(Colour) evolution

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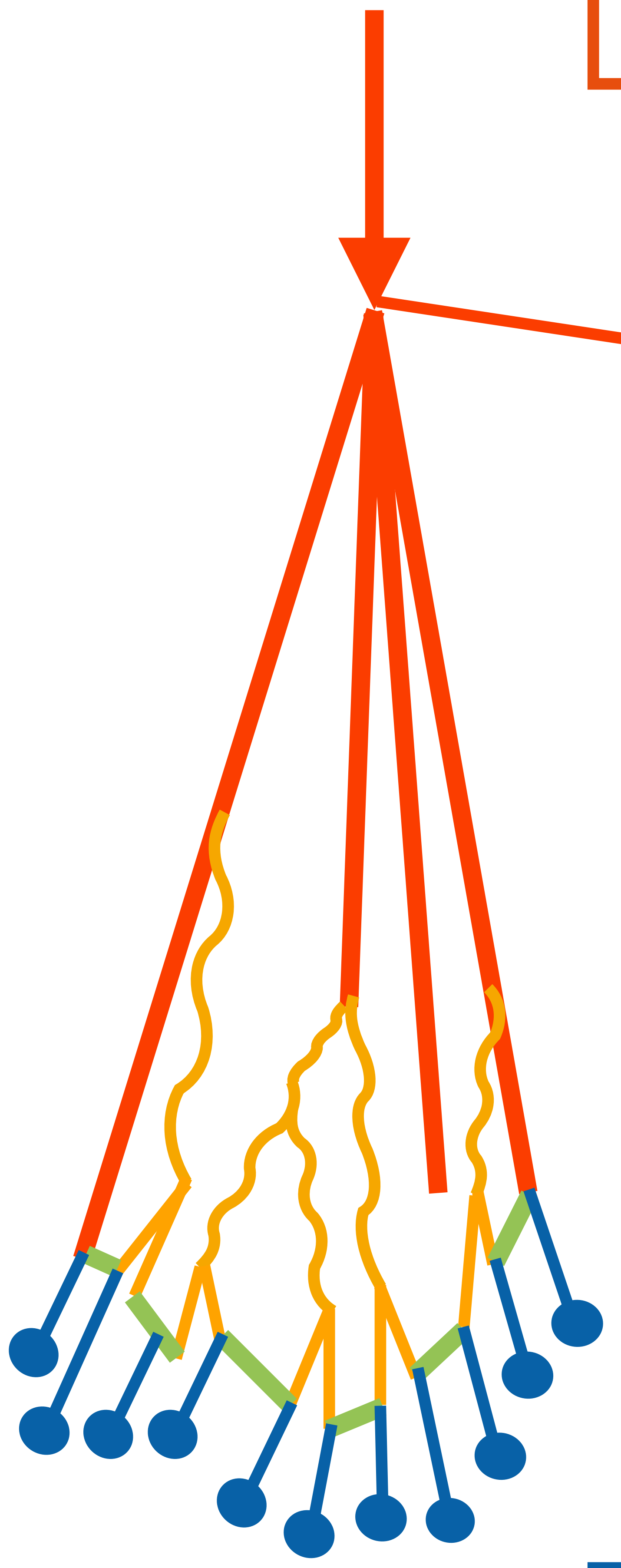
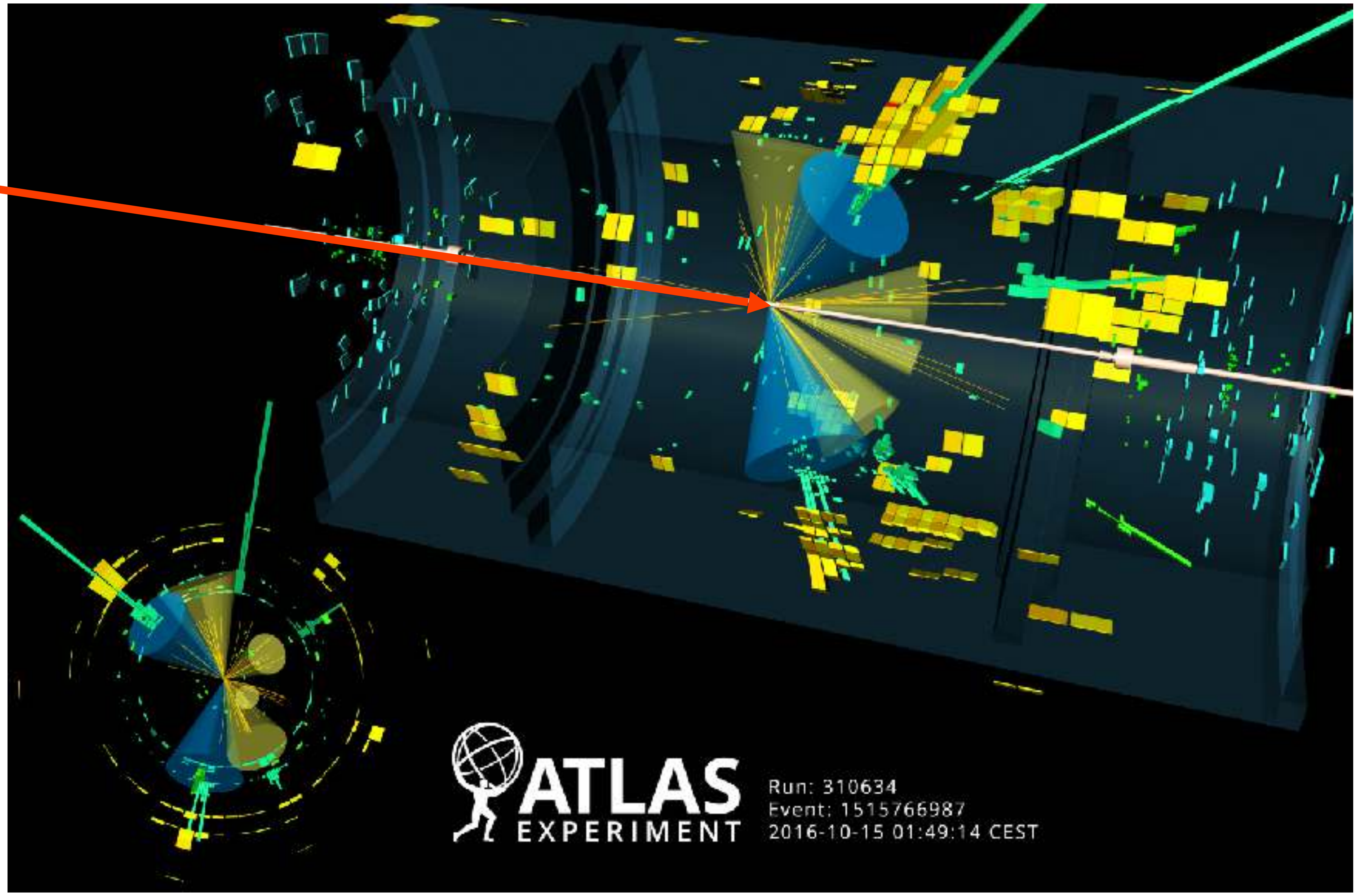
At the
Birdtrack Meeting
Online | 26 February 2024

The Complexity of Observations

New physics?



Experimental Detection



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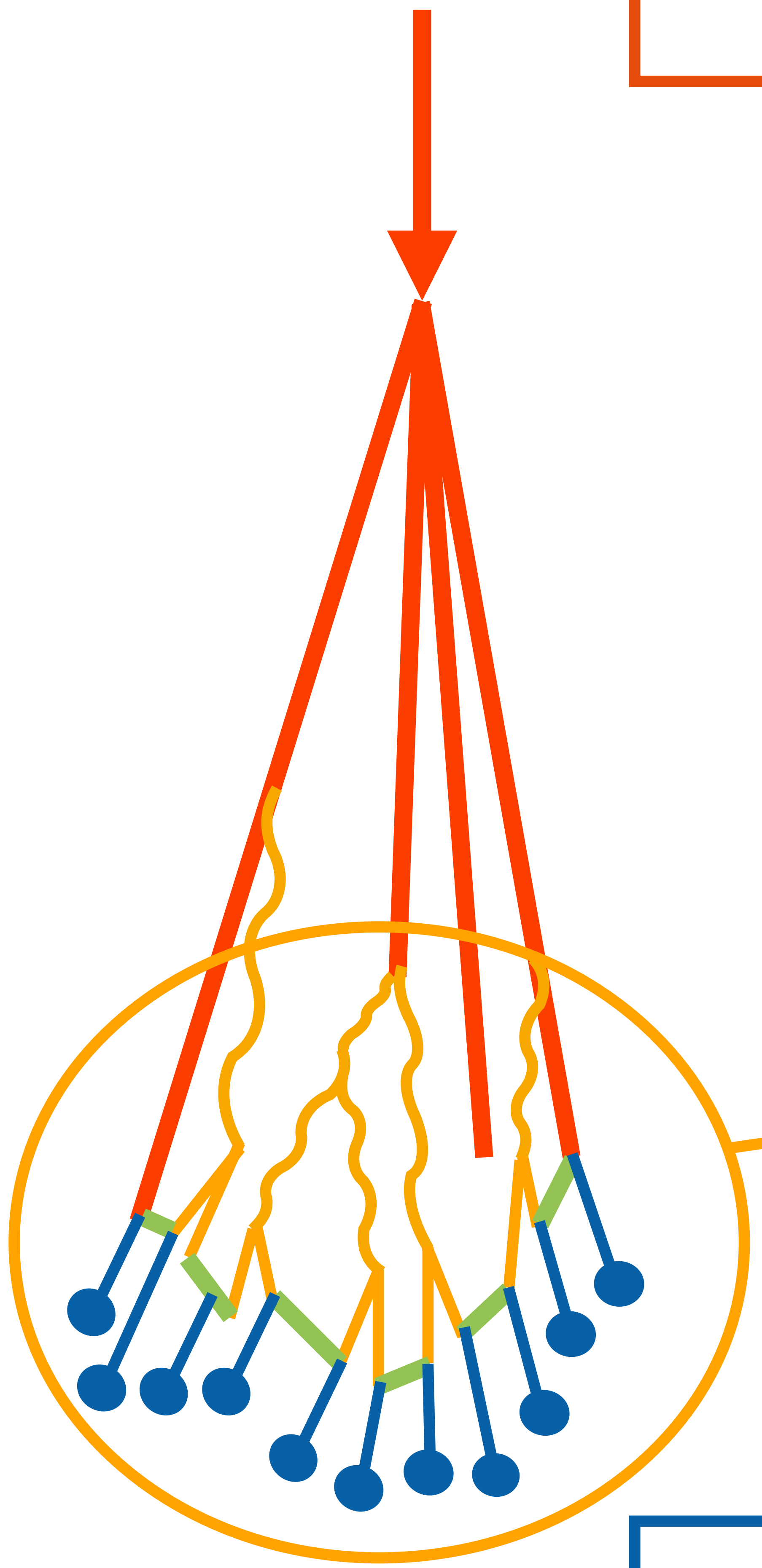
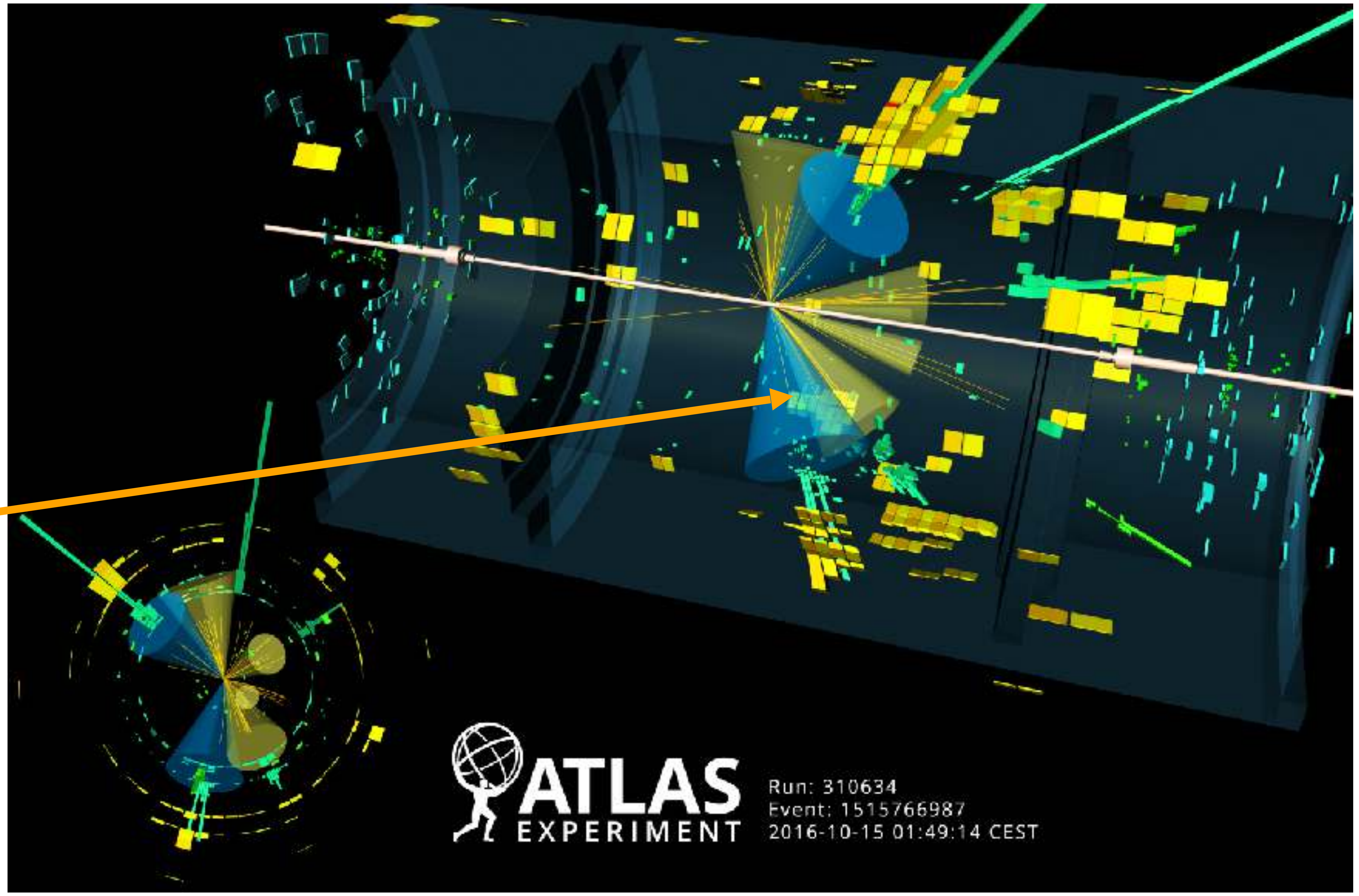


jet evolution:

parton shower
hadronization

Experimental Detection



The Complexity of Observations

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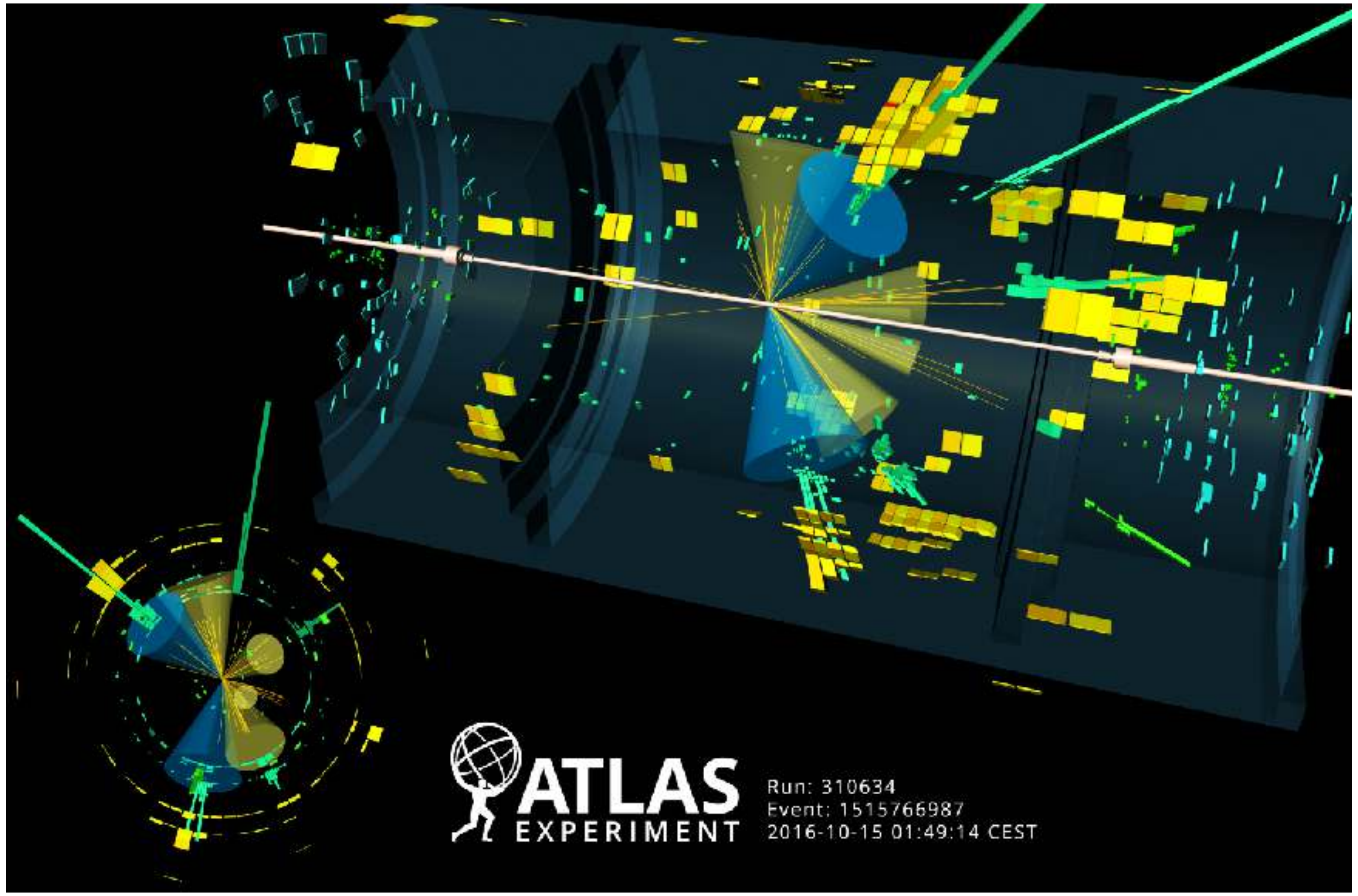
Confirmed! We ~~Live in~~ use a Simulation



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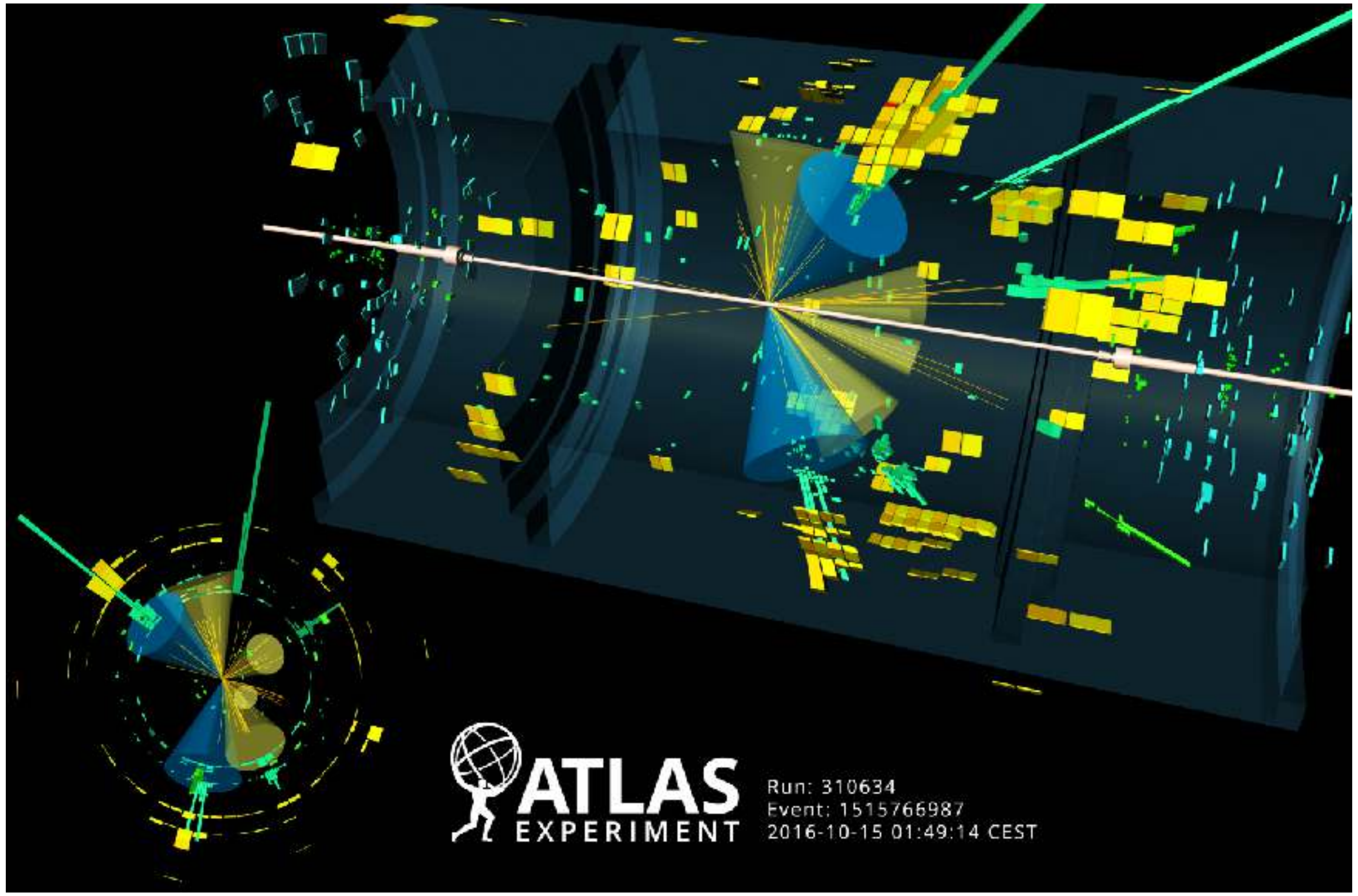
- as an event generator



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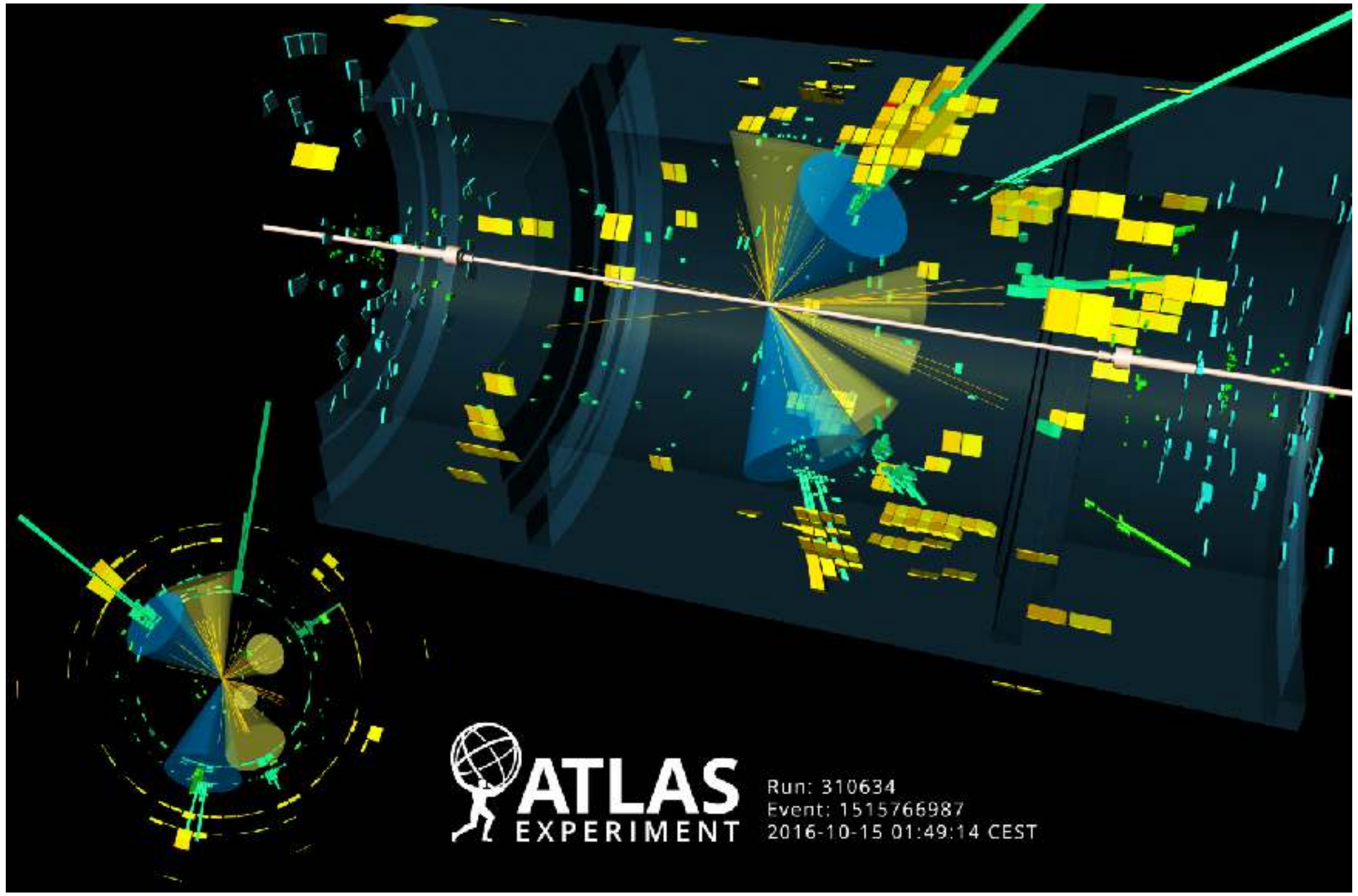

The Complexity of Observations


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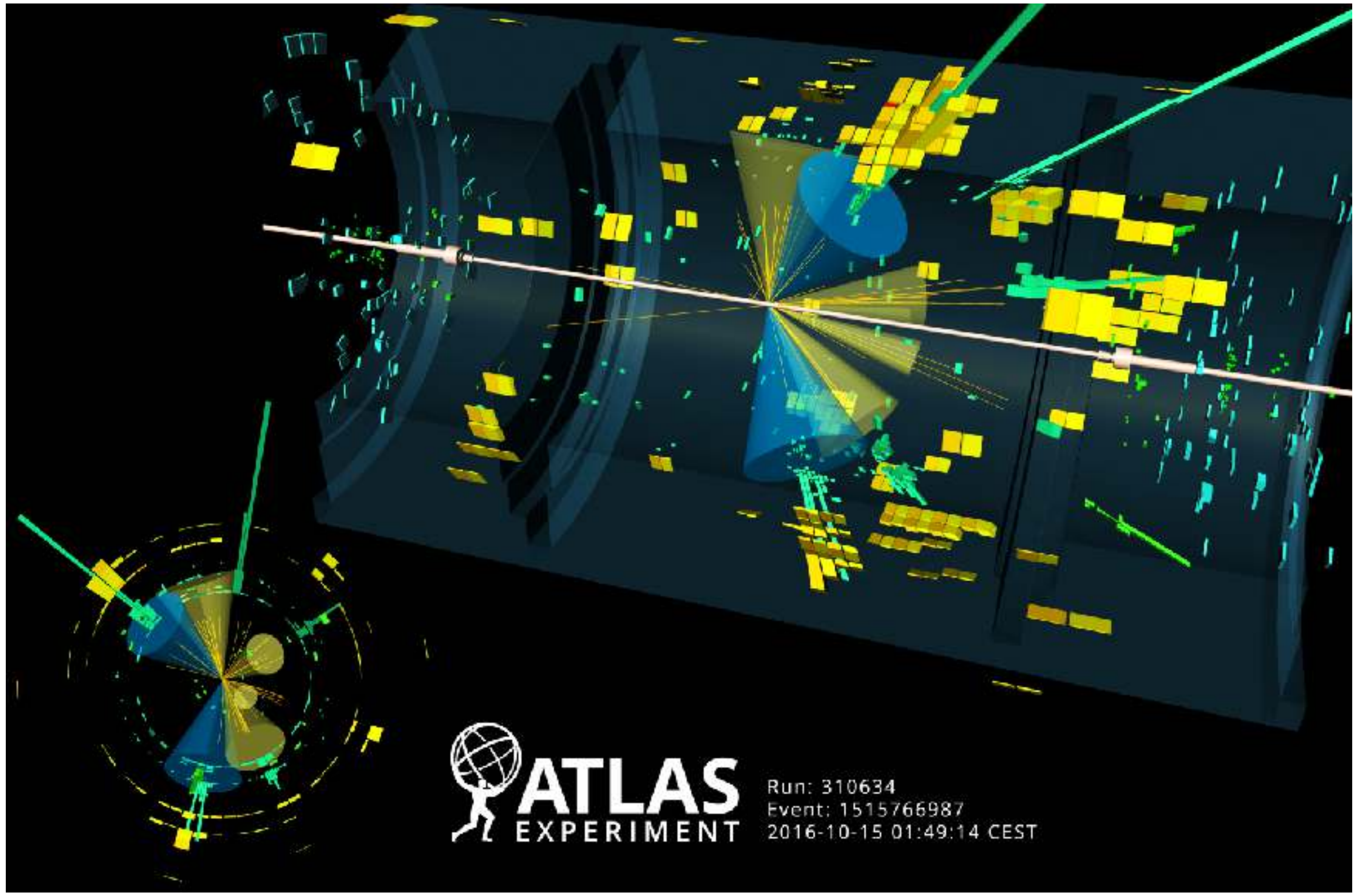
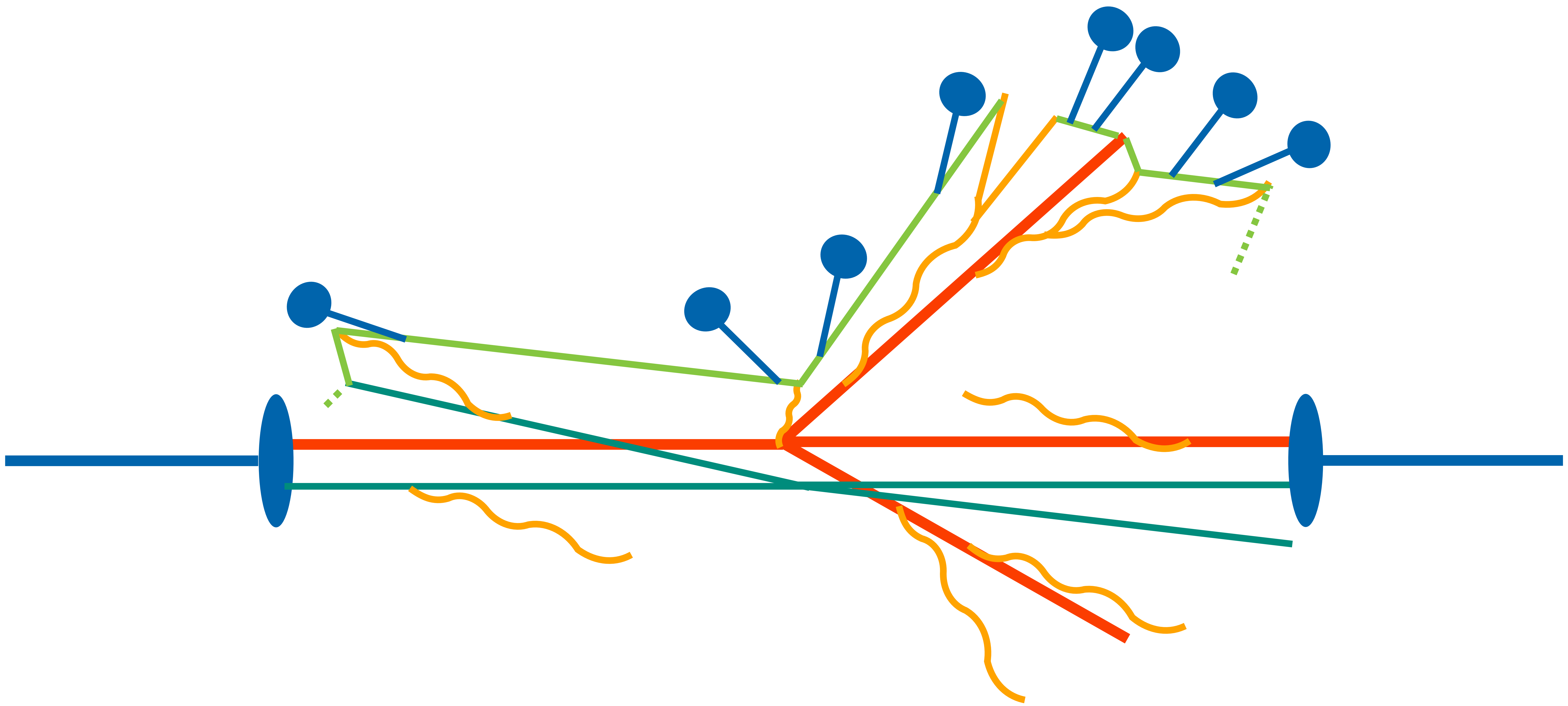
Confirmed! We ~~Live in~~ use a Simulation

- as an event generator
- as an exact tool for resummation
- as a means to explore amplitudes and structures in QFT



Event generators

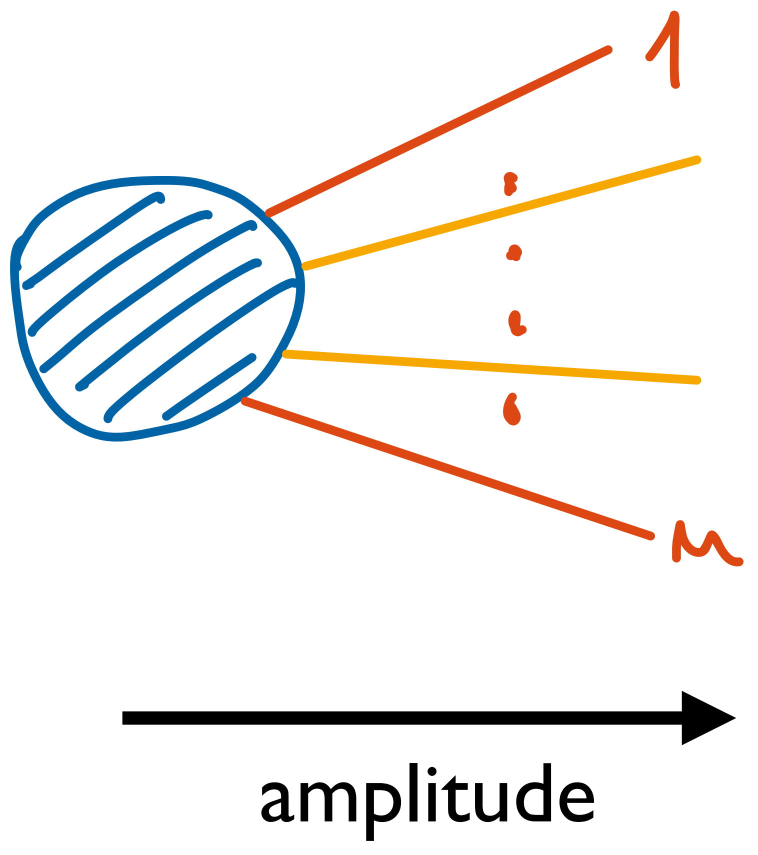


$$d\sigma \sim L \times d\sigma_H(Q) \times PS(Q \to \mu) \times MPI \times Had(\mu \to \Lambda) \times \ldots$$

Basic objects: Partonic scattering amplitudes in QCD

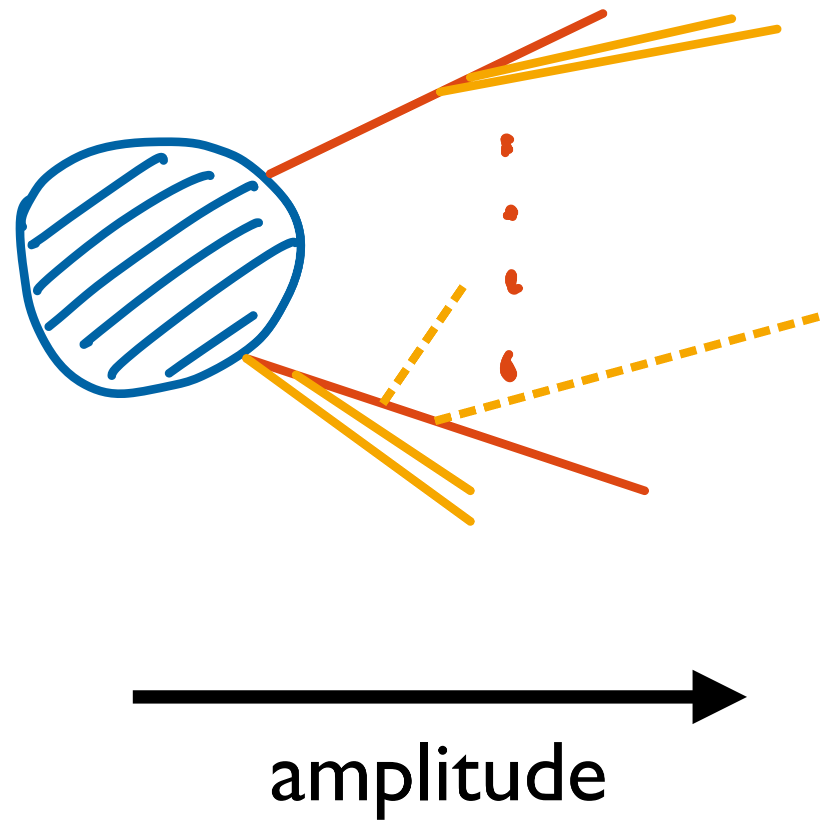
Each external leg carries a colour index ((anti-)fundamental and adjoint in SU(N)) and a momentum.

Fixed-order calculations



Calculate in fixed order in perturbation theory, including all contributions.

All-order calculations



Exploit factorisation and calculate kinematically dominant contributions at all perturbative orders.

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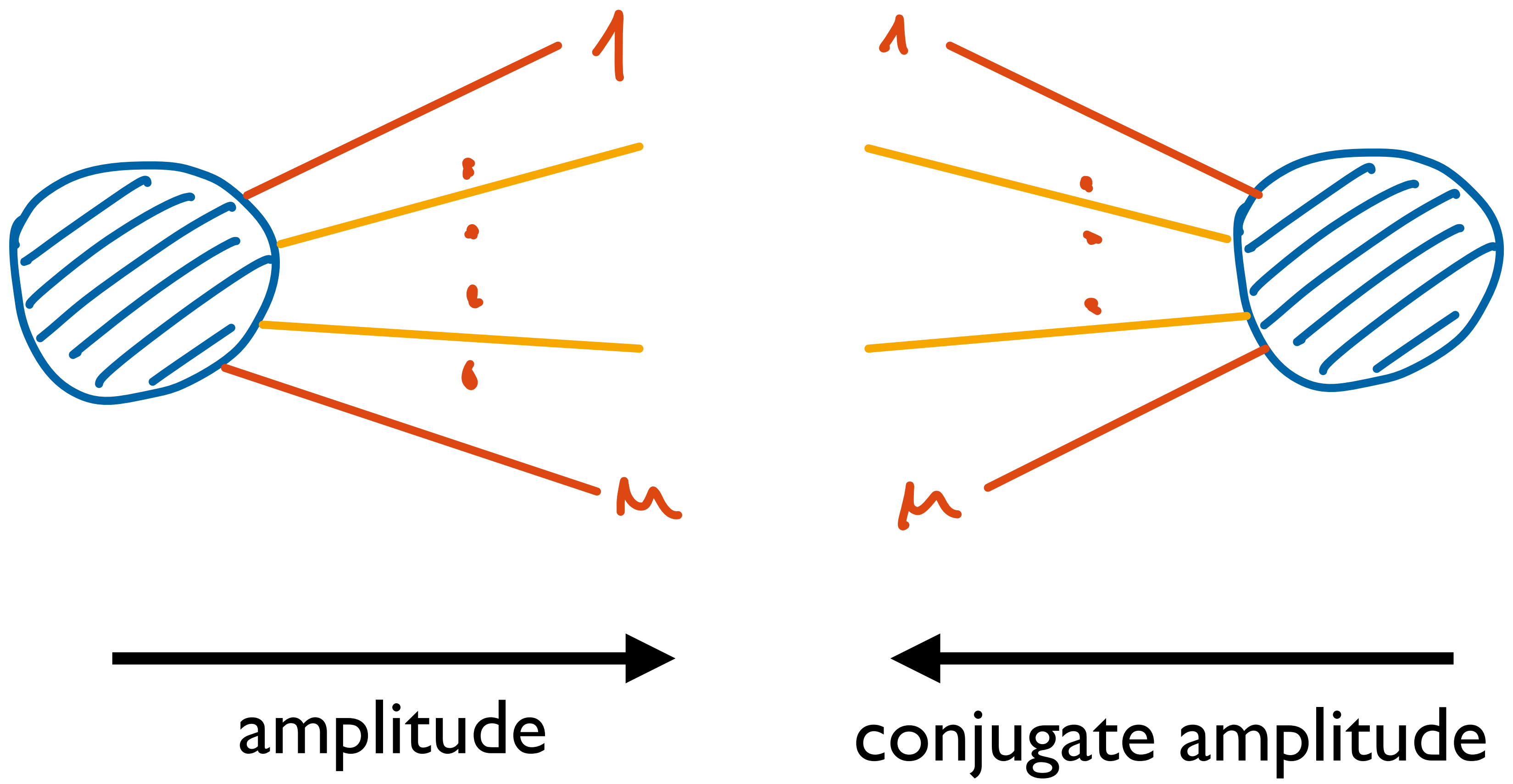

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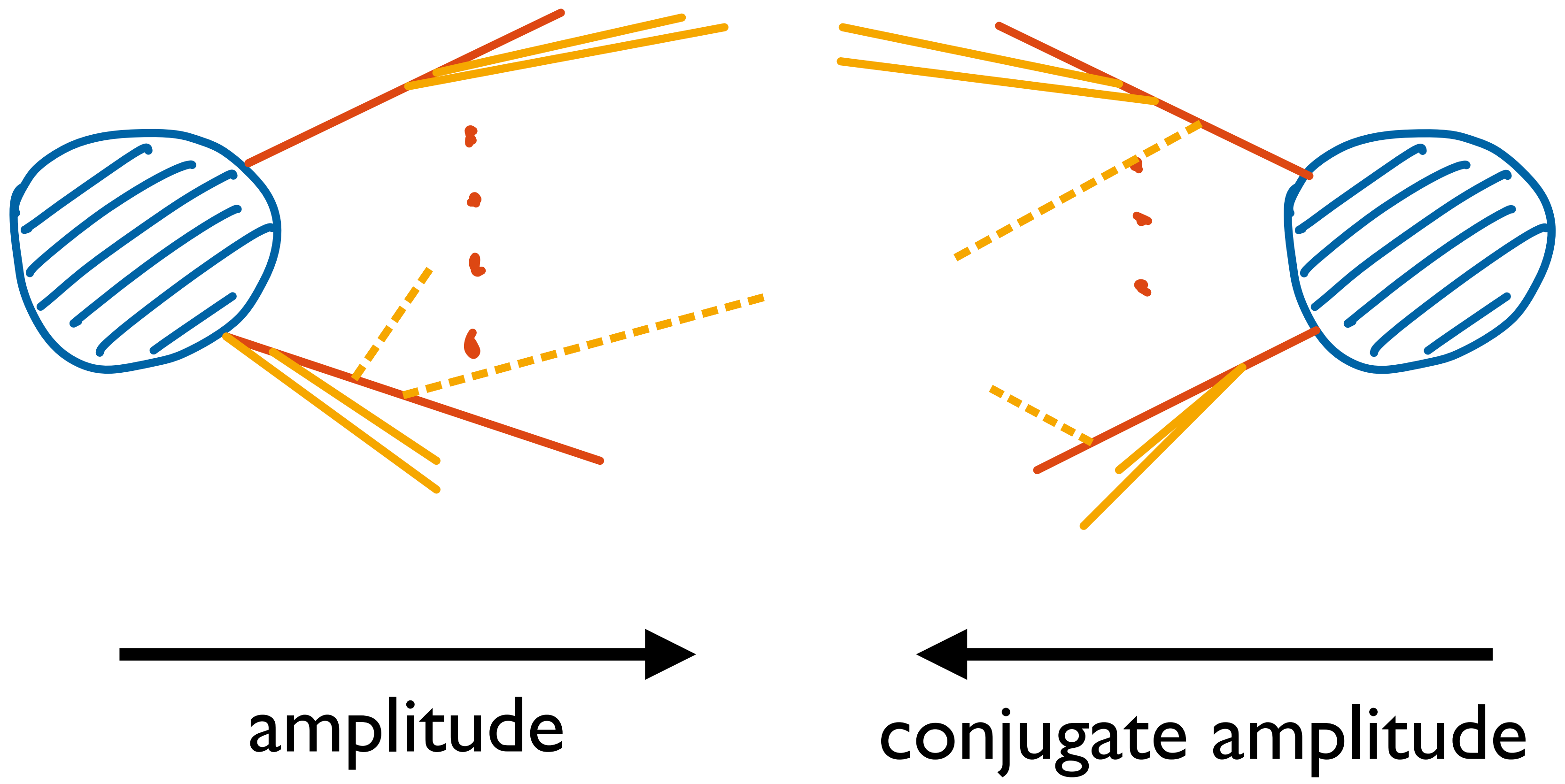
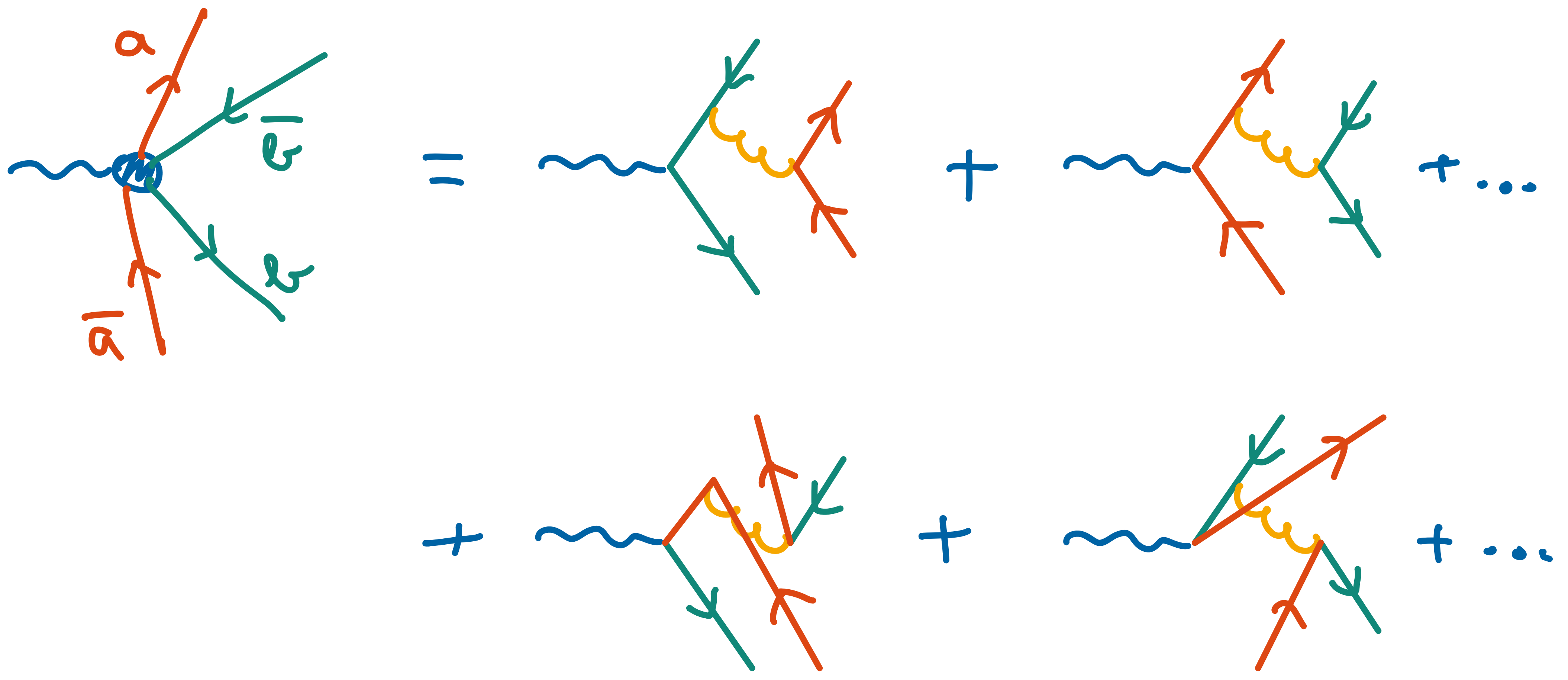


Exploit factorisation and calculate kinematically dominant contributions at all perturbative orders.

Cross section: squared amplitudes contract all open colour indices of the amplitude.

Colour space

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$



$$(t^a)^i{}_k (t^a)^j{}_l \quad \longleftrightarrow \quad |a\rangle$$

$$(t^a)^i{}_l (t^a)^j{}_k \quad \longleftrightarrow \quad |b\rangle$$

SU(N) tensors of high rank

invariance under global colour rotations

$$\sum_i \mathbf{T}_i |\mathcal{M}\rangle = 0 \qquad \sum_{i\neq j} \mathbf{T}_i\cdot\mathbf{T}_j \;\hat{=}\; -\mathbf{T}_i\cdot\mathbf{T}_i = -C_i \mathbb{1}$$

Notation

Abstract basis vectors ("basis independent notation")

$$\mathbf{A}_n = \sum_{\sigma,\bar{\sigma}} \mathcal{A}_n^{\sigma\bar{\sigma}} |\sigma_n\rangle\langle\bar{\sigma}_n|$$

Choice of basis — actual tensor structures: $\mathcal{S}_\sigma^{a_1,\ldots,a_n} = \langle a_1, \ldots, a_n|\sigma_n\rangle$ $\quad \mathcal{S}_{\bar{\sigma}}^{\dagger,\bar{a}_1,\ldots,\bar{a}_n} = \langle\bar{\sigma}_n|a_1, \ldots, a_n\rangle$

Colour charges extend tensor structures, satisfy group algebra.

$$\langle a_1, \ldots, a_n, a_{n+1}, \ldots, a_{n+k}|\mathbf{T}_{i_1}\cdots\mathbf{T}_{i_k}|\sigma_n\rangle = \mathbf{T}_{i_1}^{a_{n+1}}\cdots\mathbf{T}_{i_k}^{a_{n+k}}\mathcal{S}_\sigma^{a_1,\ldots,a_n} \qquad [\mathbf{T}_i^a, \mathbf{T}_j^b] = if^{abc}\mathbf{T}_{i,c}\delta_{ij}$$

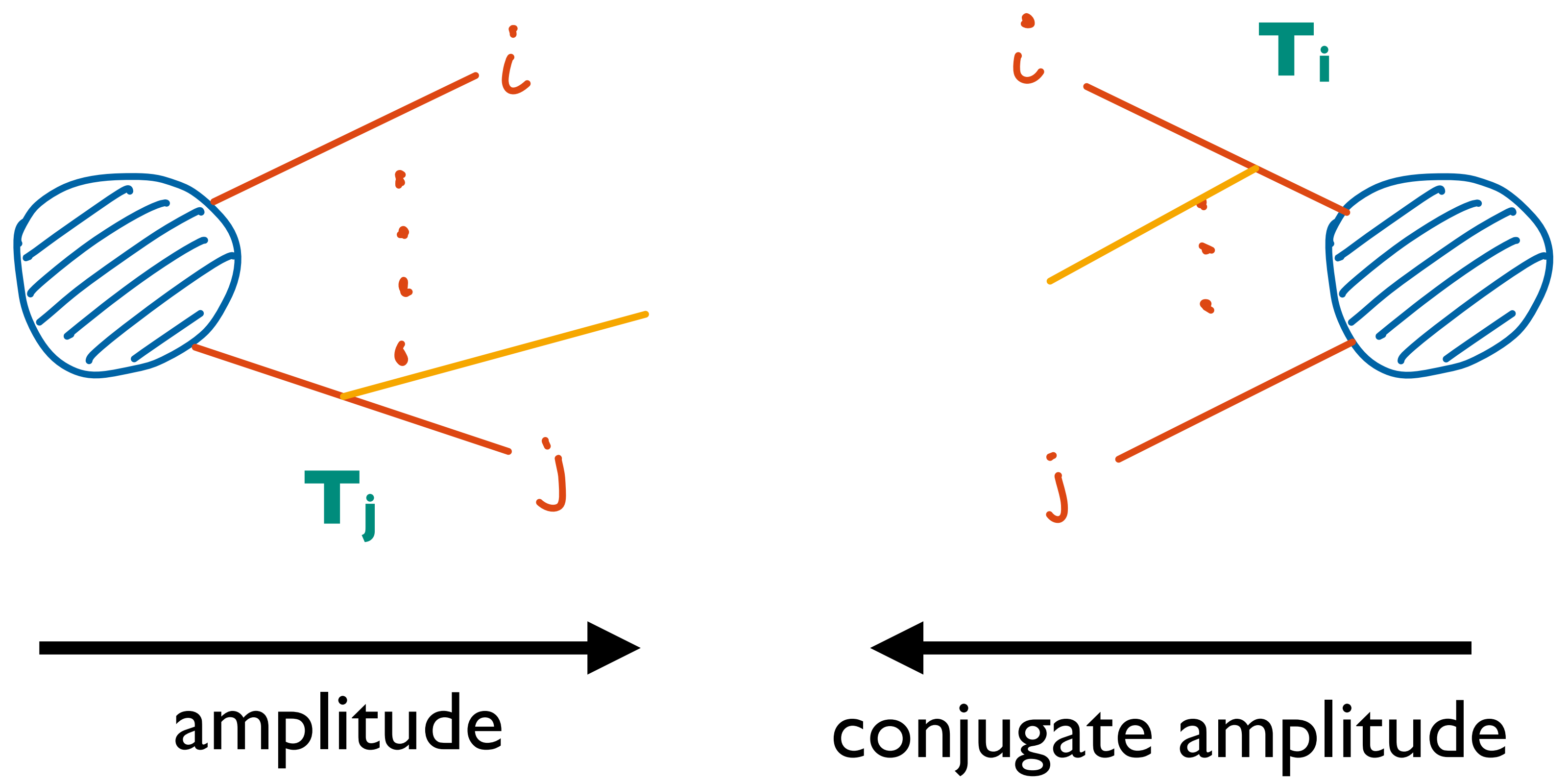
$$\mathbf{T}_i^a\mathcal{S}_\sigma^{a_1,\ldots,a_n} = \left(T_{R_i}^a\right)^{a_i}{}_{b_i}\mathcal{S}_\sigma^{a_1,\ldots,b_i,\ldots,a_n} \qquad \mathbf{T}_i\cdot\mathbf{T}_j = \mathbf{T}_i^a\mathbf{T}_j^a$$

Detailed relation to JIMWLK formalism: [Plätzer, Weigert — in progress]

$$\int \mathrm{d}k_1\cdots\mathrm{d}k_m\, G_{c_1,\ldots,c_r;\bar{c}_{r+1},\ldots,\bar{c}_m}(k_1,\ldots,k_m)\, \nabla_{k_1}^{c_1}\cdots\nabla_{k_r}^{c_r}\bar{\nabla}_{k_{r+1}}^{\bar{c}_{r+1}}\cdots\bar{\nabla}_{k_m}^{\bar{c}_m} \quad \longleftrightarrow \quad \mathcal{G}[\circ] = \sum_{i_1,\ldots,i_m} G_{c_1,\ldots,c_r;\bar{c}_{r+1},\ldots,\bar{c}_m}(p_{i_1},\ldots,p_{i_m})\, \mathbf{T}_{i_1}^{c_1}\cdots\mathbf{T}_{i_r}^{c_r}\circ\mathbf{T}_{i_{r+1}}^{\dagger,\bar{c}_{r+1}}\cdots\mathbf{T}_{i_m}^{\dagger,\bar{c}_m}$$

Building parton showers

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$



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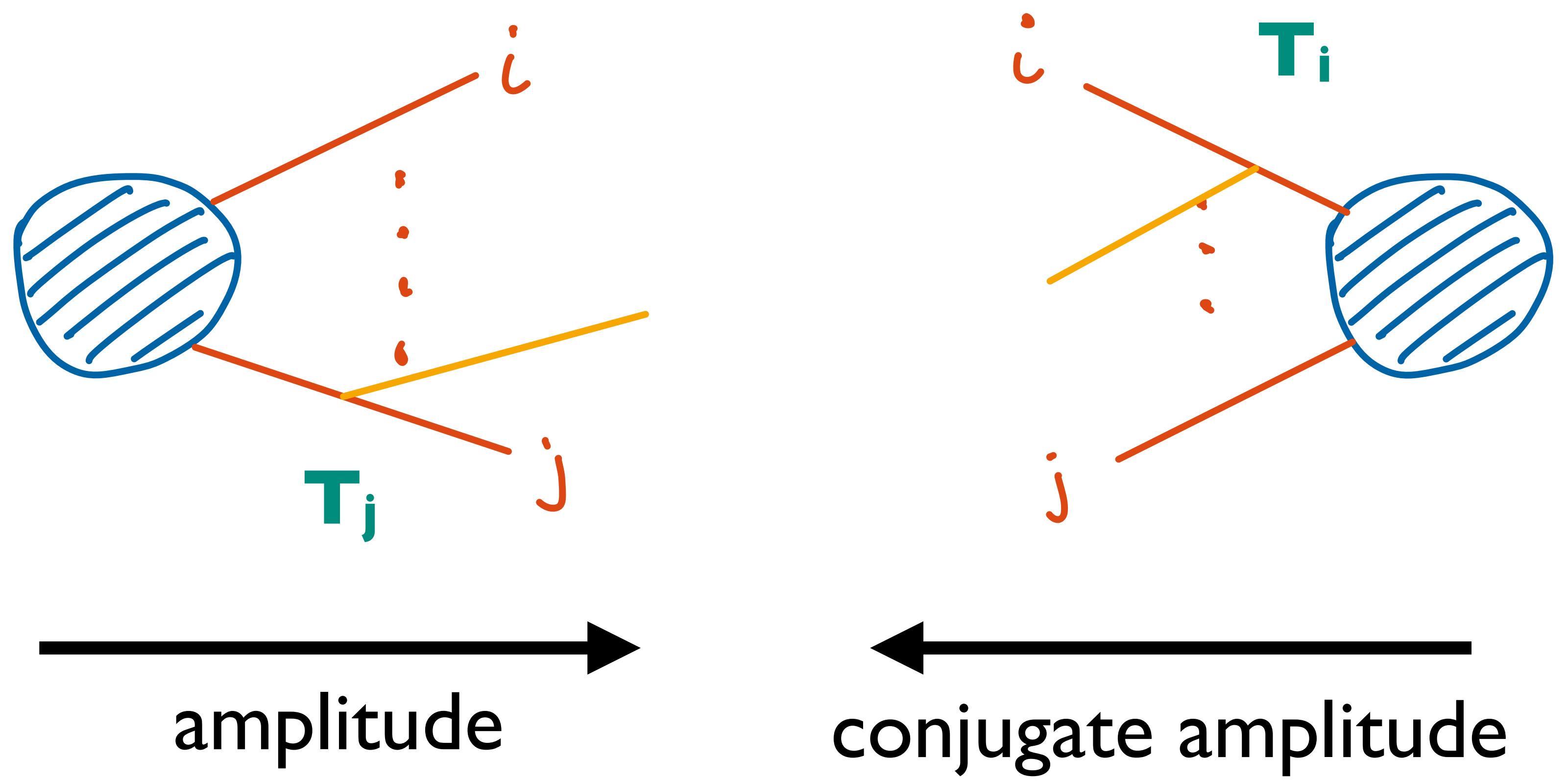


$$\left| \mathcal{M}_{n+m}(p_1, \ldots, p_n; q_1, \ldots, q_m) \right|^2$$

colour charge correlations

soft $$= \sum_{i,j} \frac{p_i \cdot p_j}{p_i \cdot q_m \, q_m \cdot p_j} \left(\mathcal{M}_{n+m-1}(\ldots \ldots) \right)^\dagger \mathbf{T}_i \cdot \mathbf{T}_j \, \mathcal{M}_{n+m-1}(\ldots \ldots)$$

collinear $$= \sum_{i} \frac{P_i(p_i, q_m)}{2 p_i \cdot q_m} \left(\mathcal{M}_{n+m-1}(\ldots \ldots) \right)^\dagger \mathbf{T}_i \cdot \mathbf{T}_i \, \mathcal{M}_{n+m-1}(\ldots \ldots)$$

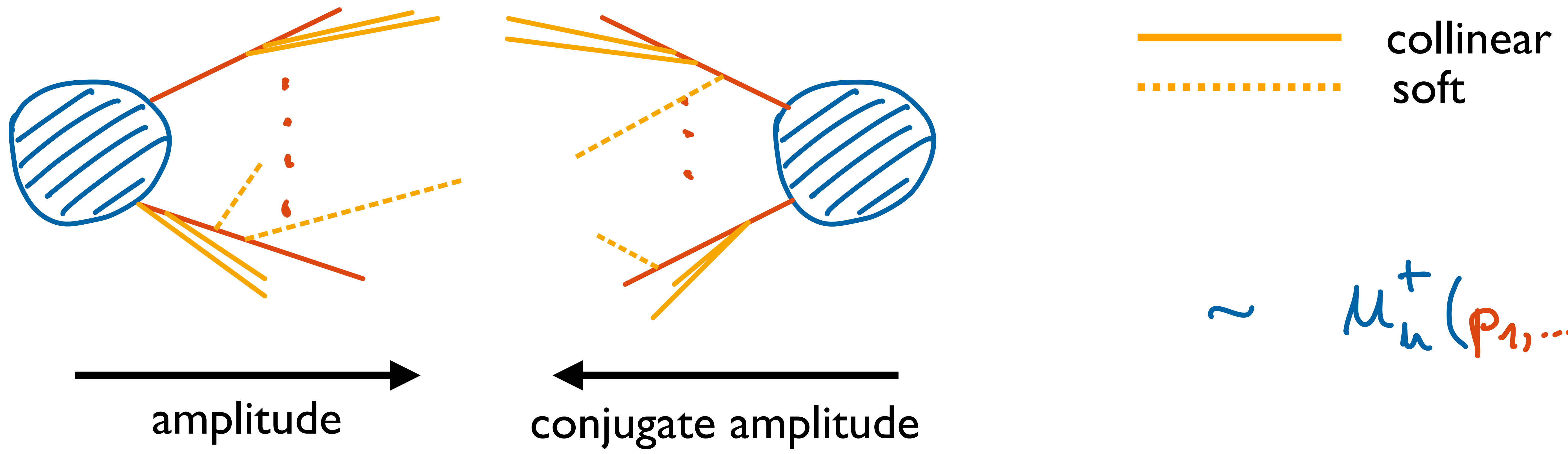
$$= C_i \mathbb{1}$$

colour charge squared

Emission of small energy and/or collinear gluons factorize from the amplitude.
Colour correlations are simple in the collinear limit.

Building parton showers

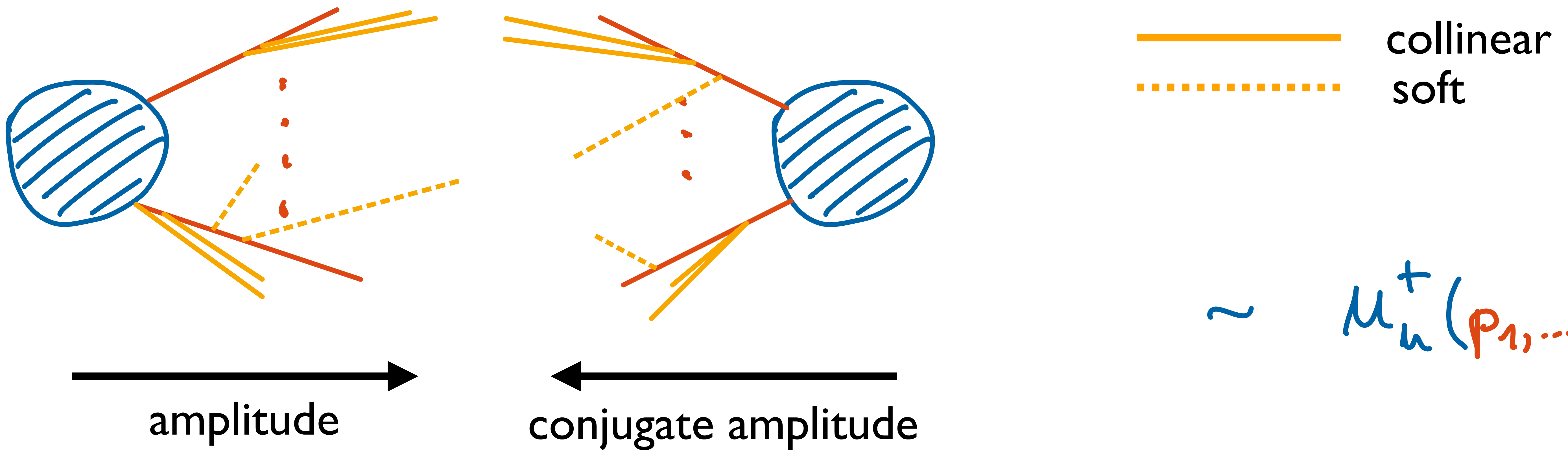
$$d\sigma \sim \mathrm{L} \times d\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$



$$\sim \quad \mathcal{M}_n^+(p_1,\ldots,p_n)\, \mathbf{T} \cdots \mathbf{T} \circ \mathbf{T} \cdots \mathbf{T}\, \mathcal{M}_n(p_1,\ldots,p_n)$$

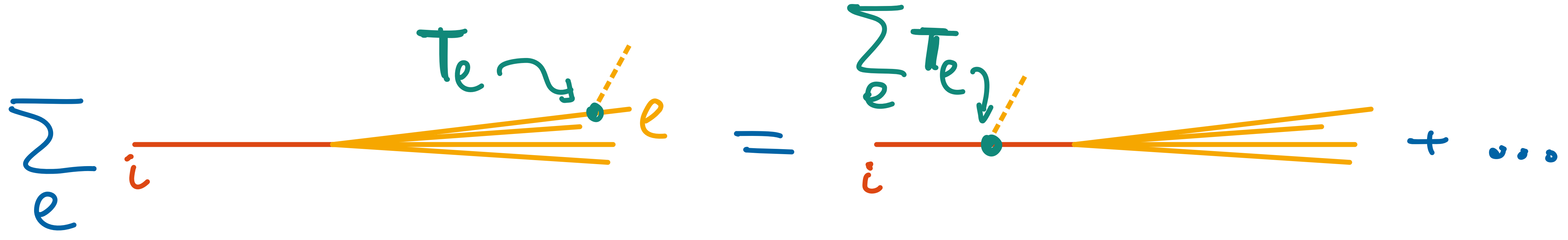
Building parton showers

$$d\sigma \sim L \times d\sigma_H(Q) \times PS(Q \to \mu) \times MPI \times Had(\mu \to \Lambda) \times \ldots$$



$$\sim \mathcal{M}_n^\dagger(p_1,\ldots,p_n)\, \mathbf{T} \cdots \mathbf{T} \cdot \mathbf{T} \cdots \mathbf{T}\, \mathcal{M}_n(p_1,\ldots,p_n)$$

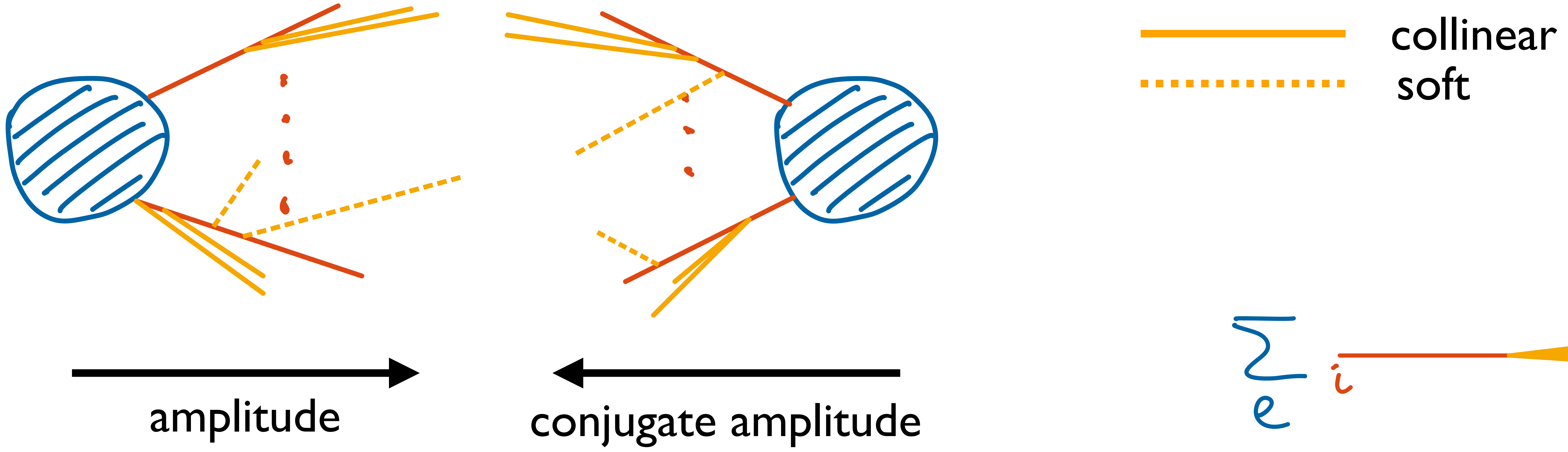
Exploit QCD coherence:

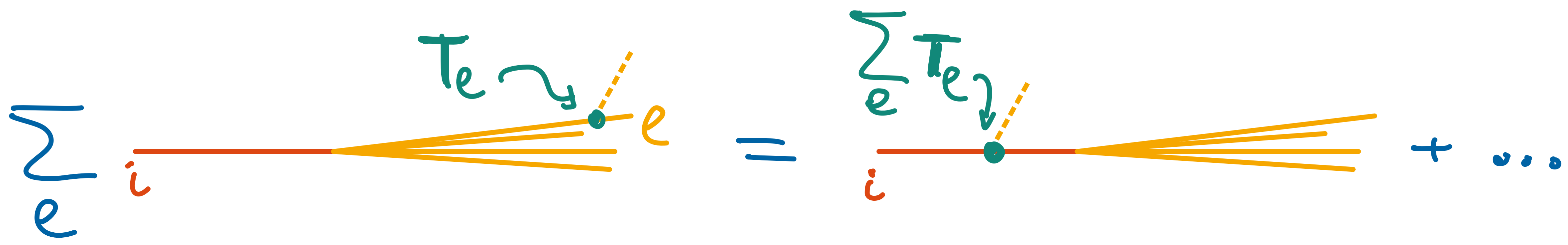


$$\mathbf{T}_j \mathbf{T}_l \mathbf{T}_i \cdot \mathbf{T}_i \mathbf{T}_m \mathbf{T}_j = C_i\, \mathbf{T}_j \mathbf{T}_l \cdot \mathbf{T}_m \mathbf{T}_j$$

Coherent branching parton showers

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$





$$\mathrm{d}S = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\tilde{q}_i^2}{\tilde{q}_i^2}\mathrm{d}z P(z_i) \exp\left(-\int_{\tilde{q}_i^2}^{Q^2}\frac{\mathrm{d}q^2}{q^2}\int_{z_-(k^2)}^{z_+(k^2)}\mathrm{d}\xi \frac{\alpha_s}{2\pi}P(z)\right)$$

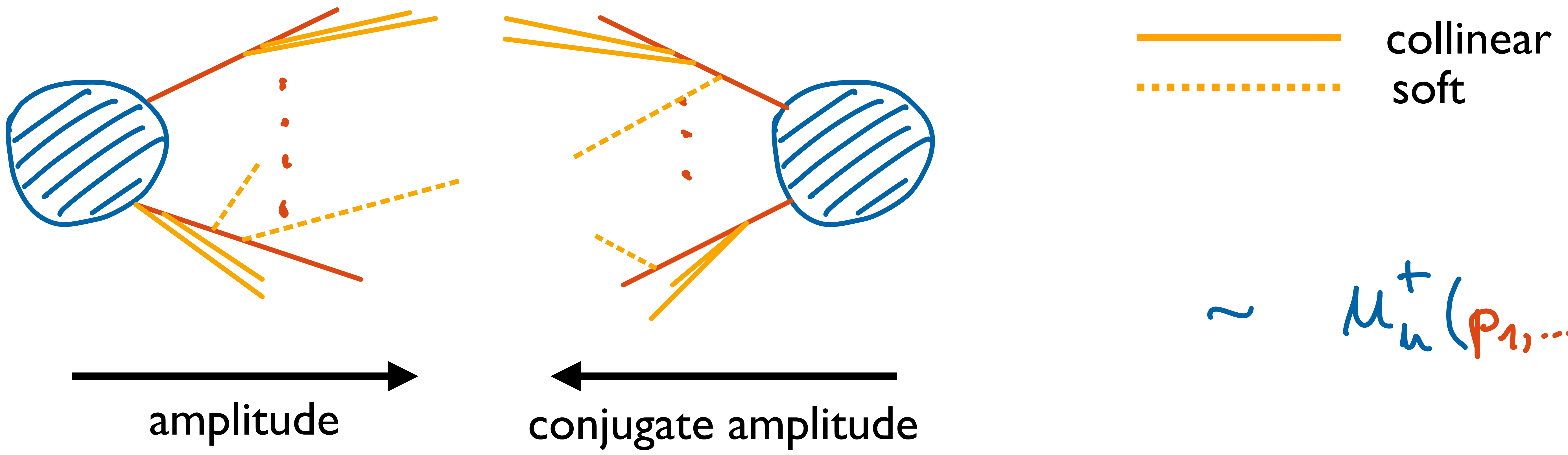
emission rate

no emission probability

All probabilistic algorithms determine the effect of gluon exchange and virtual corrections by unitarity.

Beyond coherence: amplitude evolution

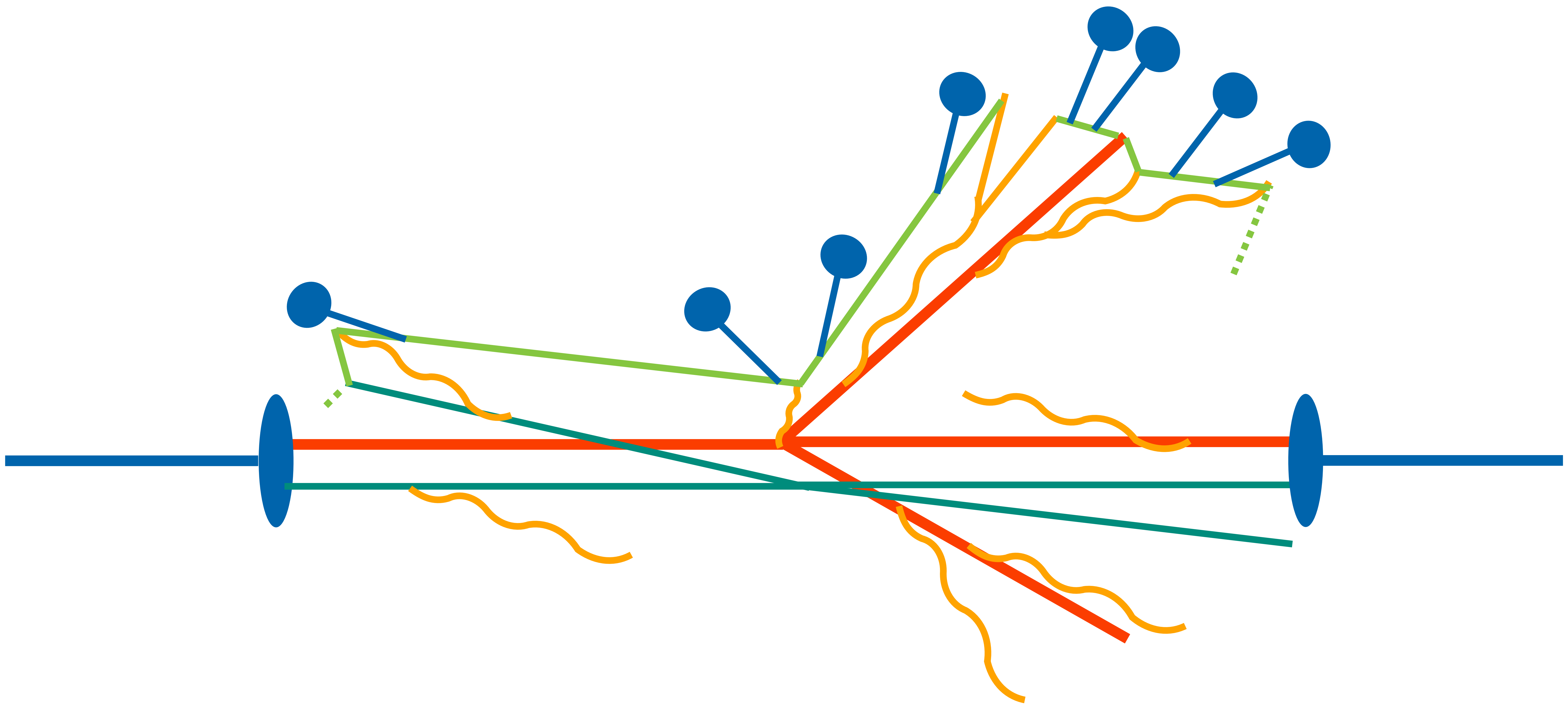
$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$



$$\sim \mathcal{M}_n^\dagger(p_1, \ldots, p_n)\, \mathbf{T} \cdots \mathbf{T} \,\vdots\, \mathbf{T} \cdots \mathbf{T}\, \mathcal{M}_n(p_1, \ldots, p_n)$$

Suggests an iterative procedure to build amplitude and conjugate amplitude with many emissions.

The full picture



$$d\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q \to \mu)\mathrm{d}\mathbf{H}(Q)\mathbf{PS}^{\dagger}(Q \to \mu)\mathbf{Had}(\mu \to \Lambda)\right]$$

Full colour and interferences are central

Colour reconnection and hadronization is about subleading-N.
So are shower accuracy and interference terms.

Colour factor algorithms

Coherent, NLL-accurate dipole showers

[Gustafson] [PanScales '21]
[Forshaw, Holguin, Plätzer '21]

Colour ME corrections

Colour-exact real emissions as far as possible

[Plätzer, Sjödahl '12, '18]
[Höche, Reichelt '20]

Full amplitude evolution

Colour-exact real and virtual corrections

[Forshaw, Plätzer + … '13 …]
[Nagy, Soper '12 …]

Colour matrix element corrections

Colour matrix element corrections:
Real emissions only amplitude evolution —
first implementation in a shower algorithm.

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha}|\alpha_n\rangle$$

$$\mathcal{M}_n = (c_{n,1}, \ldots, c_{n,d_n})^T$$

[Plätzer, Sjödahl '12]
[Plätzer, Sjödahl, Thoren '18]

$$|\mathcal{M}_n|^2 = \mathcal{M}_n^\dagger S_n \mathcal{M}_n = \mathrm{Tr}\left(S_n \times \mathcal{M}_n \mathcal{M}_n^\dagger\right)$$

$$\langle \mathcal{M}_n|\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}}|\mathcal{M}_n\rangle = \mathrm{Tr}\left(S_{n+1} \times T_{\tilde{k},n} \mathcal{M}_n \mathcal{M}_n^\dagger T_{\tilde{ij},n}^\dagger\right)$$

approximation — correction factor

$$V_{ij,k}(p_\perp^2, z; p_{\tilde{ij}}, p_{\tilde{k}}) \times \frac{-1}{\mathbf{T}_{\tilde{ij}}^2} \frac{\langle \mathcal{M}_n|\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_k|\mathcal{M}_n\rangle}{|\mathcal{M}_n|^2}$$

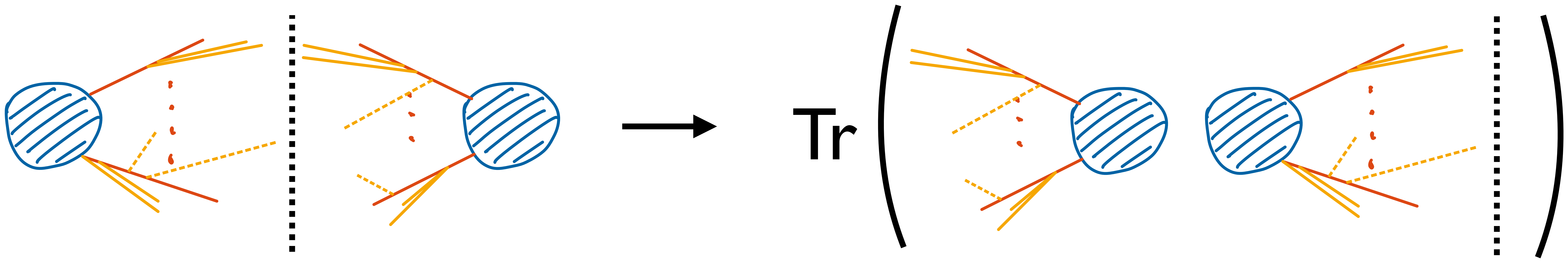
$$M_{n+1} = -\sum_{i\neq j} \sum_{k\neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}_{\tilde{ij}}^2} T_{\tilde{k},n} M_n T_{\tilde{ij},n}^\dagger$$

Full colour and interferences are central

Colour reconnection and hadronization is about subleading-N.
So are shower accuracy and interference terms.

Colour factor algorithms	Colour ME corrections	Full amplitude evolution
Coherent, NLL-accurate dipole showers	Colour-exact real emissions as far as possible	Colour-exact real and virtual corrections
[Gustafson] [PanScales '21] [Forshaw, Holguin, Plätzer '21]	[Plätzer, Sjödahl '12, '18] [Höche, Reichelt '20]	[Forshaw, Plätzer + … '13 …] [Nagy, Soper '12 …]

Amplitude evolution



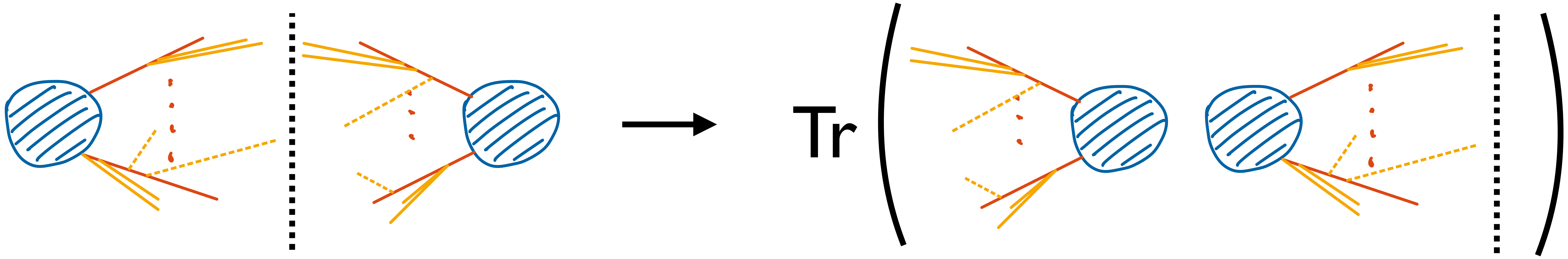
$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\ \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\ \mathbf{D}_n(k)\ \mathbf{A}_{n-1}(k)\ \mathbf{D}_n^\dagger(k)\ \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level: Iterate gluon exchanges and emission.
Different histories in amplitude and conjugate amplitude needed to include interference.

CVolver solves evolution equations in colour flow space. Flexible for dedicated resummation and new parton showers.

One-loop structures … [Plätzer '13]
Soft evolution … [Angeles, De Angelis, Forshaw, Plätzer, Seymour – '18]
Soft + collinear evolution … [Forshaw, Holguin, Plätzer – '19]
Two-loop structures … [Plätzer, Ruffa — '21]
First Monte Carlo implementation … [De Angelis, Forshaw, Plätzer — '21]
Emissions beyond leading order … [Löschner, Plätzer, Simpson-Dore — '20]

Amplitude evolution



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k} \, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')} \, \mathbf{D}_n(k) \, \mathbf{A}_{n-1}(k) \, \mathbf{D}_n^\dagger(k) \, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$

Simplest case: Eikonal current

$$\mathbf{D}^{(1,0)} \circ \mathbf{D}^{(1,0)\dagger} = \frac{\alpha_S}{2\pi} \sum_{i,j} \omega_{ij} \, \mathbf{T}_i \circ \mathbf{T}_j^\dagger$$

Simplest case: soft exchanges

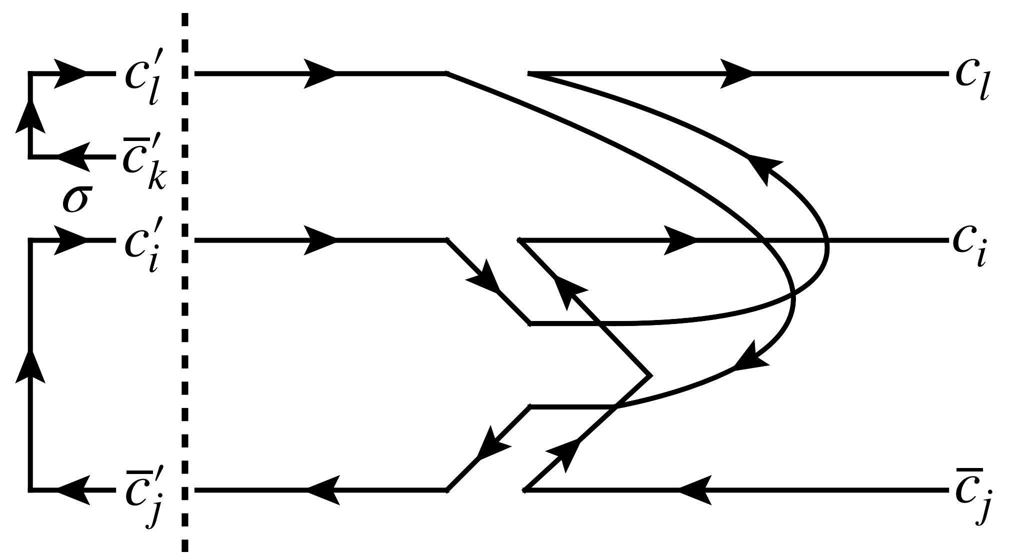
$$\mathbf{\Gamma}^{(1)} = \frac{\alpha_s}{2\pi} \sum_{i<j} \int \mathrm{d}\Omega \, \omega_{ij} \mathbf{T}_i \cdot \mathbf{T}_j$$

Colour bases

Basis choice not unique, typical "bases" are merely spanning sets, non-orthogonal …
Advantages and disadvantages not clear from the beginning, colour also intertwined with kinematics.

Colour flows

Reveal algorithmic structures for (Monte Carlo) approach.



[Forshaw, Plätzer, Ruffa, Löschner, … – '18+ & in progress]

Trace bases

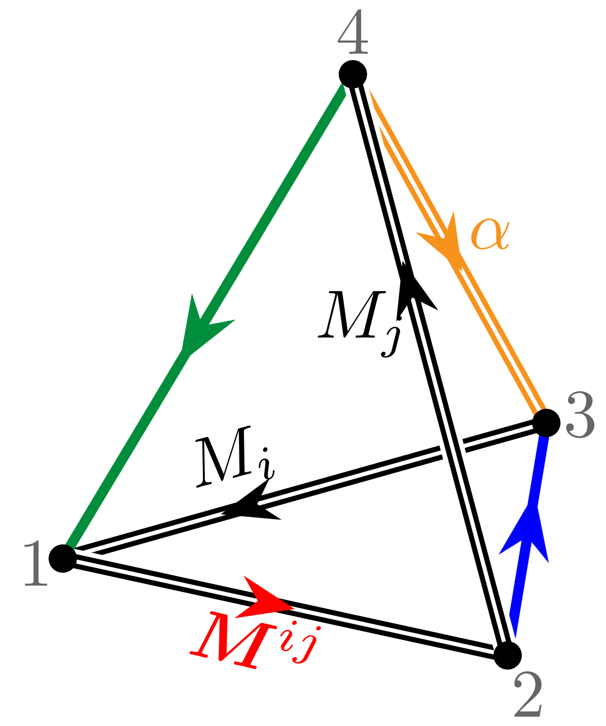
Those had been used at some point.

$$(t^a)^i{}_l \, (t^a)^j{}_k$$

e.g. used in fixed-order calculations [Plätzer, Sjödahl — '12 to '18]

Proper bases

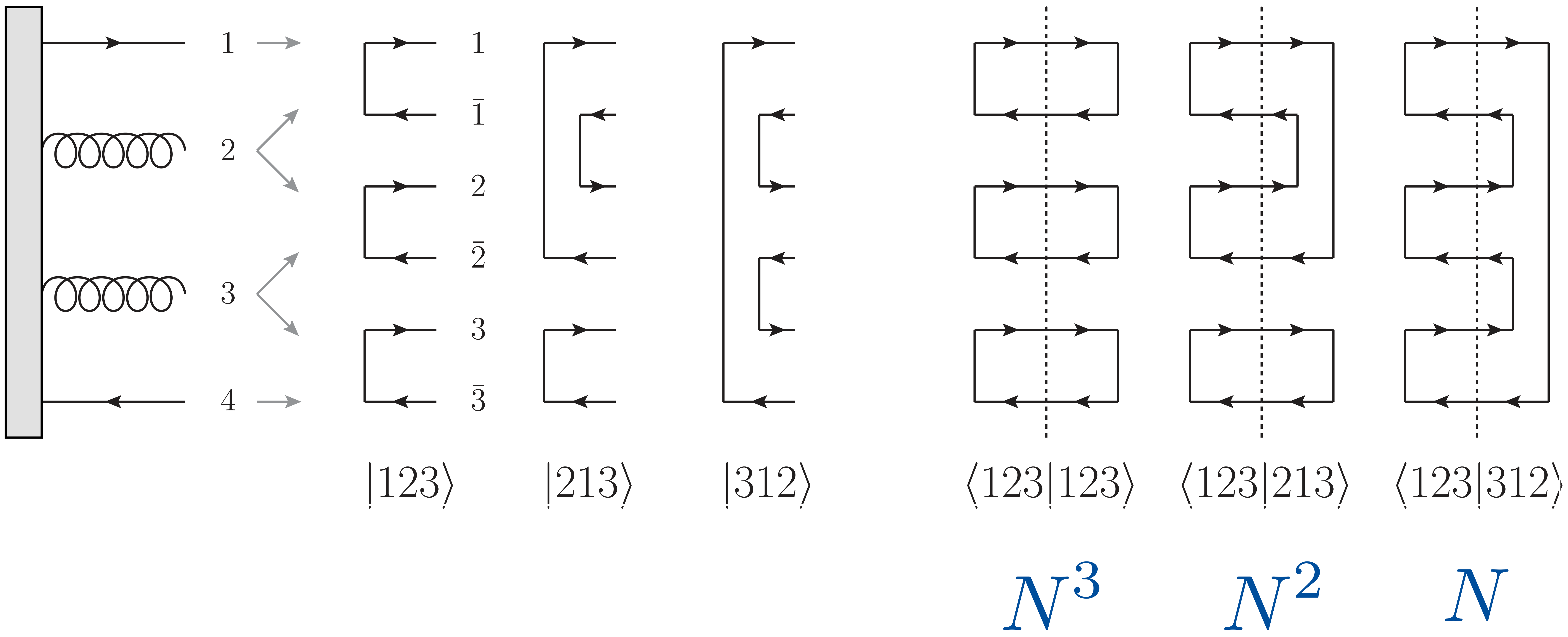
Understand colour multiplets for many legs.



[Alcock-Zeilinger, Keppeler, Plätzer, Sjödahl – '22,'23 & in progress]

Tracking colour flow

Decompose amplitudes in flow of colour charge. $(t^a)^i{}_k (t^a)^j{}_l = T_R \left(\delta^i_l \delta^j_k - \frac{1}{N} \delta^i_k \delta^j_l \right)$



Suppression of interferences outside of colour connected dipoles.

[Plätzer '13]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour '18]

Non-orthogonal, spanning set, …

Define orthogonal basis:

$$S_{\tau\sigma} = \langle\tau|\sigma\rangle \qquad |\sigma] = \sum_\tau S^{-1}_{\tau\sigma}|\tau\rangle \qquad [\tau|\sigma\rangle = \langle\tau|\sigma] = \delta_{\tau\sigma} \qquad \sum_\sigma |\sigma]\langle\sigma| = \sum_\sigma |\sigma\rangle[\sigma| = \mathbf{1}$$

Singular for a critical N in SU(N) or for a critical number of external legs.

Definition of matrix elements formally and algorithmically possible:

$$\mathbf{A}|\sigma\rangle = \sum_\tau \mathcal{A}_{\tau\sigma}|\tau\rangle \qquad [\tau|\mathbf{A}|\sigma\rangle = \mathcal{A}_{\tau\sigma}$$

[Plätzer '13]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour '18]

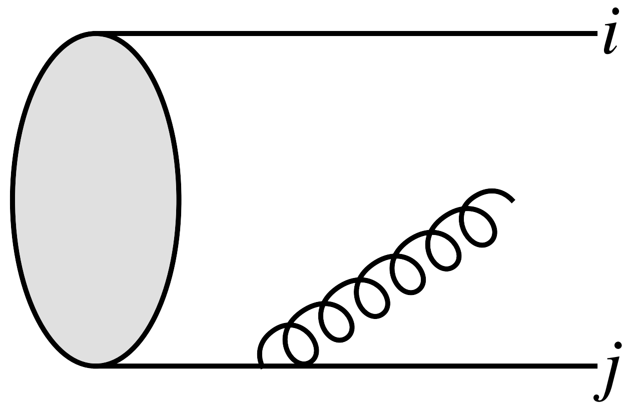
After tracing, any perturbative calculation will only give poles in 1/N from the trace condition, but is otherwise an analytic function in N. If we are algorithmically never forced to pick a value of N or to evaluate the inverse Gram, we can equally well **assume any non-critical N > 0 as a regulator.**

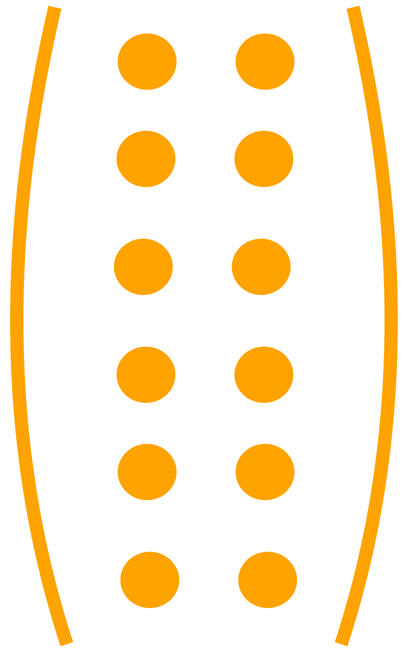
This might have further implications — [Plätzer — wip]

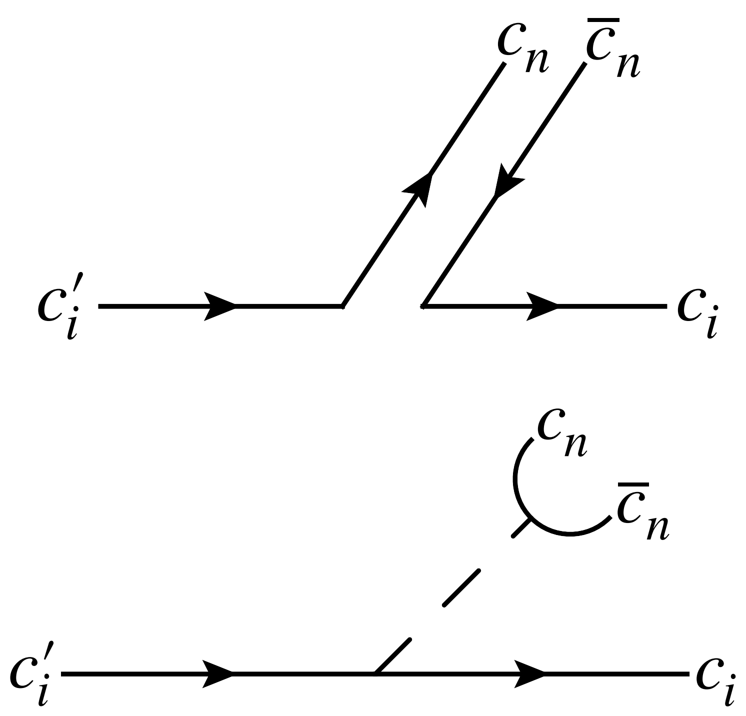
Colour flows

Gluon emission

$$\mathbf{D}_n(k)$$



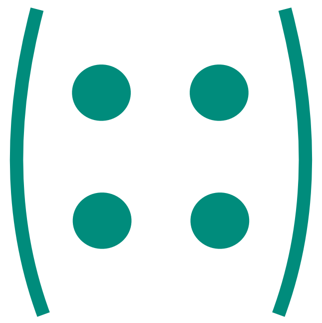


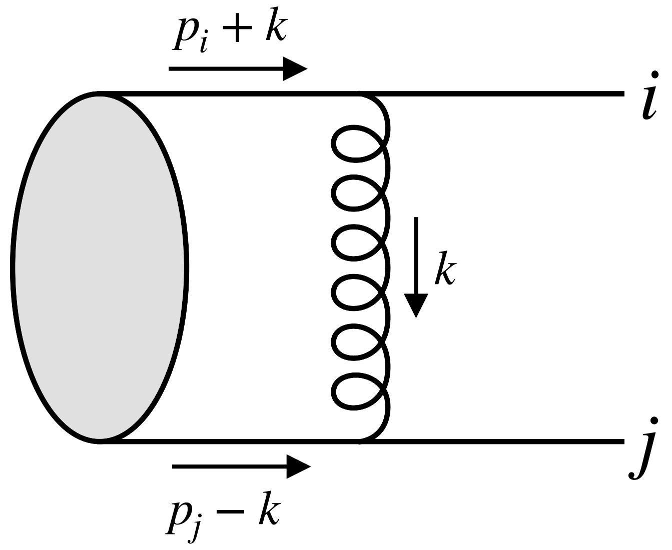


Explicit suppression in 1/N

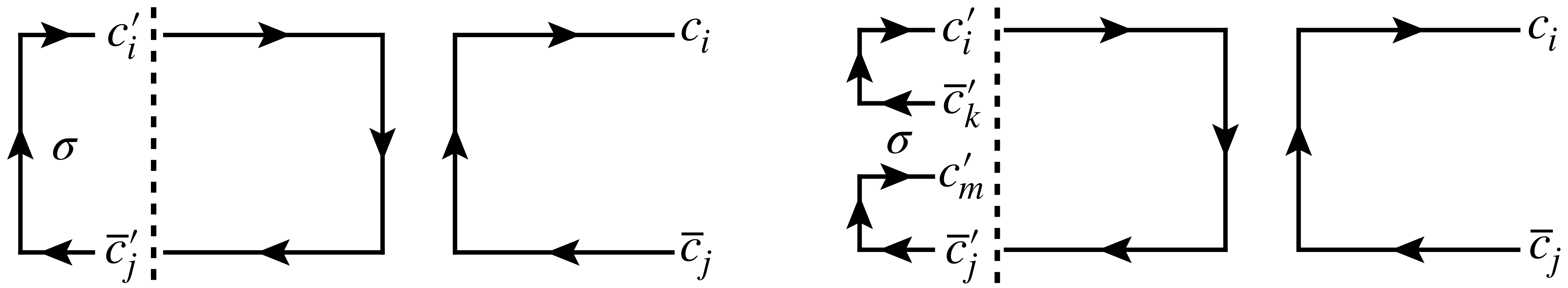
Gluon exchange

$$\mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$





$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$



$$[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma_\sigma^{(1)} + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma_{\sigma\tau}^{(1)}$$

dipole flips — implicit suppression in 1/N

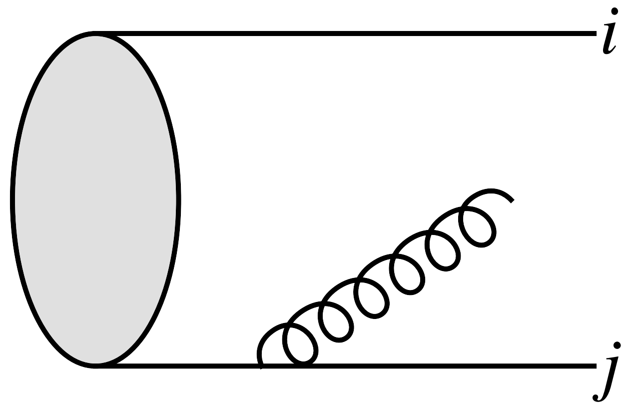
Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

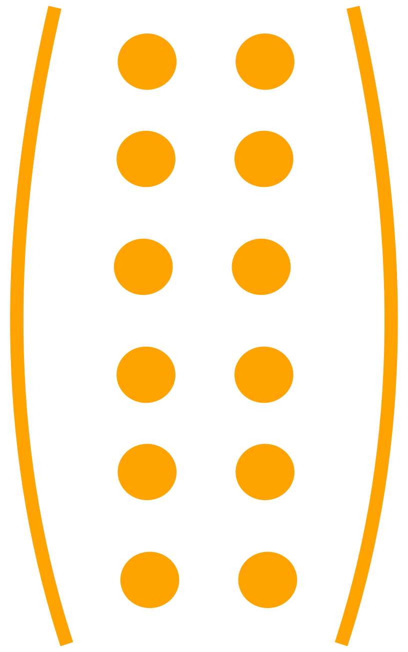
[Plätzer – '13] — diagrams from [Ruffa, MSc thesis 2020]

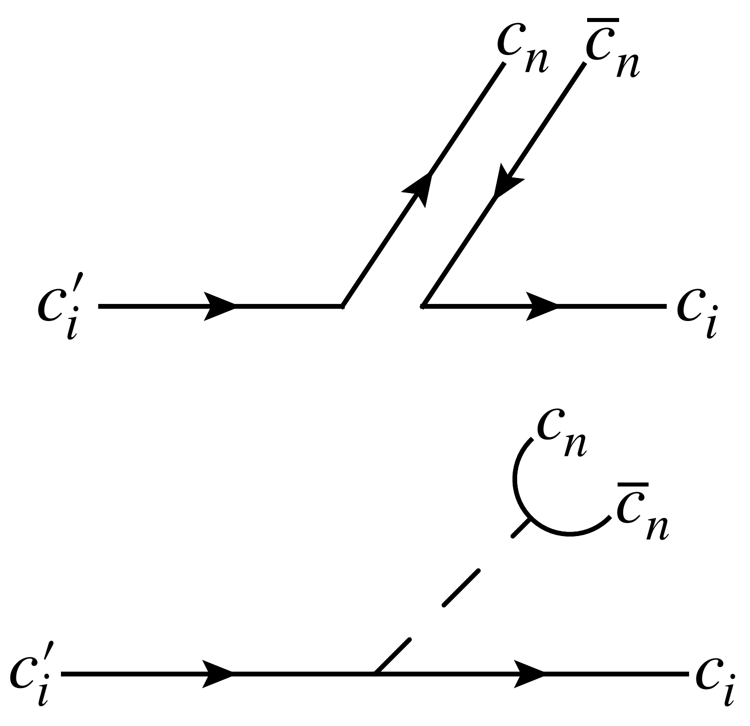
Colour flows

Gluon emission

$$\mathbf{D}_n(k)$$



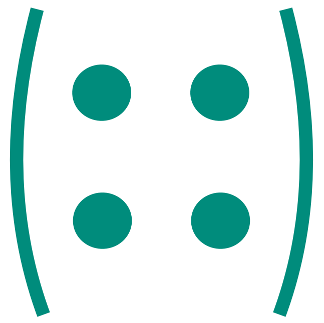


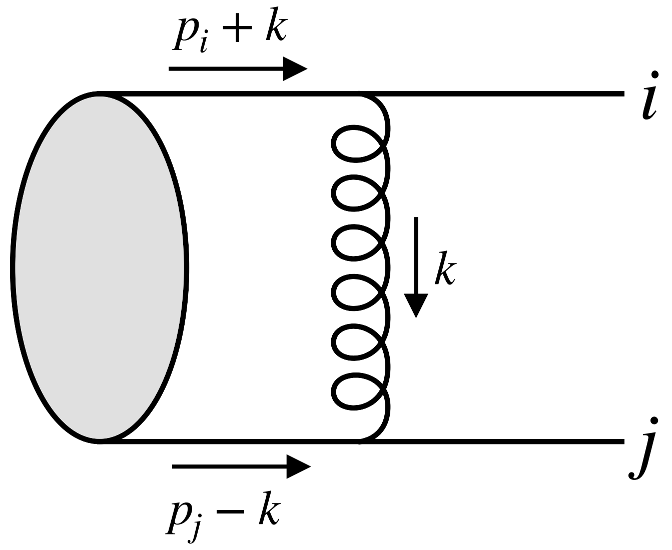


Explicit suppression in 1/N

Gluon exchange

$$P e^{-\int_q^k \frac{\mathrm{d}k'}{k'} \mathbf{\Gamma}(k')}$$





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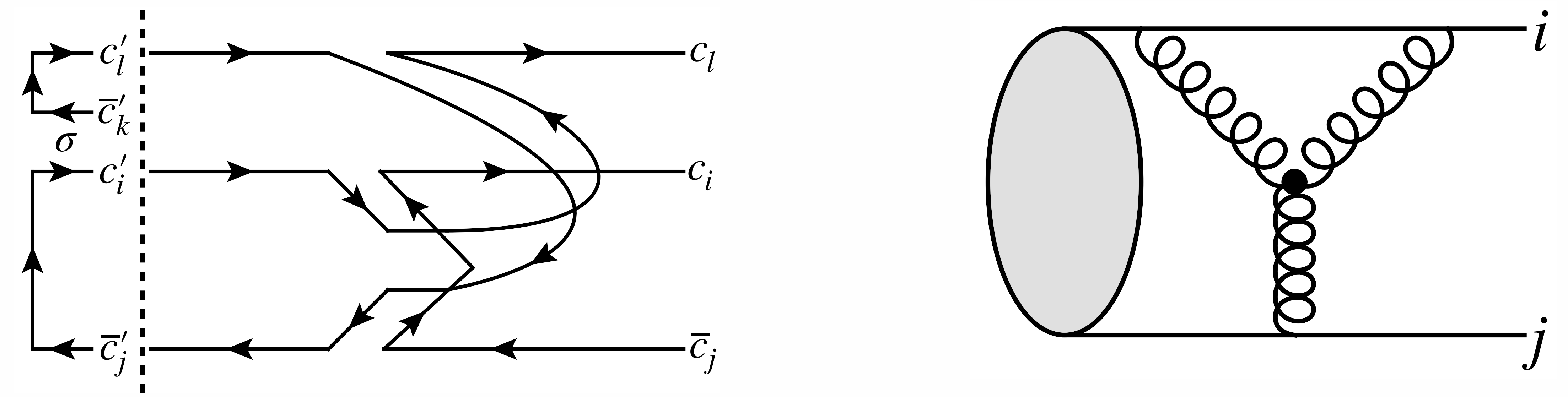

[Plätzer, Ruffa — '21]

dipole flips — implicit suppression in 1/N

Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

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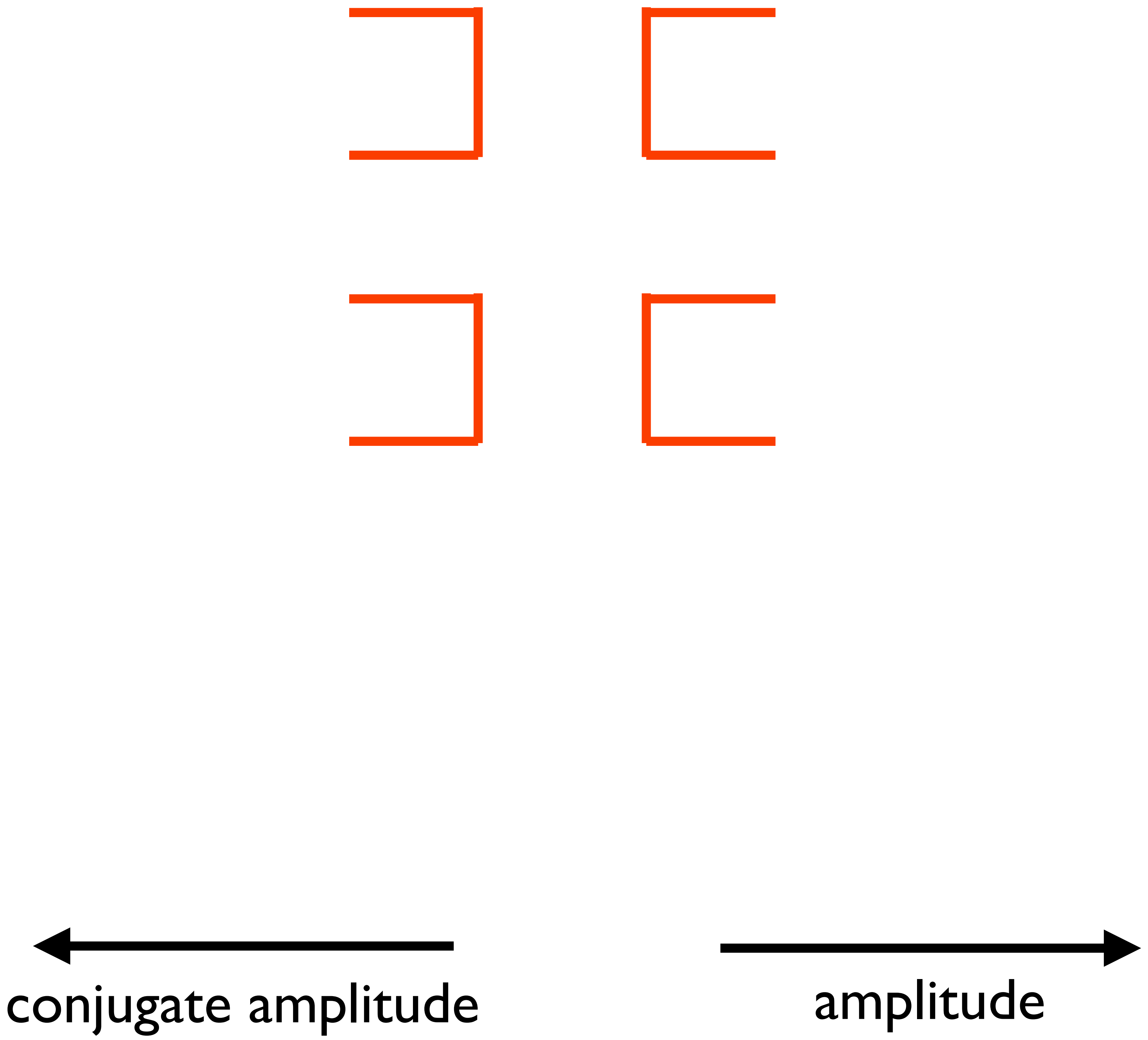
Amplitude evolution: Colour diagonal piece

$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$



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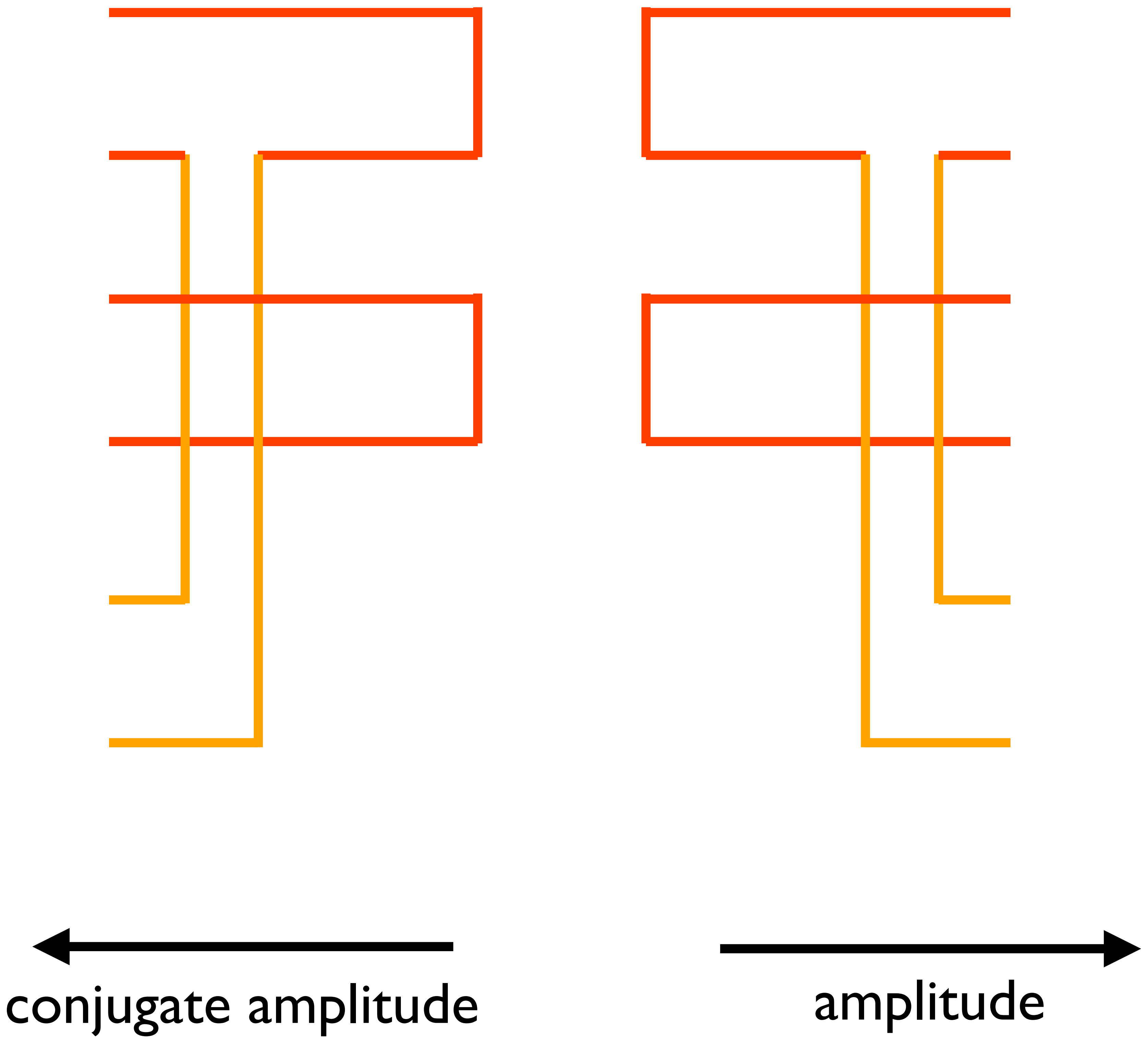

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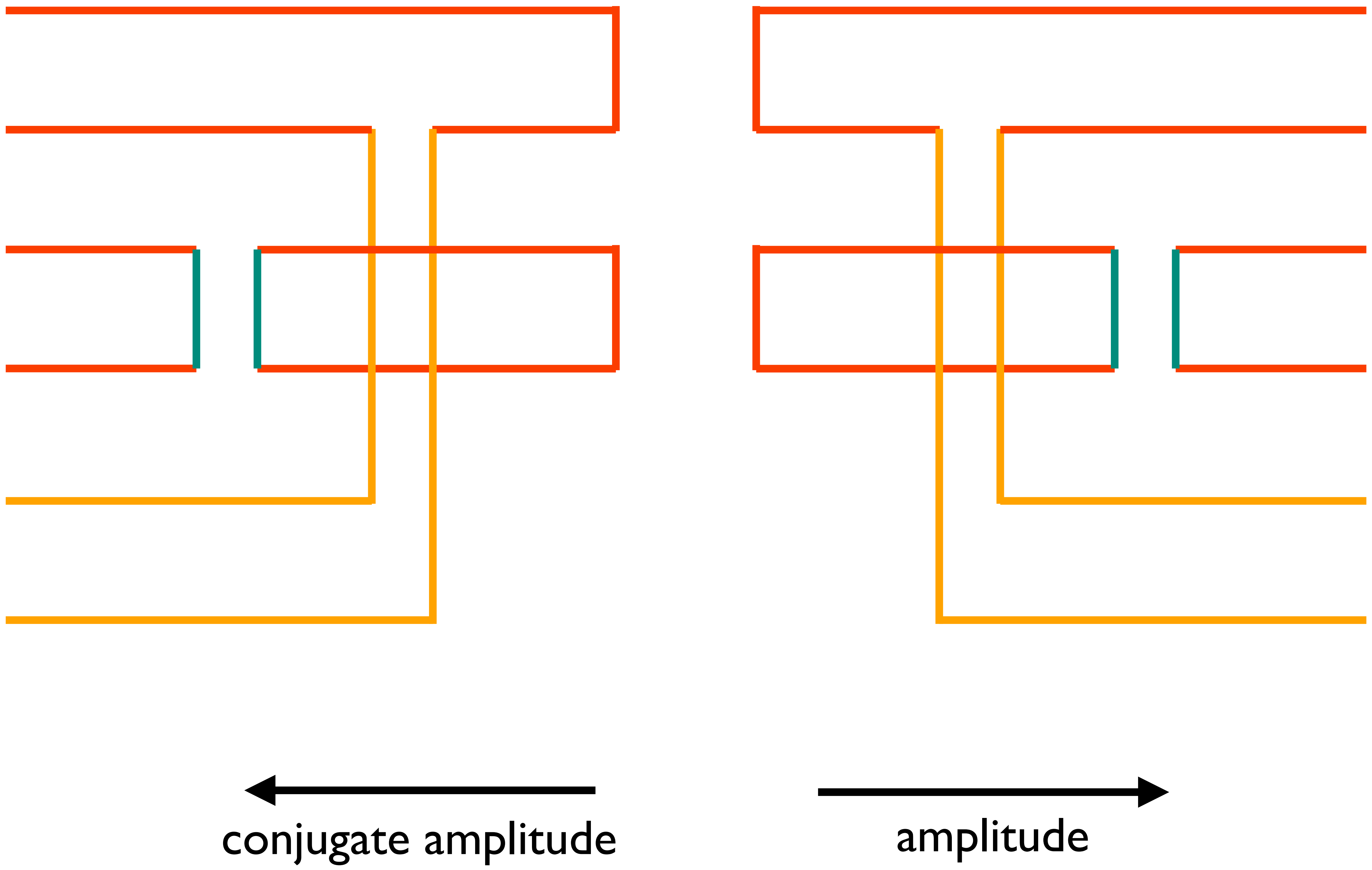

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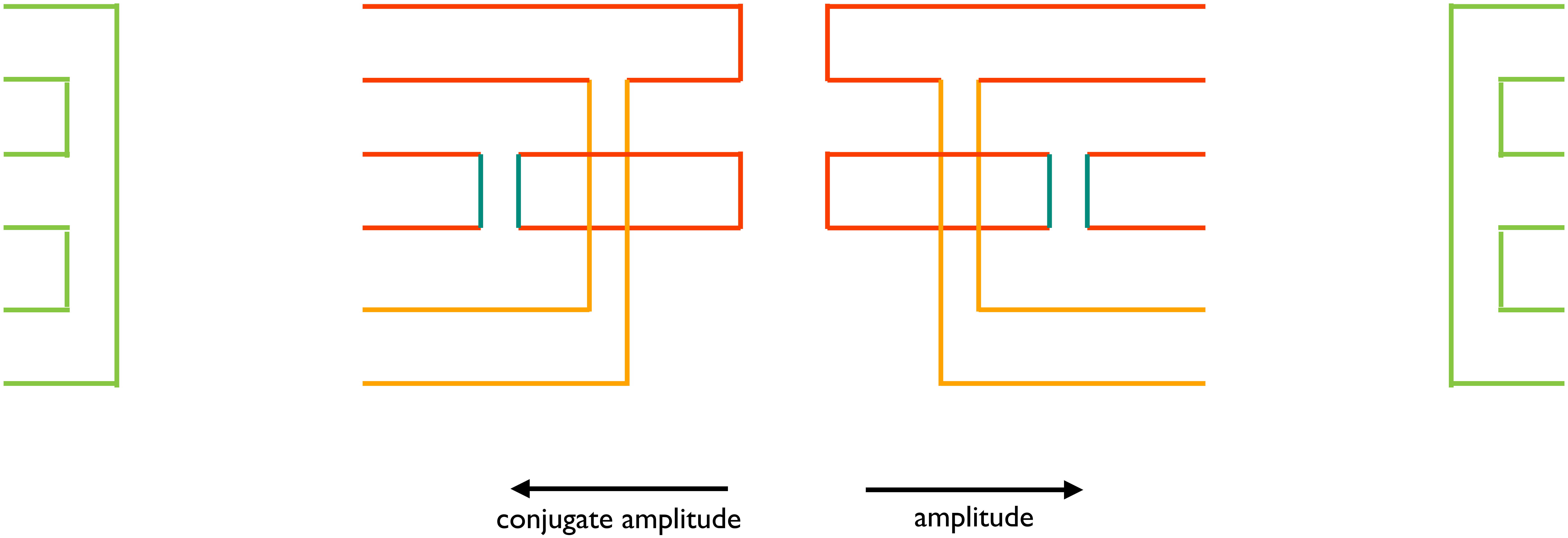

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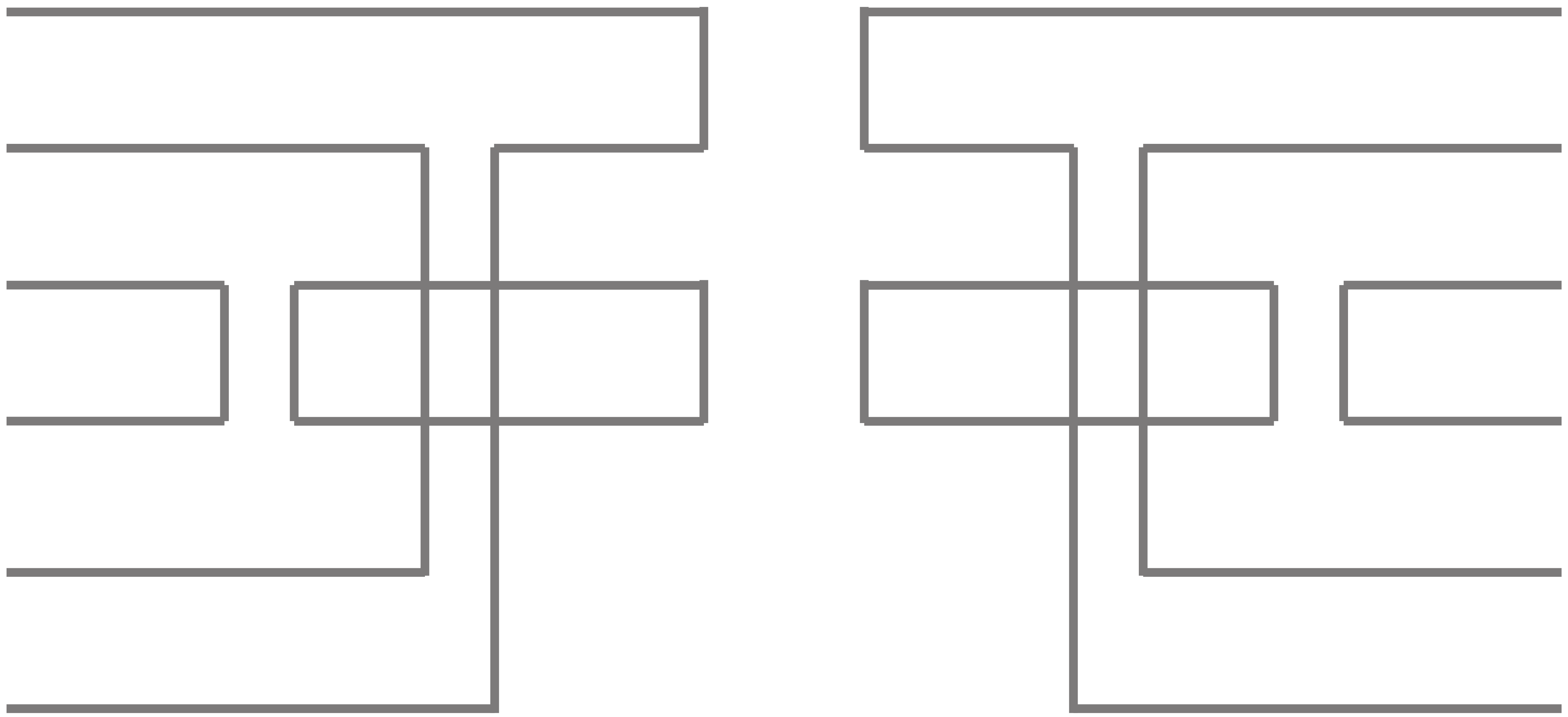
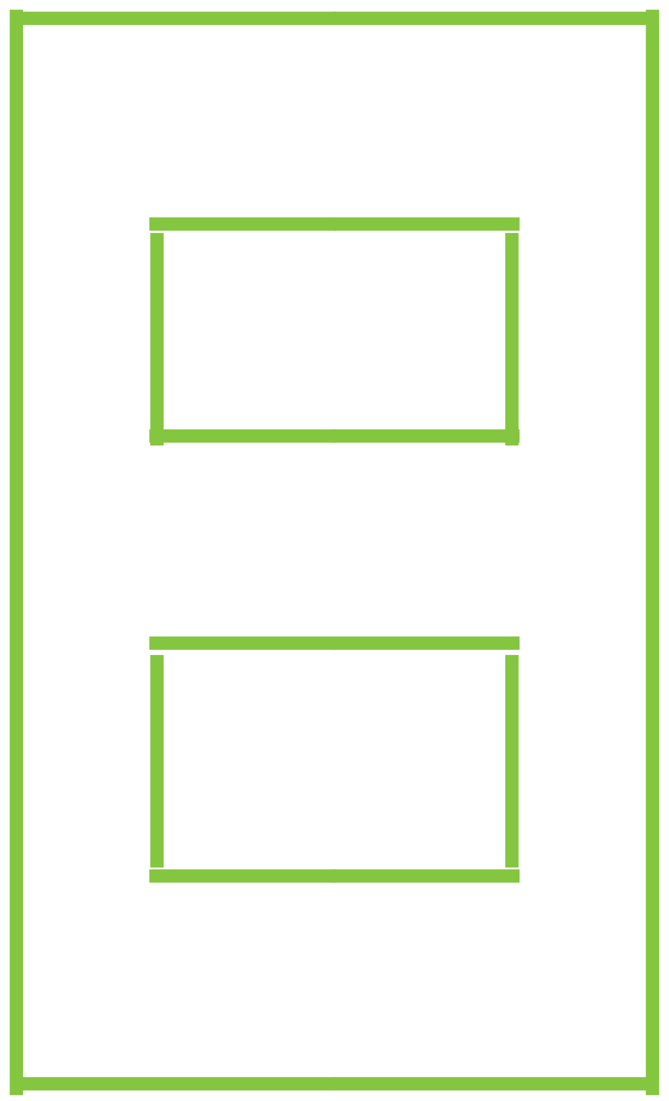
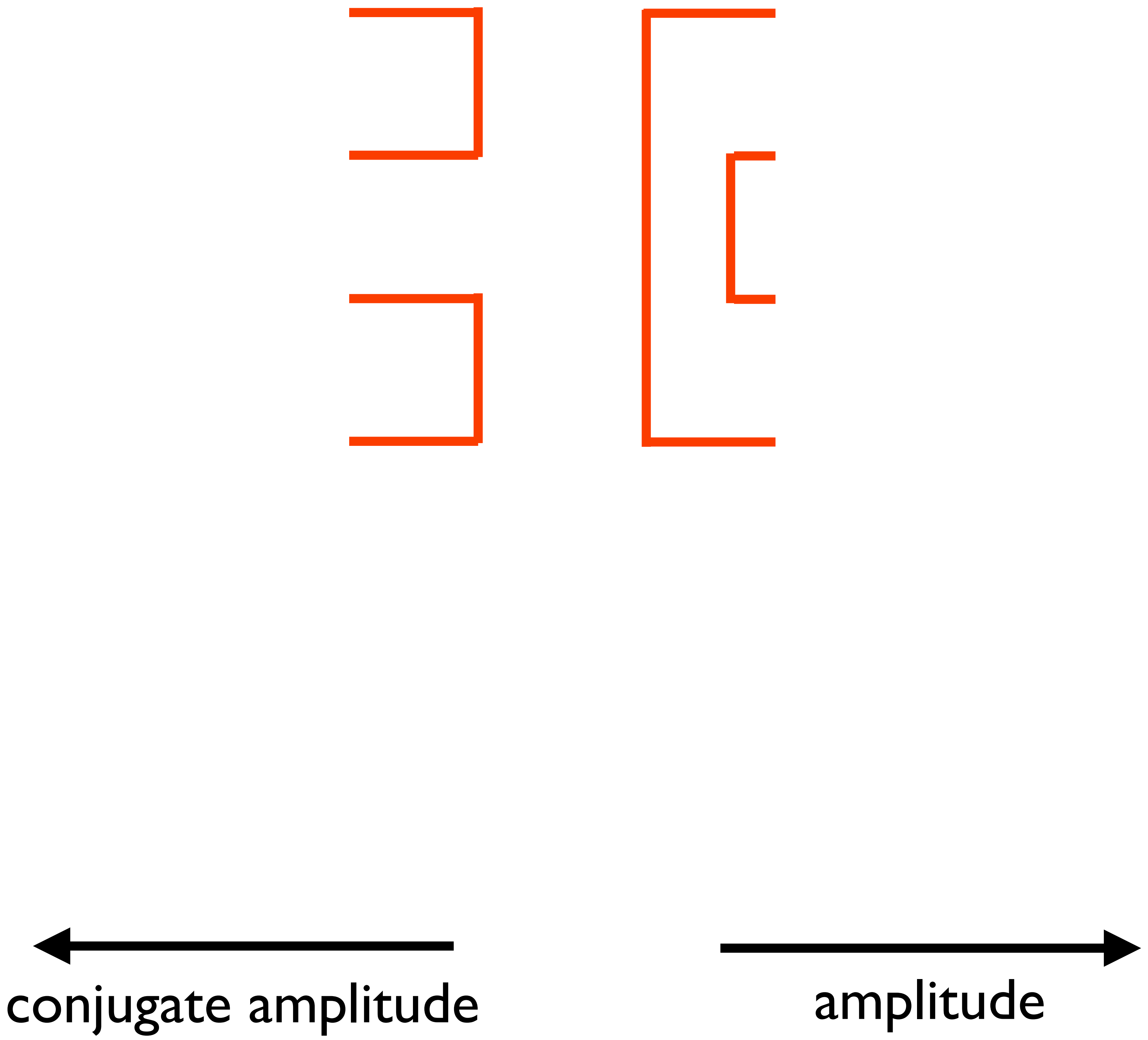




N^3

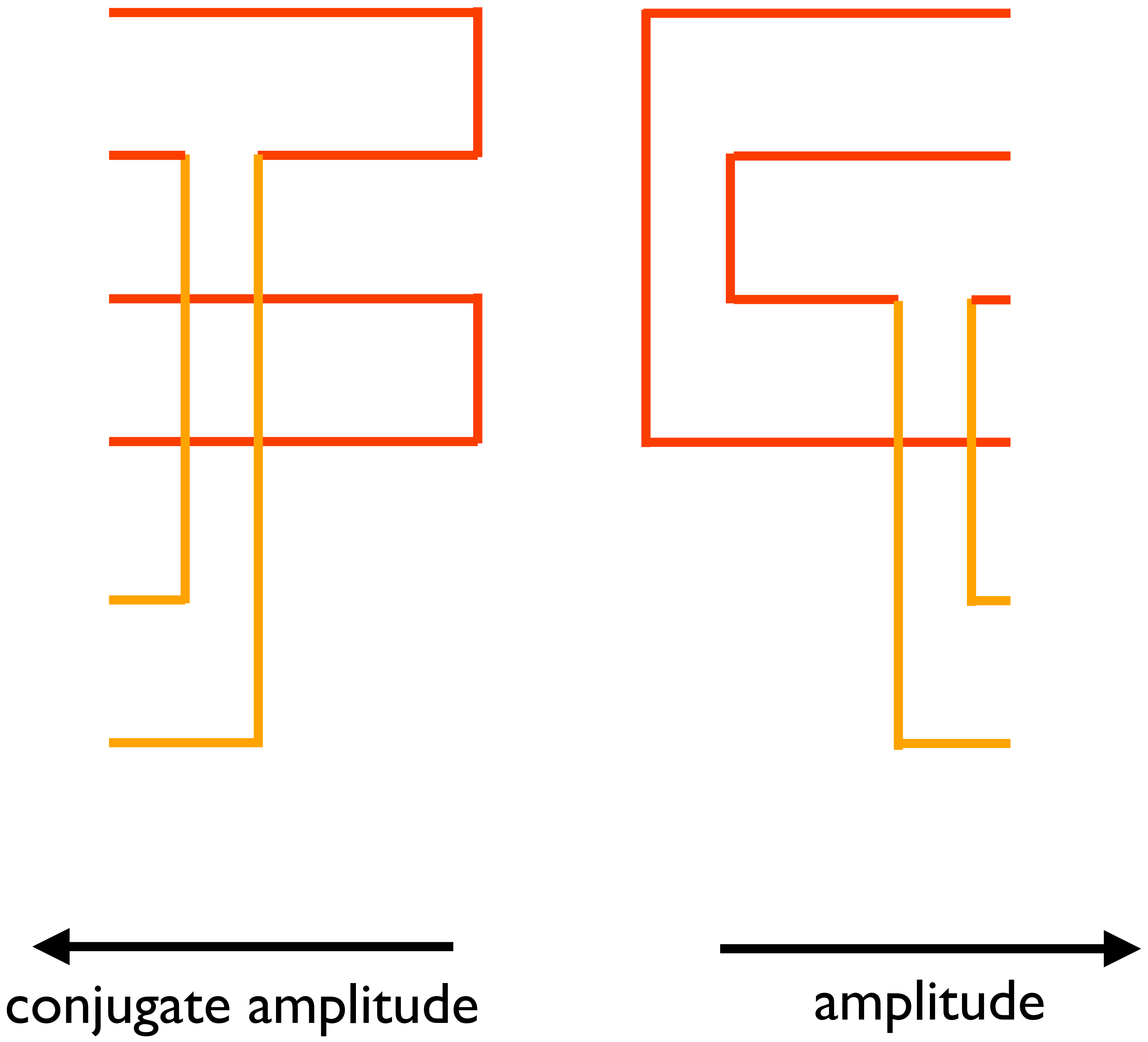
Amplitude evolution: interferences

$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\ \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\ \mathbf{D}_n(k)\ \mathbf{A}_{n-1}(k)\ \mathbf{D}_n^\dagger(k)\ \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$



Amplitude evolution: interferences

$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\; \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}(k')}\; \mathbf{D}_n(k)\; \mathbf{A}_{n-1}(k)\; \mathbf{D}_n^\dagger(k)\; \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}^\dagger(k')}$$



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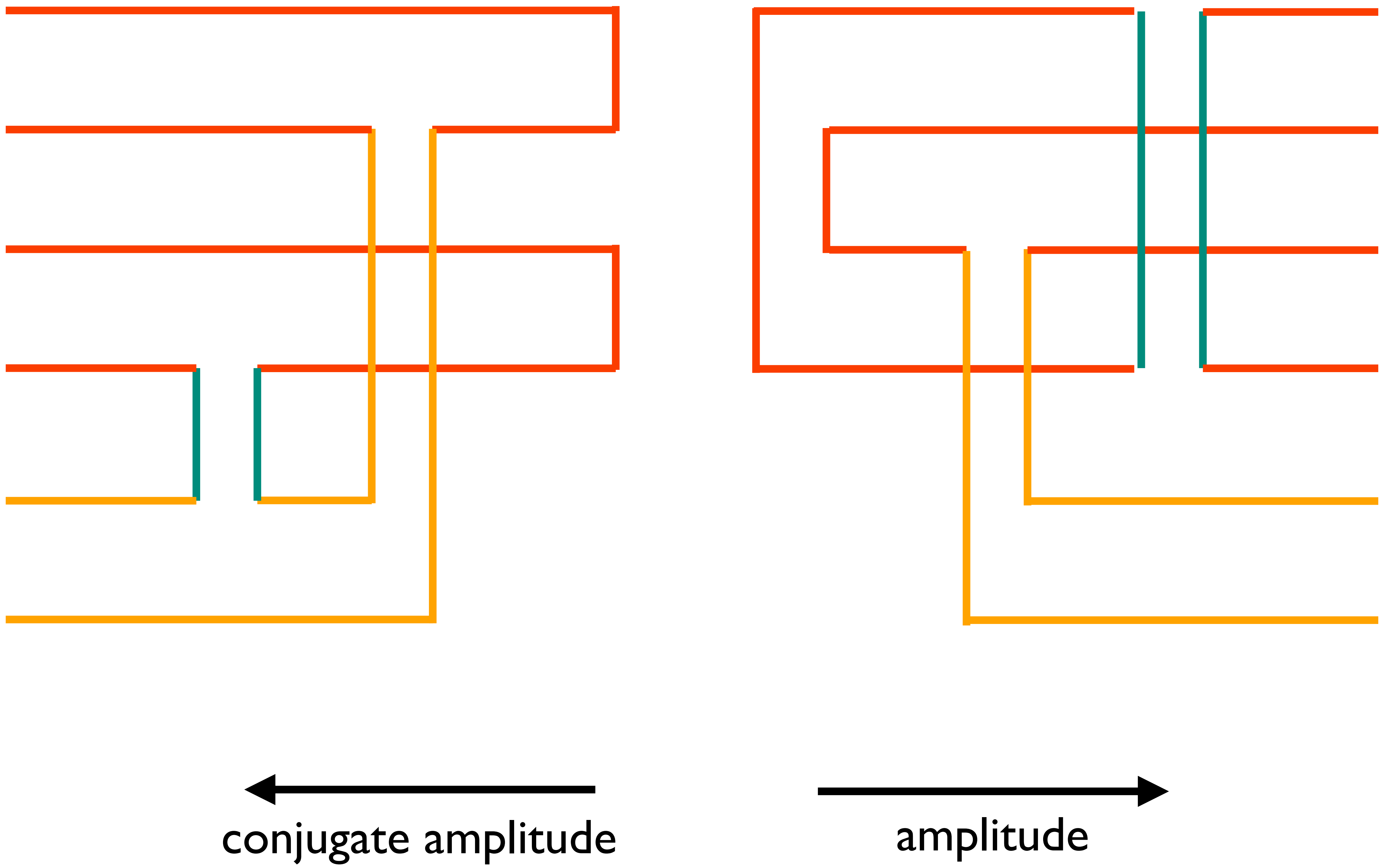

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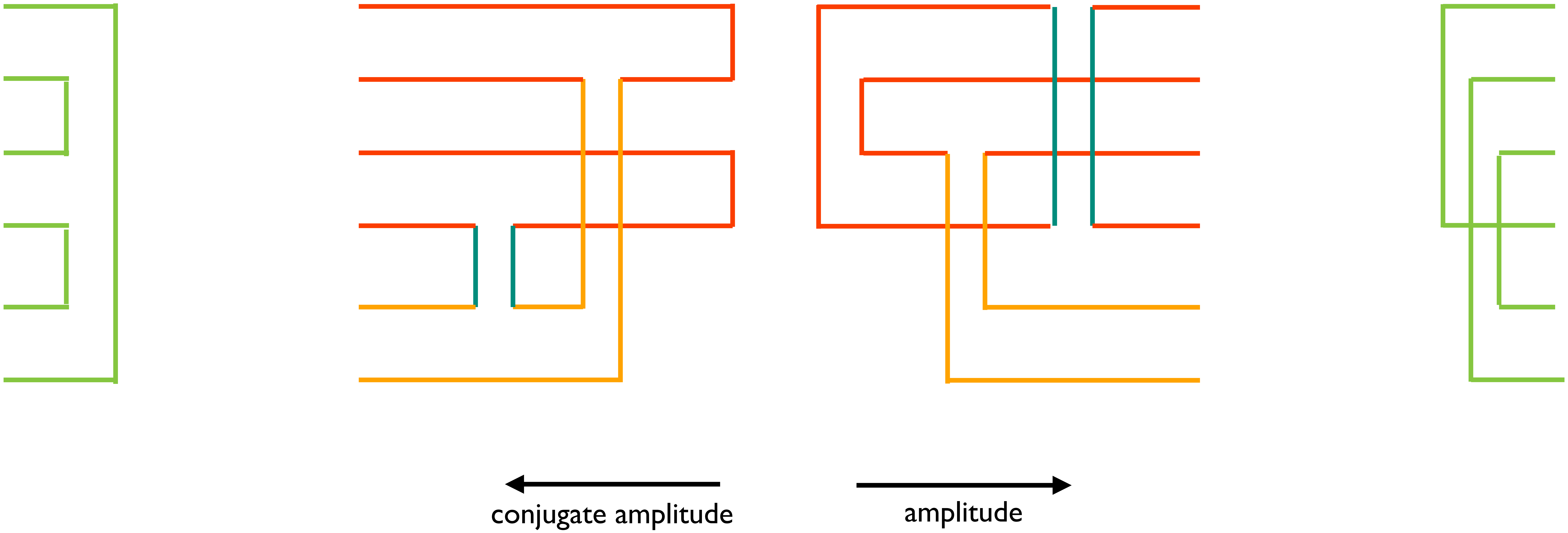

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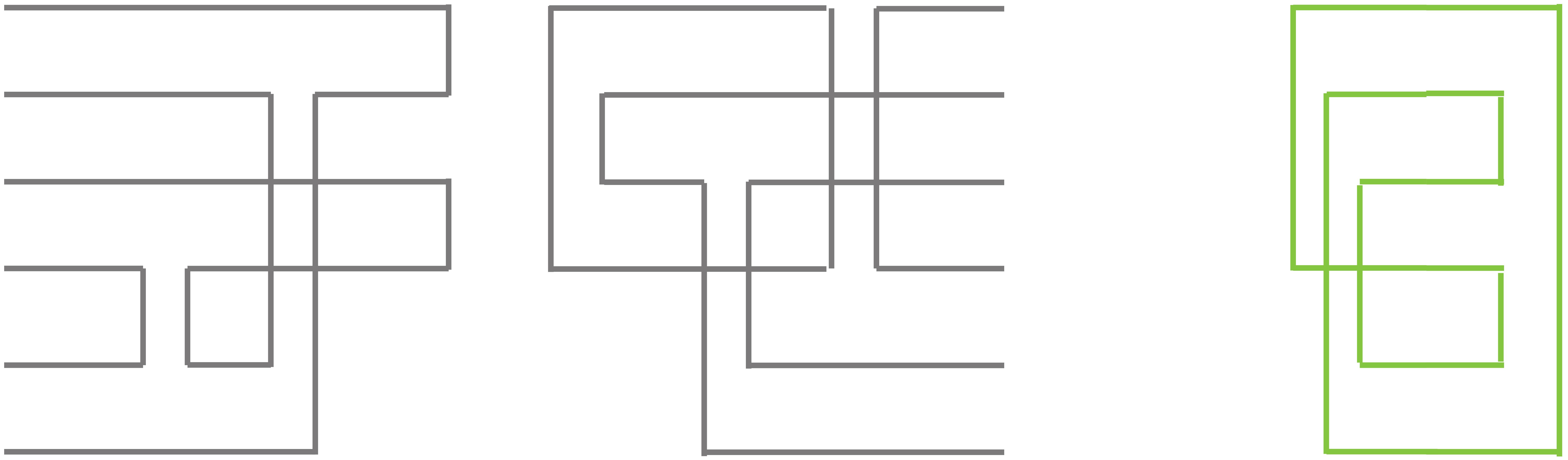
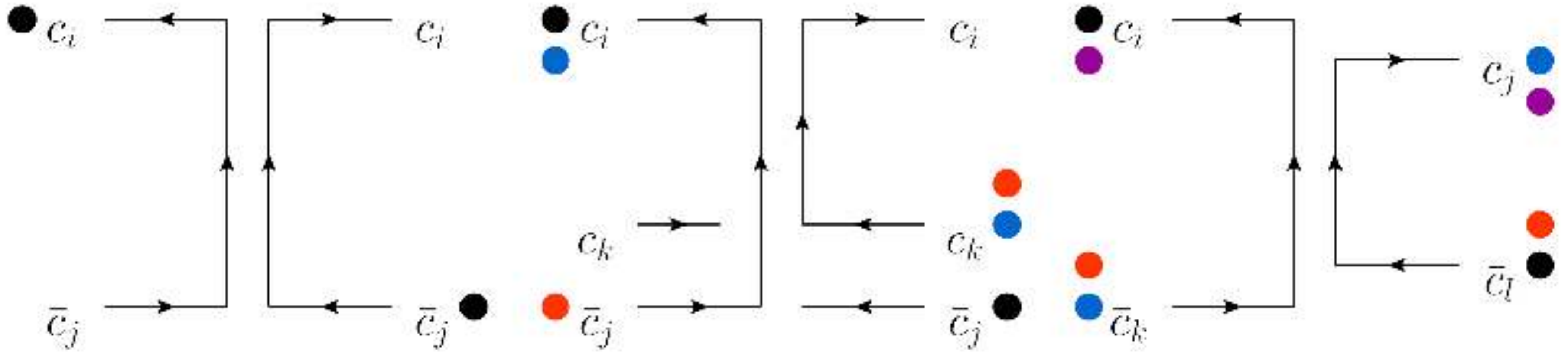


N

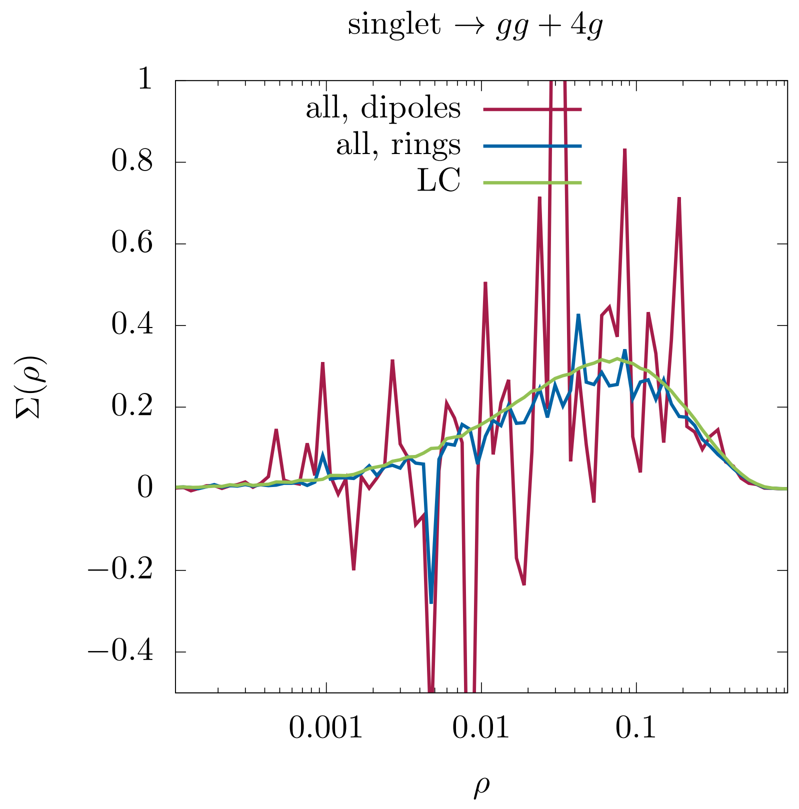
Colour/kinematic cross talk

$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\boldsymbol{\Gamma}^\dagger(k')}$$

Understand basis functions beyond large-N.
Shows how to sample colour flows.



Same "ring" & "string" patterns present in gluon exchanges — subleading or free of collinear singularities.



$$\omega_{ij}$$

$$\omega_{ij} + \omega_{ik} - \omega_{jk}$$

$$\omega_{il} + \omega_{kj} - \omega_{kl} - \omega_{ij}$$

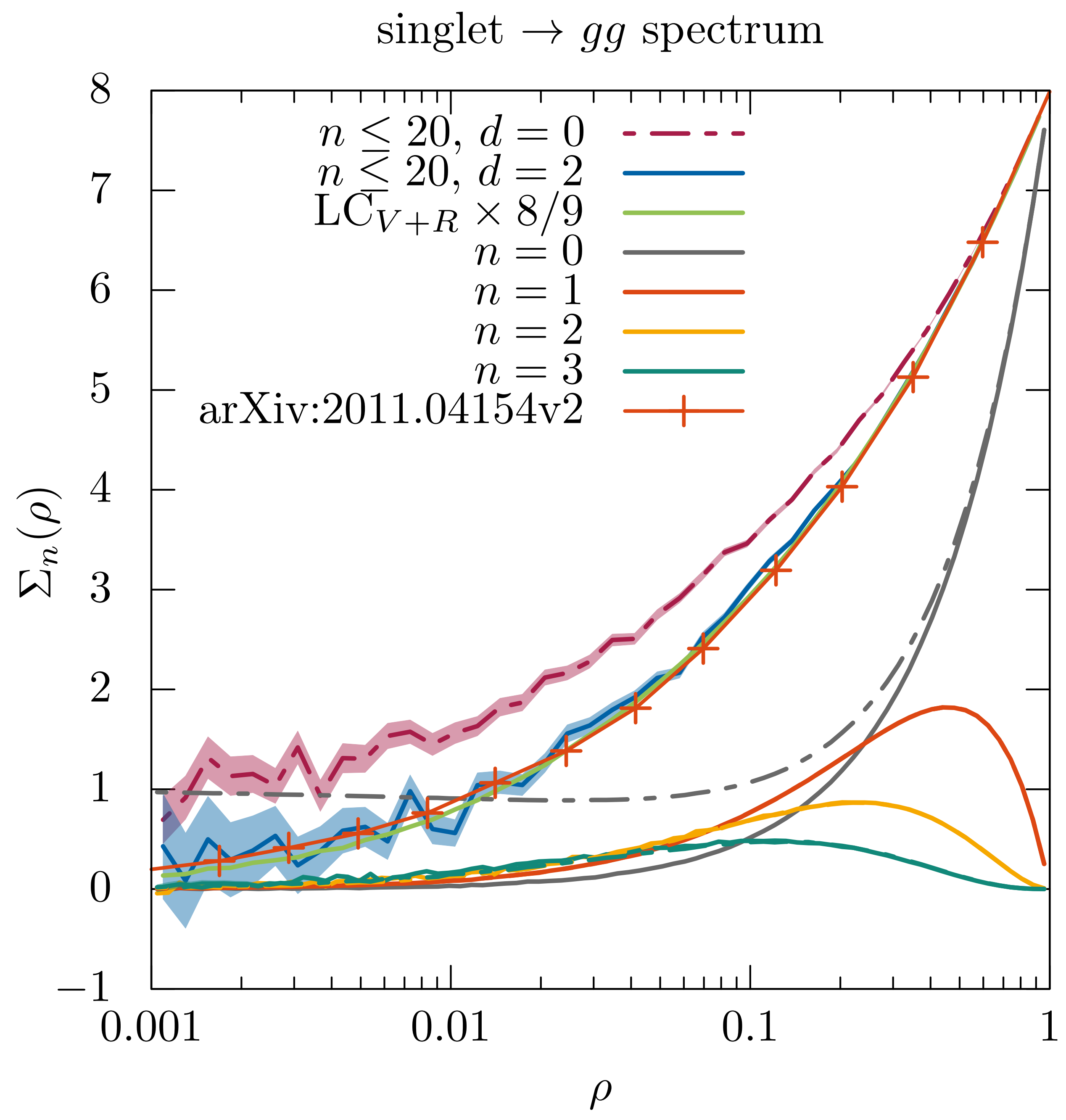
[Plätzer — '13] [Holguin, Forshaw, Plätzer — '21]

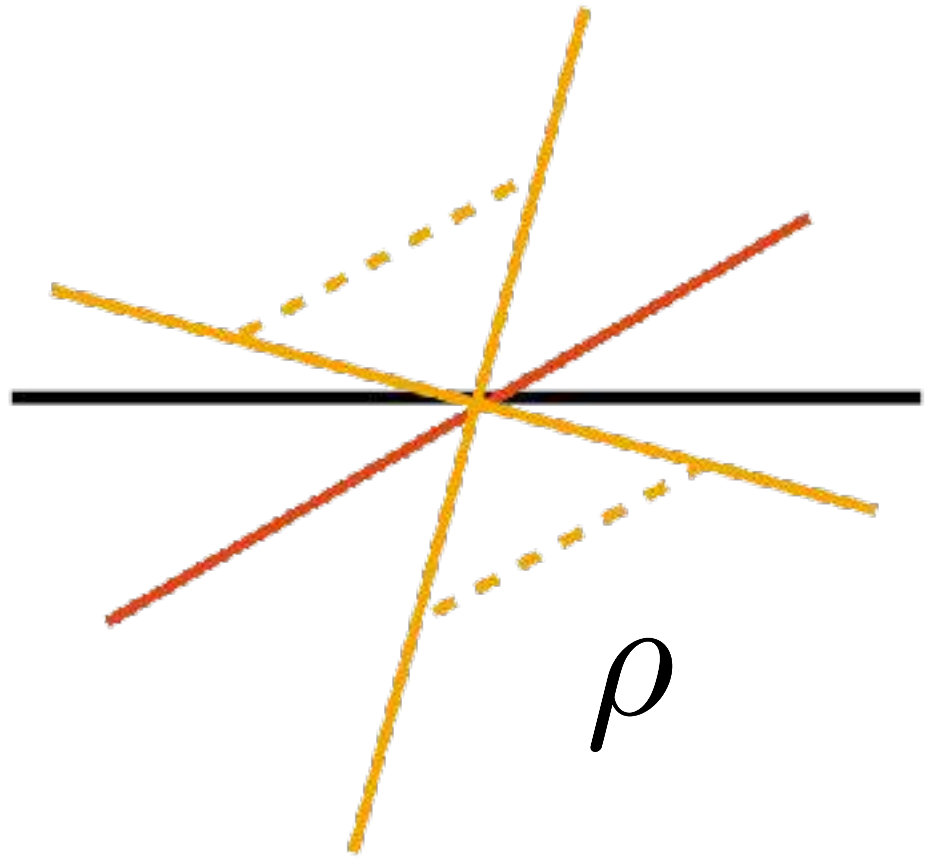
Amplitude evolution

CVolver solves evolution equations in colour flow space

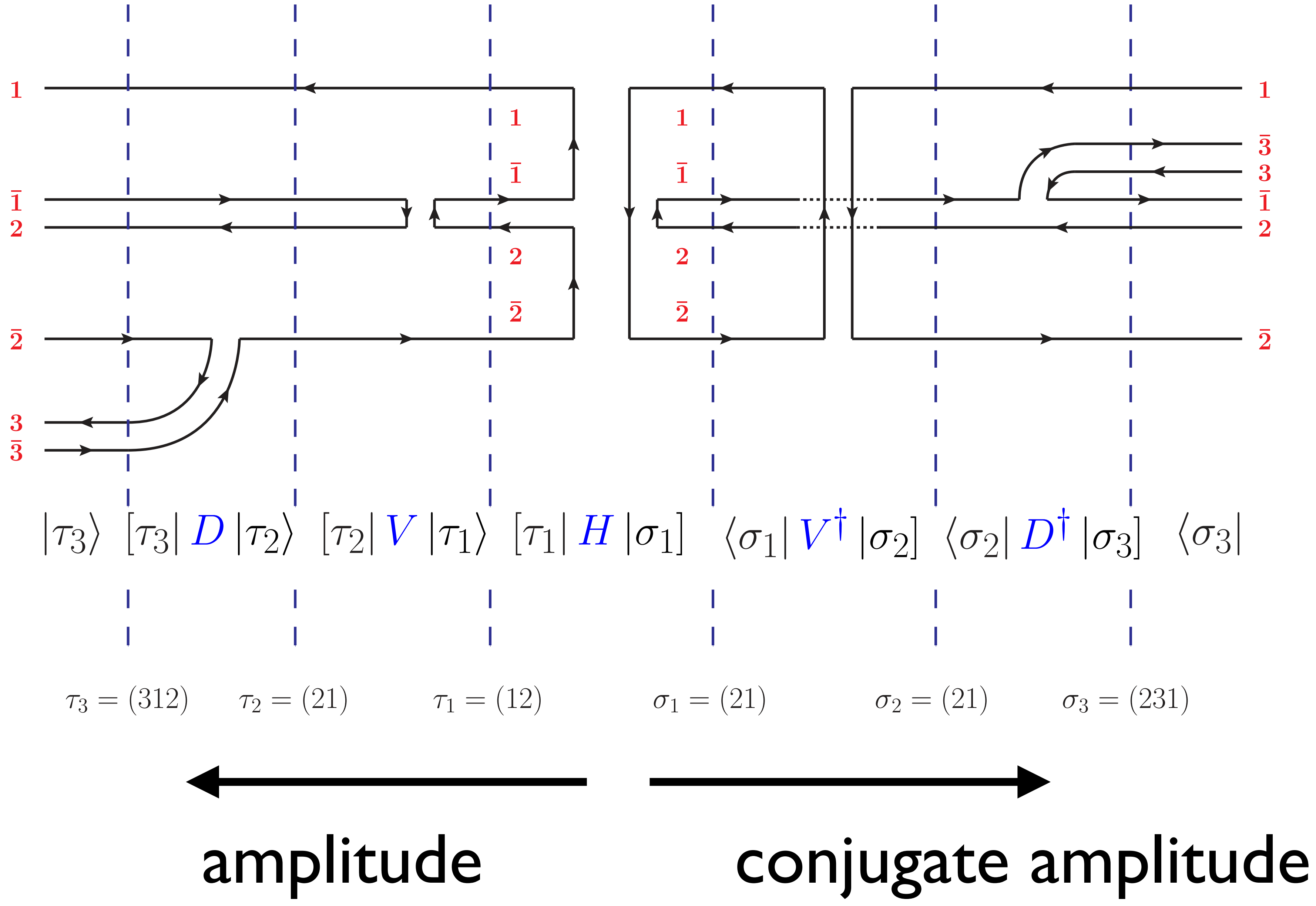
[De Angelis, Forshaw, Plätzer '21]
[Plätzer '13]

$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$





$$\Sigma(\rho) = \sum_n \int \mathrm{d}\sigma(\{p_i\}) \prod_i \theta_{\mathrm{in}}(\rho - E_i)$$



Agrees with Hatta & Ueda using equivalent Langevin formulation by Weigert.

Amplitude evolution

CVolver solves evolution equations in co

er '21]
er '13]

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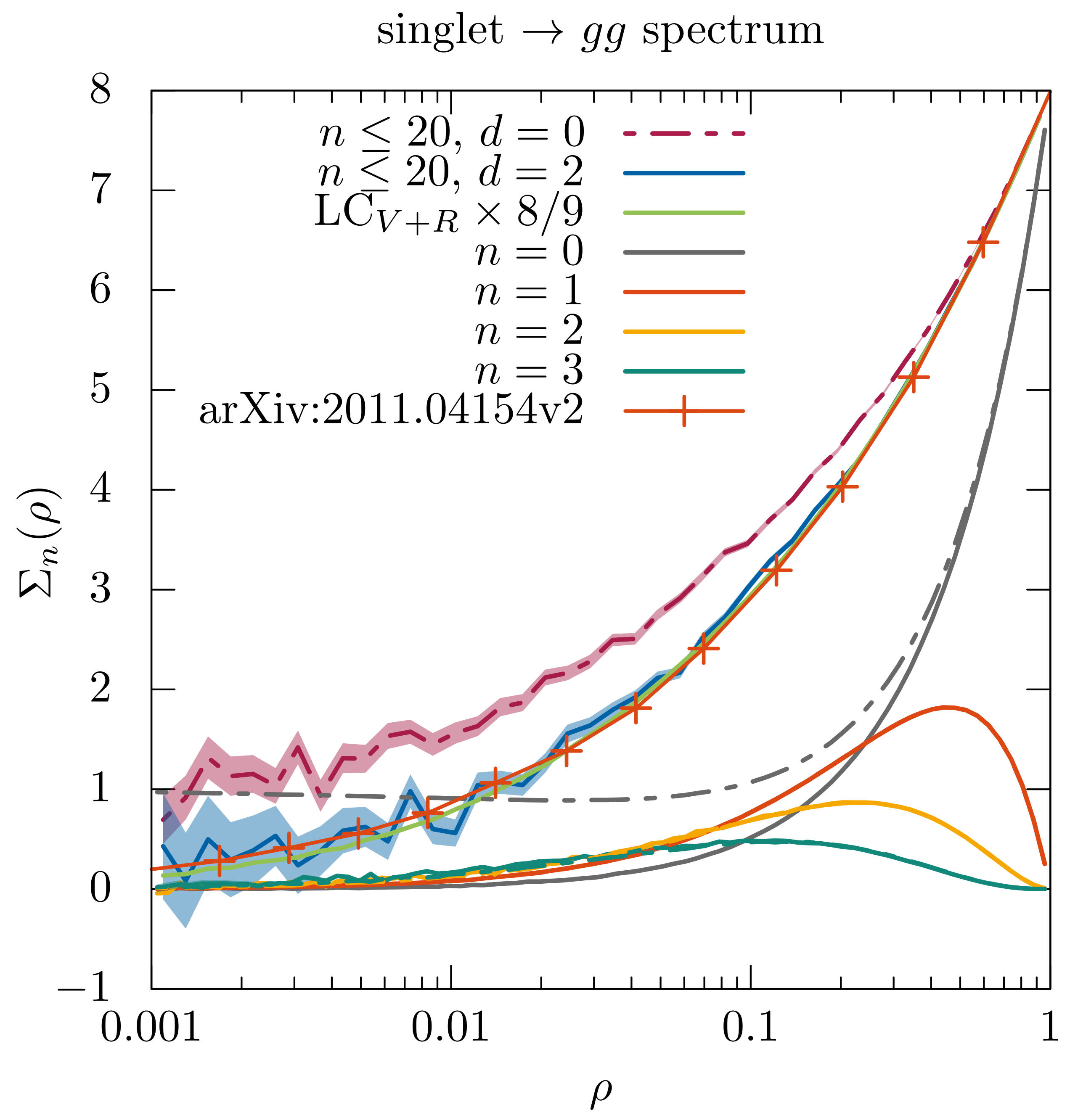
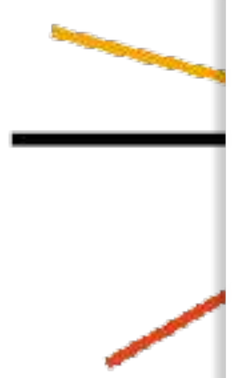
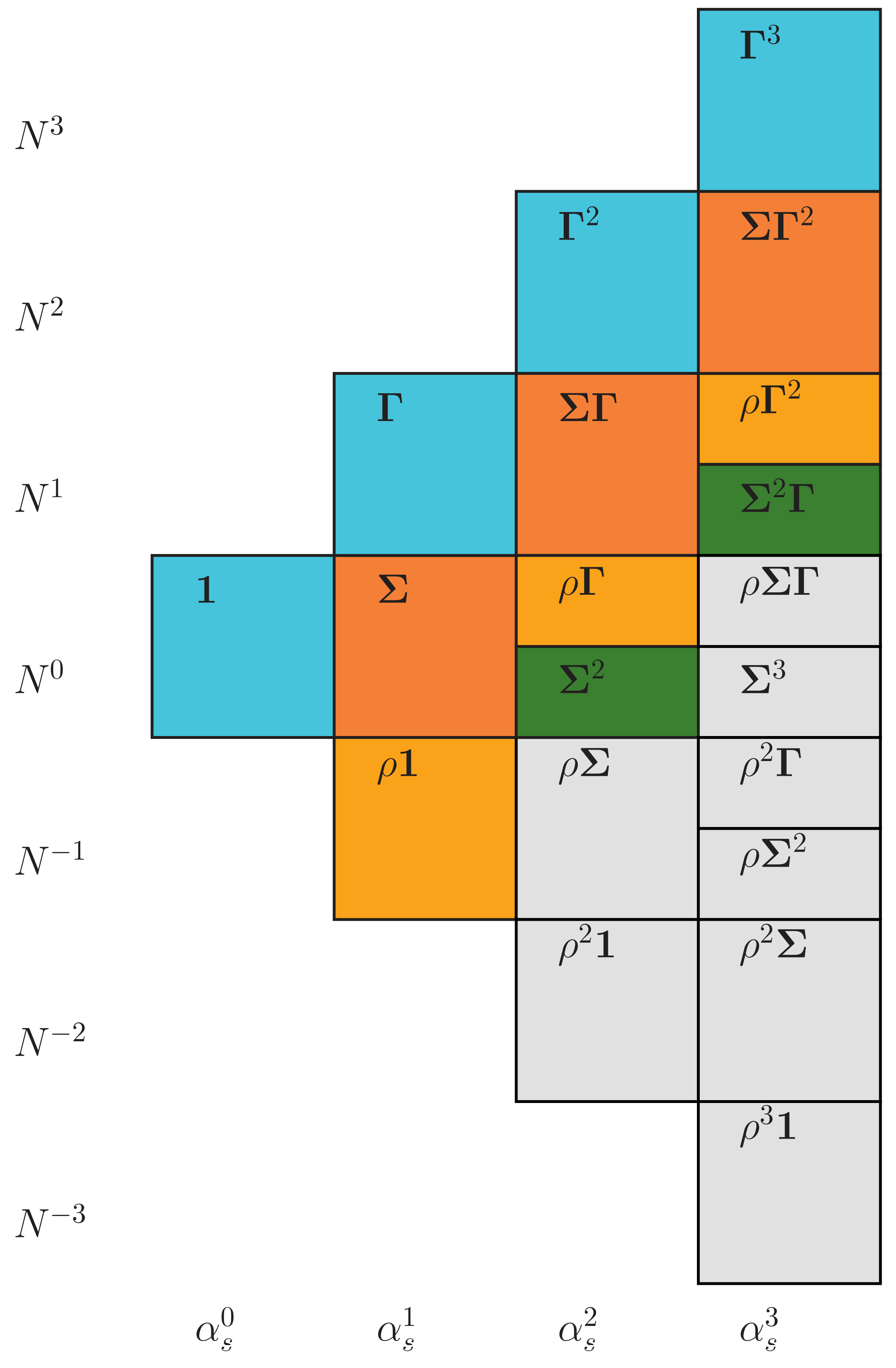




$\Sigma(\rho) =$



	virtuals	reals
(blue)	(0 flips) $\times\, 1 \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[...]\mathbf{t}\vert_{2\text{ flips}} \times 1$ $(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[...]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$ $(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{s}[...]\mathbf{s}\vert_{0\text{ flips}} \times N^{-2}$
(orange)	(1 flip) $\times\, \alpha_s \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[...]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$
(yellow)	(0 flips) $\times\, \alpha_s N^{-1} \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r}$
(green)	(0 flips) $\times\, \alpha_s^2 \times (\alpha_s N)^n$ (2 flips) $\times\, \alpha_s^2 \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\ \mathbf{t}[...]\mathbf{t}\vert_{2\text{ flips}}$

d=0
d=1
d=2

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – '18]

Agrees with Hatta & Ueda using equivalent Langevin formulation by Weigert.

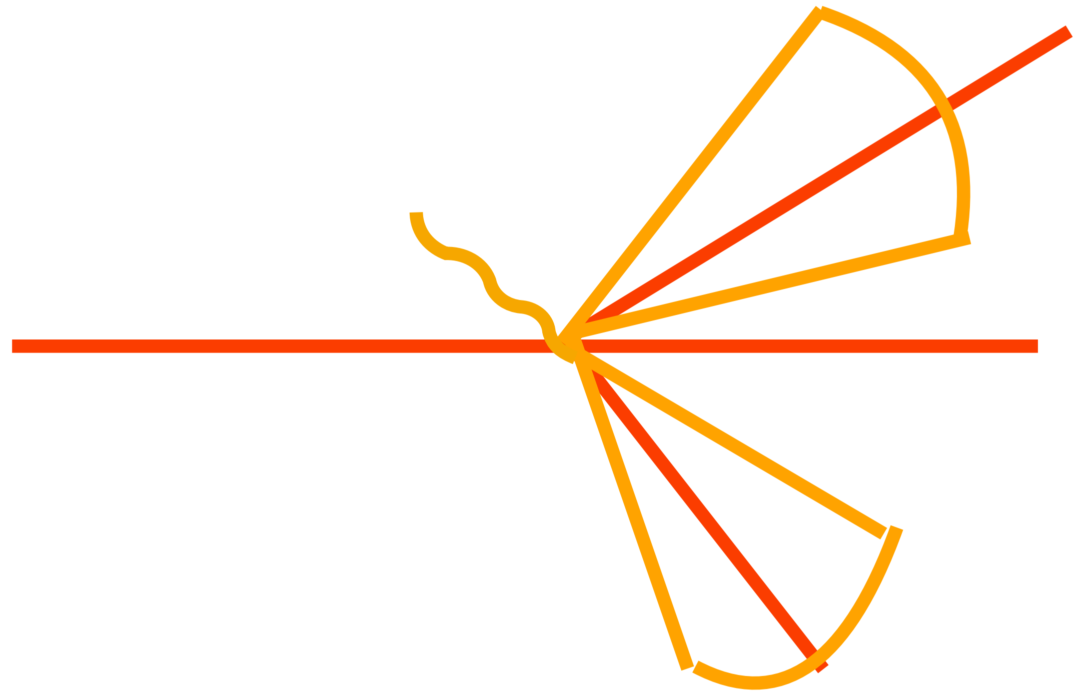
Amplitude evolution: new results

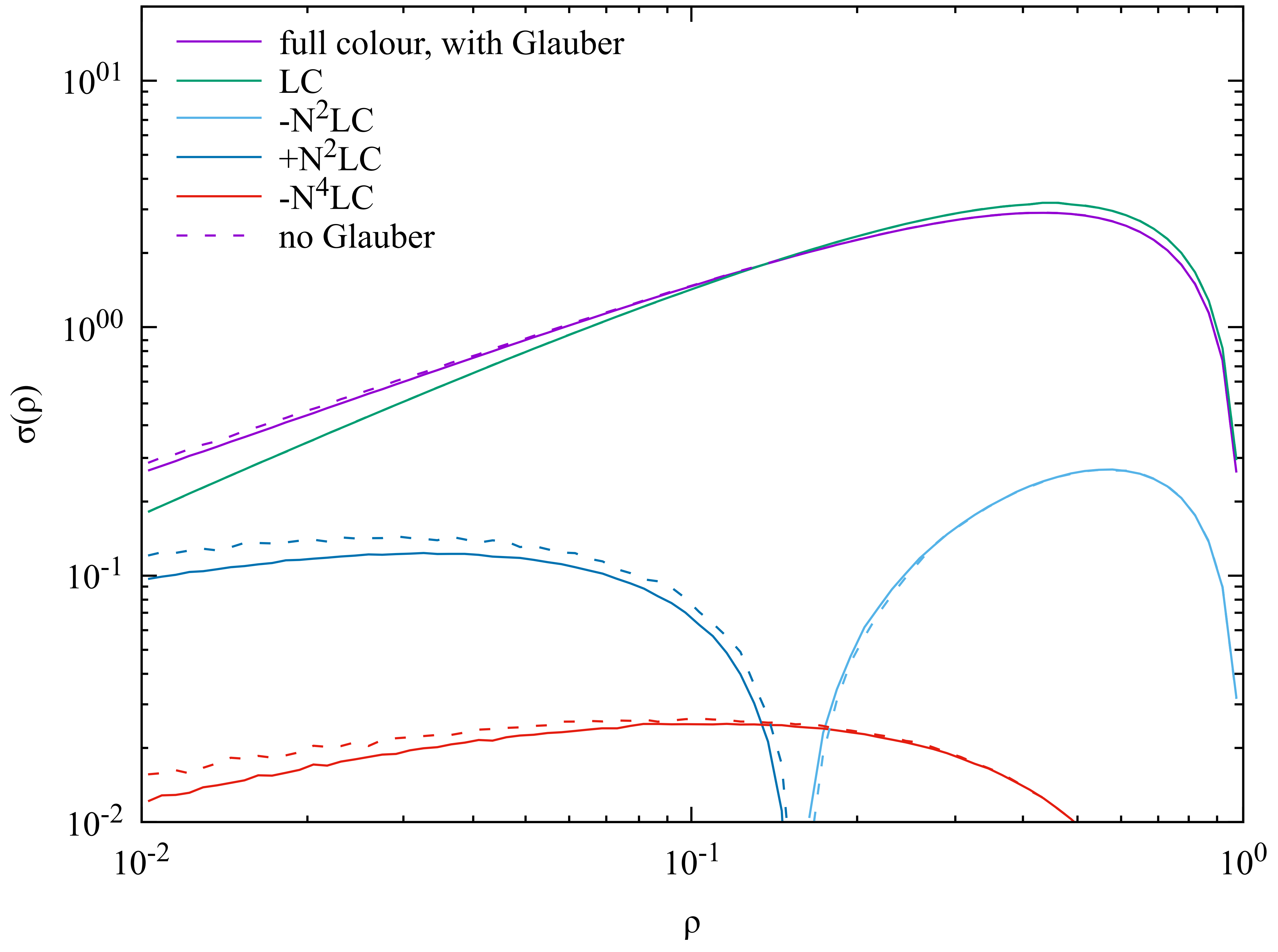
Full hadron collider and multi-jet campaign:

- QCD jet production and vetoes
- VBF including all interferences
- e+e- to hadronic WW — demand for FCC

Physics questions:

- Impact of Glauber exchanges
- Recoil to (inter-)jet radiation
- Impact of interference terms





[Forshaw, Kirchgaesser, Plätzer, Torre– wip]

[QCD jets with additional emissions — also relevant to top]

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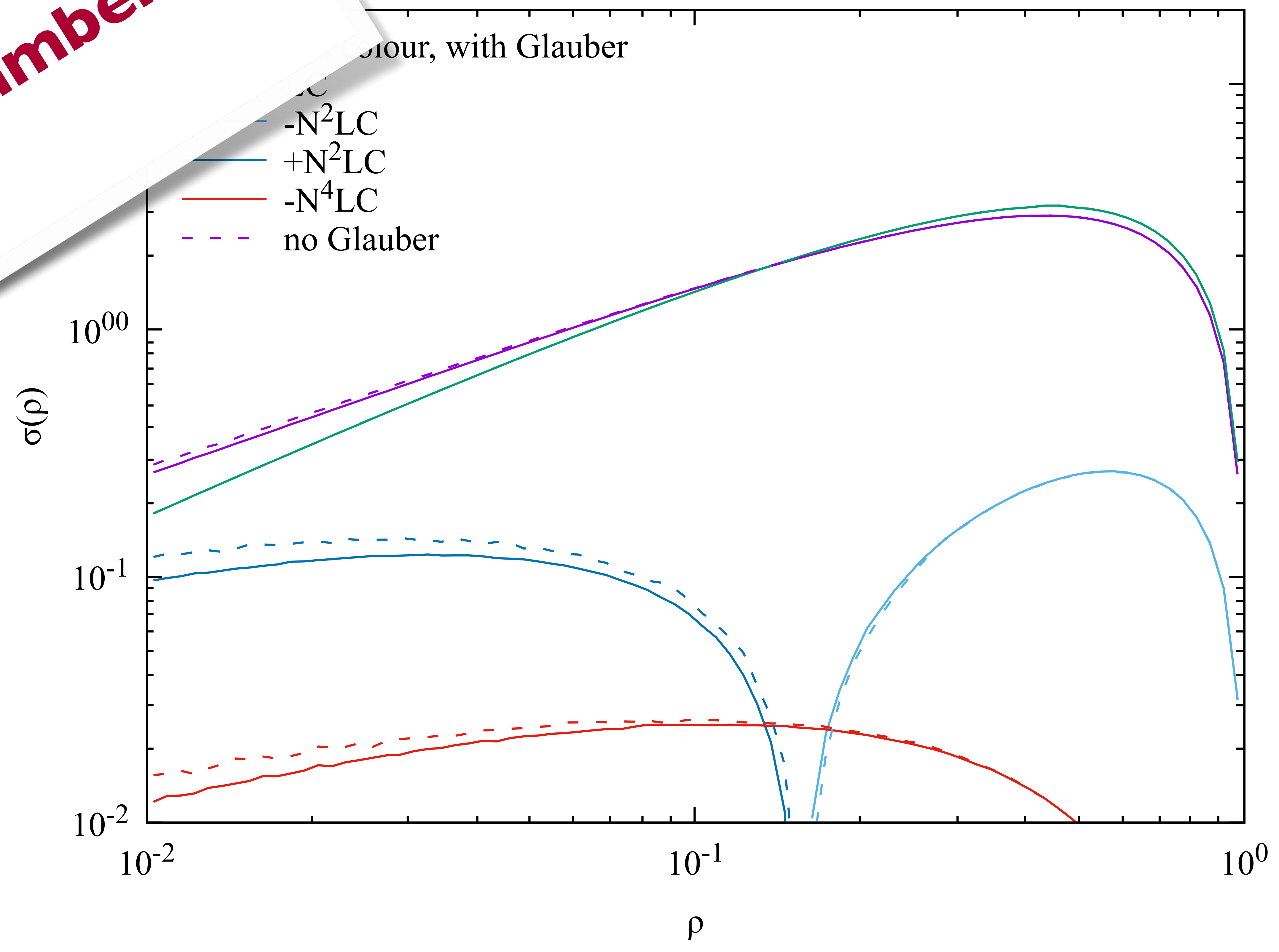
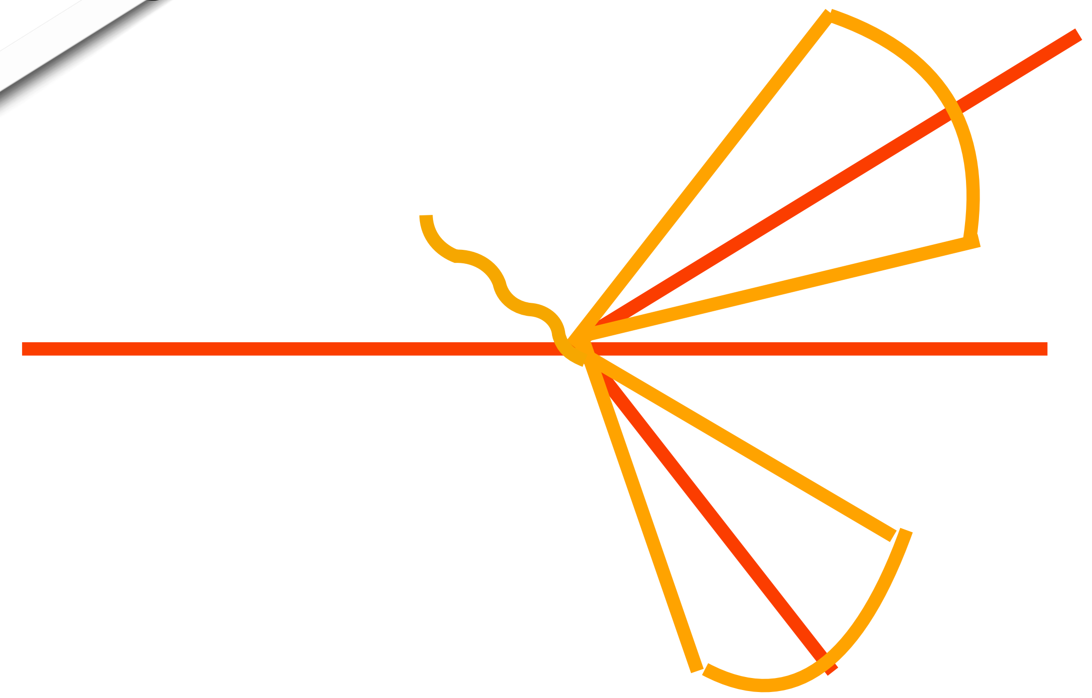
Inaccessible to inclusive approaches with fixed number of emissions as well as normal parton showers.





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Factorisation and evolution

[Plätzer – JHEP 07 (2023) 126]

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Factorisation and evolution

[Plätzer – JHEP 07 (2023) 126]

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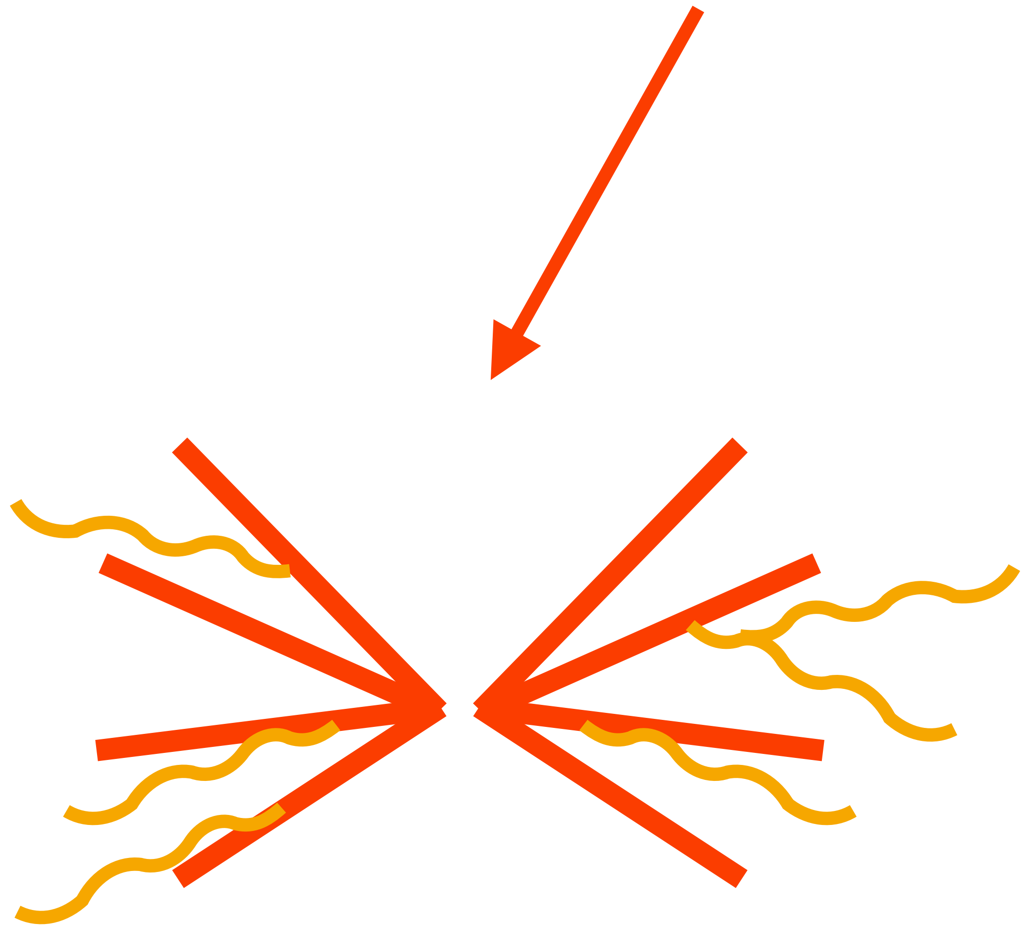

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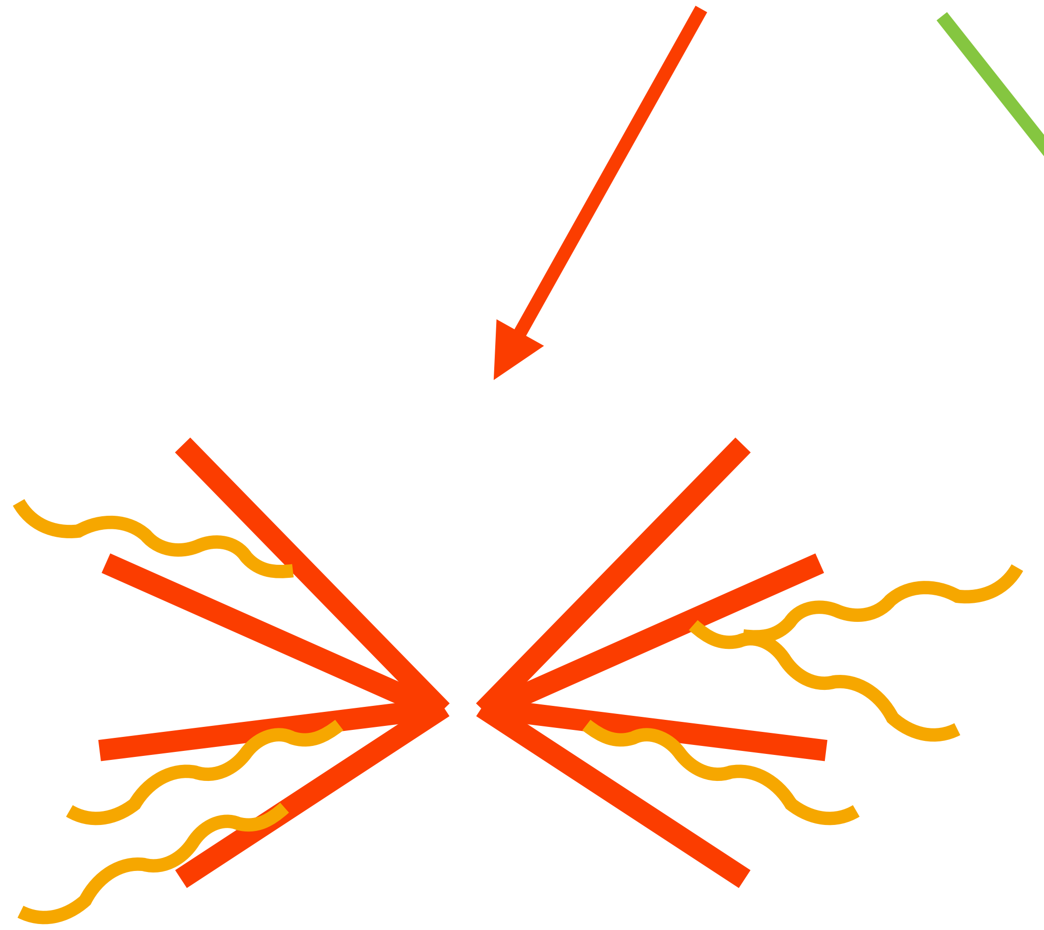

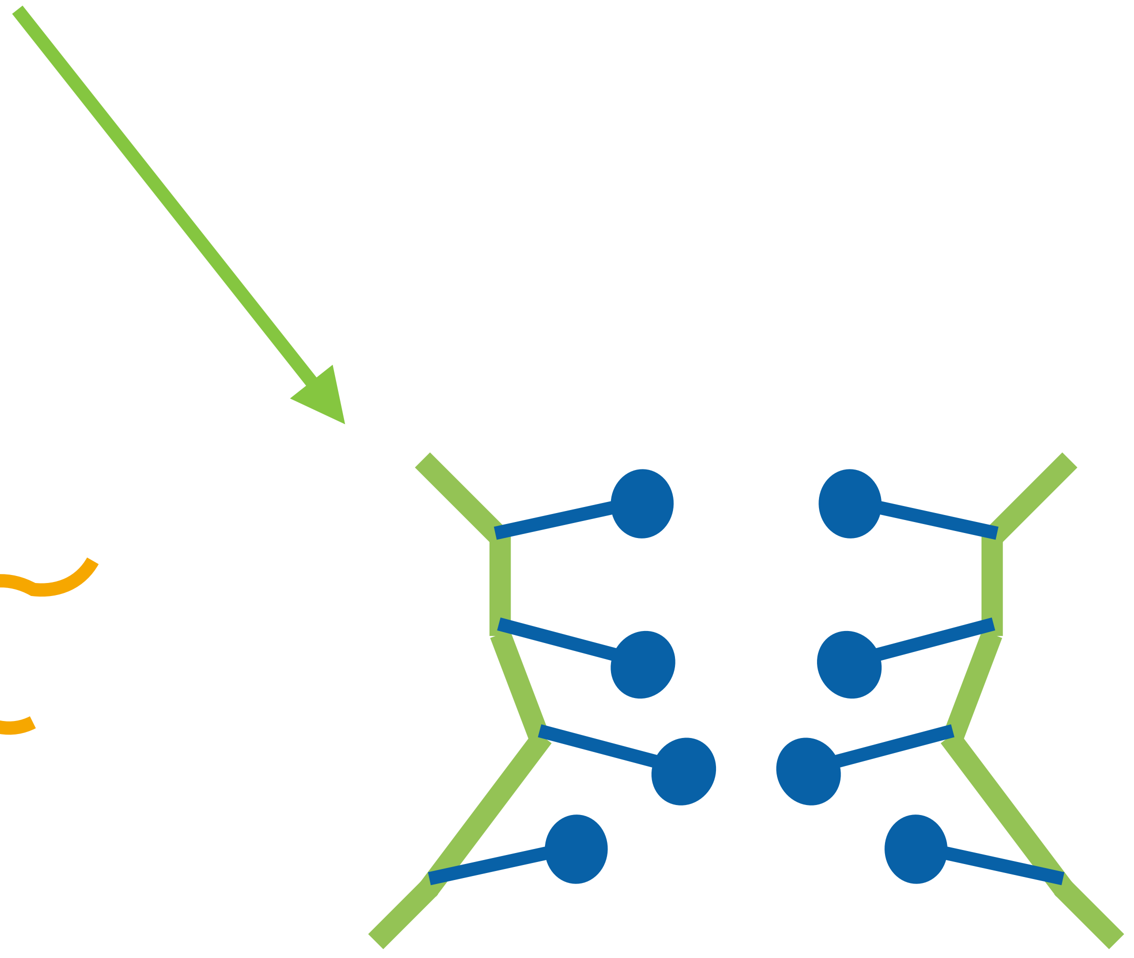

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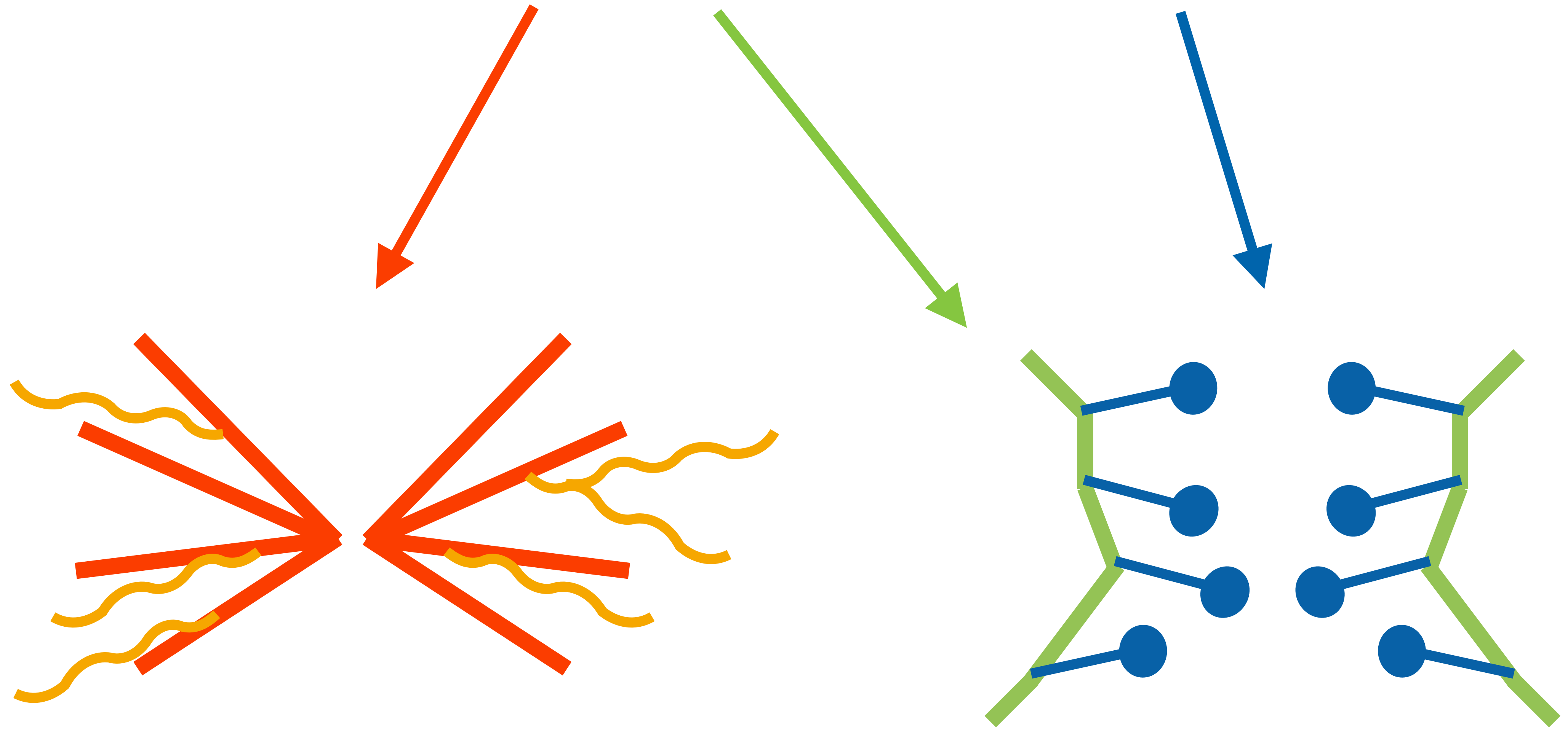

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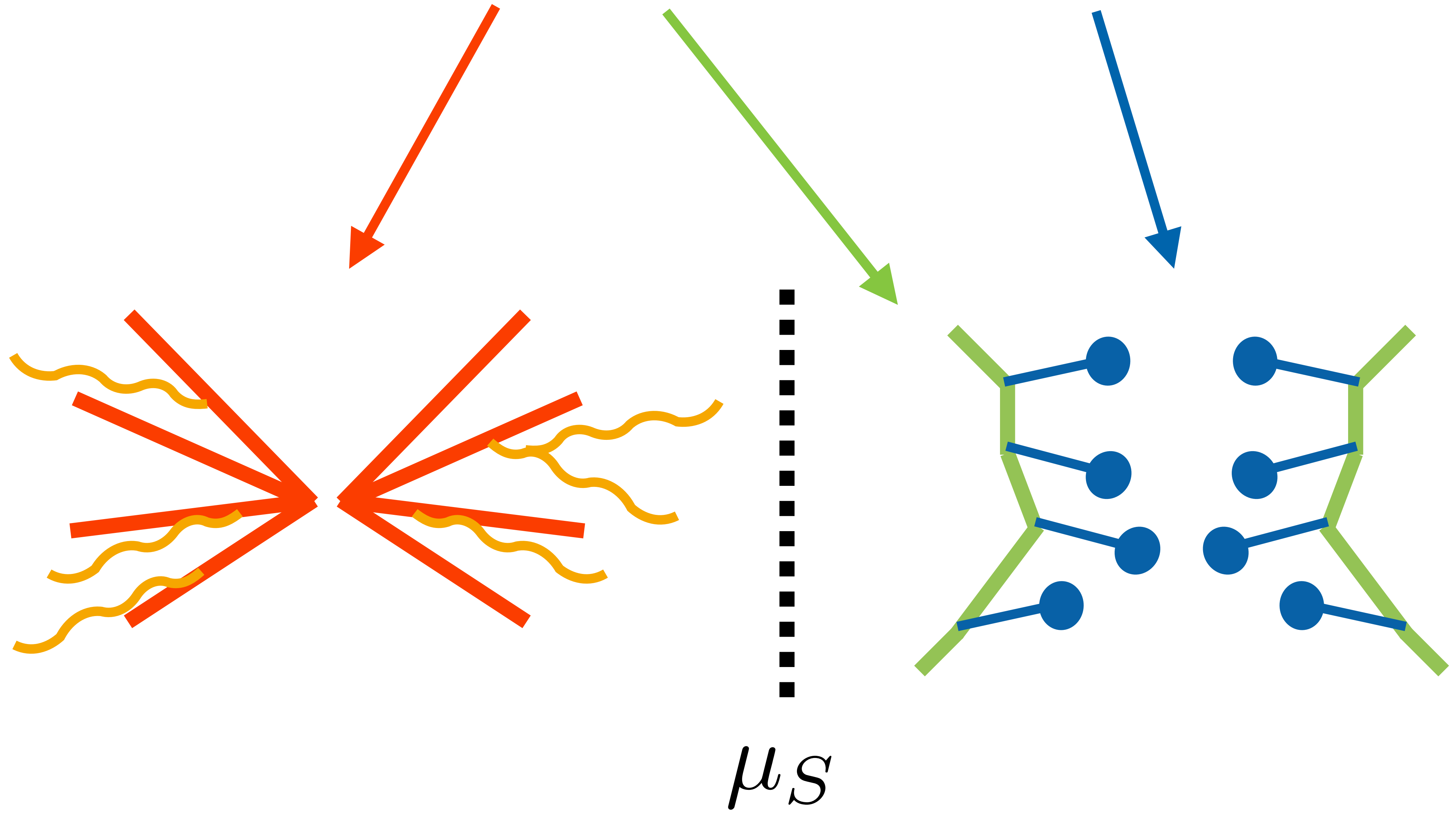
$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



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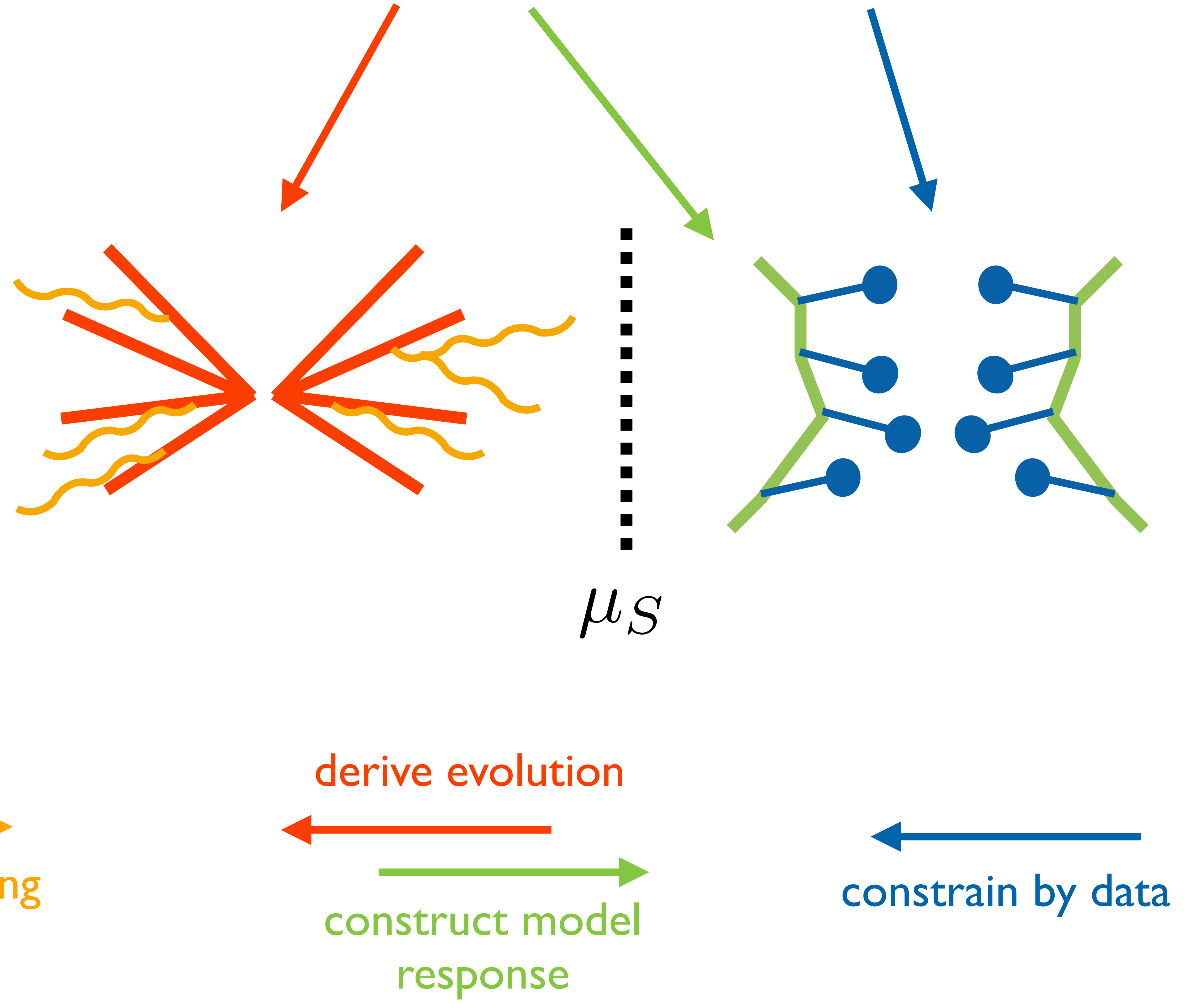
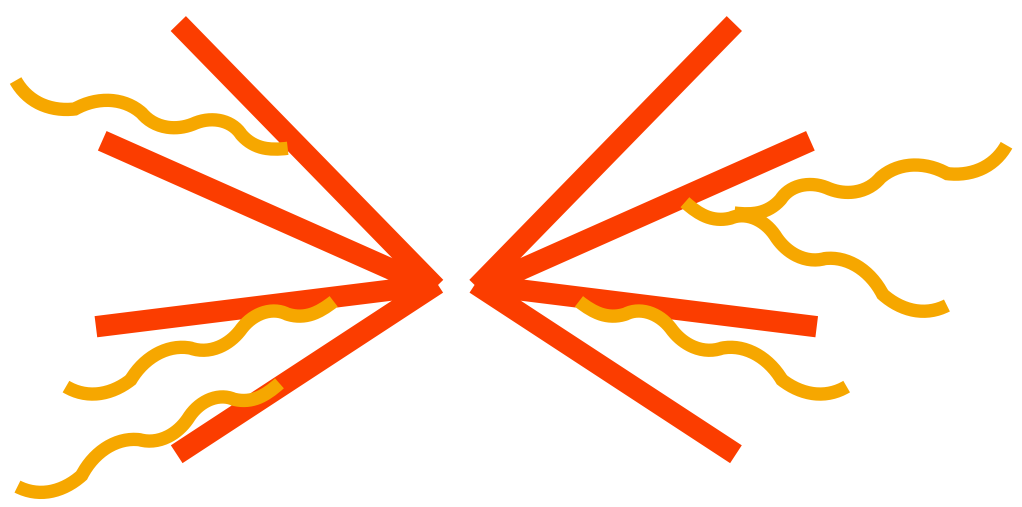
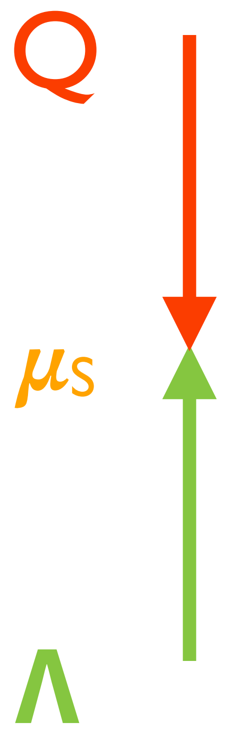
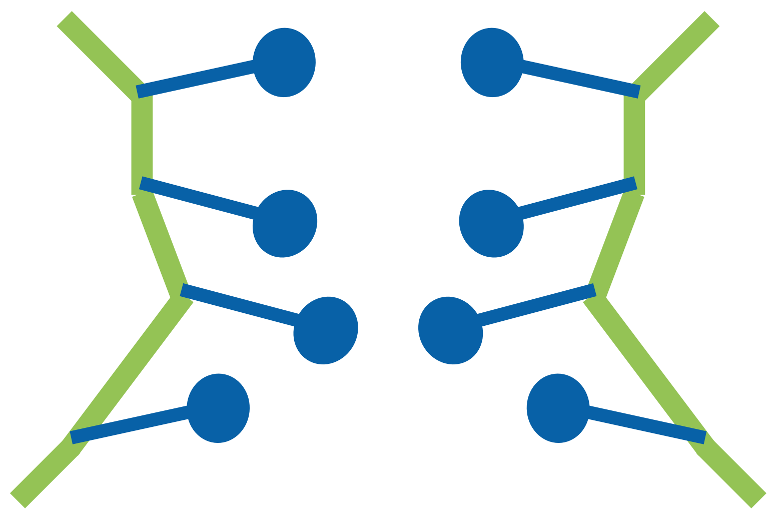


The origin of the IR cutoff







Just a technical parameter?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

Starting point: "renormalise" bare colour operators. [Plätzer – JHEP 07 (2023) 126]

Subtract IR divergencies
in unresolved regions

Re-arrange to resum
IR enhancements

$$\mathbf{U}_n = \mathcal{X}_n \left[\mathbf{S}(\mu_S), \mu_S \right]$$

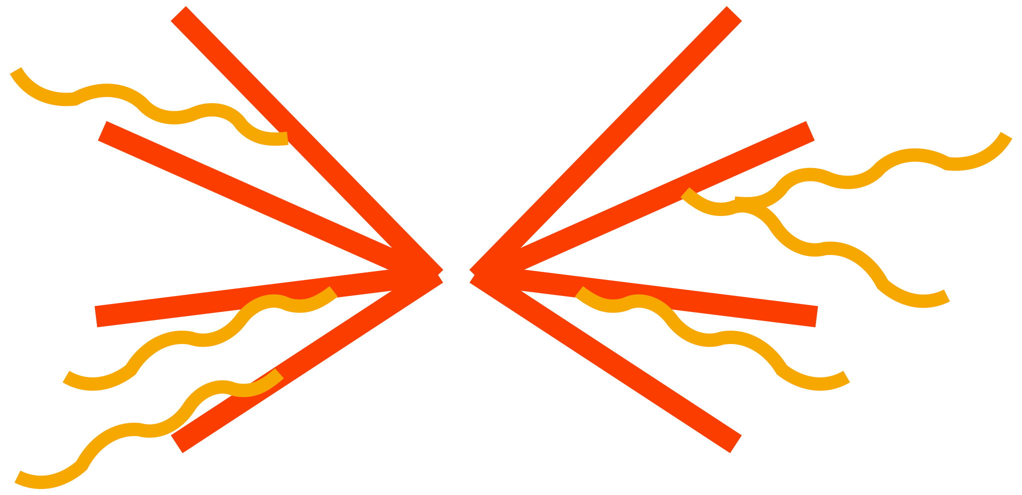
$$\sigma = \sum_n \alpha_S^n \int \mathrm{Tr} \left[\mathbf{A}_n(\mu_S) \mathbf{S}_n(\mu_S) \right] \mathrm{d}\phi_n$$

$$\mathbf{M}_n Z_g^n = \mathcal{Z}_n \left[\mathbf{A}(\mu_S), \mu_S \right]$$

- Even if we include explicit virtual contributions, a cutoff is present. This is independent of the UV renormalisation scheme, and impacts the structure of the resummation.

[Plätzer – (slow) progress]

The role of the IR cutoff

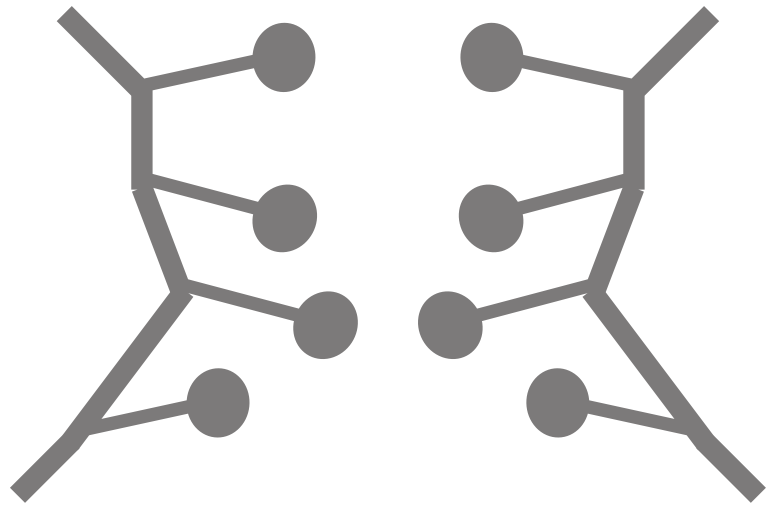


Just a technical parameter?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

- Provides factorisation and subtractions for renormalised matrix elements in unresolved regions, consistency enforced by overall power counting

$$\mathbf{M}_n \to \alpha_s^n (\mathbf{M}_n^{(0)} + \alpha_s [\mathbf{M}_n^{(1)} - \mathbf{X}_n^{(1)} \mathbf{M}_n^{(0)} - \mathbf{M}_n^{(0)} \mathbf{X}_n^{(1)\dagger} - \mathbf{F}^{(1,0)} \mathbf{M}_{n-1}^{(0)} \mathbf{F}_n^{(1,0)\dagger}] + \mathcal{O}(\alpha_s^2))$$



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- At higher orders we find appropriate removal of iterated subtractions.

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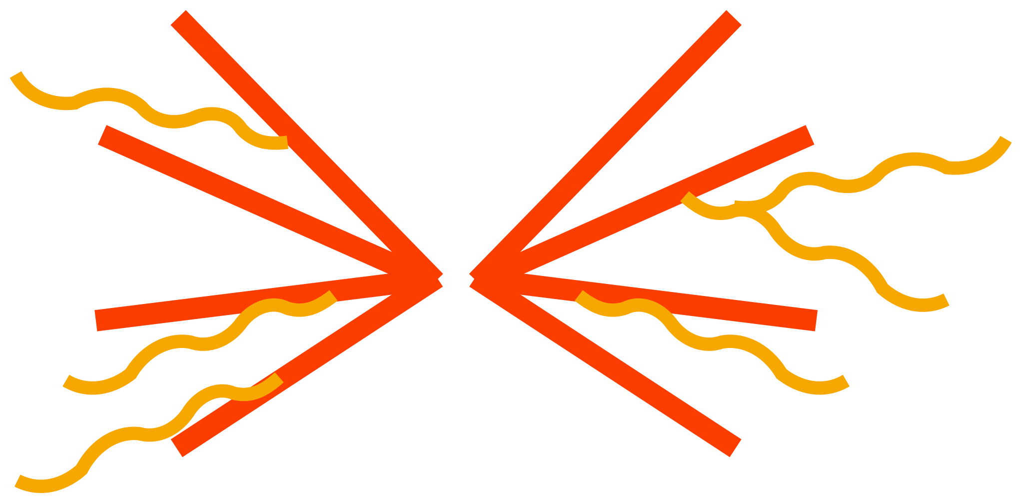

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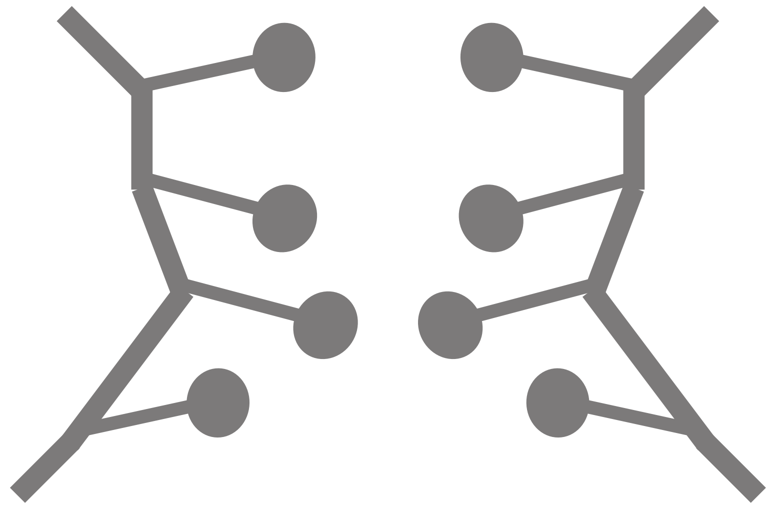
unresolved emission
at leading power



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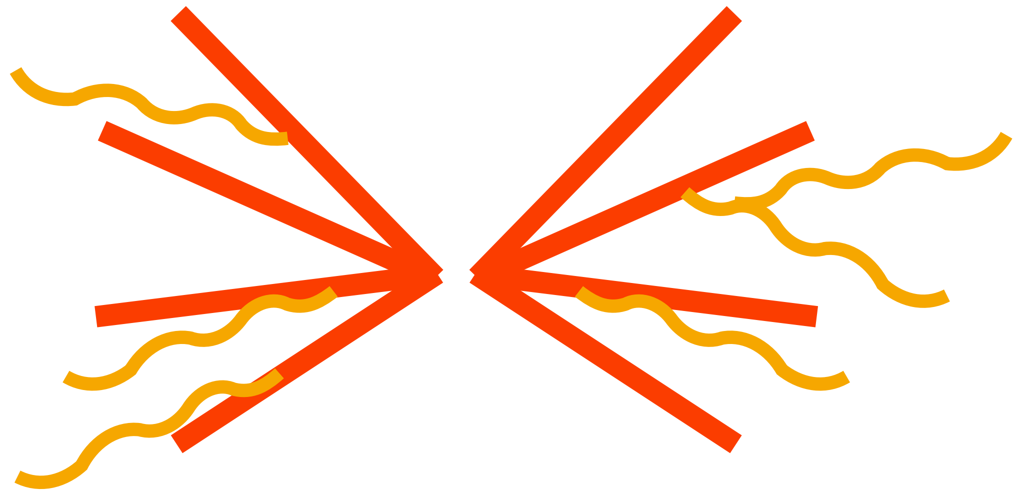

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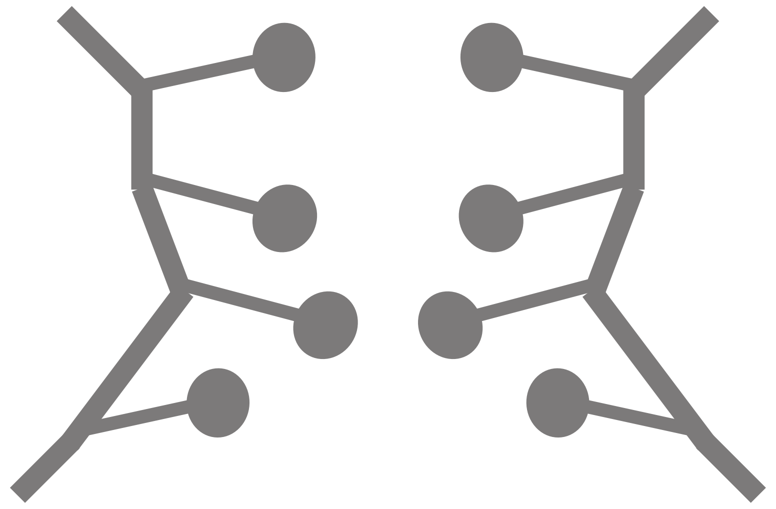
loop divergence at leading power, no unresolved emission



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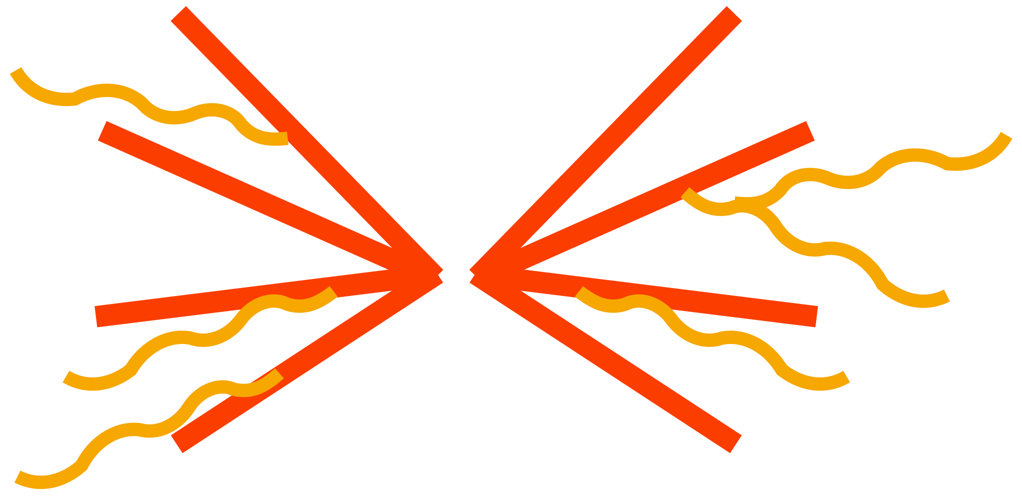

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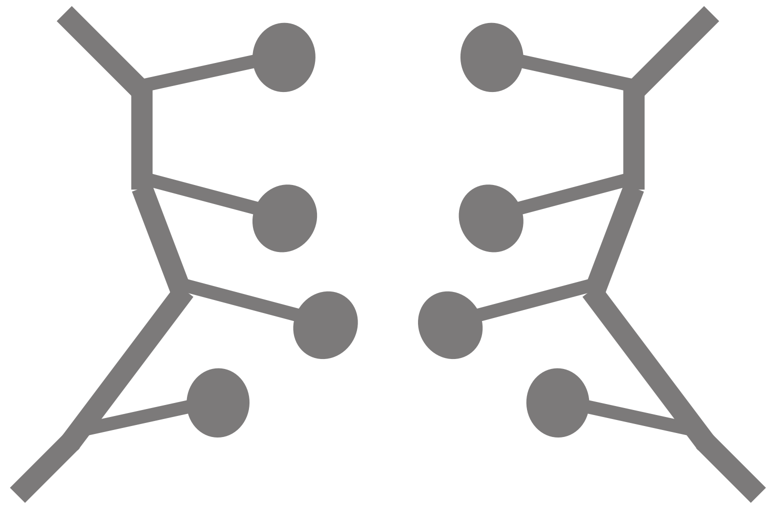
subtraction for loop divergence



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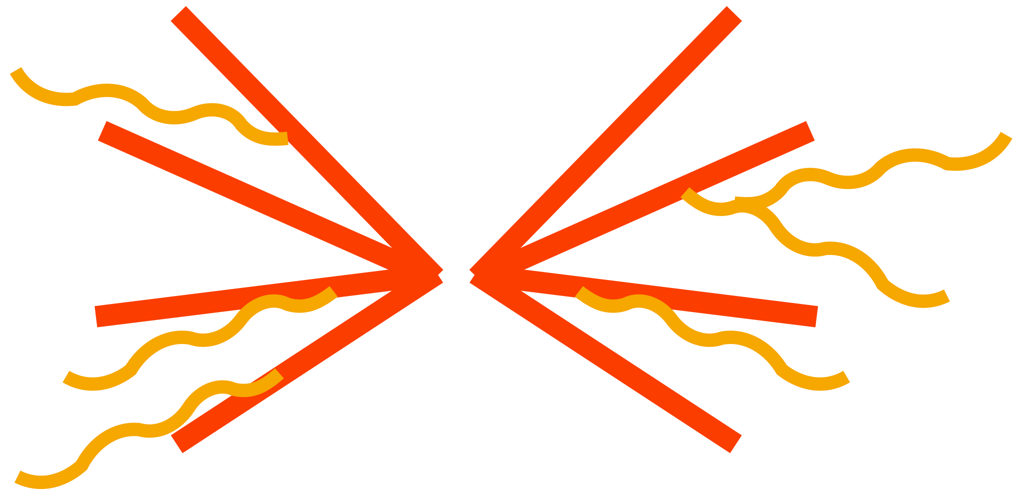

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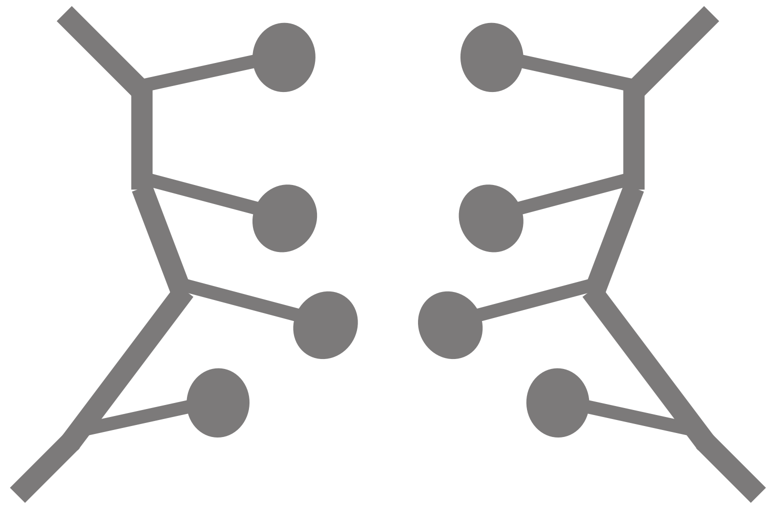
subtraction for unresolved emission



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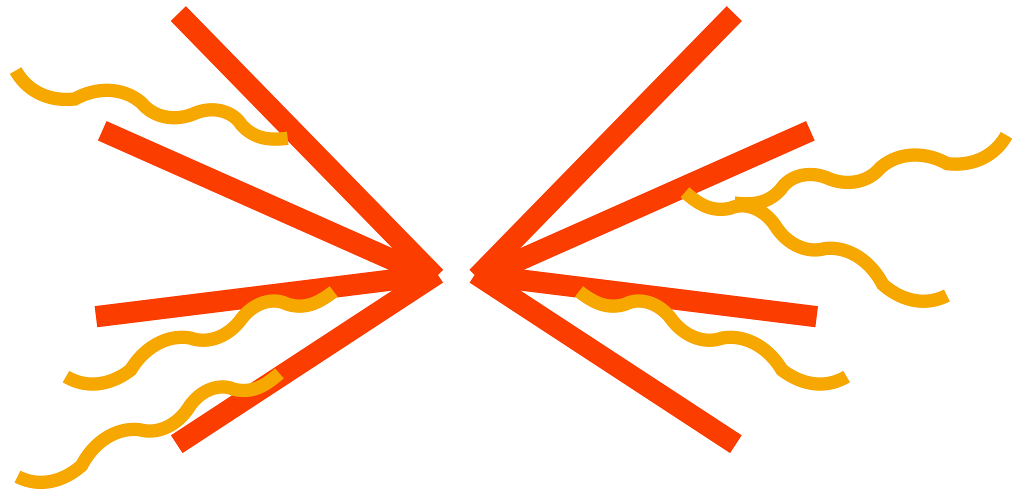
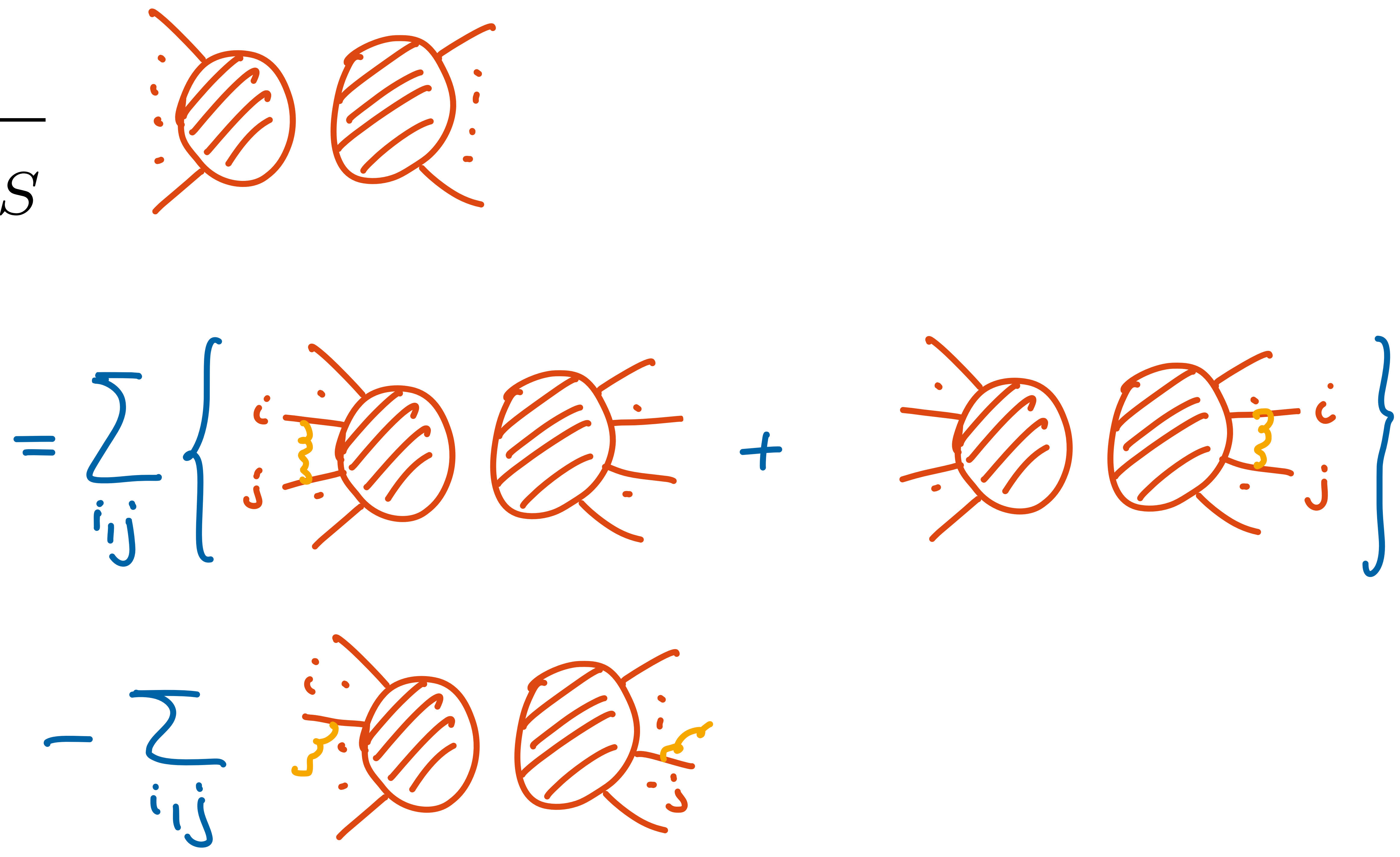
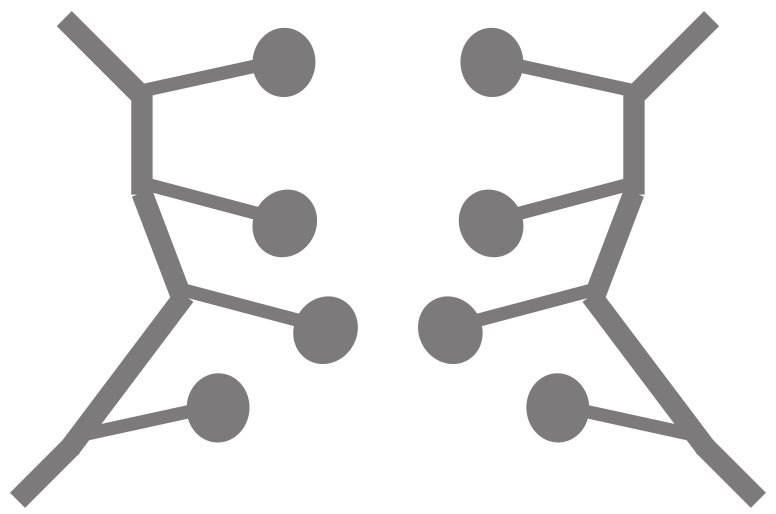
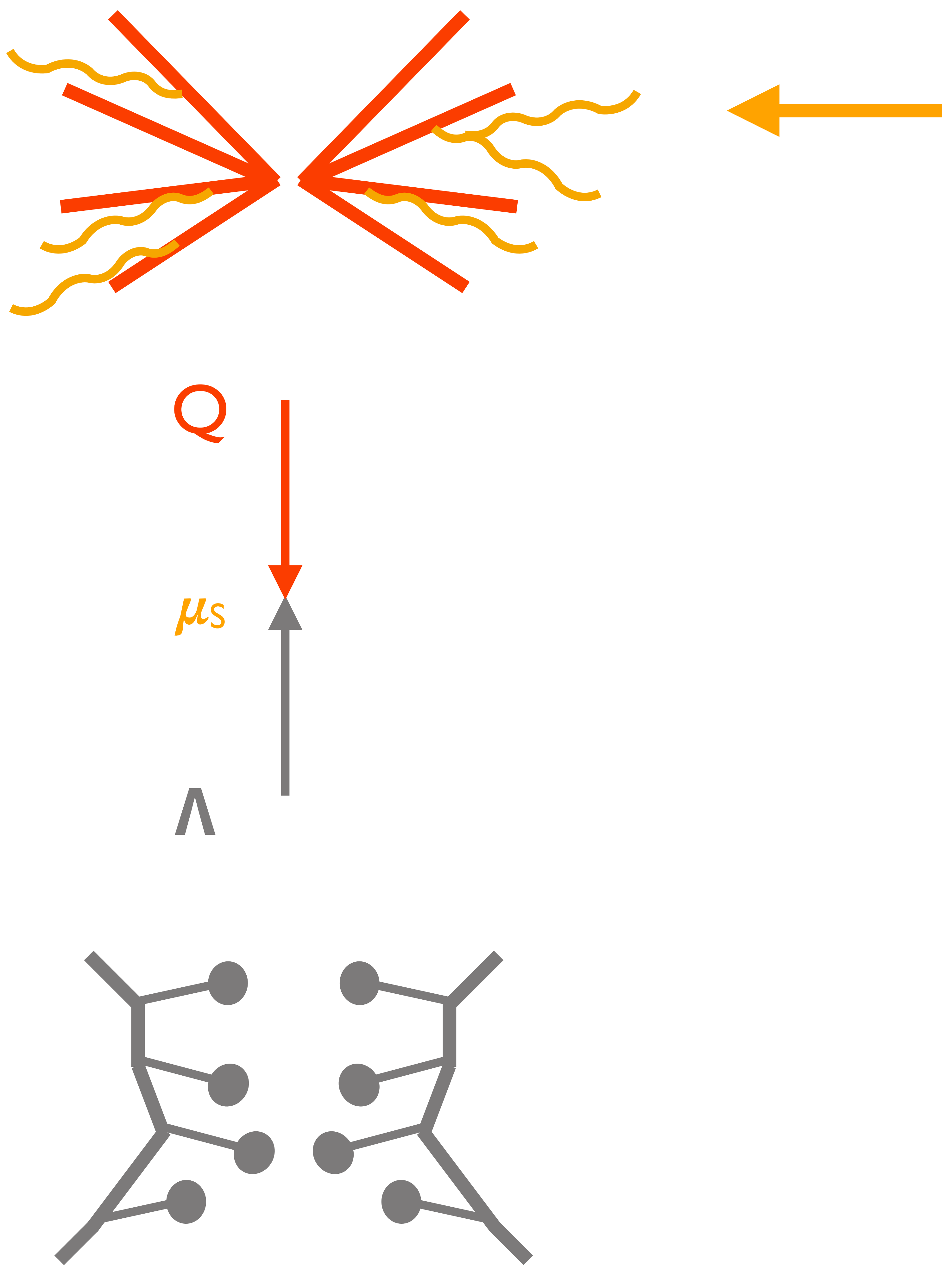


$$\frac{\mathrm{d}}{\mathrm{d}\mu_S}$$

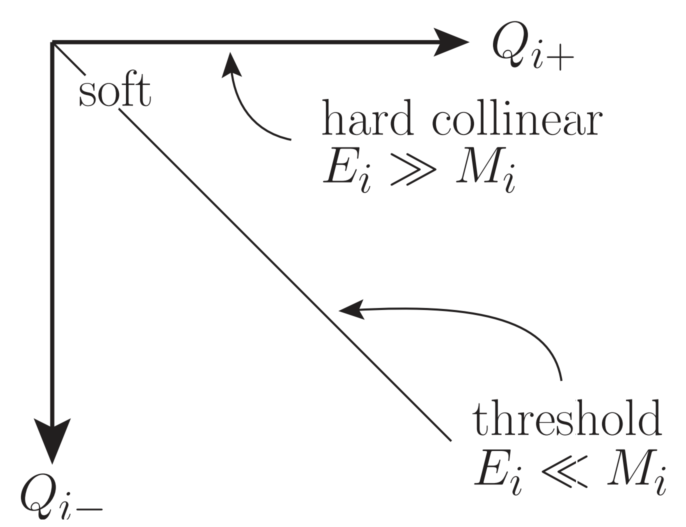


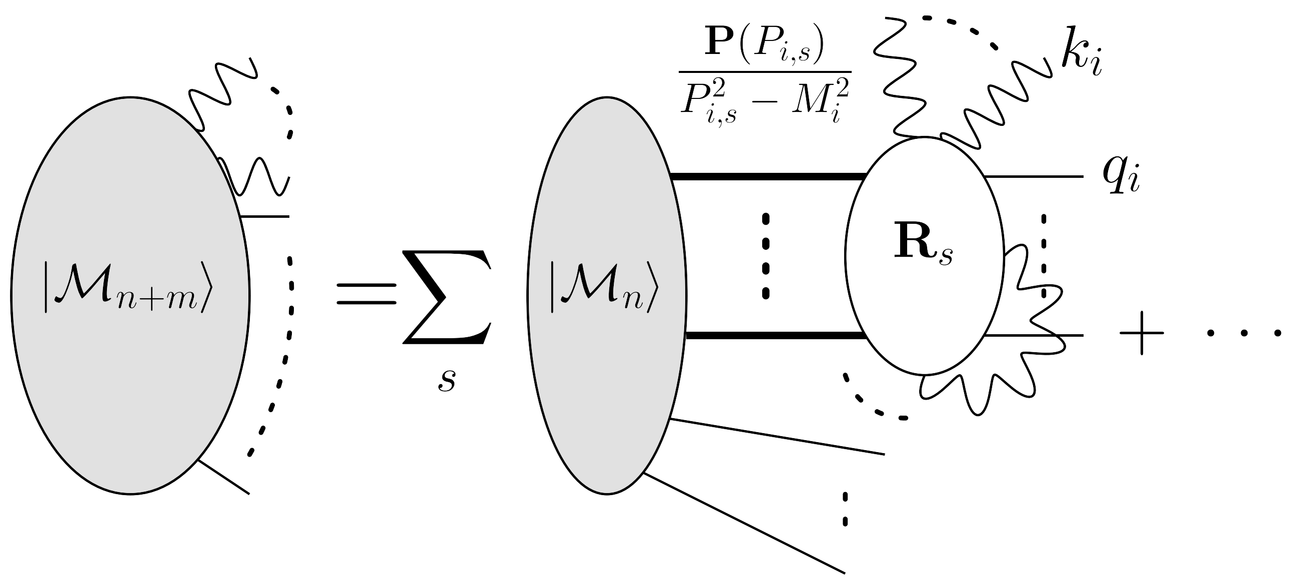


Building shower and resummation algorithms



Factorisation of amplitudes and power expansions.





[Löschner, Plätzer, Ruffa, Sjödahl — '20+] [Plätzer & Weigert – wip]

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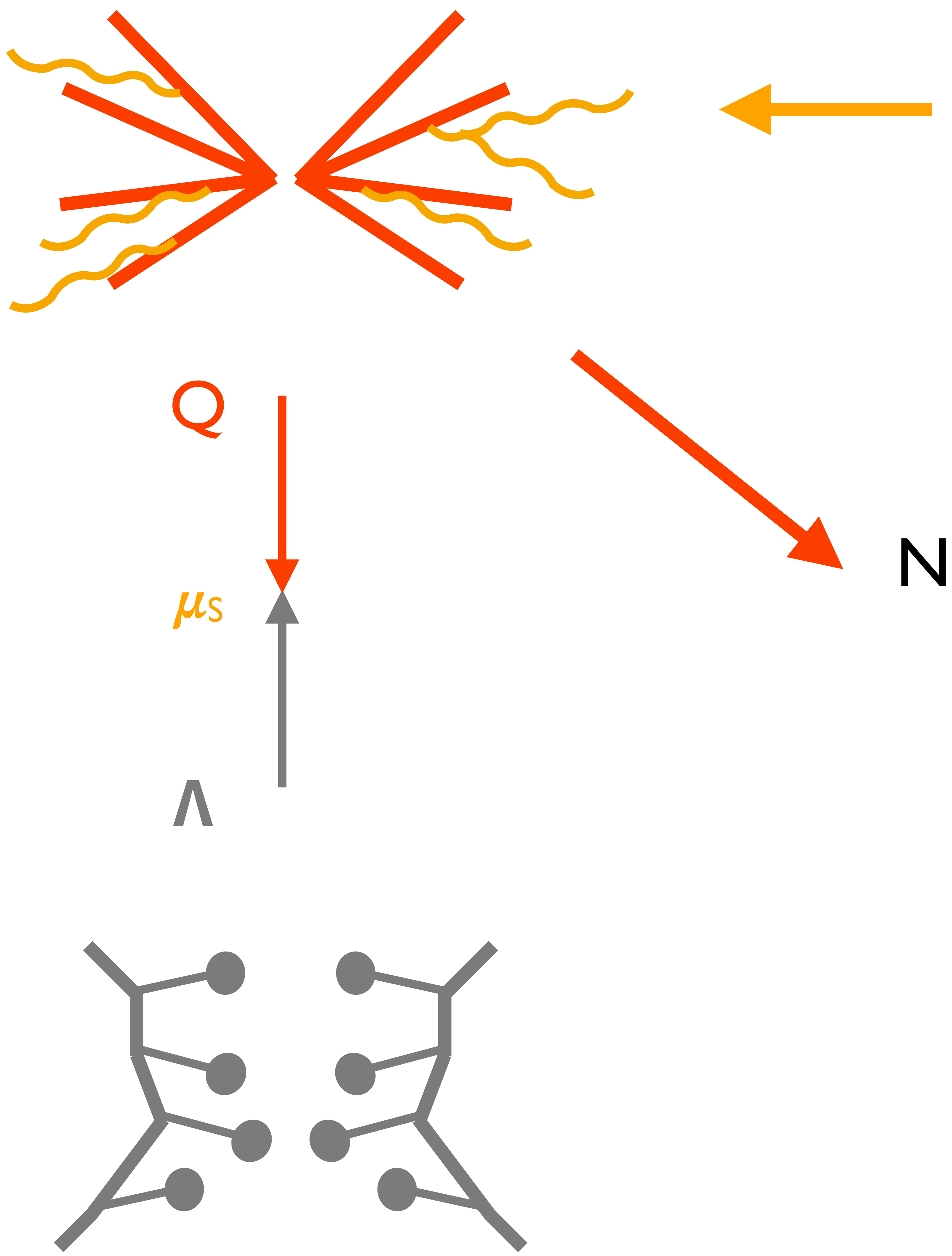

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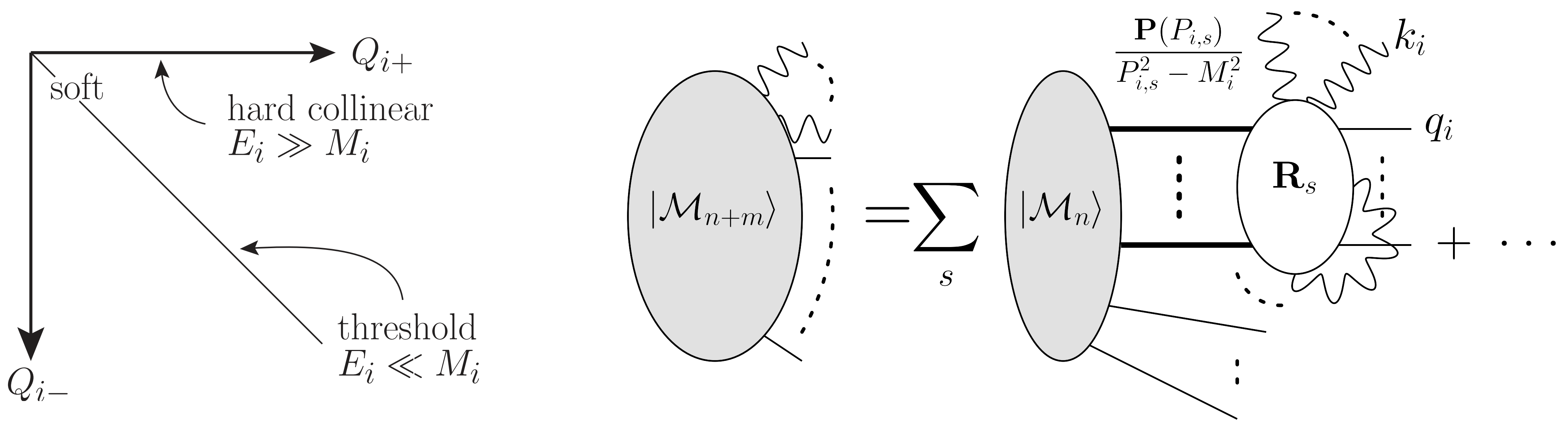

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NLL parton showers — Herwig 7 dipole shower

[Forshaw, Holguin, Plätzer — '20+] [Duncan, Holguin, Plätzer, Sule – wip]

$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

LL — qualitative NLL — quantitative NNLL — precision

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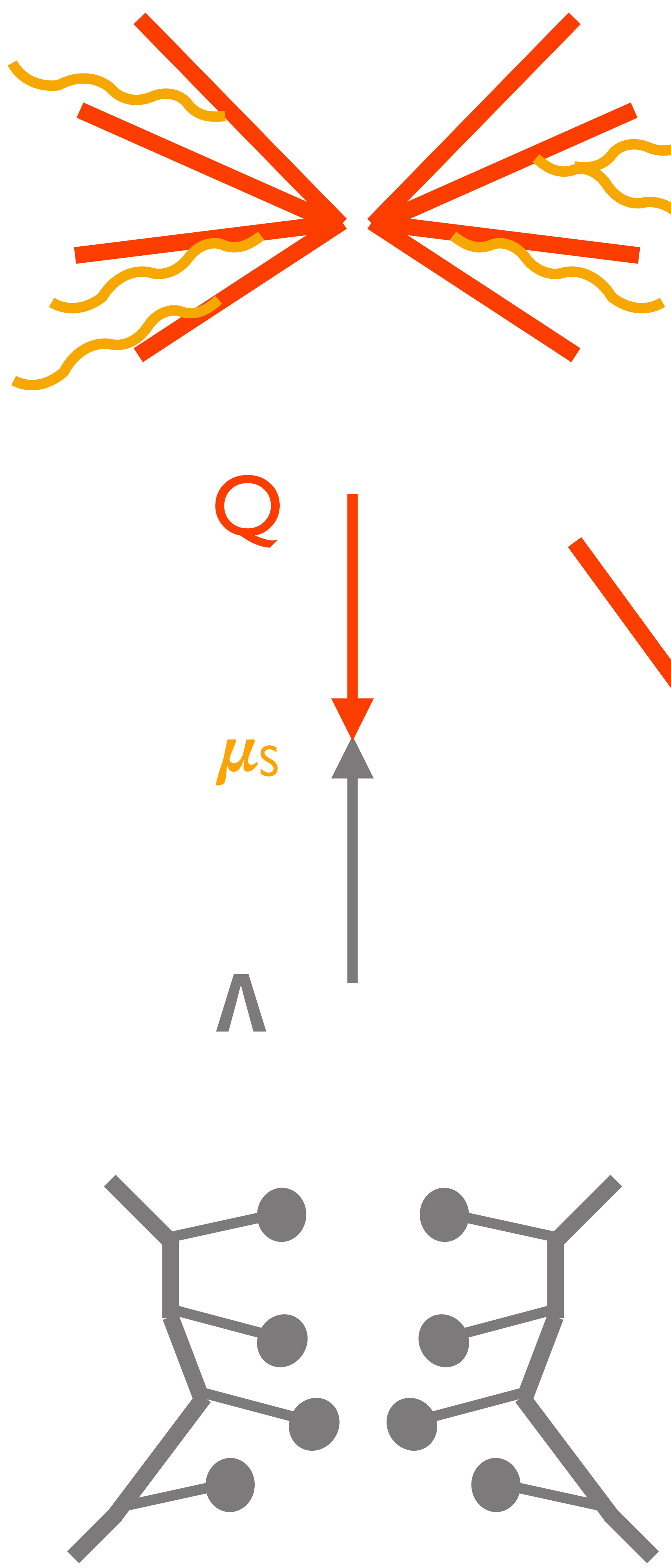

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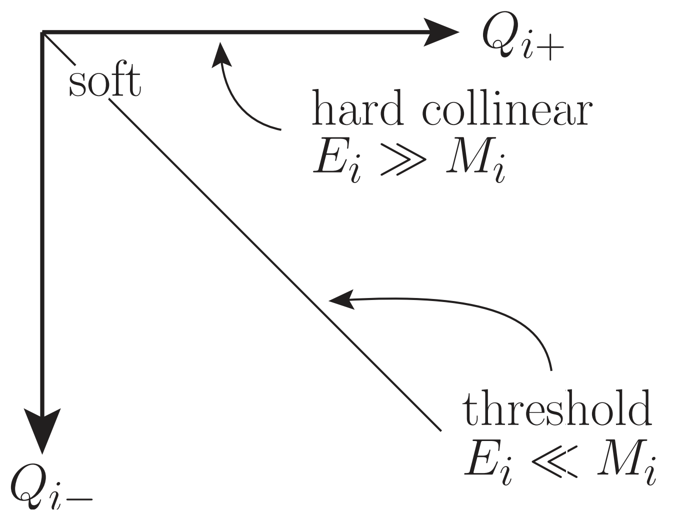

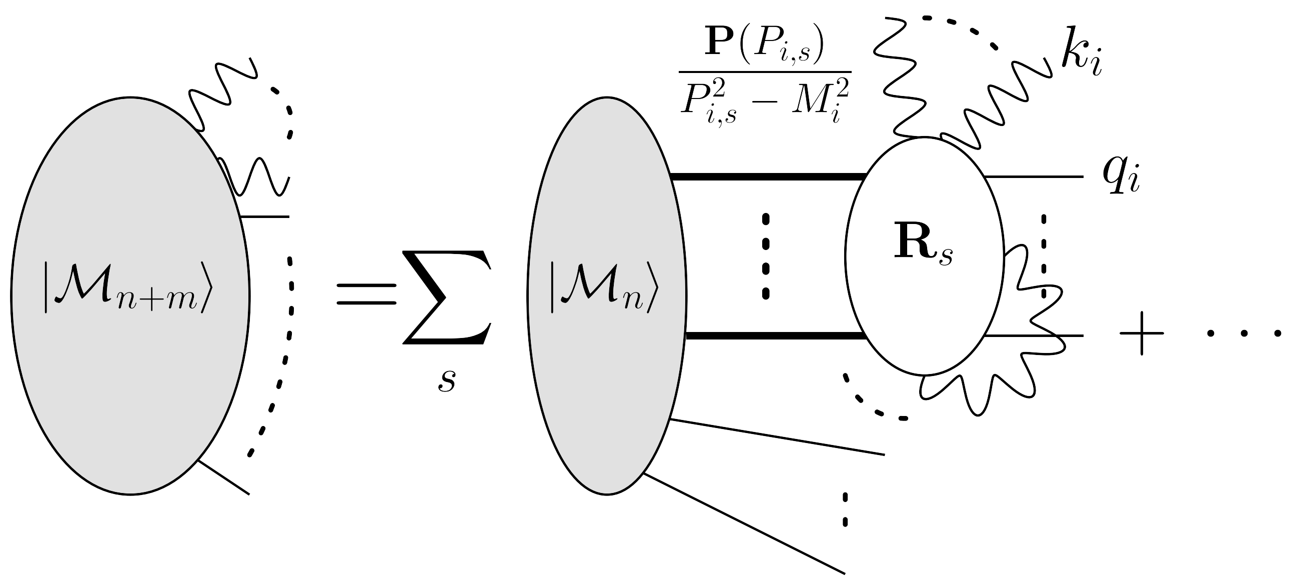

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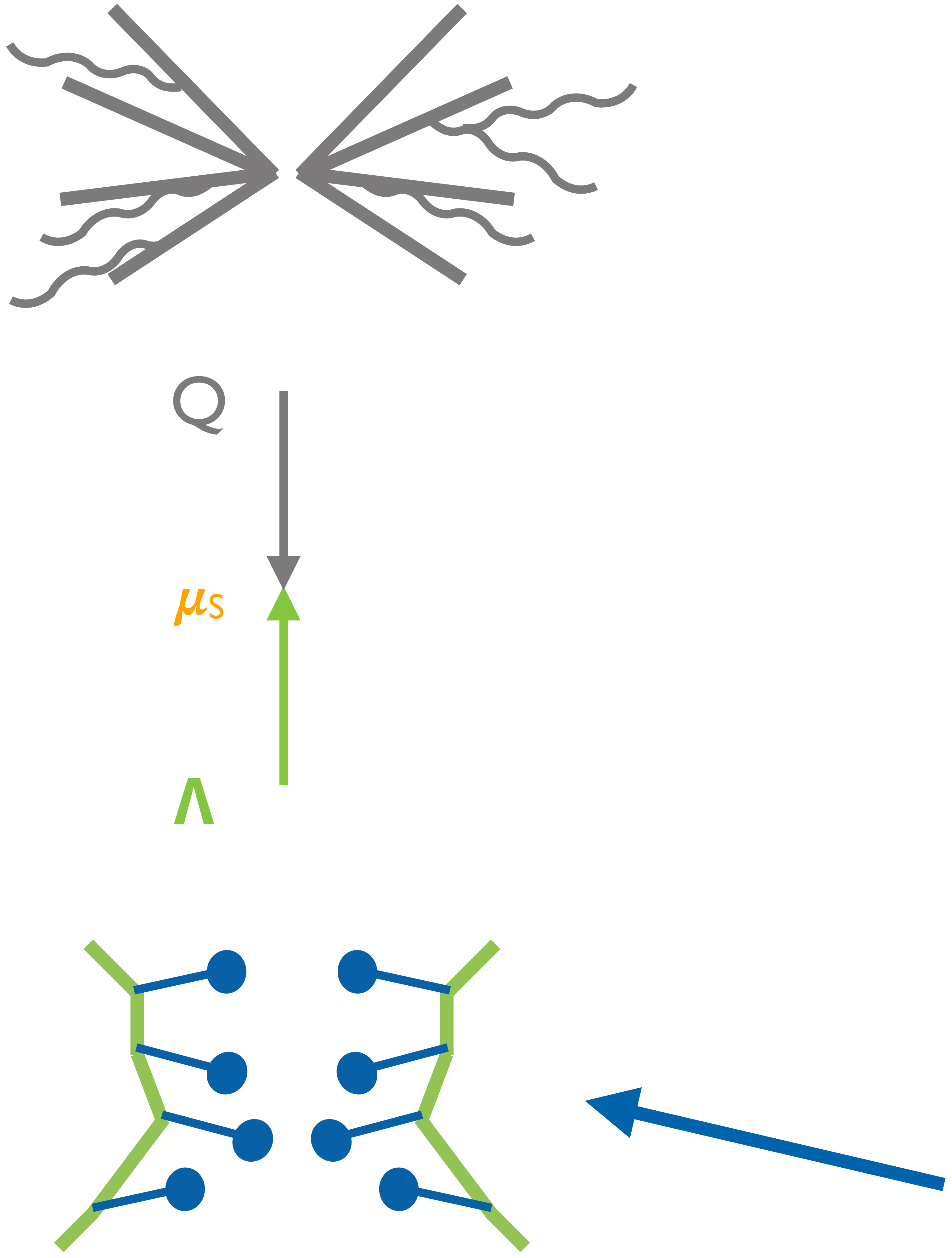
LL — qualitative   NLL — quantitative   NNLL — precision

Amplitude evolution and resummation algorithms.

- Started with non-global logarithms. [Forshaw, Plätzer et al. — '18+]
- Establishing links to JIMWLK, EFT, direct QCD resummation.

[Forshaw & Plätzer – wip] [Plätzer & Weigert – wip]

Building and constraining hadronization models



Hadron correlations not predicted or simplistic, baryon rates not right, cutoff dependence uncontrolled (fitted together with strong coupling), uncertainties not quantified … impact to ML more than worrying.

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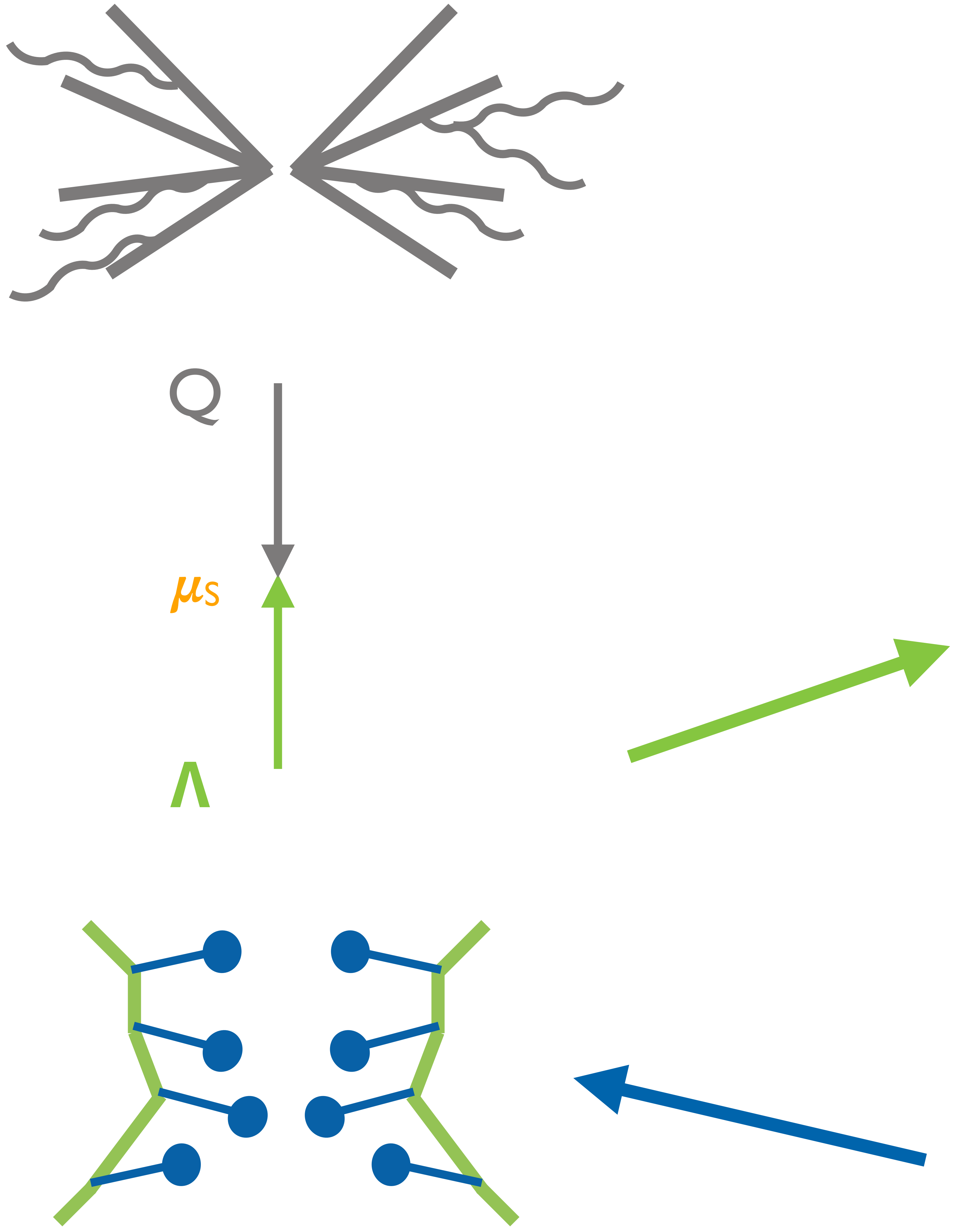
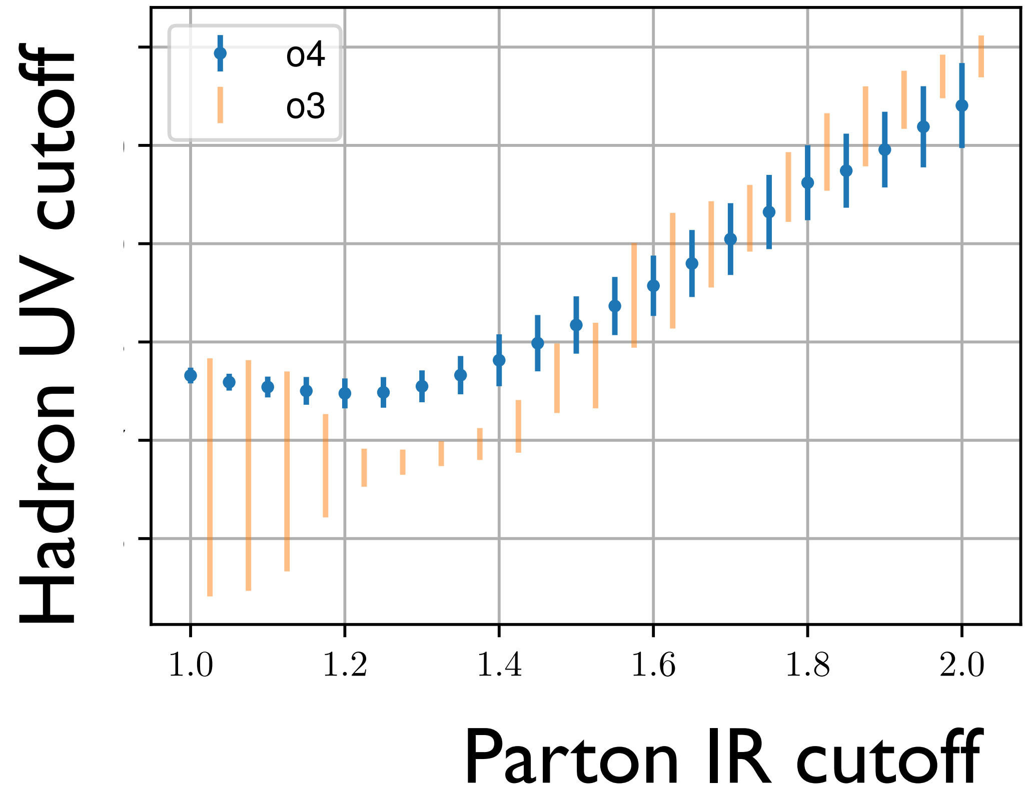


Towards a smooth matching of shower and hadronization at the infrared cutoff — inspired by coherent branching.

[Hoang, Jin, Plätzer, Samitz — in preparation]



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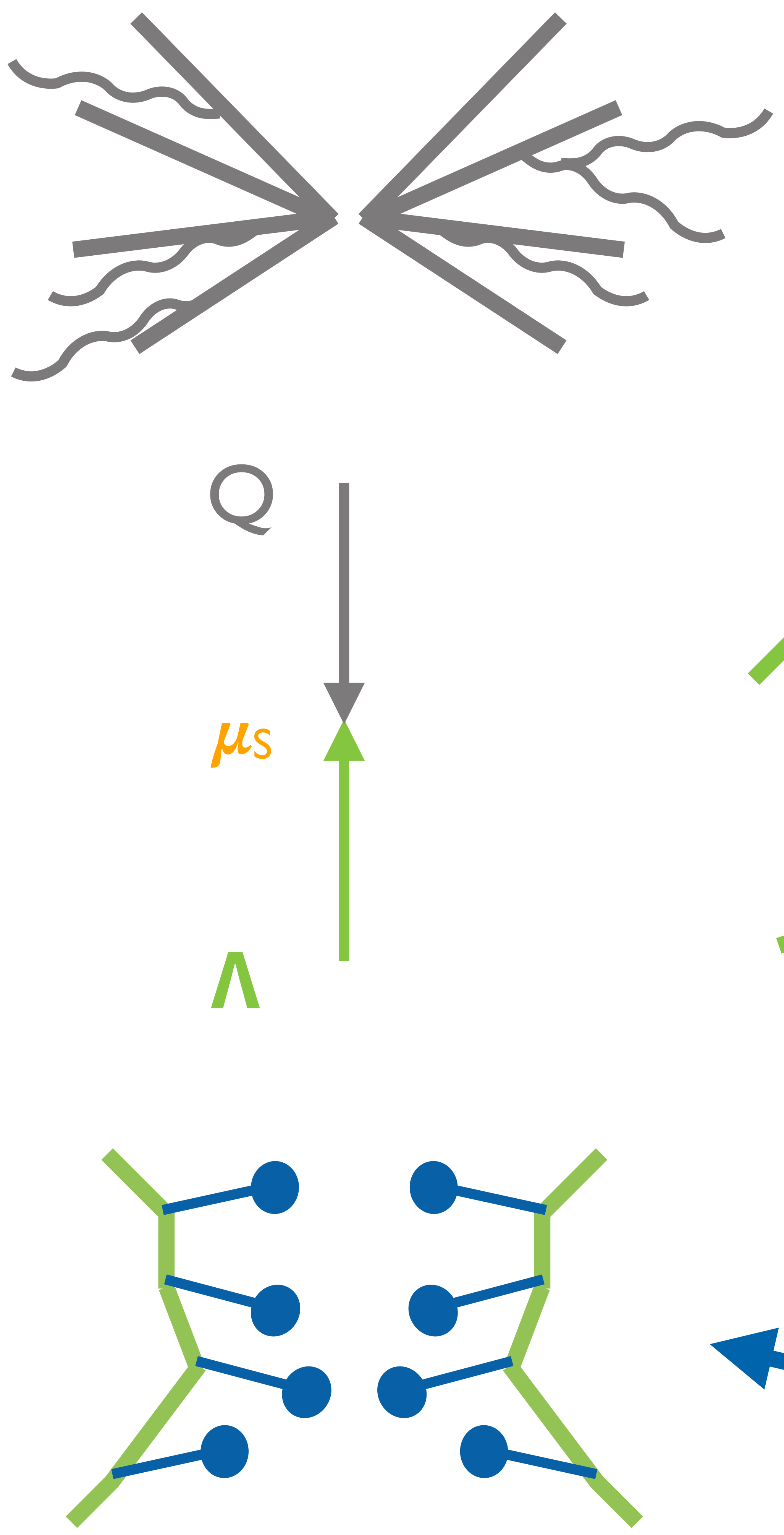
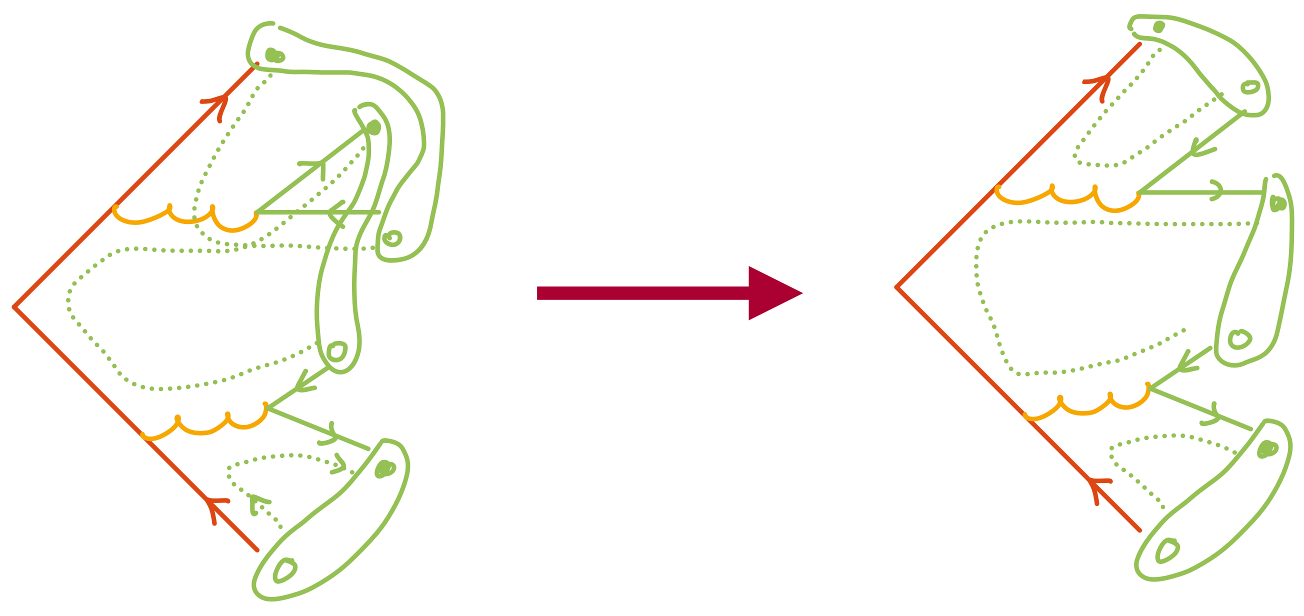


Towards a full model of cluster evolution with fission and colour reconnection informed by perturbative evolution.



[Gieseke, Kiebacher, Plätzer, Priedigkeit — in progress]

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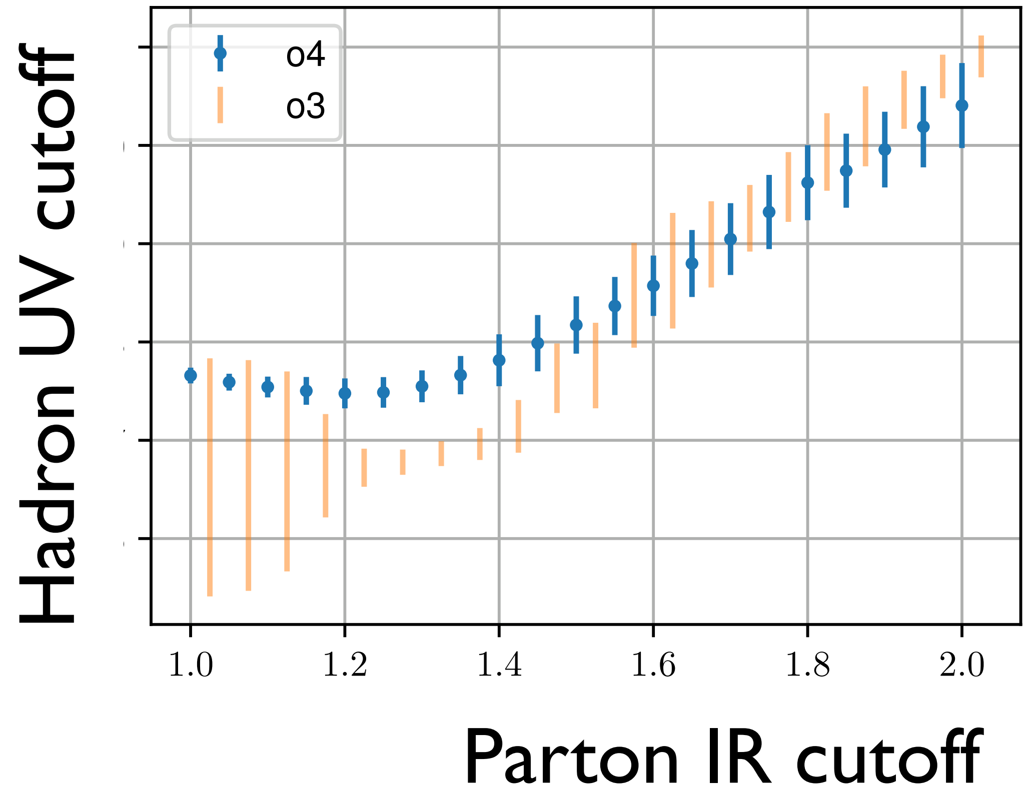

[Hoang, Jin, Plätzer, Samitz — in preparation]

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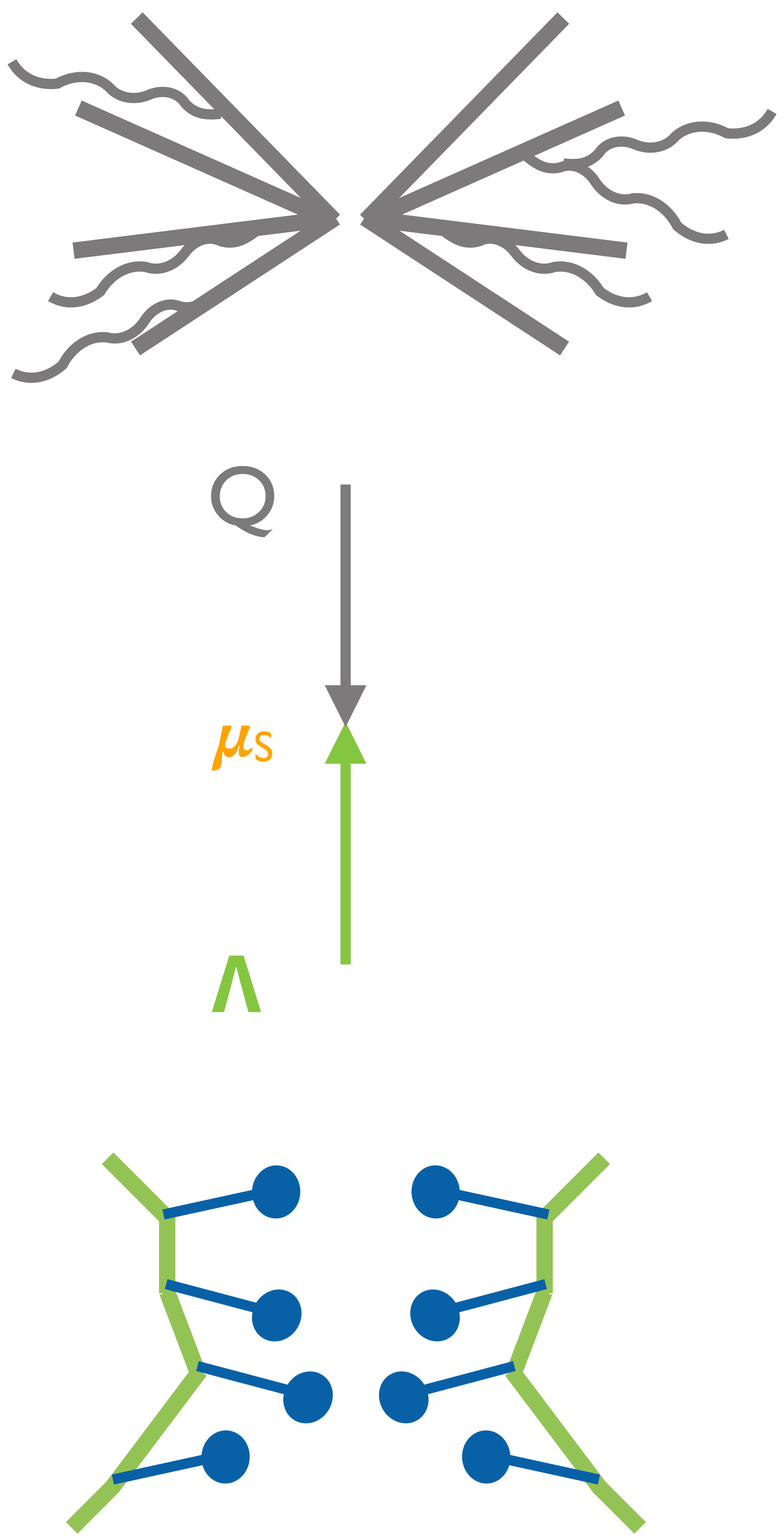
What structures are admissible?



Subtracted ("renormalised") observable defines a very general criterion of infrared safety: finiteness means the bare observable must admit **cancellations local in momentum and colour space.**

$$\mathbf{S}_n = \mathbf{Z}_n^\dagger \mathbf{U}_n \mathbf{Z}_n + \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{E}_{n+s}^{(s)\dagger} \mathbf{U}_{n+s} \mathbf{E}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

This structure is ubiquitous if we talk about **electroweak final states** (in isospin space) and if we want to predict fully detailed and exclusive final states as needed for an event generator.

Observables singular at his level are genuine non-perturbative.

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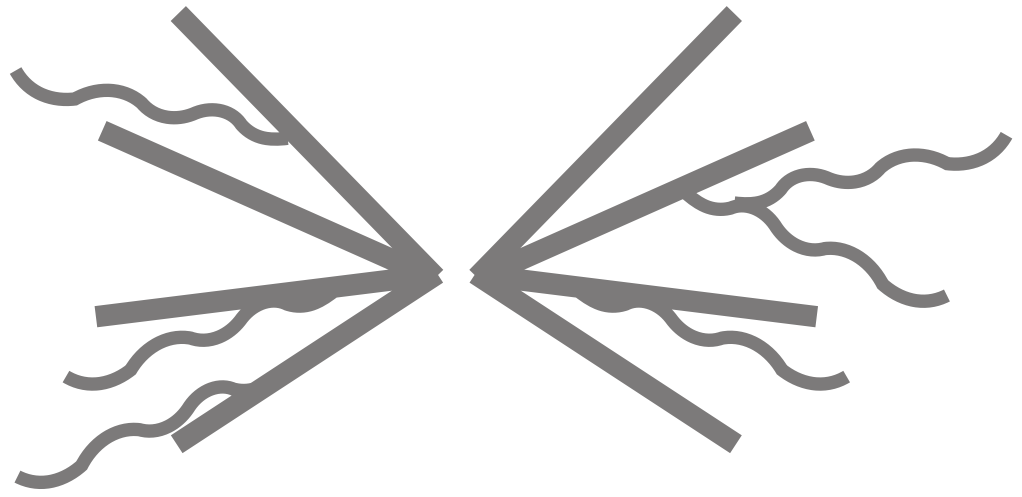

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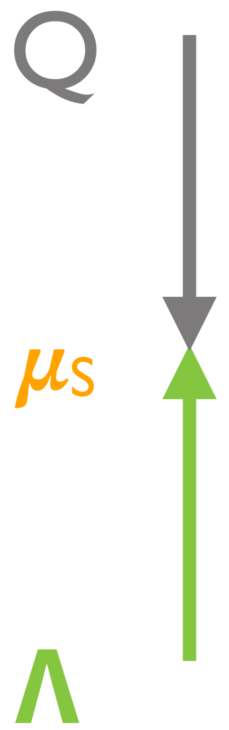
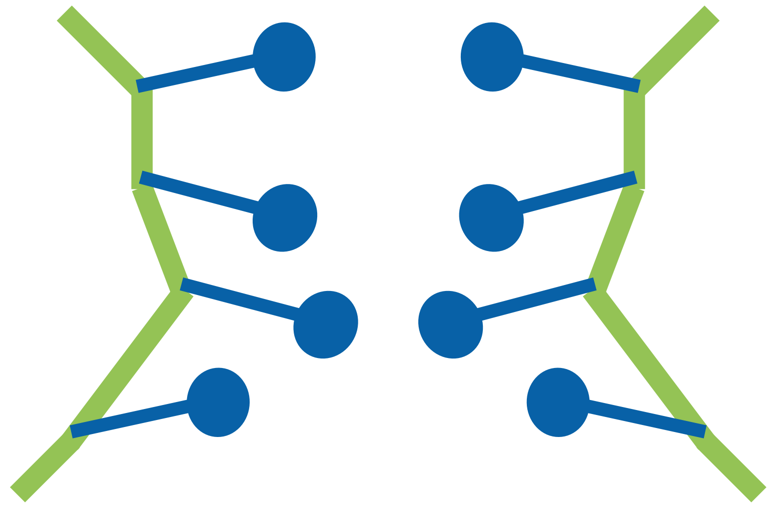


infrared resolution vs observable

$$\mathbf{S}_n = \mathbf{1}_n u(p_1, \ldots, p_n) - \alpha_s \int \mu_R^{2\epsilon}[\mathrm{d}p_{n+1}]\tilde{\delta}(p_{n+1})\hat{\mathbf{D}}^{(1,0)\dagger}_{n+1}\hat{\mathbf{D}}^{(1,0)}_{n+1}\Theta_{n,1}\left[u(p_1, \ldots, p_n, p_{n+1}) - u(p_1, \ldots, p_n)\right] + \mathcal{O}(\alpha_s^2)$$



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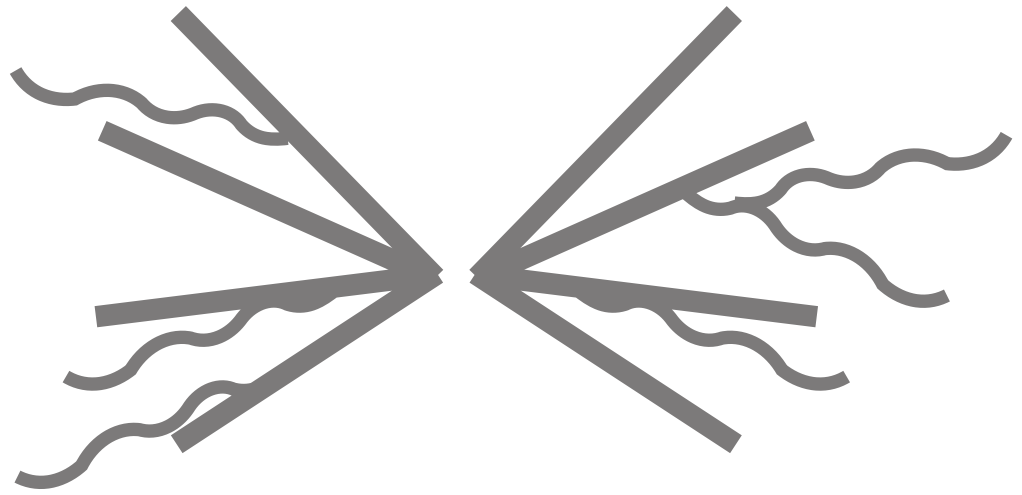
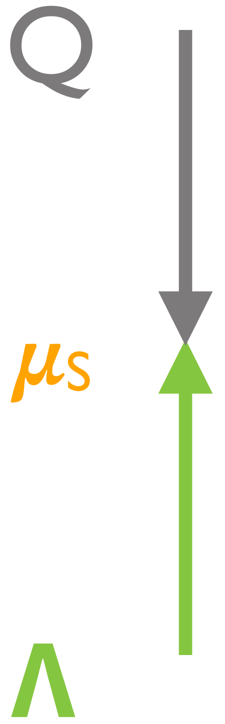
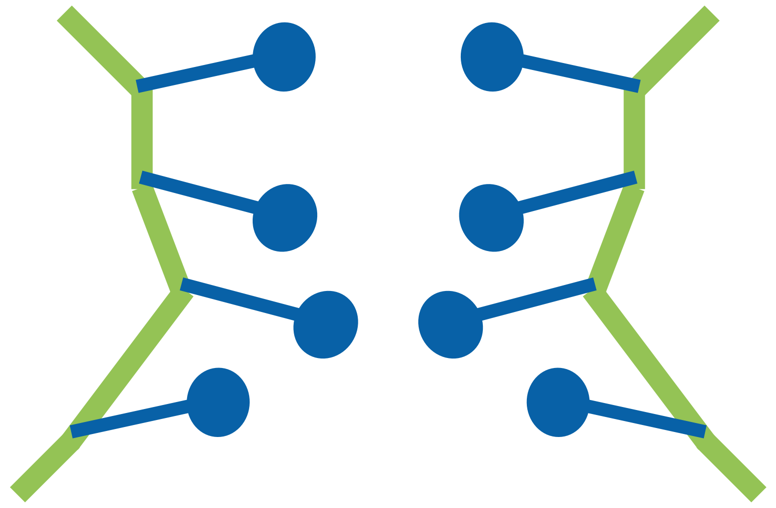
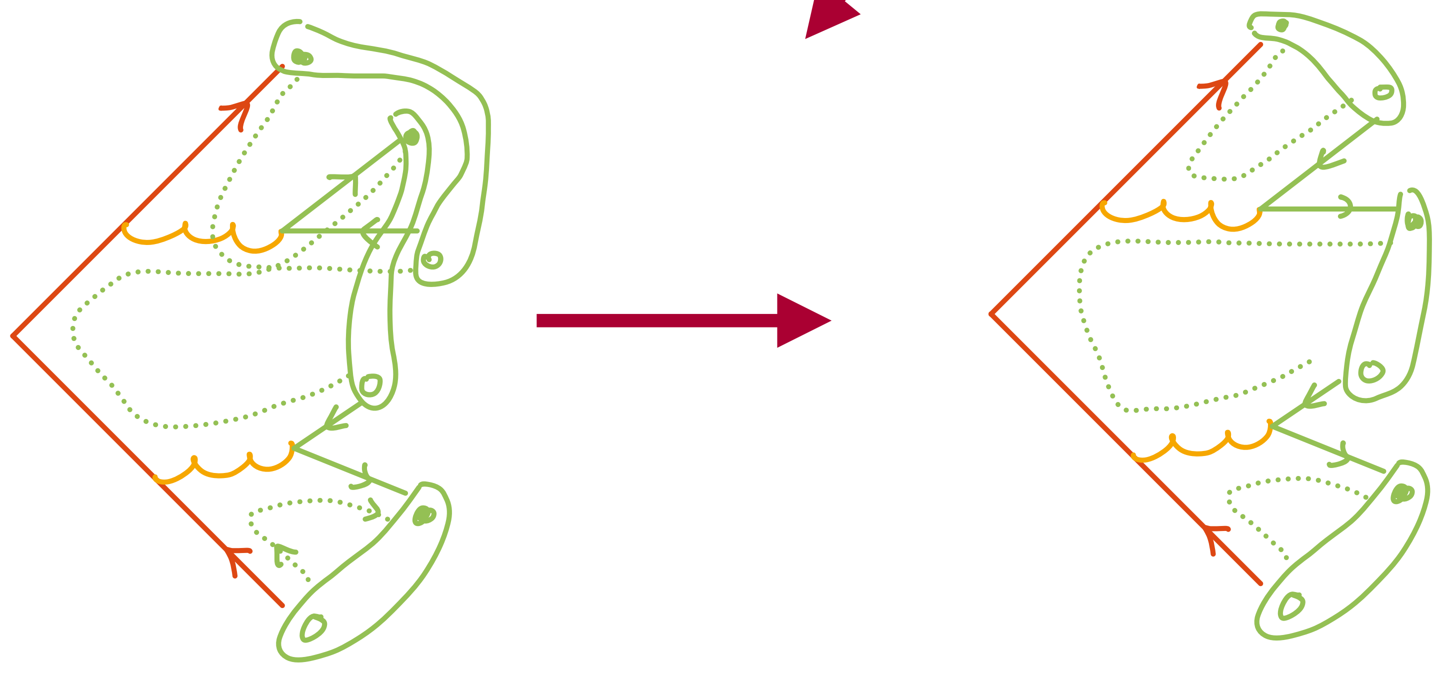


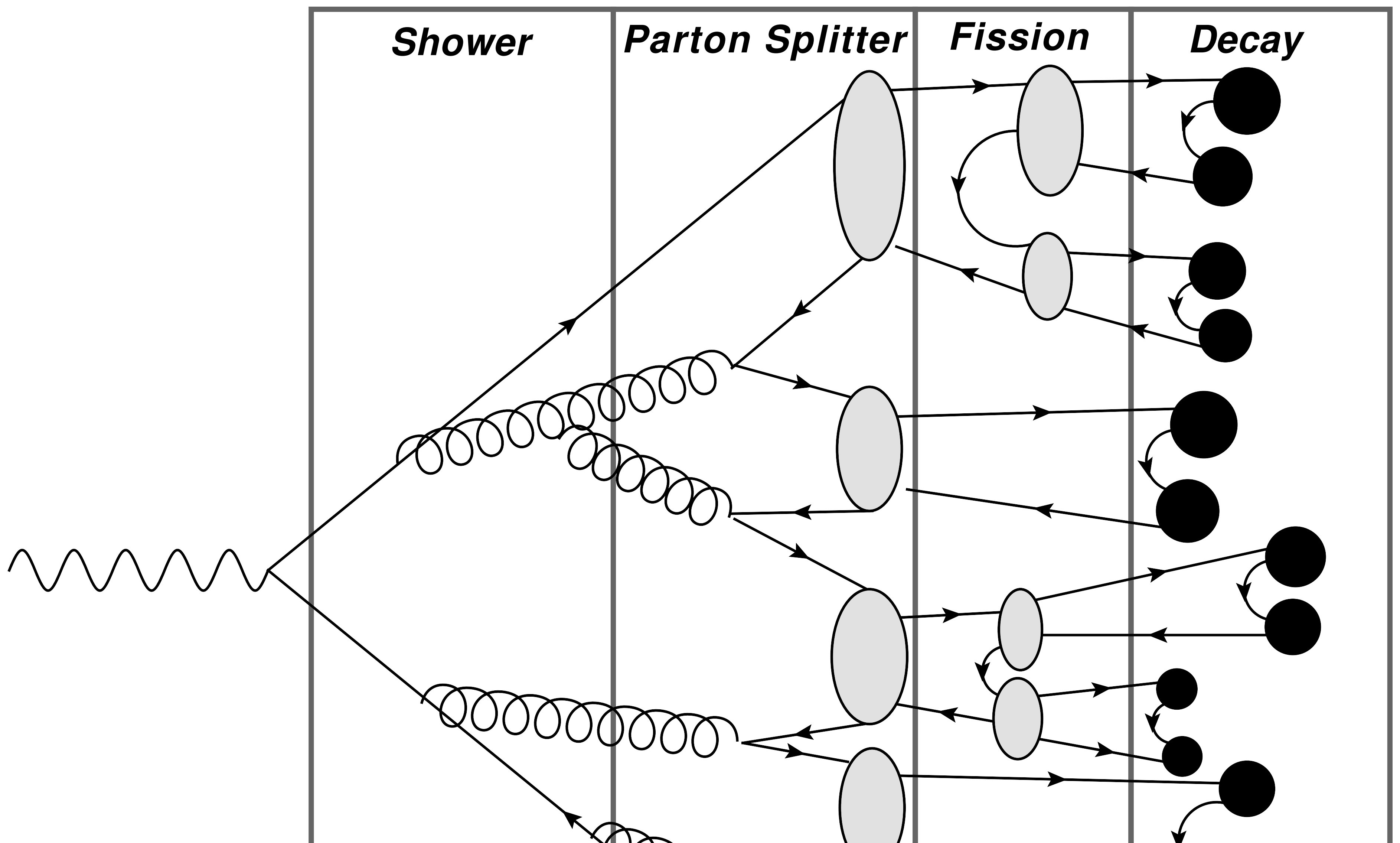




Hadronization models would start by studying clusters.
Possible relation to amplitudes from functional methods.

$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$





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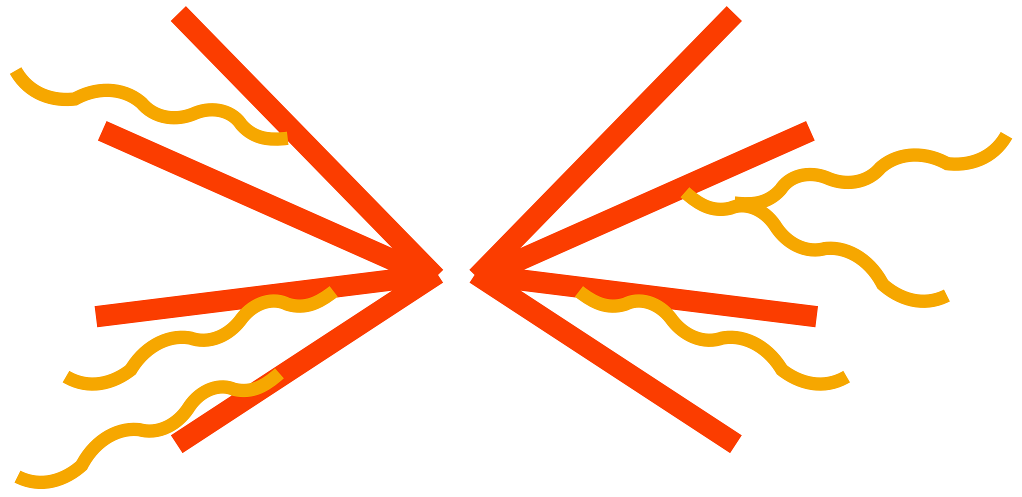
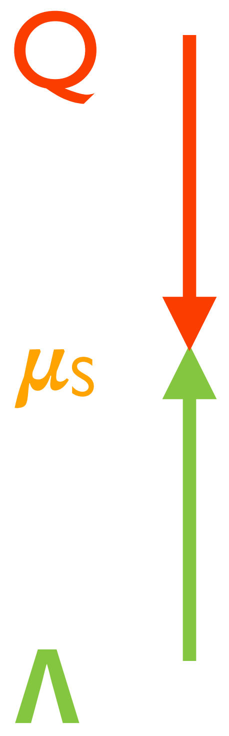
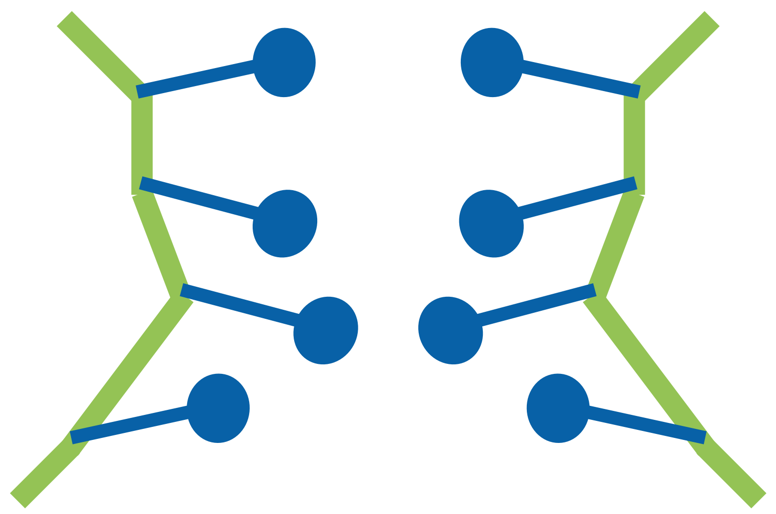


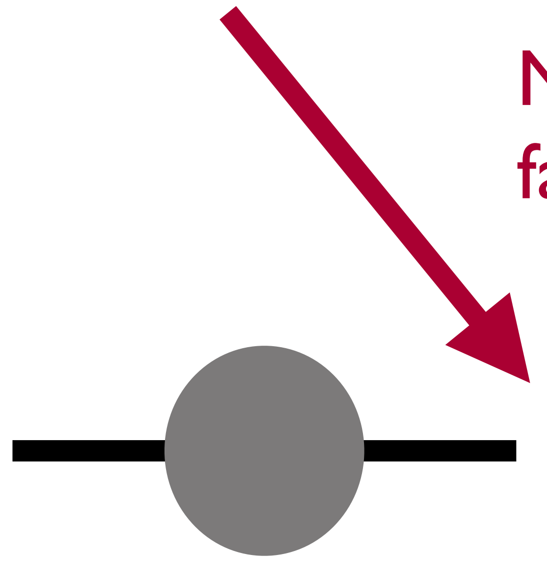




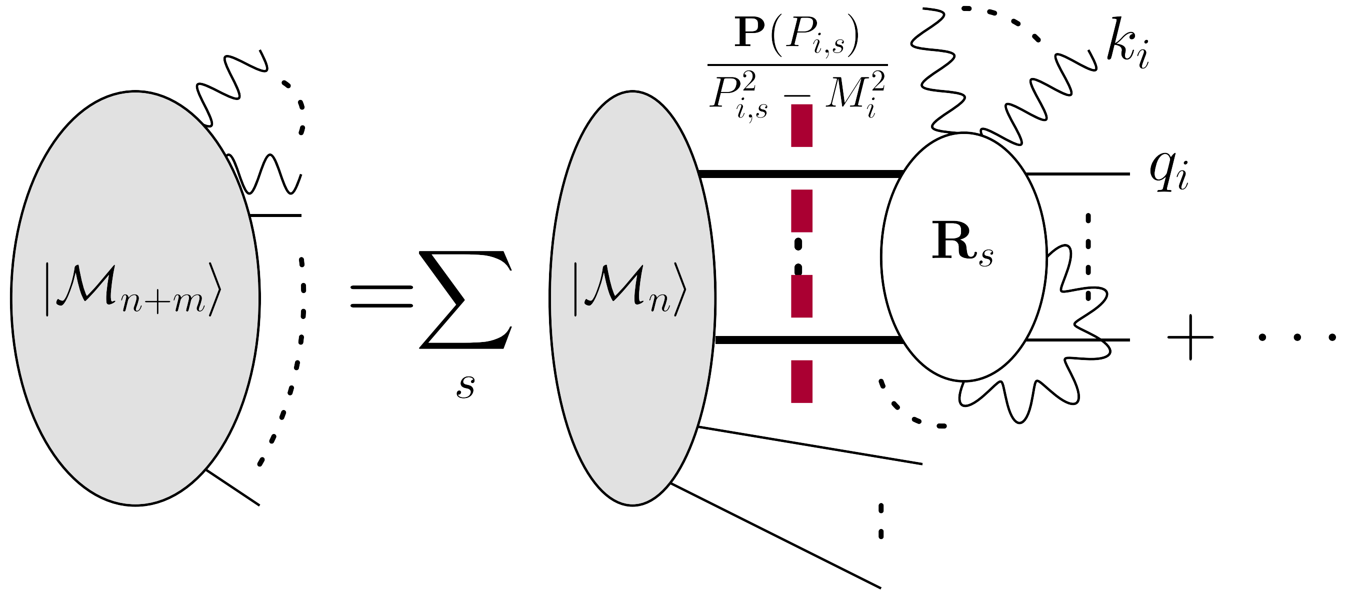
Generally we need to understand exclusive processes and factorisation and projections onto physical (singlet) final states — including spin.

$$K^\mu_{i,s} = \Lambda^\mu{}_\nu \left(Q^\nu_{i,s} + \delta_{i,s}\, n^\nu_{i,s}\right)$$
$$q^\mu_i = \Lambda^\mu{}_\nu \left(\alpha p^\nu_i + \frac{(1-\alpha^2)M_i^2 + p_i\cdot Q_{i,s}}{2\alpha\, n_{i,s}\cdot p_i} n^\nu_{i,s}\right) - K^\mu_{i,s}$$

Momentum mappings to systematically factor renormalised matrix elements.

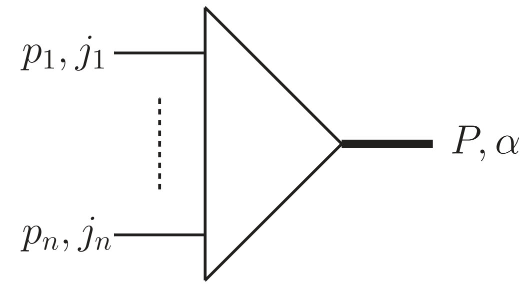


$$= \frac{1}{2p_i\cdot Q_{i,s}} \frac{\Psi(\Lambda p_i, M_i)\bar\Psi(\Lambda p_i, M_i)}{1-\Sigma'(M_i^2)} + \mathcal{O}(\lambda)$$

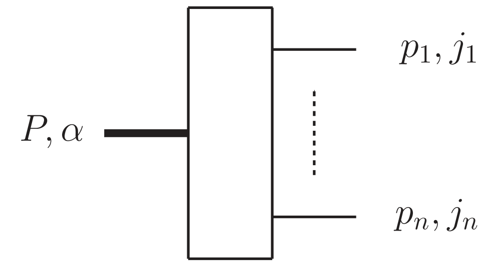


[Plätzer, Sjödahl — '22]

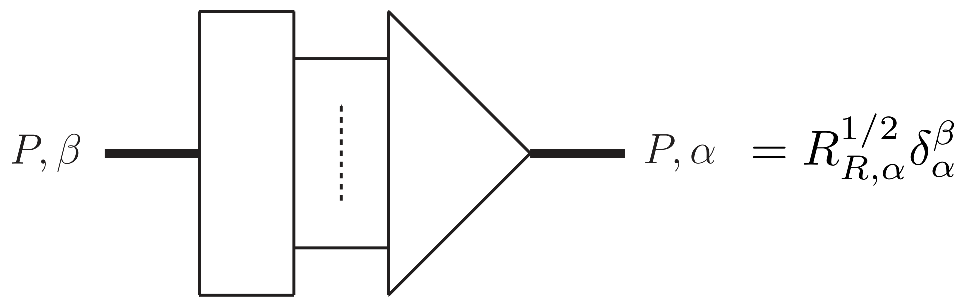
$$\chi_{\alpha|j_1,\dots,j_n}(\vec P, M|p_1,\dots,p_n)\,\delta\left(P - \sum_{i=1}^n p_i\right) =$$

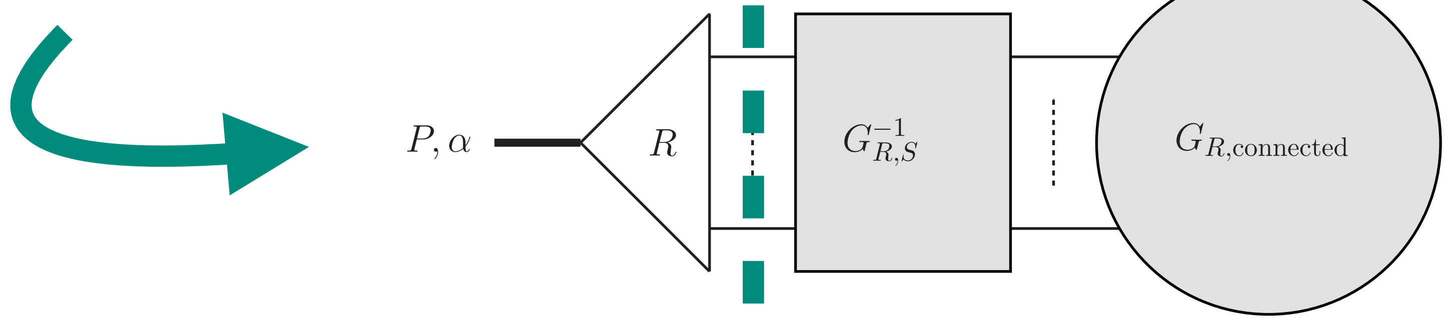


$$\left(Z_\Phi^{-1/2}\prod_{i=1}^n Z_{\phi_i}^{-1/2}\right)\bar X^\alpha(\vec P, M|p_1,\dots,p_n)\,u_\alpha^{j_1,\dots,j_n} =$$



Composite particle scattering — for FMS as well as to study exclusive processes.





[Maas, Plätzer — in progress]

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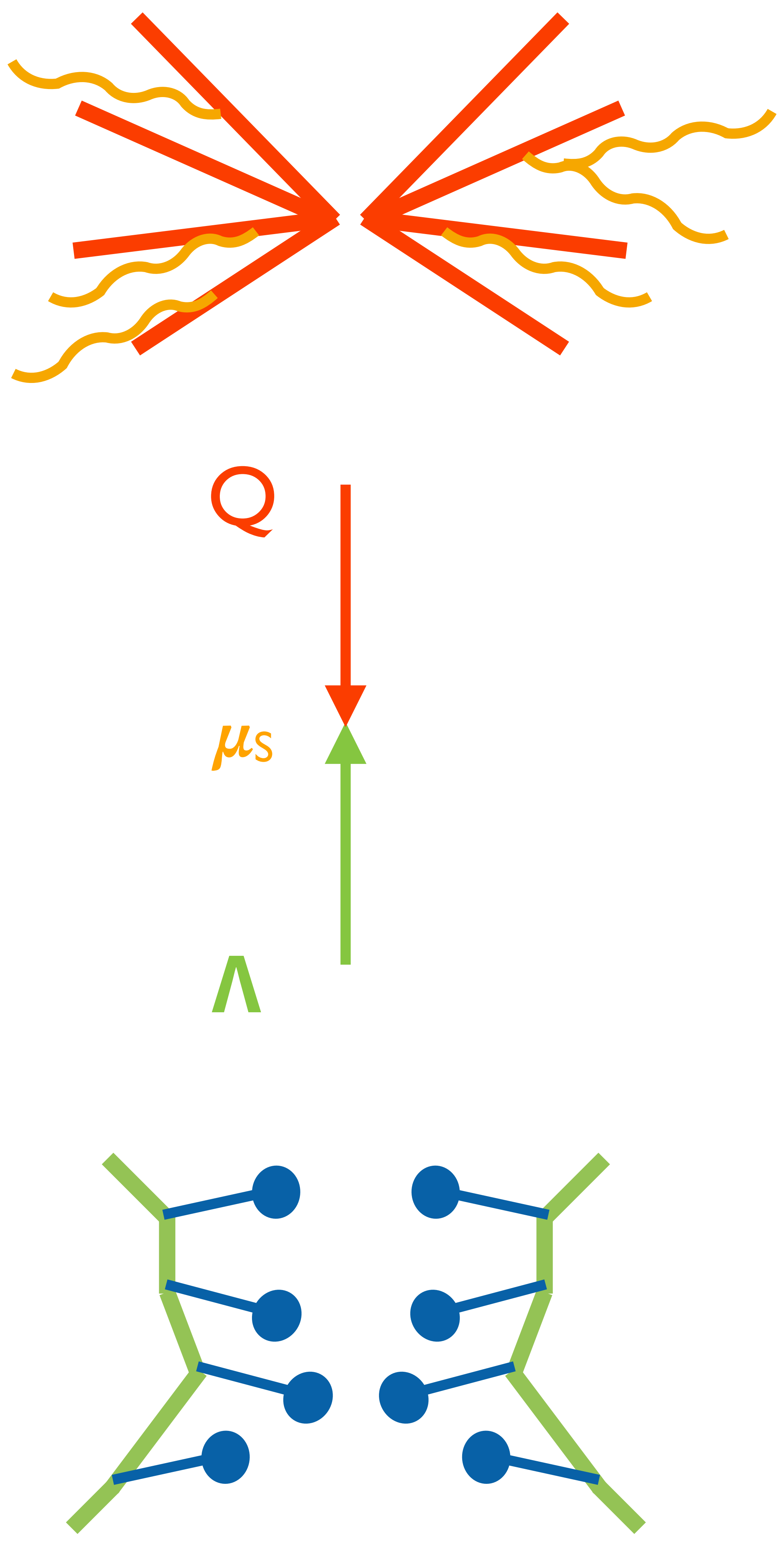
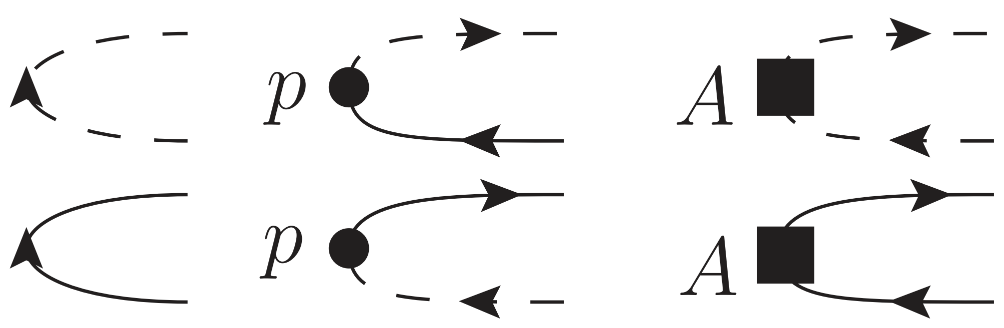


Generally we need to understand exclusive processes and factorisation and projections onto physical (singlet) final states — including spin.

Find a basis of spin structures, together with isospin and colour.

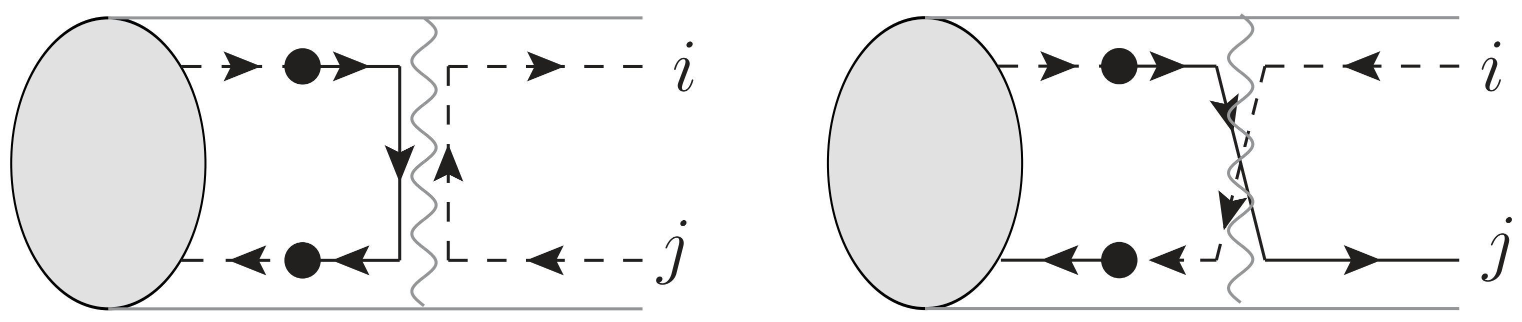


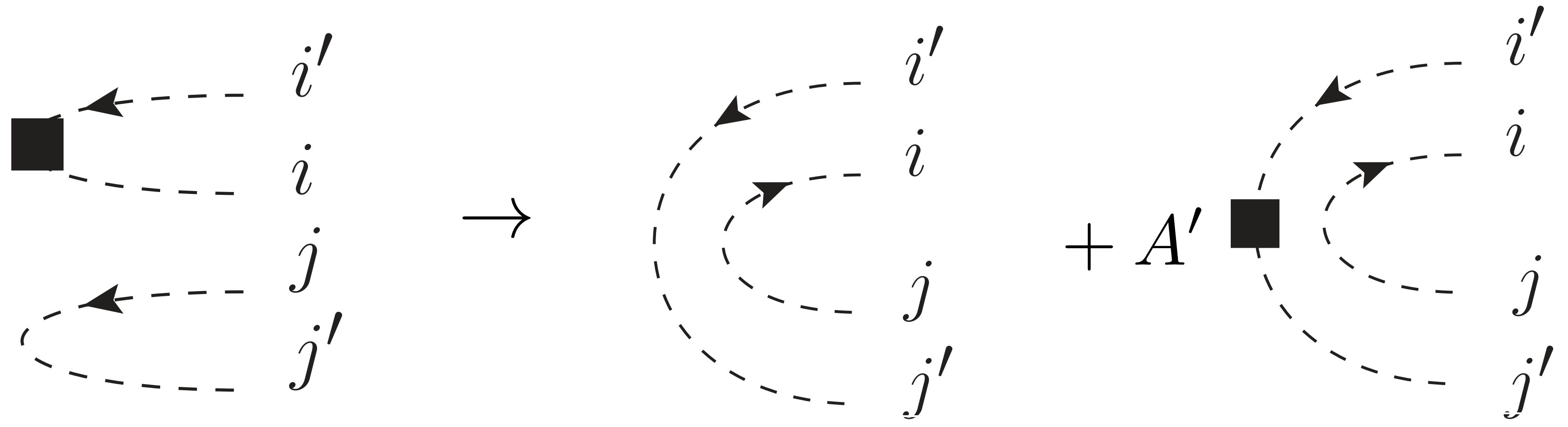
Essentially a basis of

$$\mathbf{1} \qquad \sigma^\mu \qquad \frac{1}{2}\left(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu\right)$$

Could this point to a more general version of setting up graphical tensor calculus?

Electroweak bosons now mix different chiral basis states.





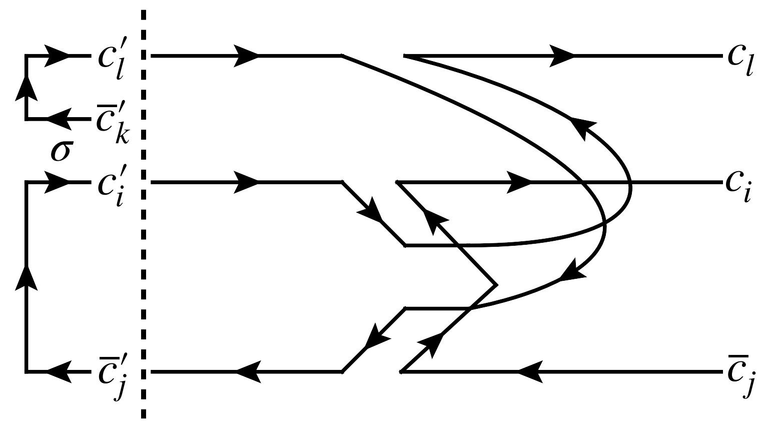
[Plätzer, Sjödahl — '22]

Summary

Colour space evolution equations:

- exiting theoretical tool to build parton shower and resummation algorithms,
- important subject in their own right to study structures in (QCD) amplitudes.

Graphical methods for tensor calculus 🤔 are crucial to reveal structure, to design algorithms, to perform explicit analytic calculations, …



Definitions of measurements and completeness of (asymptotic) final states will become an even more interesting tool in developing a comprehensive understanding of how we predict exclusive cross sections and how we can built simulations.

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

A lot of work in progress I wasn't able to talk about … including some interesting constructions of lattice operators, which complement the technology we use for perturbative calculations.

Advertisement (1)

MIAPbP

EVENT GENERATORS AT COLLIDERS AND BEYOND COLLIDERS

28 July - 22 August 2025

Simon Plätzer, Leif Lönnblad, Anita Reimer, Stefan Söldner-Rembold, Laura Fabbietti


C.Stadler/Bwag

July 1 Room for informal meetings

2024 Graz July 2-4

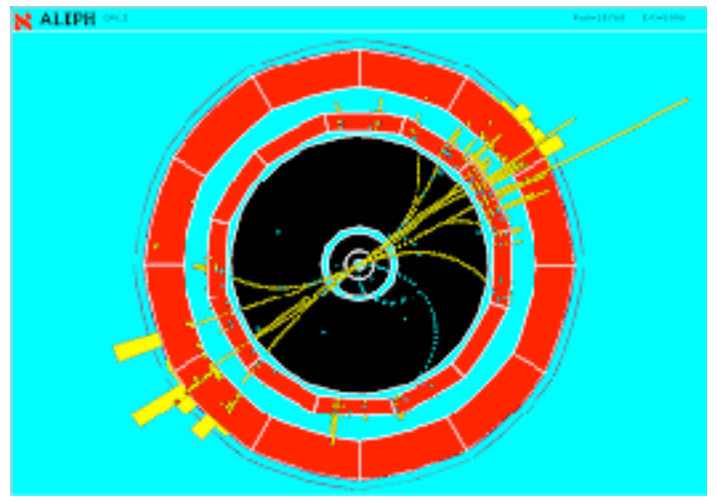
Parton Showers and Resummation

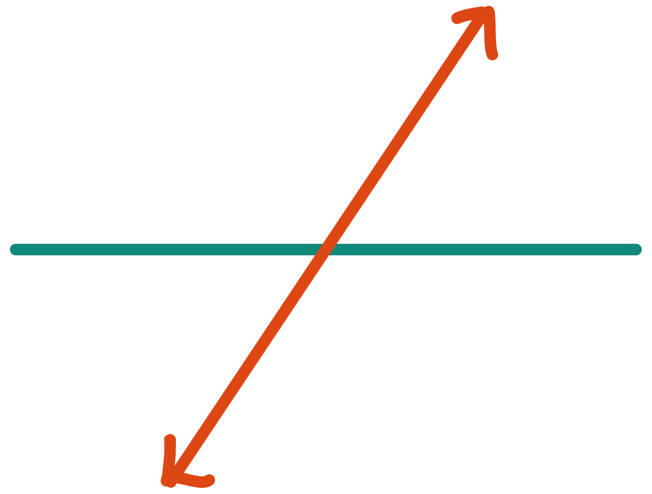
July 5 A fresh look at hadronization

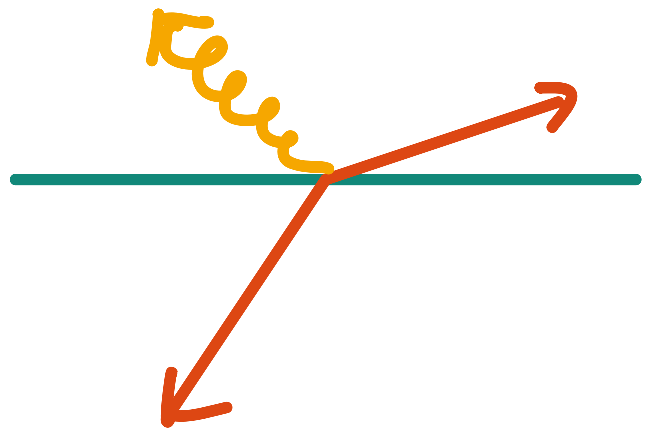
Organised by J. Forshaw, A. Maas, S. Plätzer and M. Sjödahl

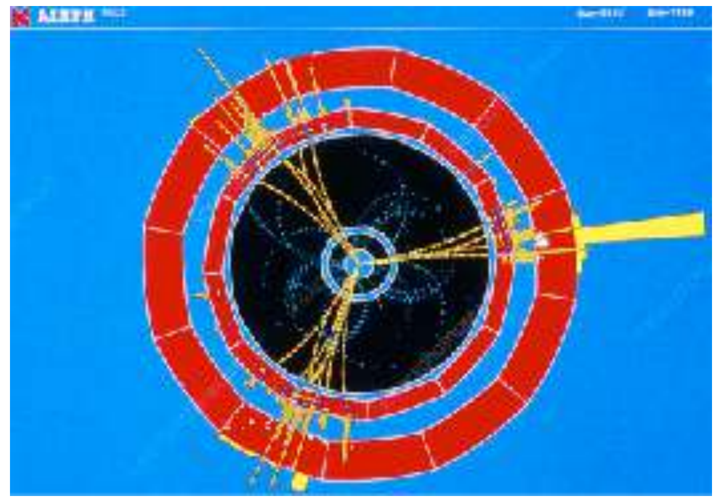
Thank you!

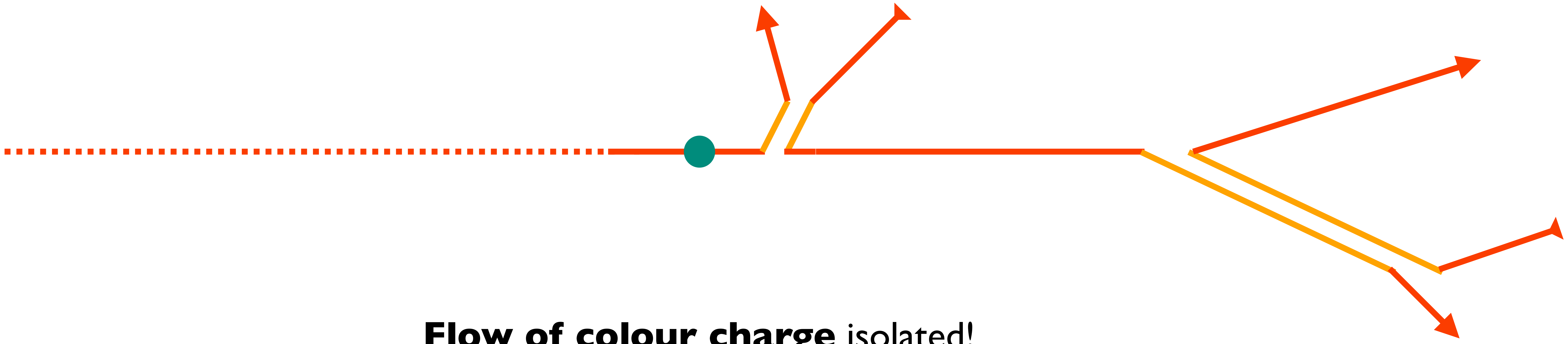
The formation of jets







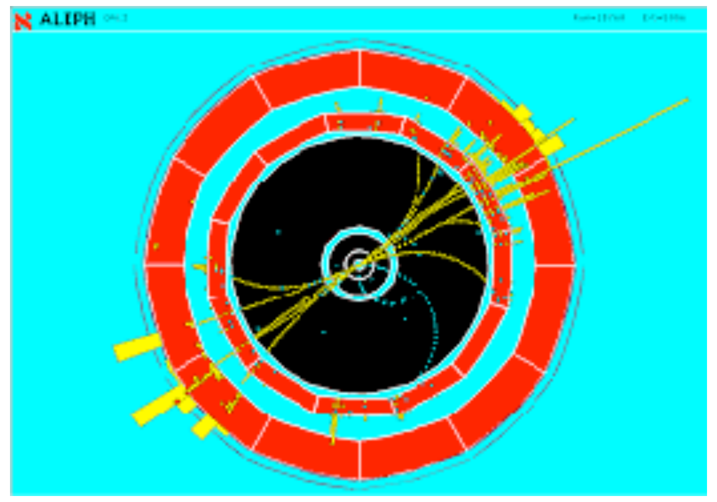


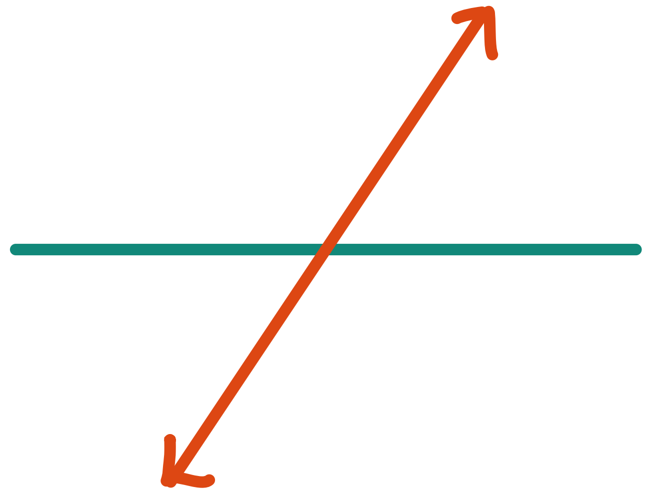


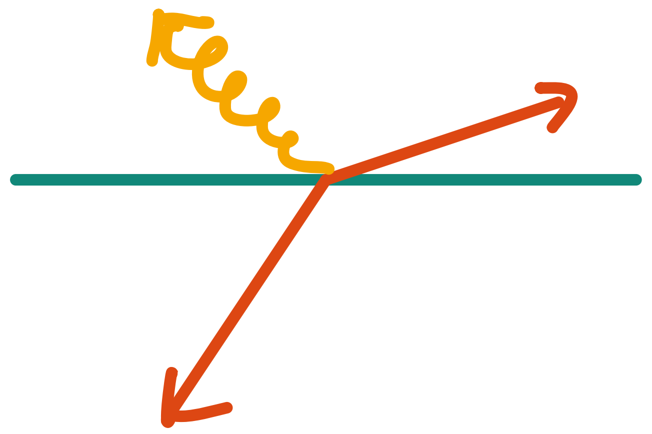
Flow of colour charge isolated!

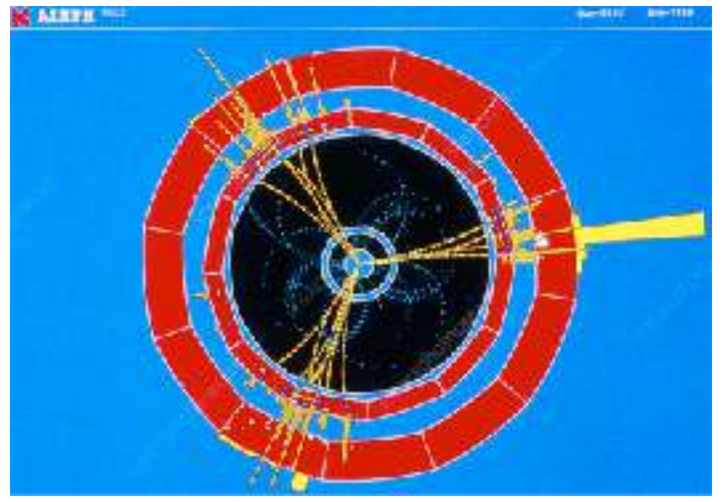
t ~ 1/Q

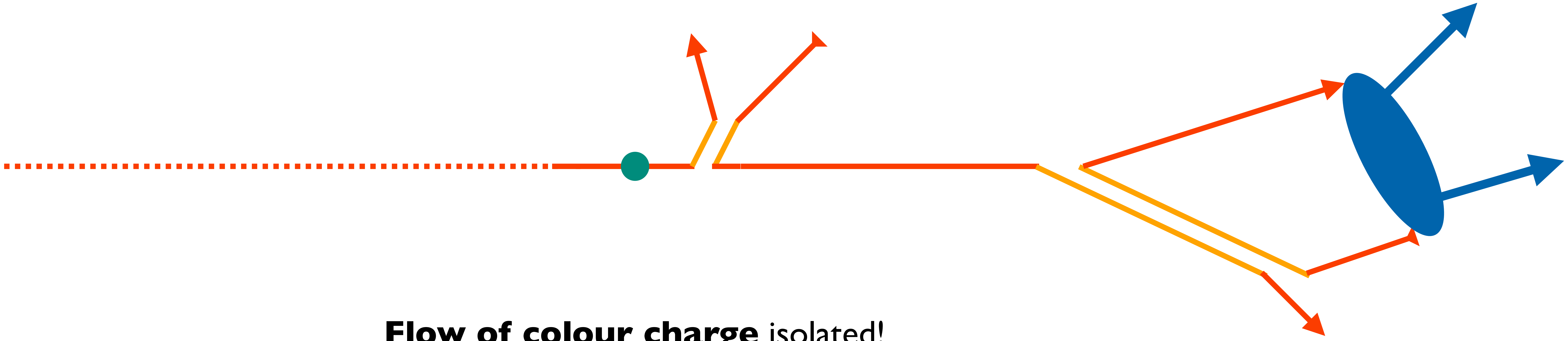
The formation of jets











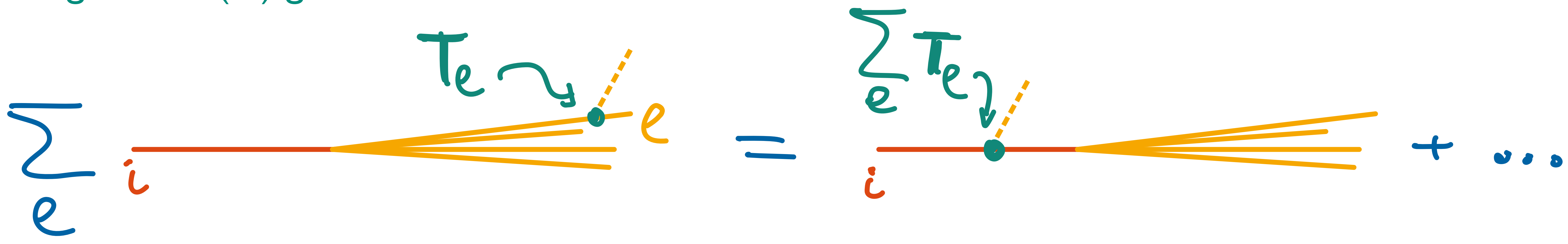
Flow of colour charge isolated!

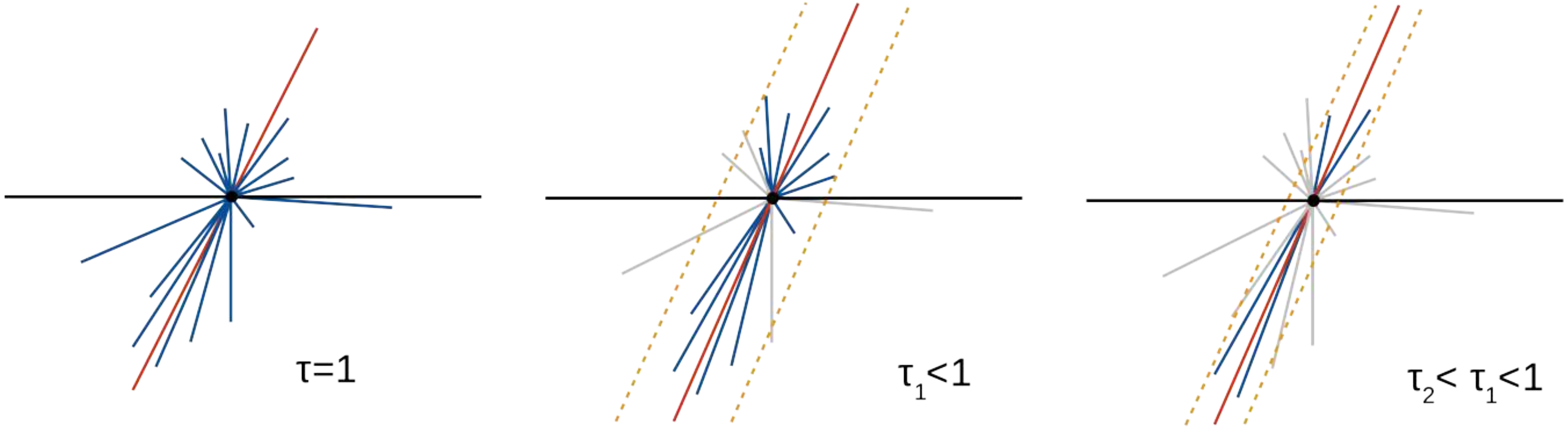
t ~ 1/Q

Jets in momentum space: coherence

Flow of colour charge is a statement at the level of scattering amplitudes.

Colour charge — SU(N) generator



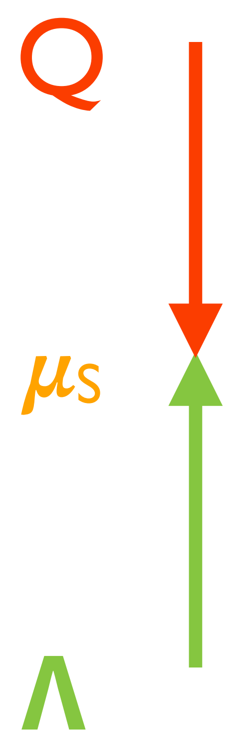


Redefinitions of "bare" operators

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Redefinitions of hard and soft factor **inverse** to each other:

$$\mathbf{Z}_n = \mathbf{X}_n^{-1} \qquad \mathbf{X}_n \mathbf{E}_n^{(s)} \circ \mathbf{E}_n^{(s)\dagger} \mathbf{X}_n^\dagger - \mathbf{F}_n^{(s)} \mathbf{Z}_{n-s} \circ \mathbf{Z}_{n-s}^\dagger \mathbf{F}_n^{(s)\dagger} - \sum_{t=1}^{s-1} \mathbf{F}_n^{(t)} \mathbf{E}_{n-t}^{(s-t)} \circ \mathbf{E}_{n-t}^{(s-t)\dagger} \mathbf{F}_n^{(t)\dagger} = 0$$

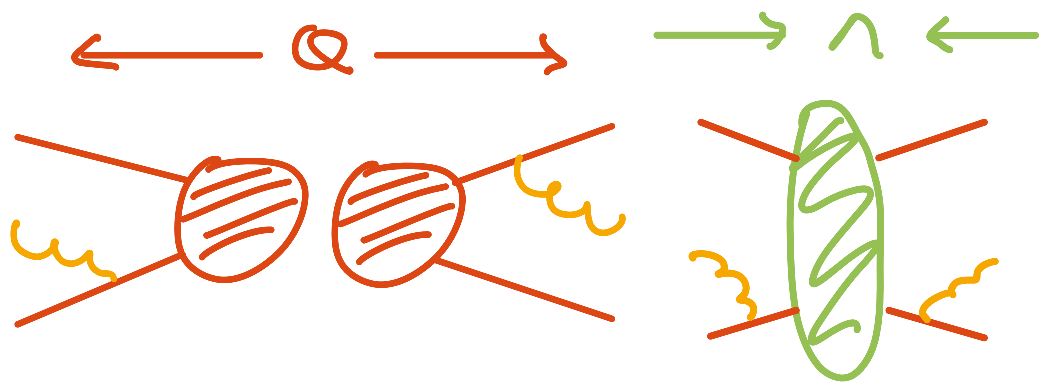


dressing of hard process ~ parton shower

soft evolution ~ hadronization model

$$\sum_n \int \alpha_S^n \ \mathrm{Tr}\left[(\mathbf{A}_n + \mathbf{\Delta}_n)\mathbf{S}_n\right] \mathrm{d}\phi(Q) \prod_{i=1}^n \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

α_s corrections to tower of logarithms in A —
truncation error of relation of Z factors

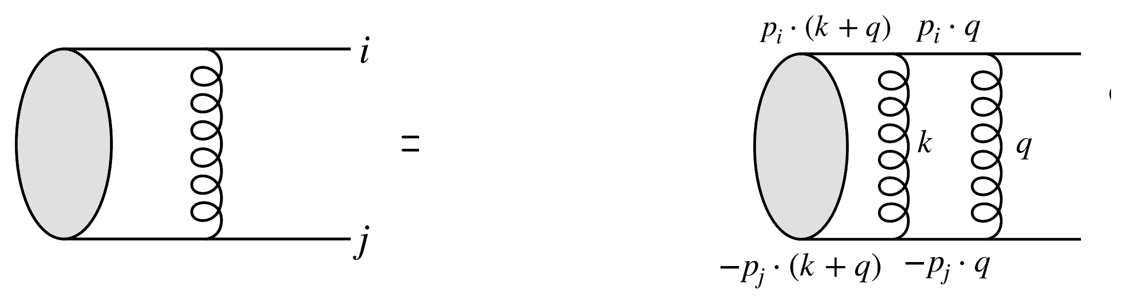


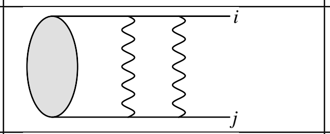
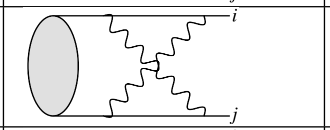
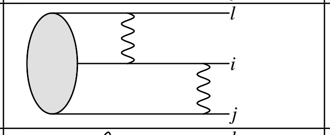
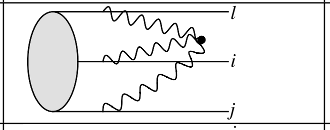
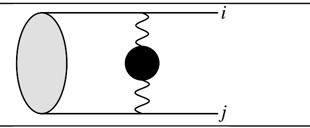
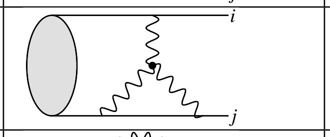
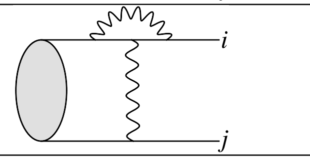
(Soft) factorisation of amplitudes

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Factorisation of virtual contributions

$$\mathbf{M}_n^{(l)} = \mathbf{V}^{(1)}\mathbf{M}_n^{(l-1)} + \mathbf{M}_n^{(l-1)}\mathbf{V}^{(1)\dagger} + \mathbf{V}^{(1)}\mathbf{M}_n^{(l-2)}\mathbf{V}^{(1)\dagger} + \mathbf{V}^{(2)}\mathbf{M}_n^{(l-2)} + \mathbf{M}_n^{(l-2)}\mathbf{V}^{(2)\dagger} + \ldots$$

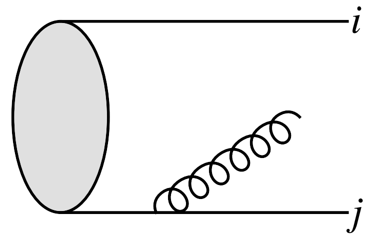


Coefficient	Diagram	Colour-factor
$\Omega_{ij}^{(2)}$		$(\mathbf{T}_i \cdot \mathbf{T}_j)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\tilde{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^a\mathbf{T}_i^b\mathbf{T}_j^b\mathbf{T}_j^a$
$\Omega_{ijl}^{(2)}$		$(\mathbf{T}_i \cdot \mathbf{T}_l)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}_{ijl}^{(2)}$		$if^{abc}\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_l^c$
$\Omega_{ij,\mathrm{self-en.}}^{(2)}$		$T_R(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\Omega_{ij,\mathrm{vertex-corr.}}^{(2)}$		$T_R(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^b\mathbf{T}_i^a\mathbf{T}_i^b\mathbf{T}_j^a$

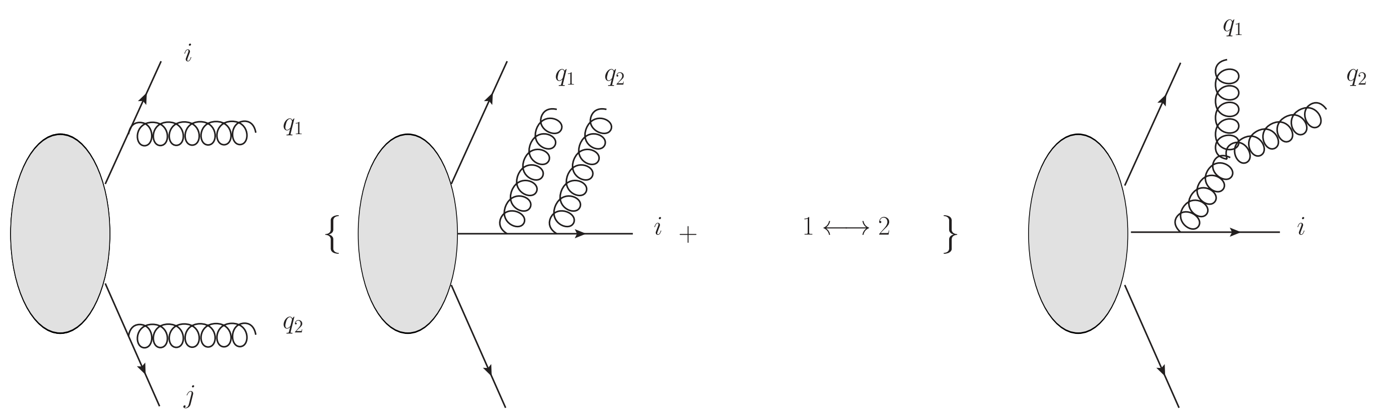
Handle virtual as phase-space type integrals to remove divergencies with subtractions.

[Plätzer, Ruffa — '21]

Factorisation of real contributions



$$\mathbf{M}_n^{(l)} = \mathbf{D}_n^{(1,0)}\mathbf{M}_{n-1}^{(l)}\mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,1)}\mathbf{M}_{n-1}^{(l-1)}\mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)}\mathbf{M}_{n-1}^{(l-1)}\mathbf{D}_n^{(1,1)\dagger} + \mathbf{D}_n^{(2,0)}\mathbf{M}_{n-2}^{(l)}\mathbf{D}_n^{(2,0)\dagger} + \ldots$$



$$\sum_{(a,b),(c,d)} \sum_{i,j,k,l=1}^{n} \omega_{ijkl}^{abcd}\, \mathrm{T}_i^{(a)}\mathrm{T}_j^{(b)} \circ \mathrm{T}_k^{(c),\dagger}\mathrm{T}_l^{(d),\dagger}$$

[Majcen — M.Sc. thesis 2022]
based on Catani & Grazzini

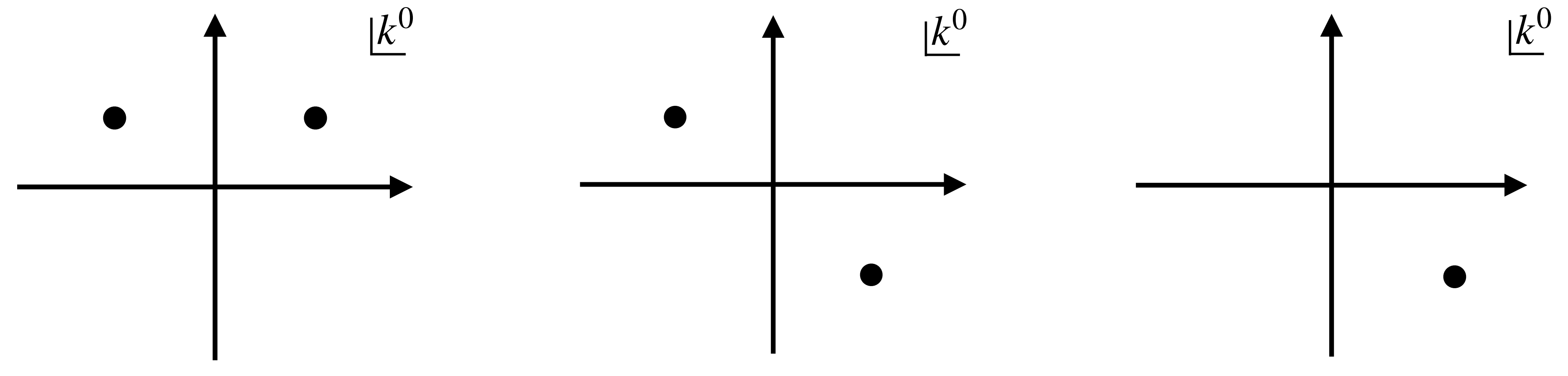
Cutting rules

[Plätzer, Ruffa — '21]

Algorithmic treatment of virtual corrections needed

$$\frac{1}{k^2 + i0(T \cdot k)^2} = \frac{1}{(k^0)^2 - \vec{k}^2 + 2i0(k^0)^2}$$

$(T^\mu) = (\sqrt{2}, \vec{0})$



Advanced Feynman

Feynman tree theorem:

$$\frac{1}{[q^2 - i0(T \cdot q)|T \cdot q|]} = \frac{1}{[q^2 + i0(T \cdot q)^2]} + 2\pi i \delta(q^2)\theta(T \cdot q)$$

Cutting rules

[Plätzer, Ruffa — '21]

Algorithmic treatment of virtual corrections needed

$$\frac{1}{k^2 + i0(T\cdot k)^2} = \frac{1}{(k^0)^2 - \vec{k}^2 + 2i0(k^0)^2}$$

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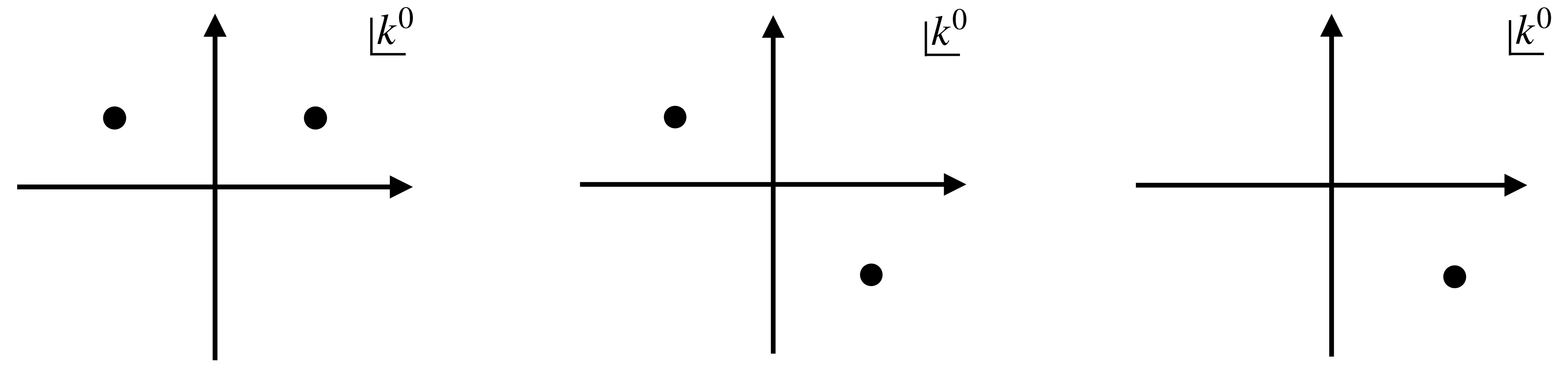
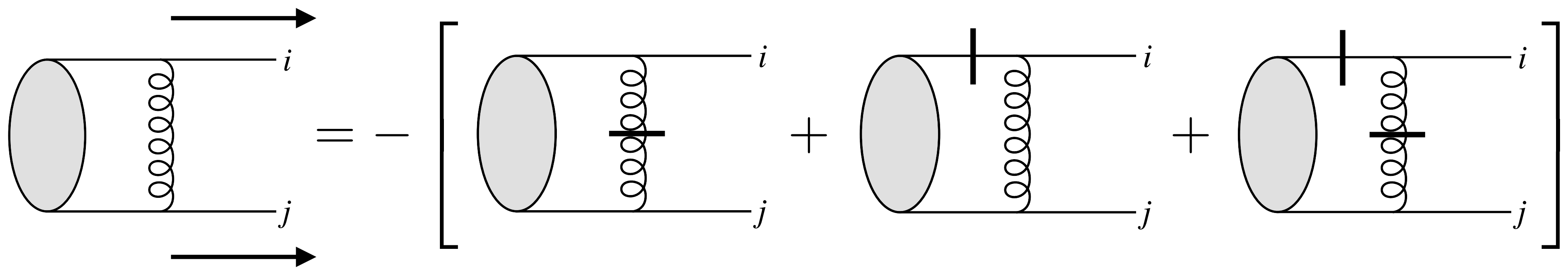


Advanced   Feynman   Eikonal

Feynman tree theorem:

$$\frac{1}{[q^2 - i0(T\cdot q)|T\cdot q|]} = \frac{1}{[q^2 + i0(T\cdot q)^2]} + 2\pi i \delta(q^2)\theta(T\cdot q)$$



$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int \frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\frac{n_i\cdot n_j}{n_i\cdot n\ n\cdot n_j} - i\pi\int\frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

Extend to Eikonal and higher-power propagators:

$$\frac{1}{2p_i\cdot k - i0(T\cdot p_i)^2} = \frac{1}{2p_i\cdot k + i0(T\cdot p_i)^2} + 2\pi i\ \delta(2p_i\cdot k)$$

$$\frac{1}{[q^2 - i0(T\cdot q)|T\cdot q|]^2} - \frac{1}{[q^2 + i0(T\cdot q)^2]^2} = -2i\pi\theta(T\cdot q)\delta'(q^2)$$

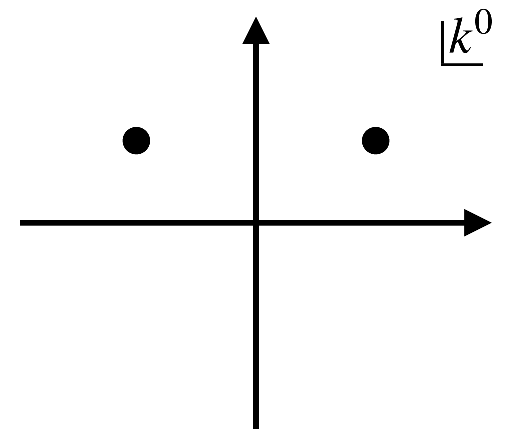
Cutting rules

[Plätzer, Ruffa — '21]

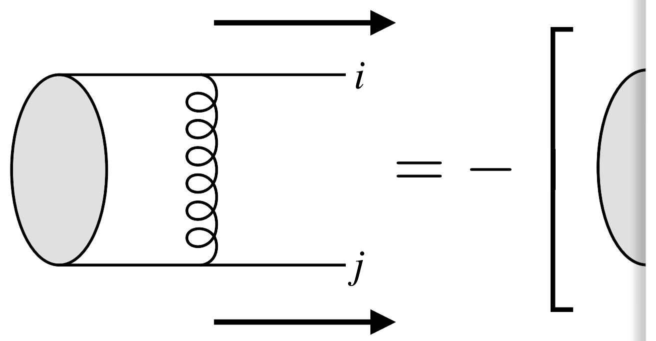
Algorithmic treatment of virtual corrections needed

$$\frac{1}{k^2 + i0(T\cdot k)^2} = \frac{1}{(k^0)^2 - \vec{k}^2 + 2i0(k^0)^2}$$

$(T^\mu) = (\sqrt{2}, \vec{0})$



Advanced



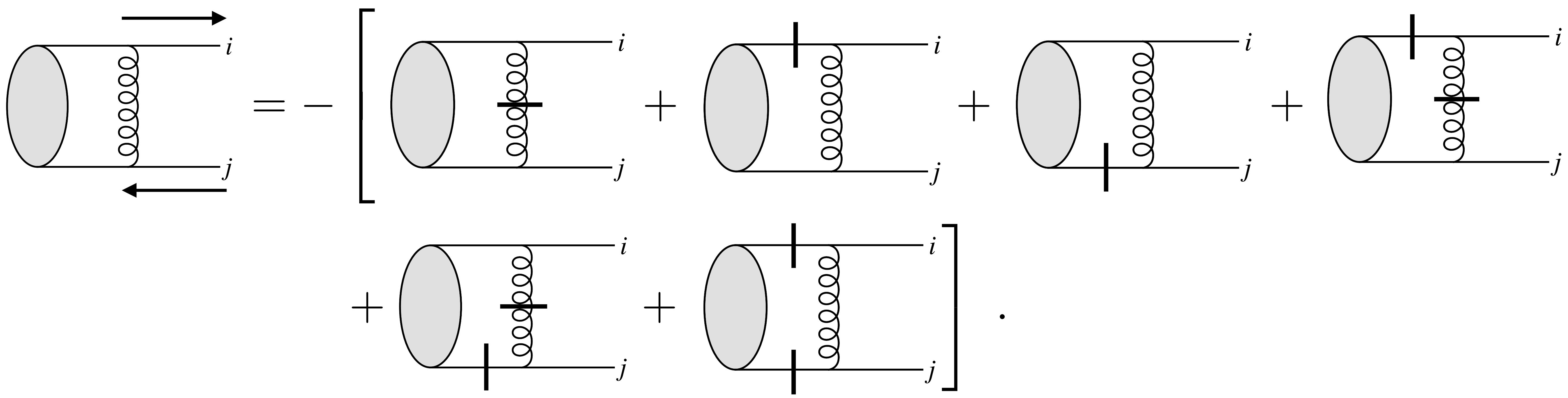
$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int \frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j} - i\pi\int\frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

$$T\cdot q)^2] + 2\pi i\delta(q^2)\theta(T\cdot q)$$

pagators:

$$(T\cdot p_i)^2 + 2\pi i\,\delta(2p_i\cdot k)$$

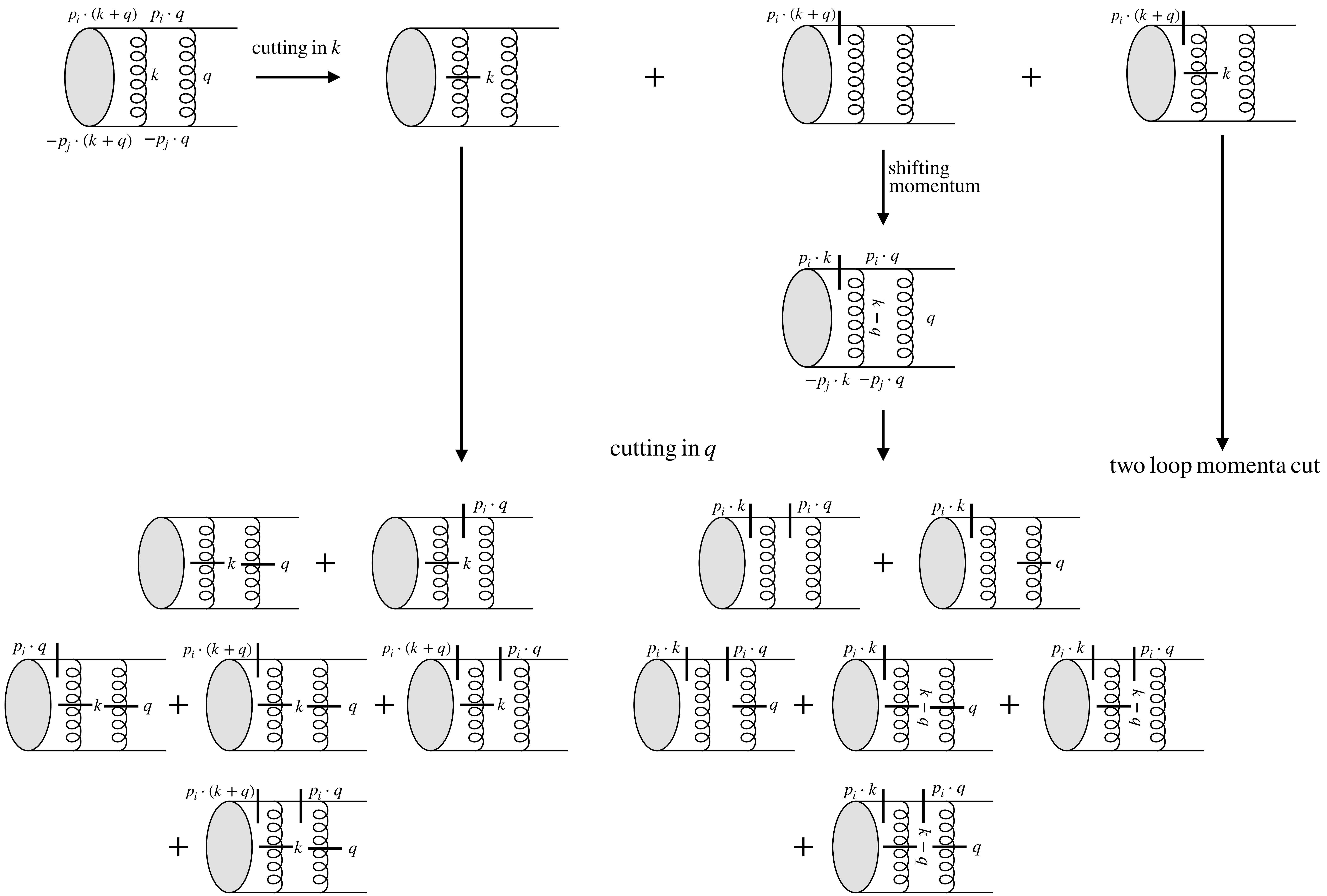
$$[q^2 - i0(T\cdot q)|T\cdot q|]^2 \quad [q^2 + i0(T\cdot q)^2]^2 = -2i\pi\theta(T\cdot q)\delta'(q^2)$$



$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int \frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j} - i\pi\int\frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

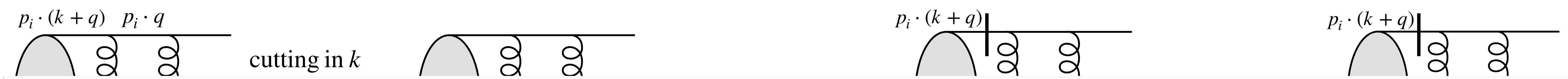
Cutting rules

[Plätzer, Ruffa — '21]

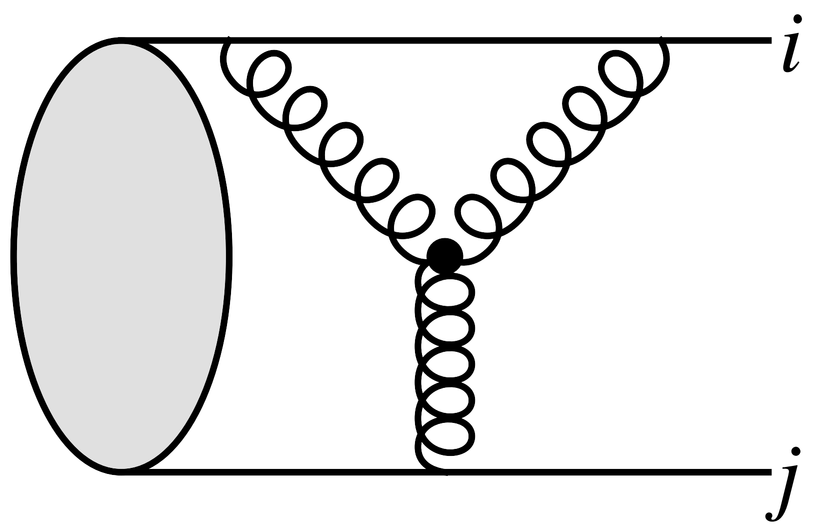


Cutting rules

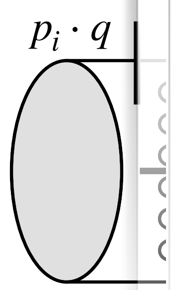
[Plätzer, Ruffa — '21]



$$\mu^{4\varepsilon}\int \frac{\bar{d}^d k}{i\pi^{d/2}}\frac{\bar{d}^d q}{i\pi^{d/2}}\frac{1}{[p_i\cdot k+i0][-p_j\cdot(k+q)+i0][k^2+i0][q^2+i0][(k+q)^2+i0]}$$



$$\mu^{4\varepsilon}\int \frac{\bar{d}^d k}{i\pi^{d/2}}\frac{\bar{d}^d q}{i\pi^{d/2}}\left\{\frac{(2\pi i)^2\tilde{\delta}(k)\tilde{\delta}(q)}{[p_i\cdot k+i0][(k+q)^2+i0][-p_j\cdot(k+q)+i0]}\right.$$
$$+\frac{(2\pi i)^2\tilde{\delta}(q)\delta(p_i\cdot k)}{[k^2+i0][(k+q)^2+i0][-p_j\cdot(k+q)+i0]}$$
$$+\frac{(2\pi i)^2\tilde{\delta}(k)\tilde{\delta}(q)}{[p_i\cdot k+i0][(q-k)^2+i0][-p_j\cdot q+i0]}$$
$$+\frac{(2\pi i)^2\tilde{\delta}(q)\delta(p_i\cdot k)}{[k^2+i0][(q-k)^2+i0][-p_j\cdot q+i0]}$$
$$-\frac{(2\pi i)^2\tilde{\delta}(q)\tilde{\delta}(k)}{[p_i\cdot(k-q)+i0][(k-q)^2+i0][-p_j\cdot k+i0]}$$
$$+\frac{(2\pi i)^3\tilde{\delta}(k)\tilde{\delta}(q)\tilde{\delta}(k+q)}{[p_i\cdot k+i0][-p_j\cdot(k+q)+i0]}$$
$$+\frac{(2\pi i)^3\tilde{\delta}(q)\tilde{\delta}(k+q)\delta(p_i\cdot k)}{[k^2+i0][-p_j\cdot(k+q)+i0]}+\frac{(2\pi i)^3\tilde{\delta}(q)\tilde{\delta}(k)\tilde{\delta}(q-k)}{[p_i\cdot k+i0][-p_j\cdot q+i0]}$$
$$\left.+\frac{(2\pi i)^3\tilde{\delta}(q-k)\tilde{\delta}(q)\delta(p_i\cdot k)}{[k^2+i0][-p_j\cdot q+i0]}\right\}.$$

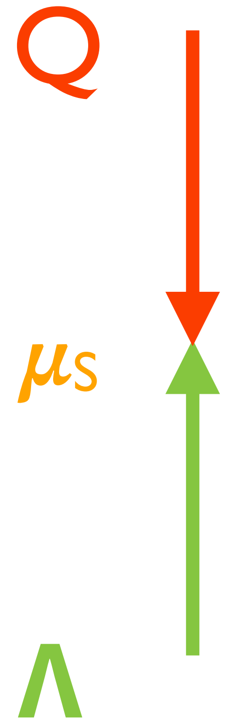


Eikonal coupling only to hard lines!

see also [Angeles, Forshaw, Seymour – '15]

Infrared subtractions

$$\sigma = \sum_{n,m} \int\int \mathrm{Tr}_n\left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$



Subtractions necessitate a resolution:
what is it we call ‘unresolved’?
Encompass all singular regions!

$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

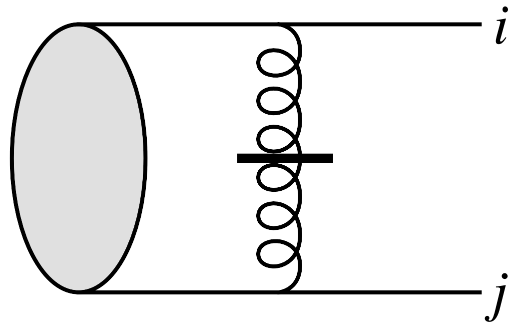
resolution function for (cut) loop momenta

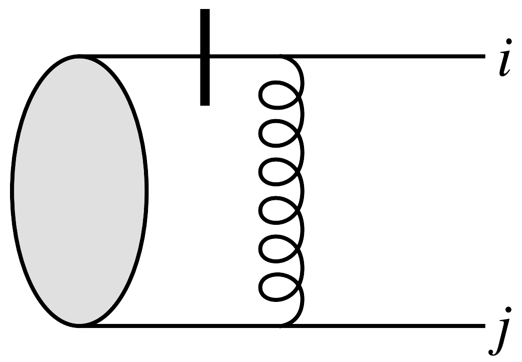
$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}]$$

$$\hat{\mathbf{V}}_n^{(l)}[\Xi_{n,l}] = \sum_\alpha \int \mathcal{I}_{n,\alpha}^{(l)}(p_1, ..., p_n; k_1, ..., k_l)\ \Xi_{n,l}^{(\alpha)} \prod_{i=1}^{l} \mu_R^{2\epsilon}[\mathrm{d}k_i]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

resolution function for real emission



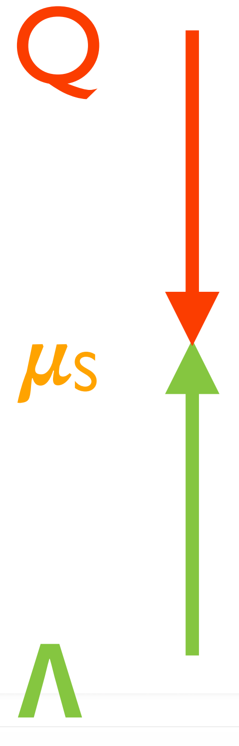


Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

Subtractions necessitate a resolution:
what is it we call 'unresolved'?
Encompass all singular regions!

$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$



resolution function for (cut) loop mome

$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)} [\Xi_{n,1}]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

resolution function for real emission

Continues to higher orders …

$$\mathbf{X}_n^{(2)} = \hat{\mathbf{V}}_n^{(2)} [\Xi_{n,2}] - \hat{\mathbf{V}}_n^{(1)} [\Xi_{n,1}] \hat{\mathbf{V}}_n^{(1)}$$

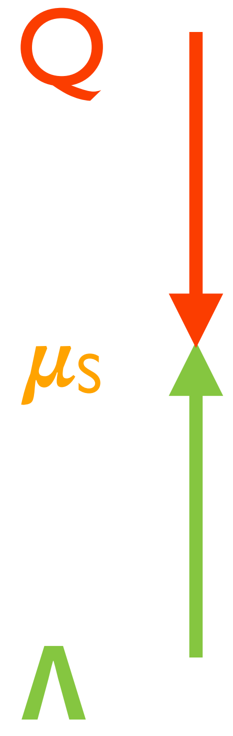
$$\begin{aligned} \mathbf{F}_n^{(1,1)} \circ \mathbf{F}_n^{(1,0)\dagger} = {} & \mathbf{D}_n^{(1,1)} [\Xi_{n-1,1}] \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} \\ & + \mathbf{D}_n^{(1,1)} [1 - \Xi_{n-1,1}] \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} + \mathbf{D}_n^{(1,1)} [\Xi_{n-1,1}] \circ \mathbf{D}_n^{(1,0)\dagger} (1 - \Theta_{n,1}) \\ & - \hat{\mathbf{V}}_n^{(1)} [\Xi_{n-1,1}] \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)} \hat{\mathbf{V}}_{n-1}^{(1)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1} \end{aligned}$$

$$\mathbf{F}_n^{(2,0)} \circ \mathbf{F}_n^{(2,0)\dagger} = \mathbf{D}_n^{(2,0)} \circ \mathbf{D}_n^{(2,0)\dagger} \Theta_{n,2} - \mathbf{D}_n^{(1,0)} \mathbf{D}_{n-1}^{(1,0)} \circ \mathbf{D}_{n-1}^{(1,0)\dagger} \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

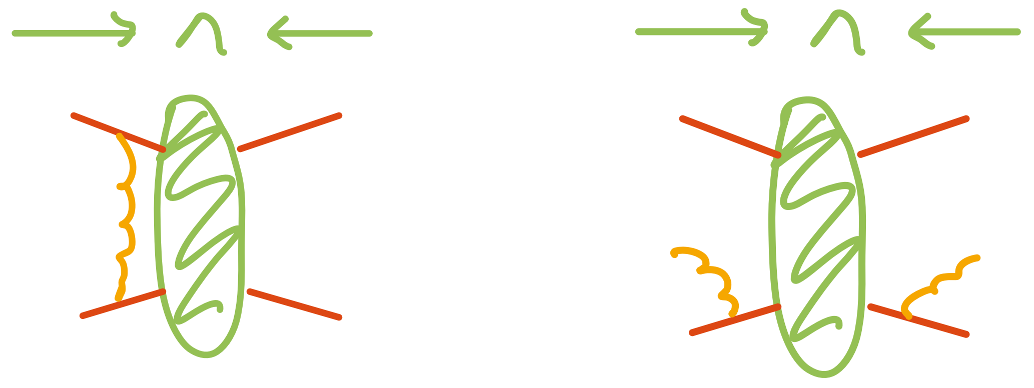
Redefinitions of “bare” operators

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.



$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$

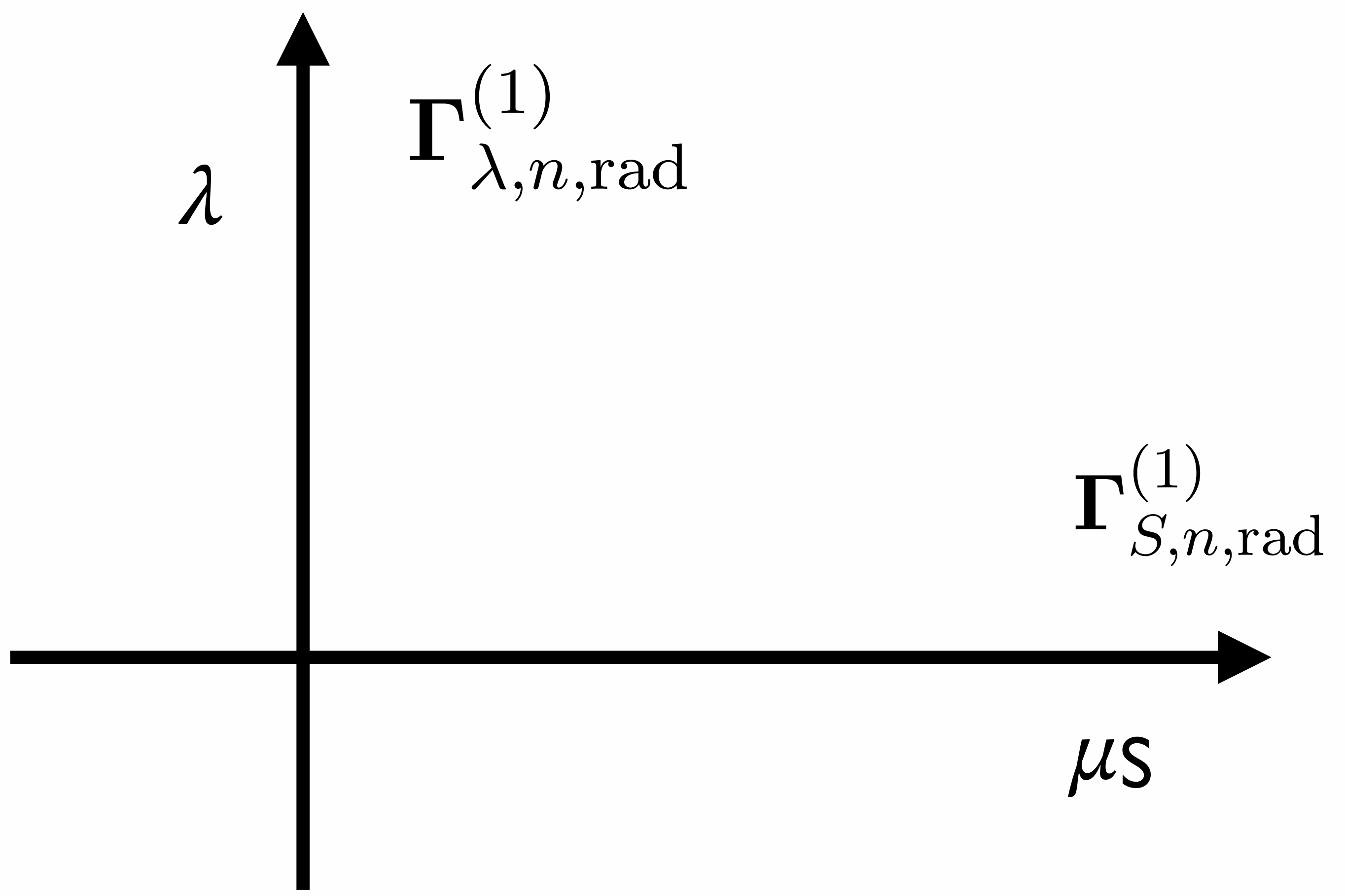


$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S}\mathbf{A}_n + \mathbf{A}_n\mathbf{\Gamma}^\dagger_{n,S} - \sum_{s\geq 1}\alpha_S^s \mathbf{R}^{(s)}_{S,n}\mathbf{A}_{n-s}\mathbf{R}^{(s)\dagger}_{S,n}$$

Redefinitions of "bare" operators

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

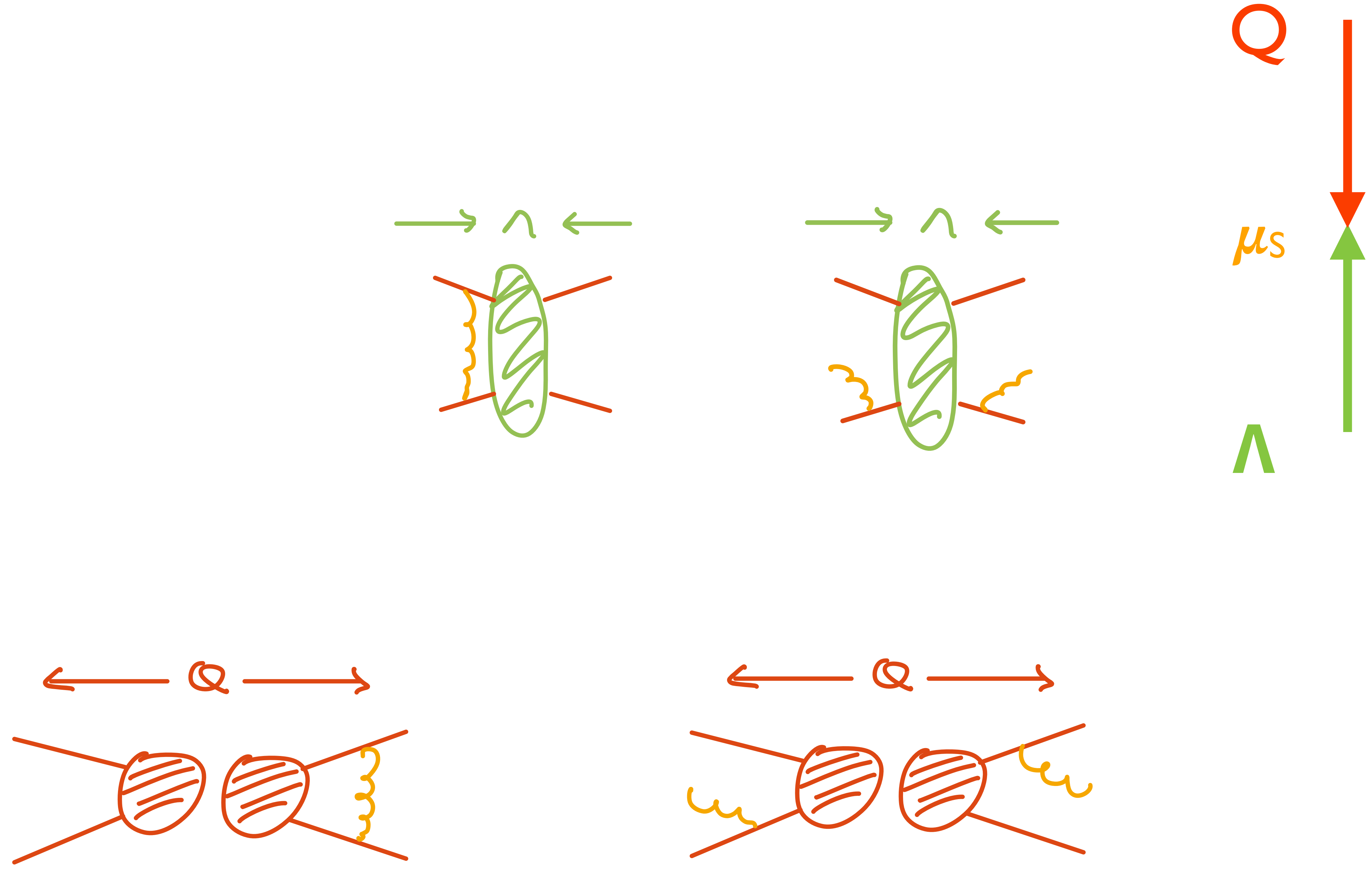
Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.



$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1}\theta(E_n - \mu_S)$$

$[\mathrm{d}p_i]\tilde{\delta}(p_i)$

1



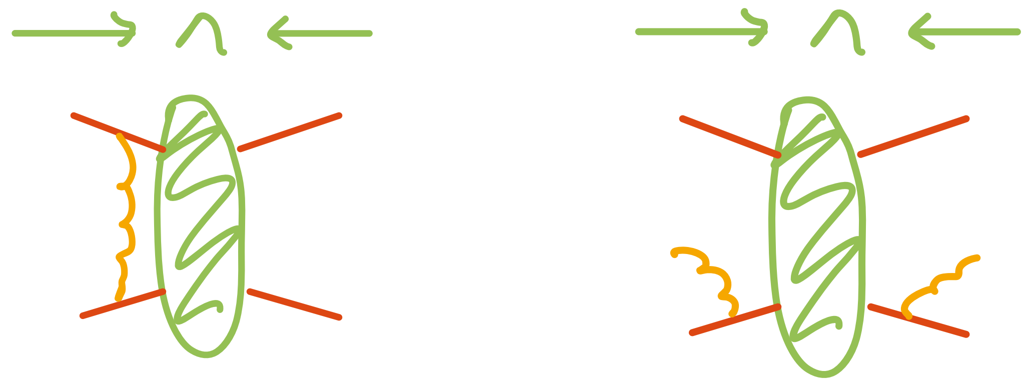
Constructing evolution algorithms

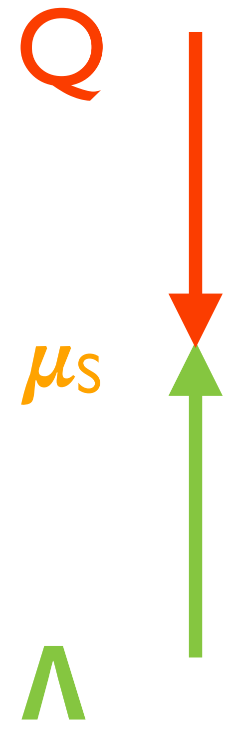
$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.

Subtract iterated contribution in ordered phase space.

$$\mathbf{R}_n^{(2,0)} \circ \mathbf{R}_n^{(2,0)\dagger} = \left(\hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger}\hat{\Theta}_{n,2} - \hat{\mathbf{D}}_n^{(0,1)}\hat{\mathbf{D}}_{n-1}^{(0,1)} \circ \hat{\mathbf{D}}_{n-1}^{(0,1)\dagger}\hat{\mathbf{D}}_n^{(0,1)\dagger}\hat{\Theta}_{n-1,1}\hat{\Theta}_{n,1}\right) \times\theta(E_{n-1} - \mu_S)\delta(E_n - \mu_s) + \hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger}\hat{\Theta}_{n,2}\theta(E_n - \mu_S)\delta(E_{n-1} - \mu_S)$$





Use full double gluon matrix element outside.

Similar consequences for virtual corrections.