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The magic of entangled top quarks

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Overview

Which quantities from Quantum Information / Computing could be useful for collider physics?

- Brief introduction to Quantum Computing / Information.
- The property of *magic* of quantum states.
- Does nature produce magic top quarks?
- What might this be useful for?

Motivation

- In recent years, many people have looked at high energy tests of quantum theory.
- One such test involves **entanglement** (e.g. Bell inequalities) of top quarks at the LHC (Afik, de Nova; Dong, Gonçalves, Kong, Navarro; Fabbrichesi, Floreanini, Panizzo; Aoude, Madge, Maltoni, Mantani, Severi, Boschi, Sioli; Aguilar-Saavedra, Casas).
- Entanglement is not the only special property of quantum states.
- Lots of other things are studied in Quantum Computation / Information theory, for interesting reasons…
- …might these also be useful in high energy physics?

A bit of quantum computing

- In quantum computers, classical bits (with values {0,1}) are replaced by *qubits*:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Orthogonal basis states

where the complex coefficients satisfy $|\alpha|^2 + |\beta|^2 = 1$.

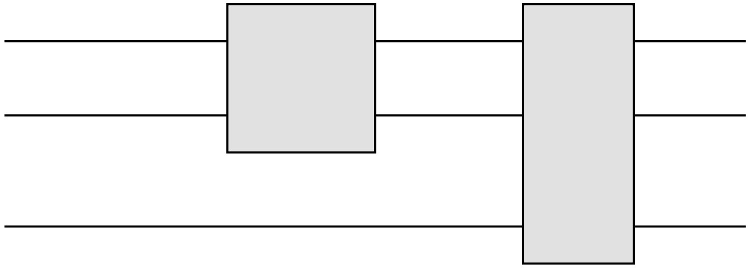
- Example: a spin-1/2 particle is a single “qubit”, where the above states are spin states.

- For multi-qubit systems, a choice of basis states is

$$|\psi_1\psi_2 \ldots \psi_n\rangle \equiv |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_n\rangle$$

Quantum computers

- Quantum computers take qubits, and subject them to unitary transformations.
- We can draw circuit diagrams, with fancy symbols to represent the transformations (“quantum gates”):



- These are the equivalent of logic gates in classical computers…
- …and change the quantum state at each intermediate step.

- The gates have names like *Hadamard*, *phase*, *CNOT*, *Pauli* etc.
- We will not need the precise details.

Why use quantum computers?

- Quantum computers are expected to vastly outperform classical computers.
- Naïvely, this is due to quantum *superposition* and *entanglement*.
- However, this not quite true.
- To see why, we need the concept of a *stabiliser state.*
- These are states that give a simple spectrum for *Pauli string* operators*:*

$$\mathcal{P}_n = P_1 \otimes P_2 \otimes \ldots \otimes P_N , \quad P_a \in \{\sigma_1^{(a)}, \sigma_2^{(a)}, \sigma_3^{(a)}, I^{(a)}\}$$

Pauli matrix
acting on qubit *a*

Identity matrix
acting on qubit *a*

- Can make such states by acting on $|0\rangle \otimes |0\rangle \otimes \ldots \otimes |0\rangle$ with Hadamard, phase, CNOT and Pauli gates.

The Gottesman-Knill theorem

- Given a state $|\psi\rangle$, we can consider the *Pauli spectrum*

$$\mathrm{spec}(|\psi\rangle) = \{\langle\psi|P|\psi\rangle, \quad P \in \mathcal{P}_n\}$$

(i.e. expectation values of each Pauli string).

- Stabiliser states have 2^n values +1 or -1, and the rest zero.
- These states are important because of the *Gottesman-Knill theorem*:

For every quantum computer containing stabiliser states only, there is a classical computer that is just as efficient! 😅

- Stabiliser states include certain maximally entangled states.
- Something other than entanglement is needed for efficient quantum computers!

Magic

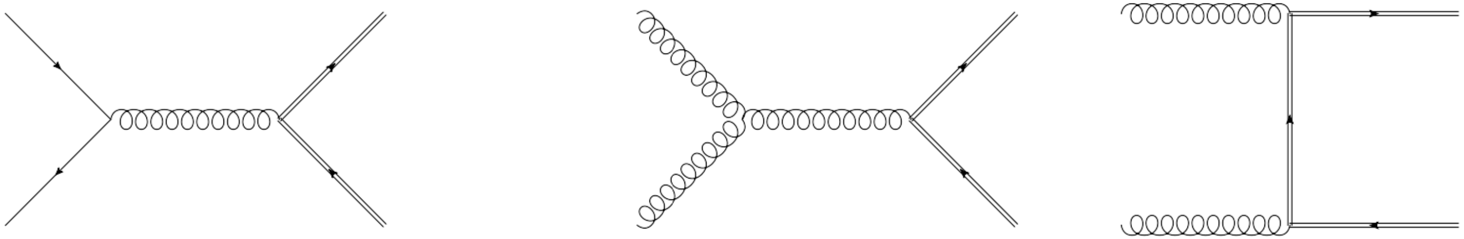
- The “something else” has been called *magic* in the literature…
- …and basically means “non-stabiliserness” of a quantum state.
- Different definitions exist. We use *Stabilizer Rényi Entropies*: (Leone, Oliviero, Hamma)

$$M_q = \frac{1}{1-q} \log_2\left(\zeta_q\right), \quad \zeta_q \equiv \sum_{P \in \mathcal{P}_n} \frac{\langle\psi|P|\psi\rangle^{2q}}{2^n}$$

- Each (integer) *q* corresponds to a higher moment of the Pauli spectrum.
- The magic is additive, **vanishes** for stabiliser states, and is crucial for making fault-tolerant quantum computers.
- In what follows, examining *q*=2 is enough: the *Second Stabilizer Rényi Entropy (SSRE).*
- We can now ask: do top quarks provide a nice system for studying magic?

Are top quarks magic?

- Top quarks are produced in pairs at the LHC…



- …such that the final state is a two-qubit system!
- However, the final state is a *mixed state* (superposition of many different *pure states*), where the SM tells us what this is in principle.
- Mixed states can be described in terms of their *density matrix*:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Probability of being
in state *I*

Top quark spin density matrix

- On general grounds, the top quark spin density matrix has decomposition:

$$\rho^I \sim \tilde{A}^I I_4 + \sum_i \left(\tilde{B}_i^{I+} \sigma_i \otimes I_2 + \tilde{B}_i^{I-} I_2 \otimes \sigma_i + \sum_{i,j} \tilde{C}_{ij}^I \sigma_i \otimes \sigma_j \right)$$

Contribution from partonic channel *I*

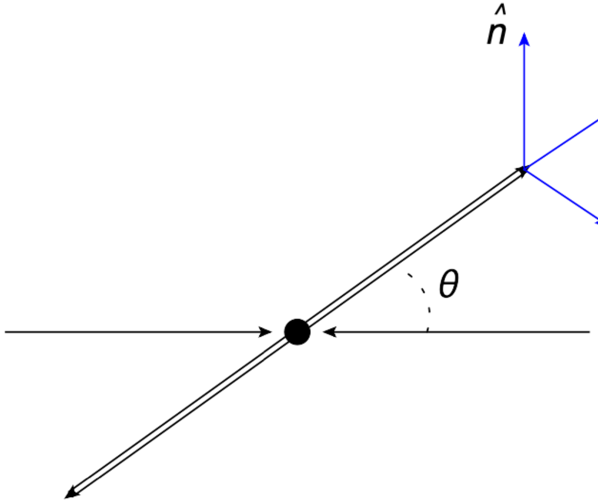
Identity matrix

Identity matrix

Pauli matrix

- The *Fano coefficients* $\{\tilde{A}^I, \tilde{B}_i^{I\pm}, \tilde{C}_{ij}^I\}$ depend on the top quark kinematics…
- …as well as the basis relating spin directions (1,2,3) to physical space.
- A common choice is the *helicity basis*.

The helicity basis



- In the helicity basis, one chooses an axis parallel to the top quark direction and two transverse directions (Baumgart, Tweedie).
- Each Fano coefficient is then a function of

$$z = \cos\theta, \quad \beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}.$$

- At LO in the SM, one has:

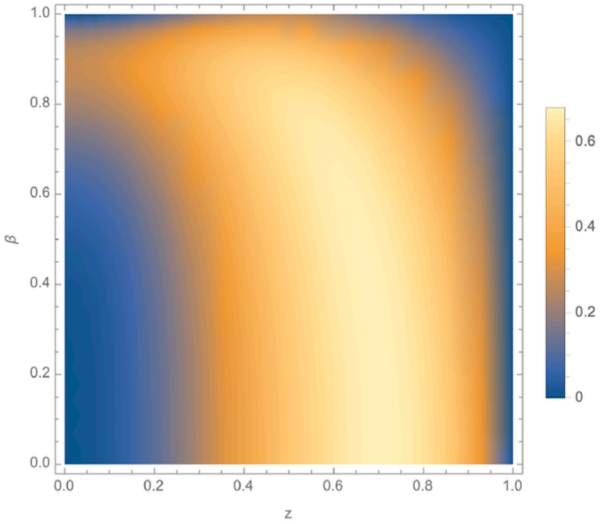
$$\tilde{B}_i^{I+} = \tilde{B}_i^{I-} = \tilde{C}_{nr}^{I} = \tilde{C}_{nk}^{I} = 0, \quad \tilde{C}_{ij}^{I} = \tilde{C}_{ji}^{I}$$

- The SSRE can be corrected for mixed states (Leone, Oliviero, Hamma), and yields

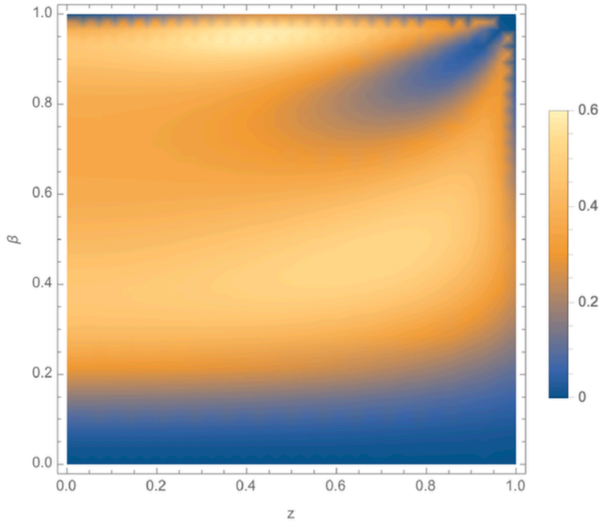
$$\tilde{M}_2(\rho^I) = -\log_2\left(\frac{(\tilde{A}^I)^4 + (\tilde{C}_{nn}^I)^4 + (\tilde{C}_{kk}^I)^4 + (\tilde{C}_{rr}^I)^4 + 2(\tilde{C}_{rk}^I)^4}{(\tilde{A}^I)^2[(\tilde{A}^I)^2 + (\tilde{C}_{nn}^I)^2 + (\tilde{C}_{kk}^I)^2 + (\tilde{C}_{rr}^I)^2 + 2(\tilde{C}_{rk}^I)^2]}\right)$$

Results: parton level

- We can now see how magic top quarks are! 😀
- The magic is concentrated away from extreme kinematic limits (e.g. threshold, high energy).



$q\bar{q}$



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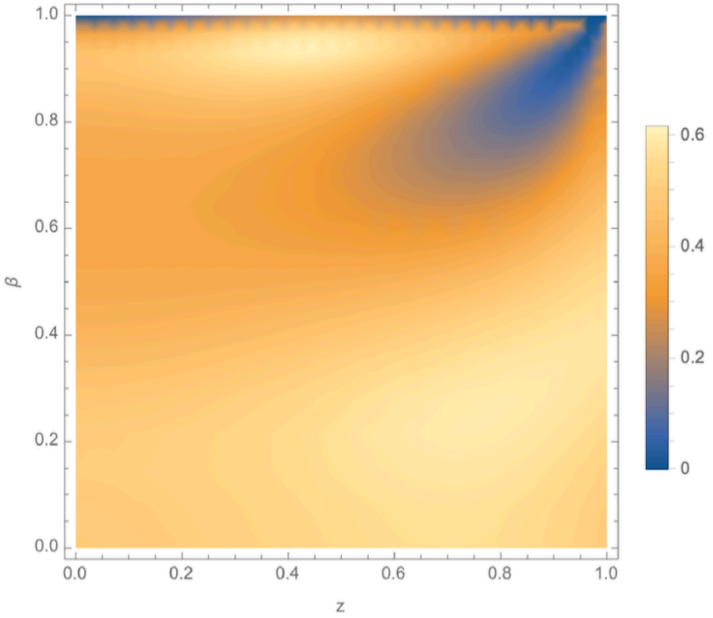
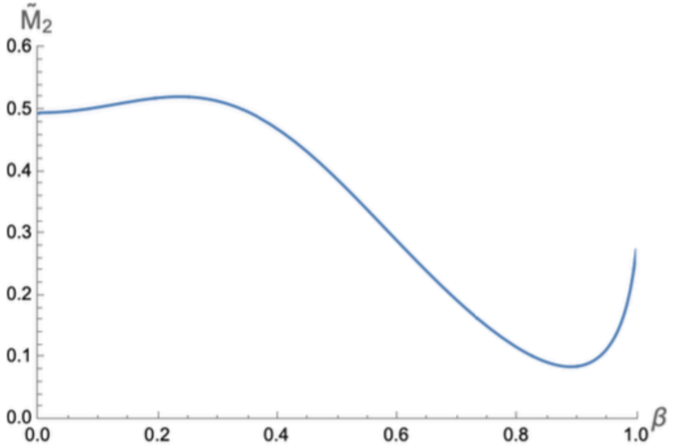
- It is known that the top quark final state becomes separable and / or maximally entangled in these regions.
- These happen to be stabiliser states, and hence the magic vanishes.
- Magic offers more information than entanglement, as expected.

Results: hadron level

- Can also calculate results at hadron level, upon which some regions of zero magic disappear.
- This is not surprising: combining different channels leads to more of a mixed state, which can increase the magic.

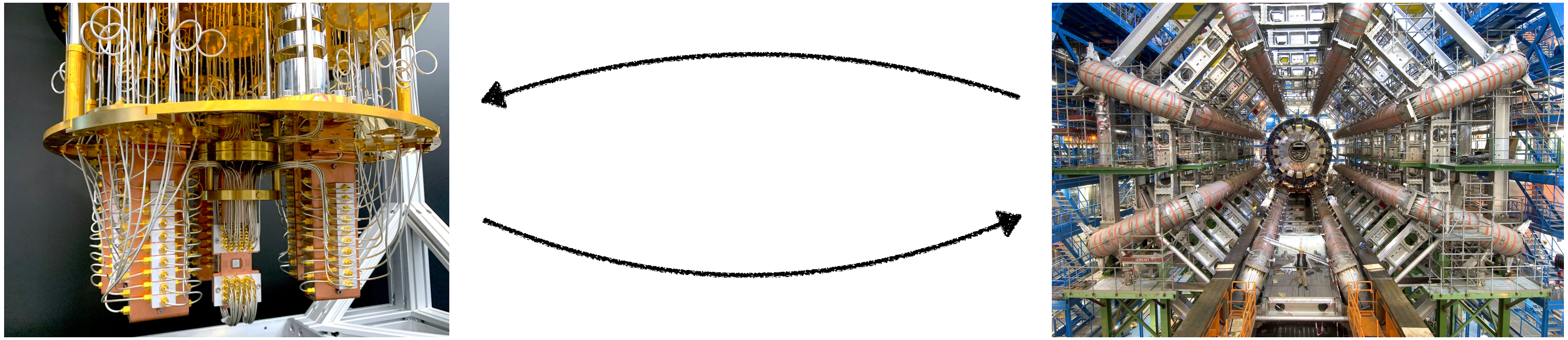




- Other increases in magic are observed after averaging over scattering angles.

What’s the use?

- Top quarks provide a system in which magic can be produced and studied…
- …and is **tuneable** using event selection.
- Might it provide useful insights into how to make magic in other systems?
- Can one use magic as a useful observable for new physics?
- Or strengthen the dialogue between Quantum Computing / Collider Physics?



Conclusions

- Magic is a property of quantum states that distinguishes computational advantage over classical computers.
- It might also be useful for collider physics systems.
- We have shown that top quark pairs are naturally magic…
- …and that this provides complementary information to entanglement alone.
- Our results create new links between Quantum Computation / Collider Physics.
- This is just a start - there is much more that can be done.

Open Questions

- Can magic be a useful probe of BSM physics? (Aoude, Banks, White^2)
- What about the other Rényi entropies? Are these useful?
- How about magic in other collider processes?
- Are there useful insights for Quantum Computation / Information theory?
- What other quantities or concepts from QC / QI are useful for colliders, and vice versa?