The top doesn't just spin, it also orbits!

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Rescuing L
from oblivion

Rescuing *L* from oblivion

We have been hearing about measuring spin of top, *W*, *Z*, even τ, for decades.

But probably, last time you studied orbital angular momentum (OAM) was in your degree. Why is it so?

OAM cannot be directly measured from angular distributions!

Yet, it is there. Consider for example $H \to VV$, with $V = W,Z$

Initial state J = 0 → Final state J = 0

total spin of *VV* pair: 0,1,2 ⇒ $\ell = 0,1,2$

Rescuing *L* from oblivion

Let's introduce a reference system (x,y,z) in the *H* rest frame. Decay amplitudes using a fixed spin quantisation axis $\hat{z}$ [whatever] look like

$$A^c_{11} = [\cdots]\, Y_2^{-2}(\Omega)$$

$$A^c_{10} = [\cdots]\, Y_1^{-1}(\Omega) + [\cdots]\, Y_2^{-1}(\Omega)$$

$$A^c_{1-1} = [\cdots]\, Y_0^{0}(\Omega) + [\cdots]\, Y_1^{0}(\Omega) + [\cdots]\, Y_2^{0}(\Omega)$$

$$\cdots$$

s_1, s_2: 3rd spin components for V_1, V_2 in $\hat{z}$ axis

There are 9 amplitudes. c superscript stands for `canonical´ as opposed to the commonly-used helicity amplitudes

with Ω=(θ,ϕ) the angles of [say] V_1 in *H* rest frame

Note:

- spherical harmonics up to $\ell = 2$
- $s_1 + s_2 + m = 0$

OAM is there! And there it is, just like it should be!

Rescuing *L* from oblivion

Hmm... ok... but how do the amplitudes get an angular dependence?

$$A^c_{11} = [\cdots]\, Y_2^{-2}(\Omega)$$

$$A^c_{10} = [\cdots]\, Y_1^{-1}(\Omega) + [\cdots]\, Y_2^{-1}(\Omega)$$

$$A^c_{1-1} = [\cdots]\, Y_0^{0}(\Omega) + [\cdots]\, Y_1^{0}(\Omega) + [\cdots]\, Y_2^{0}(\Omega)$$

$$\cdots$$

Setting a particular value for the third spin component along a particular direction $\hat{z}$ breaks isotropy in the Higgs decay.

This is in contrast with helicity amplitudes which are just numbers

$$A^h_{11} = a_{11}$$

$$A^h_{00} = a_{00}$$

$$A^h_{-1-1} = a_{-1-1}$$

Only 3 amplitudes

Rescuing *L* from oblivion

But wait... the Higgs is a scalar, and scalar decays are isotropic, how is this possible?

density operator for *L*

$\ell = 2$ ...

$\ell = 0$

$$\rho_L = \begin{pmatrix} 0.021 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.021 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.021 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.021 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.021 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.895 \end{pmatrix}$$

$$\sum_{m=-2}^{m=2} |Y_m^2(\theta,\phi)|^2 = \frac{5}{4\pi}$$

→ all (θ,ϕ) dependence is lost

OAM cannot be directly measured from angular distributions!

[of course, you can calculate it, from initial state and decay amplitudes]

Rescuing *L* from oblivion

Q: OK, but I have seen helicity amplitudes, and I never saw OAM there.

A: Sure, in the helicity direction $\vec{p}$ the third component of OAM vanishes

$$m = 0$$

because $\vec{L} = \vec{r} \times \vec{p}$. This does not imply

$$\ell = 0$$

Actually, this property makes helicity amplitudes simpler in order to extract parameters from experimental measurements.

For $H \to VV$ there are 3 helicity amplitudes vs 9 canonical amplitudes.
For $t \to bW$ it is 4 vs 26!

Rescuing *L* from oblivion

Entanglement measurements involving OAM are rare — and never done in HEP!

HOME > SCIENCE > VOL. 361, NO. 6407 > QUANTUM ENTANGLEMENT OF THE SPIN AND ORBITAL ANGULAR MOMENTUM OF PHOTONS USING METAMATERIALS

REPORT

Quantum entanglement of the spin and orbital angular momentum of photons using metamaterials

TOMER STAV, ARKADY FAERMAN, ELHANAN MAGUID, DIKLA OREN, VLADIMIR KLEINER, EREZ HASMAN, AND MORDECHAI SEGEV Authors Info & Affiliations

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nature > letters > article

Letter | Published: 19 July 2001

Entanglement of the orbital angular momentum states of photons

Alois Mair, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger

Nature **412**, 313–316 (2001) | Cite this article

take your new physics hats off, enjoy new SM measurements!

The trick

The trick

If one is able to experimentally measure

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- initial spin state ρ_t ← already done, I will skip it
- canonical decay amplitudes $A_{M s_1 s_2; lm}$

then, one is able to determine the full density operator ρ_{LWb} that fully describes the top decay, also including OAM!

$$(\rho_{LWb})_{s_1 s_2; lm}^{s_1' s_2'; l' m'} = (\rho_t)_{MM'} A_{M s_1 s_2; lm} A^*_{M' s_1' s_2'; l' m'}$$

Fortunately, most of these quantities can be obtained from observables already measured by ATLAS.

The trick

$$A_{\frac{1}{2}1\frac{1}{2};1-1} = A_{-\frac{1}{2}-1-\frac{1}{2};11} = \sqrt{\frac{\pi}{6}}\mathcal{F}_1^- , \qquad A_{\frac{1}{2}1\frac{1}{2};2-1} = -A_{-\frac{1}{2}-1-\frac{1}{2};21} = \sqrt{\frac{\pi}{30}}\mathcal{F}_2 ,$$

$$A_{\frac{1}{2}0\frac{1}{2};00} = -A_{-\frac{1}{2}0-\frac{1}{2};00} = \sqrt{\frac{\pi}{9}}\mathcal{F}_0 , \qquad A_{\frac{1}{2}0\frac{1}{2};10} = A_{-\frac{1}{2}0-\frac{1}{2};10} = \sqrt{\frac{\pi}{3}}\mathcal{F}_1^0 ,$$

$$A_{\frac{1}{2}0\frac{1}{2};20} = -A_{-\frac{1}{2}0-\frac{1}{2};20} = -\sqrt{\frac{2\pi}{45}}\mathcal{F}_2 , \qquad A_{\frac{1}{2}-1\frac{1}{2};11} = A_{-\frac{1}{2}1-\frac{1}{2};1-1} = \sqrt{\frac{\pi}{6}}\mathcal{F}_1^+ ,$$

$$A_{\frac{1}{2}-1\frac{1}{2};21} = -A_{-\frac{1}{2}1-\frac{1}{2};2-1} = \sqrt{\frac{\pi}{30}}\mathcal{F}_2 , \qquad A_{\frac{1}{2}1-\frac{1}{2};00} = -A_{-\frac{1}{2}-1\frac{1}{2};00} = -\sqrt{\frac{2\pi}{9}}\mathcal{F}_0 ,$$

$$A_{\frac{1}{2}1-\frac{1}{2};10} = A_{-\frac{1}{2}-1\frac{1}{2};10} = \sqrt{\frac{\pi}{3}}\mathcal{F} , \qquad A_{\frac{1}{2}1-\frac{1}{2};20} = -A_{-\frac{1}{2}-1\frac{1}{2};20} = -\sqrt{\frac{\pi}{45}}\mathcal{F}_2 ,$$

$$A_{\frac{1}{2}0-\frac{1}{2};11} = A_{-\frac{1}{2}0\frac{1}{2};1-1} = -\sqrt{\frac{\pi}{3}}\mathcal{F} , \qquad A_{\frac{1}{2}0-\frac{1}{2};21} = -A_{-\frac{1}{2}0\frac{1}{2};2-1} = \sqrt{\frac{\pi}{15}}\mathcal{F}_2 ,$$

$$A_{\frac{1}{2}-1-\frac{1}{2};22} = -A_{-\frac{1}{2}1\frac{1}{2};2-2} = -\sqrt{\frac{2\pi}{15}}\mathcal{F}_2 .$$

$$\mathcal{F} = a_{-1-\frac{1}{2}} - a_{1\frac{1}{2}} , \quad \mathcal{F}_1^0 = a_{0-\frac{1}{2}} - a_{0\frac{1}{2}} ,$$

$$\mathcal{F}_0 = \sqrt{2}(a_{1\frac{1}{2}} + a_{-1-\frac{1}{2}}) + a_{0\frac{1}{2}} + a_{0-\frac{1}{2}} ,$$

$$\mathcal{F}_1^\pm = \pm(a_{-1-\frac{1}{2}} - a_{1\frac{1}{2}}) + \sqrt{2}(a_{0\frac{1}{2}} - a_{0-\frac{1}{2}}) ,$$

$$\mathcal{F}_2 = a_{-1-\frac{1}{2}} + a_{1\frac{1}{2}} - \sqrt{2}(a_{0\frac{1}{2}} + a_{0-\frac{1}{2}})$$

$a_{\lambda_1\lambda_2}$ are the helicity amplitudes

The trick

Use helicity basis to <u>measure</u> $a_{\lambda_1\lambda_2}$ in data

model-independent

obtain canonical amplitudes from $a_{\lambda_1\lambda_2}$

model-independent

obtain ρ_{LWb} from data

Measuring helicity amplitudes

Measuring helicity amplitudes

To obtain helicity amplitudes $a_{\lambda_1 \lambda_2}$ we ignore OAM for a moment and work in the helicity basis [of course!]

4D angular distribution → helicity amplitudes

The relevant angles are well known:

$(\theta^*_\ell, \phi^*_\ell)$ of ℓ in W rest frame

(θ, ϕ) of W in t rest frame

JAAS, Boudreau, Escobar, Müller 1702.03297

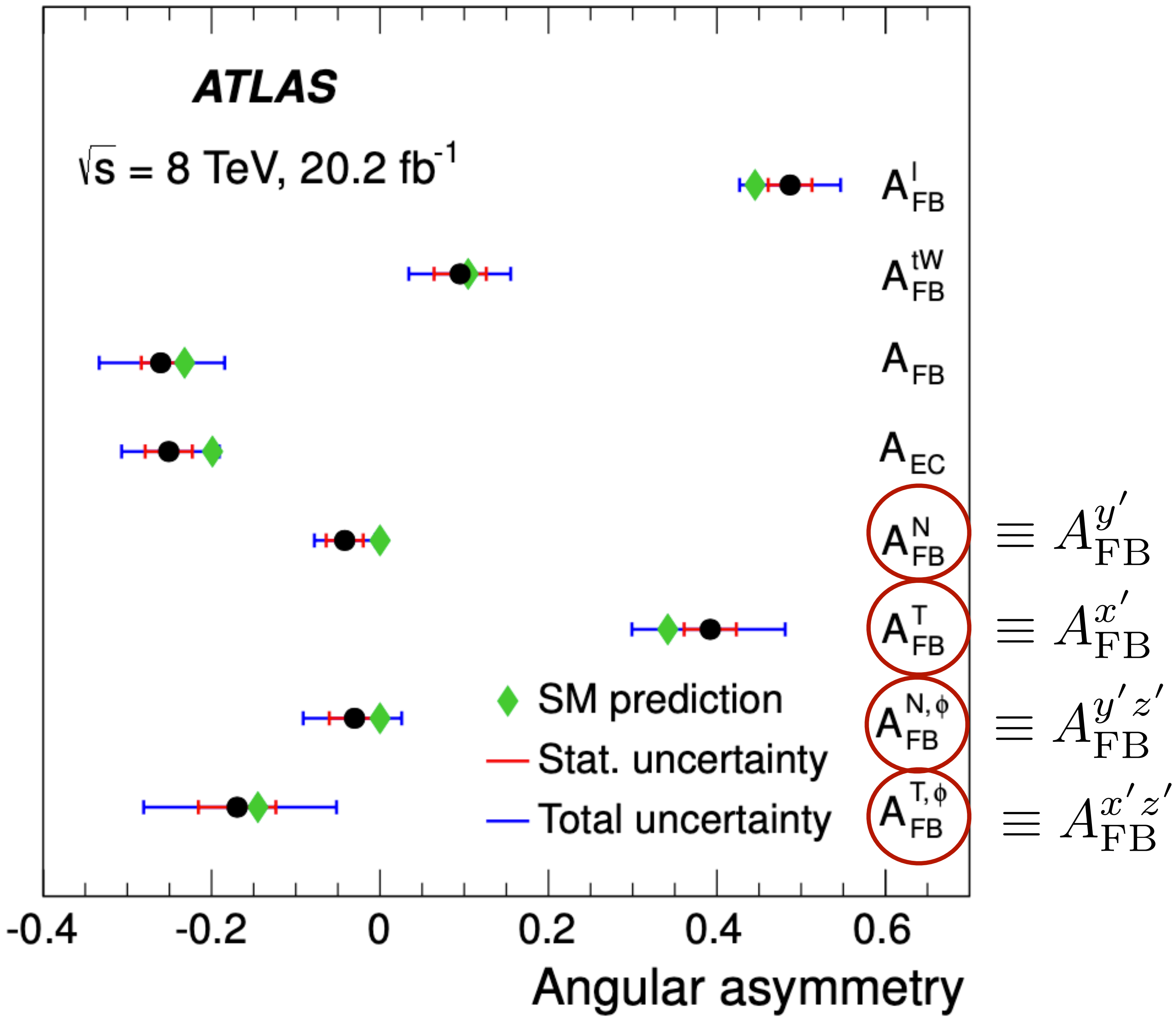
W helicity fractions

\+

$A^{z,z'}_{\mathrm{FB,EC}}$ → $|a_{-1-\frac{1}{2}}|, \; |a_{0-\frac{1}{2}}|, \; |a_{0\frac{1}{2}}|, \; |a_{1\frac{1}{2}}|$

$A^{z,z'}_{\mathrm{FB,EC}}$ → *not yet measured*

Measuring helicity amplitudes



$\arg a_{0\frac{1}{2}} a^*_{1\frac{1}{2}}$

$\arg a_{-1-\frac{1}{2}} a^*_{0-\frac{1}{2}}$

What about $\arg a_{0\frac{1}{2}} a^*_{0-\frac{1}{2}}$? Can't be measured without measuring *b* polarisation, unknown phase.

But don't panic!

Bipartite and tripartite entanglement

Bipartite and tripartite entanglement

There are several points one can address from ρ_{LWb}:

- Is tripartite entanglement genuine?
- Are *A*, *B* subsystems entangled when *C* is marginalised?

They can be addressed using Peres-Horodecki sufficient condition for entanglement, and using the entanglement measure [negativity]

$$N(\rho) = \frac{\|\rho^{T_B}\| - 1}{2}$$

for *A*, *B* any subsystems of $\mathcal{H}_L \otimes \mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2}$

W *b*

Bipartite and tripartite entanglement

Tripartite entanglement is genuine if the state is entangled under any bipartition of $\mathcal{H}_L \otimes \mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2}$

$N > 0 \Rightarrow$ entanglement

A	B	N(ρ)
$\mathcal{H}_L$	$\mathcal{H}_W \otimes \mathcal{H}_b$	1.65
$\mathcal{H}_W$	$\mathcal{H}_L \otimes \mathcal{H}_b$	0.80
$\mathcal{H}_b$	$\mathcal{H}_L \otimes \mathcal{H}_W$	0.50

- Entanglement is very large in all cases!
- These are SM values for top [not anti-top] with $P_z = 1$
- For single-top production, slightly smaller

Bipartite and tripartite entanglement

Given the three subsystems *A*, *B*, *C*, we can marginalise *C* [trace over its space] and obtain the entanglement between *A* and *B*

A	B	N(ρ)
$\mathcal{H}_L$	$\mathcal{H}_W$	0.62
$\mathcal{H}_L$	$\mathcal{H}_b$	0.40
$\mathcal{H}_W$	$\mathcal{H}_b$	0.01

- *C* is the unlisted subsystem in all cases.
- These are SM values for top [not anti-top] with $P_z = 1$

Bipartite and tripartite entanglement

What about $\arg a_{0\frac{1}{2}} a^*_{0-\frac{1}{2}}$ which in principle is not measured?

This is the phase between amplitudes with $\lambda = 1/2$ and $\lambda = -1/2$ for *b*.

Since the former are quite suppressed by the small m_b, this phase is fairly irrelevant in order to establish entanglement.

Entanglement	SM phase	[0,2π]
L-(*Wb*)	1.653	[1.653,1.678]
W-(L*b*)	0.795	diff = 10^{-6}
b-(L*W*)	0.496	[0.495,0.496]

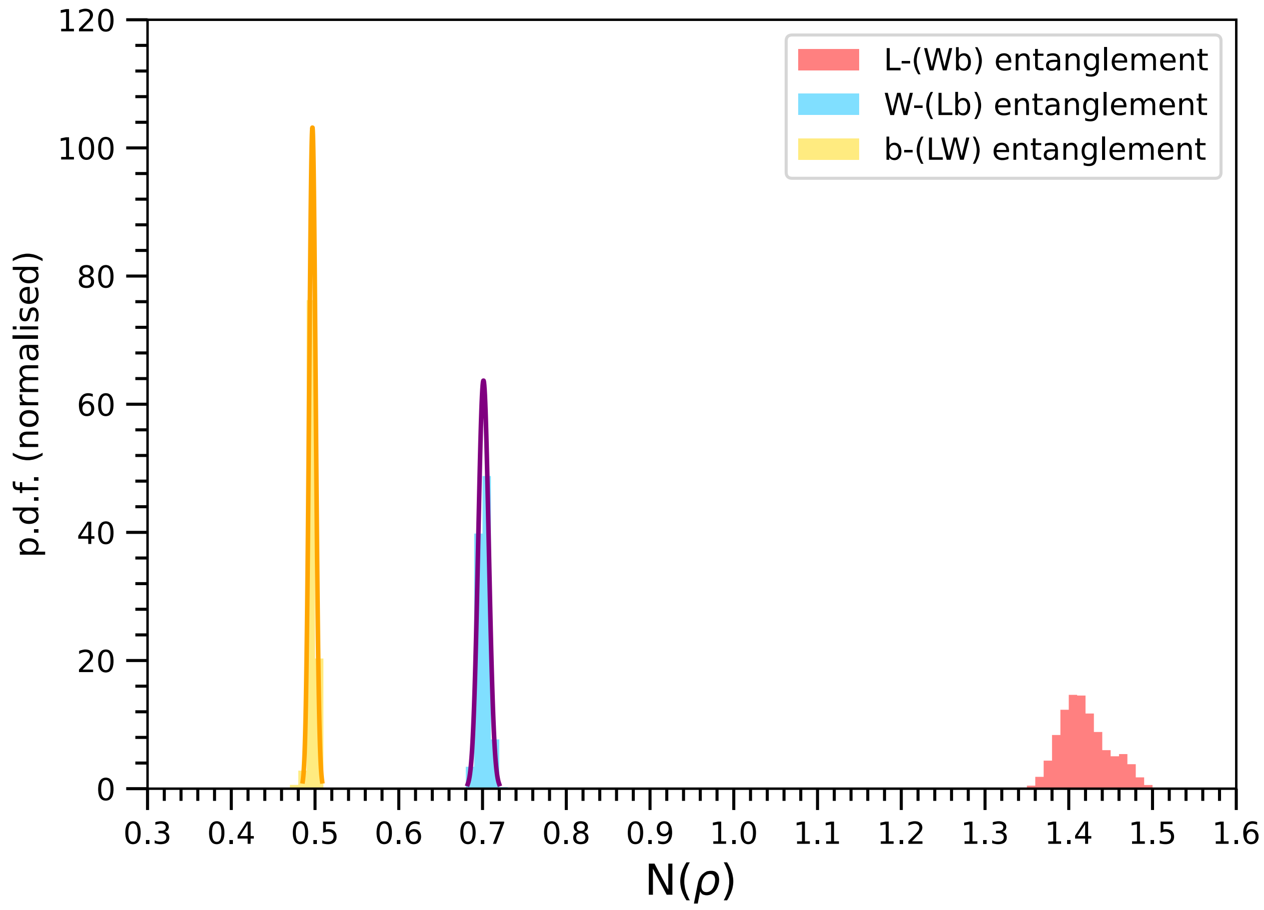
Entanglement	SM phase	[0,2π]
L-*W*	0.615	[0.615,0.628]
L-*b*	0.396	[0.396,0.411]

Expected uncertainty

Expected uncertainty in *t*-channel single top

Pseudo-experiments performed assuming 50K events

[number obtained from Run 2 ATLAS measurement]



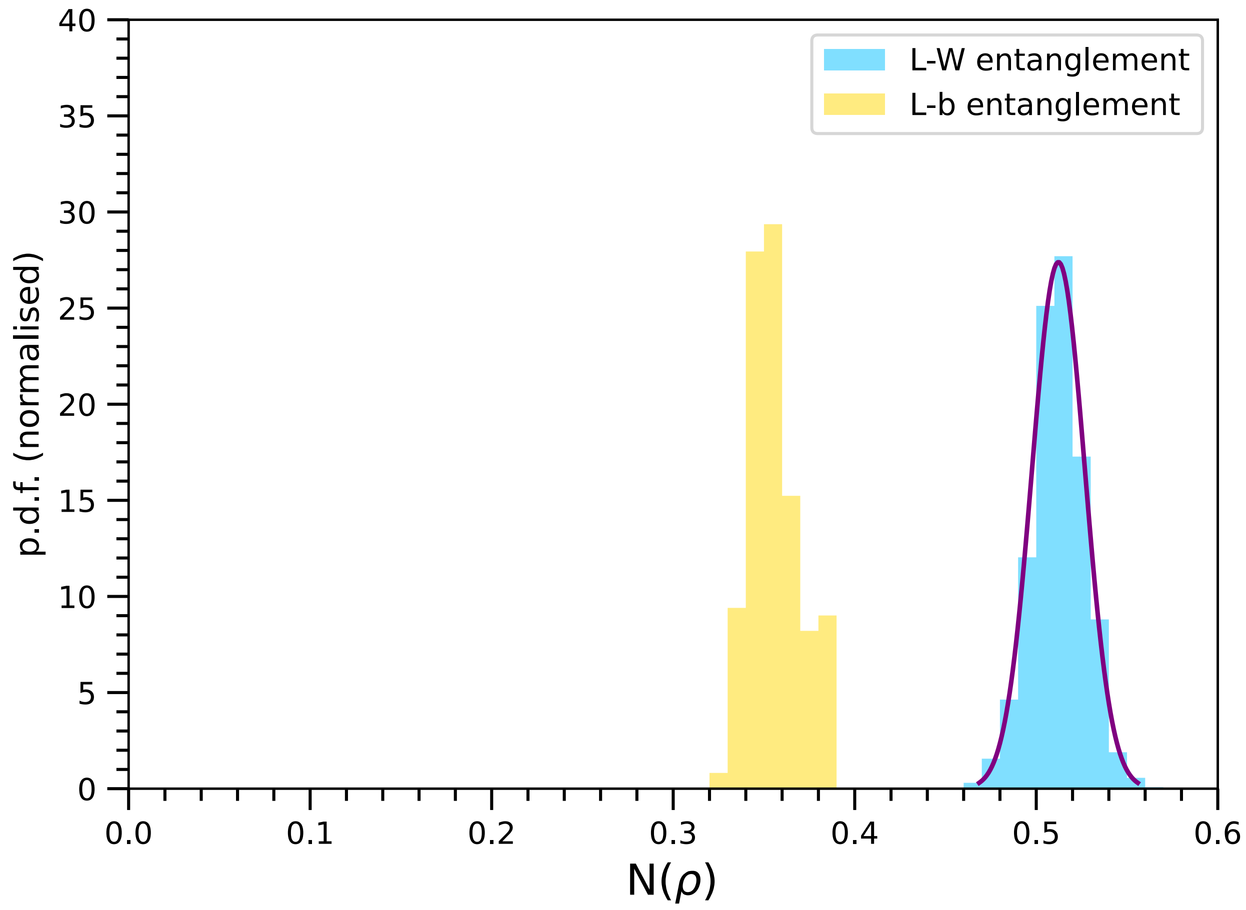
Tripartite entanglement

	Run 2
L-(*Wb*)	≫5σ
W-(L*b*)	≫5σ
b-(L*W*)	≫5σ

Expected uncertainty in *t*-channel single top

Pseudo-experiments performed assuming 50K events

[number obtained from Run 2 ATLAS measurement]



Bipartite entanglement

	Run 2
L-*W*	≫5σ
L-*b*	≫5σ

Expected uncertainty in *t*-channel single top

Systematics in entanglement measurements can be estimated using actual uncertainties in Run 1 / Run 2 measurements of the input observables

	$N(\rho_{LW})$	$N(\rho_{Lb})$	$N(\rho_{L[Wb]})$	$N(\rho_{W[Lb]})$	$N(\rho_{b[LW]})$
P_1	$^{+0.01}_{-0}$	$^{+0.003}_{-0}$	$^{+0.03}_{-0}$	$^{+0.044}_{-0.003}$	$^{+0.021}_{-0.003}$
P_2	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0.001}_{-0}$	$^{+0}_{-0}$
P_3	$^{+0.015}_{-0.024}$	$^{+0}_{-0.013}$	$^{+0.091}_{-0.063}$	$^{+0.057}_{-0.027}$	$^{+0.008}_{-0.003}$
F_+	$^{+0}_{-0.02}$	$^{+0}_{-0.034}$	$^{+0}_{-0.054}$	$^{+0.004}_{-0}$	$^{+0}_{-0}$
F_0	$^{+0.003}_{-0.023}$	$^{+0.007}_{-0.017}$	$^{+0.036}_{-0.033}$	$^{+0.028}_{-0.025}$	$^{+0.008}_{-0}$
$A_{\mathrm{FB,EC}}^{z,z'}$	$^{+0.05}_{-0.042}$	$^{+0}_{-0.1}$	$^{+0.078}_{-0.006}$	$^{+0.054}_{-0.003}$	$^{+0.008}_{-0.041}$
$A_{\mathrm{FB}}^{x'}$	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0}_{-0}$
$A_{\mathrm{FB}}^{y'}$	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0.002}_{-0}$	$^{+0}_{-0}$	$^{+0}_{-0}$
$A_{\mathrm{FB}}^{x'z'}$	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0}_{-0}$	$^{+0}_{-0}$
$A_{\mathrm{FB}}^{y'z'}$	$^{+0.008}_{-0}$	$^{+0}_{-0.002}$	$^{+0.023}_{-0}$	$^{+0.01}_{-0}$	$^{+0}_{-0}$
Total	$^{+0.054}_{-0.057}$	$^{+0.008}_{-0.11}$	$^{+0.13}_{-0.09}$	$^{+0.095}_{-0.038}$	$^{+0.026}_{-0.041}$

educated guess for this observable

Expected uncertainty in *t*-channel single top

Entanglement significance including systematics

	Run 2
L-(*Wb*)	15σ
W-(L*b*)	18σ
b-(L*W*)	12σ
L-*W*	8.7σ
L-*b*	3.2σ

Remarks

- ☑ Possible to measure right now entanglement between OAM and spin
- ☑ Possible to measure right now tripartite entanglement
- ☑ Doing the same as previously done, using available data

End

Quantum Entanglement: basics

Quantum entanglement: basics

The state of a system composed by two sub-systems A and B is separable if it can be written as

$$|\psi\rangle = |a\rangle_A \otimes |b\rangle_B$$

Otherwise, it is entangled, e.g. something like

$$|\psi\rangle = |a_1\rangle_A \otimes |b_1\rangle_B + |a_2\rangle_A \otimes |b_2\rangle_B$$

A typical example of entanglement is the combination of two spin-1/2 systems in the spin-0 configuration

$$|\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B$$

General systems are not described by pure states $|\psi\rangle$ but by density operators ρ.

Quantum entanglement: basics

Any operator cannot be a density operator. A valid density operator has several characteristics:

- Unit trace
- Hermitian
- Positive semidefinite: eigenvalues ≥ 0

A density operator describing a composite system is separable if it can be written as

$$\rho_{\mathrm{sep}} = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

Note: in general, one has something like

$$\rho = \sum_{ijkl} p_{ij}^{kl} |\psi_i\rangle\langle\psi_j| \otimes |\psi_k\rangle\langle\psi_l|$$

Quantum entanglement: basics

Necessary criterion for separability:

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

taking the partial transpose in subspace of B [for example] the resulting density operator is valid.

→ it has non-negative eigenvalues [unit trace and hermicity automatic]

Example: composite system A ⊗ B with dim $\mathcal{H}_A$ = n, dim $\mathcal{H}_B$ = m

P_{ij} are m x m matrices, $(P_{ij})^{kl} = p_{ij}^{kl}$

$$\rho = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & & \\ \vdots & & \ddots & \\ P_{n1} & & & P_{nn} \end{pmatrix} \longrightarrow \rho^{T_2} = \begin{pmatrix} P_{11}^T & P_{12}^T & \cdots & P_{1n}^T \\ P_{21}^T & P_{22}^T & & \\ \vdots & & \ddots & \\ P_{n1}^T & & & P_{nn}^T \end{pmatrix}$$

(n∗m) × (n∗m) matrix

Quantum entanglement: basics

To take away:

- It is quite complicated to prove [analytically] that a composite system is in a separable state.
- However, we are interested in showing that the system is entangled.
- To prove that, in some systems there are simple sufficient conditions that do the work
    - two spin-1/2 particles
    - $H \to VV$ [bipartite]
- Otherwise, use directly the counter-reciprocal of Peres-Horodecki necessary condition

$$\rho^{T2} \text{ non-positive} \Rightarrow \rho^{T2} \text{ not valid} \Rightarrow \text{system entangled}$$