



Pinning Down A New Approach To Cluster Hadronization

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Introduction to Monte Carlo Event Generators

Monte Carlo event generators:

- Simulation of high energy particle collisions
- Different codes such as Herwig, Sherpa and Pythia

Stages of hadronization model in Herwig:

- 1. Gluon splitting to $q\bar{q}$
- 2. Primordial cluster formation
- 3. Colour Reconnection (CR)
- 4. Cluster Fission (CF)
- 5. Cluster Decay (CD)





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Cluster Hadronization

- Cluster model [Marchesini and Webber 1984] is based on preconfinement [Amati and Veneziano 1979] was designed to be so simple to be easily excluded by data
 - \Rightarrow Still around and **unreasonably good** at describing data
- Many ad-hoc phenomenological modelling choices compared to Lund string model
- In the context of colour evolution it is not infra-red safe [Platzer 2022]
- In the cluster model we need to impose kinematics. How can we test these imposed kinematics?

\rightarrow Correlations of hadrons!

Outline:

- Describe the current cluster model
- Show the current failures of the cluster model in correlation data
- Present parts of a new cluster model, which show interesting features
- Show improvements/effects on observables at e^+e^- at different \sqrt{s}
- Show sensitivity of some observables to different parts of the cluster model

Cluster Fission (CF)



- Fission all clusters $M > M_{\max}(q_1, \bar{q}_2)$ above a threshold $M_{\max}(q_1, \bar{q}_2)$ recursively
- 1. Draw a light $q \bar{q}$ pair from the vacuum with probability P_q (no diquarks currently allowed!)
- 2. Draw new masses $M_1, M_2 \sim (M_1 M_2)^{\alpha}$ for the fission products C_1, C_2
- 3. Choose Direction of decay \Rightarrow Currently aligned with the original constituent momenta



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Cluster Decay (CD)

- Clusters decay to two hadrons
- Essentially the same as Cluster fission with a few differences:
 - 1. The masses are fixed by the hadron masses
 - 2. Mesonic clusters $C(q_1, \bar{q}_2)$ can decay to a baryon-antibaryon pair $B(q_1, (q, q')), \bar{B}'(\bar{q}_2, (\bar{q}, \bar{q}'))$
 - 3. The direction of decay is chosen **isotropically** in the cluster rest frame (except for *b*, *c* containing clusters)
- Note: High-mass clusters can produce baryons!





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Baryon-Antibaryon Angular Correlations

- Cluster Decay (CD) baryons are responsible for unphysical far-side peak
- Cluster Fission (CF) baryons solve the unphysical far-side peak
- Switching CF baryons on and CD baryons off is not the full story:
 - Soft peak of p_T spectra not present [Adam et al. 2015]
 - At LEP meson in the middle pMp
 is ill-modelled [Abreu et al. 2000]



Figure: measured by ALICE [Adam et al. 2017]



Hadronization From Colour Evolution

Idea of colour evolution picture [Plätzer 2023]:

- Shower Cut-off is a factorization scale (see also [Hoang et al. 2024])
- Parton shower, Colour Reconnection and Hadronization (Cluster Fission) should be matched
- Final non-perturbative scale (NP scale) interfaces to the initial condition

Consequence: Cluster Fission is a perturbative process similar to the parton shower. Only Cluster Decay is non-perturbative (could be obtained from lattice QCD)



New Cluster Fission Kinematics



Consequence: CF is a partonic 2 \rightarrow 4 process [Plater 2023]. CF should continue the "job" of the parton shower. • Factorize the process $C(p_1, p_2) \rightarrow C_1(q_1, q), C_2(q_2, \bar{q})$ (see Jan Priedigkeit's Bachelor thesis Graz):

$$d\Gamma(C \to C_1, C_2) = \int d\Phi_4(P|q_1, q, \bar{q}, q_2) |\mathcal{M}(p_1, p_2 \to q_1, q, \bar{q}, q_2)|^2$$
(1)
$$d\Gamma(C \to C_1, C_2) \propto \int dM_1^2 dM_2^2 d\Phi_2(P|Q_1, Q_2) d\Phi_2(Q_1|q_1, q) d\Phi_2(Q_2|q_2, \bar{q}) |\mathcal{M}(p_1, p_2 \to q_1, q, \bar{q}, q_2)|^2$$
(2)



New Cluster Fission Kinematics



 Sample Masses M₁, M₂ from flat Phase Space weight dΦ₄ ∝ dM₁dM₂√λ(M, M₁, M₂)√λ(M₁, m₁, m)√λ(M₂, m₂, m)/(M₁M₂)
 Importance sampling of soft qq̄ emission diagram, which in the soft limit factorizes [Catani and Grazzini 2000]:

$$|\mathcal{M}(p_1, p_2 \to q_1, q, \bar{q}, q_2)|^2 \propto |\mathcal{M}(p_1, p_2 \to q_1, q_2)|^2 \frac{2(q_1 \cdot q_2)(q \cdot \bar{q}) + [q_1 \cdot (q - \bar{q})][q_2 \cdot (q - \bar{q})]}{2(q \cdot \bar{q})^2[q_1 \cdot (q + \bar{q})][q_2 \cdot (q + \bar{q})]}$$



New Cluster Decay Kinematics



- Idea: Again describe cluster decay by an "effective" matrix element
- Which matrix element? \Rightarrow t-channel like to preserve local parton-hadron duality
- \blacksquare Avoid hard transition from $\mathsf{CF} \to \mathsf{CD}$

$$egin{aligned} \mathcal{M}_{ ext{CD}}(p_1,p_2 o p_{h_1},p_{h_2})|^2 &\propto rac{1}{[(p_1-p_3)^2-M_S^2]^2} &\propto rac{1}{[A-\cos(heta)]^2} \ ext{with } A>1 ext{ if } M_S = \max\{(m_1-m_{h_1}),(m_2-m_{h_2})\} \end{aligned}$$





Di-hadron mass spectra

- Observable probes final cluster mass distribution m_{h_1,h_2} for $z \lesssim 1$
- Default gives unphysical plateau
- Mass phase space resolves the behaviour
- Cluster Decay (CD) has impact
- Missing resonance due to hard cutoff for CF



Figure: measured by BELLE [Seidl et al. 2017a]



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pT in/out spectra

- LEP at the Z pole
- Default isotropic CD spits multiplicity in/out of plane
- Aligned CD overcorrects for this
- t-channel CD gives most reasonable prediction



Figure: measured by DELPHI [Abreu et al. 1996]



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Generalized Angularities

- LEP at the Z pole
- Generalized angularities $\lambda_{\beta}^{\alpha} = \sum_{i \in jet} z_i^{\beta} \theta_{i,jet}^{\alpha}$ with $z_i = \frac{E_i}{E_{vis}}$

[Gras et al. 2017; Larkoski, Moult, and Neill 2014]

 Discriminative power between CF and CD



Figure: Generalized angularities $\alpha = {\rm 0.3}, \beta = {\rm 1.25}$



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Figure: Generalized angularities $\alpha=\mathbf{2},\beta=\mathbf{1}$



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Figure: Generalized angularities $\alpha = \mathbf{2}, \beta = \mathbf{0.1}$



Energy-weighted Correlations

- LEP at the Z pole
- Energy-weighted correlations $EEC^{\gamma}(\rho) = \frac{1}{N_{\gamma}} \sum_{i < j \in jet} (E_i E_j)^{\gamma} \delta(\Delta R_{ij} - \rho)$ with $N_{\gamma} = \sum_{i < j \in jet} (E_i E_j)^{\gamma}$
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Figure: Correlations for $\gamma = 2.0$



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- Discriminative power between CF and CD



Figure: Correlations for $\gamma = -2.0 \Rightarrow$ "Infrared dangerous"

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Summary and Outlook

Summary:

- Found shortcomings of the kinematics of the cluster model in the correlations
- Try to address these issues with a new cluster hadronization model, motivated by colour evolution
- Found observables which may be able to discriminate between different stages of hadronization

Outlook for hadronization in Herwig:

- Examine the model with a dynamic CF threshold as is the default in Herwig 7.3.x [Bewick et al. 2023]
- Merge our work with dynamic gluon constituent masses and cluster fission [Hoang et al. 2024]
- Make Colour Reconnection dynamic via soft gluon evolution [Gieseke et al. 2018; Plätzer 2023] (WIP with S. Plätzer, S. Gieseke)

Thank You For Your Attention!

Questions? Remarks? Comments?

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Backup: Meson in the middle

If we only consider baryon production due to CF we ill-model rapidity rank structure



Figure: Rapidity rank structure by DELPHI only CD or only CF baryon mechanisms [Abreu et al. 2000]



Backup: Spectra of Protons

• Proton p_T -spectra are badly modelled



Figure: Compare p_T - spectra of p for only BCR, only CD or only CF baryon mechanisms [Adam et al. 2015]



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Figure: Compare p_T - spectra of p for only new DCR baryon mechanism with different probabilities [Adam et al. 2015]



Backup: Cluster Fission vs Cluster Decay Baryons

- CD baryon mechanism vs new Cluster Fission (CF) mechanism
- Near-side depletion not reproduced
 - \Rightarrow CD and CF are oblivious to other baryons





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 Near-side depletion not reproduced

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Preliminary: New Cluster Fission Kinematics applied

 BELLE data for di-hadron mass spectrum improve for large z with Preliminary results



Figure: Thrust $T>0.8, \vec{p}_1\cdot\vec{p_2}>0$ and $z=rac{2(E_1+E_2)}{\sqrt{s}}$ [Seidl et al. 2017b]



Preliminary: New Cluster Fission Kinematics applied

- BELLE data for di-hadron mass spectrum improve for large z with Preliminary results
- However still very inefficient rejection sampling for large mass clusters (LEP manageable; LHC way too slow)



Figure: Thrust $T>0.8, \vec{p}_1\cdot\vec{p_2}>0$ and $z=rac{2(E_1+E_2)}{\sqrt{s}}$ [Seidl et al. 2017b]

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Backup: Tuning







Strategy:

1. Perform a dedicated tune to LEP multiplicities, event shapes and momentum spectra for CF, CR parameters



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- 2. Keep CF and flavour parameters fixed and tune CR and MPI parameters to LHC multiplicities, *p*_T-spectra and angular correlations



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 - For a large set of observables χ² as a measure to minimize is not neccesarily the most suitable one (large deviations in some bins may drive the system to odd regions of parameter space)



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Possible solutions: Use of a different "Loss function" than regular χ^2 e.g. $\chi^2 \rightarrow \frac{\chi^2}{1+\chi^2}$ or tanh(χ^2)

Backup: Cluster Fission Details



- A cluster of mass *M* is fissioned if M^{Cl_{pow}} ≥ Cl^{Cl_{pow}_{max} + (m₁ + m₂)^{Cl_{pow}, where m₁, m₂ are the masses of the constituents of the cluster}}
- Currently masses are sampled as follows, where $r_1, r_2 \in [0, 1]$ are uniform random numbers [Bahr et al. 2008]:

$$M_{1} = m_{1} + (M - m_{1} - m_{q})r_{1}^{\frac{1}{P_{\text{split}}}}$$
(3)

$$M_2 = m_2 + (M - m_2 - m_q) r_2^{\frac{1}{P_{\text{split}}}}$$
(4)

- Reject samples if $M_1 + M_2 > M$
- Problems: huge dependence on parameters Cl_{max} and especially P_{split}
- Work in progress: Sample masses according to phase space

Backup: Angular Correlations



- The shown plots are showing correlations integrated in $\Delta\eta$ up to $\Delta\eta_{max}=$ 1.3
- The angular correlations are measured via the event mixing [Adam et al. 2017]:

$$C_i(\Delta\phi, \Delta\eta) = \frac{S(\Delta\phi, \Delta\eta)}{B(\Delta\phi, \Delta\eta)}$$
(5)

$$S_{i}(\Delta\phi,\Delta\eta) = \frac{1}{N_{\text{pairs}}^{\text{same}}} \frac{d^{2}N_{\text{pairs}}^{\text{same}}}{d\Delta\eta d\Delta\phi}$$
(6)

$$B_{i}(\Delta\phi,\Delta\eta) = \frac{1}{N_{\text{pairs}}^{\text{mixed}}} \frac{d^{2} N_{\text{pairs}}^{\text{mixed}}}{d\Delta\eta d\Delta\phi}$$
(7)

$$C_{i}(\Delta\phi) = \int_{0}^{\Delta\eta_{\max}} C_{i}(\Delta\phi, \Delta\eta) d\Delta\eta$$
(8)

Spectra of Protons





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Figure: Compare p_T – spectra of Λ , \equiv for only BCR, only CD or only CF baryon mechanisms [Khachatryan et al. 2011]





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Consistent Two Particle Boost



- If we boost a two particle system P = (p_i + p_j) into its rest frame P̂ = (p̂_i + p̂_j) one needs to be careful to tranform the relative momentum correctly P̂_{rel} = (p̂_i p̂_j)
- The naive transformation would be to just use Λ_(-P), but this would give in general P̂_{rel} = (p̂_i p̂_j + 2k), because Λp̂_i = p_i + k and Λp̂_j = p_j k.
- Intuitively the momentum P is completely oblivious to its components and therefore Λ must depend on both the consituents p_i, p_j
- Want a Lorentz Tranformation (matrix or tensor) $\Lambda(p_i, p_j | \hat{p}_i, \hat{p}_j)$ such that $\Lambda \hat{p}_i = p_i$ and $\Lambda \hat{p}_j = p_j$
- Found solution for $\Lambda(p_i, p_j | \hat{p}_i, \hat{p}_j)$, but numerically not very easy
- Work in Progress: Tensor for this trafo $\Lambda^{\nu}_{\ \mu}$