International Conference on Neutrinos and Dark Matter

The Inert Doublet Model: Insights into Dark Matter

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Outline

- Cosmological Standard Model
- Inert Doublet Model (IDM)
- Bridge between particle physics and cosmology
- Extensions of IDM

From Gravitation to Cosmology

Infinite Universe

Uniform Mass Density ρ

$$g_{uv}(r \to \infty) = \eta_{uv}$$

$$G_{uv} = \frac{8\pi G}{c^4} T_{uv}$$

$$G_{uv} = 0$$

$$T_{uv} = 0!$$

First Proposed Solution

Finite Universe with positive curvature

Non-Static Universe!

Second Proposed Solution

$$G_{uv} = \frac{8\pi G}{c^4} T_{uv} \xrightarrow[\text{solution}]{} G_{uv} - \Lambda g_{uv} = \frac{8\pi G}{c^4} T_{uv}$$

trace reversed form of the einstein field equations

$$R_{uv} = \frac{8\pi G}{c^4} \left(T_{uv} - \frac{1}{2} \left(T + \underbrace{\frac{c^4}{4\pi G} \Lambda}_{\text{A form of energy}} \right) g_{uv} \right)$$

Dark Energy

FLRW (unified) metric

$$g_{uv} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -\frac{a^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & -a^2r^2 & 0 \\ 0 & 0 & 0 & -a^2r^2\sin^2\theta \end{pmatrix}$$

Perfect fluid assumption

$$T_{uv} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & \frac{a^2p}{1-kr^2} & 0 & 0 \\ 0 & 0 & pa^2r^2 & 0 \\ 0 & 0 & 0 & pa^2r^2\sin^2\theta \end{pmatrix}$$

Einstein equations

$$G_{uv} - \Lambda g_{uv} = \frac{8\pi G}{c^4} T_{uv}$$

Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

$$\frac{\dot{a}^2 + c^2k}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3}$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right)$$

$$\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\Lambda) - \frac{c^2k}{a^2}$$

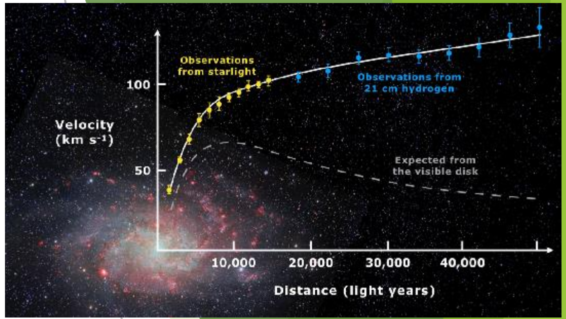
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + 2\rho_r + 2\rho_\Lambda)$$

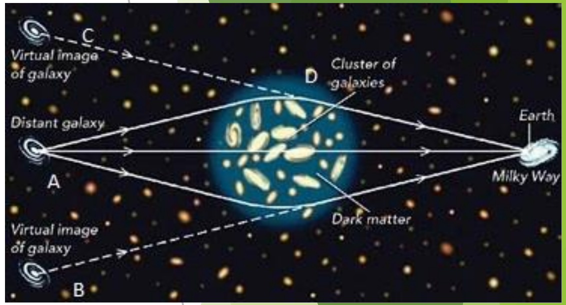
Dimensionless Friedmann equation

$$1 = \underbrace{\frac{8\pi G\rho}{3H^2}}_{\Omega_{M/R}} + \underbrace{\frac{\Lambda c^2}{3H^2}}_{\Omega_\Lambda} - \underbrace{\frac{c^2k}{a^2H^2}}_{\Omega_k}$$

$$\Omega_{total} = \underset{\approx 0.31}{\Omega_m} + \underset{\approx 5\times10^{-5}}{\Omega_r} + \underset{\approx 0.69}{\Omega_\Lambda} + \underset{\approx 0}{\Omega_k} \approx 1$$

- **Galactic Rotation Curves:** The observed rotation speed of stars in galaxies remains constant far from the center, contrary to predictions based on visible matter alone.
- **Gravitational Lensing:** Light from distant galaxies is bent more than can be explained by visible matter, indicating the presence of unseen mass.
- **Cosmic Microwave Background (CMB):** The patterns in the CMB require dark matter to explain the observed density fluctuations.
- **Galaxy Cluster Dynamics:** The motion of galaxies within clusters shows more mass than can be accounted for by visible matter.
- **Structure Formation:** The formation of galaxies and large-scale structures in the universe is best explained by cold dark matter models.





$$\Omega_{total} = \underbrace{\Omega_m}_{\underset{\approx 0.05 \quad \approx 0.26}{\Omega_{bm} + \Omega_{DM}}} + \Omega_r + \Omega_\Lambda + \Omega_k$$

Dark Matter

Good DM candidate

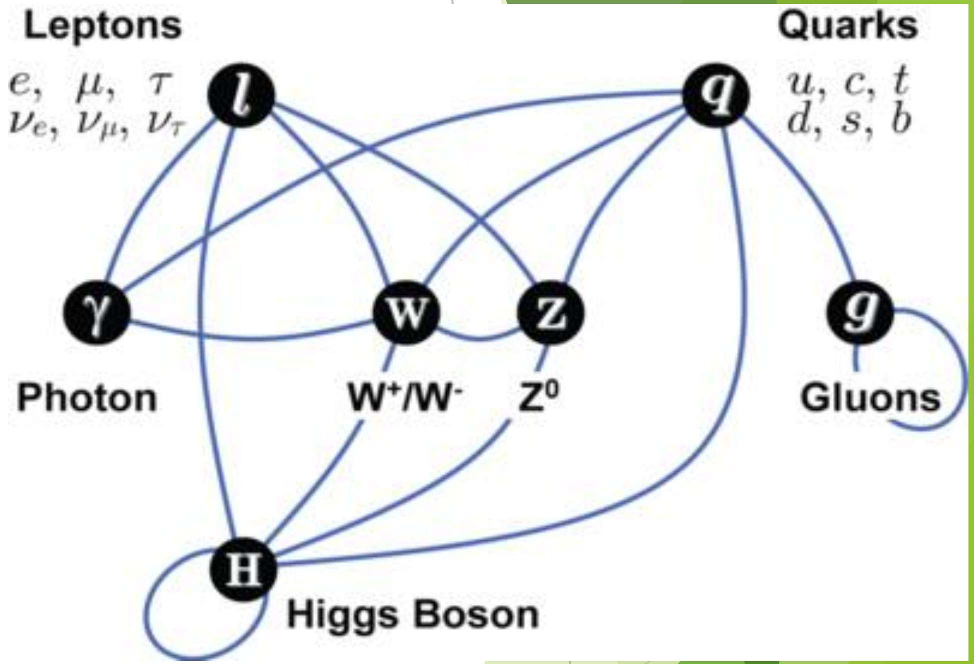
- Dark matter is a form of matter that does not emit, absorb, or reflect electromagnetic radiation, making it **invisible to telescopes**. Its presence is inferred through its gravitational effects on visible matter, and on the cosmic microwave background (CMB).
- It constitutes approximately **84% of the matter in the universe** and 27% of the total mass-energy content of the universe, while ordinary (baryonic) matter makes up about 5%.
- Dark matter is described as being **cold** (velocity is non-relativistic at the epoch of radiation-matter equality),
- **Non-baryonic** (consisting of matter other than protons and neutrons),
- **Dissipationless** (cannot cool by radiating photons)

- **Collisionless** (i.e., the dark matter particles interact with each other and other particles only through gravity and possibly the weak force).
- Evidence suggests it does **not interact strongly with itself or baryonic matter**, apart from weak interactions. (because such interactions would produce observable effects that are inconsistent with astrophysical and cosmological observations.). **Strong self-interactions would lead to core collapse in dark matter halos or the formation of dense cores**, which are not consistent with observations.
- **Stable**: If dark matter were unstable and decayed, **Decay products would be observable**; and alter the anisotropies in the CMB, which are well-matched to current models of stable dark matter.

Call for Particle Physics

Looking in the Standard Model

- Quarks carry color charges (red, blue, or green) so they participate in strong interactions.
- All particles have electric charge participate in electromagnetic interaction.
- Quarks and leptons carry flavor charges and participate in weak nuclear interactions.



No DM!

The Inert Doublet Model

- One of the **simplest** and most studied BSM scenario is the inert doublet model (IDM) where the Higgs sector of the SM is extended by an **additional electroweak doublet** with a stabilizing discrete **Z_2 symmetry**.
- The **lightest neutral component** of the new doublet serves as a DM candidate.
- In this scenario, DM interacts with the SM sector via its interaction with the Higgs boson, and weak gauge interactions.

$$\Phi_h = \begin{pmatrix} \Phi_h^+ \\ \Phi_h^0 \end{pmatrix} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix} \qquad \Phi_{inert} = \begin{pmatrix} \Phi_{inert}^+ \\ \Phi_{inert}^0 \end{pmatrix} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H^0 + iA^0) \end{pmatrix}$$

This Z2 symmetry ensures that:

- Φ_{inert} **does not participate in electroweak symmetry breaking**,
- **does not couple to the SM fermion**
- the lightest neutral component of Φ_{inert} is **stable** and can serve as a dark matter candidate.

Z_2 symmetry

$$\Phi_h \longrightarrow \Phi_h\,, \qquad \Phi_{inert} \longrightarrow -\Phi_{inert}$$

The scalar sector of the model is described by the potential

$$V(\Phi_h, \Phi_{inert}) = \left\{ \begin{array}{c} -\mu_h^2|\Phi_h|^2 + \lambda_h|\Phi_h|^4 + \mu_{inert}^2|\Phi_{inert}|^2 + \lambda_{inert}|\Phi_{inert}|^4 \\ +\lambda_3|\Phi_h|^2|\Phi_{inert}|^2 + \lambda_4|\Phi_h^\dagger\Phi_{inert}|^2 + \frac{\lambda_5}{2}\left[(\Phi_h^\dagger\Phi_{inert})^2 + h.c.\right] \end{array} \right\}$$

Symmetry Breaking

$$\Phi_h = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix}, \quad \Phi_{inert} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H^0 + iA^0) \end{pmatrix}$$

H^0 and A^0 are lighter than $H^\pm$

H^0 the lightest particle in the inert doublet

$$V(\Phi_h, \Phi_{inert}) = \left\{ \begin{array}{c} \mu_1^2|\Phi_h|^2 + \mu_2^2|\Phi_{inert}|^2 + \frac{1}{2}\lambda_1|\Phi_h|^4 + \frac{1}{2}\lambda_2|\Phi_{inert}|^4 \\ +\lambda_3|\Phi_h|^2|\Phi_{inert}|^2 + \lambda_4|\Phi_h^\dagger\Phi_{inert}|^2 + \frac{\lambda_5}{2}[(\Phi_h^\dagger\Phi_{inert})^2 + h.c.] \end{array} \right\}$$

Mass Spectrum

$$M_h^2 = 2\lambda_h v^2 = 2\mu_h^2$$
$$M_{H^0}^2 = \mu_{inert}^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)v^2$$
$$M_{A^0}^2 = \mu_{inert}^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5)v^2$$
$$M_{H^\pm}^2 = \mu_{inert}^2 + \frac{1}{2}\lambda_3 v^2$$

Theoretical constraints

- Regarding the theoretical constraints, we incorporate considerations of perturbativity, stability, and tree-level unitarity in the scalar sector

vacuum stability

$$\lambda_1 > 0,$$
$$\lambda_2 > 0,$$
$$\lambda_3 > -\sqrt{\lambda_1 \lambda_2}\,,$$
$$\lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1 \lambda_2}\,,$$

unitarity

$$3(\lambda_1 + \lambda_2) + \sqrt{9(\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2} < 16\pi,$$
$$\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2} < 16\pi,$$
$$\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2} < 16\pi,$$
$$|\lambda_3 + 2\lambda_4 \pm 3\lambda_5| < 8\pi,$$
$$|\lambda_3 \pm \lambda_4| < 8\pi,$$
$$|\lambda_3 \pm \lambda_5| < 8\pi.$$

perturbativity

$$|\lambda_i| < 4\pi$$

Experimental constraints

- Production of inert scalars at the LEP: No signals have been reported by LEP for processes like :

$$Z \longrightarrow (H^0 A^0, H^+ H^-) \qquad W^\pm \longrightarrow (H^0 H^\pm, A^0 H^\pm)$$

- This leads to the conditions

$$m_{H^0,A^0} + m_{H^\pm} > m_W$$
$$m_{A^0} + m_{H^0} > m_Z$$
$$2m_{H^\pm} > m_Z$$

https://arxiv.org/pdf/2410.00638

- **Higgs invisible decays:** When the inert scalars have masses lower than half of Higgs masse, certain decay modes become kinematically possible. These include:

$$h \rightarrow H^0 H^0$$

$$h \rightarrow A^0 A^0, H^+ H^-$$

- . . . .

- **DM relic density? How?**

From particle physics to cosmology

- The Boltzmann equation **connects microphysical interactions** (annihilation rates) **with cosmological dynamics** (expansion of the Universe).

$$\frac{dn_\chi}{dt} = \underbrace{-3Hn_\chi} - \underbrace{\langle \sigma_{eff} v \rangle}[n_\chi^2 - (n_\chi^{eq})^2]$$

Dilution of the number density due to the **expansion of the Universe**

Thermally averaged annihilation **cross-section** times relative velocity.

$$\langle \sigma v \rangle = \frac{\int \sigma(v) v e^{-\frac{E}{K_B T}} d^3 v}{\int e^{-\frac{E}{K_B T}} d^3 v}$$

The product σv describes the rate of particle annihilation per unit pair of particles.

it ensures that all possible velocities and energy distributions in the thermal bath are considered.

- The effective annihilation cross-section for a system with multiple processes is a sum over all possible final states:

$$H^0H^0 \longrightarrow W^+W^-, ZZ, hh, f\bar{f}, \ldots$$

$$A^0A^0 \rightarrow W^+W^-, ZZ, hh, f\bar{f}, \ldots$$

$$H^+H^- \rightarrow W^+W^-, ZZ, hh, f\bar{f}, \ldots$$

$$\sigma_{eff} v = \sum_i \sigma_i v_{rel;i}$$

- For co-annihilation, the effective σv becomes

$$H^0A^0 \rightarrow Zh, \quad H^0H^+ \rightarrow W^+h, \quad A^0H^+ \rightarrow W^+Z, \ldots$$

$$\langle \sigma v \rangle_{eff} = \sum_{ij} \langle \sigma_{ij} v \rangle \frac{n_i n_j}{(n_{total})^2}$$

- Approximation for Non-Relativistic Dark Matter

$$\sigma v = a + bv^2 + O(v^4)$$

$$\sigma \sim \sigma_0 + \sigma_2 v^2$$

$$\sigma \sim \sigma_0$$

cold dark matter

- To quantify these processes, we numerically solve the Boltzmann equation, accounting for all possible annihilation and co-annihilation contributions.
- The relic density of DM is required to align with the PLANCK measurement

$$\Omega_{DM}h^2 = 0.1200 \pm 0.0012$$

Aghanim, N., et al. "Erratum: Planck 2018 results: VI. Cosmological parameters (Astronomy and Astrophysics (2020) 641 (A6." *Astronomy & Astrophysics* 652 (2021): 1-3.

Problems of the Inert Doublet Model:

1. Ensuring correct dark matter (DM) relic density requires **tuning parameters** like the mass of the DM candidate and scalar couplings.
2. Indirect detection faces issues in probing high-mass regions due to limited experimental sensitivity.
3. The model's ability to achieve a strong first-order **EWPT** for baryogenesis is sensitive to parameter choices.
4. The IDM does not inherently explain the smallness of **neutrino masses** or their observed mixing patterns.
5. The model lacks a built-in mechanism for explaining the **baryon asymmetry** of the universe.
6. . . .

Extensions of the Inert Doublet Model:

- **Adding singlet or triplet scalars or fermions** can address neutrino masses (via seesaw mechanisms) and baryogenesis.
- Adding **right-handed neutrinos** allows for neutrino mass generation via the seesaw mechanism and provides new interactions with the inert doublet.
- **Enlarging the gauge group** (e.g., adding U(1) symmetry) can stabilize dark matter and introduce new interactions that modify IDM phenomenology.
- Introducing **additional discrete symmetries**, to stabilize multi-component DM or allow for new decay channels.
- Embedding IDM within a **supersymmetric** framework (e.g., SUSY) can address fine-tuning and provide additional dark matter candidates.
- Adding **new gauge bosons or interactions** to the model to enhance DM annihilation channels or address indirect detection signals.
- Studying **loop corrections** to the scalar potential can provide more precise constraints and potentially resolve vacuum stability issues.
- Incorporating IDM into a framework with modifications to **gravity**, such as extra dimensions, may provide connections to cosmology.