Neutrinos as possible probes for quantum gravity

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Study motivation

1. **Testing the Planck scale structure of spacetime is essential to formulate a complete QG theory**
   - **Necessity of testing the universality of the supposed QG corrections**
2. **Advantages of using astroparticles in a multimessenger approach:**
   - **High energies** (useful for testing the Planck scale)
   - **Long propagation path** (allowing accumulation of tiny QG perturbations during propagation)
3. **Neutrinos can be ideal candidates**
   - **Possibility of testing different combinations of energies and baselines**
   - **Neutrino weak interactions: advantages in pointing to the sources**

Quantum Gravity (QG) testable scenarios

Kinematical symmetry group modification scenario	Lorentz Invariance Violation scenario CPT symmetry testable
DSR (Doubly Special Relativity) HMSR (Homogeneous Modified Special Relativity)	SME (Standard Model Extension)

The main phenomenological effects introduced by the QG models pertain the modification of the dispersion relations:

- **Universal modifications scenario - differences in the time of flight of astrophysics neutrinos:**
  QG perturbations may affect the in-vacuum dispersion relations, leading to an energy dependence of the particle velocity. This effect can be detected in
  - **GRB candidate accelerated neutrinos**
  - **Supernova neutrinos**
- **Non universal modifications scenario – differences in the envisaged oscillation pattern of atmospheric neutrinos:**
  Introduction of mass eigenstate-dependent QG perturbations may alter the oscillation probability.
- **CPT-odd scenario –** challenges in distinguish QG CPT-odd perturbations from Non Standard Interactions.

Doubly Special Relativity (DSR)

Universal and non universal QG scenario testable in the context of modified symmetry group

Real geometry is defined in momentum space → **Spacetime geometry (coordinate space) is a projection of the real one**

Modified momenta composition rule → **Modified momentum space geometry**

$$p_1 \oplus p_2 = p_1 + p_2 + F(p_1, p_2)$$

$$\frac{\partial}{\partial p_{(1)\mu}} \frac{\partial}{\partial p_{(2)\nu}} (p_1 \oplus p_2)_\alpha = -\Gamma^{\mu\nu}_\alpha$$

- The underlying symmetry group is the **κ-Poincarè**, associated to a **Hopf algebra** structure
- The resulting **momentum space** has **a non trivial geometric structure**
- The **Lorentz invariance is promoted to diffeomorphism invariance**

In this context it is **possible** to consider **corrections that are particle species depending**

The different Hopf algebra associated to the different particles species are related to a support algebra via a projection

Homogeneously Modified Special Relativity (HMSR)

Bibliography [3]

Universal and non universal QG scenario testable in the context of modified symmetry group

Starting point to introduce LIV – kinematic modifications → **Modified Dispersion Relations (MDR)**

Homogeneity of the perturbation function → **Geometrical origin of the MDR**

Finsler Geometry

$$f_{(i)}(p_{(i)}) = \sum_{j=2}^{n} \alpha_j \frac{\left|\vec{p}_{(i)}\right|^j}{E_{(i)}{}^j}$$

→

$$F_{(i)}{}^2(p_{(i)}) = E_{(i)}{}^2 - \left|\vec{p}_{(i)}\right|^2 \left(1 - f_{(i)}(p_{(i)})\right) = m_{(i)}{}^2$$

Modifications depending on (i) particle species

Lorentz invariance is modified
the Lorentz group is amended in order to preserve the geometric structure

Minimal extension of the Standard Model of particle physics

High energy limit of the model is compatible with the Coleman-Glashow modified special relativity

Standard Model Extension (SME)

Universal, non universal QG scenario and CPT violation testable in the context of Lorentz invariance violation

Interactions in string theories could lead to the breaking of Lorentz symmetry

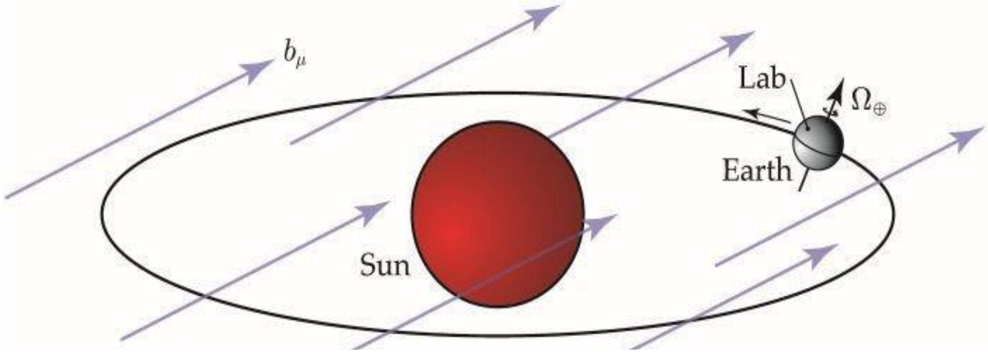
SME includes operators that both break and preserve CPT symmetry

$$L_{SME} = L_{SM} + L_{LIV}$$

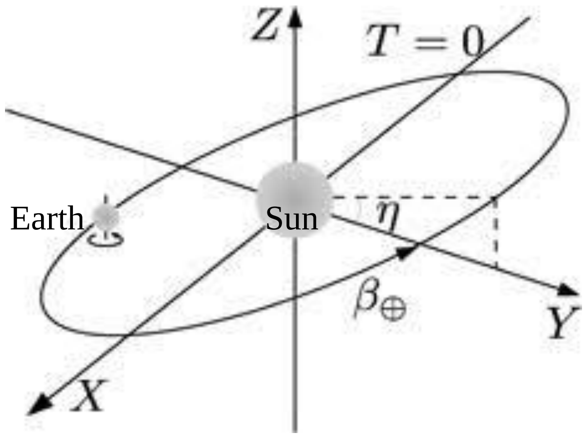
→ LIV introduced perturbation in the Standard Model Lagrangian

Standard model extension preserves $SU(3)\times SU(2)\times U(1)$ internal symmetry

Lorentz Invariance is broken → Isotropy not preserved → Necessity to introduce a privileged reference frame



Background fixed tensorial fields



Neutrinos and classical gravity

The curvature of spacetime predicted by General Relativity can influene Neutrino propagation

Oscillation probability as a function of propagation length

$$P(\nu_\alpha \longrightarrow \nu_\beta) = \left|\langle \nu_\beta | \nu_\alpha(L) \rangle\right|^2$$

Evolution of a state as a function of the background spacetime metric

$$|\nu_\alpha(L)\rangle = U_{\alpha k} exp\left[-i \int_{r_A}^{r_B} \left(\eta_{\mu\nu} + \frac{1}{2} h_{\mu\nu}\right) p_k^\mu dx^\nu\right] |\nu_k\rangle$$

General Relativity foreseen modification of the oscillation phase

$$\phi_{ij} = \phi_{ij}^0 + \phi_{ij}^{GR}$$

Oscillation phase perturbation introduced by the gravity interaction in a Schwarzschild spacetime

$$\phi_{ij}^{GR} = \frac{\Delta m_{ij}^2}{2E}\left[\frac{G \cdot M}{r_B} - \frac{G \cdot M}{L} log\left(\frac{r_B}{r_A}\right)\right]$$

Predicted QG perturbations on neutrino propagation must be disentangled from the modification induced by the interaction with classical gravity

Neutrino oscillations – non universal scenario

Bibliography [6, 8, 9, 10]

DSR, HMSR and SME theories can foresee particle species depending QG perturbations

Introducing mass eigenstates depending QG perturbations **the oscillation phenomenon can be modified – investigation sector: atmospheric neutrinos**

Modified oscillation phase

DSR scenario: $$\phi_{ij} = \left(\frac{\Delta m^2_{ij}}{2E} - \delta_{ij}E^2\right)L$$

Modified oscillation phase

HMSR scenario: $$\phi_{ij} = \left(\frac{\Delta m^2_{ij}}{2E} - \delta_{ij}E\right)L$$

Integrated probability $$P_{\nu_\mu \to \nu_e} = \frac{\int_{E_{min}}^{E_{max}} \phi_\nu(E) P_{\nu_\mu \to \nu_e}(E) dE}{\int_{E_{min}}^{E_{max}} \phi_\nu(E) dE}$$

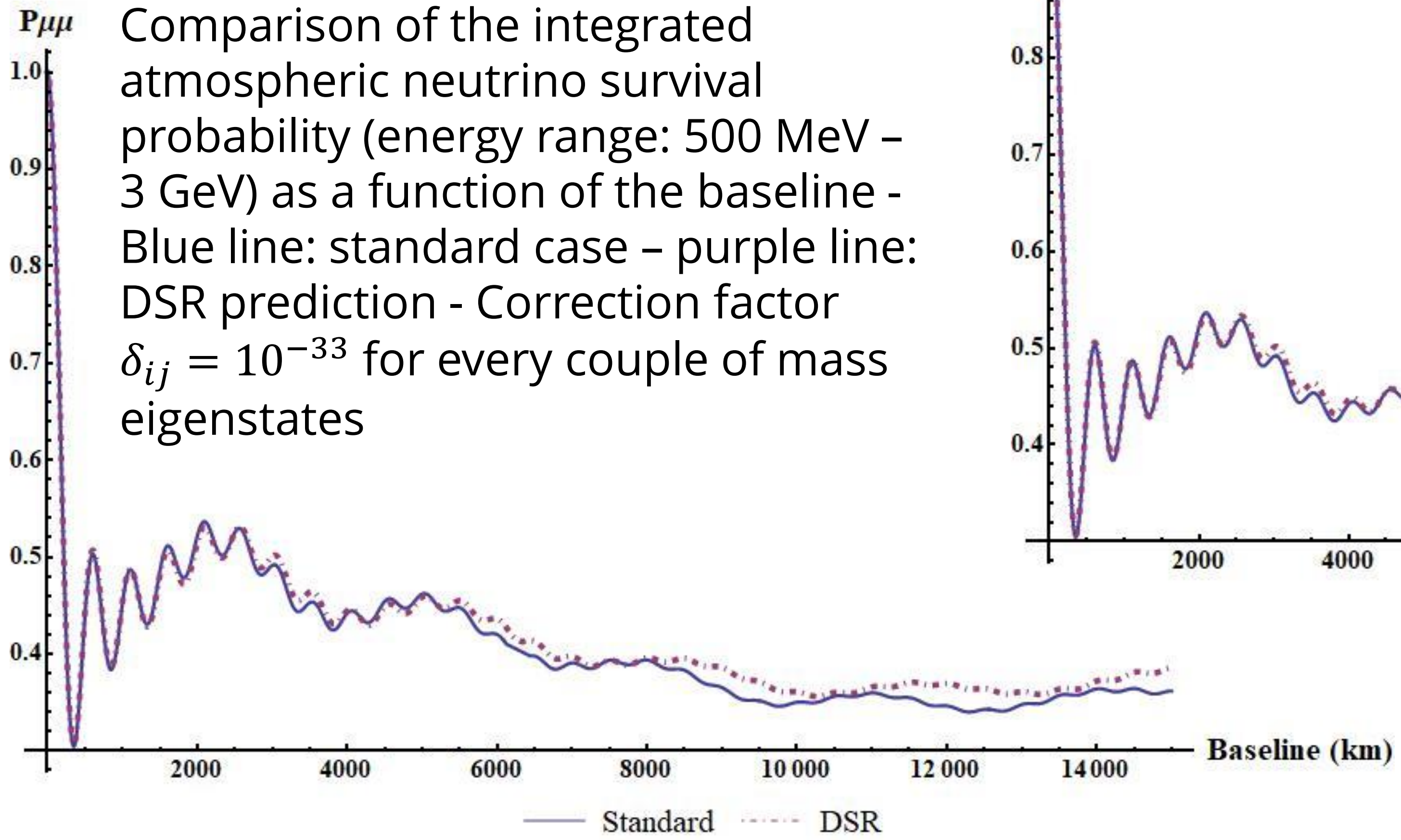
This effect can be detected in the atmospheric sector –
Modification of the expected events for different flavor neutrino beams

Neutrino oscillations – non universal scenario

Bibliography [6, 8, 9, 10]

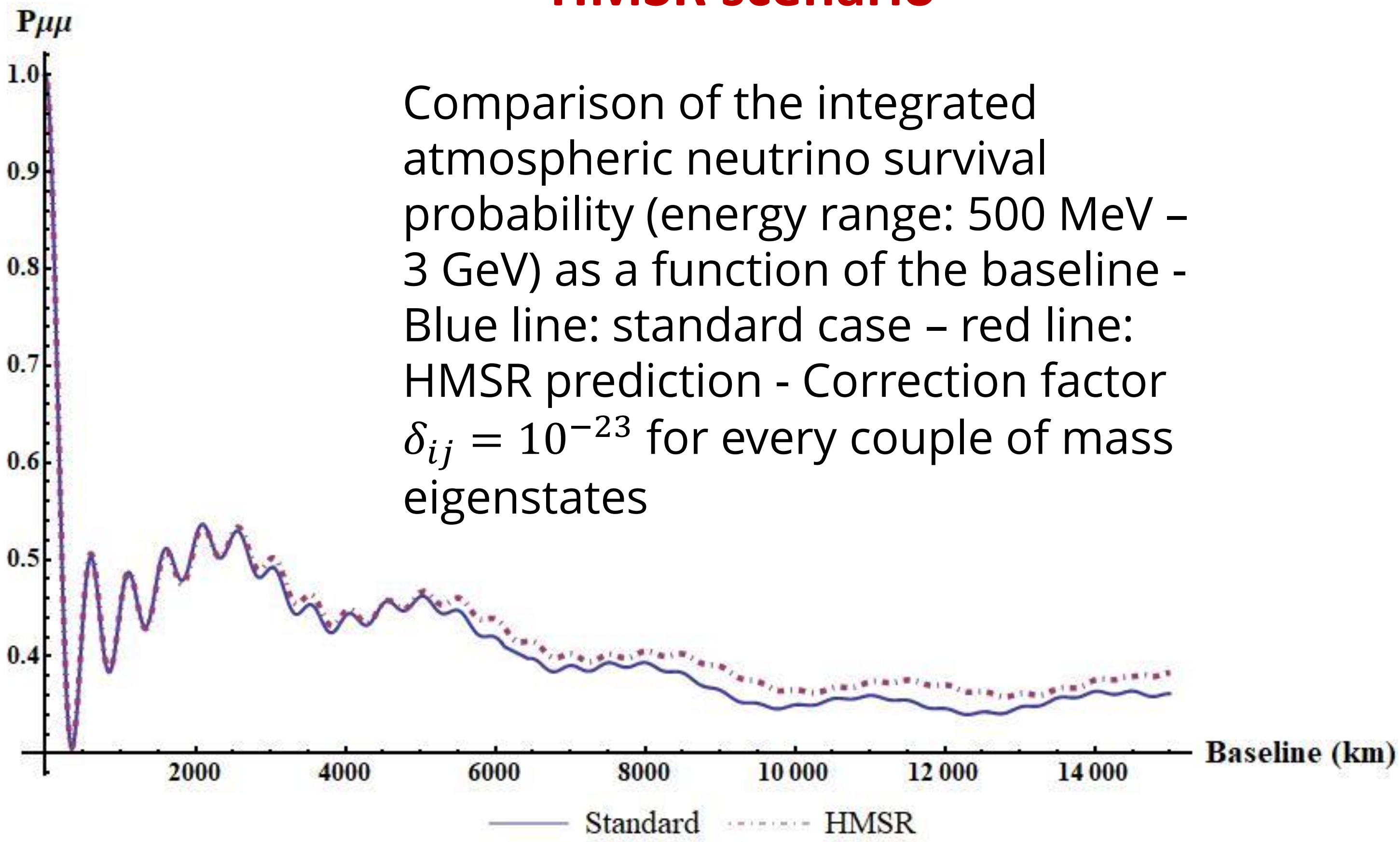
DSR scenario

Comparison of the integrated atmospheric neutrino survival probability (energy range: 500 MeV – 3 GeV) as a function of the baseline - Blue line: standard case – purple line: DSR prediction - Correction factor $\delta_{ij} = 10^{-33}$ for every couple of mass eigenstates



HMSR scenario

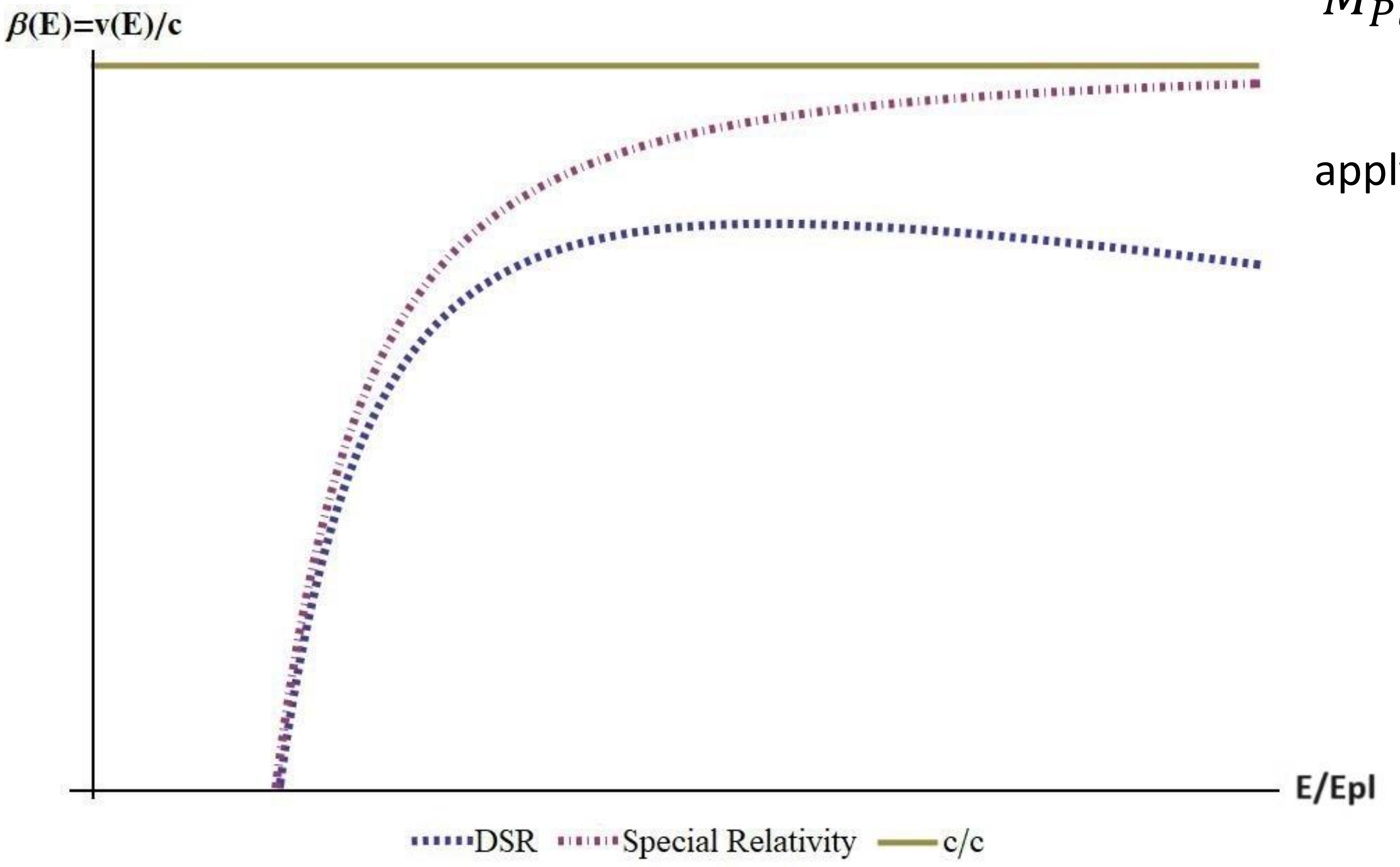
Comparison of the integrated atmospheric neutrino survival probability (energy range: 500 MeV – 3 GeV) as a function of the baseline - Blue line: standard case – red line: HMSR prediction - Correction factor $\delta_{ij} = 10^{-23}$ for every couple of mass eigenstates



Modified Dispersion Relations (MDR)

The main phenomenological effects introduced by the QG models pertain the modification of the dispersion relations:

DSR scenario MDR $E^2 - e^{\lambda E} p^2 \approx m^2 \Longrightarrow E^2 - \frac{\delta}{M_{Pl}} E p^2 \approx m^2 \Longrightarrow E \approx \sqrt{p^2 + \frac{\delta\, p}{M_{Pl}} + m^2}$



applying the Hamilton's equation the velocity is computed:

$$v(E) = \frac{\partial E}{\partial p} \approx \frac{p}{\sqrt{p^2 + \frac{\delta\, p^3}{M_{Pl}} + m^2}}$$

Comparison of the Lorentz factor β = v/c as a function of the energy with the factor β = c/c (green continuous line):
DSR (blue dashed line) vs Special Relativity (red dash dotted line).

Time of flight – universal scenario

Bibliography [6, 7]

One of the main channels for the QG signatures detection involves measuring the **time of flight of astrophysics neutrinos.**

QG perturbations can affect the in-vacuum dispersion relations introducing an energy dependence of the particle velocities.

QG contribution to the flight time as a function of the particle energy:

$$\Delta t = lE \int_0^z \frac{1+\varsigma}{H_0\sqrt{\Omega_\Lambda + \Omega_m (1+\varsigma)^3}} d\varsigma$$

H_0, Ω_Λ and Ω_m denote respectively the Hubble and the cosmological constants and the matter fraction, z is the redshift parameter.

The time delay is proportional to the particle energy.

A dependence on the particle energy of the time of flight can be detected in the energy spectrum of

- **GRB candidate accelerated neutrinos**
- **Supernova spectrum**

CPT scenario

Bibliography [5, 11, 12]

In the SME framework CPT-odd Lorentz violations can be tested

In the hamiltonian picture: $H_{SME} = H_0 + H_{even}^{CPT} + H_{odd}^{CPT}$

The CPT-odd Hamiltonian has the same form of the **Non Standard Interactions** effective Hamiltonian:

$$H_{odd}^{CPT} = 2\sqrt{2}G_F \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu} & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau} & \varepsilon_{\mu\tau} & \varepsilon_{\tau\tau} \end{pmatrix} \sim H_{NSI}$$

The perturbations induced by the supposed NSI can mimic the effect of CPT-odd QG Lorentz violating contributions

Necessity of complementary studies to disentangle the different contributions

Conclusions

1. **Neutrinos can be ideal probes for Planck scale physics in a multimessenger approach in the context of astroparticle physics.**

   - **Testing the universality of modifications of the oscillation pattern induced by QG (atmospheric neutrinos).**

   - **Testing QG perturbations of dispersion relations, the modification of the time of flight and the resulting spectrum of candidate GRB and supernova neutrinos.**

   - **Test of the CPT symmetry.**

2. **Challenges remain in distinguishing QG-induced perturbations from those pertaining to other causes, such as NSI and classical gravity interaction (General Relativity).**

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Thank you very much for your attention!!!!

Backup slides

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3. **Neutrinos and classical gravity**
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   - Astroparticle time of flight (universal QG perturbation scenario)
   - CPT symmetry violation scenario
5. **Conclusions**

Doubly Special Relativity (DSR)

Bibliography [1, 2, 6]

Modified composition rule of momenta

$$(p \oplus q)_0 = p_0 + q_0 \qquad\qquad (p \oplus q)_j = p_j + e^{lp_0} q_j$$

Modified momentum space geometry – non trivial with torsion

$$\Gamma^{\mu\nu}_{\alpha} = -\frac{\partial}{\partial p_\mu}\frac{\partial}{\partial q_\nu}(p \oplus q)_\alpha\Big|_{p=q=0}. \qquad -T^{\mu\nu}_{\alpha} = \frac{\partial}{\partial p_\mu}\frac{\partial}{\partial q_\nu}(p \oplus q - q \oplus p)_\alpha\Big|_{p=q=0}.$$

$$R^{\mu\nu\alpha}_{\beta} = \frac{\partial}{\partial p_{[\mu}}\frac{\partial}{\partial q_{\nu]}}\frac{\partial}{\partial k_\alpha}((p \oplus q) \oplus k - p \oplus (q \oplus k))_\beta\Big|_{p=q=k=0} = 0.$$

K-Poincarè generators algebra

$$[P_\mu, P_\nu] = 0, \qquad [R_i, R_j] = i\,\epsilon_{ijk}R_k, \qquad [N_i, N_j] = -i\,\epsilon_{ijk}R_k,$$

$$[R_i, P_0] = 0, \qquad [R_i, P_j] = i\,\epsilon_{ijk}P_k, \qquad [R_i, N_j] = -i\,\epsilon_{ijk}N_k,$$

$$[N_i, P_0] = i\,P_i, \quad [N_i, P_j] = i\,\delta_{ij}\left(\frac{1}{2l}\left(1 - e^{-2P_0 l}\right) + \frac{l}{2}|\vec{P}|^2\right) - i\,l P_i P_j$$

Doubly Special Relativity (DSR)

Bibliography [6]

Projection from a starting algebra to a support one

$$\phi_j : H_j \to H'.$$

$$\phi(P_\mu) = \frac{l'}{l} P'_\mu, \quad \phi(R_i) = R'_i, \quad \phi(N_i) = N'_i$$

Resulting Hopf algebra associated to interaction of 2 particles

$$\Delta' : H' \otimes H' = \phi_1(H_1) \otimes \phi_2(H_2)$$

$$H' \xrightarrow{\Delta'} H' \otimes H' \xrightarrow{\phi_1^{-1} \otimes \phi_2^{-1}} H_1 \otimes H_2.$$

The projection respects the κ-Poincarè commutation rules

$$[\phi(R_i), \phi(P_j)] = \frac{l'}{l}[R'_i, P'_j] = i\,\epsilon_{ijk}\frac{l'}{l}P'_k = \phi([R_i, P_j]),$$

$$[\phi(N_i), \phi(P_0)] = i\frac{l'}{l}P'_i = \frac{l'}{l}[N'_i, P'_0] = \phi([N_i, P_0])$$

$$[\phi(N_i), \phi(P_j)] = i\,\delta_{ij}\left(\frac{1 - e^{-2l'P'_0}}{2l} + l\left(\frac{l'}{l}\right)^2 |\vec{P}'|^2\right) - i\,l\left(\frac{l'}{l}\right)^2 P'_i P'_j =$$

$$= \frac{l'}{l}\left(i\,\delta_{ij}\left(\frac{1 - e^{-2l'P'_0}}{2l'} + l'|\vec{P}'|^2\right) - i\,l' P'_i P'_j\right) = \phi([N_i, P_j]).$$

$$[\phi(P_\mu), \phi(P_\nu)] = 0 = \phi([P_\mu, P_\nu]),$$

$$[\phi(R_i), \phi(R_j)] = i\,\epsilon_{ijk}R'_k = \phi([R_i, R_j]),$$

$$[\phi(N_i), \phi(N_j)] = -i\,\epsilon_{ijk}R'_k = \phi([N_i, N_j]),$$

$$[\phi(R_i), \phi(N_j)] = i\,\epsilon_{ijk}N'_k = \phi([N_i, P_j]),$$

$$[\phi(R_i), \phi(P_0)] = 0 = \phi([R_i, P_0]).$$

Doubly Special Relativity (DSR)

Projection from a starting algebra to a support one

$$\phi_j : H_j \to H'.$$

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Resulting Hopf algebra associated to interaction of 2 particles

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$$H' \xrightarrow{\Delta'} H' \otimes H' \xrightarrow{\phi_1^{-1} \otimes \phi_2^{-1}} H_1 \otimes H_2.$$

Modified composition rule of momenta for different particle species

$$p_0^{(1)} \oplus'_{l_1\, l_2} q_0^{(2)} = \frac{l'}{l_1} p_0^{(1)} + \frac{l'}{l_2} q_0^{(2)},$$

$$p_i^{(1)} \oplus'_{l_1\, l_2} q_i^{(2)} = \frac{l'}{l_1} p_i^{(1)} + \frac{l'}{l_2} e^{-l p_0^{(1)}} q_i^{(2)}.$$

Homogeneously Modified Special Relativity (HMSR)

Bibliography [3, 6]

Geometry introduced from the Modified Dispersion Relation

$$F(\mathbf{p}) = E^2 - \left(1 - f\left(\frac{|\vec{p}|}{E}\right)\right) = m^2. \quad \longrightarrow \quad g^{\mu\nu}(p) = \begin{pmatrix} 1 & 0 \\ 0 & -\mathbb{I}_{3\times 3}(1 - f(|\vec{p}|/E)) \end{pmatrix}.$$

$$g_{\mu\nu}(x) = e^{\alpha}_{\mu}(p)\,\eta_{\alpha\beta}\,e^{\beta}_{\nu}(p) = \begin{pmatrix} 1 & 0 \\ 0 & -\mathbb{I}_{3\times 3}(1 + f(|\vec{p}|/E)) \end{pmatrix}.$$

Using the vierbein as projectors for coordinates, it is possible to obtain the non-commutativity of the spacetime

$$\tilde{x}^{\mu} = e^{\mu}_{\nu}(p)x^{\nu} \text{ and } \tilde{p}_{\mu} = e^{\nu}_{\mu}(p)p_{\nu}. \quad \longrightarrow$$

$$[\tilde{x}^{\mu}, \tilde{x}^{\nu}] = \theta^{\mu\nu}$$

$$[\tilde{p}_{\mu}, \tilde{p}_{\nu}] = 0$$

$$[\tilde{x}^{\mu}, \tilde{p}_{\nu}] = \delta^{\mu}_{\nu} - \frac{\epsilon}{3}\left(\frac{|\vec{p}|}{E}\right)^2$$

Homogeneously Modified Special Relativity (HMSR)

Bibliography [3, 6]

Relating different particle species interaction

$$\begin{array}{ccc}
(TM,\, \eta_{ab},\, p) & \xrightarrow[\Lambda]{\qquad\qquad} & (TM,\, \eta_{ab},\, q) \\
\Big\downarrow e(p) & & \bar{e}(q) \Big\downarrow \\
(T_xM,\, g_{\mu\nu}(p)) & \xrightarrow[\tilde{e} \circ \Lambda \circ e^{-1}]{\qquad\qquad} & (T_xM,\, \tilde{g}_{\mu\nu}(q))
\end{array}$$

Generalization of the internal product of different states

$$\langle p+q|p+q\rangle = \begin{pmatrix} p & q \end{pmatrix} \begin{pmatrix} g^{\mu\nu}(p) & e^{a\mu}(p)\tilde{e}_a^{\ \beta}(q) \\ \tilde{e}^{a\alpha}(q)e_a^{\ \nu}(p) & \tilde{g}^{\alpha\beta}(q) \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} =$$

$$= (p_\mu\, e_a^\mu(p) + q_\mu\, \tilde{e}_a^\mu(q))\, \eta^{ab}\, (p_\nu\, e_b^\nu(p) + q_\nu\, \tilde{e}_b^\nu(q)).$$

GRB accelerated neutrinos

The emission spectrum of neutrinos accelerated in a GRB is described by a double broken power law

$$\phi_\nu(E_\nu) = \phi_0 \cdot \begin{cases} \epsilon_b^{-1} E_\nu^{-1} & E_\nu \leq \epsilon_b \\ E_\nu^{-2} & \epsilon_b \leq E_\nu \leq 10 \cdot \epsilon b \\ (10\,\epsilon_b)^2 E_\nu^{-4} & E_\nu \geq 10 \cdot \epsilon_b \end{cases}$$

where ϕ_0 is the spectral normalization flux and ϵ_0 is the break energy.

Some properties of the GRB prompt phase of the neutrino emission are not well known and still under debate, such as the time dependence of the emission spectrum.

Despite this dependence on the model considered for neutrino emission phase, some research works foresee the possibility to conduct this kind of investigation, finding a good agreement with the expected linear dependence of the time delay.

Supernova neutrinos

Bibliography [6]

The different neutrino flavors exhibit distinct differential fluxes, which can be expressed as functions of time and energy:

$$\frac{d^2\Phi_\alpha}{dt\,dE} = \frac{L_\alpha(t)}{4\,\pi\,d^2}\,\frac{f_\alpha(t,\,E)}{\langle E_\alpha(t)\rangle}$$

where ϕ_α represents the original neutrino flux, $L_\alpha\ (t)$ is the luminosity, *d* is the SN-detector distance and $f_\alpha\ (t, E)$ is the energy spectrum for each α neutrino flavor.

$$f_\alpha(t,\,E) = \lambda_\alpha \left(\frac{E}{\langle E_\alpha(t)\rangle}\right)^{\beta_\alpha} \exp\left(-\frac{(\beta_\alpha(t)+1)\,E}{\langle E_\alpha(t)\rangle}\right)$$

β_α represents the model-dependent pinching parameter, accounting for the thermal neutrino spectrum, and $\langle E_\alpha(t)\rangle$ is the mean neutrino energy.

Neutrinos with varying energies can accumulate an energy-dependent time delay
This effect can modify the expected neutrino energy spectrum