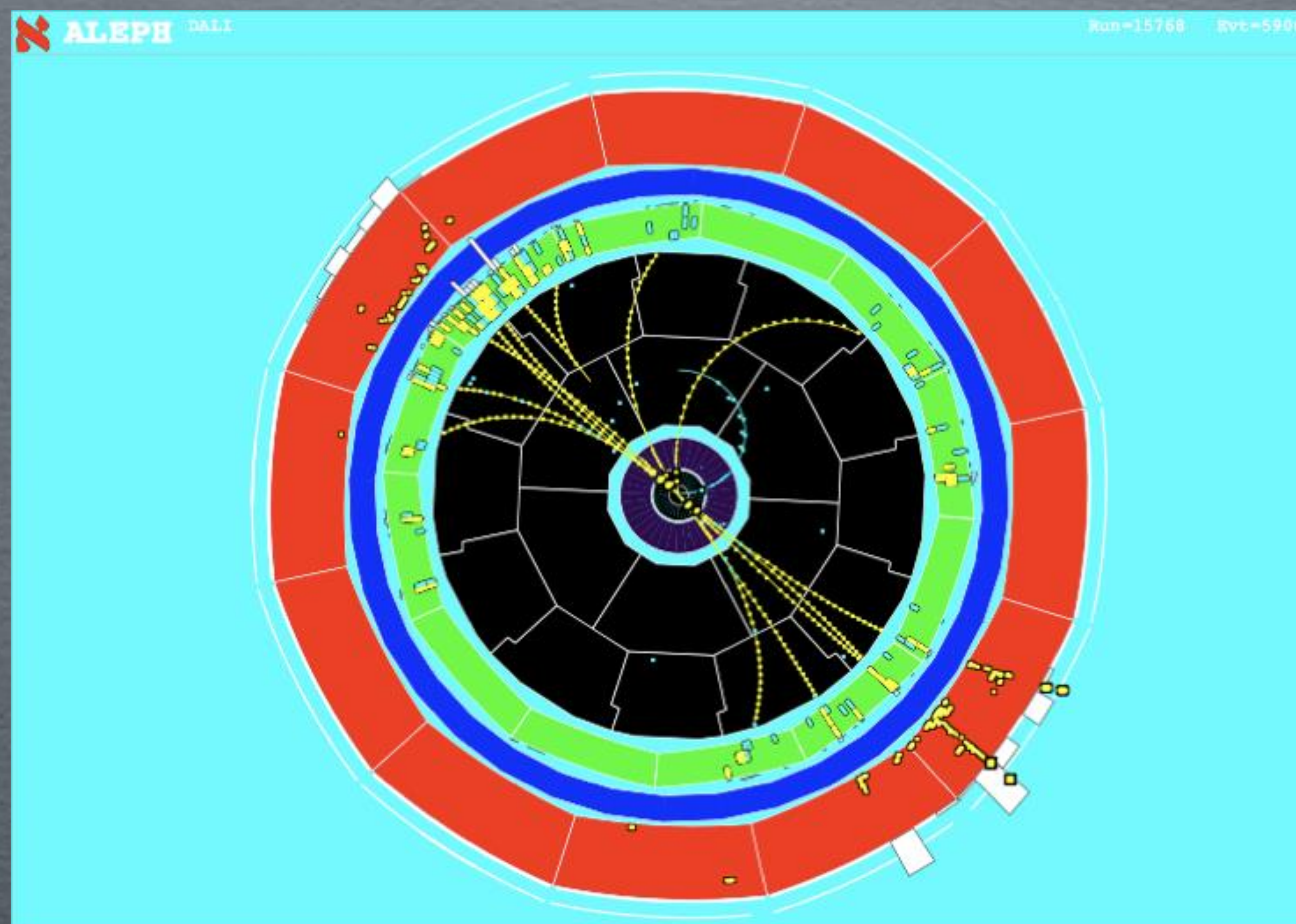
Hadronisation corrections with the ARES method

Andrea Banfi



University of Sussex

PSR2024 – 5 July 2024 – Graz

Simultaneous PT-NP fits

- Leading 1/Q hadronisation corrections can be theoretically modelled in terms of the emission of a single ultra-soft gluon $\Rightarrow$ simultaneous fit of α_s and a single NP parameter for different event shapes

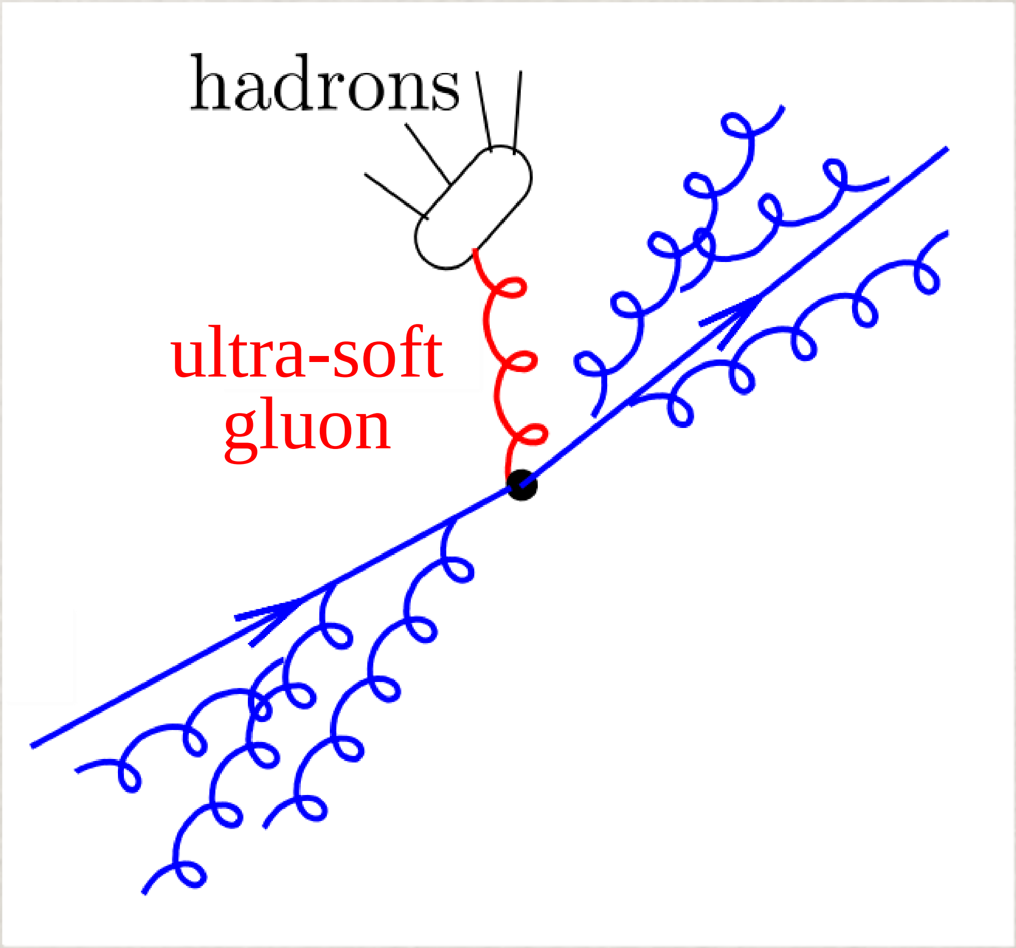
Universal (?) NP parameter

Observable dependent but calculable

$$\text{shift} = \frac{\langle k_t \rangle_{\rm NP}}{Q} \langle h_V \rangle_{\rm PT}$$

$$\langle h_V \rangle_{\rm PT} \equiv \int d\eta \frac{d\phi}{2\pi} \langle h_V(\eta,\phi) \rangle$$

Average over PT configurations



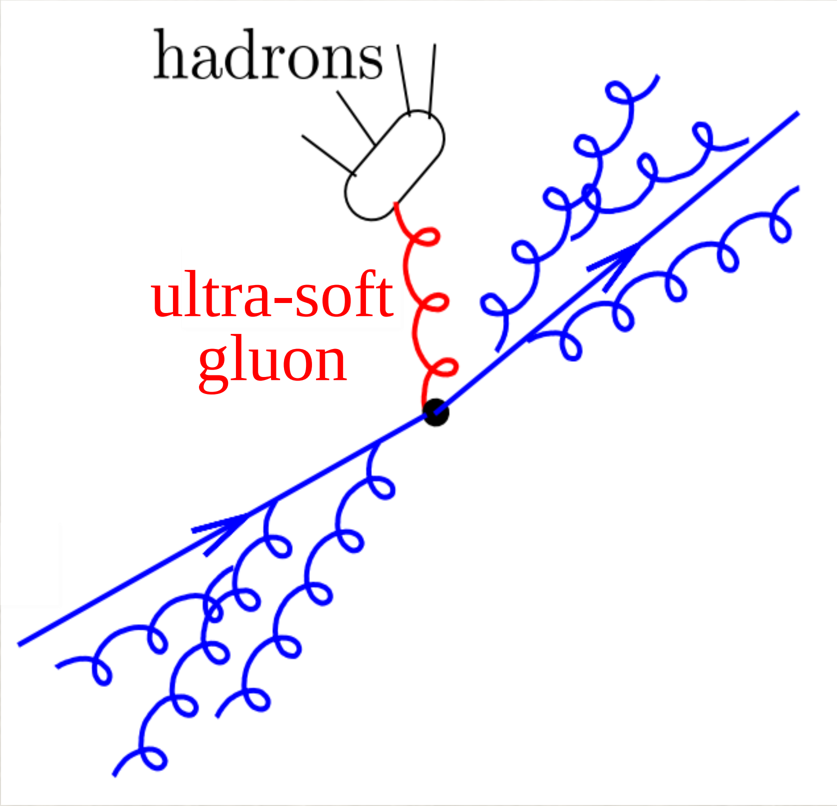
Origin of the NP shift

- Leading hadronisation corrections can be modelled in terms of the emission of a non-perturbative (NP) ultra-soft gluon k

$$\Sigma(v) \simeq \Sigma_{\rm PT}(v) + \delta\Sigma_{\rm NP}(v)$$

$$\delta\Sigma_{\rm NP}(v) = \int \underbrace{dZ[\{k_i\}]}_{=\text{PT gluons}} [dk]\mathcal{M}^2_{\rm NP}(k)\times$$

$$\times\left[\underbrace{\Theta\left(V(\{\tilde p\},\{k_i\},k)-v\right)}_{k\ \text{real}} - \underbrace{\Theta\left(V(\{\tilde p\},\{k_i\})-v\right)}_{k\ \text{virtual}}\right]$$



- In the two-jet region, the effect of k is much smaller than $V(\{\tilde p\},\{k_i\})$

$$\Theta\left(V(\{\tilde p\},\{k_i\},k)-v\right)-\Theta\left(V(\{\tilde p\},\{k_i\})-v\right) \simeq -\delta V_{\rm NP}(\{\tilde p\},\{k_i\},k)\,\delta\left(V(\{\tilde p\},\{k_i\})-v\right)$$

$$\delta\Sigma_{\rm NP}(v) \simeq -\int dZ[\{k_i\}][dk]M^2_{\rm NP}(k)\delta V_{\rm NP}(\{\tilde p\},\{k_i\},k)\delta\left(V(\{\tilde p\},\{k_i\})-v\right)$$

$$= -\langle\delta V_{\rm NP}\rangle\frac{d\Sigma_{\rm PT}}{dv} \implies \boxed{\Sigma(v) \simeq \Sigma\left(v-\langle\delta V_{\rm NP}\rangle\right)}$$

Calculation of the NP shift

- For most observables, $\delta V_{\rm NP}(\{\tilde{p}\},\{k_i\},k) \sim k_t/Q \ll v$, more precisely

$$\delta V_{\rm NP}(\{\tilde{p}\},\{k_i\},k) = \frac{k_t}{Q} h_V(\{p\},\{k_i\},\eta,\phi)$$

- Local parton-hadron duality: emission of ultra-soft gluons is uniform in rapidity and azimuth, same as PT soft gluons

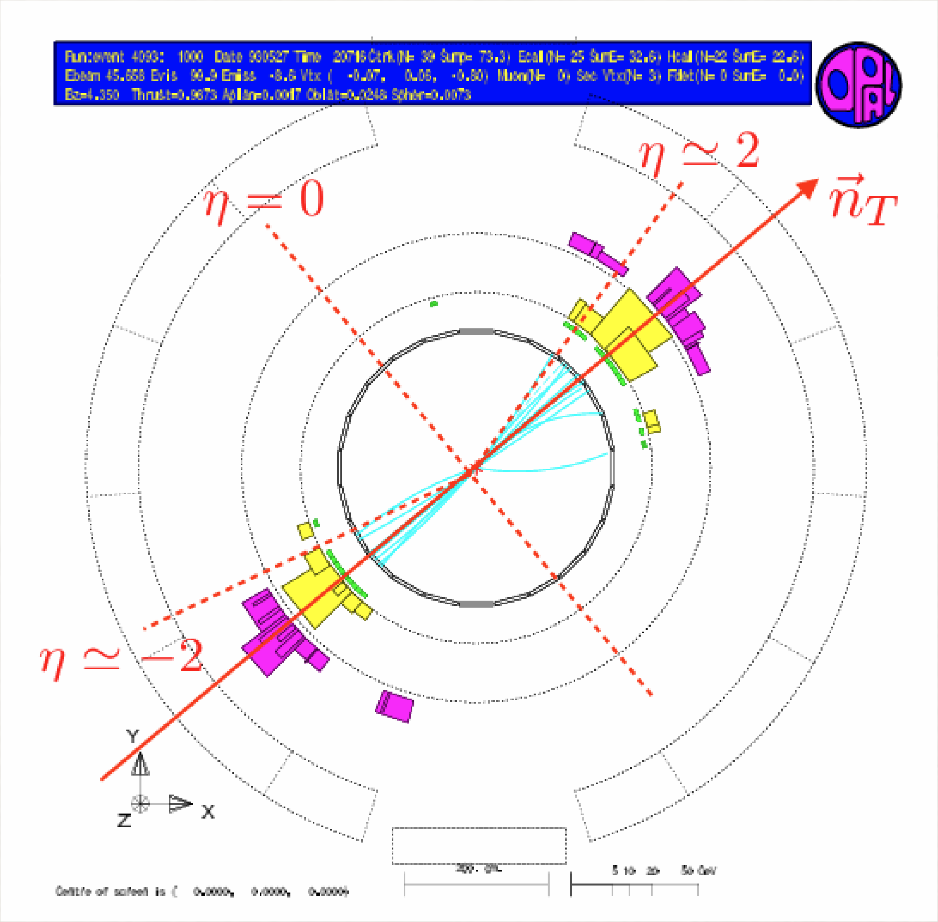
$$[dk]\mathcal{M}^2_{\rm NP}(k) = \frac{dk_t}{k_t} M^2_{\rm NP}(k_t) d\eta \frac{d\phi}{2\pi}$$

$$\Downarrow$$

$$\langle \delta V_{\rm NP} \rangle = \frac{\langle k_t \rangle_{\rm NP}}{Q} \langle h_V \rangle$$

$$\langle k_t \rangle_{\rm NP} \equiv \int dk_t \, M^2_{\rm NP}(k_t)$$

$$\langle h_V \rangle = \frac{\left\langle \int d\eta \frac{d\phi}{2\pi} h_V(\{\tilde{p}\},\{k_i\},\eta,\phi) \delta\left(1 - \frac{V_{\rm sc}^{\rm NLL}(\{\tilde{p}\},\{k_i\})}{v}\right) \right\rangle}{\left\langle \delta\left(1 - \frac{V_{\rm sc}^{\rm NLL}(\{\tilde{p}\},\{k_i\})}{v}\right) \right\rangle}$$

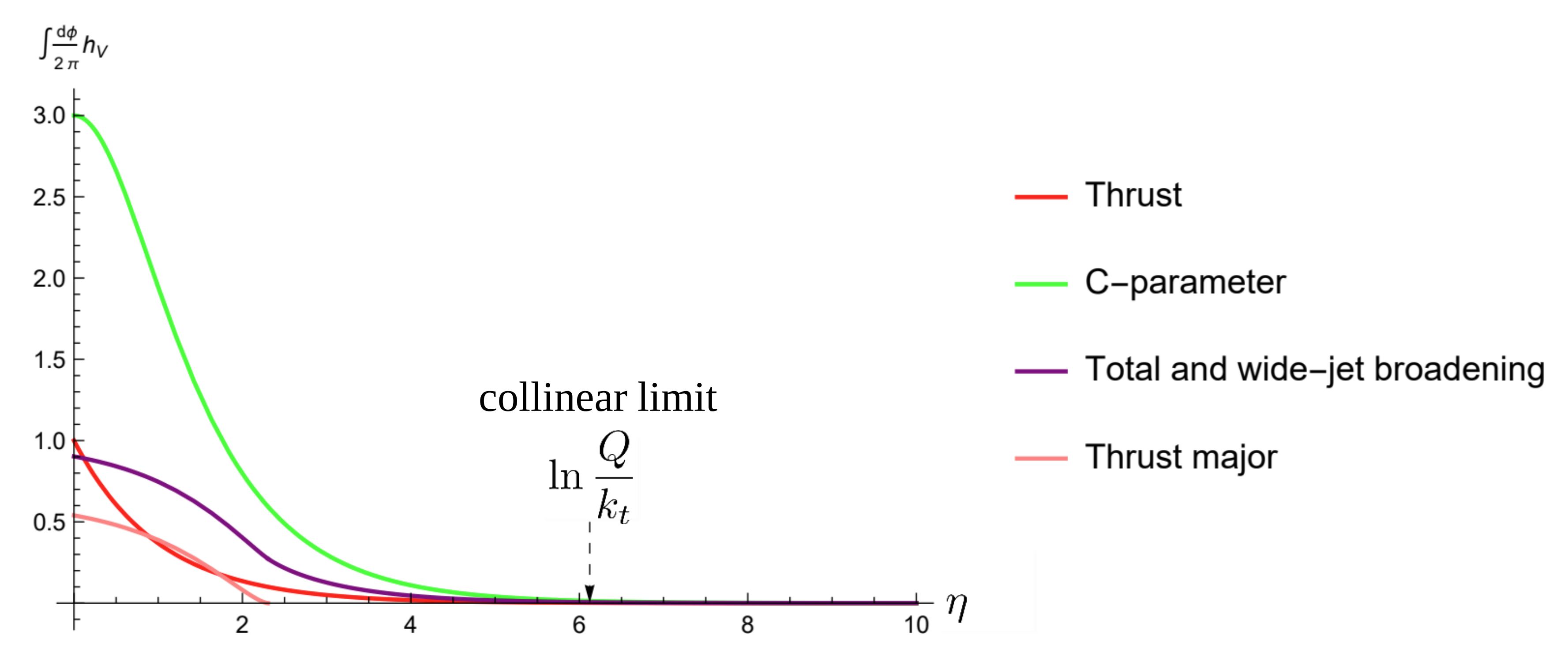


BEHAVIOUR OF NP CORRECTION

- Let us look in more detail at the expression for $\langle h_V \rangle$

$$\Sigma_{\mathrm{PT}}(v) \simeq e^{-R(v)}\mathcal{F}_{\mathrm{NLL}}(R')$$
$$\Downarrow$$
$$\langle h_V \rangle = \frac{\left\langle \int d\eta \frac{d\phi}{2\pi} h_V(\{\tilde{p}\},\{k_i\},\eta,\phi)\delta\left(1-\frac{V_{\mathrm{sc}}^{\mathrm{NLL}}(\{\tilde{p}\},\{k_i\})}{v}\right)\right\rangle}{R'\mathcal{F}_{\mathrm{NLL}}(R')}$$

- The rapidity integral implicitly extends up to $\pm\infty$: is it convergent?



- The answer is YES for all known event shapes, no extra k_t dependence from $\langle h_V \rangle$

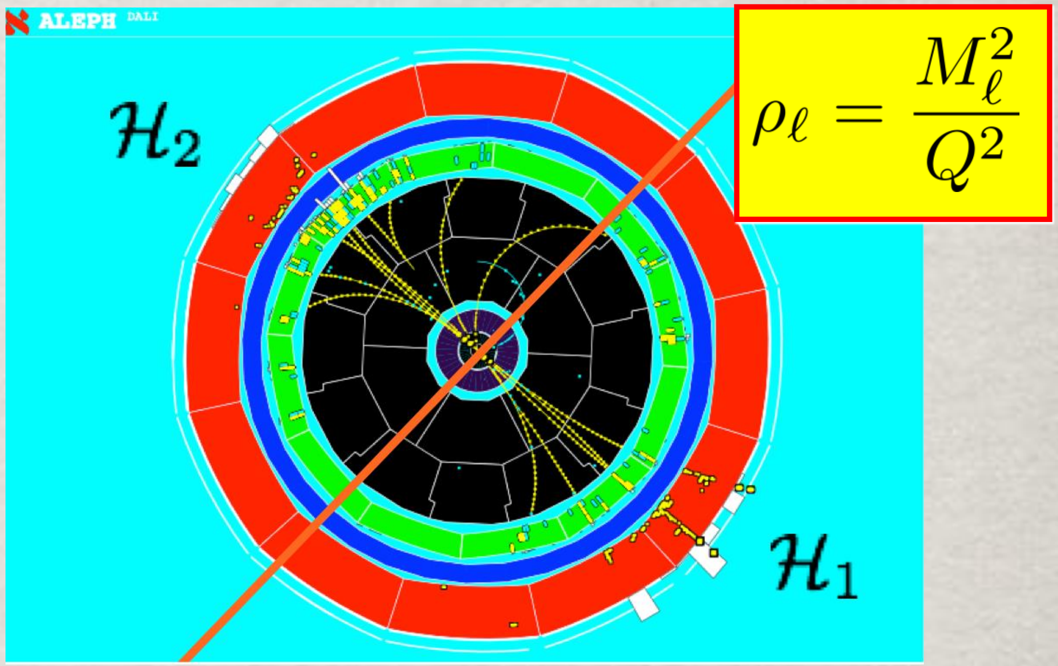
Notable examples

- Thrust and C-parameter

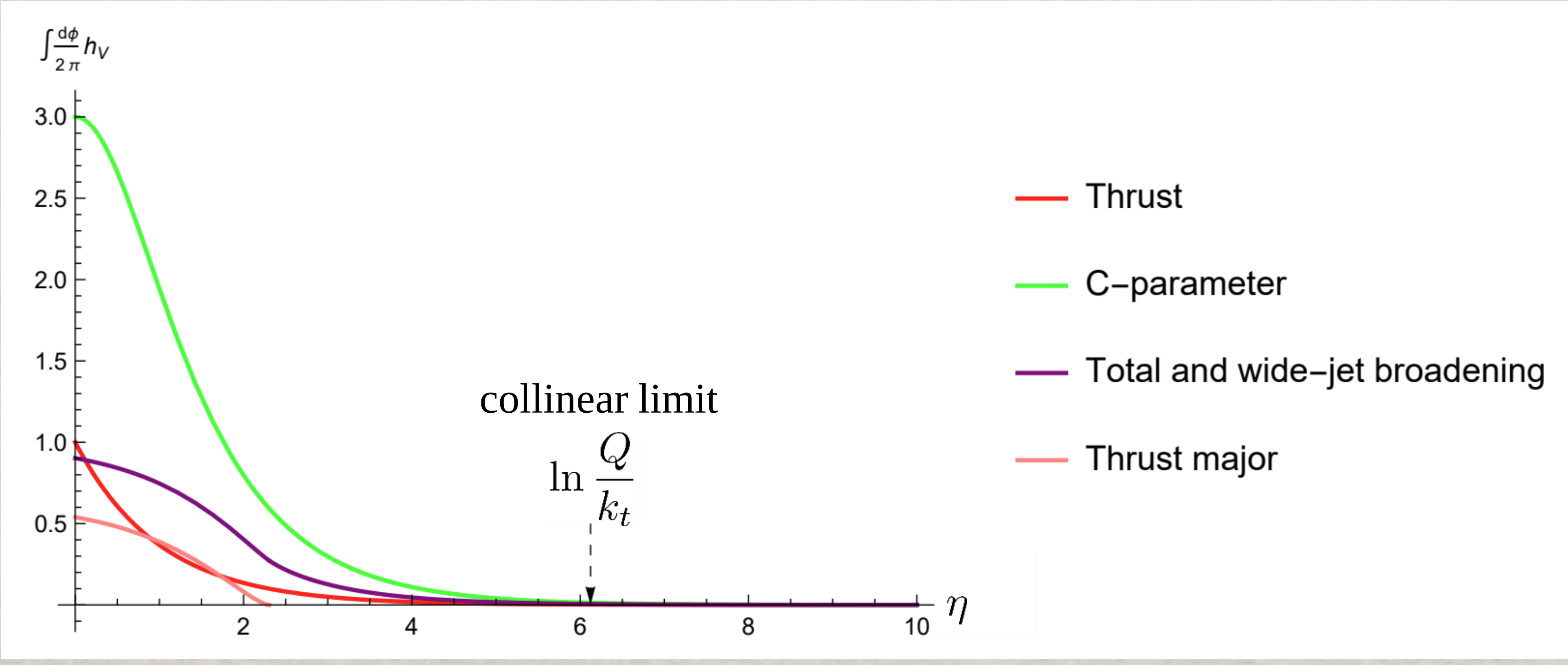
$$h_{1-T} = e^{-|\eta|} \qquad h_C = \frac{3}{\cosh\eta}$$

- Heavy-jet mass: in the presence of PT emissions, an ultra-soft gluon cannot change which hemisphere is the heavier

$$h_{\rho_H} = e^{-\eta}\Theta(\rho_1 - \rho_2) + e^{\eta}\Theta(\rho_2 - \rho_1)$$



- The heavy-jet mass is the simplest example of PT-NP interplay

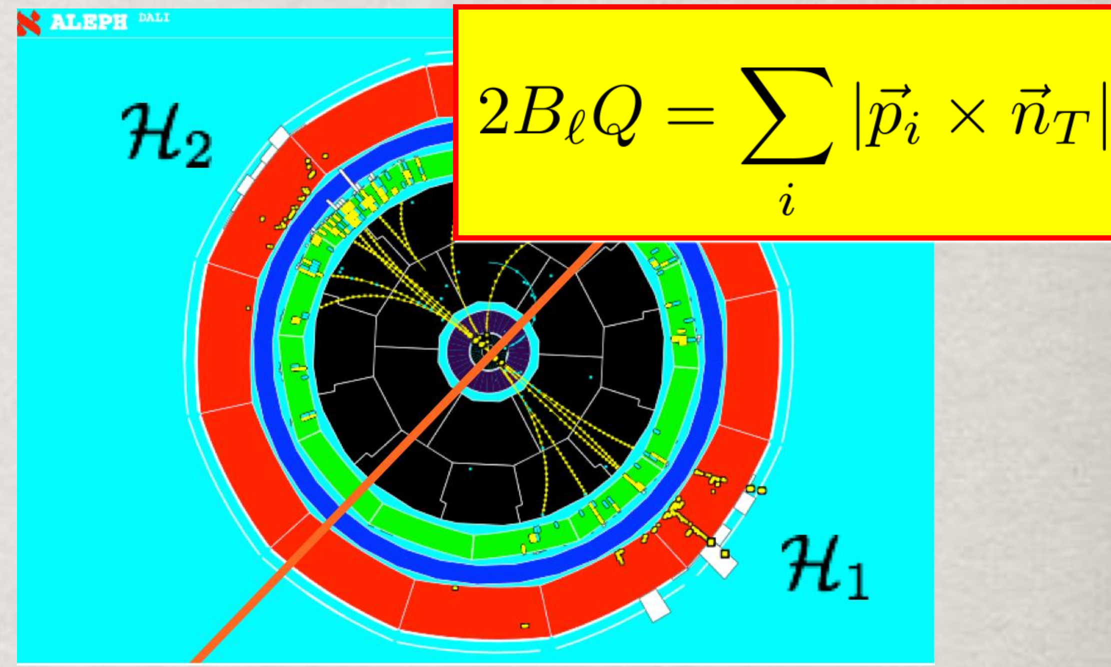


Notable examples

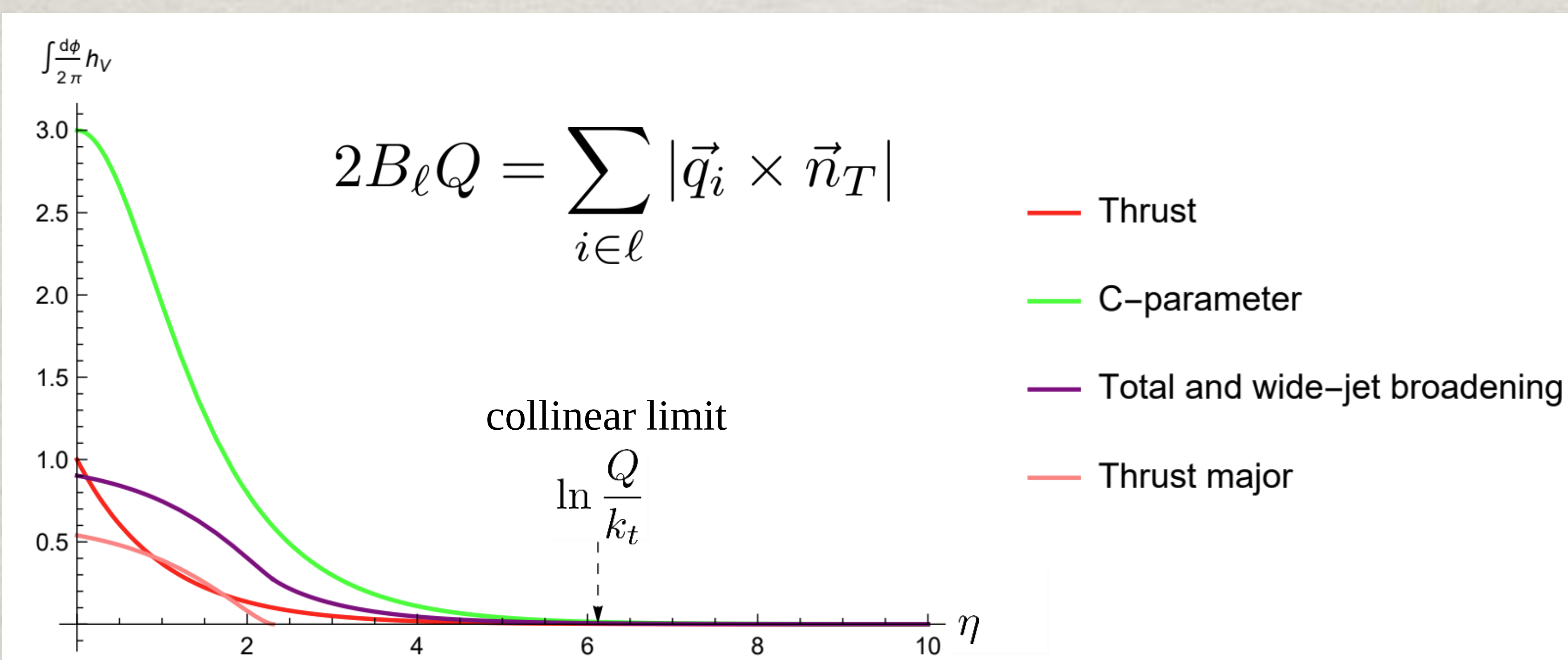
- Broadenings

$$2\int d\eta \int \frac{d\phi}{2\pi} h_{B_T} = \ln \frac{Q e^{\eta_0^{(B)}}}{p_{t,1}} + \ln \frac{Q e^{\eta_0^{(B)}}}{p_{t,2}}$$

$$2\int d\eta \int \frac{d\phi}{2\pi} h_{B_W} = \ln \frac{Q e^{\eta_0^{(B)}}}{p_{t,1}} \Theta(B_1 - B_2) + \ln \frac{Q e^{\eta_0^{(B)}}}{p_{t,2}} \Theta(B_2 - B_1)$$



- The dependence on $p_{t,1}$ and $p_{t,2}$ is due to recoil effects, k_t is different from $|\vec{k} \times \vec{n}_T|$, the transverse momentum of the ultra-soft gluon with respect to the thrust axis

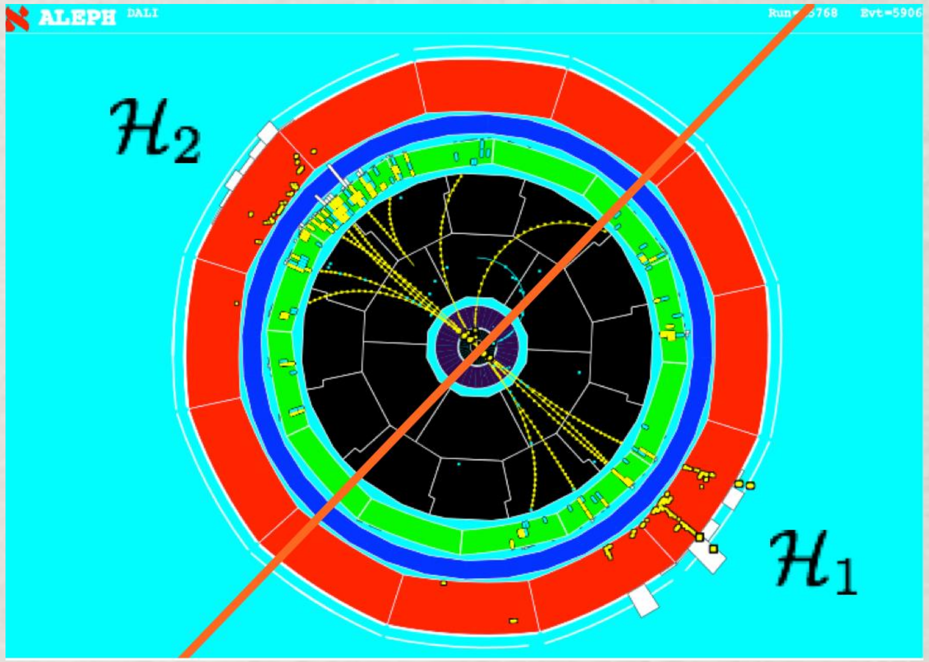


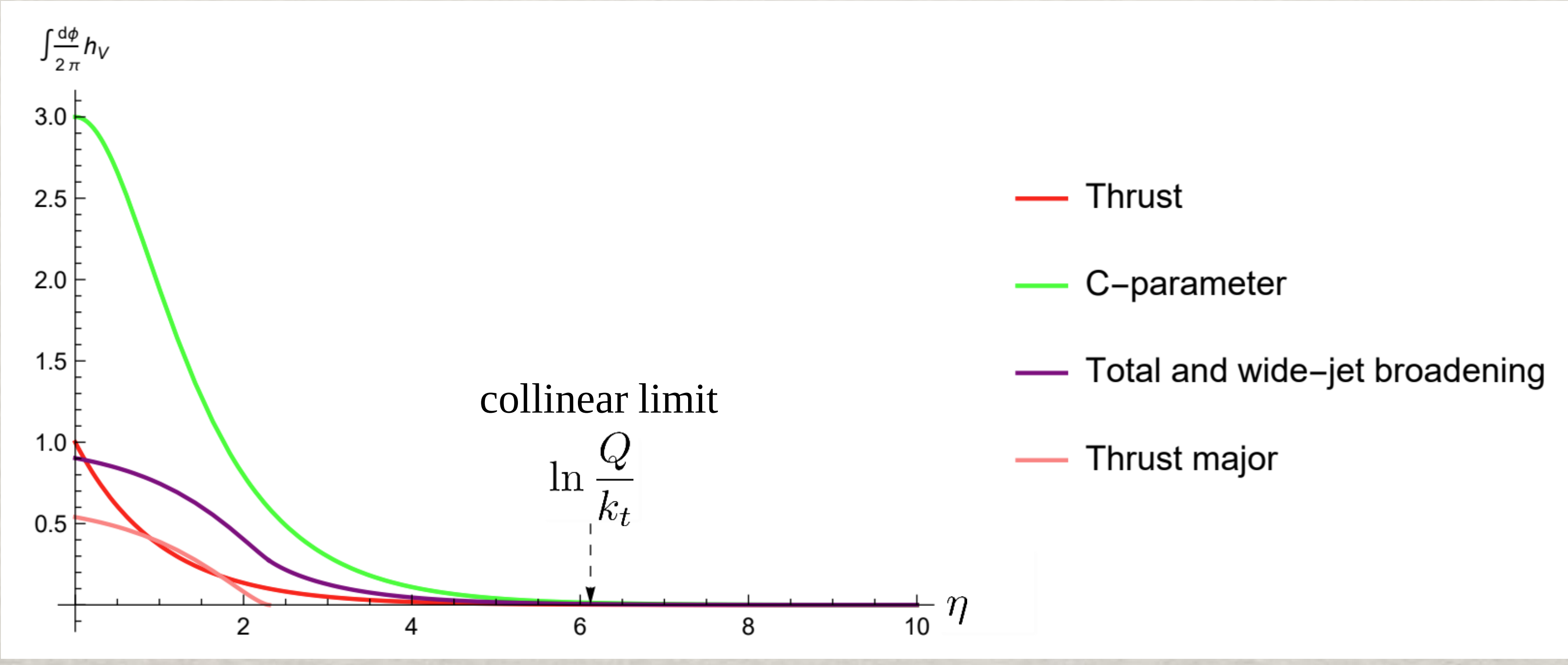
New result: thrust major

- Thrust major is the analogous of thrust but for momenta transverse to the thrust axis

$$T_M Q = \max_{\vec{n}_M \perp \vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_M| = \sum_i |p_{y,i}|$$

$$\frac{\pi}{2}\int d\eta \int \frac{d\phi}{2\pi} h_{T_M} = \ln\frac{2Qe^{-2}}{|p_{y,1}|} + \ln\frac{2Qe^{-2}}{|p_{y,2}|}$$



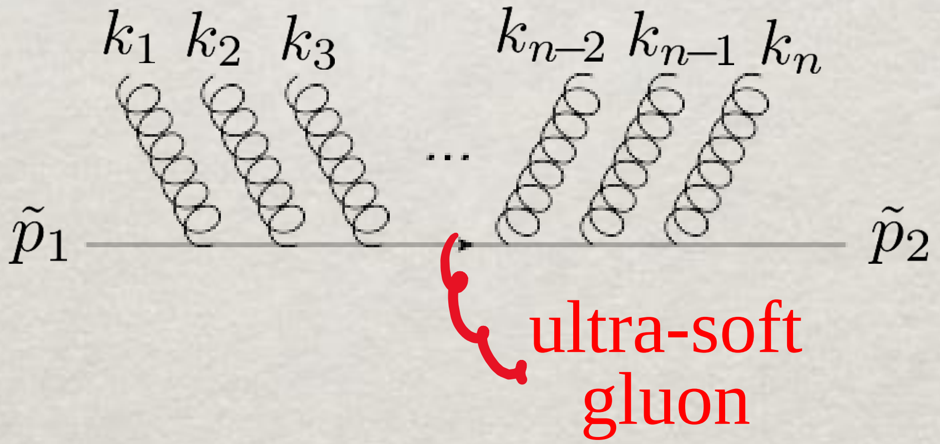


REMARKS ABOUT SCALING

- Let us look in more detail at the expression for $\langle h_V \rangle$

$$\langle h_V \rangle = \frac{\left\langle \int d\eta \frac{d\phi}{2\pi} h_V(\{\tilde{p}\},\{k_i\},\eta,\phi)\delta\left(1 - \frac{V_{\rm sc}^{\rm NLL}(\{\tilde{p}\},\{k_i\})}{v}\right) \right\rangle}{R'\mathcal{F}_{\rm NLL}(R')}$$

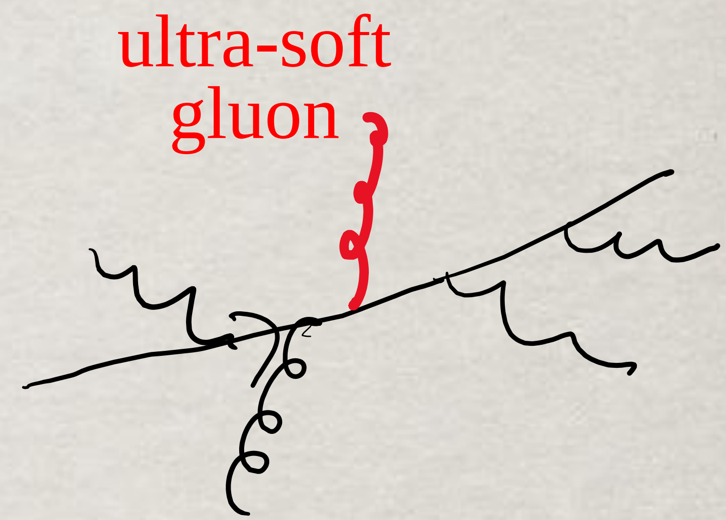
- For all final-state observables, $\mathcal{F}_{\rm NLL}(R') \to 1$ for $R' \to 0 \Rightarrow$ the numerator needs to vanish at least as fast as R' for the calculation of $\langle h_V \rangle$ to be finite

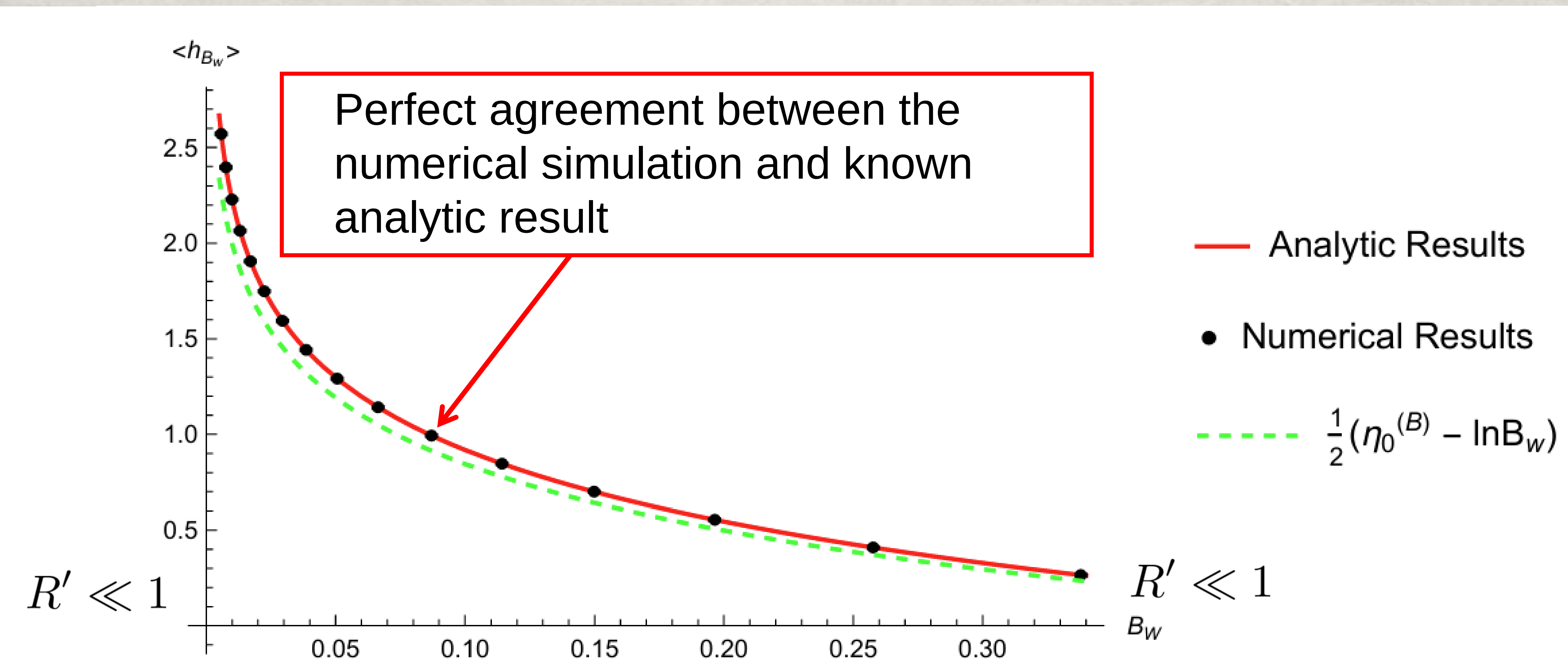


- This should be always guaranteed because the phase space for emissions closes when $R' \to 0$ (i.e. when the observables is large)

VALIDATION FOR B_W

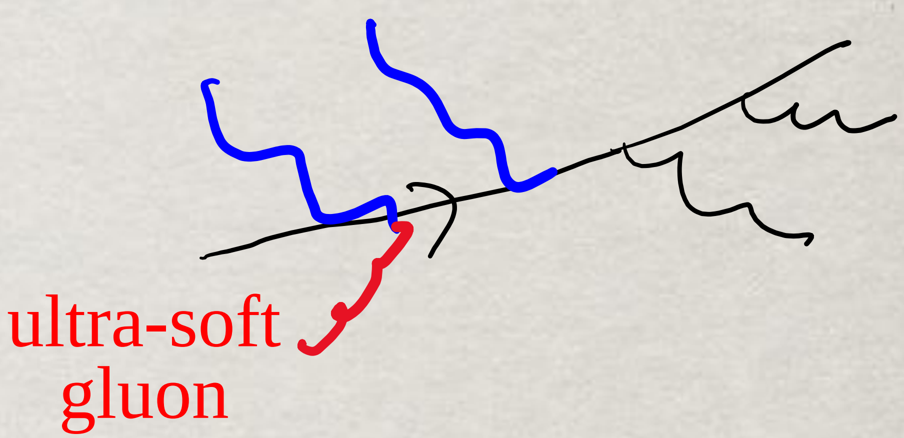
- For the wide-jet broadening, $p_{t,\ell} \sim B_W$
- The basic PT configurations needed for $\langle h_{B_W} \rangle$ are soft and collinear gluons with $B_W(\{\tilde{p}\}, k_i)$ of the same order
- These configurations are efficiently simulated numerically with the ARES method

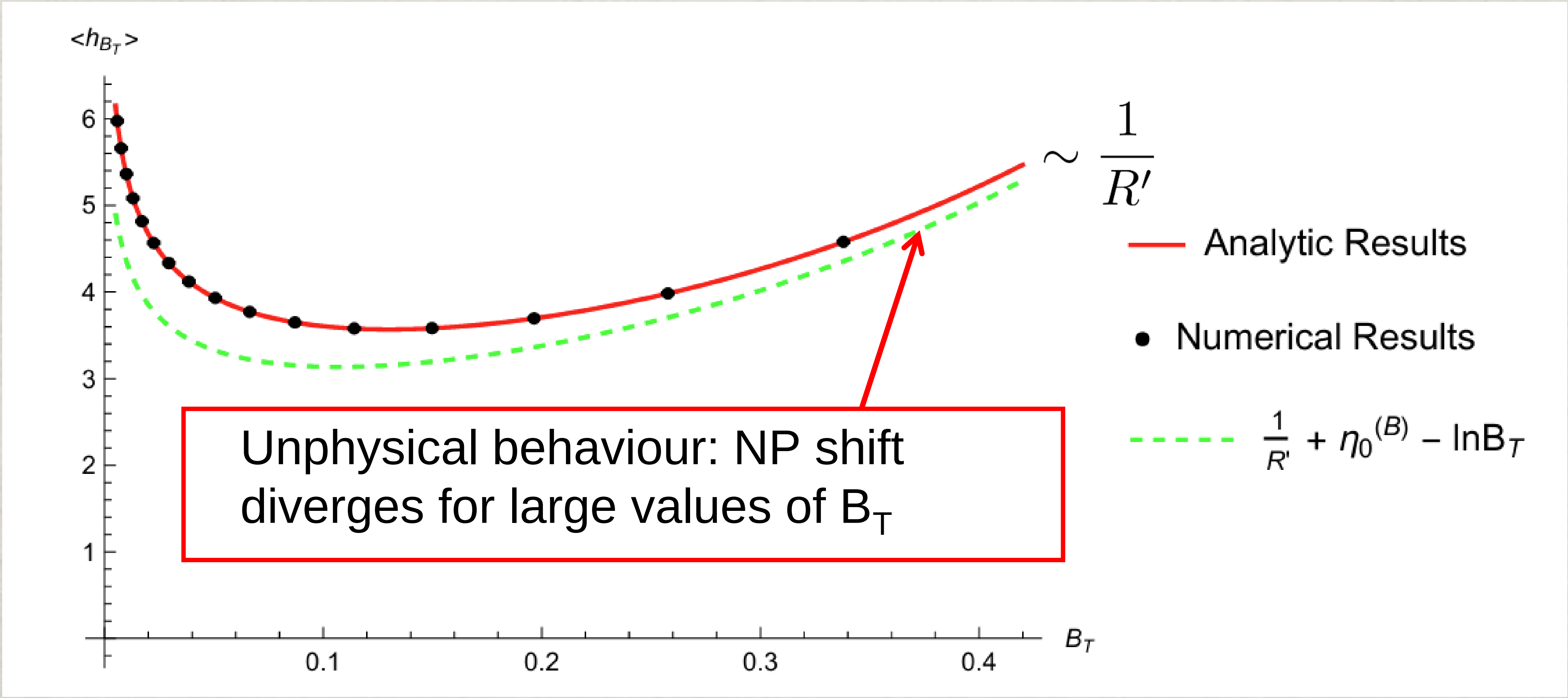




Problems for B_T …

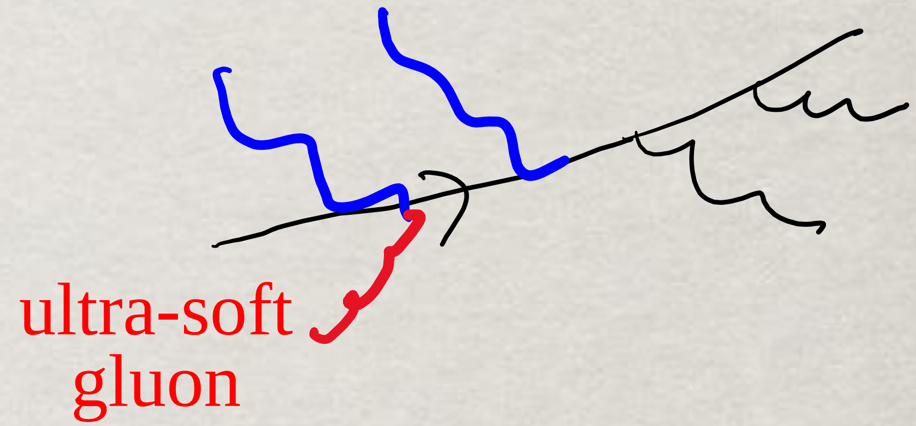
- For the total jet broadening, $\max[p_{t,1}, p_{t,2}] \sim B_T$ but $\min[p_{t,1}, p_{t,2}] \ll B_T$
- Configurations with $B_T(\{\tilde{p}\}, k_i) \ll B_T$ are important for the calculation of $\langle h_{B_T} \rangle$
- These configurations can be generated in ARES only by artificially lowering the internal cutoff for soft and collinear emissions

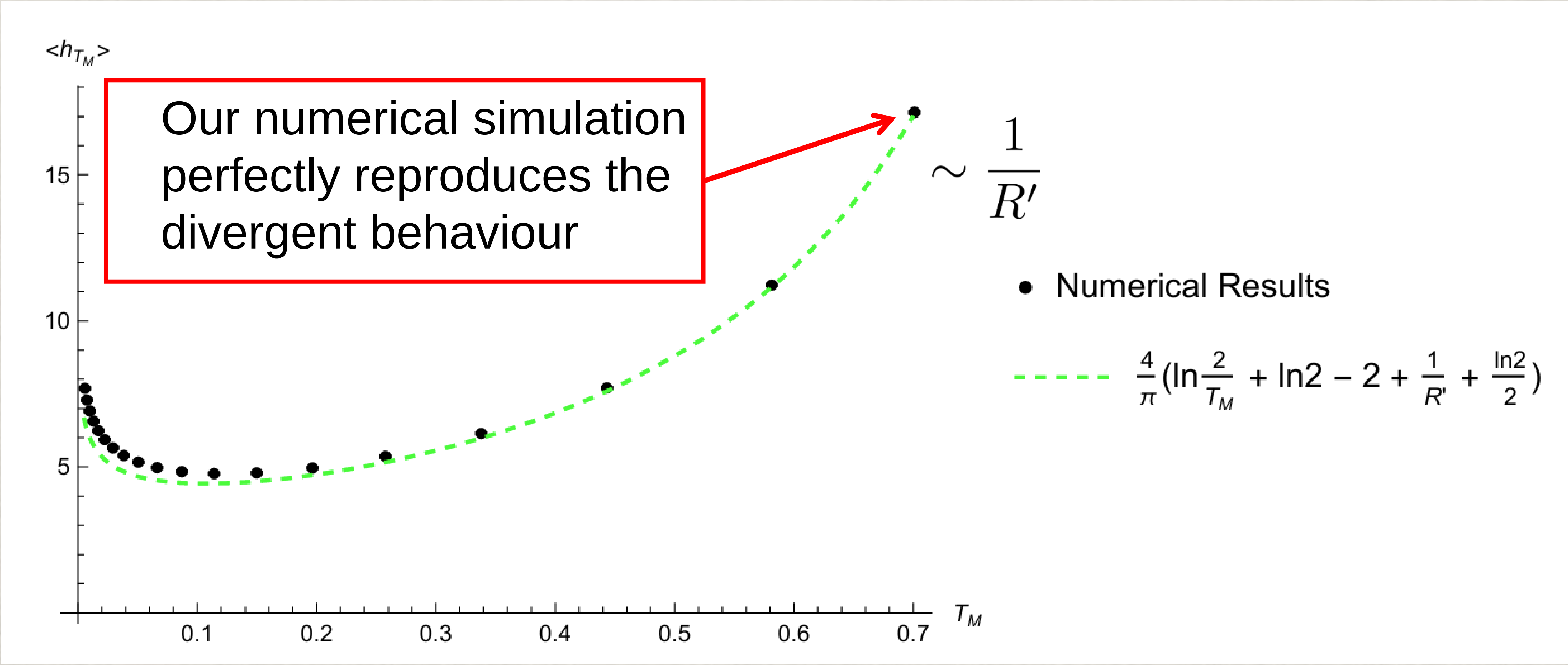




... AND FOR T_M

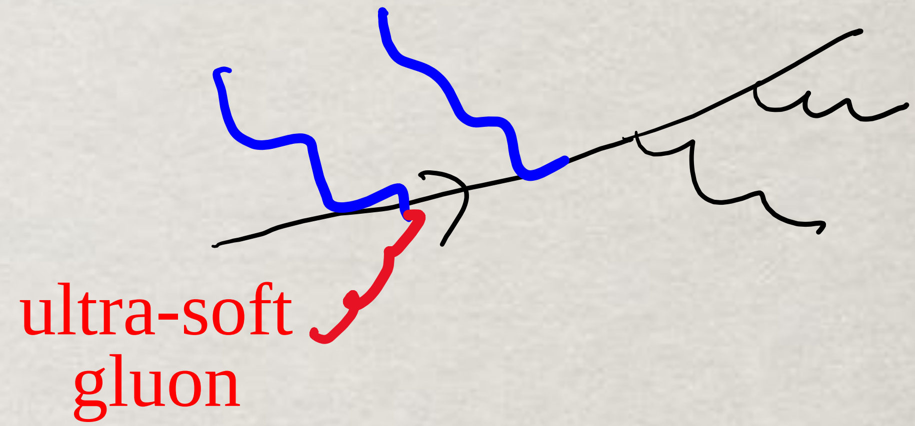
- Also for the thrust major, $\max[|p_{y,1}|, |p_{y,2}|] \sim T_M$ but $\min[|p_{y,1}|, |p_{y,2}|] \ll T_M$
- Again, the shift diverges like $1/R'$ for large T_M
- As the phase space for emissions closes for vanishing R', one expects that the shift stays finite for $R' \to 0$, instead of diverging





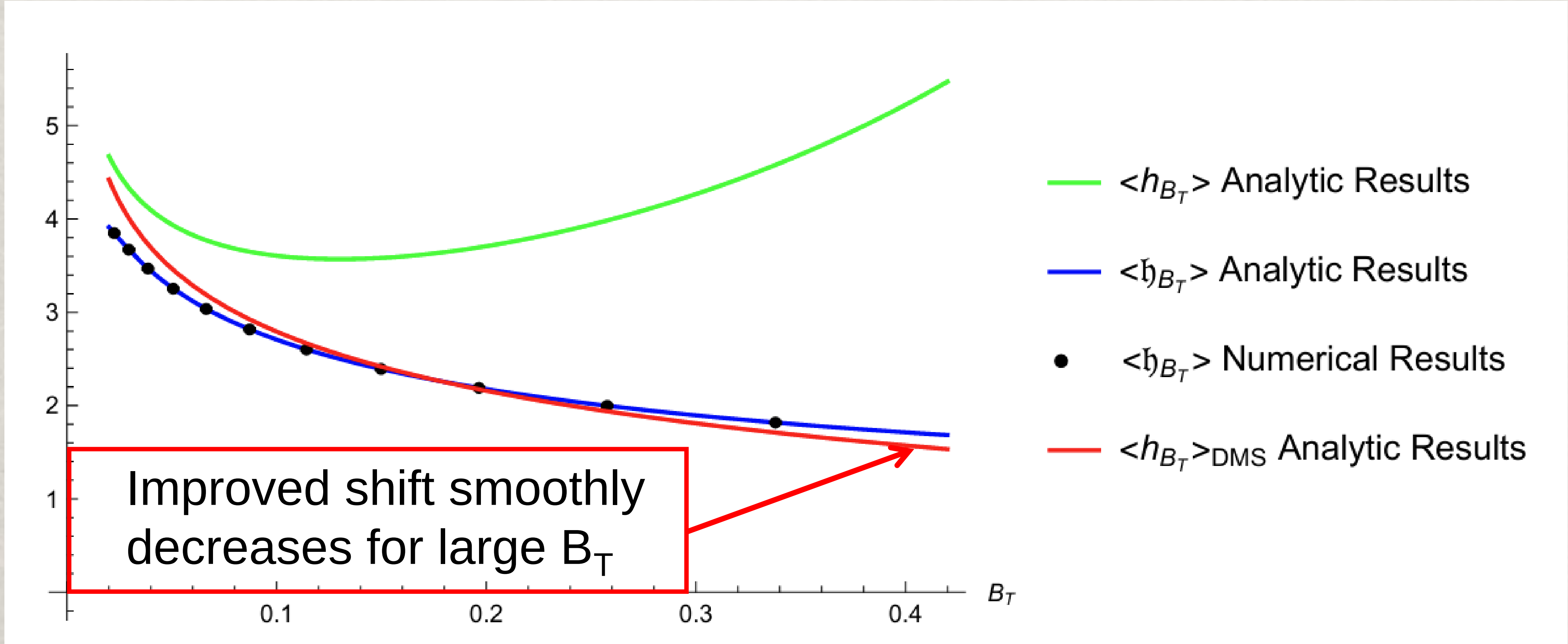
ANALYTIC SOLUTION FOR B_T

- Hemispheres are never empty: there must always be soft and collinear radiation
- Soft emissions with $B_T(\{\tilde{p}\}, k_i) \ll B_T$ give rise to a Sudakov suppression for the minimum transverse momentum p_t



$$\langle h_{B_T} \rangle \sim \int_0^\infty dp_t \ln \frac{Q}{p_t} \frac{d}{dp_t} e^{-\alpha_s \ln^2 \frac{Q}{p_t}} \sim \frac{1}{\sqrt{\alpha_s}}$$

[Dokshitzer Marchesini Salam hep-ph/9812487]



Numerical solution

The analytic calculation of $\langle h_{B_T} \rangle$ causes some problems for ARES

- It requires improved matrix elements for soft emissions which cannot be implemented in ARES without a major rewriting of the Monte Carlo procedure to generate soft and collinear emissions
- It is very difficult to generalise to observables like T_M which don't admit analytic treatment

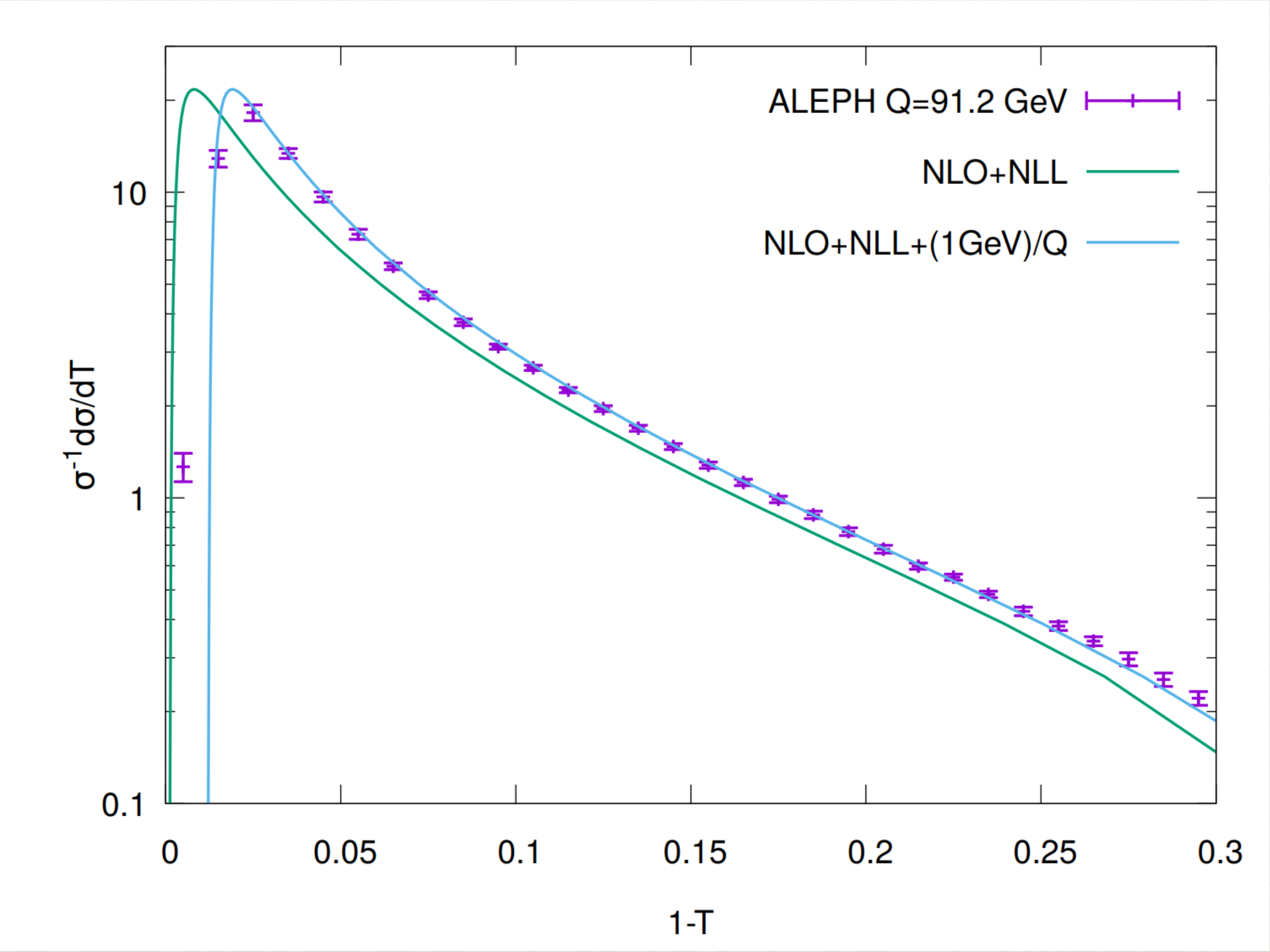
Solution: devise counterterms that

- reproduce the $1/R'$ behaviour of $\langle h_V \rangle$
- cancel the $1/R'$ singularity of the ARES Monte Carlo procedure "locally"
- can be computed analytically

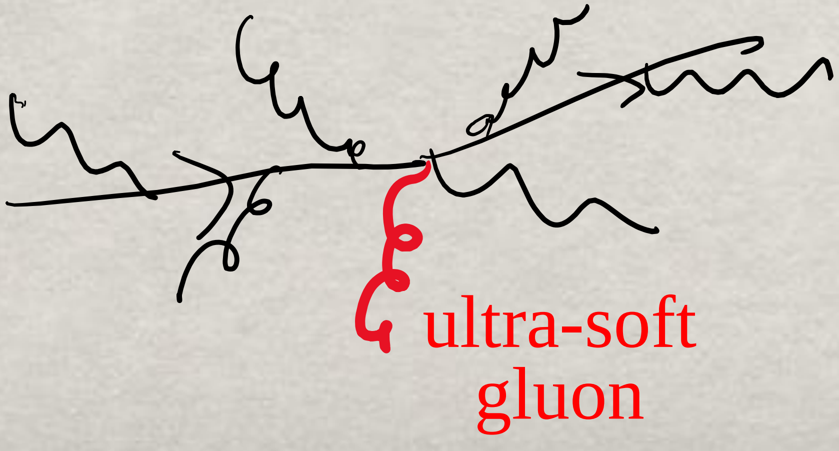
[AB El-Menoufi Wood 2303.01534]

Numerical solution

The key observation that allows the calculation of the counterterms is that for $R' \to 0$ there is only one PT emission k_1 with $V(\{\tilde{p}\}, k_1) \sim v$: this simplifies the kinematics at the point that the counterterms can always be computed analytically



$R' \gg 1$



$R' \ll 1$

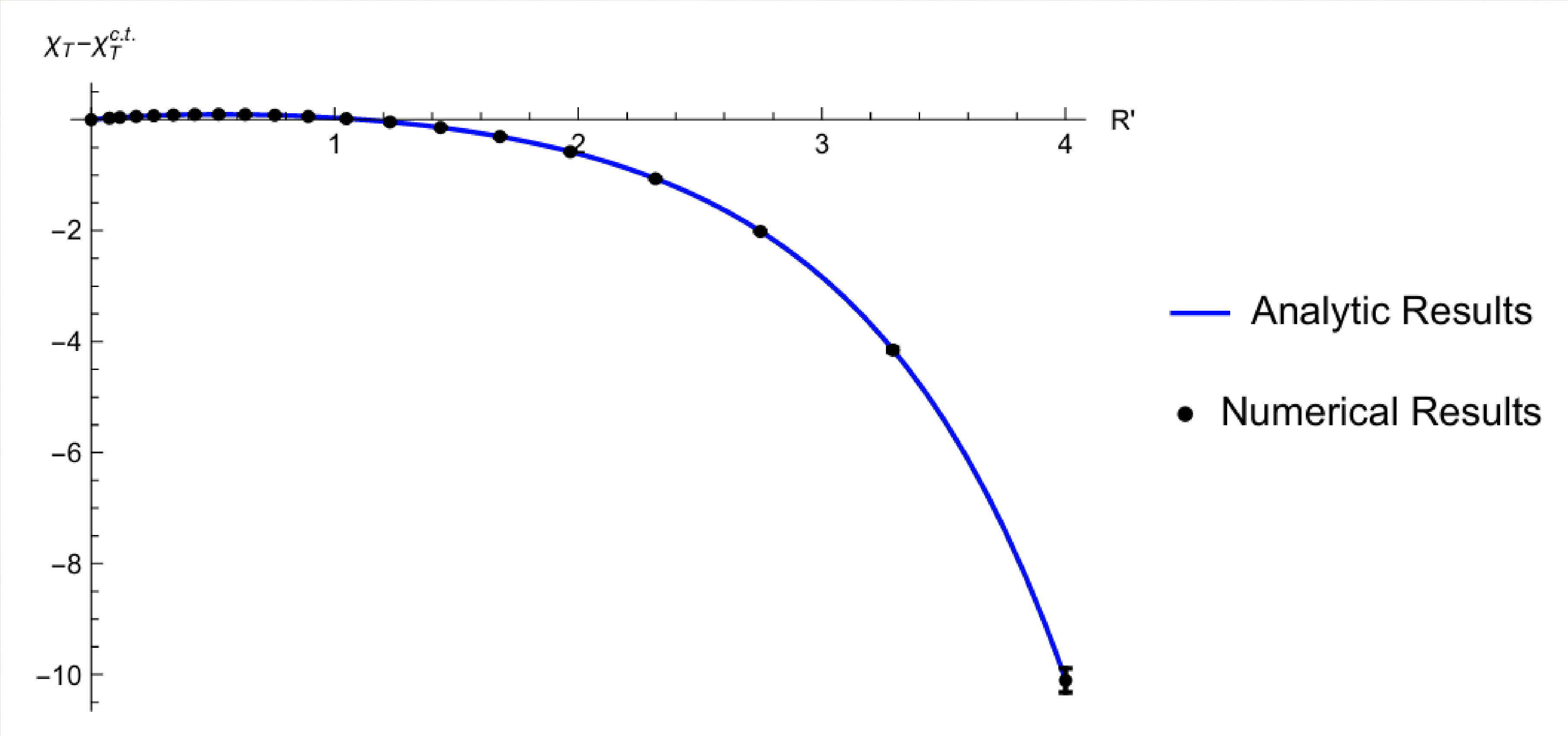
Numerical solution for B_T

The introduction of the counterterm gives rise to an improved version of the NP shift

$$\langle h_{B_T} \rangle \to \langle \mathfrak{h}_{B_T} \rangle \equiv \langle h_{B_T} - h_{B_T}^{\text{c.t.}} \rangle + \langle h_{B_T}^{\text{c.t.}} \rangle_{\text{imp.}}$$

- The subtracted part of the shift is finite for $R' \to 0$

$$\langle h_{B_T} - h_{B_T}^{\text{c.t.}} \rangle = \left[1 - \frac{1}{2} f_T(R')\right] \ln \frac{1}{B_T} + \eta_0^{(B)} + \chi_T(R') - \chi_T^{\text{c.t.}}(R')$$

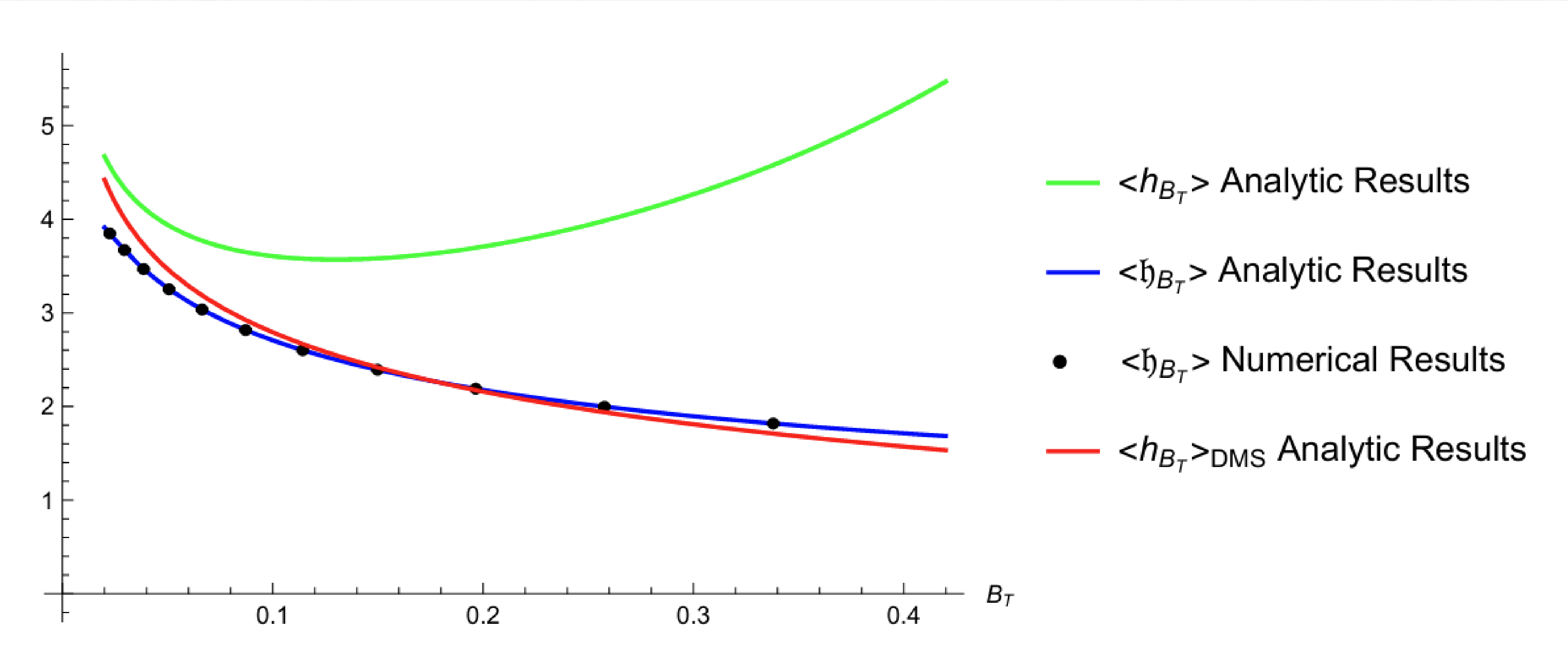


Numerical solution for B_T

The introduction of the counterterm gives rise to an improved version of the NP shift

$$\langle h_{B_T} \rangle \to \langle \mathfrak{h}_{B_T} \rangle \equiv \langle h_{B_T} - h^{\mathrm{c.t.}}_{B_T} \rangle + \langle h^{\mathrm{c.t.}}_{B_T} \rangle_{\mathrm{imp.}}$$

- The subtracted part of the shift is finite for $R' \to 0$
- The improved version of the shift agrees with the analytic solution $\langle h_{B_T} \rangle_{\mathrm{DMS}}$, up to terms of order $\sqrt{\alpha_s}$, which are beyond our control



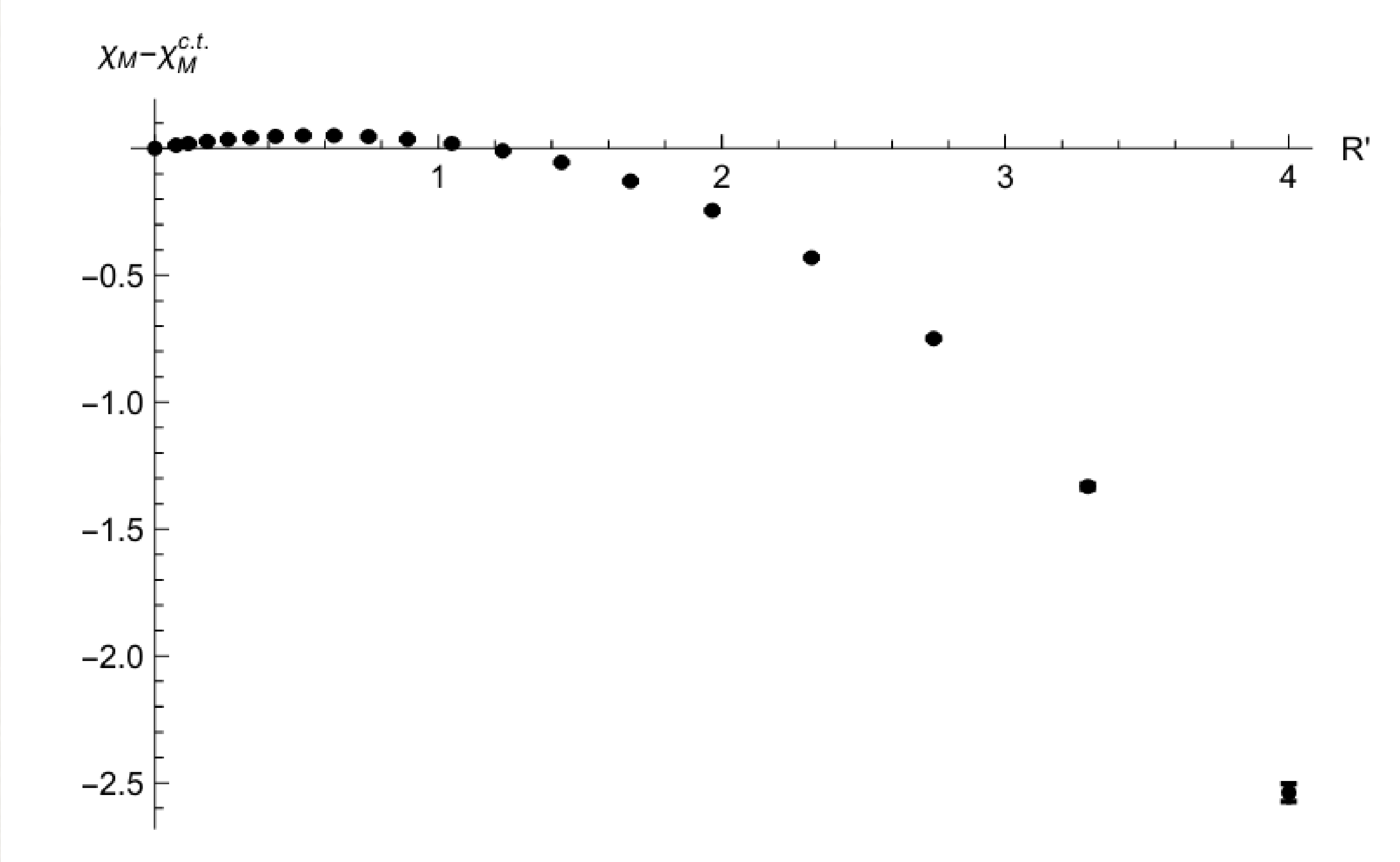
Improved NP shift for T_M

We can apply a similar counterterm to $\langle h_{T_M} \rangle$

$$\langle h_{T_M} \rangle \to \langle \mathfrak{h}_{T_M} \rangle \equiv \langle h_{T_M} - h_{T_M}^{\text{c.t.}} \rangle + \langle h_{T_M}^{\text{c.t.}} \rangle_{\text{imp.}}$$

- The subtracted part of the shift is finite for $R' \to 0$

$$\langle h_{T_M} - h_{T_M}^{\text{c.t.}} \rangle = \frac{4}{\pi} \left[1 - \frac{1}{2} f_M(R') \right] \ln \frac{2}{T_M} + \frac{4}{\pi} (\ln 2 - 2) + \chi_M(R') - \chi_M^{\text{c.t.}}(R')$$

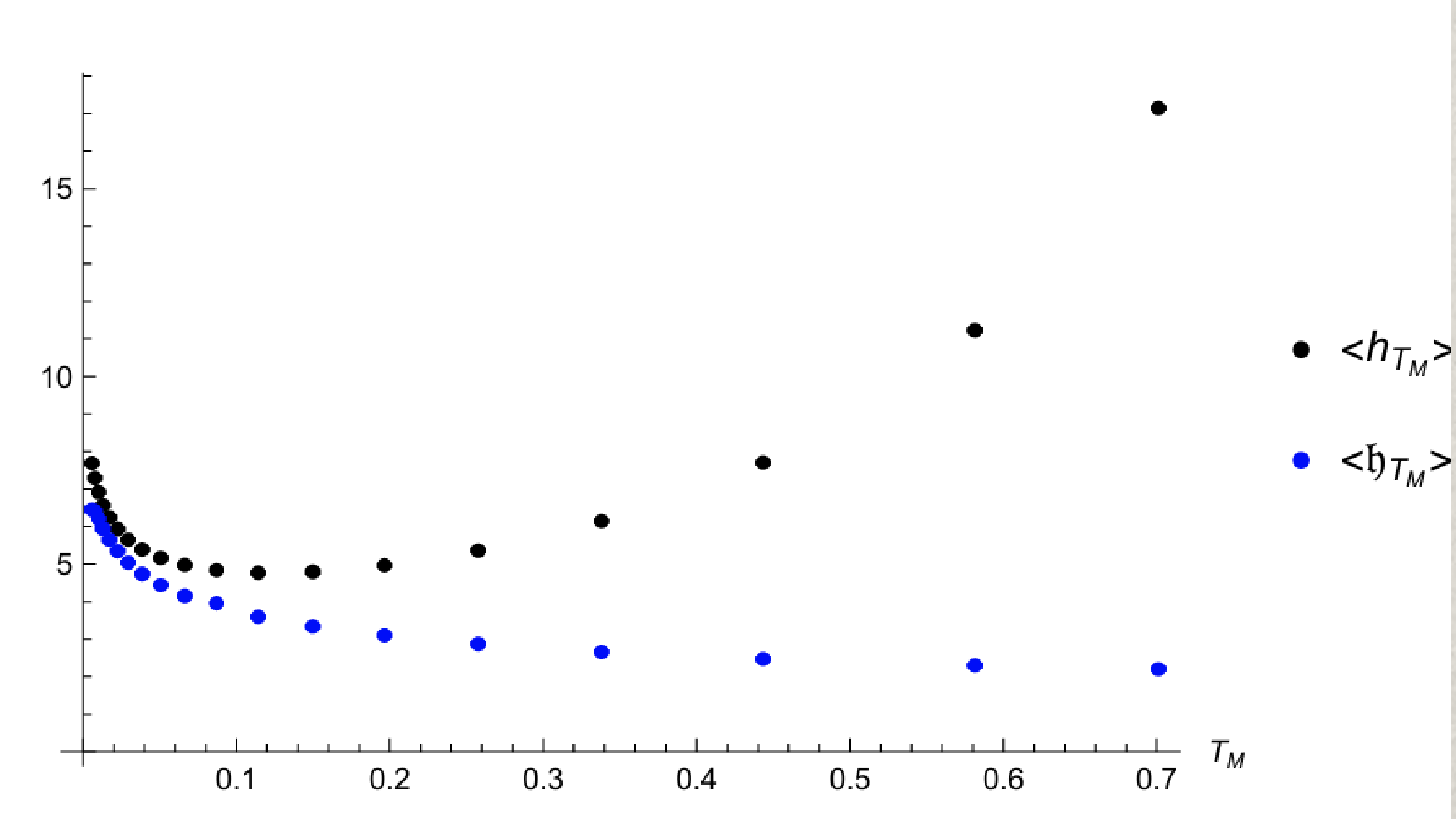


Improved NP shift for T_M

We can apply a similar counterterm to $\langle h_{T_M} \rangle$

$$\langle h_{T_M} \rangle \to \langle \mathfrak{h}_{T_M} \rangle \equiv \langle h_{T_M} - h_{T_M}^{\text{c.t.}} \rangle + \langle h_{T_M}^{\text{c.t.}} \rangle_{\text{imp.}}$$

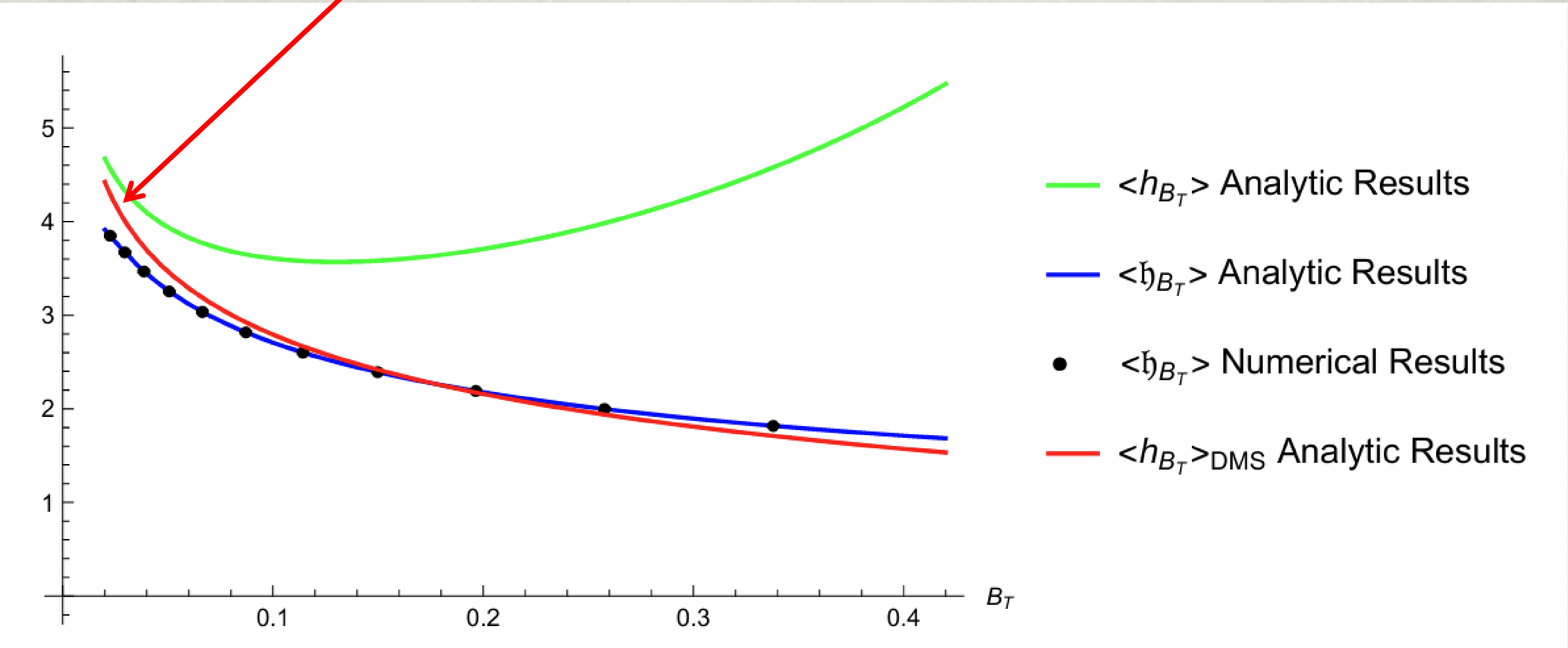
- The subtracted part of the shift is finite for $R' \to 0$
- The improved version of the shift smoothly decreases with increasing T_M



Issues with counterterms

Despite its advantages, our method shows two areas for improvement

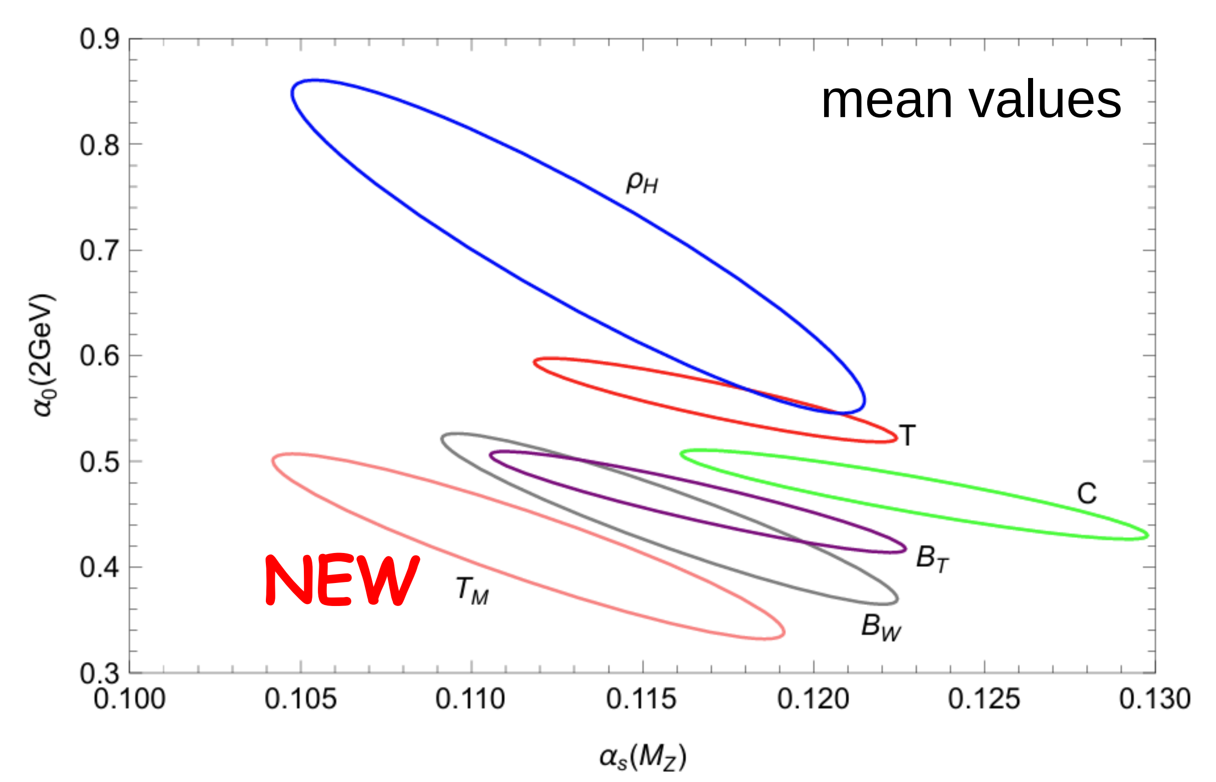
- The counterterms are to some extent observable dependent
- The residual term order $\sqrt{\alpha}_s$ does not vanish for large R' as it does in the analytic calculation for the total broadening

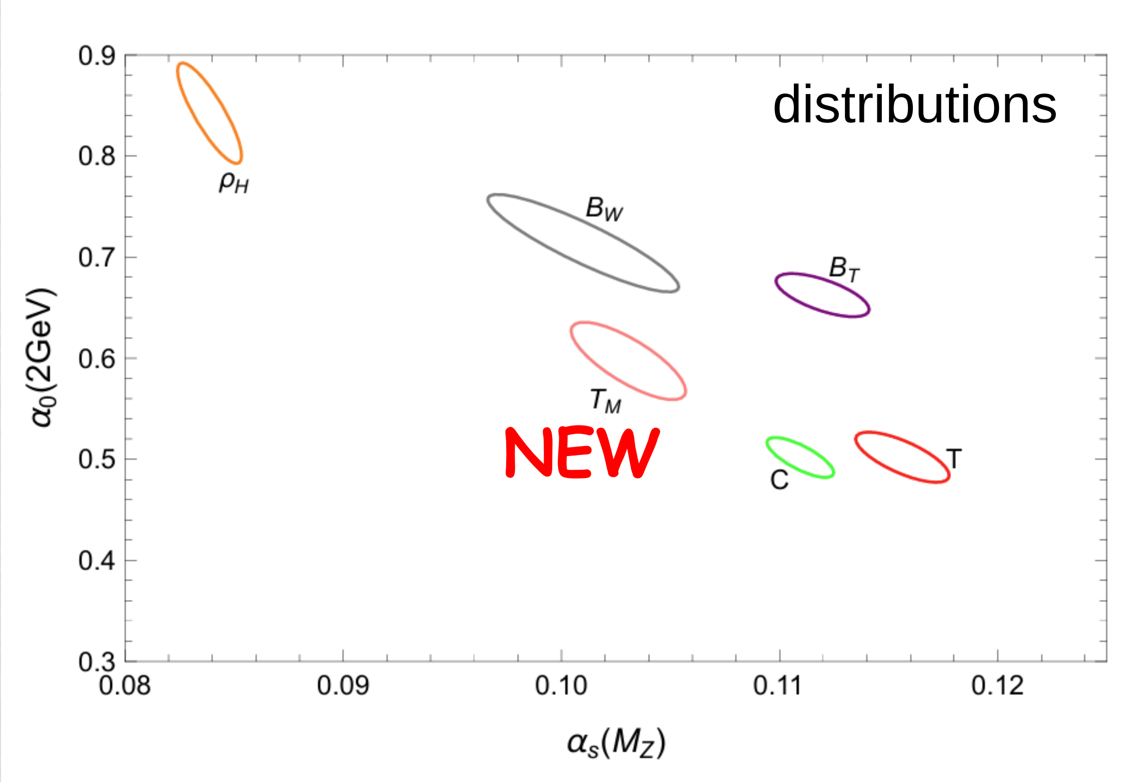


Fits of NP parameter

- We used our version of the shift to simultaneously extract $\alpha_s(M_Z)$ and the NP parameter $\alpha_0(\mu_I) \sim \langle k_t \rangle_{\mathrm{NP}}/\mu_I$ using event-shape distributions and mean values

[AB El-Menoufi Wood 2303.01534]



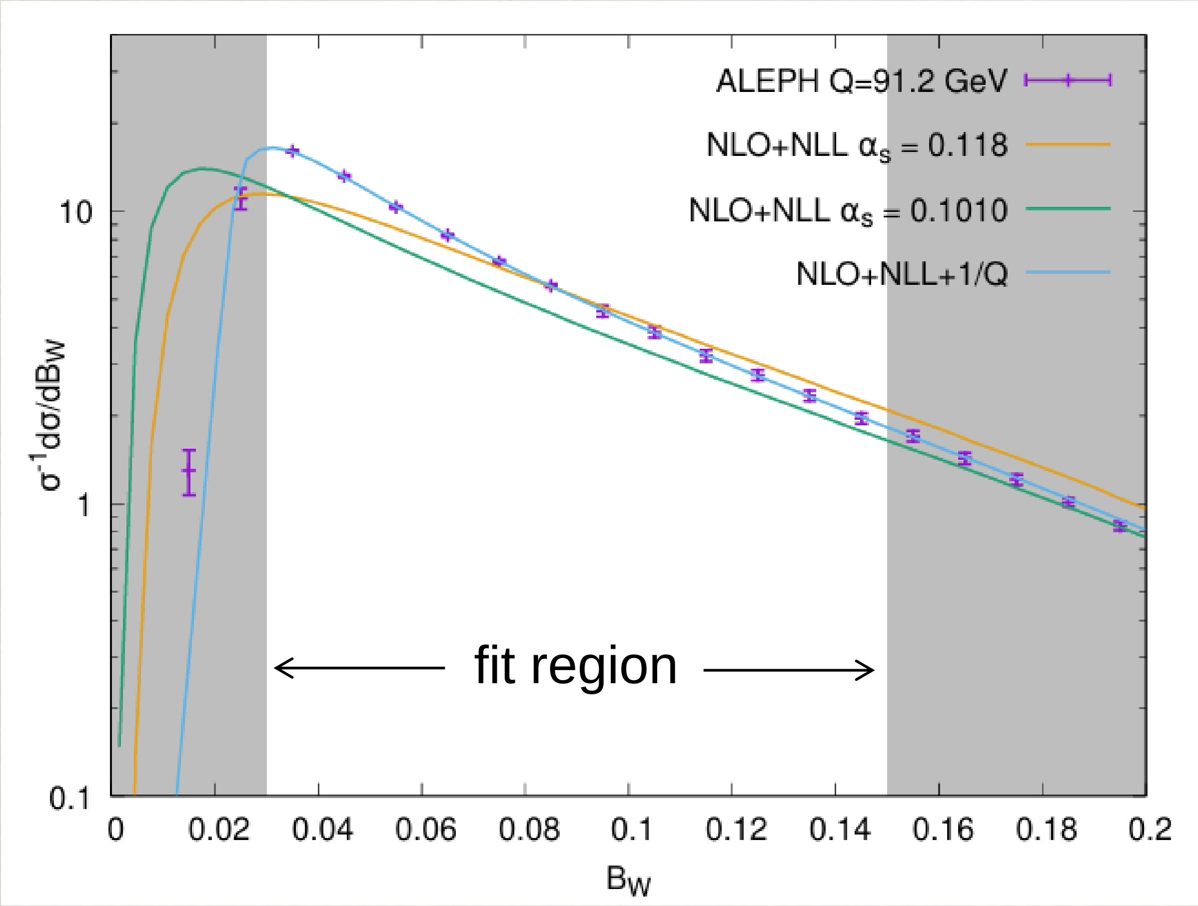


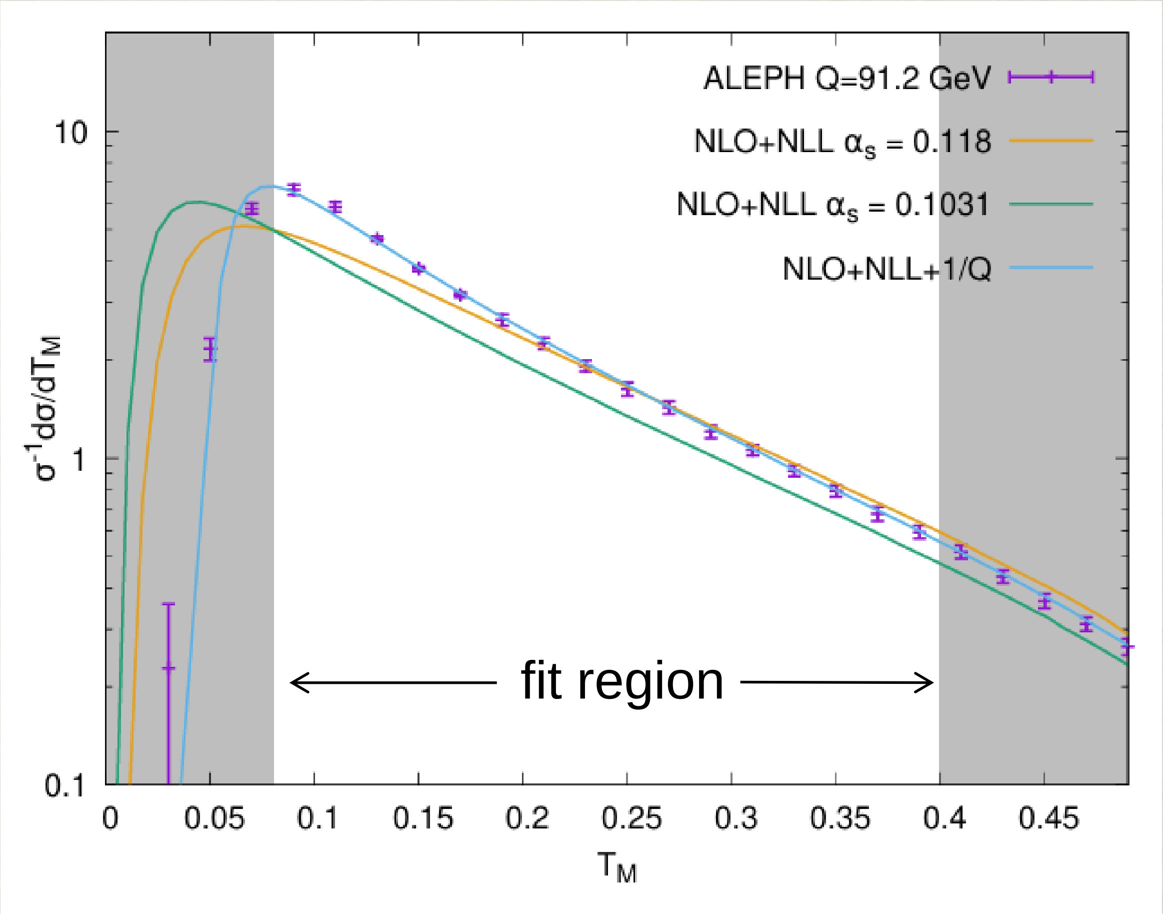
- The new results for T_M are in line with the universality pattern already seen for other event shapes in the two-jet region

- We know that NP corrections receive large modifications in the three-jet region: matching of the two and three-jet region needed

Sample distributions

- For values of α_s in line with the world average, the NP shift for B_W and T_M becomes negative in the three-jet region





- Matching the shift to the three-jet region will most likely have a huge impact on simultaneous fits of α_s and NP parameter α_0