

Vacuum Stability in the Standard Model and Beyond

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in collaboration with
Gudrun Hiller, Tim Höhne, Daniel Litim
[arXiv 2401.08811]

TOP 2024
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Motivation

- » Higgs discovery in 2012 [ATLAS,CMS 2012] → Metastability [Buttazzo et al, 2013]
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- » Can Stability be excluded?

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- » Guidance for building SM extensions?
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Outline

- » Stability in the SM – An update
- » BSM solutions

How to compute vacuum stability

1. Observables

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- Higgs mass M_h
- Top mass M_t
- Strong coupling $\alpha_s^{(5)}(M_Z)$
- Z mass M_Z
- Fermi constant G_F
- Fine structure & hadronic threshold $\alpha_e, \Delta\alpha_e^{(5),\text{had}}$
- Lepton masses $M_{e,\mu,\tau}$
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PDG 2024

2. Matching Observables to $\overline{\text{MS}}$

at least 2L + 3L QCD [Martin, Patel, 2018]

→ running couplings at a reference scale $\alpha_x(\mu_{\text{ref}})$
 $\mu_{\text{ref}} = 200 \text{ GeV}$

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3. Compute Effective Potential

3L (4L QCD) with RG improvement

[Ford, Jack, Jones, 1992] [Martin, 2013-17]

→ minima

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only interested in absolute stability

Effective Potential

– potential of classical field h & quantum effects, RG invariant, physical extrema

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4L gauge (+ 5L QCD)

[Davies, Herren, Poole, Steinhauser, Thomsen, 2019]

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- completely resum all logs $\ln h/\mu_{\text{ref}}$

What Observables impact vacuum stability?

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- Higgs mass
- Strong coupling
- Top mass

- Z mass M_Z
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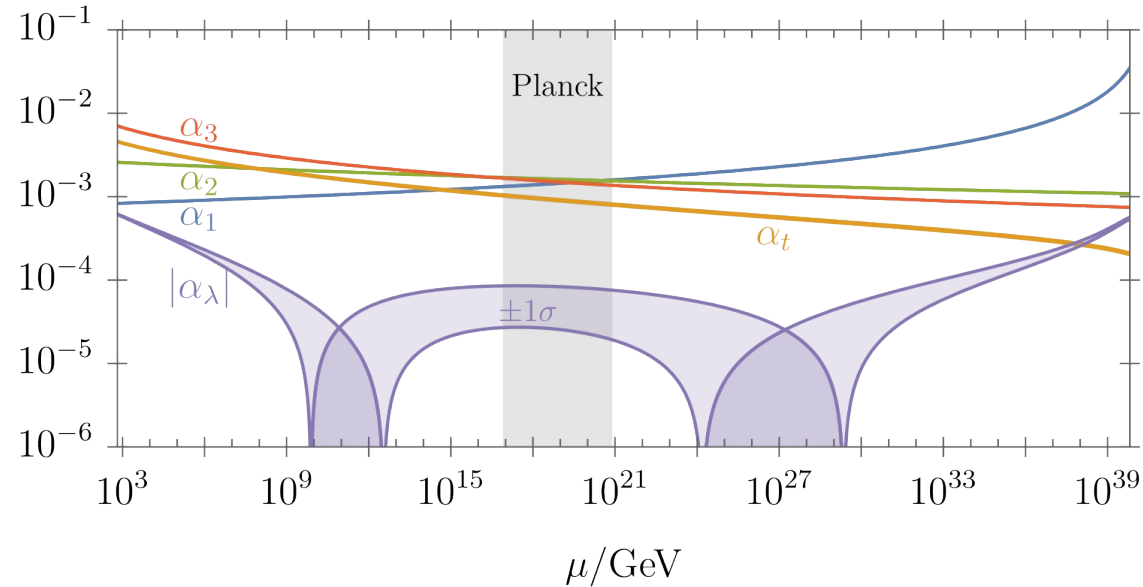
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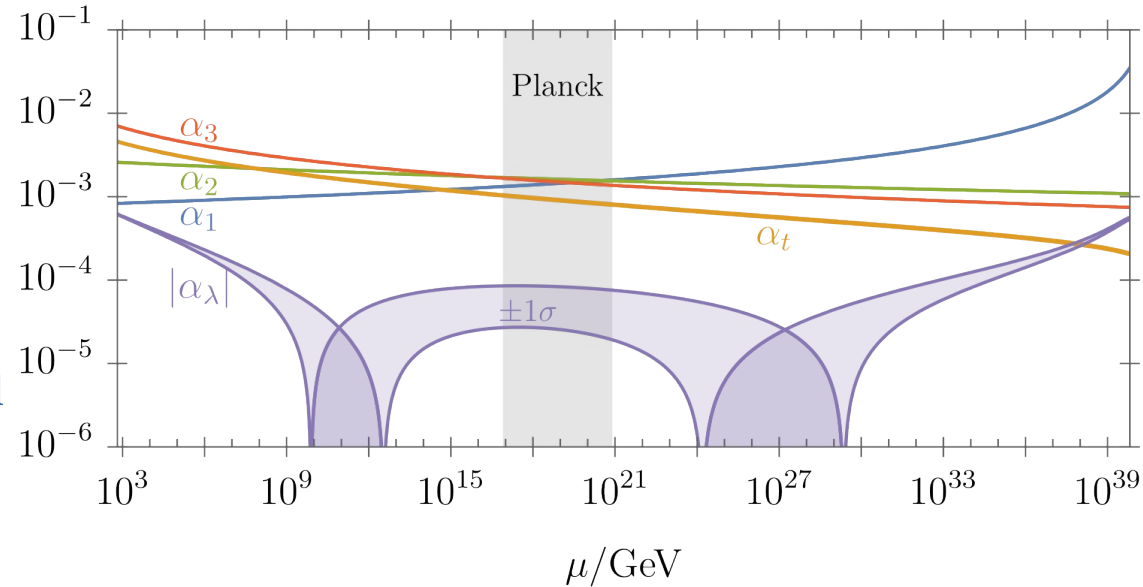
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~~$m_b(m_b), m_c(m_c), m_{u,d,s}(2\text{GeV})$~~

Impact small



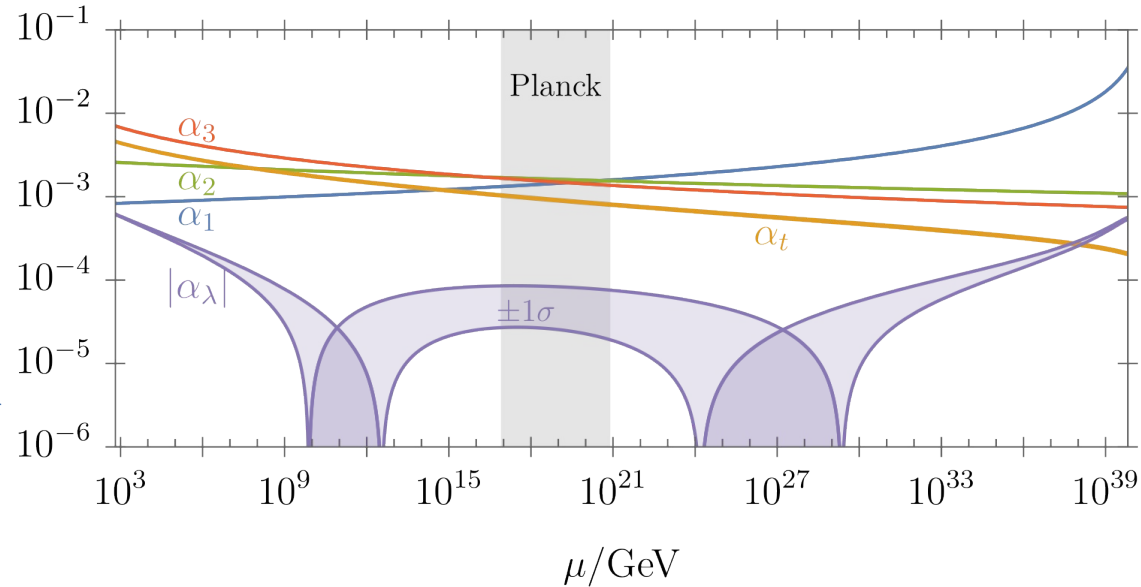
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What Observables impact vacuum stability?

1. Observables PDG 2024

- Higgs mass $M_h = 125.20(11)$ GeV
- Strong coupling
- Top mass

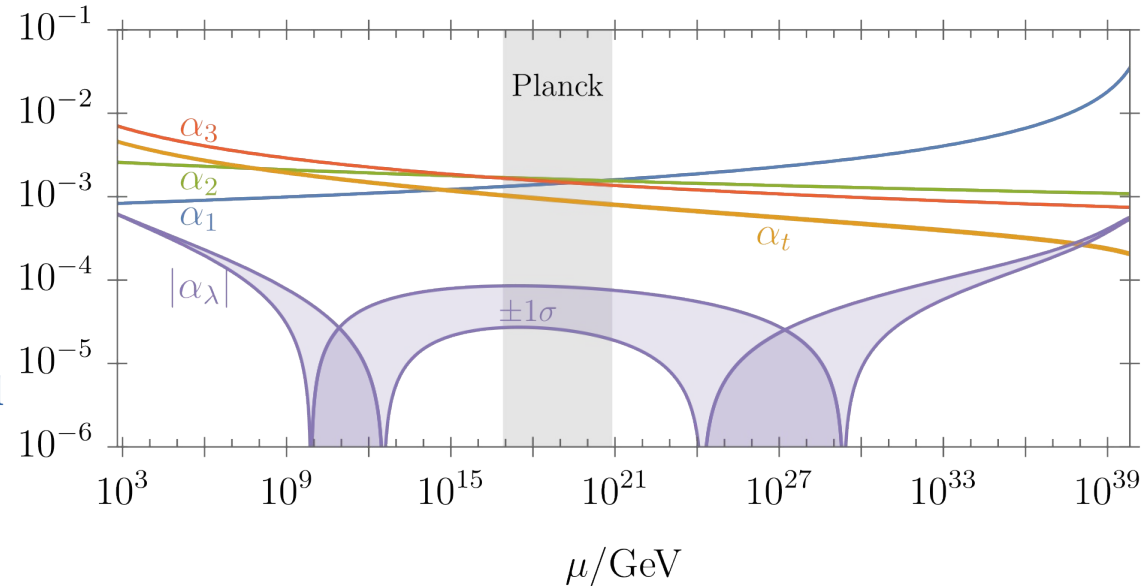
Uncertainty small $+24 \sigma$

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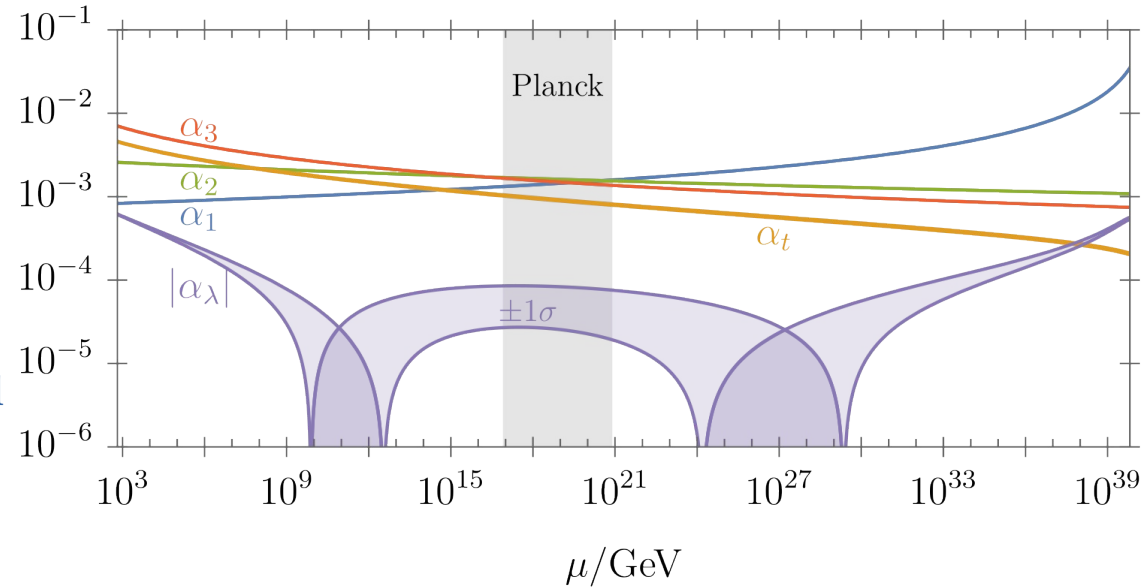
- Higgs mass $M_h = 125.20(11)$ GeV
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- Top mass

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+3.7 σ

~~- Z mass M_Z~~
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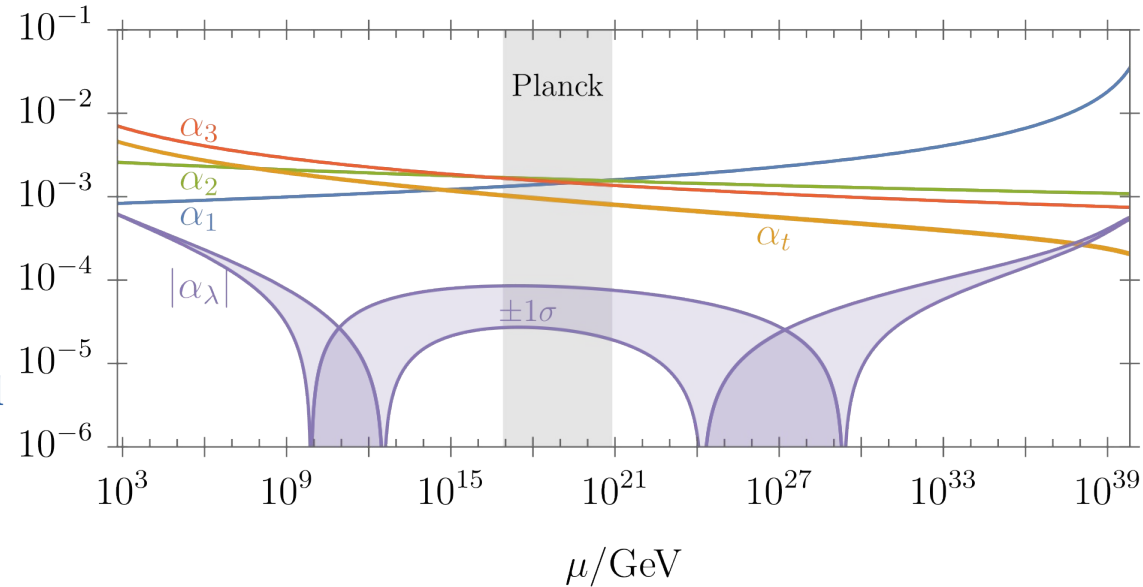
- Higgs mass $M_h = 125.20(11)$ GeV
- Strong coupling $\alpha_s^{(5)}(M_Z) = 0.1180(9)$
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 $M_t^{\text{MC}} = 172.57(29)$ GeV

Uncertainty small +24 σ
+3.7 σ
-1.9 σ
-5.1 σ

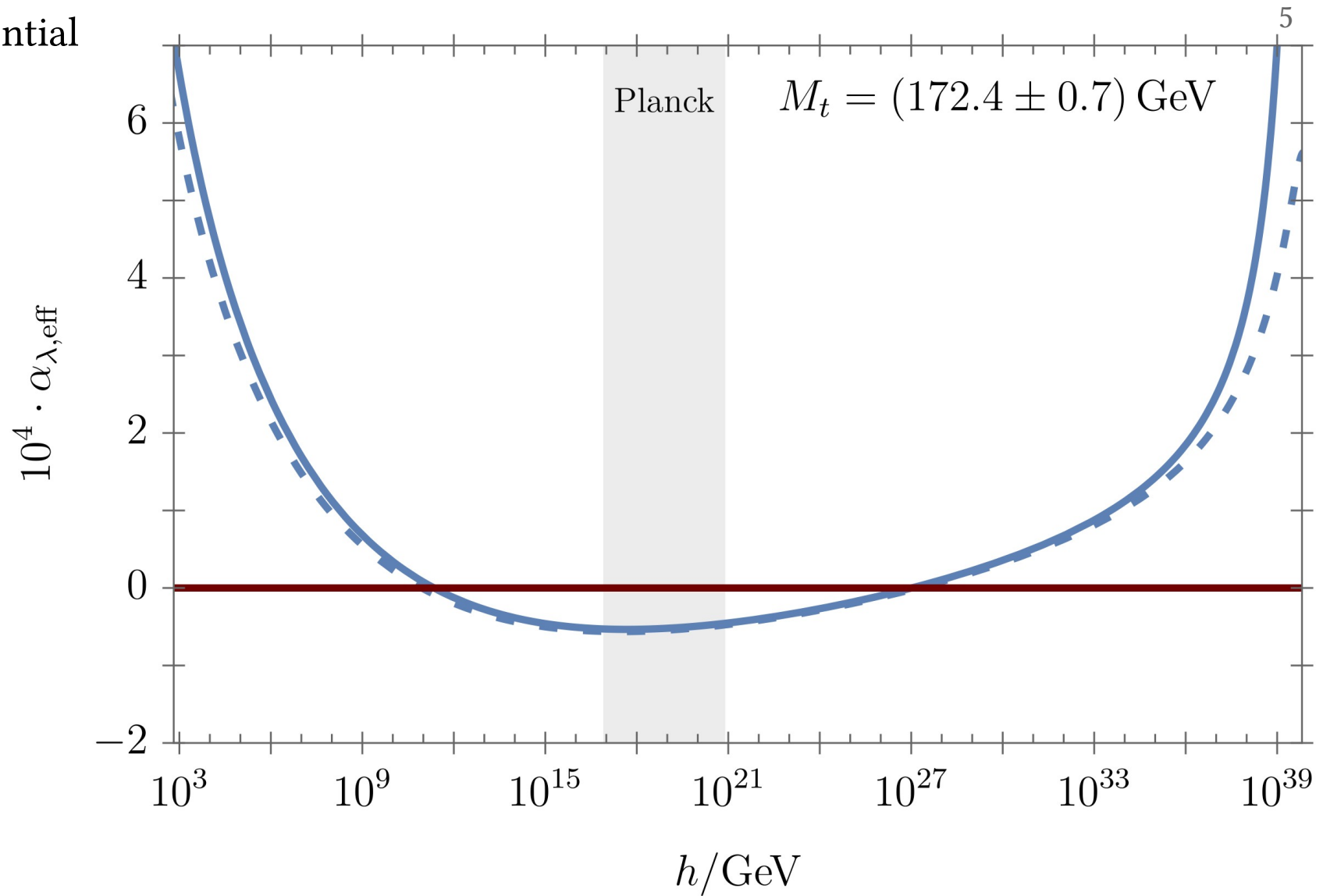
which one?

~~- Z mass M_Z~~
~~- Fermi constant G_F~~ Uncertainty small
~~- Fine structure & hadronic threshold $\alpha_e, \Delta\alpha_e^{(5), \text{had}}$~~

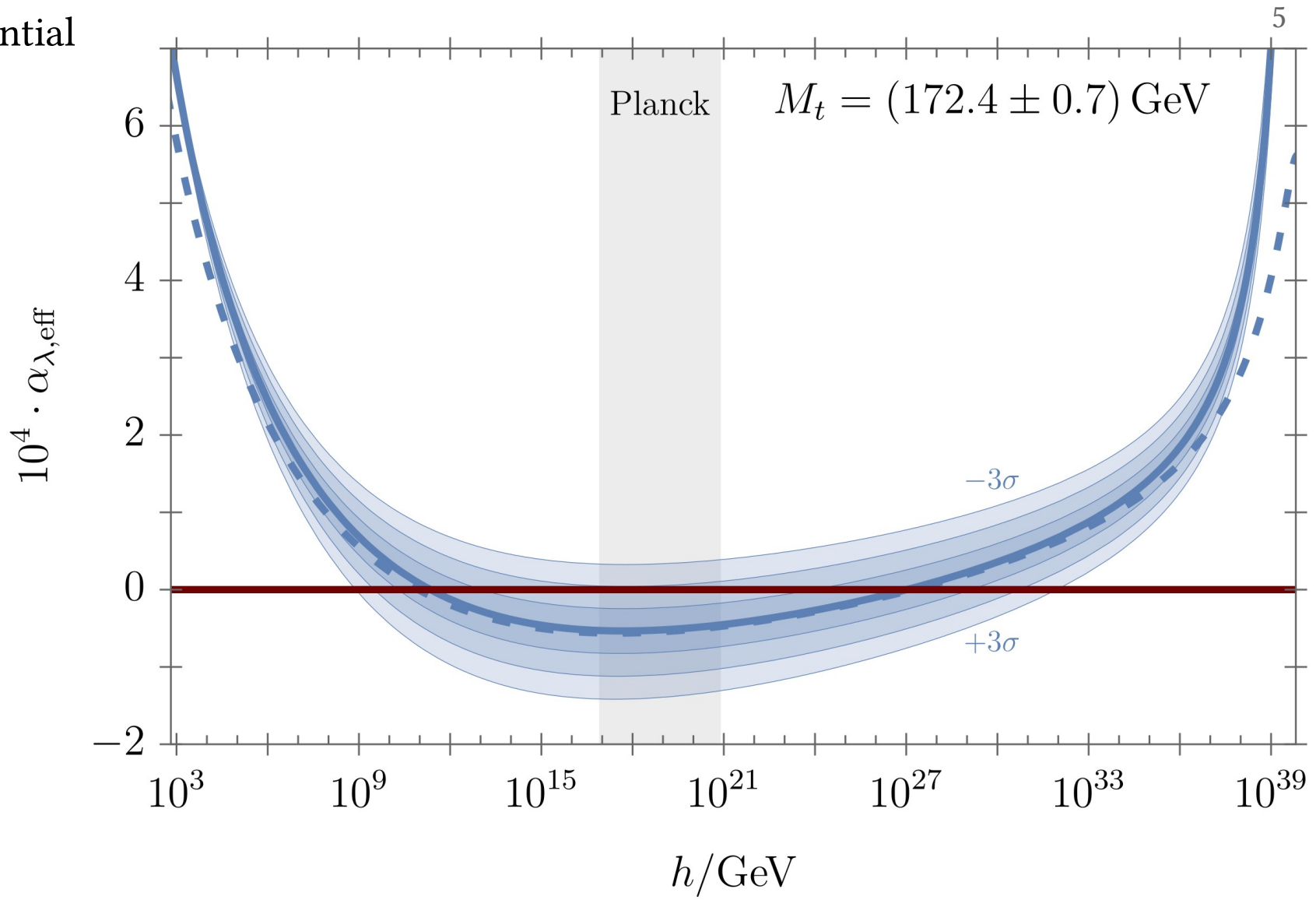
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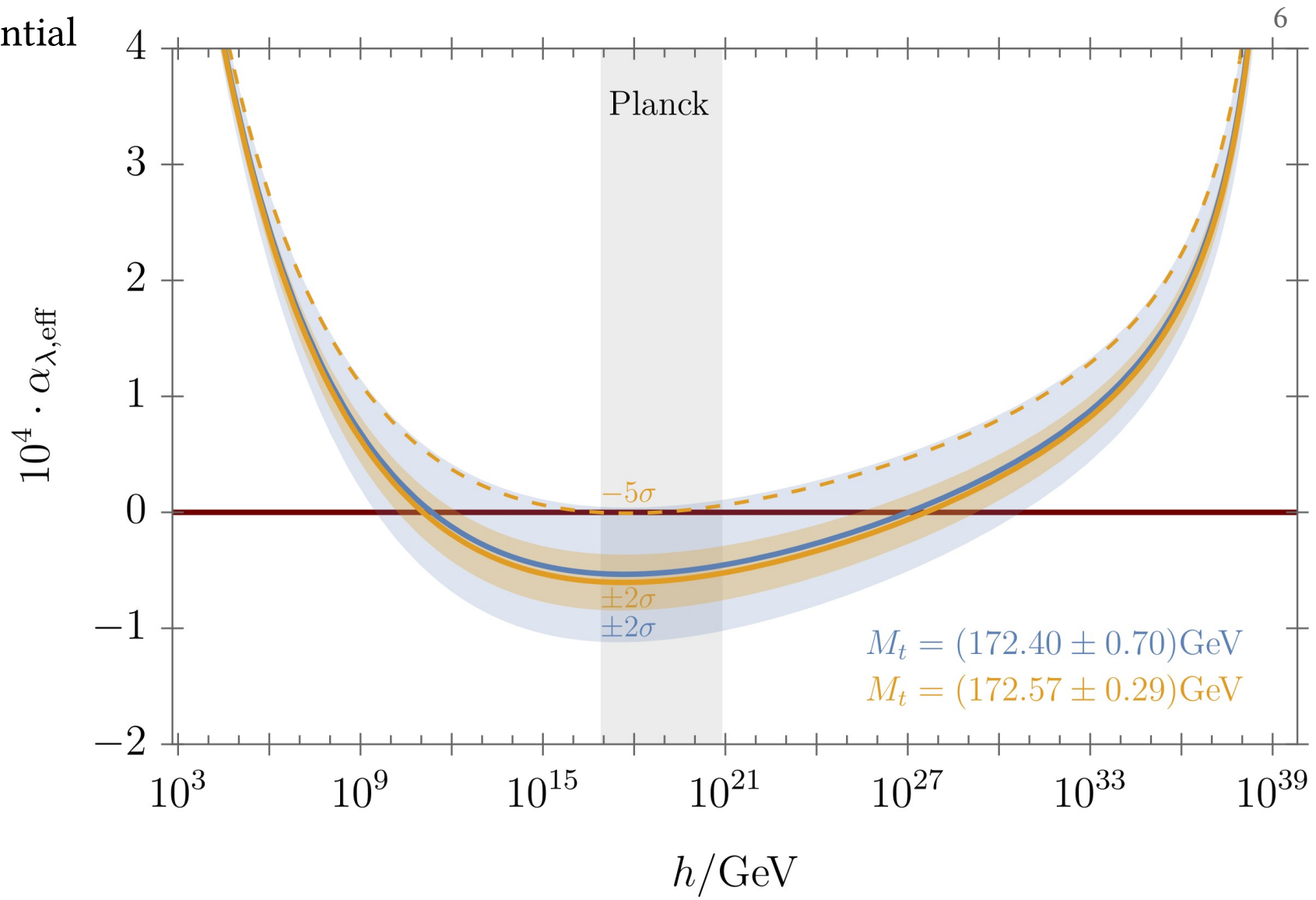
Effective Potential



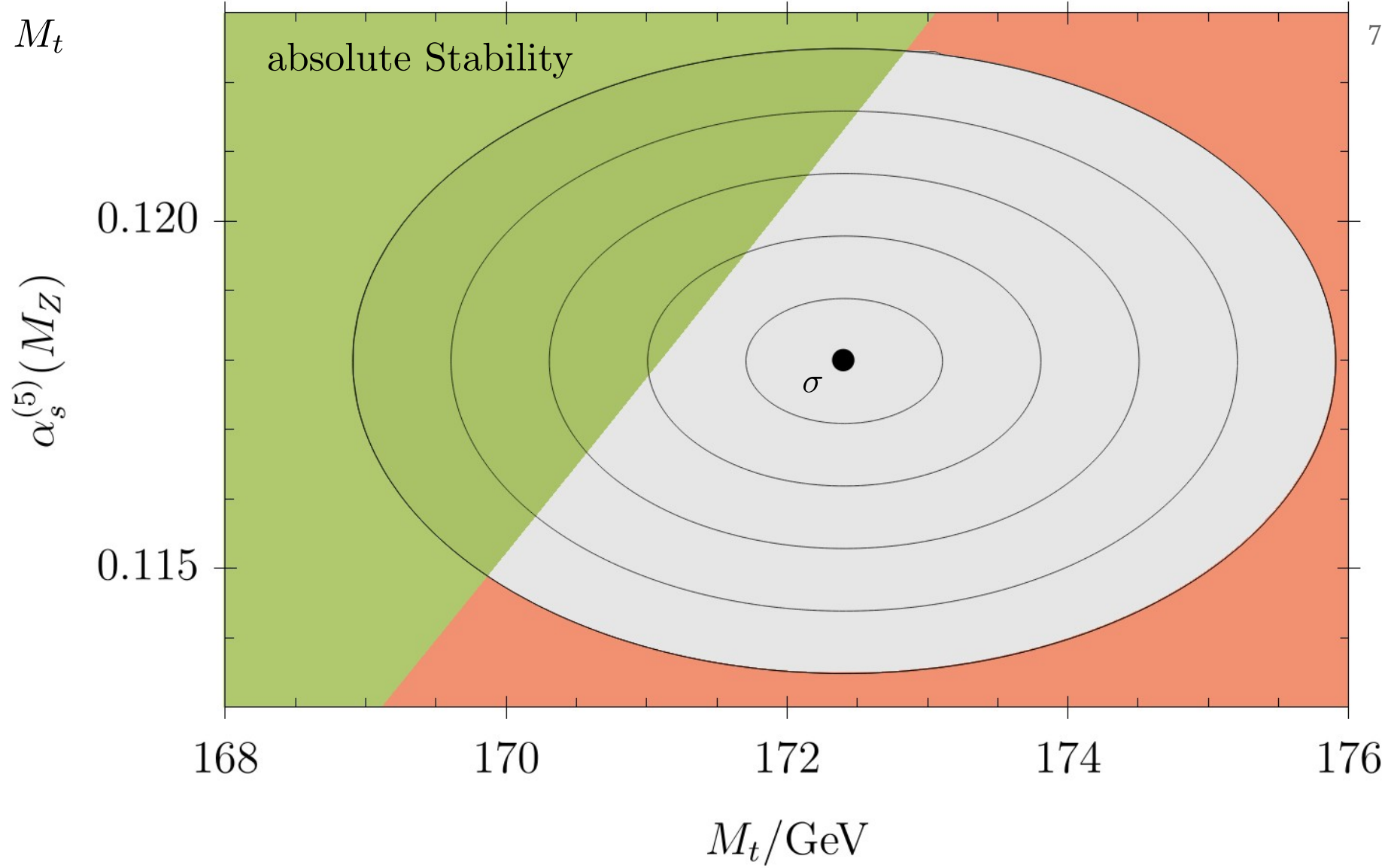
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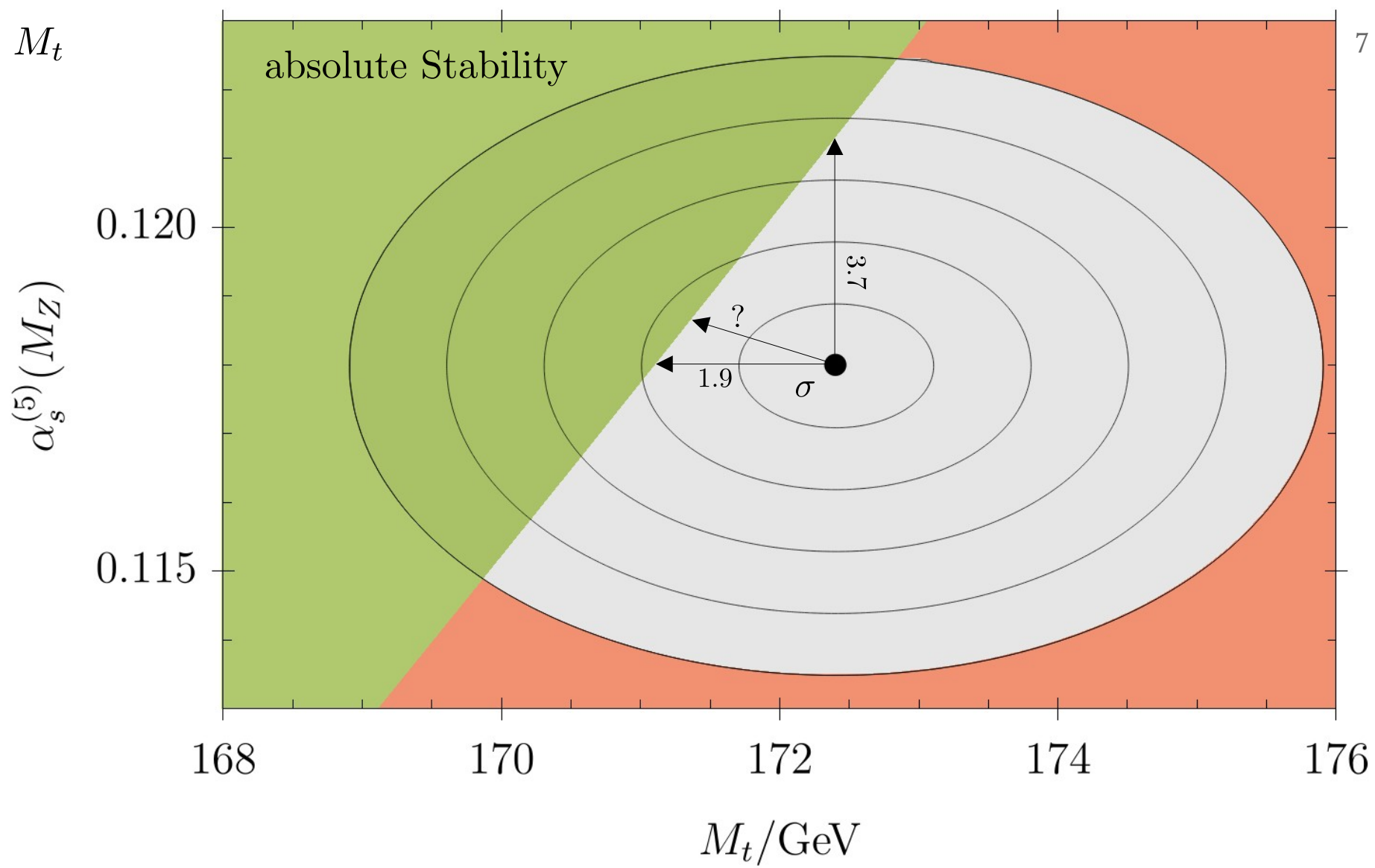
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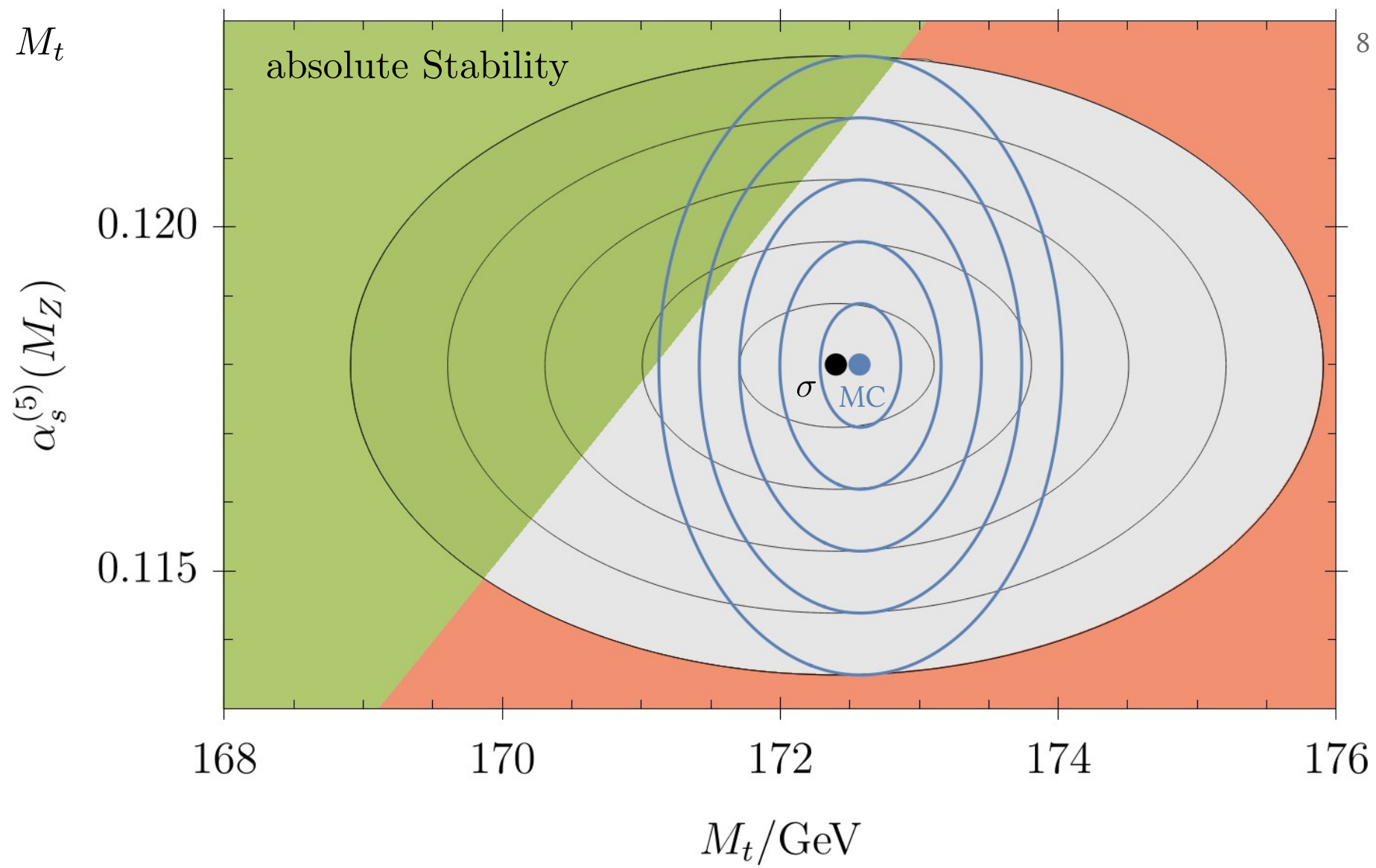
α_s vs. M_t

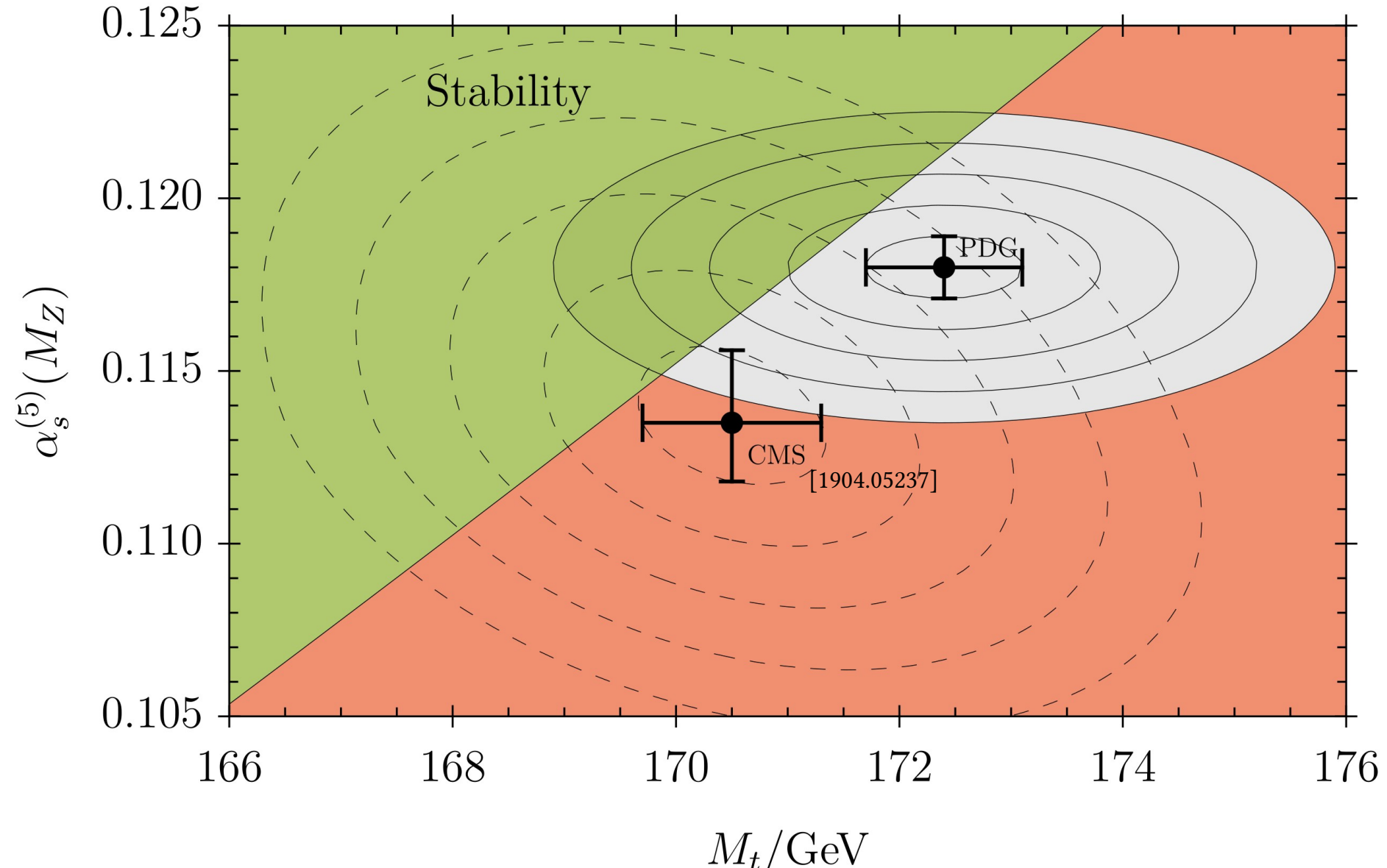


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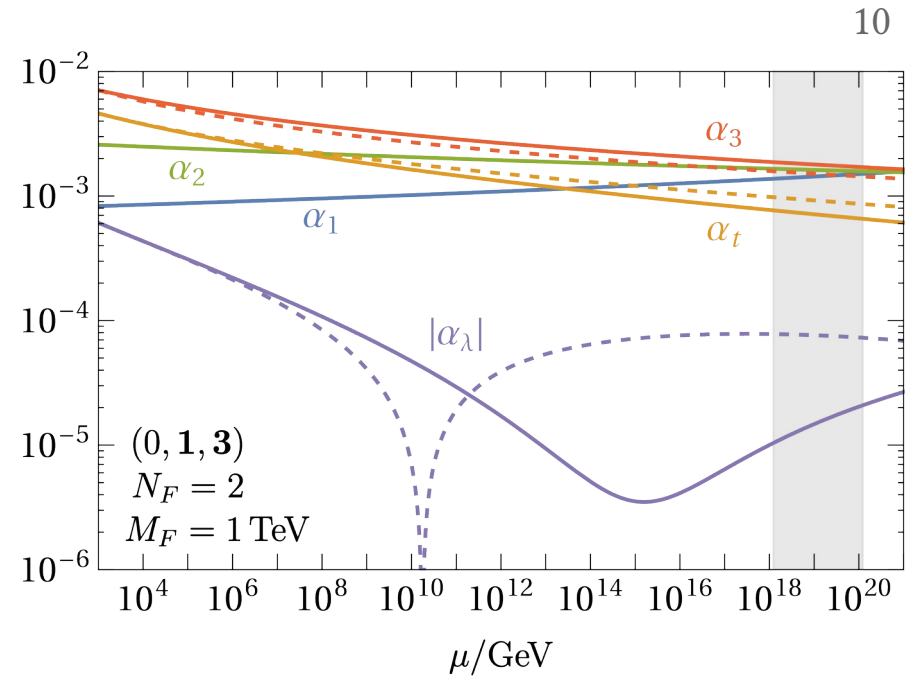
α_s vs. M_t





Stability via BSM?

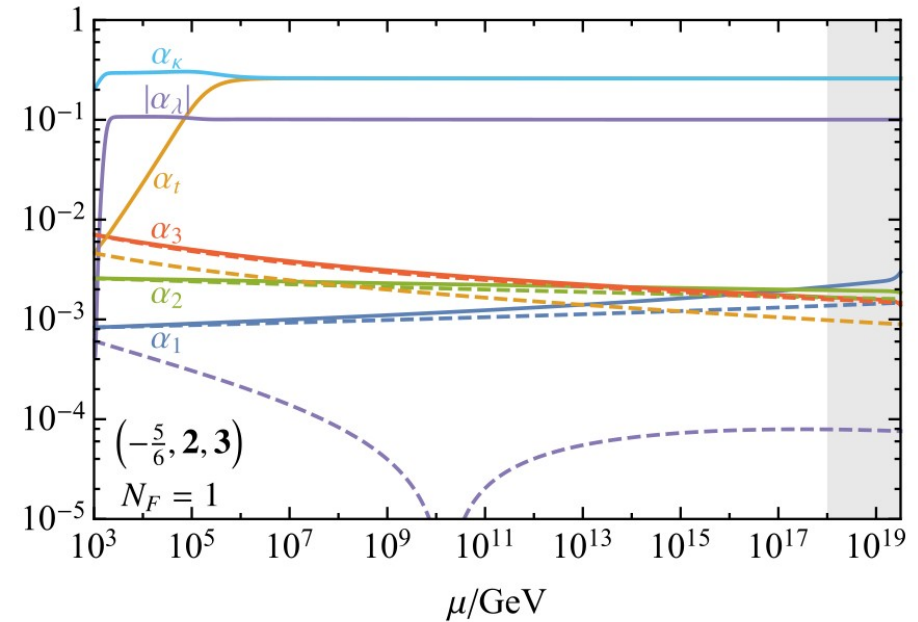
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» Gauge Portal – adding new charged fermions
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» Yukawa Portal – sizable new Yukawa interactions
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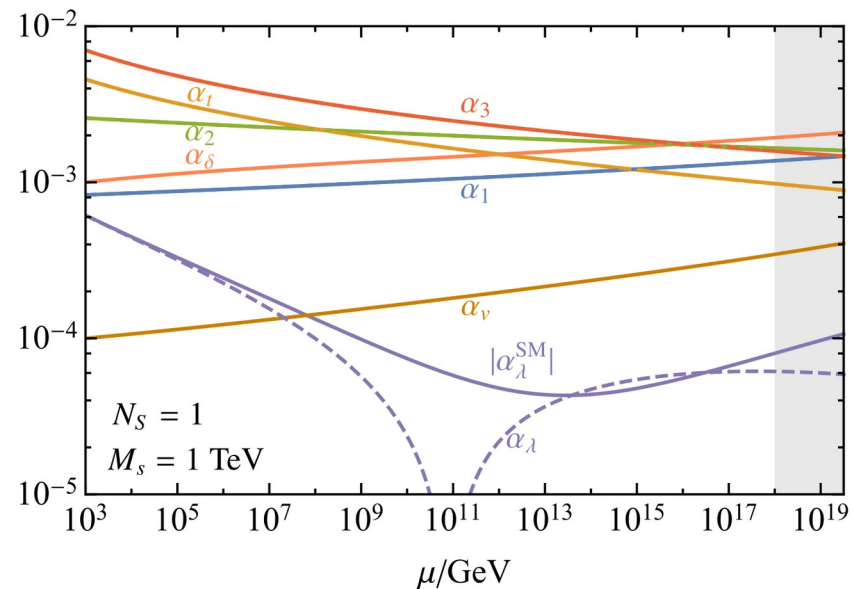
» Scalar Portal

[Hiller, Höhne, Litim, TS 2024]

$$V_{H,S} = \lambda (H^\dagger H)^2 + \delta (H^\dagger H)(S^T S) + v (S^T S)^2$$

Portal coupling

$$\beta_\lambda = \beta_\lambda^{\text{SM}} + \mathcal{N}\delta^2$$



Summary

- » evidence for metastability of SM persists
- » more precision measurements of $\alpha_s^{(5)}(M_Z)$ and M_t necessary to exclude stability at 5σ
- » correlation important
- » understanding of MC Top mass required
- » instability is RG dominated

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-
- » many BSM approaches to address SM instability
 - » can be valid until Planck scale
 - » testable at current and future colliders