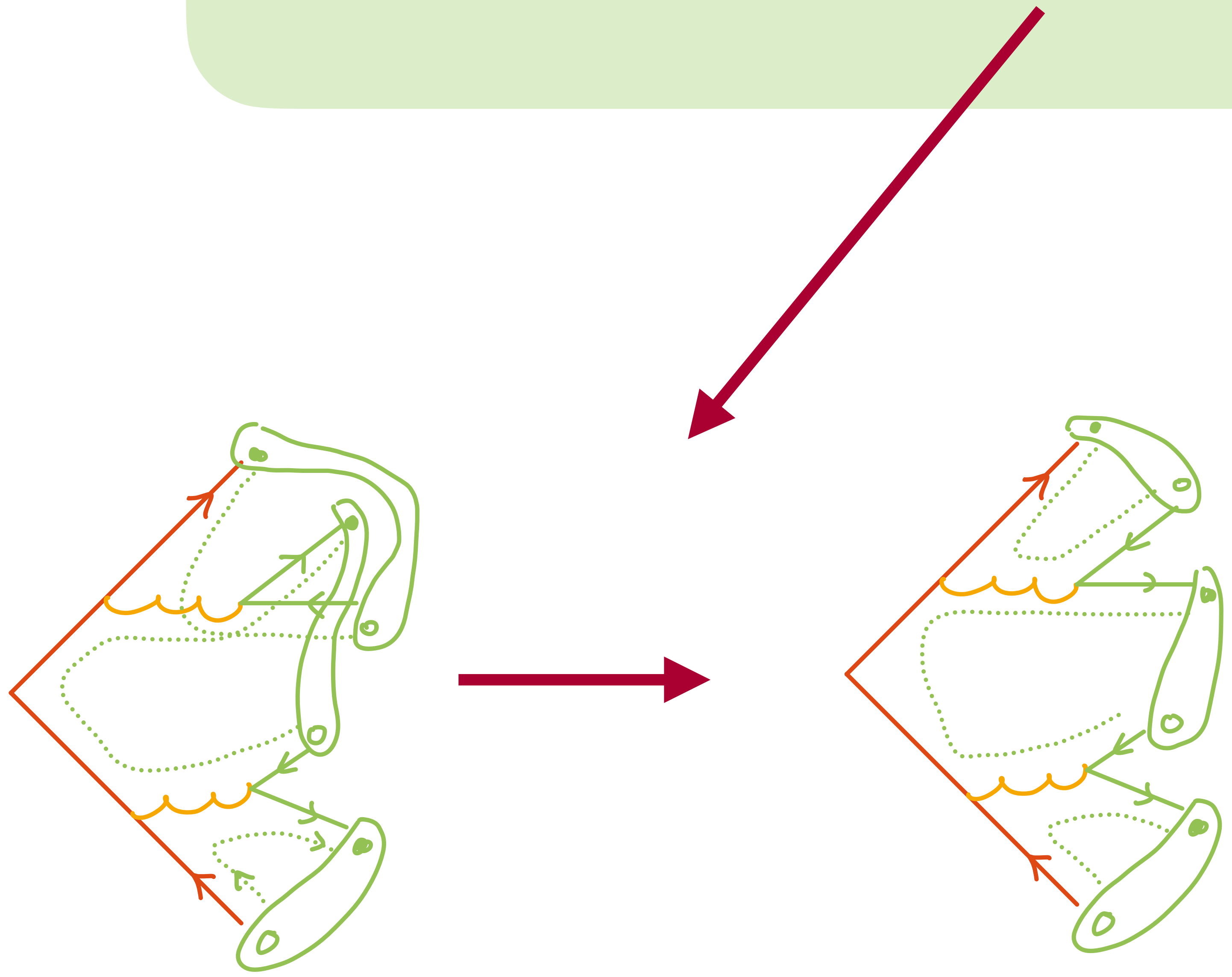
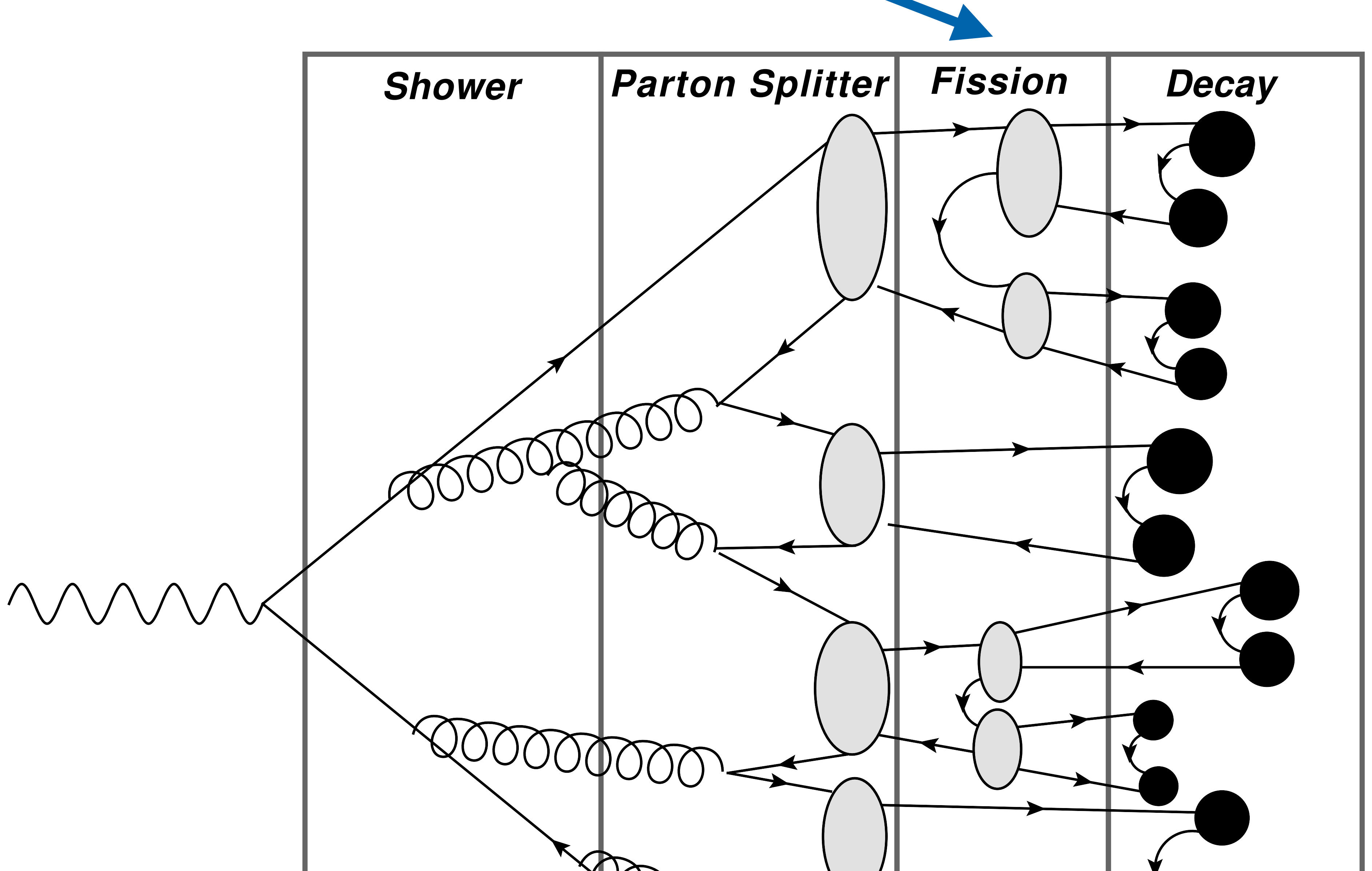


Hadronization models would start by studying clusters.
Colour reconnection and cluster fission present.

$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$





Why?

→ Kiebacher's and Stafford's talk

Hadronization and the interface to showers lacks:

- comprehensive uncertainty estimates
- predictivity
- extrapolation across different energy regimes
- links to analytic models …

NB: Machine learned models will only continue to parametrize our ignorance.

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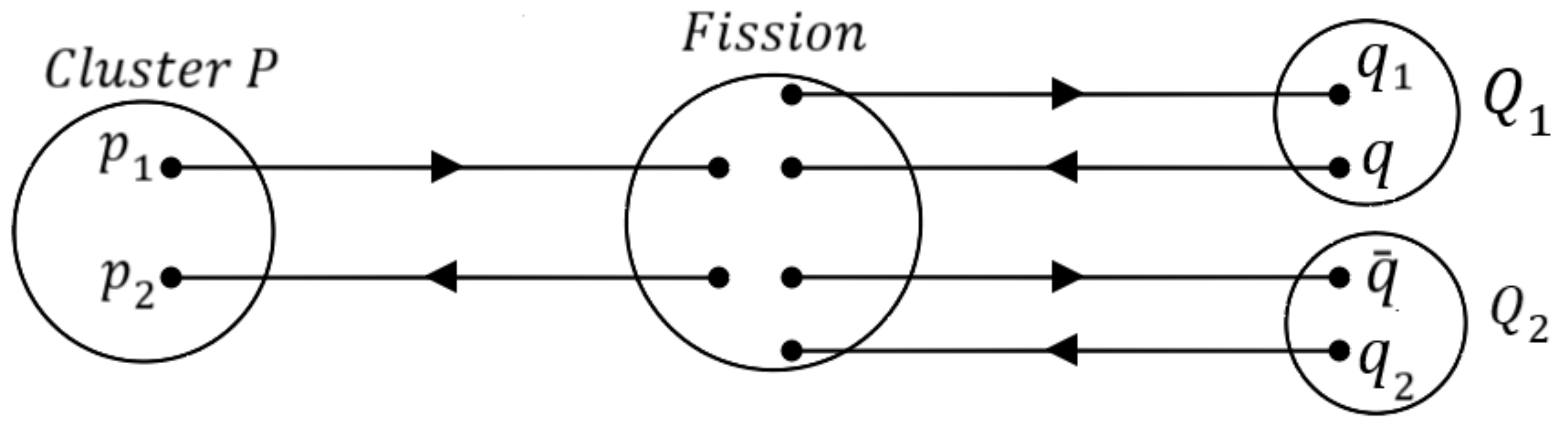
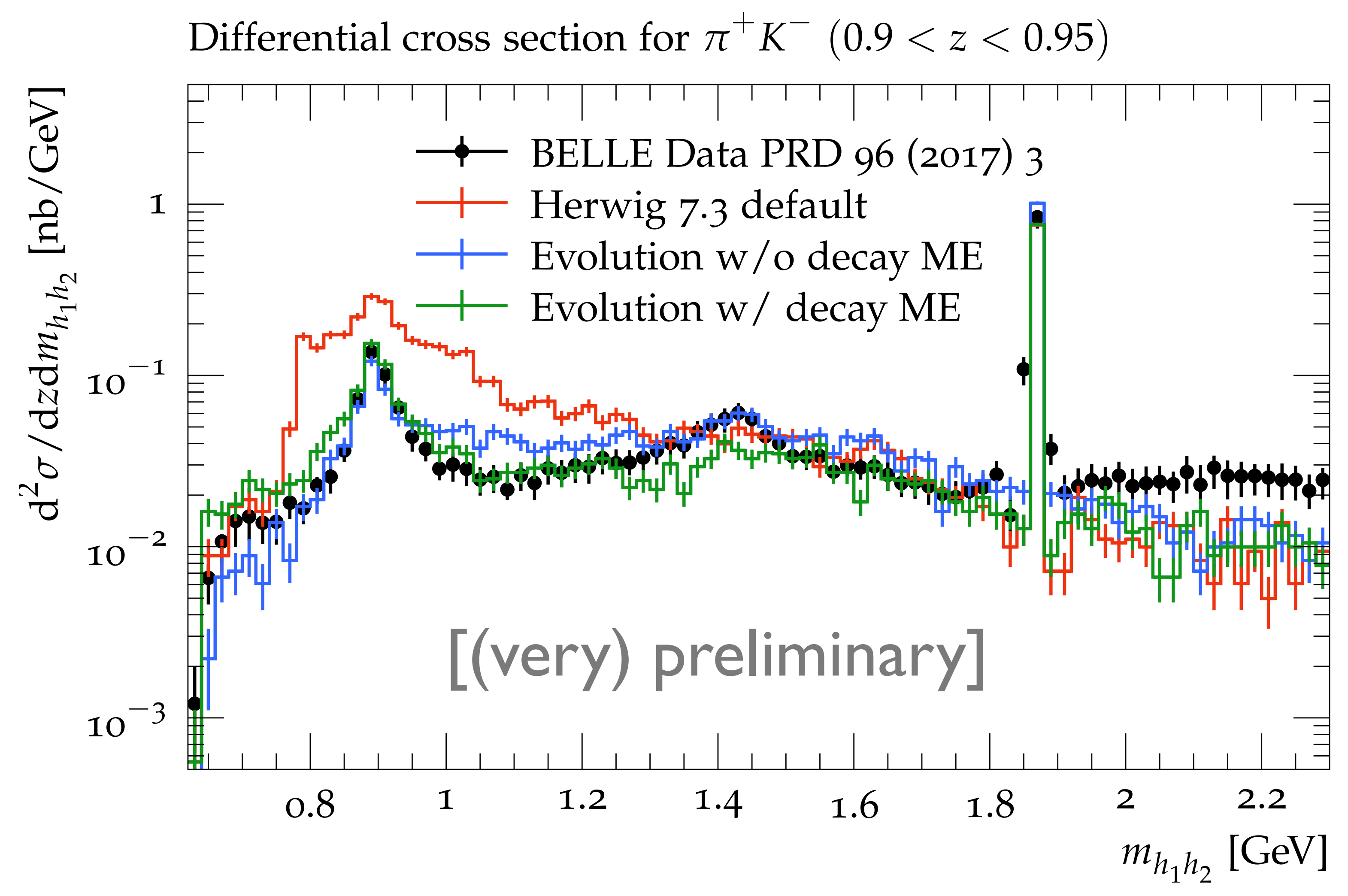


[figure by Jan Priedigkeit]



[Gieseke, Kiebacher, Plätzer, Priedigkeit — in progress]

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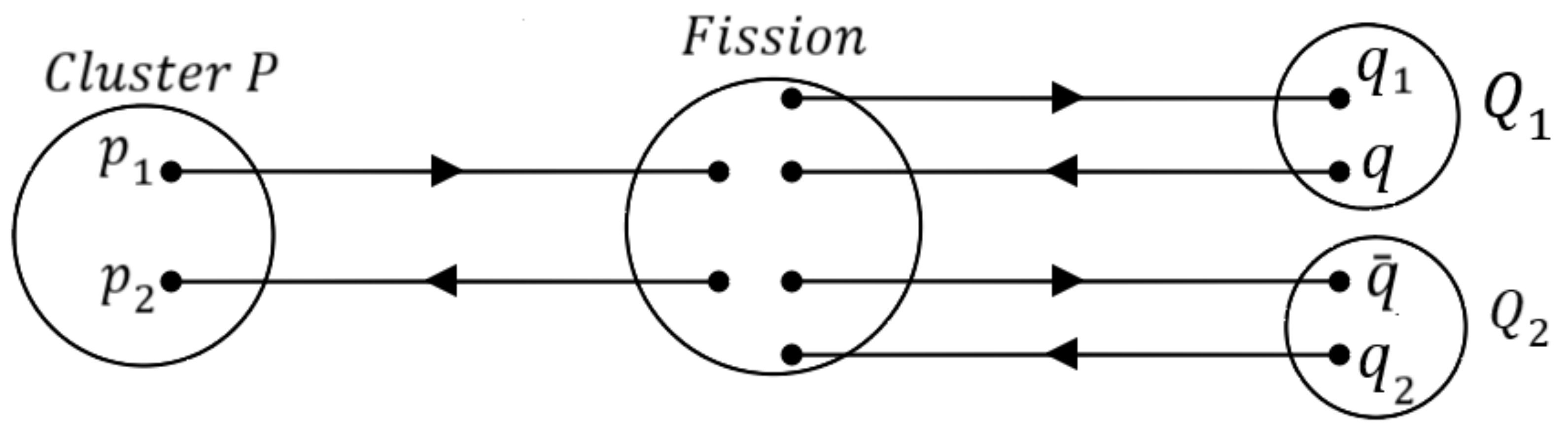

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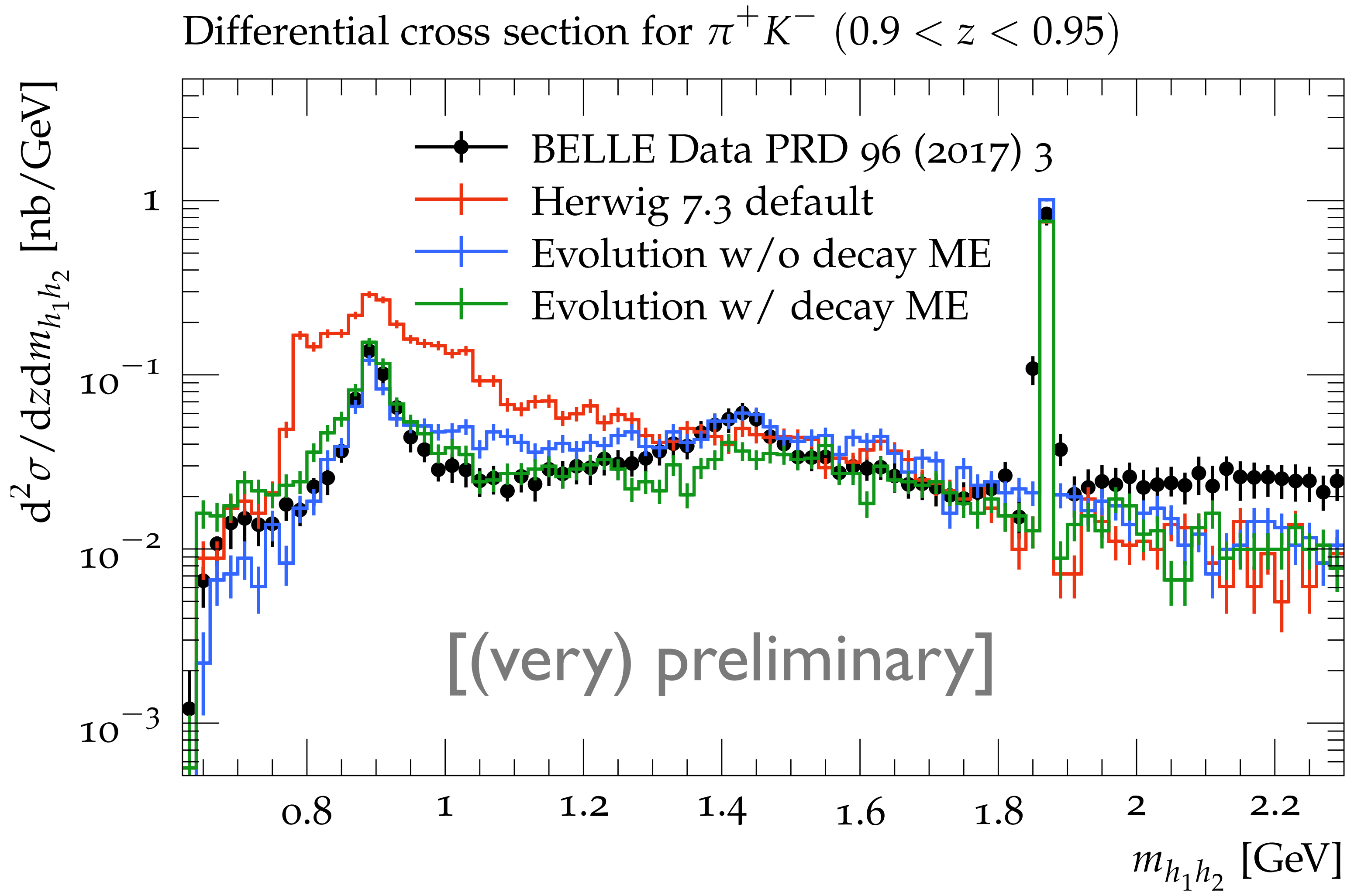
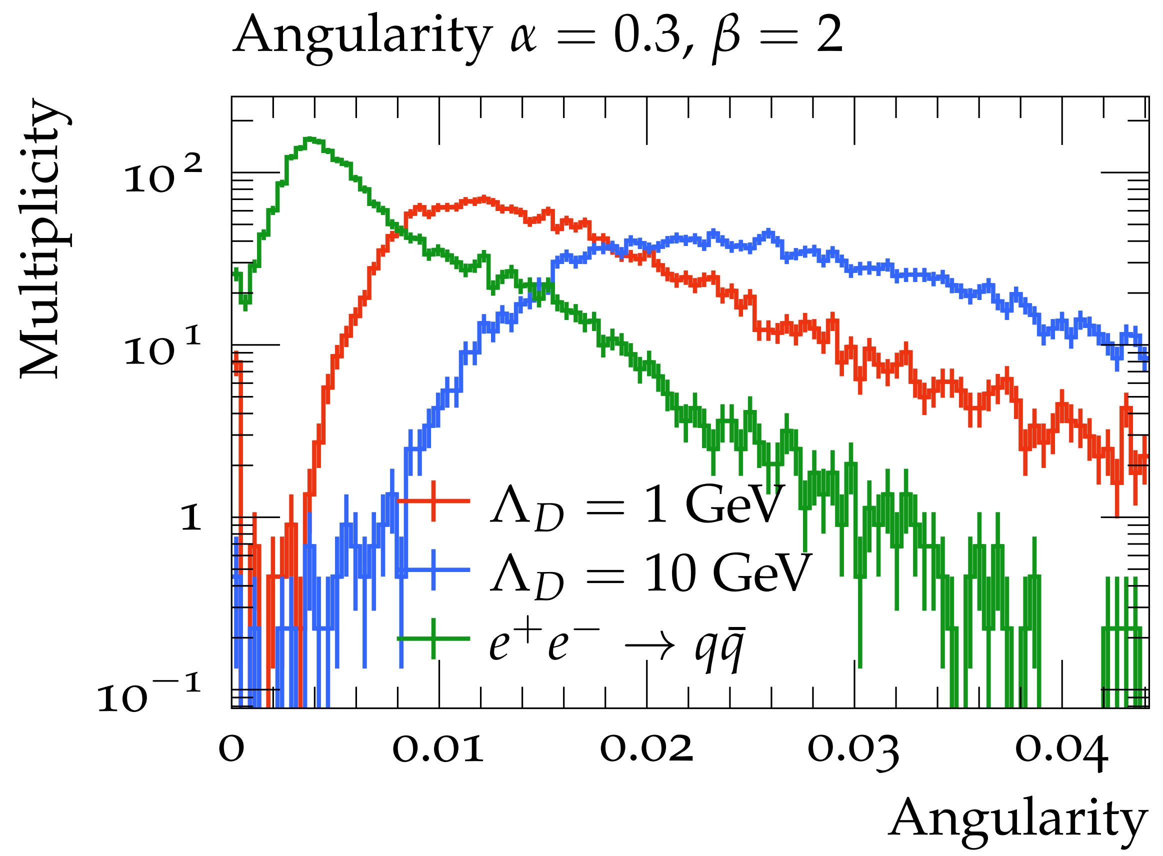


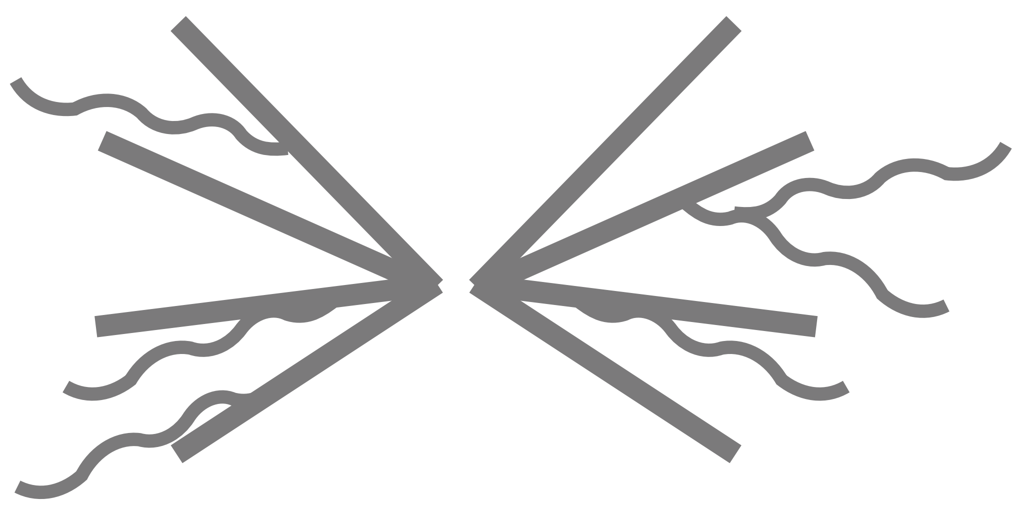
[Gieseke, Kiebacher, Plätzer, Priedigkeit — in progress]

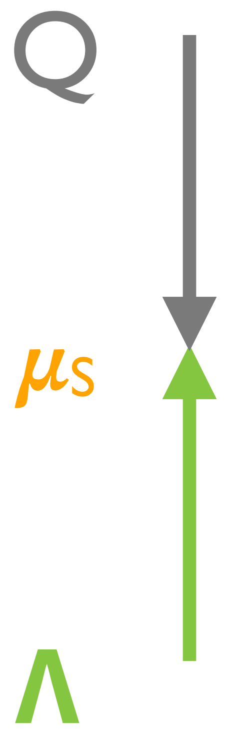
Most needed when hadronizing the unknown: strongly interacting dark matter.

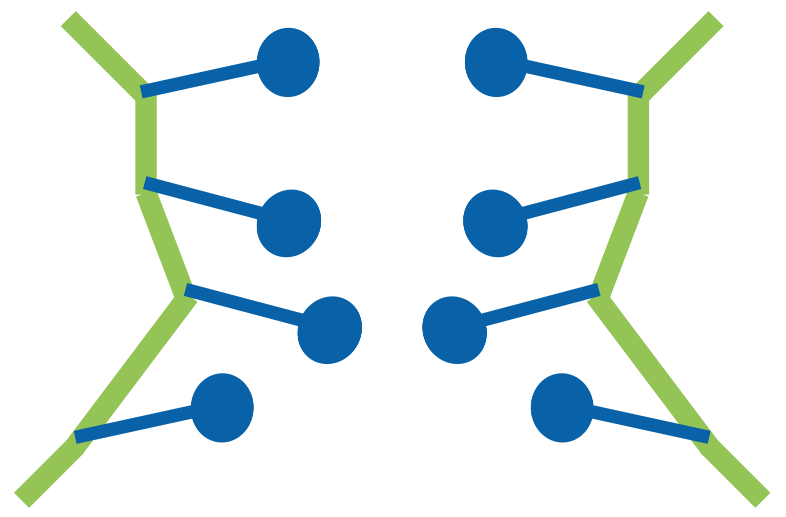


[Kulkarni, Massouminia, Plätzer, Stafford — in preparation]

Looking ahead — Foundations





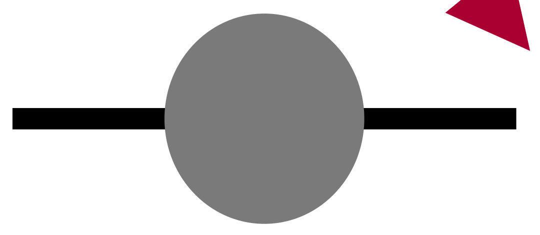


Generally we need to understand exclusive processes and factorisation, and (renormalised) LSZ and projections onto physical (singlet) final states.

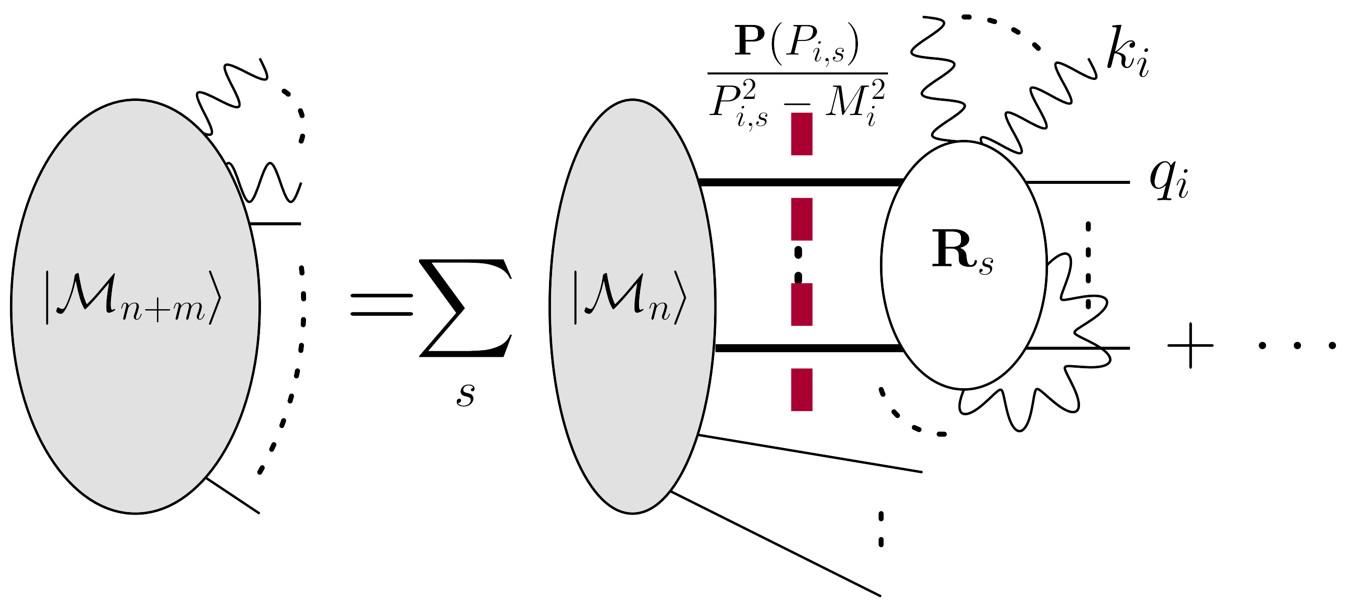
$$K^{\mu}_{i,s} = \Lambda^{\mu}{}_{\nu} \left(Q^{\nu}_{i,s} + \delta_{i,s}\, n^{\nu}_{i,s} \right)$$

$$q^{\mu}_{i} = \Lambda^{\mu}{}_{\nu} \left(\alpha p^{\nu}_{i} + \frac{(1-\alpha^2) M_i^2 + p_i \cdot Q_{i,s}}{2\alpha\, n_{i,s} \cdot p_i} n^{\nu}_{i,s} \right) - K^{\mu}_{i,s}$$

Momentum mappings to systematically factor renormalised matrix elements.

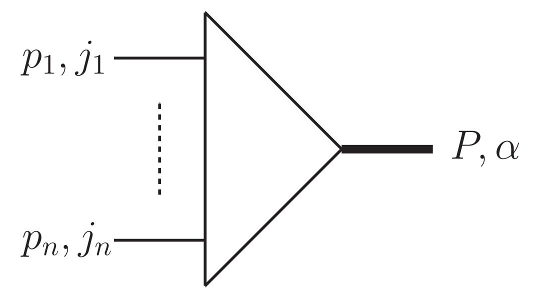


$$= \frac{1}{2 p_i \cdot Q_{i,s}} \frac{\Psi(\Lambda p_i, M_i) \bar{\Psi}(\Lambda p_i, M_i)}{1 - \Sigma'(M_i^2)} + \mathcal{O}(\lambda)$$

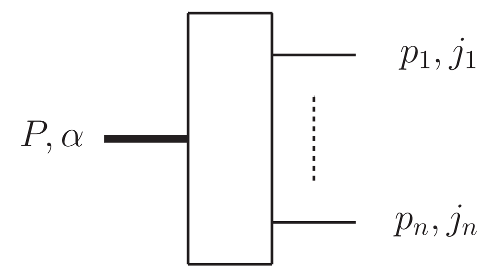


[Plätzer, Sjödahl — '22]

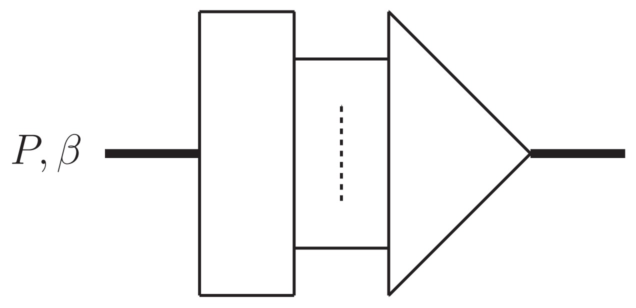
$$\chi_{\alpha|j_1,\dots,j_n}(\vec{P}, M|p_1,\dots,p_n)\,\delta\left(P - \sum_{i=1}^{n} p_i\right) =$$

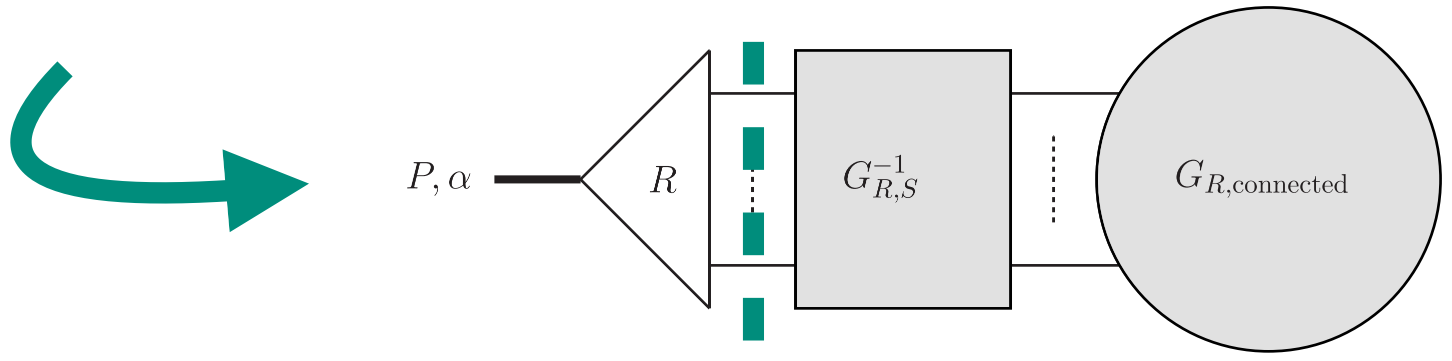


$$\left(Z_\Phi^{-1/2} \prod_{i=1}^{n} Z_{\phi_i}^{-1/2} \right) \bar{X}^{\alpha}(\vec{P}, M|p_1,\dots,p_n)\, u_{\alpha}^{j_1,\dots,j_n} =$$



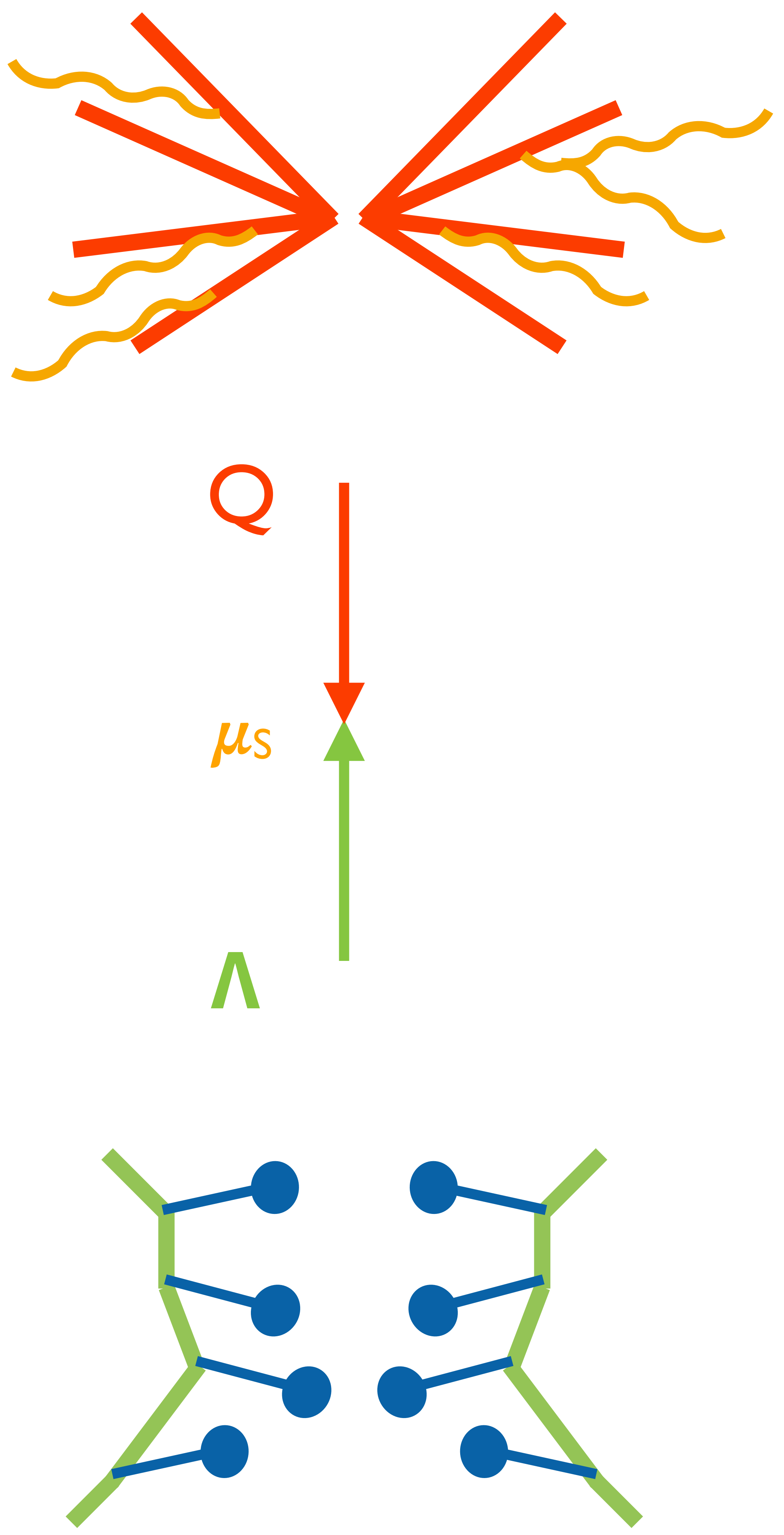
Composite particle scattering — for FMS as well as to study exclusive processes.





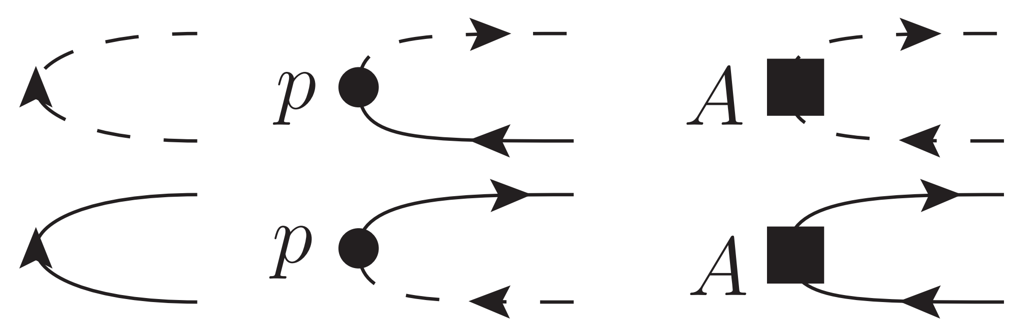
[Maas, Plätzer — in progress]

Looking ahead — Spin



Generally we need to understand exclusive processes and factorisation and projections onto physical (singlet) final states — including spin.

Find a basis of spin structures, together with isospin and colour.

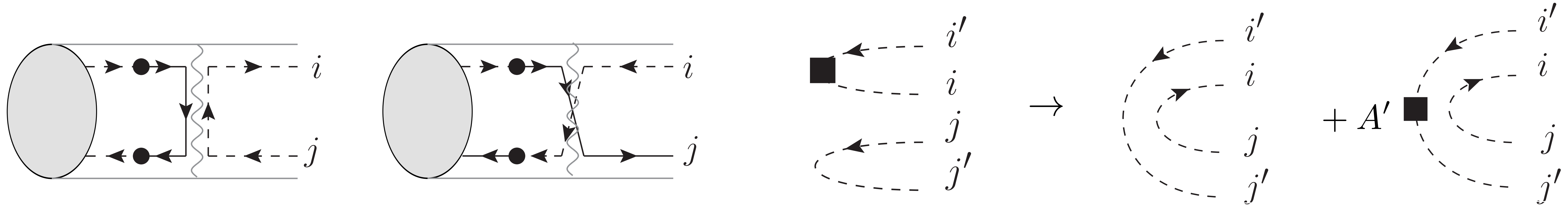


Essentially a basis of

$$\mathbf{1} \qquad \sigma^\mu \qquad \frac{1}{2}\left(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu\right)$$

Could this point to a more general version of setting up graphical tensor calculus?

Electroweak bosons now mix different chiral basis states.



[Plätzer, Sjödahl — '22]

Summary

Colour space evolution equations are an exiting theoretical tool to build parton shower and resummation algorithms, and an important subject in their own right to study structures we can expect from QCD evolution.

For event generators, parton shower accuracy is crucial, but hadronization is the elephant in the room. Together with parton showers we need to get this back on top of the agenda.

Factorisation of infrared physics will also teach us about the development of hadronization models and colour reconnection, but also the possible connections to other evolution equations like JIMWLK.

The framework outlined here has significant room for extensions and further analytic and simulation work, either in or beyond the large-N limit.

Thank you