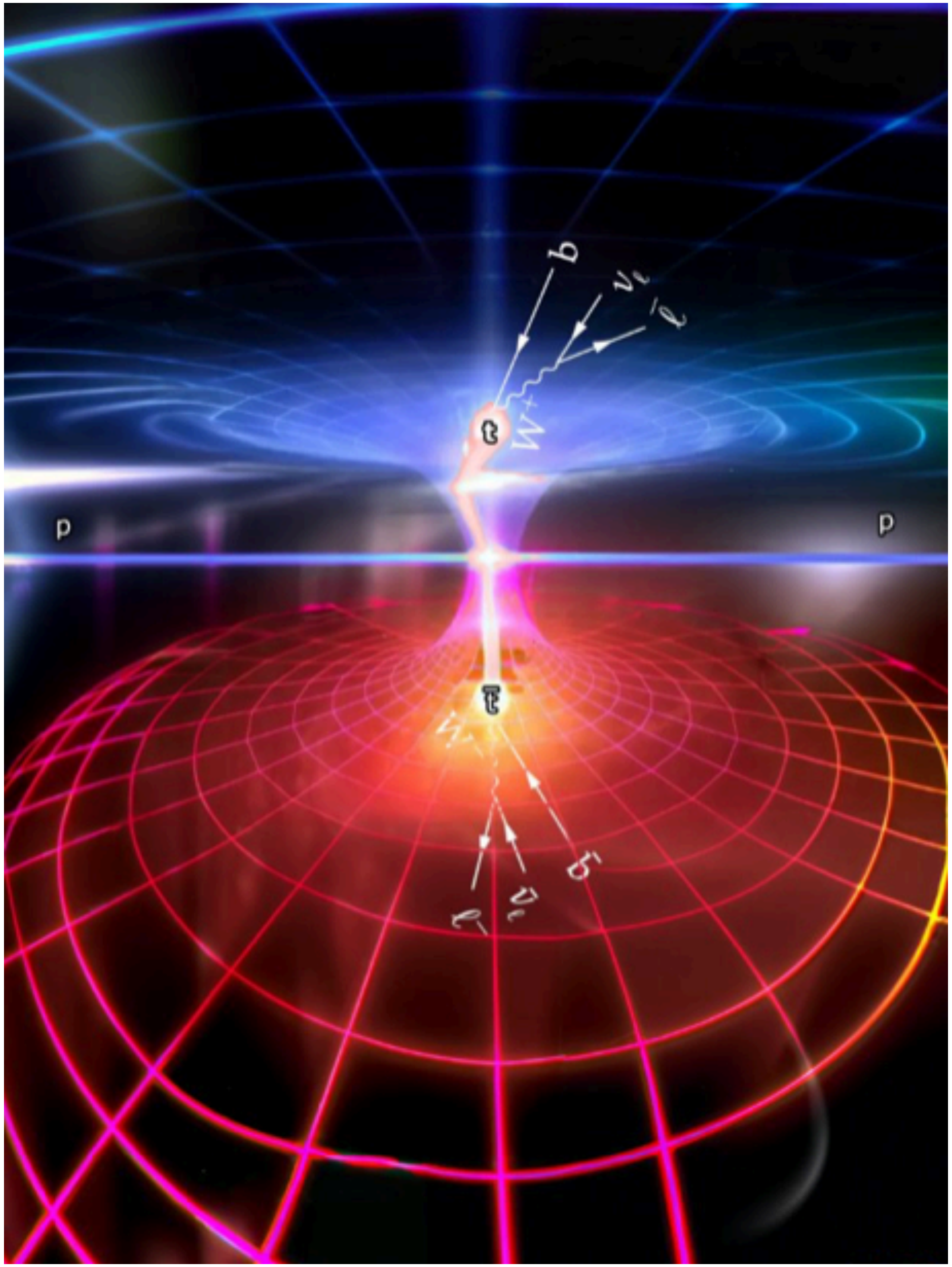
Probing entanglement in top quark production with the CMS detector

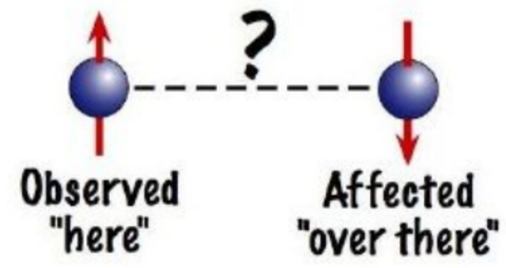
Giulia Negro
on behalf of the CMS Collaboration

17th International Workshop on Top Quark Physics (TOP2024)

26 September 2024



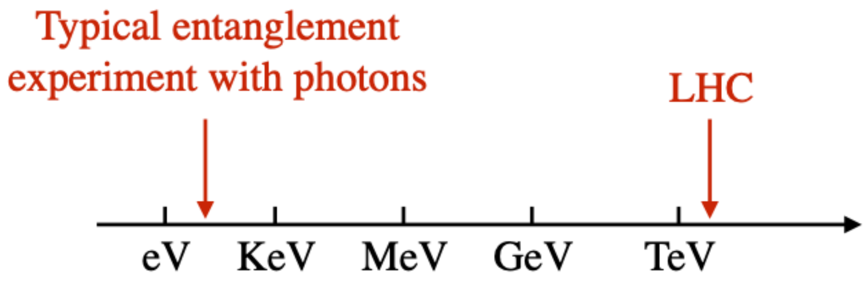
Entanglement at LHC



- **Qubit** (two-level quantum system $|0\rangle, |1\rangle$) = most simple quantum system
  → **two qubits** = most simple example of quantum correlations

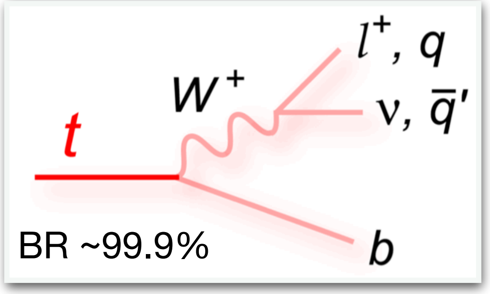
- **Entanglement = non-separability of a quantum state**

- LHC can provide a unique environment to study entanglement
  - **simplest qubits at LHC =** $t\bar{t}$ pair



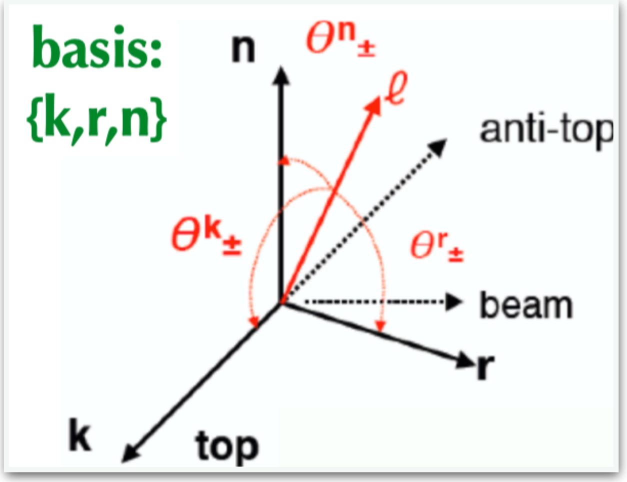
- Top quark = ideal candidate for spin measurements:
  - extremely short lifetime allows measuring polarization and spin correlation in tt̄ production
  - spin information preserved in the angular distribution of its decay products
  - spin information ~100% transmitted to charged leptons and down type quarks

$$\underbrace{\frac{1}{m_t}}_{\substack{\text{production}\\ 10^{-27}\ \text{s}}} < \underbrace{\frac{1}{\Gamma_t}}_{\substack{\text{lifetime}\\ 10^{-25}\ \text{s}}} < \underbrace{\frac{1}{\Lambda_{\text{QCD}}}}_{\substack{\text{hadronization}\\ 10^{-24}\ \text{s}}} < \underbrace{\frac{m_t}{\Lambda^2}}_{\substack{\text{spin-flip}\\ 10^{-21}\ \text{s}}}$$



BR ~99.9%

Spin correlations



- Probed by angular distribution of decay products in helicity basis:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2}\left(1 + \mathbf{B}^+\cdot\hat{\ell}^+ + \mathbf{B}^-\cdot\hat{\ell}^- - \hat{\ell}^+\cdot\mathbb{C}\cdot\hat{\ell}^-\right)$$

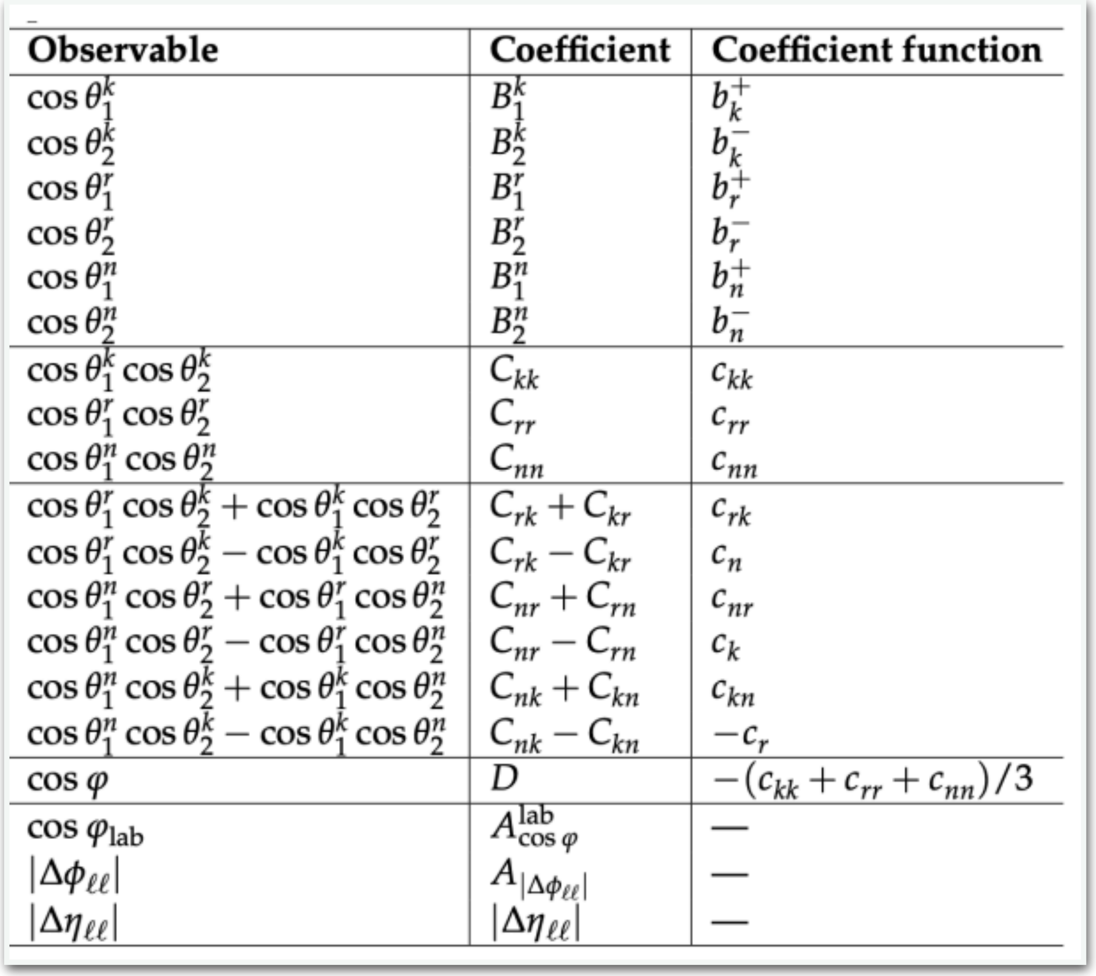
polarization

spin correlations

$$B^{+/-} = \begin{pmatrix} x \\ x \\ x \end{pmatrix} \quad C = \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}$$

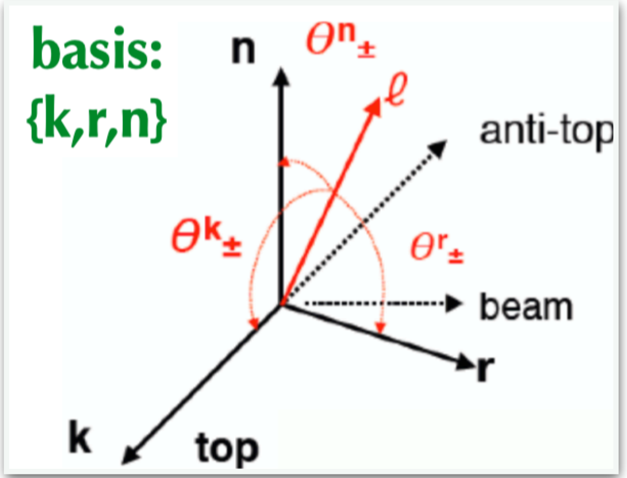
- Spin dependence of $t\bar{t}$ production **completely characterized by 15 coefficients**
  - can be probed individually by measuring 1D angular distributions

$$\boxed{\frac{1}{\sigma}\frac{d\sigma}{dx} = \frac{1}{2}\left(1 + [\mathbf{Coef.}]\, x\right) f(x)}$$



Observable	Coefficient	Coefficient function
$\cos\theta_1^k$	B_1^k	b_k^+
$\cos\theta_2^k$	B_2^k	b_k^-
$\cos\theta_1^r$	B_1^r	b_r^+
$\cos\theta_2^r$	B_2^r	b_r^-
$\cos\theta_1^n$	B_1^n	b_n^+
$\cos\theta_2^n$	B_2^n	b_n^-
$\cos\theta_1^k\cos\theta_2^k$	C_{kk}	c_{kk}
$\cos\theta_1^r\cos\theta_2^r$	C_{rr}	c_{rr}
$\cos\theta_1^n\cos\theta_2^n$	C_{nn}	c_{nn}
$\cos\theta_1^r\cos\theta_2^k + \cos\theta_1^k\cos\theta_2^r$	$C_{rk} + C_{kr}$	c_{rk}
$\cos\theta_1^r\cos\theta_2^k - \cos\theta_1^k\cos\theta_2^r$	$C_{rk} - C_{kr}$	c_n
$\cos\theta_1^n\cos\theta_2^r + \cos\theta_1^r\cos\theta_2^n$	$C_{nr} + C_{rn}$	c_{nr}
$\cos\theta_1^n\cos\theta_2^r - \cos\theta_1^r\cos\theta_2^n$	$C_{nr} - C_{rn}$	c_k
$\cos\theta_1^n\cos\theta_2^k + \cos\theta_1^k\cos\theta_2^n$	$C_{nk} + C_{kn}$	c_{kn}
$\cos\theta_1^n\cos\theta_2^k - \cos\theta_1^k\cos\theta_2^n$	$C_{nk} - C_{kn}$	$-c_r$
$\cos\varphi$	D	$-(c_{kk} + c_{rr} + c_{nn})/3$
$\cos\varphi_{\text{lab}}$	$A^{\text{lab}}_{\cos\varphi}$	—
$\lvert\Delta\phi_{\ell\ell}\rvert$	$A_{\lvert\Delta\phi_{\ell\ell}\rvert}$	—
$\lvert\Delta\eta_{\ell\ell}\rvert$	$\lvert\Delta\eta_{\ell\ell}\rvert$	—

Spin correlations



- Probed by angular distribution of decay products in helicity basis:

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polarization spin correlations

$$B^{+/-} = \begin{pmatrix} x \\ x \\ x \end{pmatrix} \quad C = \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}$$

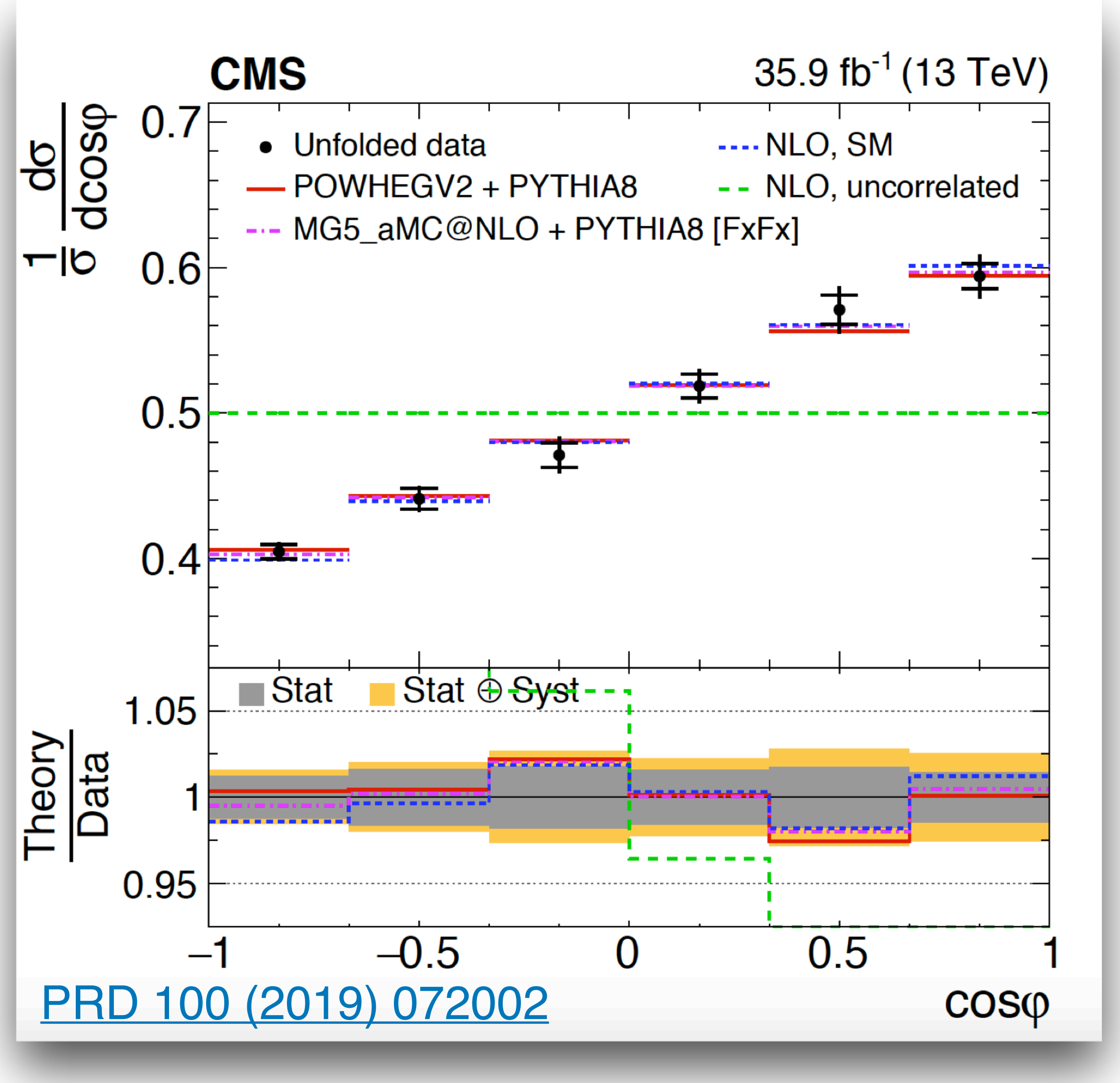
- **In SM, $t\bar{t}$ production ~ unpolarized but rich structure of spin correlations**

$$\frac{1}{\sigma}\frac{d\sigma}{dcos\varphi} = \frac{1}{2}(1 - Dcos\varphi) \text{ for}$$

$$D = -\frac{Tr[C]}{3} = -\frac{C_{kk} + C_{rr} + C_{nn}}{3}$$

$$cos\varphi = \hat{\ell}_1 \cdot \hat{\ell}_2$$

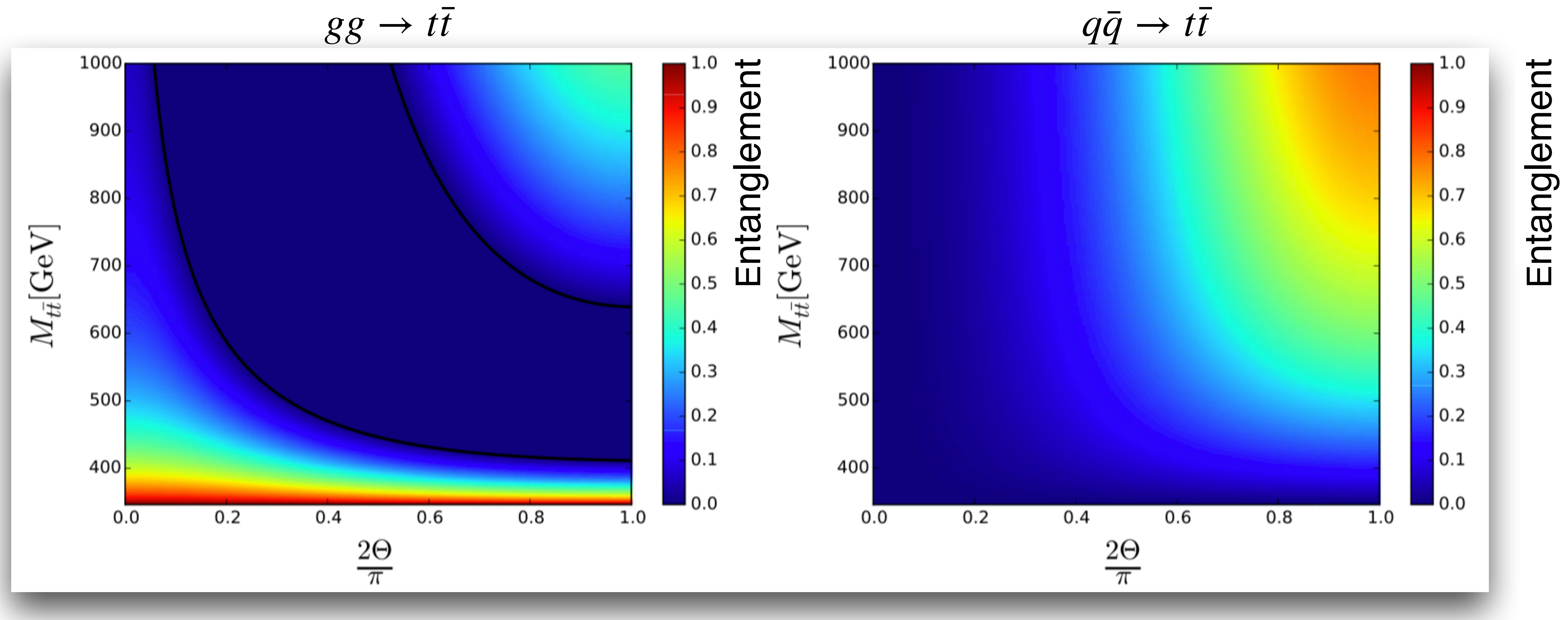
  - **most precise observable**
  - **maximal sensitivity to degree of alignment of top quark spins**
    → focus of entanglement measurement



PRD 100 (2019) 072002

Entanglement of top quarks

- **Can be measured using spin correlations variables**
- **Depends on production mode, $m_{t\bar{t}}$, scattering angle of the top quark (Θ)**



Afik, De Nova
Eur. Phys. J. Plus **136**, 907

Entanglement of top quarks

- Can be measured using spin correlations variables
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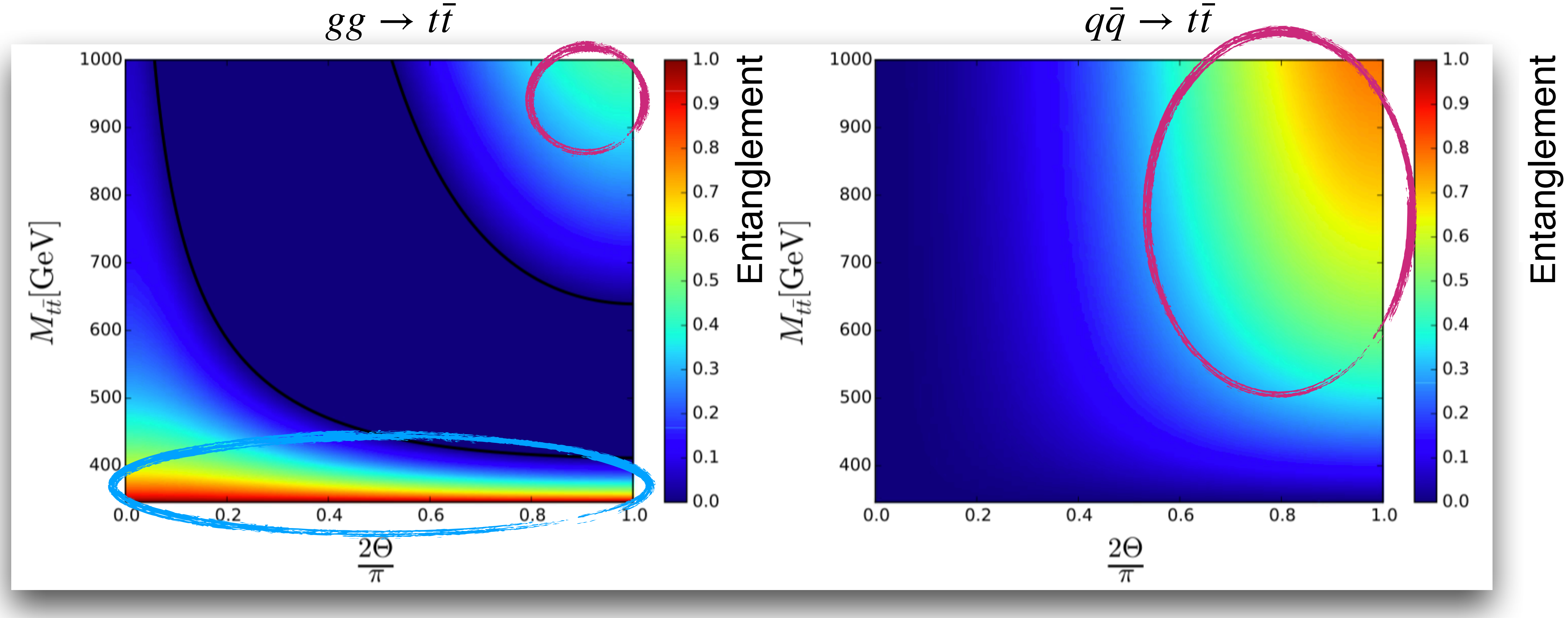
- SM predicts entangled states:
  - at the production threshold region in gg fusion production
  - at the boosted region for central production of the $t\bar{t}$ system



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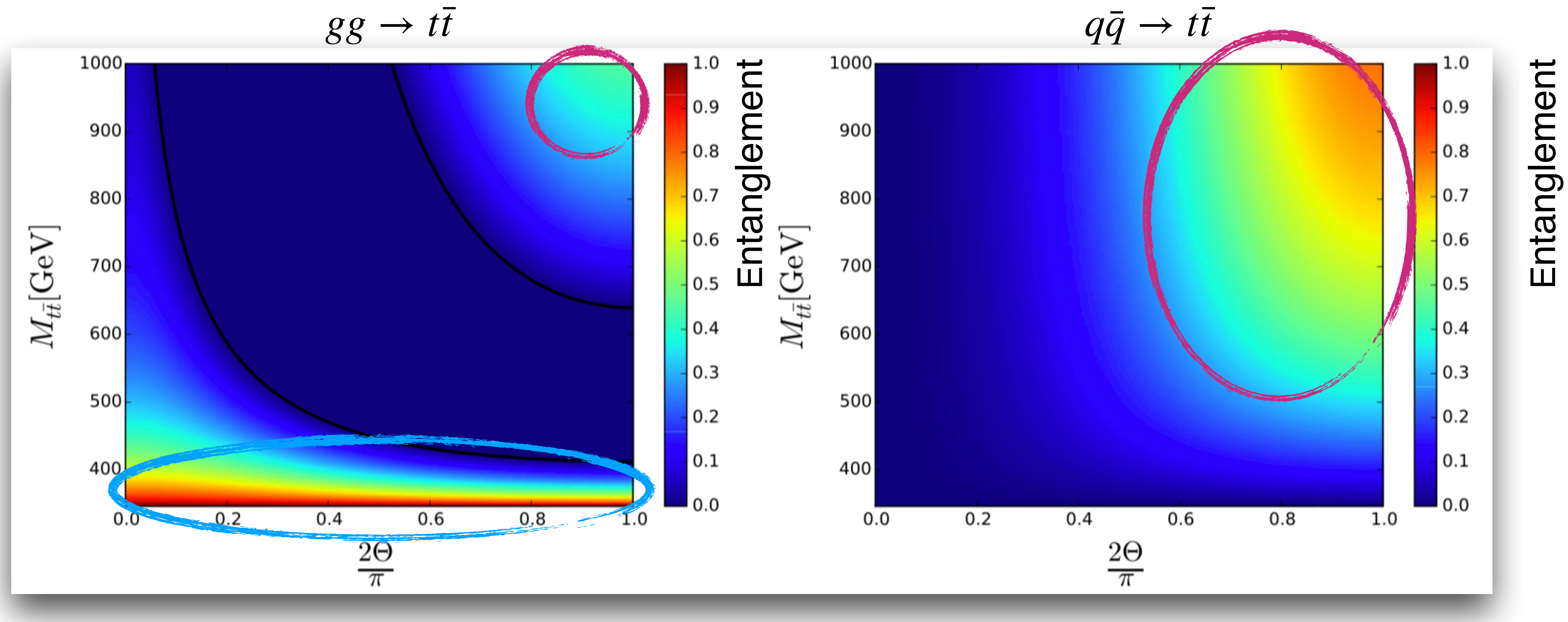
high relative velocity of top quarks → space-like separated events



low relative velocity of top quarks → time-like separated events

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How to probe entanglement

- **At the LHC, top quarks are produced in a mixed state**
  → can be represented as a density operator:

$$\rho = \frac{I_4 + \Sigma_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i\right) - \Sigma_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

$$B^{+/-} = \begin{pmatrix} x \\ x \\ x \end{pmatrix} \qquad C = \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}$$

- **Peres-Horodecki criterion**:
  if a system is separable, $\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$

  the transpose with respect to a subspace of ρ is a non-negative operator

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

  → system is entangled if at least 1 eigenvalue is negative

Peres, Phys. Rev. Lett. **77**, 1413
Horodecki, Phys. Lett. A **232**, 5

- **For $t\bar{t}$ system, spin density matrix is separable if all eigenvalues are positive**

  → **top quarks are entangled in a certain phase space if at least one eigenvalue is < 0**

How to probe entanglement

- **Peres-Horodecki criterion**:
  using simpler observables, a **sufficient condition to observe entanglement in top quarks is**:

$$\boxed{\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1}$$

Afik, De Nova
Eur. Phys. J. Plus **136**, 907

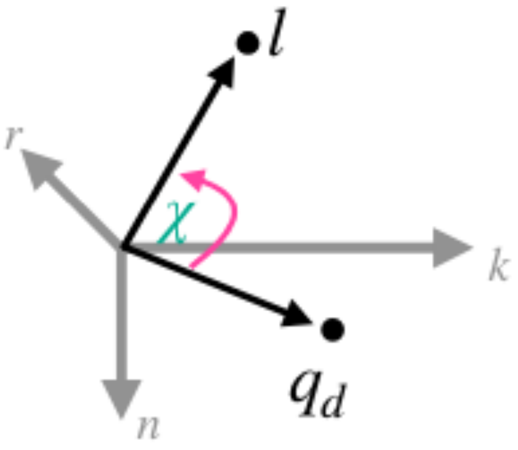
- **Two approaches:**
  - Use **full angular information of two decay products** to measure full matrix C and then construct Δ_E
  - Use **χ and $\tilde{\chi}$ distributions** to measure D and $\tilde{D}$
    - potentially improved sensitivity since we can use simpler 1D angular distributions

$$\frac{d\sigma}{d\cos\chi} = A(1 + D\cos\chi)$$

$$\frac{d\sigma}{d\cos\tilde{\chi}} = A(1 + \tilde{D}\cos\tilde{\chi})$$

χ = opening angle between the 2 decay products

$\tilde{\chi} = \chi$ with inverted sign of n-component in one of the decay products



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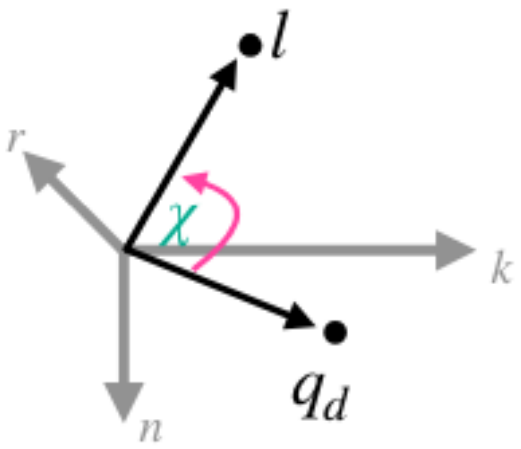
$$\frac{d\sigma}{d\cos\chi} = A(1 + D\cos\chi) \quad \rightarrow \frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$ in dilepton events

$$\frac{d\sigma}{d\cos\tilde{\chi}} = A(1 + \tilde{D}\cos\tilde{\chi})$$

χ = opening angle between
the 2 decay products



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How to probe entanglement

- Four maximally entangled states:
$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right)$$
$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right)$$

$$gg \to t\bar{t}$$

Afik, De Nova
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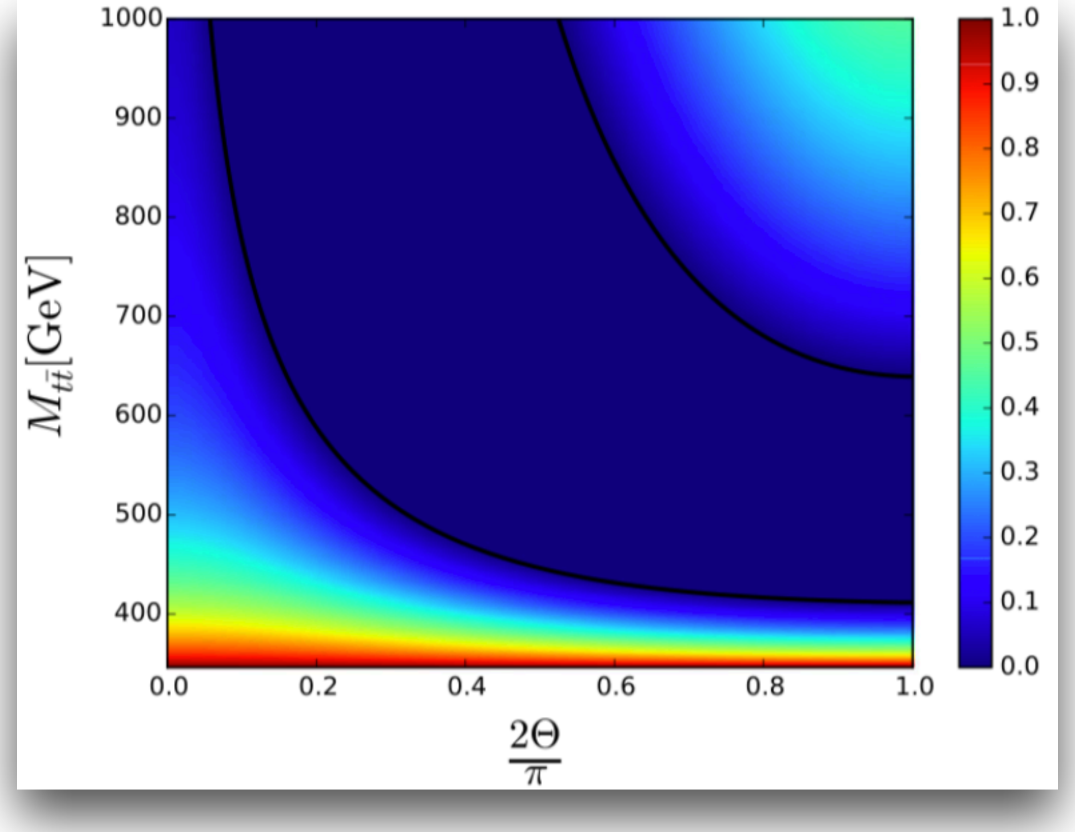

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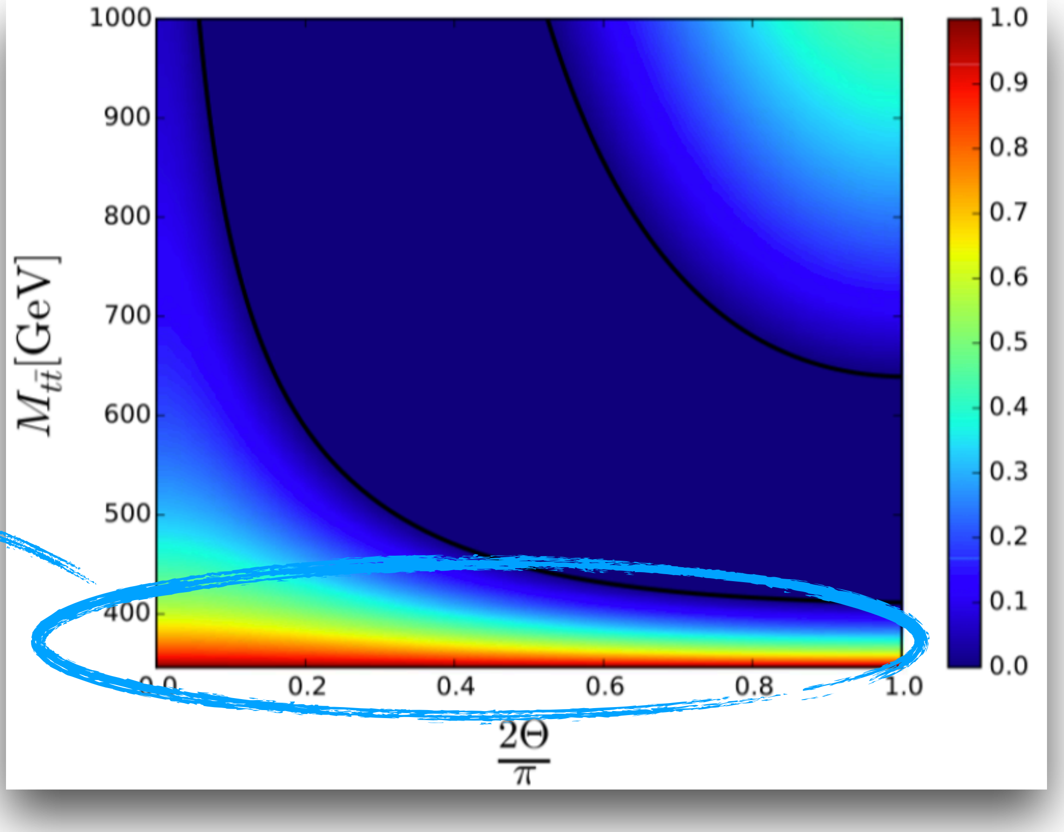
[Eur. Phys. J. Plus **136**, 907]



- Spin-singlet pseudoscalar state Ψ^-

- At low $m_{t\bar{t}}$: $C_{rr} > 0$ and $C_{kk} > 0$

$$\Delta_E = C_{nn} + C_{rr} + C_{kk} = Tr[C] = -3D > 1$$

$$D = -\frac{\text{tr}[C]}{3} \qquad \to \; D < -1/3$$

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Afik, De Nova
[Eur. Phys. J. Plus **136**, 907]

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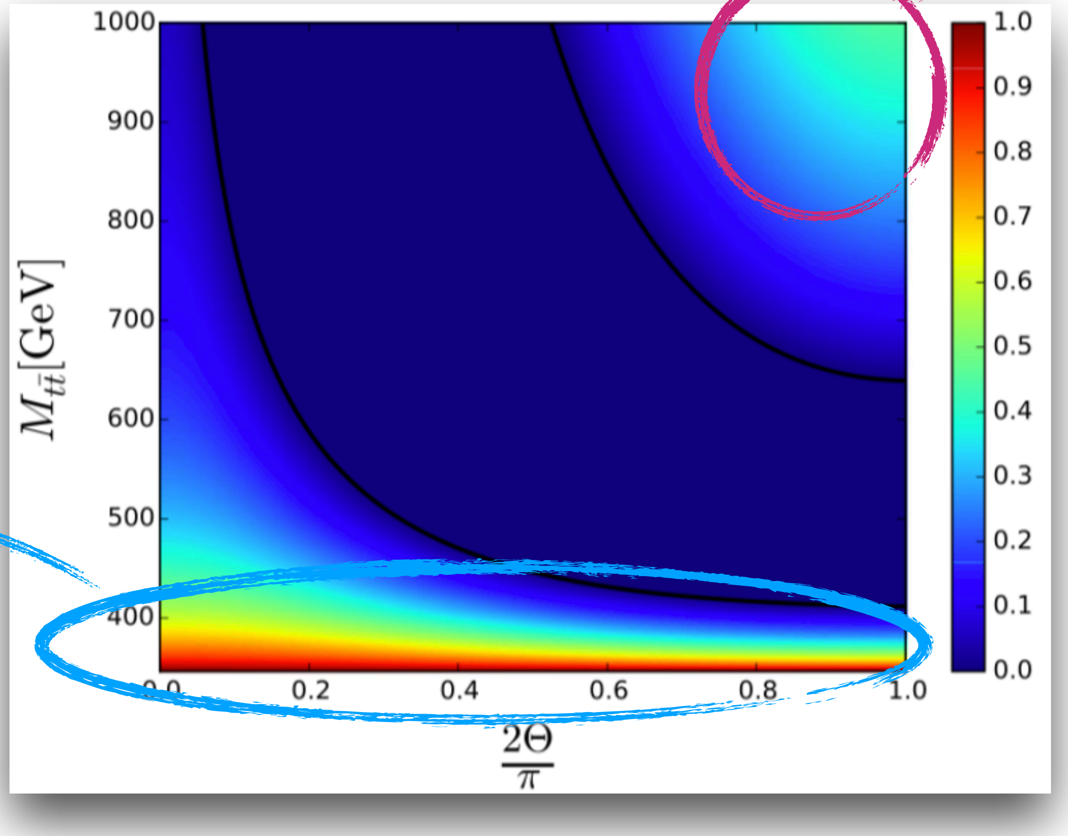

- Spin-triplet vector state ($\Phi^+ - \Phi^-$, Ψ^+, $\Phi^+ + \Phi^-$)
- At high $m_{t\bar{t}}$ and low $|\cos\Theta|$: $C_{kk} < 0$ and $C_{rr} < 0$

$$\Delta_E = C_{nn} - C_{rr} - C_{kk} = 3\tilde{D} > 1$$

$$\rightarrow \ \tilde{D} > 1/3$$

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Eur. Phys. J. Plus **136**, 907

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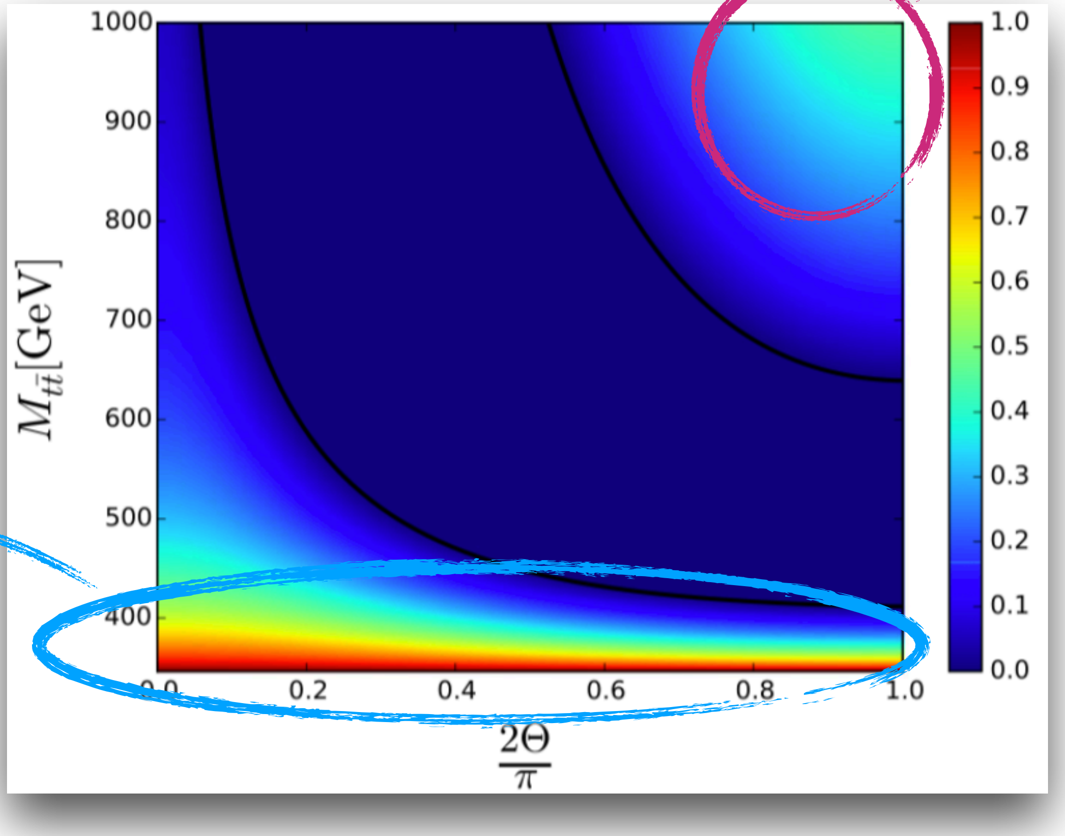

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Sufficient condition for entanglement

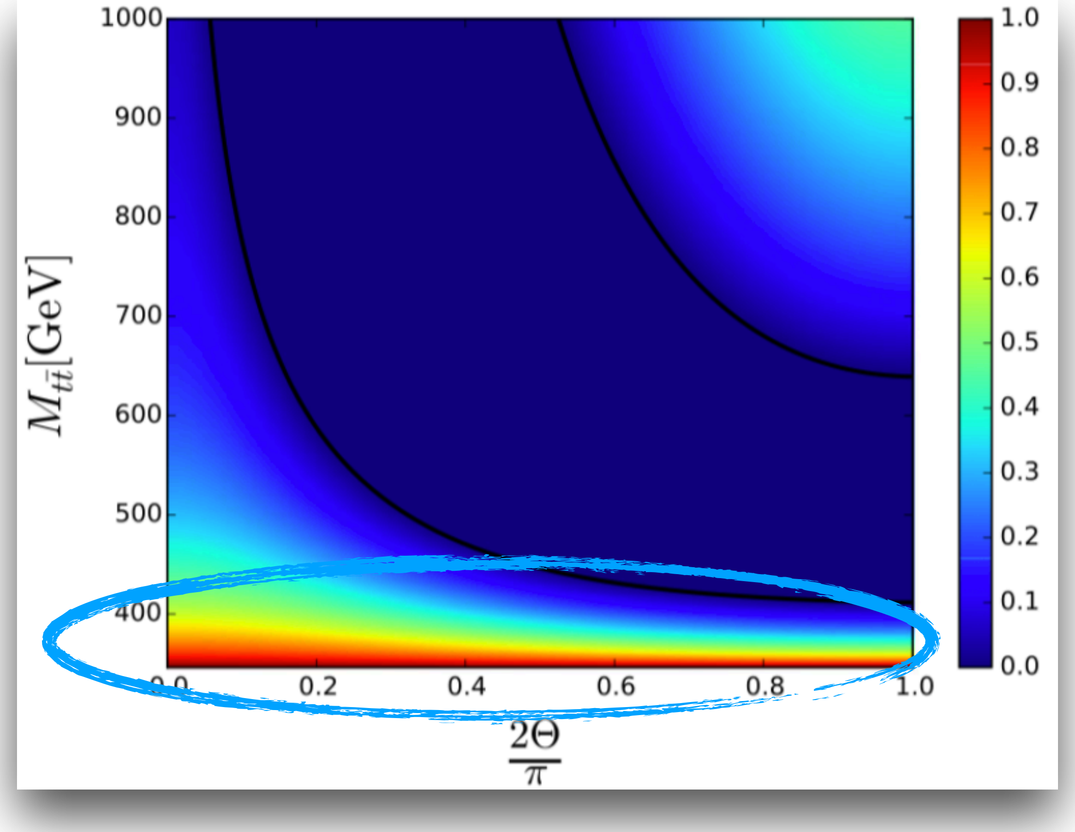
→ **measure D, $\tilde{D}$ to access entanglement information in top quark events!**

Dilepton analysis

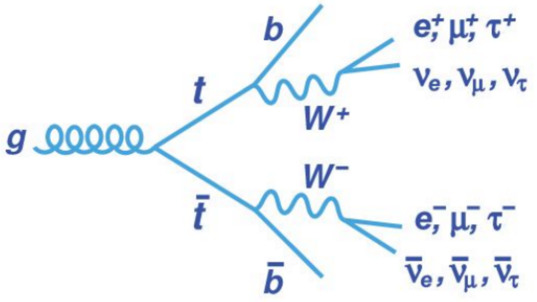
arXiv:2406.03976

accepted by ROPP

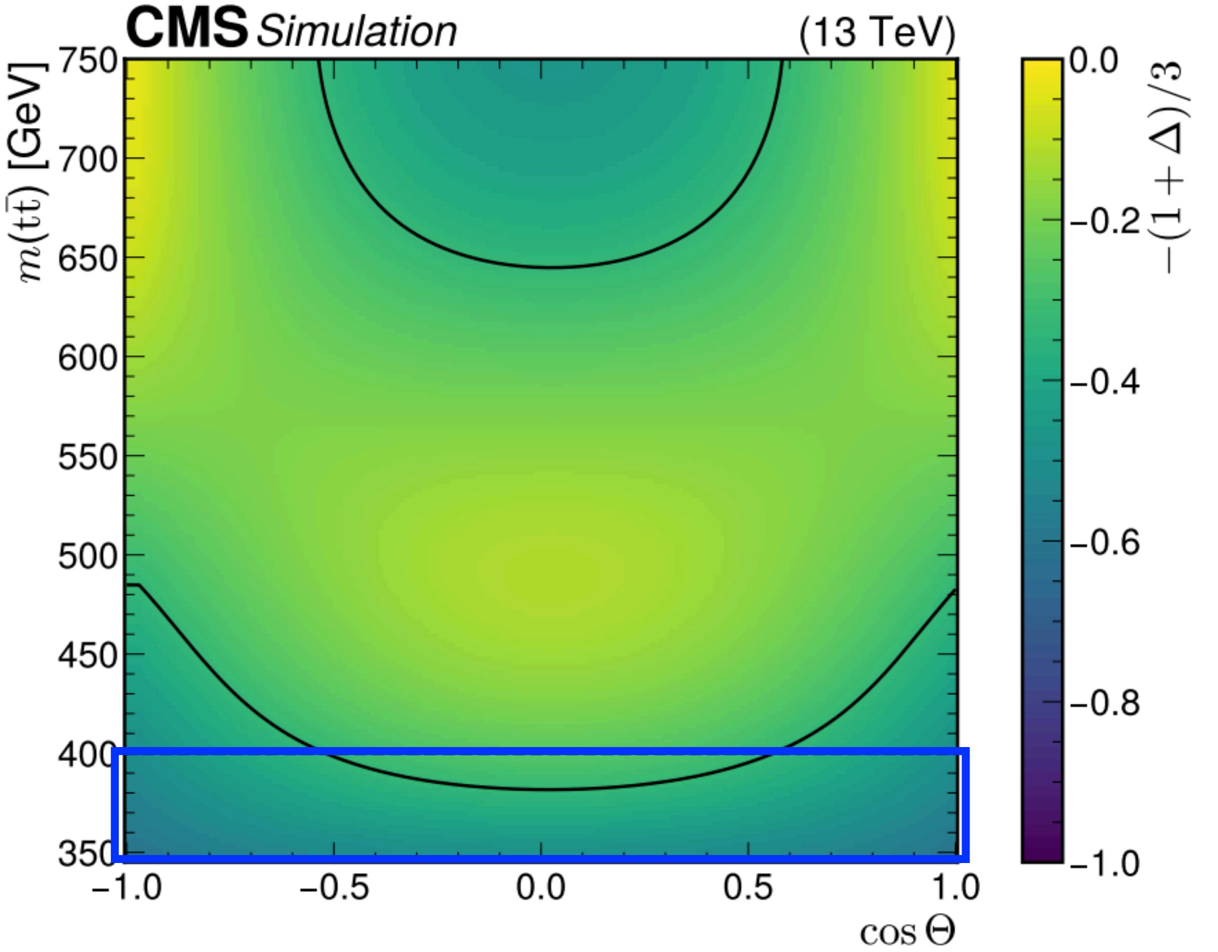
$gg \to t\bar{t}$



Dilepton analysis: strategy



- **The degree of entanglement is highly phase space-dependent**
  - scan of $\cos\Theta$ vs $m_{t\bar{t}}$ to determine most sensitive phase space while minimizing expected total uncertainties
- **Focus on low-mass region ($345 < m_{t\bar{t}} < 400$ GeV) to increase entanglement**



$D < -1/3$ → **Entangled!**

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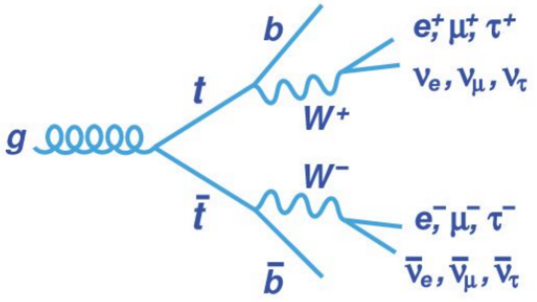

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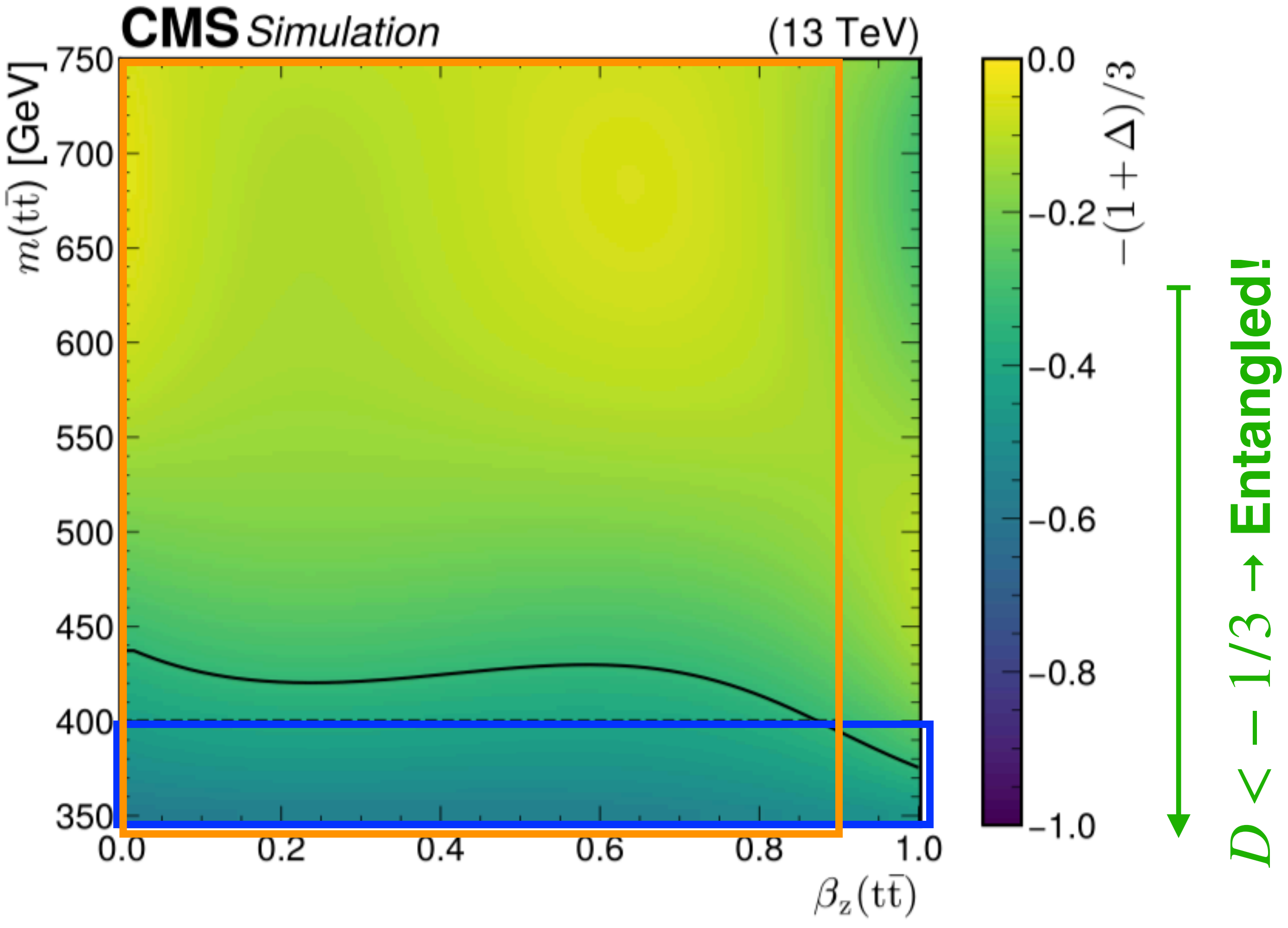
- **Cut on velocity along the beam line of the tt̄ system to increase $gg/q\bar{q}$ fraction:**

Aguilar-Saavedra,
Casas
arXiv:2205.00542

$$\boxed{\beta = \left|\frac{p_z^t + p_z^{\bar{t}}}{E^t + E^{\bar{t}}}\right| < 0.9}$$



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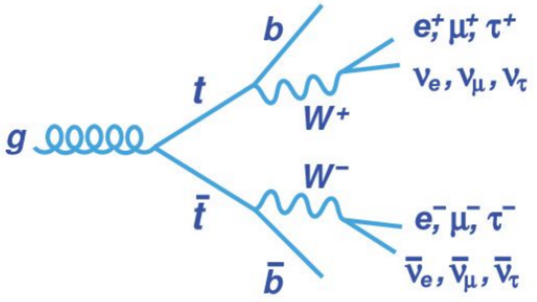

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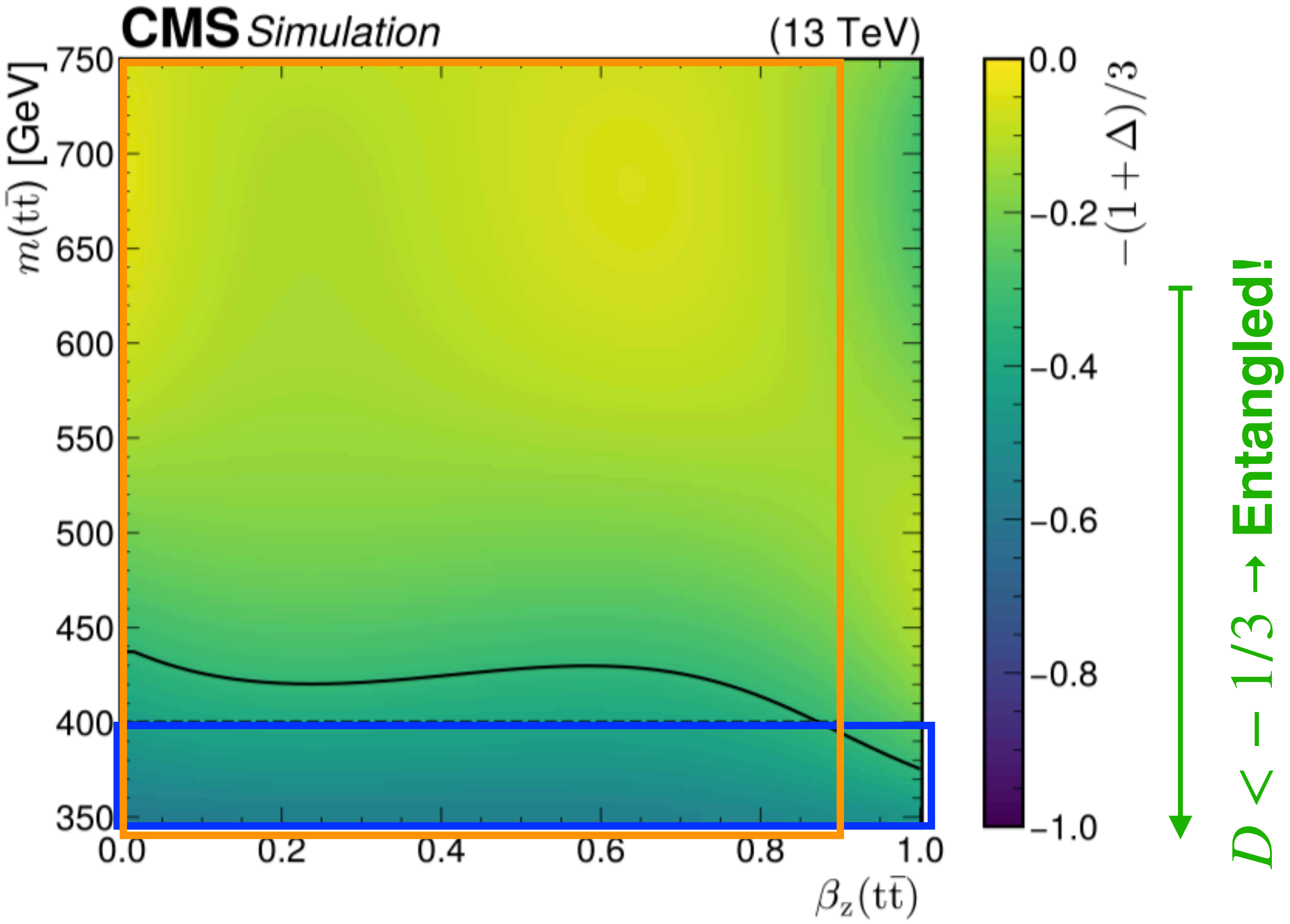
- **Top quark reconstructions with $m_{\ell b}$ weighting method**
- **Measure helicity angle $\cos\varphi = \hat{\ell}_1 \cdot \hat{\ell}_2$**
  - fully encapsulates spin correlations information for gg
- **Perform a profile maximum likelihood fit of the $\cos\varphi$ distribution in the $m_{t\bar{t}}$ - β signal region**



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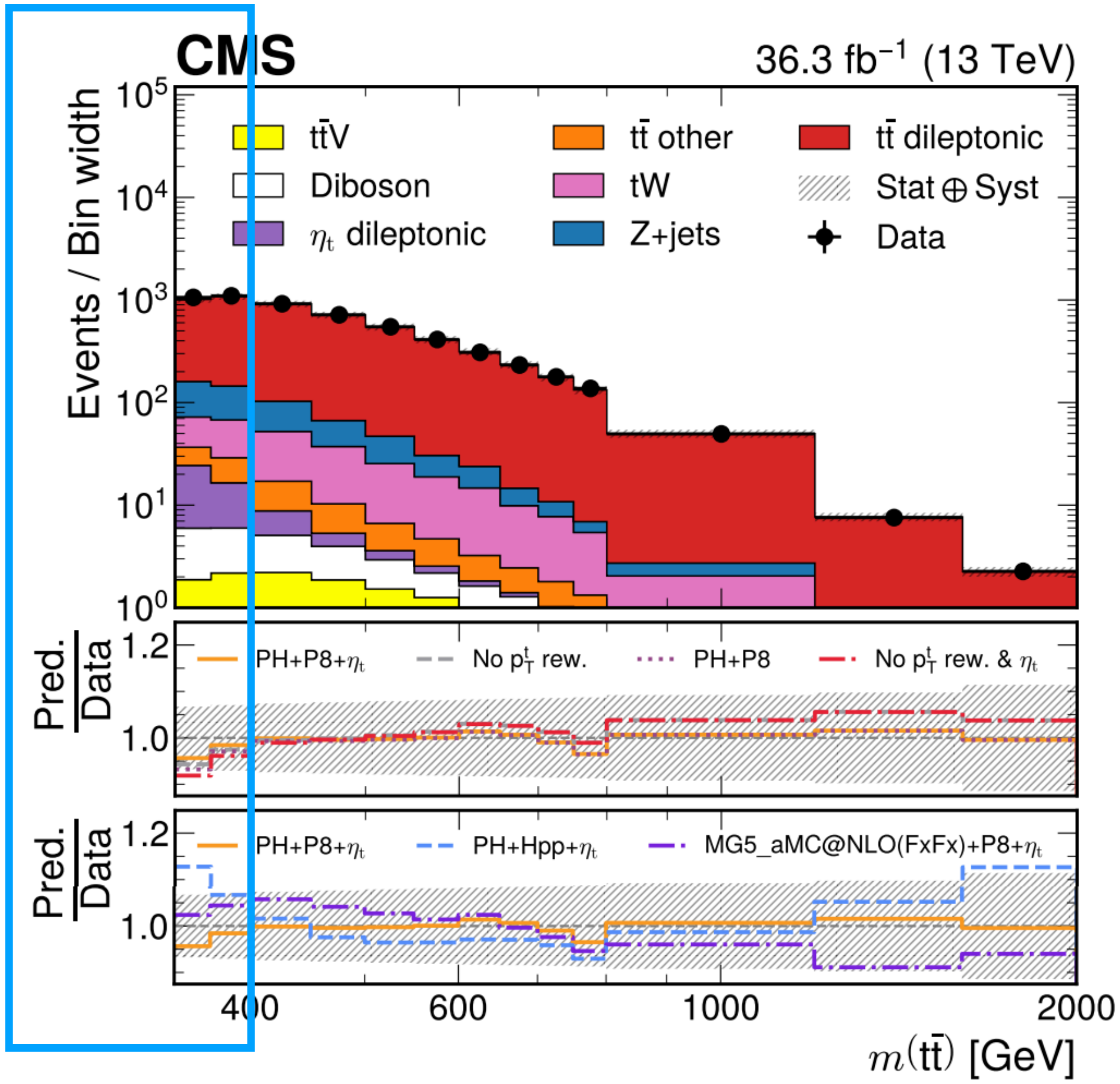
Signal model

Toponium: predicted top quark-antiquark quasi-bound state with a mass of 343 GeV and a width of 7 GeV

Combined signal model: $t\bar{t}$ + toponium (η_t)

- PowhegBox+Pythia8 (NLO) as **nominal $t\bar{t}$ sample**
    - inclusion of **EWK corrections at NLO** with HATHOR Comput. Phys. Commun. 182 (2011) 10
    - **reweighting to NNLO QCD calculations** PRL 127 (2021) 062001

- **toponium model** generated with MG5 aMC@NLO(LO)+Pythia8 B. Fuks et al. Phys Rev D 104 034023



Dilepton analysis region

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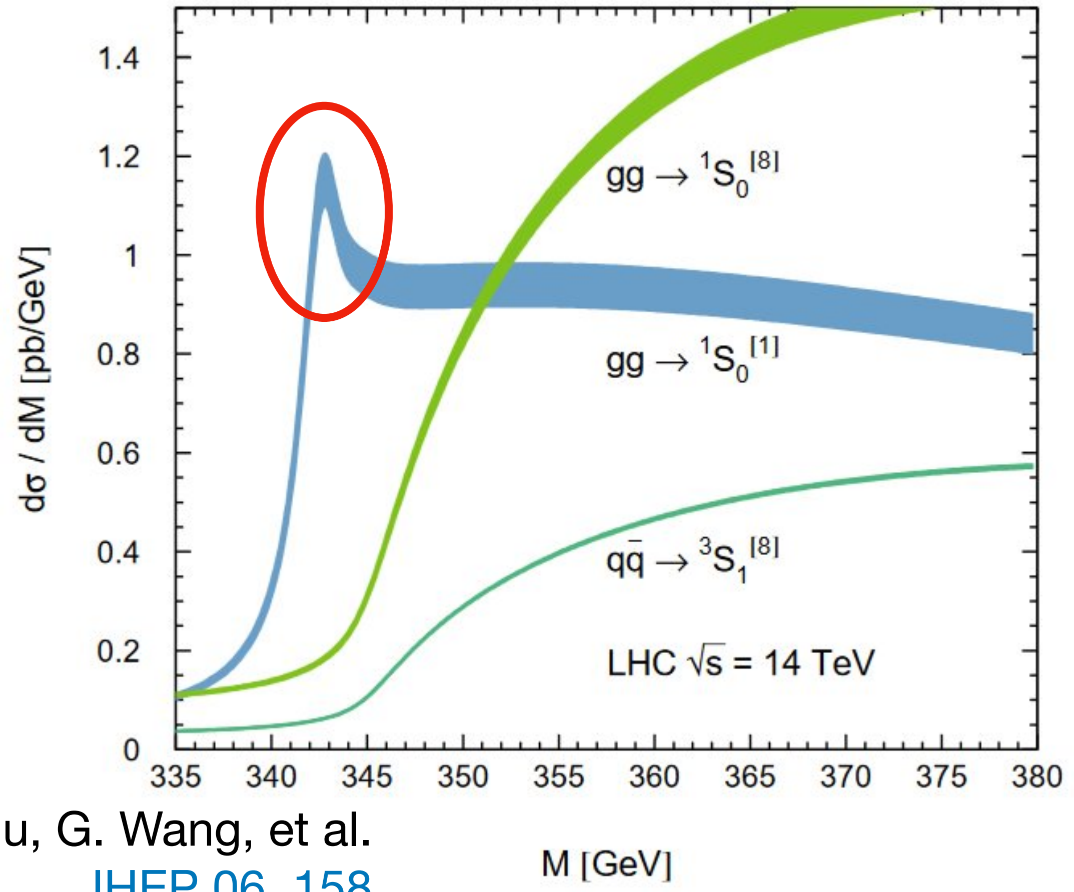
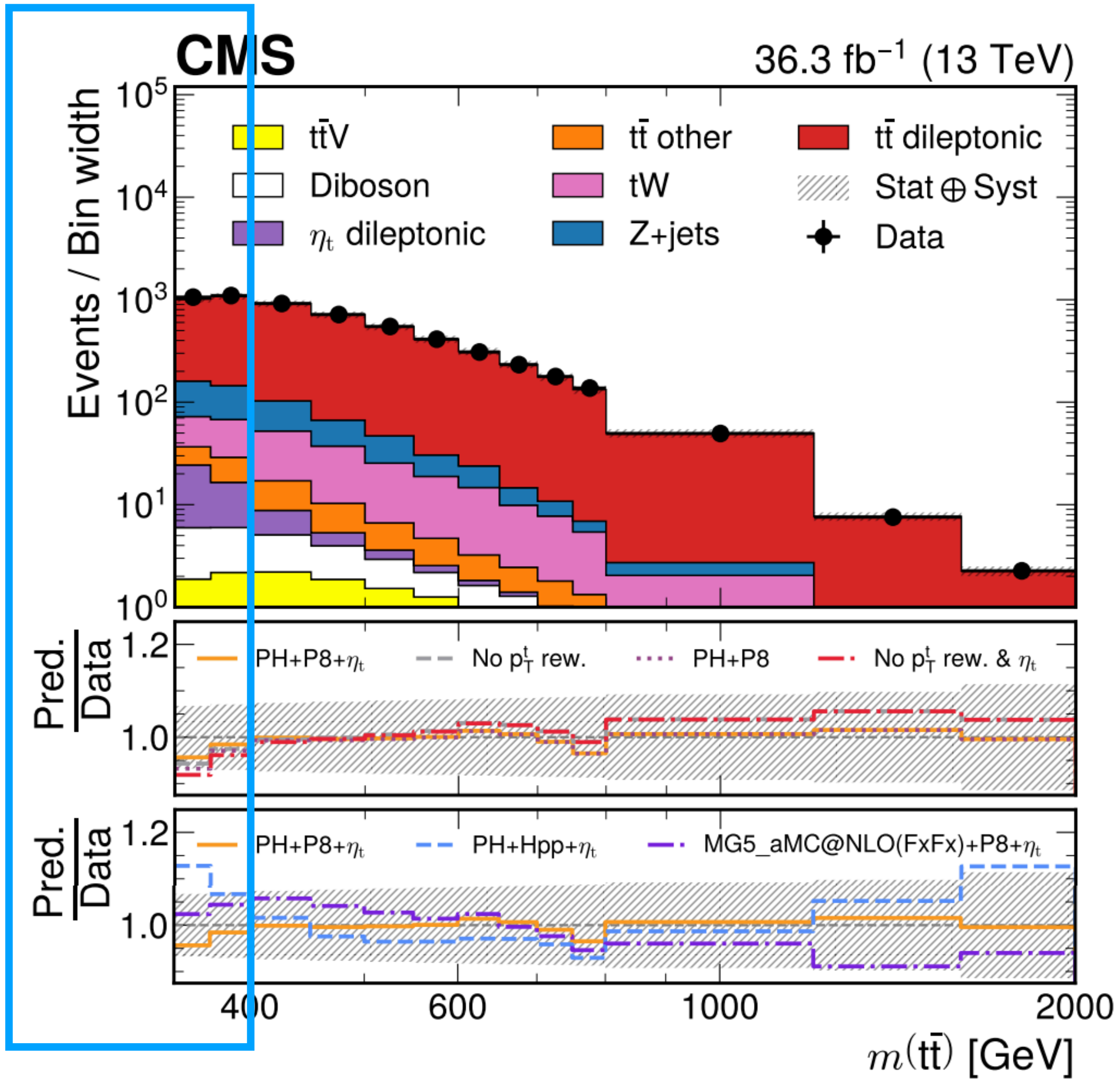
- **toponium model** generated with MG5 aMC@NLO(LO)+Pythia8 B. Fuks et al. [Phys Rev D 104 034023]
  - only pseudoscalar colour singlet and spin-0 η_t state accounted for
  - η_t improves data modeling in the threshold region



W. Ju, G. Wang, et al. [JHEP 06, 158]



Dilepton analysis region

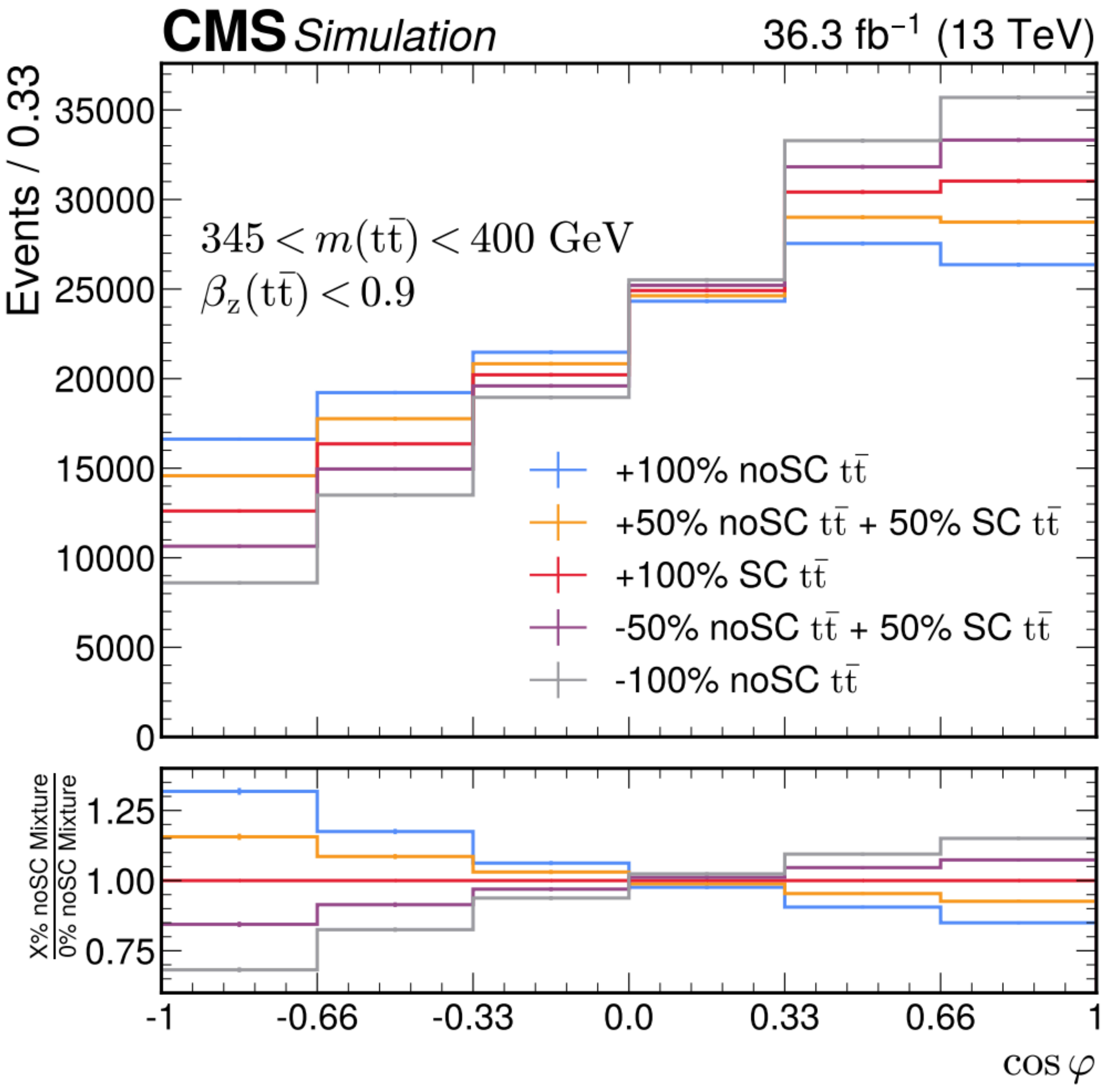
Extraction of entanglement proxy

- **The entanglement proxy D is extracted with a template fit**
  - all systematic effects included as nuisance parameters

- **Variations of D outside of SM needed to model variation of entanglement**

- **Use mixtures of SC and noSC to change fraction of $t\bar{t}$ with aligned vs opposite spins**
  ➜ any value of D between -1 and +1 can be reached

$$\boxed{D \sim \frac{\sigma(\uparrow\uparrow)+\sigma(\downarrow\downarrow)-\sigma(\uparrow\downarrow)-\sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow)+\sigma(\downarrow\downarrow)+\sigma(\uparrow\downarrow)+\sigma(\downarrow\uparrow)}}$$



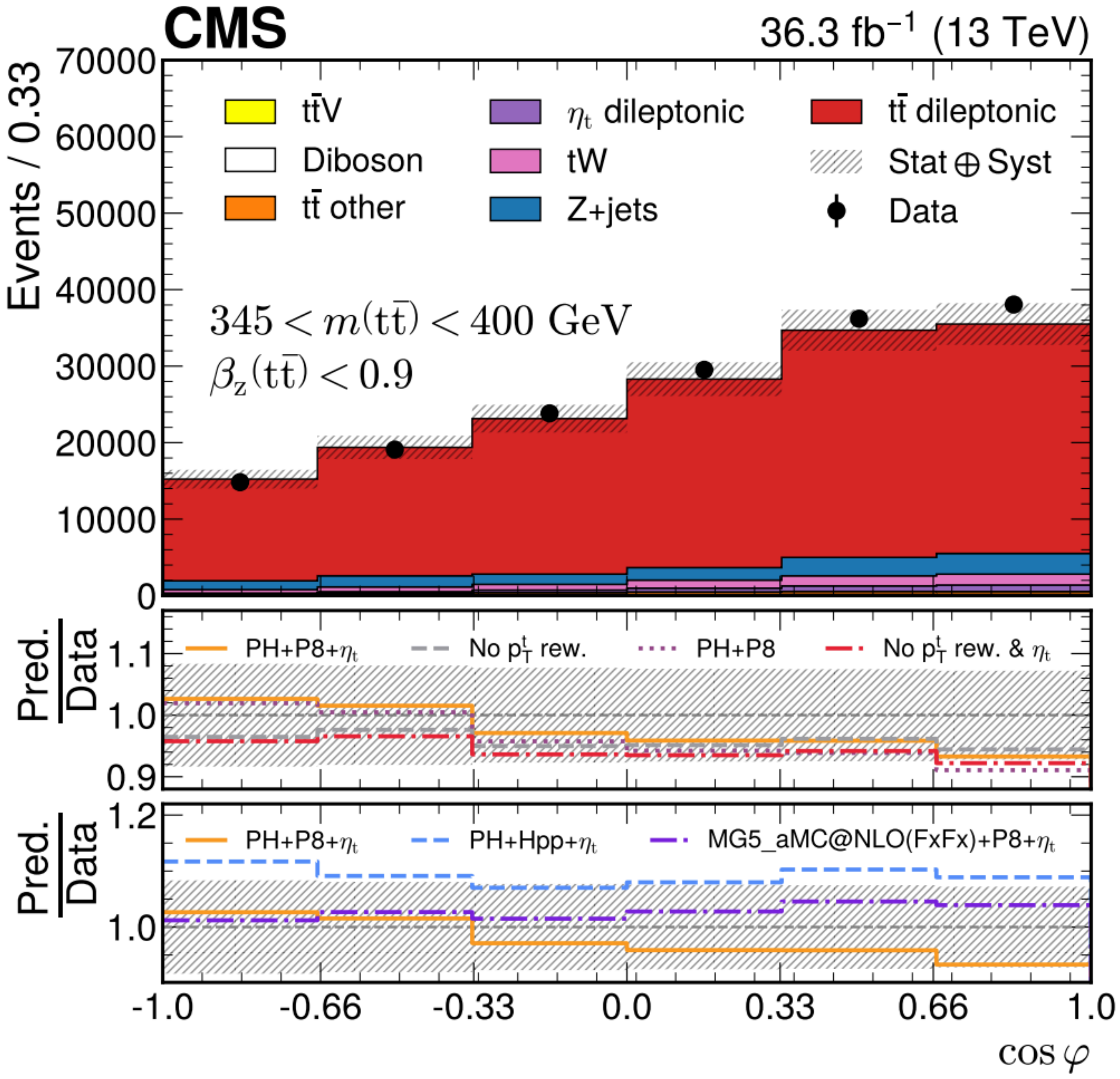
Mixed samples with (SC) and without (noSC) spin correlations

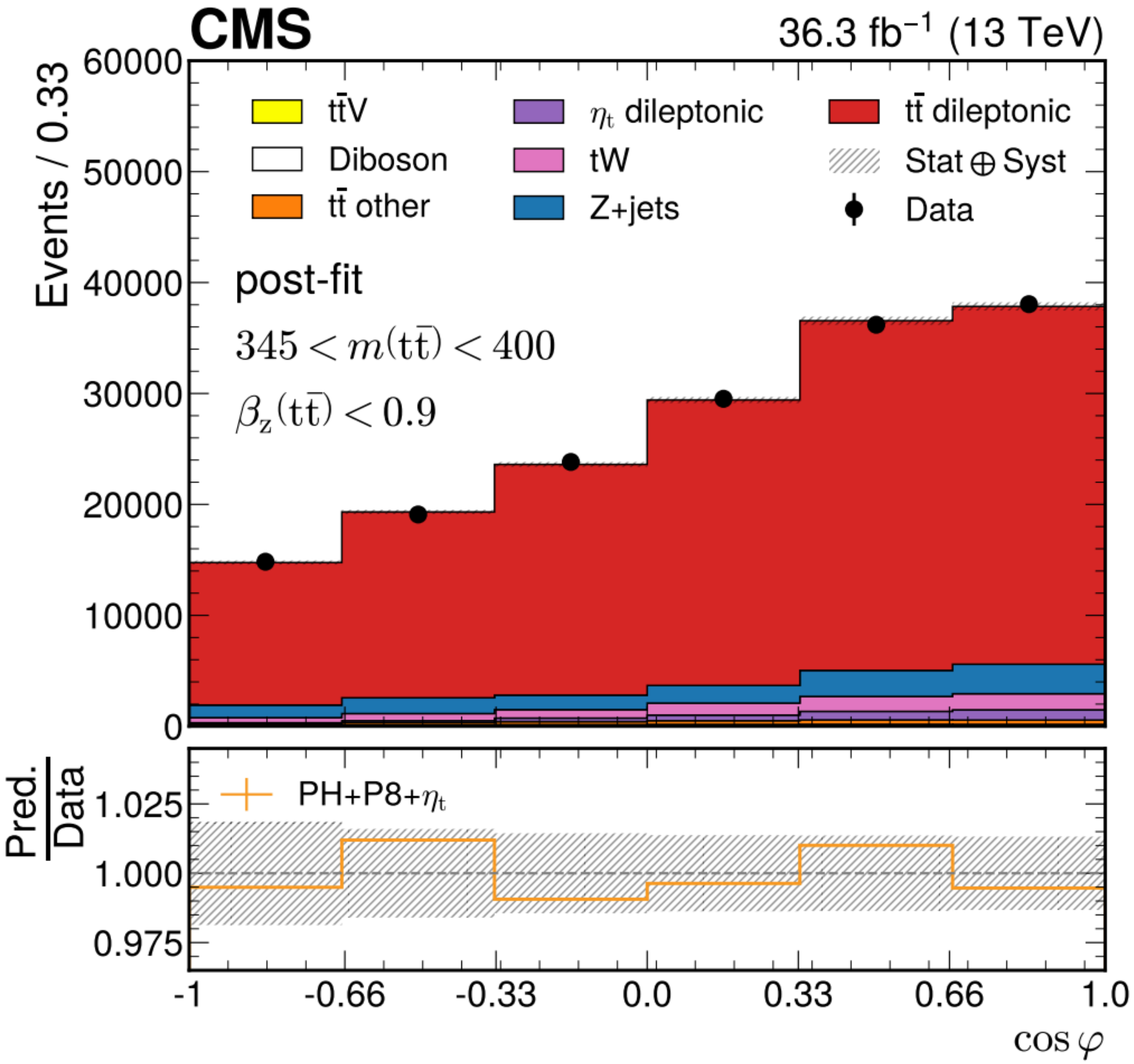
Dilepton results

- **Result of the binned profile likelihood fit of the $\cos\varphi$ distribution**
  - ~47500 signal candidates

- **Good agreement with SM predictions**

Prefit → **Postfit**





Dilepton results

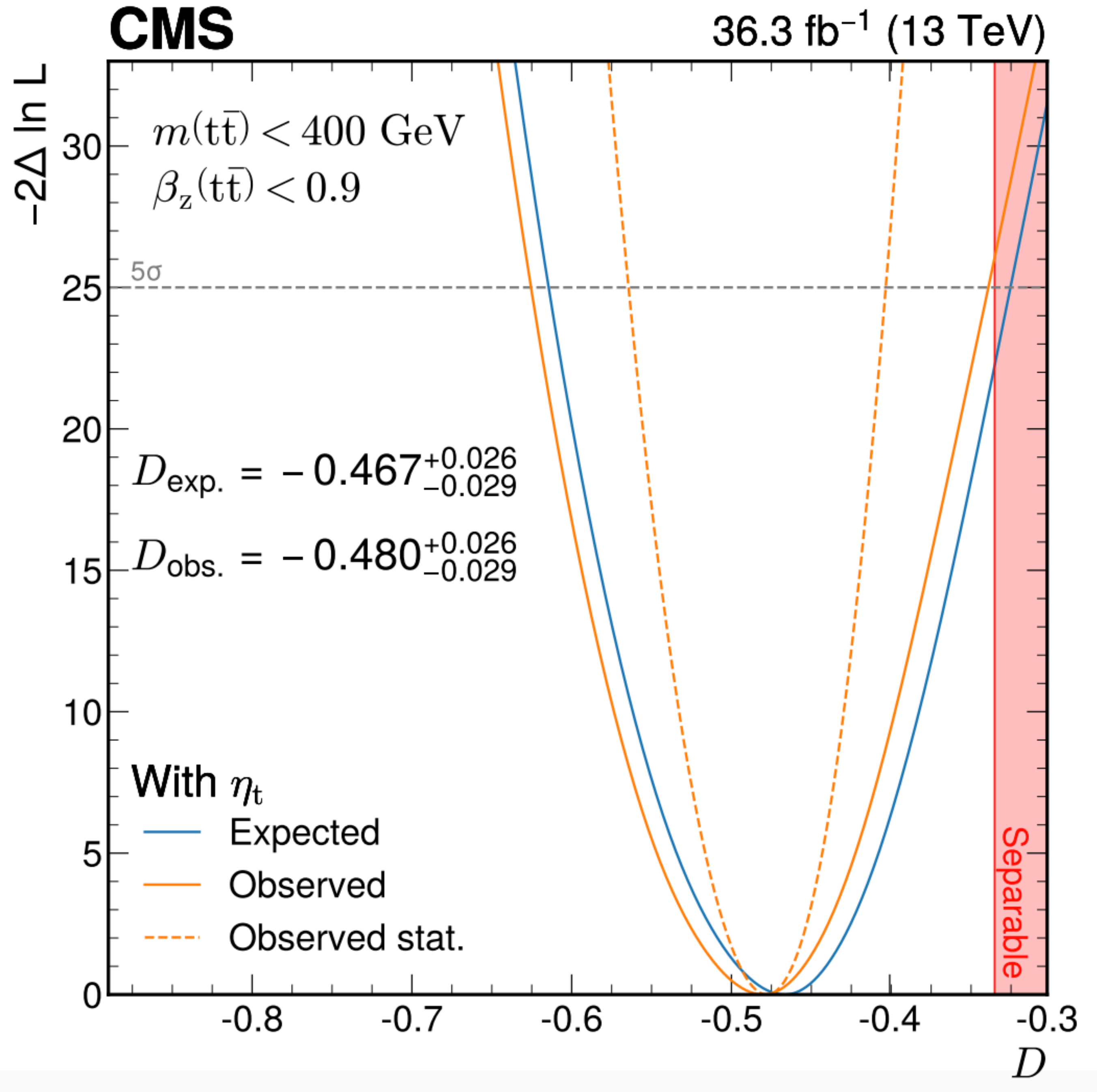
- **Scan of the $-2\Delta lnL$ distribution yields D at parton level, accounting for all detector effects**

$$\boxed{D_{obs} = -\,0.480^{+0.016}_{-0.017}(\text{stat})^{+0.020}_{-0.023}(\text{syst})}$$

$$D_{exp} = -\,0.467^{+0.016}_{-0.017}(\text{stat})^{+0.021}_{-0.024}(\text{syst})$$

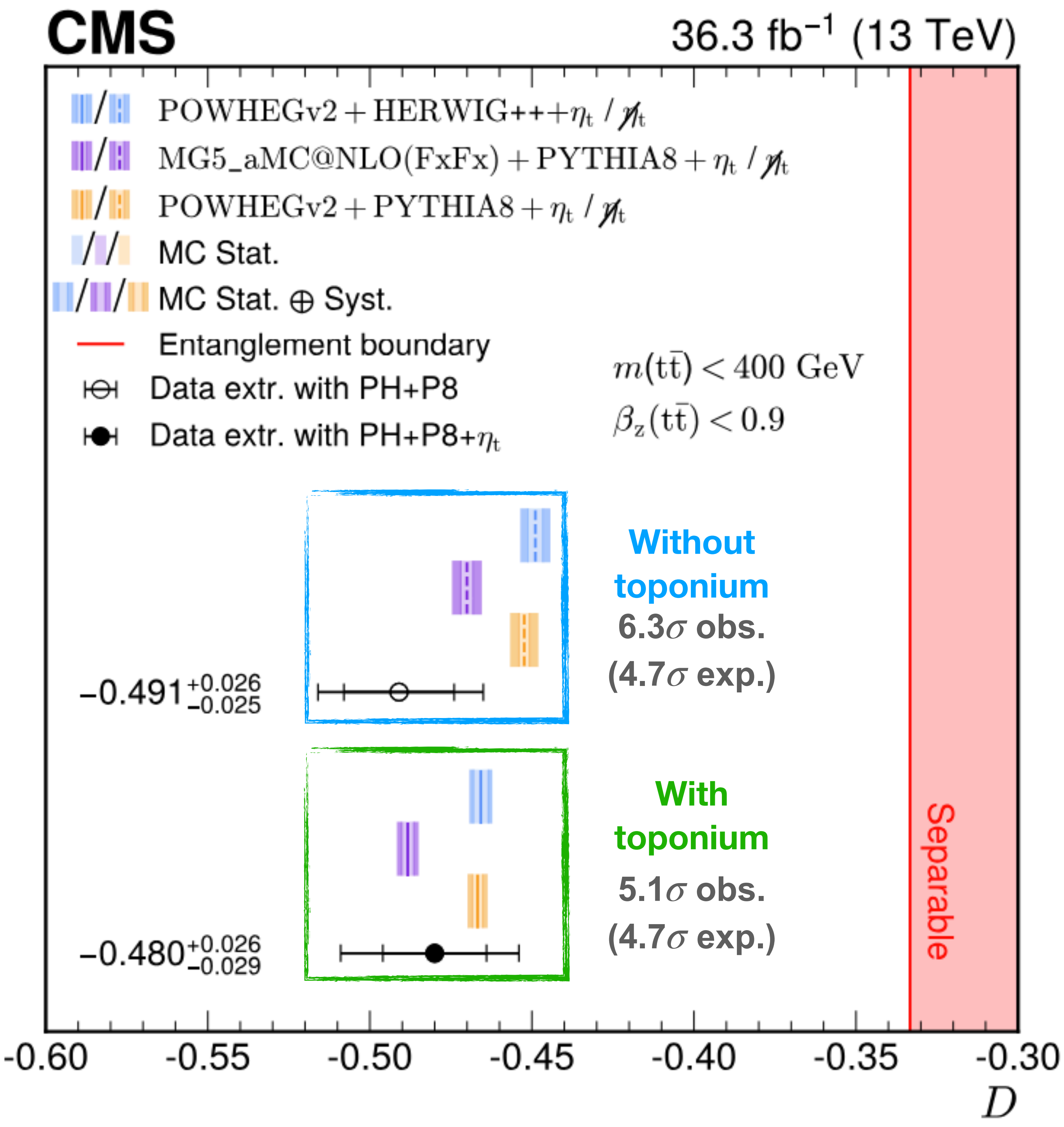
- **Significance: 5.1σ obs (4.7σ exp.)**

>5 standard deviations observation of top quarks being entangled at $t\bar{t}$ threshold !



Dilepton results with/without η_t

- **Good agreement with SM predictions**
  - **significantly improved with η_t inclusion**
  - ~1.5σ tension with the expectation if toponium is not included

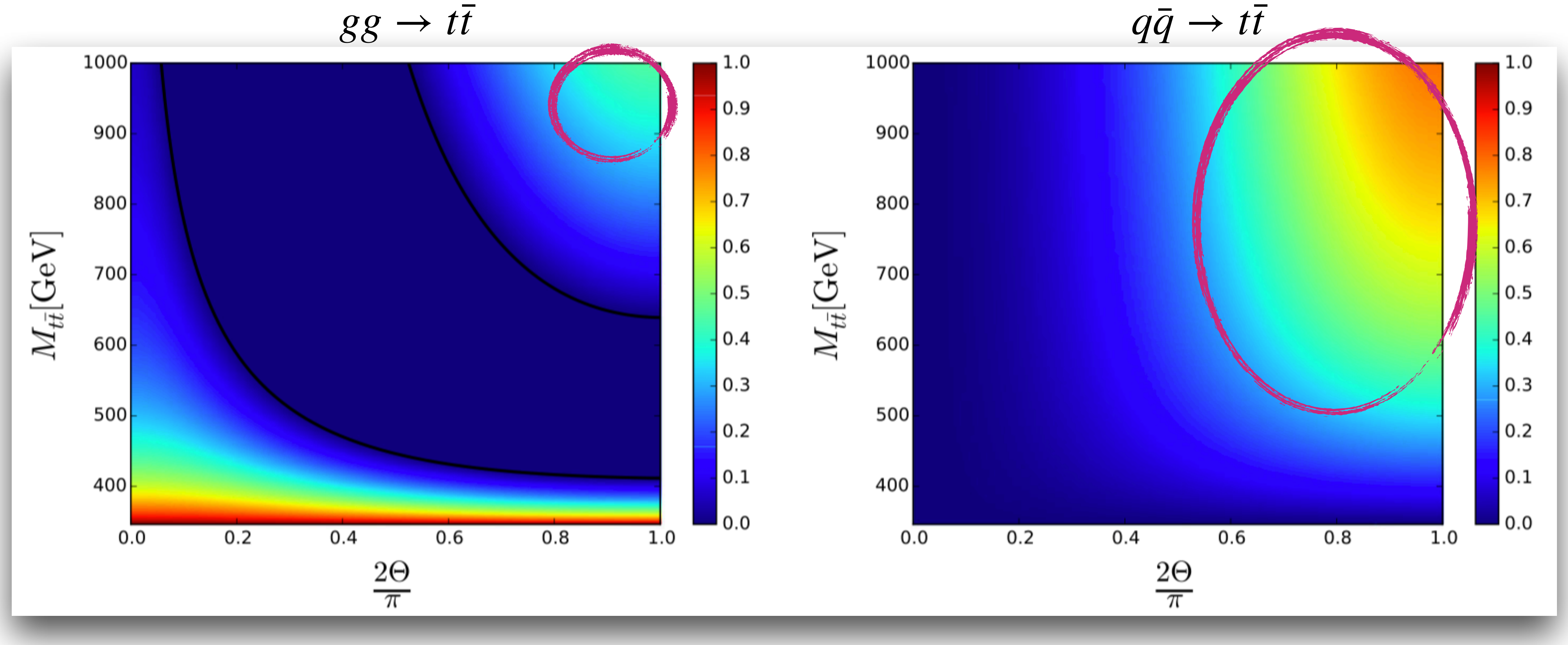


Without toponium
6.3σ obs.
(4.7σ exp.)

With toponium
5.1σ obs.
(4.7σ exp.)

Lepton + jets analysis

arXiv:2409.11067
submitted to PRD

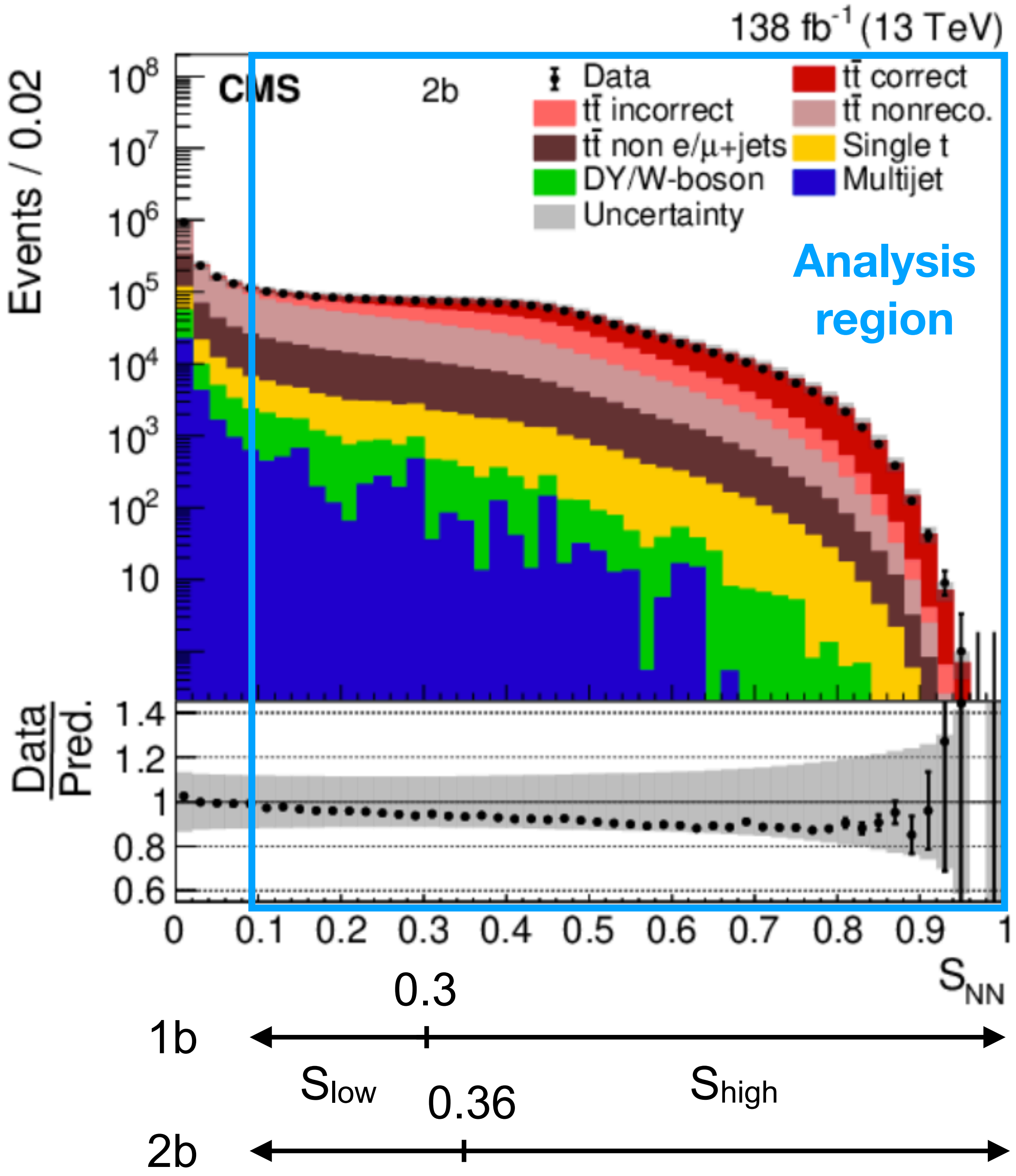


Lepton + jets: strategy

- **Artificial NN used to reconstruct the $t\bar{t}$ system in each event**
  - correctly identifying detector-level objects and up/down jet assignment

- **Remove events with NN score S_{NN}<0.1**
  - due to the low fraction of correctly reconstructed events and the large contribution of background processes

- Events divided into **categories based on lepton flavor, number of b-tags, and NN score**



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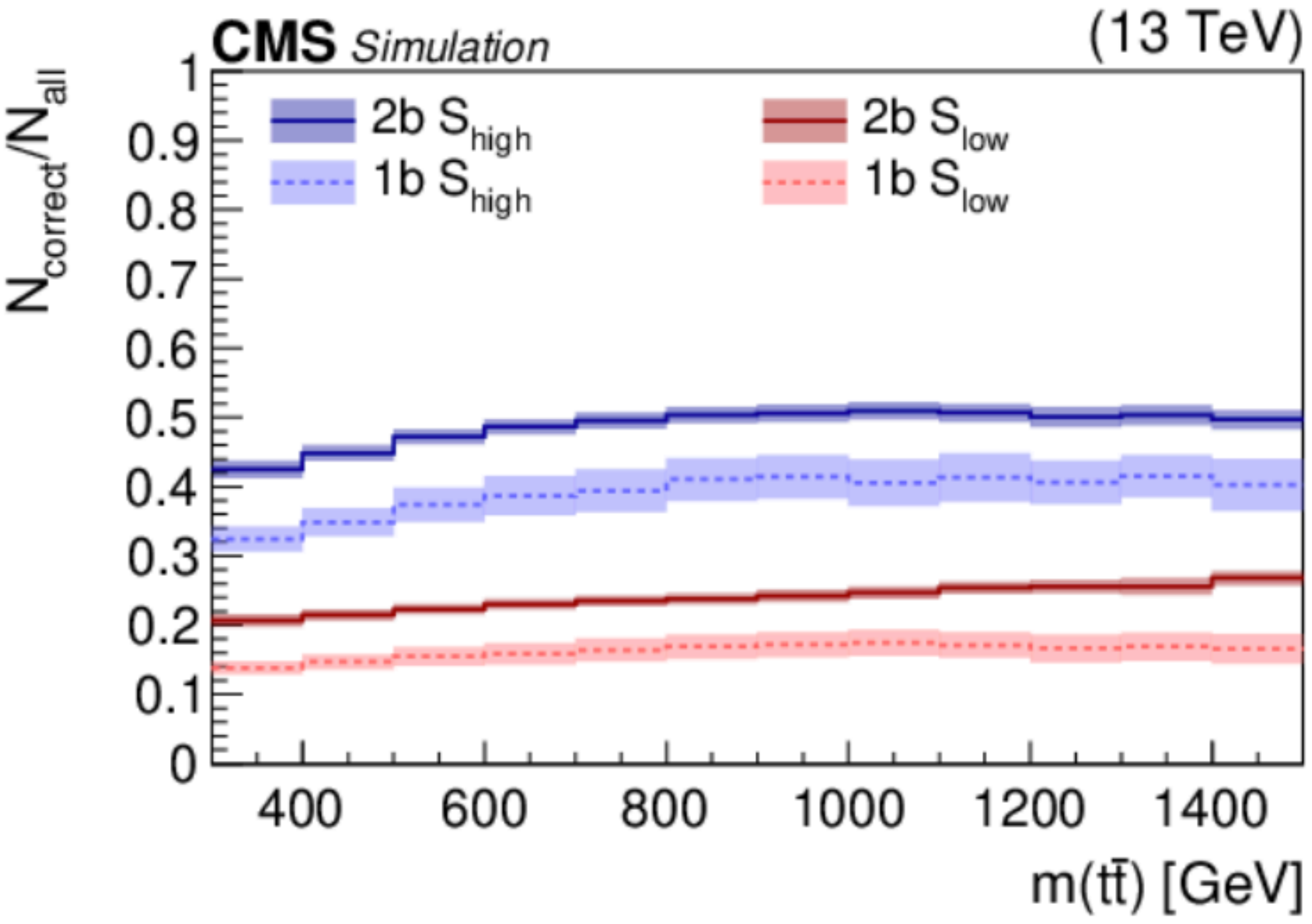
- **Remove events with NN score S_{NN}<0.1**
  - due to the low fraction of correctly reconstructed events and the large contribution of background processes

- Events divided into **categories based on lepton flavor, number of b-tags, and NN score**

- **50% correct jet assignment** (including correct d-type quark) **in 2b S_{high} region**

- All polarization and spin correlation coefficients extracted simultaneously by performing a binned **maximum likelihood fit** to the data

Fraction of events reconstructed with jets correctly assigned to partons including d-type quark



Lepton + jets: fit strategy

- The total cross section is a **linear combination of Σ_m templates** with P and C coefficients Q_m:

$$\boxed{\Sigma_{tot} = \Sigma_0 + \Sigma_{m=1}^{15} Q_m \Sigma_m}$$

$$\Sigma_m = \sigma_{norm}\{\sin\theta_p\cos\phi_p, \sin\theta_p\sin\phi_p, \ldots, \cos\theta_p\cos\theta_{\bar{p}}\}$$

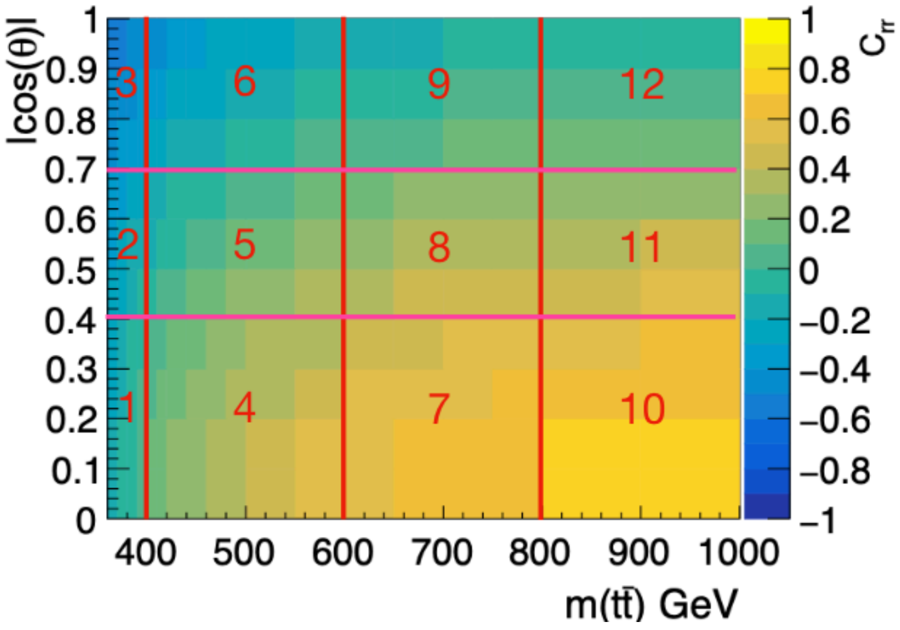
Q_m can be extracted by fitting Σ_{tot}

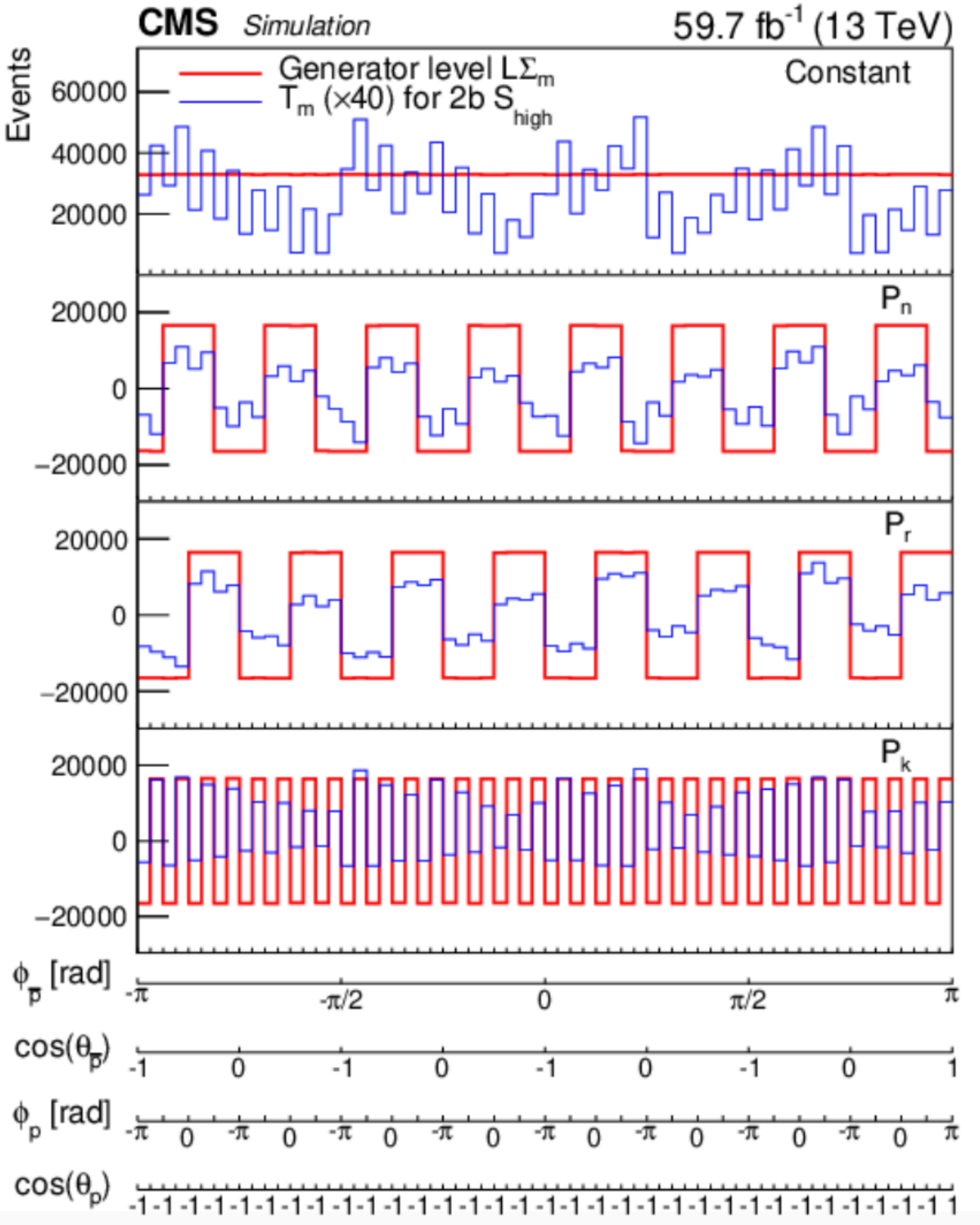
$L\Sigma_m$ = templates used at gen level
T_m = templates defined at reco level

- **Events are reweighted at the generator level**

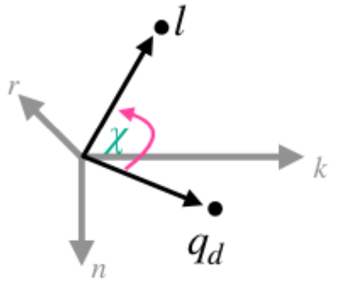
$$w_m = \frac{\Sigma_m}{\Sigma_{tot}}$$

- To minimize bias due to variations of T_m within a bin, measurements are performed in **sufficiently small bins**



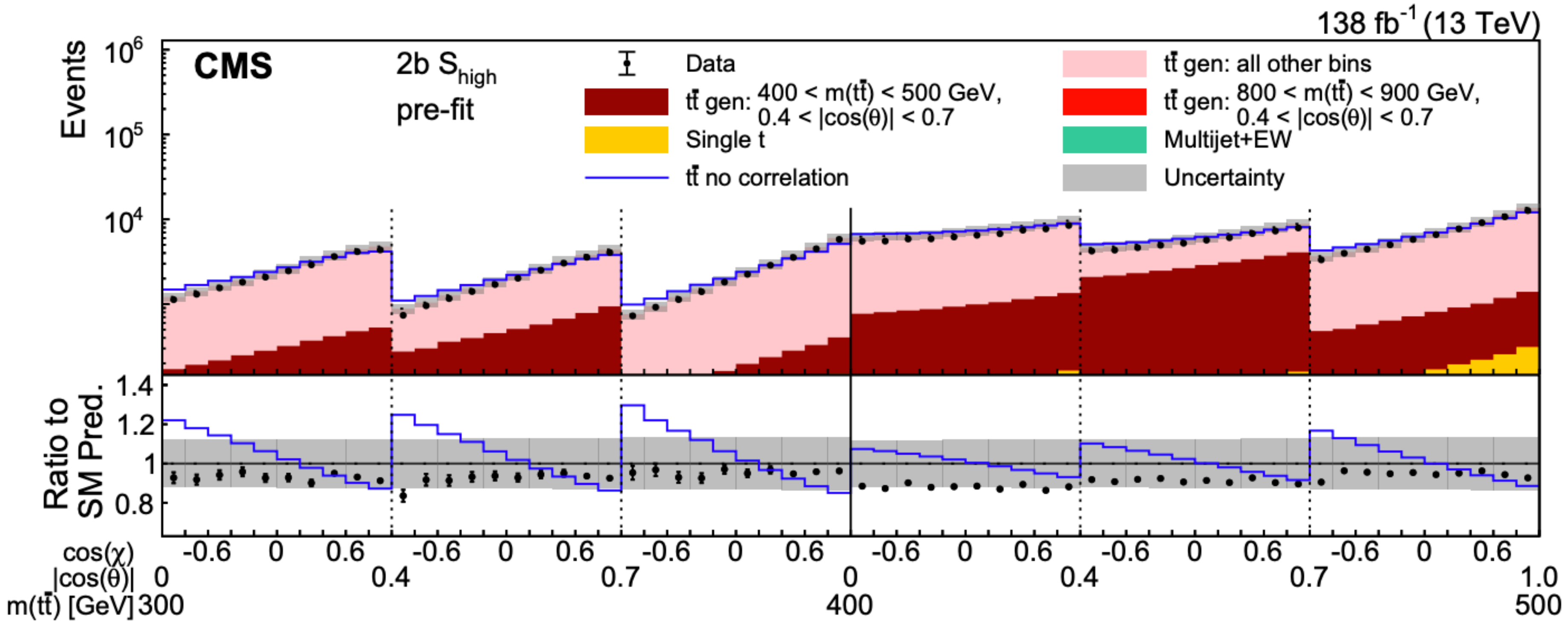


Lepton+jets: fit



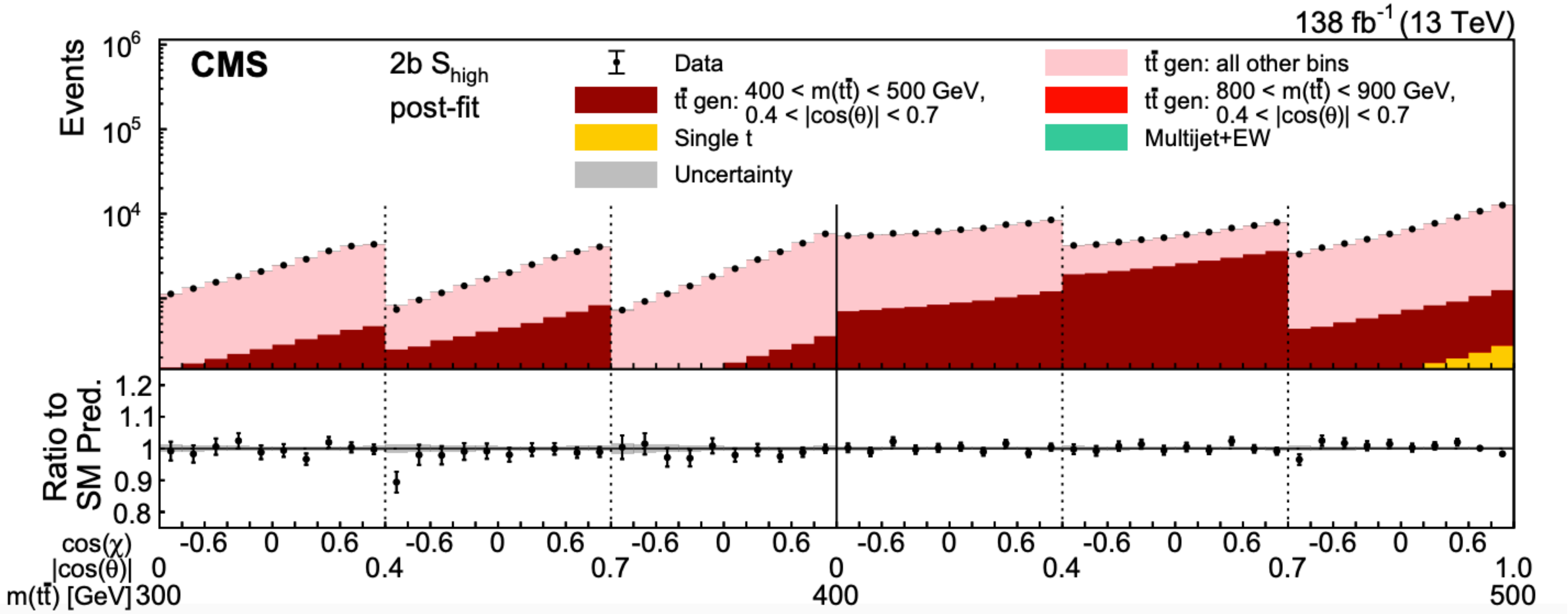
- **Maximum likelihood fit** combining information of the four categories: (2b, 1b) x (S_{high}, S_{low})
- $\cos\chi$ distribution is fit to the **reconstruction-level templates** in each $(m_{t\bar{t}}, |\cos\theta|)$ bin

Prefit



↓

Postfit

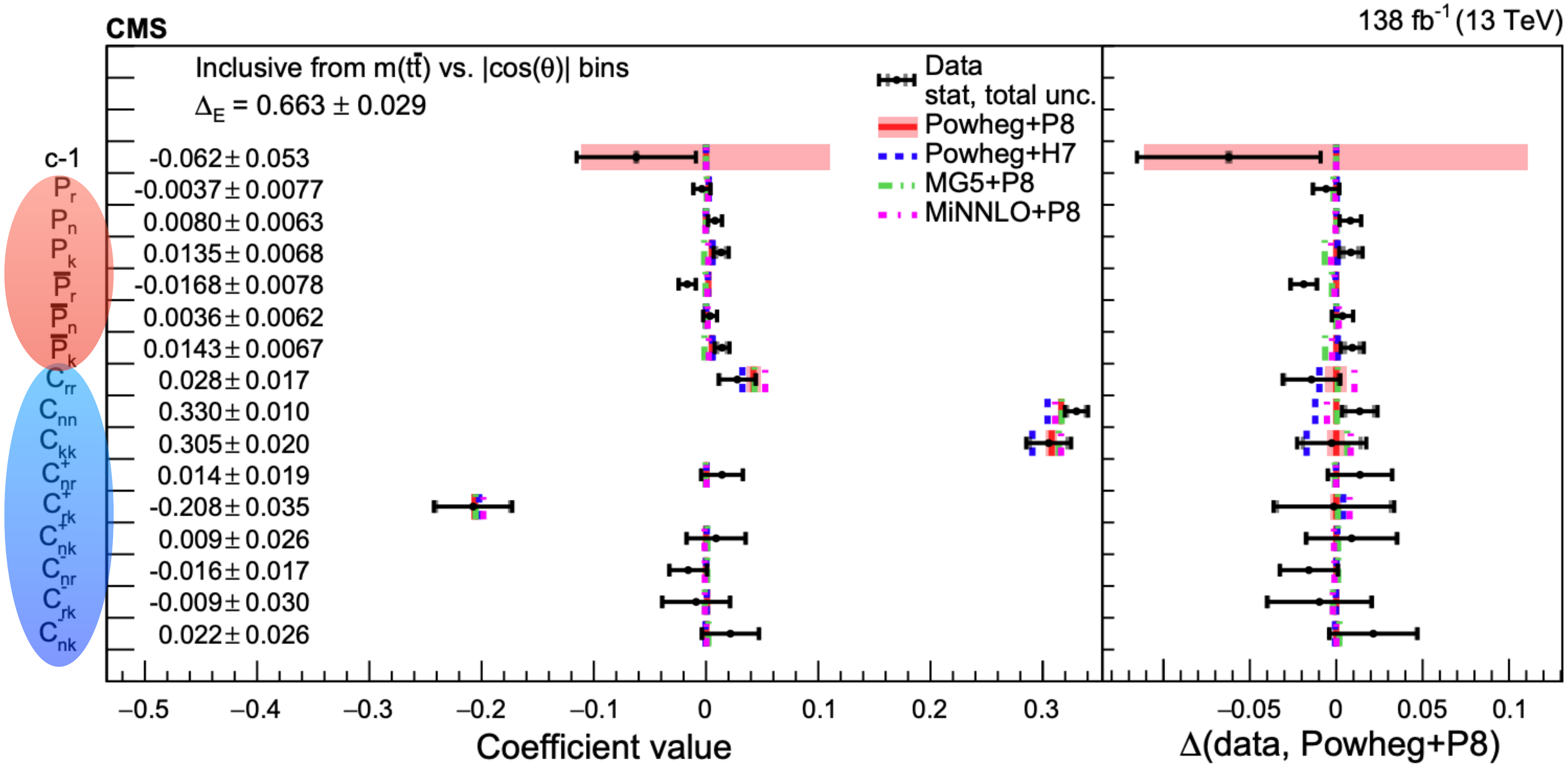


Lepton+jets results: full matrix

- Evaluation of **full correlation matrix C and polarization vectors P + measurement of D, $\tilde{D}$**
  - inclusive + differential measurements in bins of $m_{t\bar{t}}$, $|\cos\theta|$, p_T(t)
  - **first time at high $m_{t\bar{t}}$!**
- **Good agreement with SM prediction**

$$B^{+/-} = \begin{pmatrix} \times \\ \times \\ \times \end{pmatrix}$$

$$C = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$



Inclusive from $m_{t\bar{t}}$ vs $|\cos\theta|$

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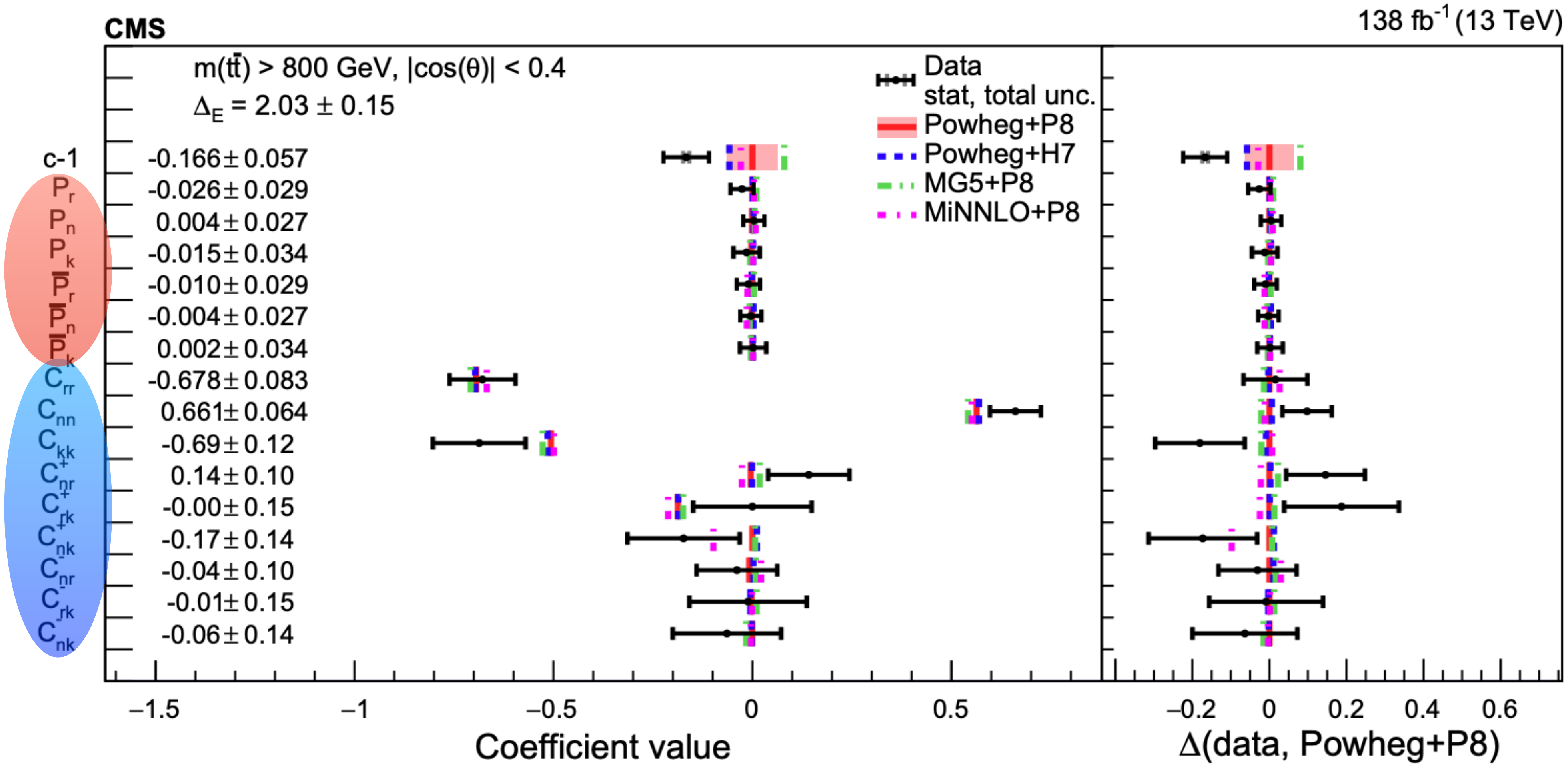

Differential in bins of $m_{t\bar{t}}$ and $|\cos\theta|$

Lepton+jets results: full matrix

- Evaluation of **full correlation matrix C and polarization vectors P + measurement of D, $\tilde{D}$**
  - inclusive + differential measurements in bins of $m_{t\bar{t}}$, $|\cos\theta|$, p_T(t)
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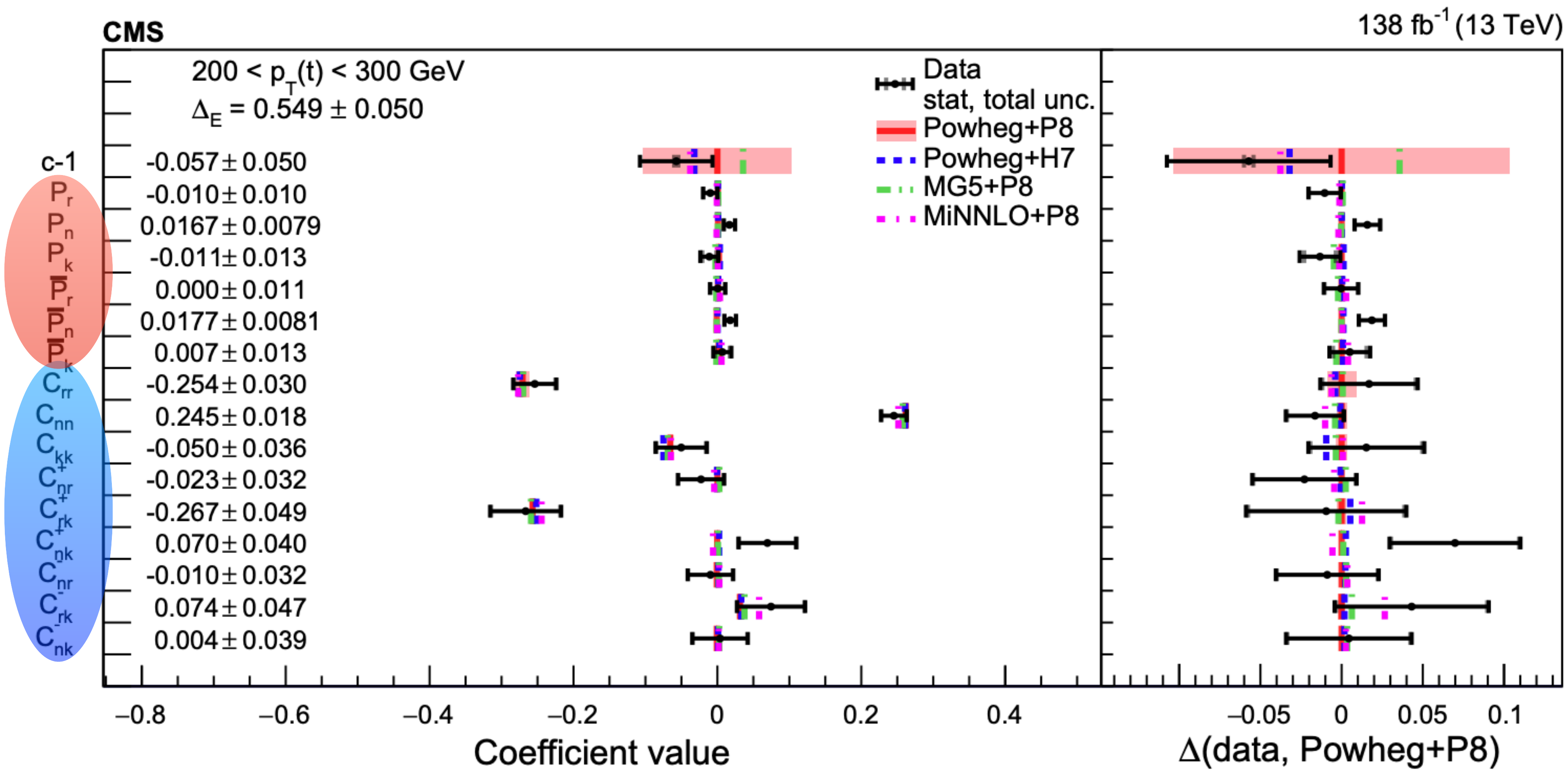
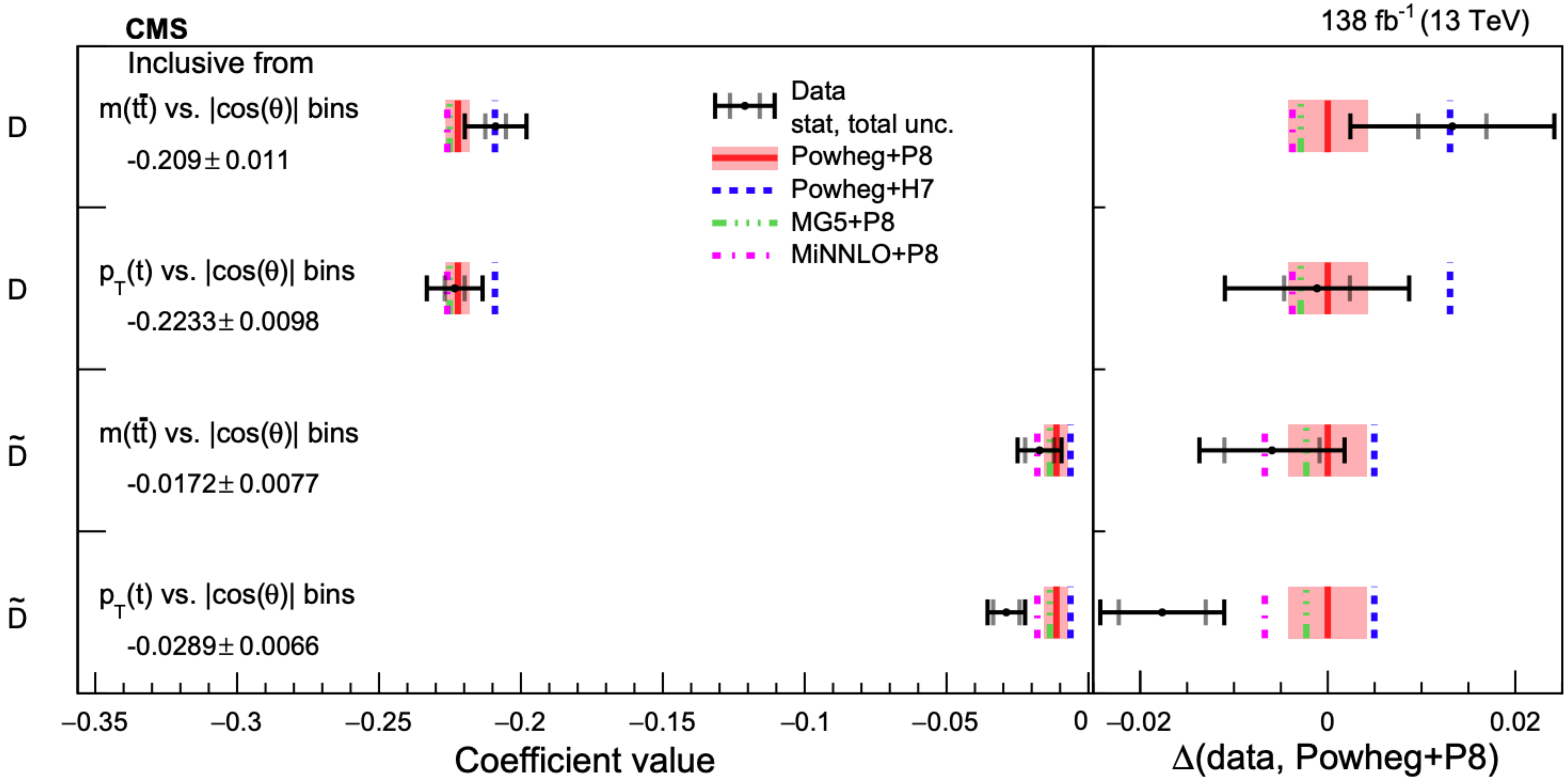


Differential in bins of p_T(t)

Lepton+jets results: D, $\tilde{D}$

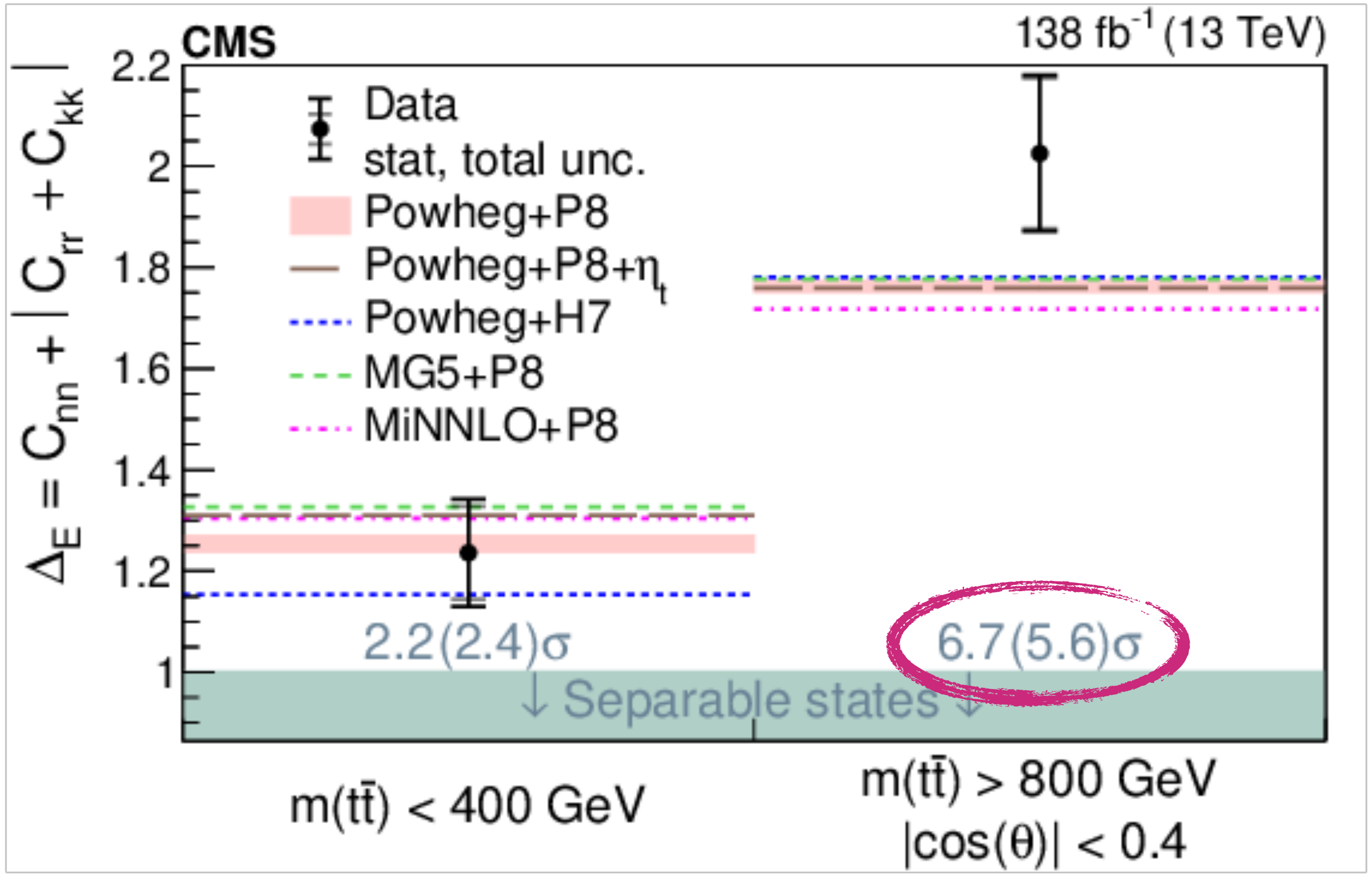
- Evaluation of full correlation matrix C and polarization vectors P + measurement of D, $\tilde{D}$
  - inclusive + differential measurements in bins of $m_{t\bar{t}}$, $|\cos\theta|$, p_T(t)
  - first time at high $m_{t\bar{t}}$!
- Good agreement with SM prediction



Lepton+jets: entanglement

- **Entanglement observed for first time in boosted region !**

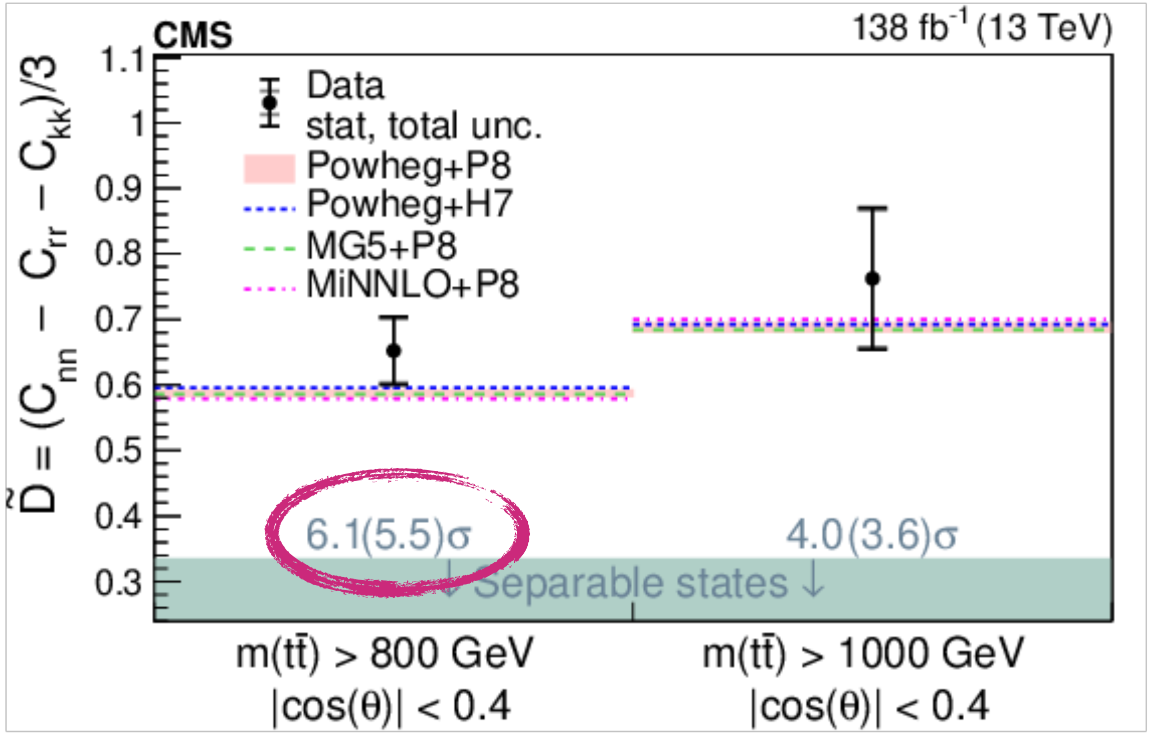
Full matrix $\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1$



Lepton+jets: entanglement

- **Entanglement observed for first time in boosted region !**
  - highest sensitivity in full matrix measurement

High $m_{t\bar{t}}$ $\tilde{D} > 1/3$



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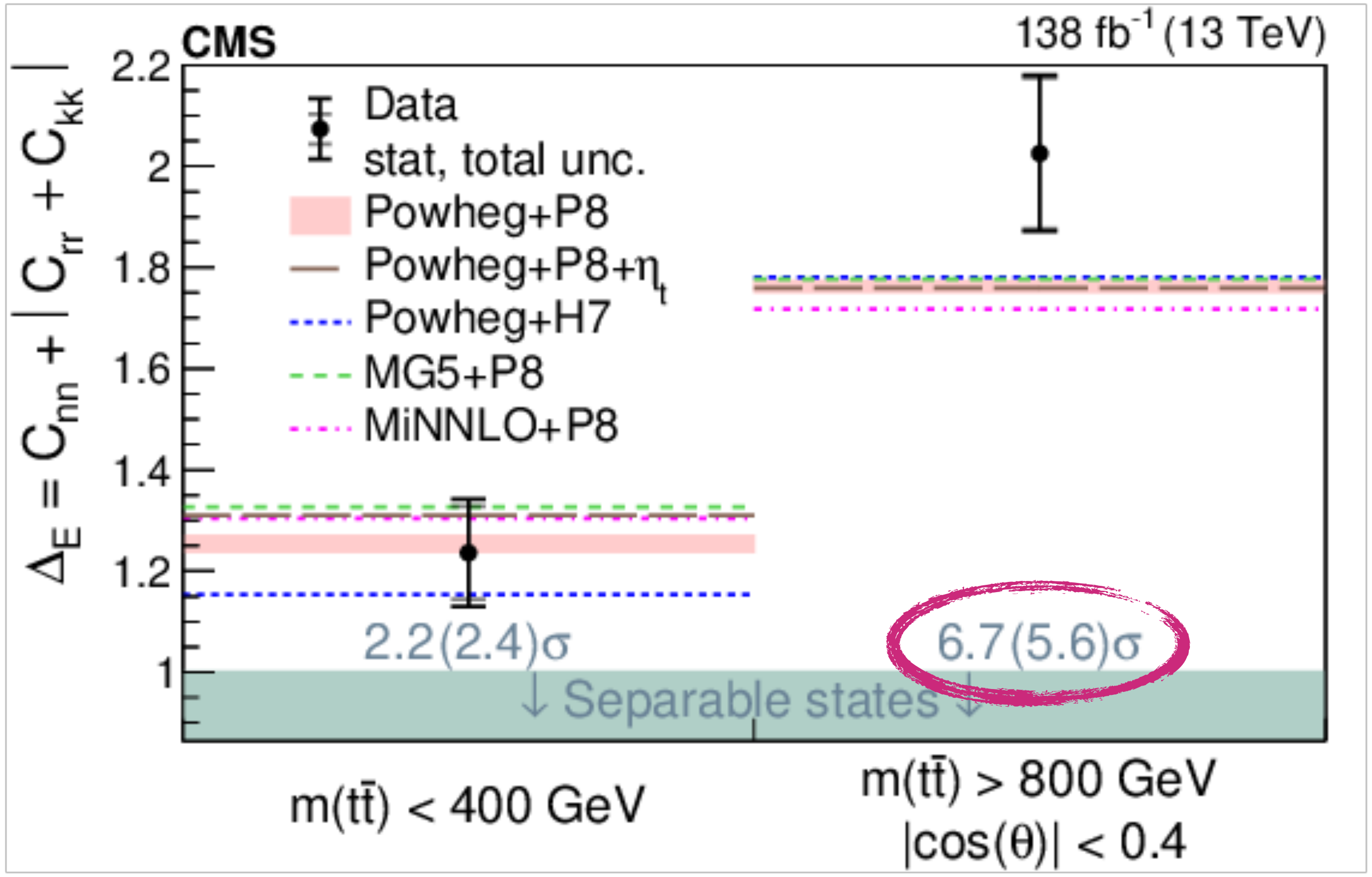
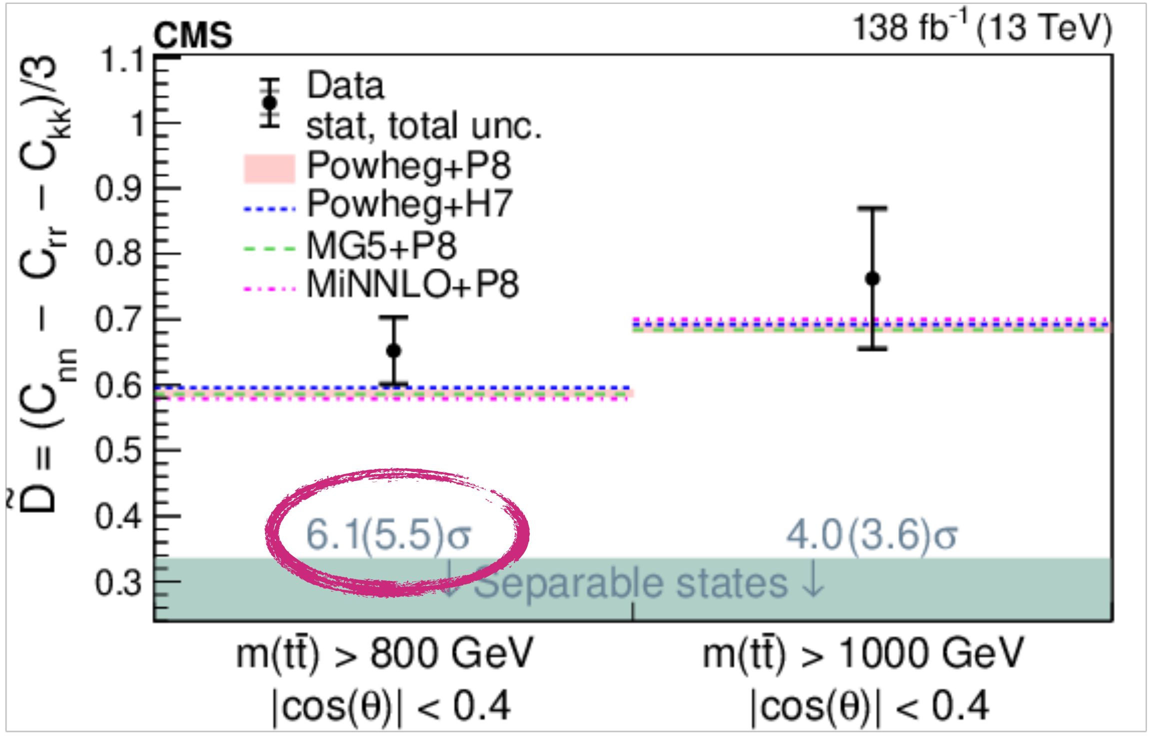


Lepton+jets: entanglement

- **Entanglement observed for first time in boosted region !**
  - highest sensitivity in full matrix measurement
- No sensitivity in low $m_{t\bar{t}}$ region

High $m_{t\bar{t}}$ $\tilde{D} > 1/3$



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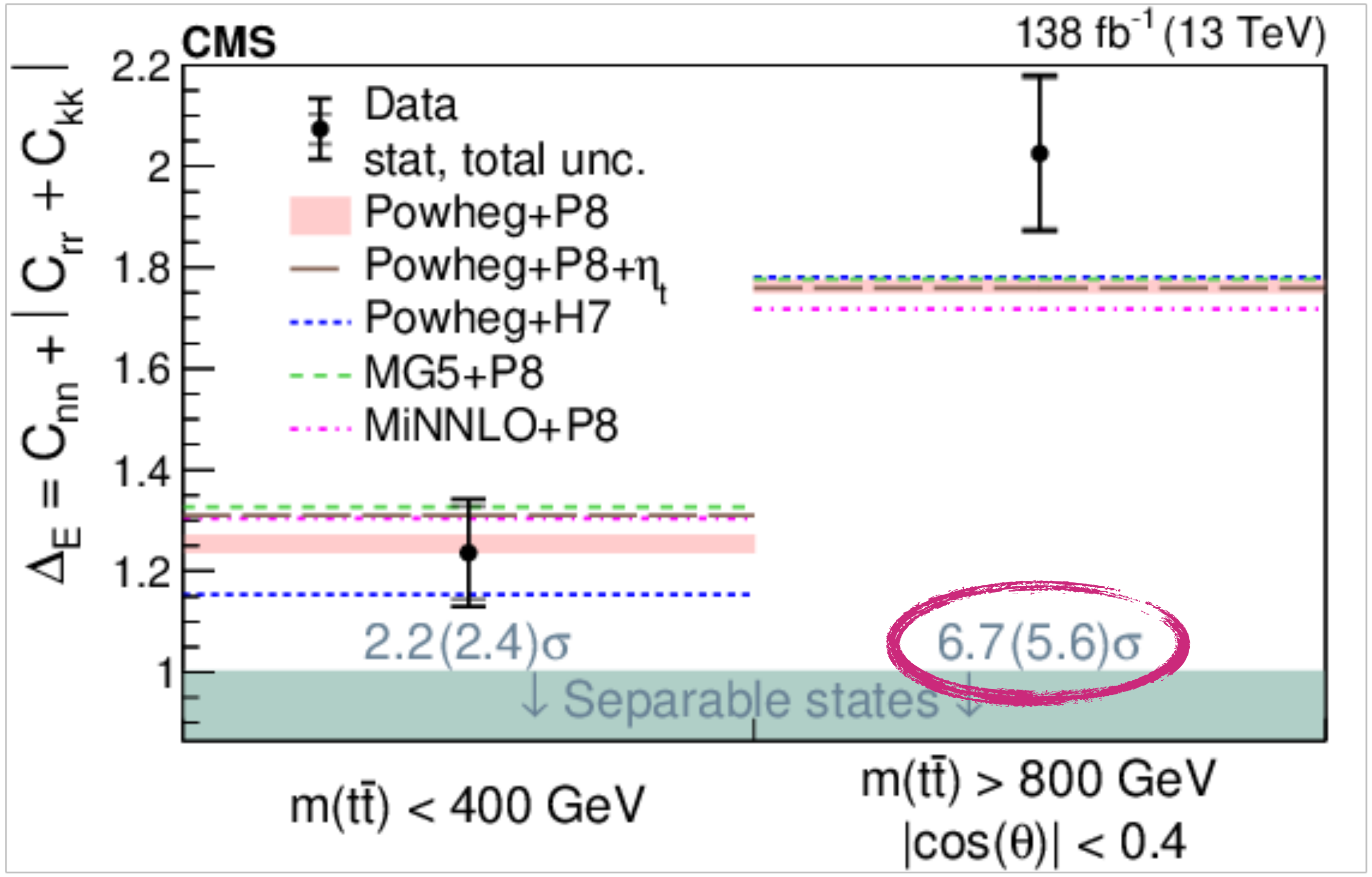
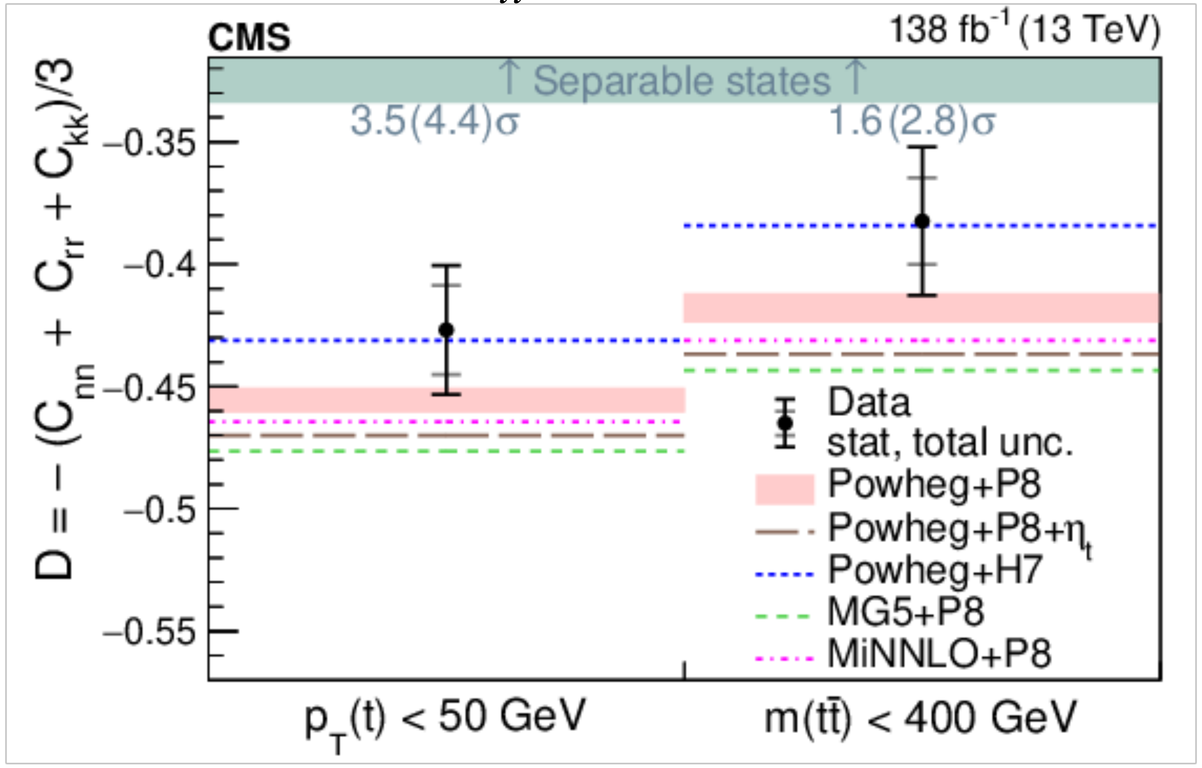


Low $m_{t\bar{t}}$ $D < -1/3$



Entanglement in space-like separated events

- **Fraction of events with space-like separation increases with $m_{t\bar{t}}$**

- In boosted region, 90% of top quark pairs are space-like separated when they decay → no interactions
  - space-like separated events: ΔE sep = 1

- Time-like separated events can interact resulting in entanglement
  - time-like separated events: ΔE max = 3

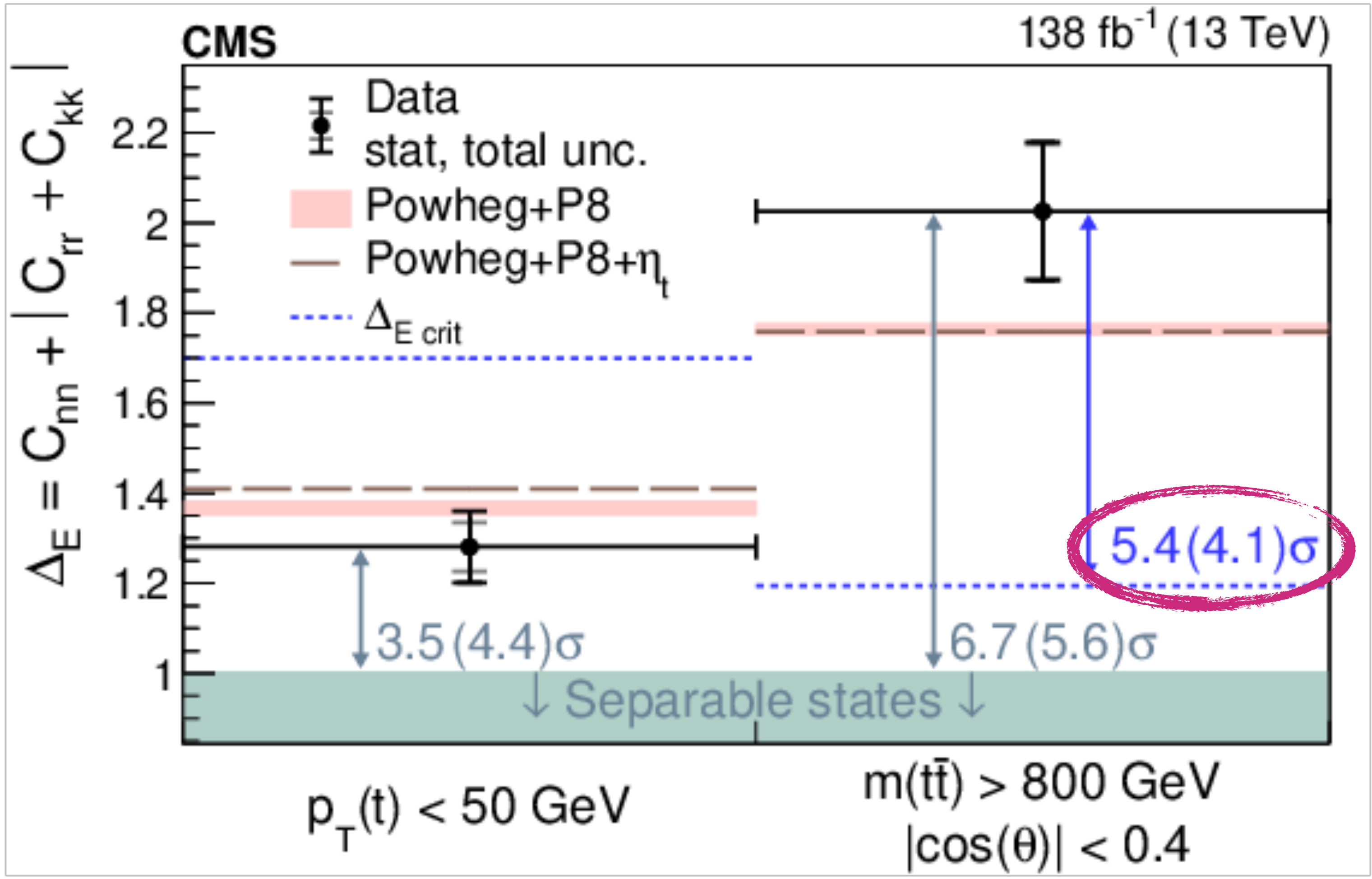
- The **boundary of critical entanglement** ($\Delta_{E\ critical}$) is defined for a given fraction f of space-like separated events as:

$$\Delta_{\mathrm{E\,crit}} = f\Delta_{\mathrm{E\ sep}} + (1-f)\Delta_{\mathrm{E\ max}}$$

$$\Delta_{E\ crit} = 0.9 \cdot 1 + 0.1 \cdot 3 = 1.2$$

Observed Δ_E exceeds $\Delta_{E\ critical}$ by >5 σ

Observation of entanglement in phase space dominated by space-like separated events !



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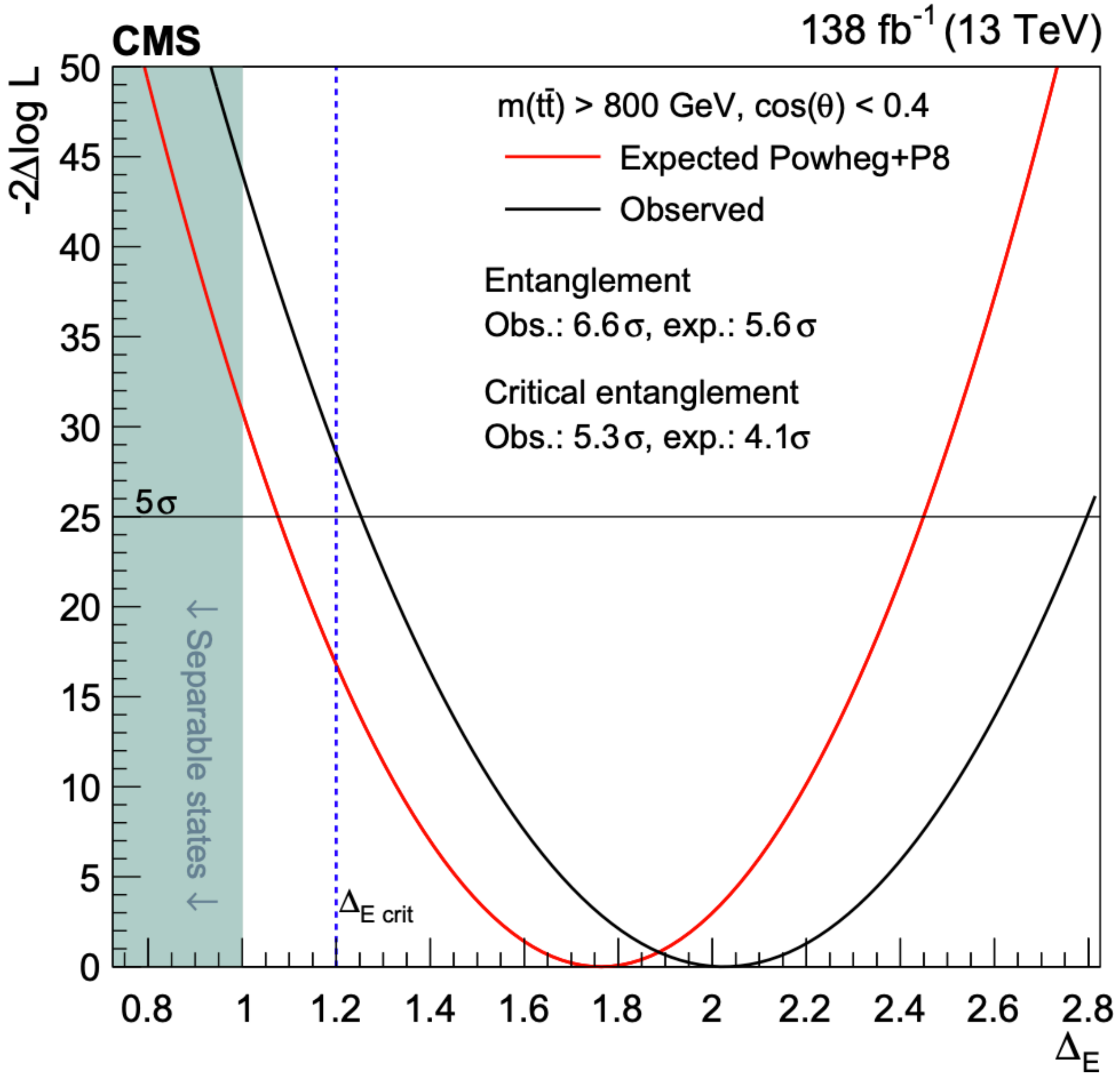
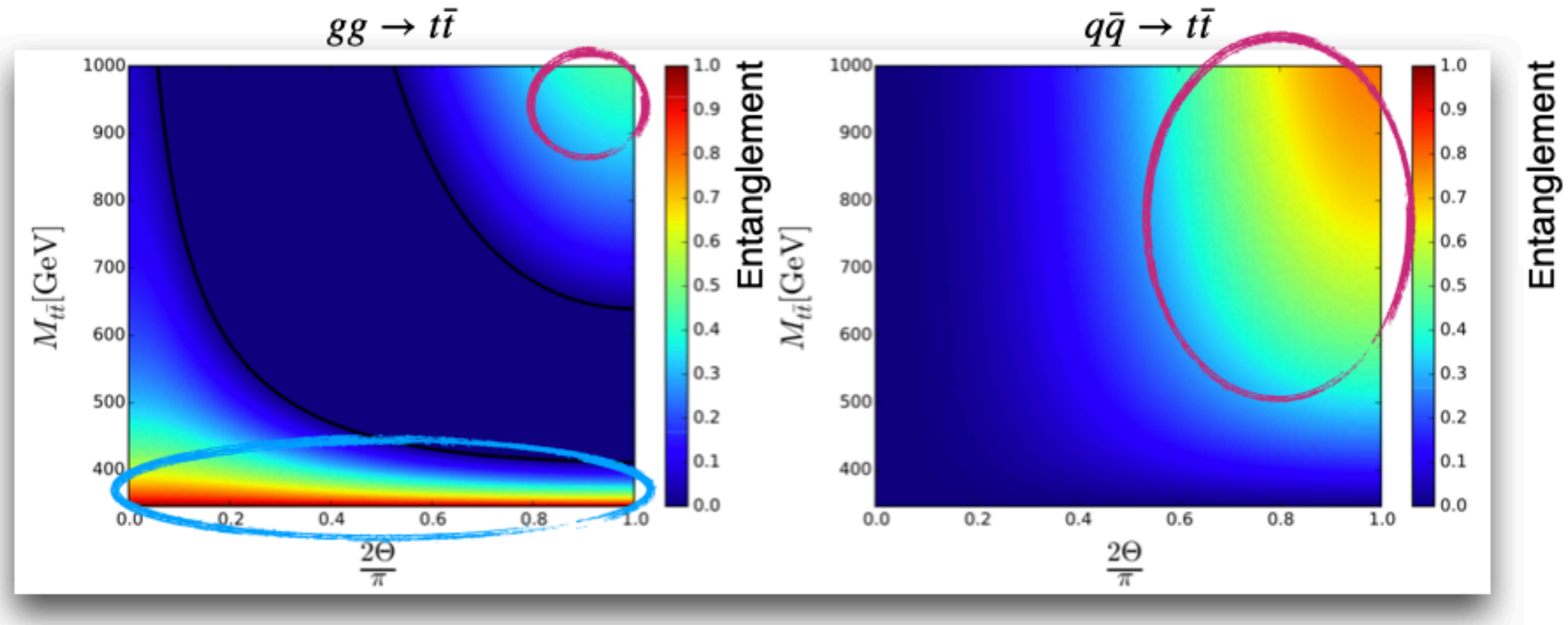


Conclusions

- **First observation of entanglement between top quarks with CMS data in dilepton analysis**
- **Even in presence of a “toponium” bound state, we confirm the existence of entanglement in the $t\bar{t}$ system with > 5 σ**
- A better modeling next to the production threshold is required → theory community is working on improving the prediction of mainstream generators for precision measurements
- **First observation of entanglement between casually separated top quarks in lepton+jets analysis**
  - full spin matrix measurement
- **More exciting results to come!**



BACKUP

Spin correlations: basis of spin quantization axes

Phys. Rev. D 100 (2019) 072002

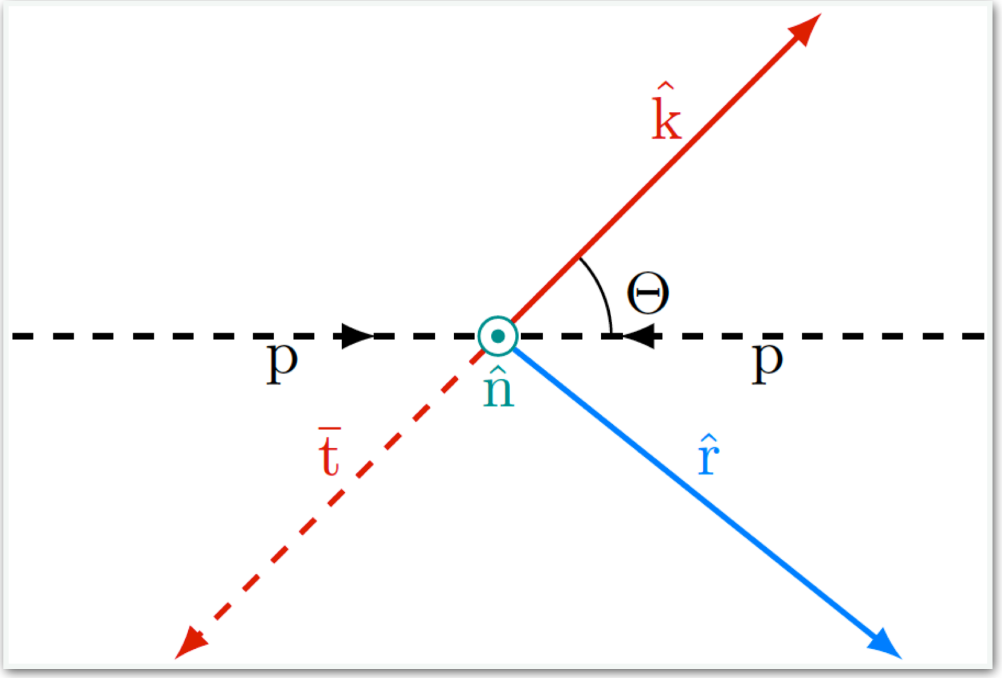
- B and C coefficients:
  - functions of $\sqrt{s}$ and of the top quark scattering angle
  - written in terms of **orthonormal basis** $\{\hat{k}, \hat{r}, \hat{n}\}$:
    - helicity $\hat{k}$-axis: top quark direction in ttbar rest frame
    - transverse $\hat{n}$-axis: transverse to production (ttbar scattering) plane

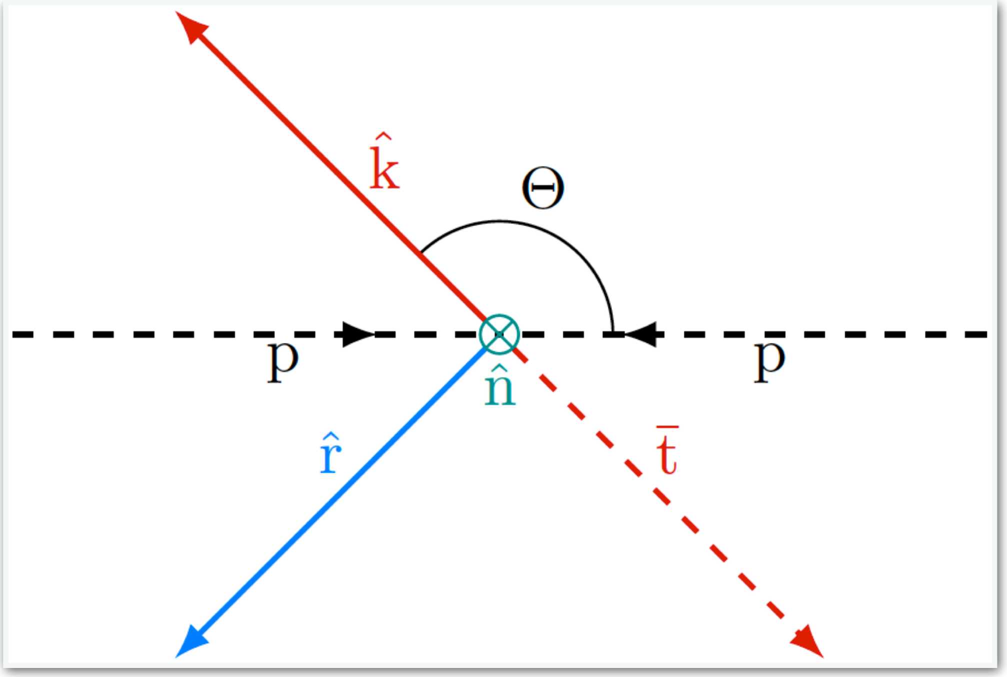
$$\boxed{\hat{n} = \frac{sign(cos\Theta)}{sin\Theta}(\hat{p} \times \hat{k})}$$

    - $\hat{r}$-axis: orthogonal to the other 2 axes (normal to k in ttbar scattering plane)

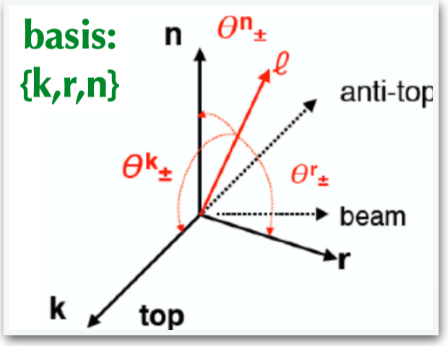
$$\boxed{\hat{r} = \frac{sign(cos\Theta)}{sin\Theta}(\hat{p} - \hat{k}cos\Theta)}$$

    - $\hat{p}$ = direction of the incoming parton, i.e. direction of the proton beam (z-direction in the laboratory frame)
    - Θ = top quark scattering angle in ttbar rest frame



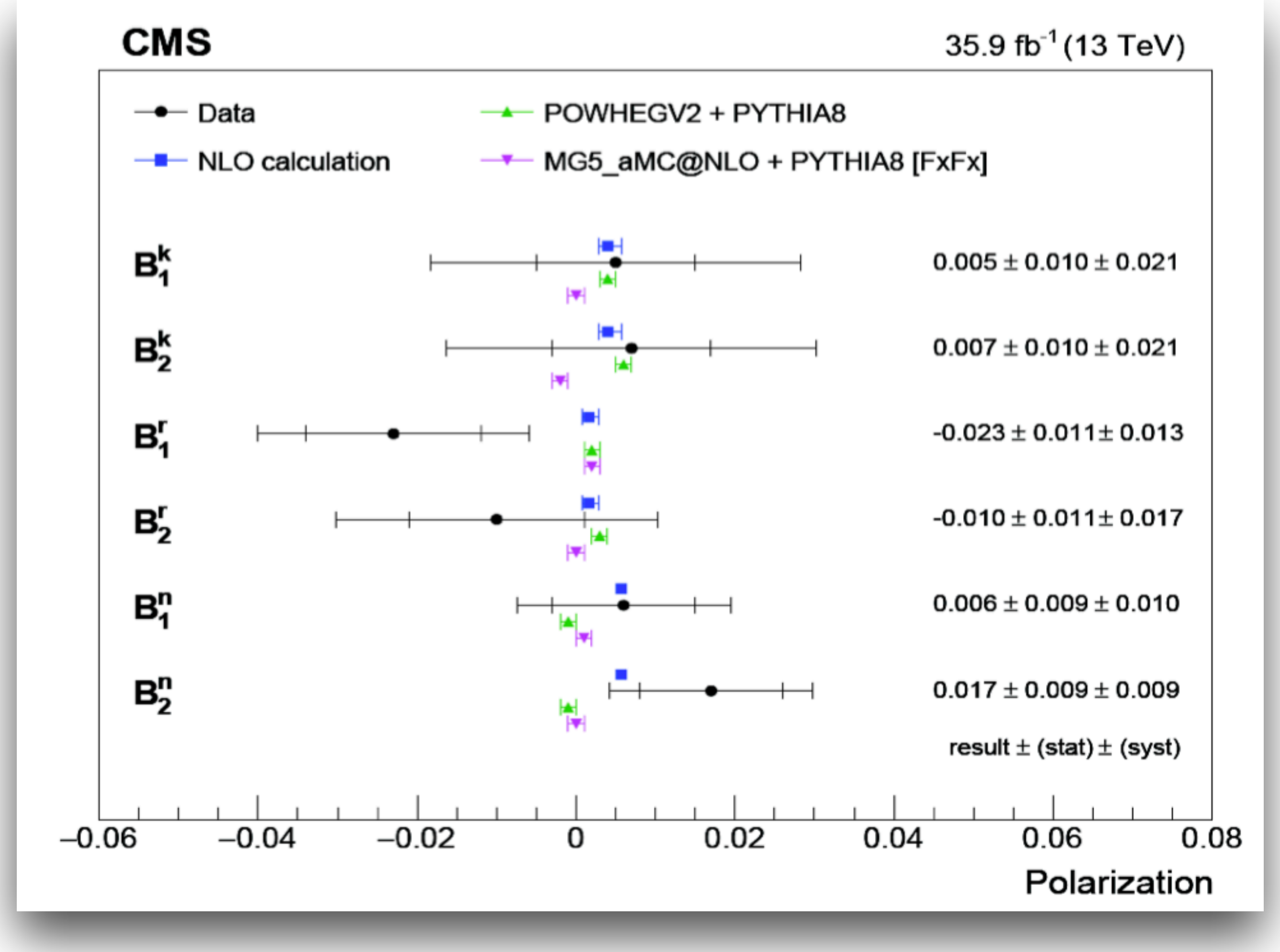


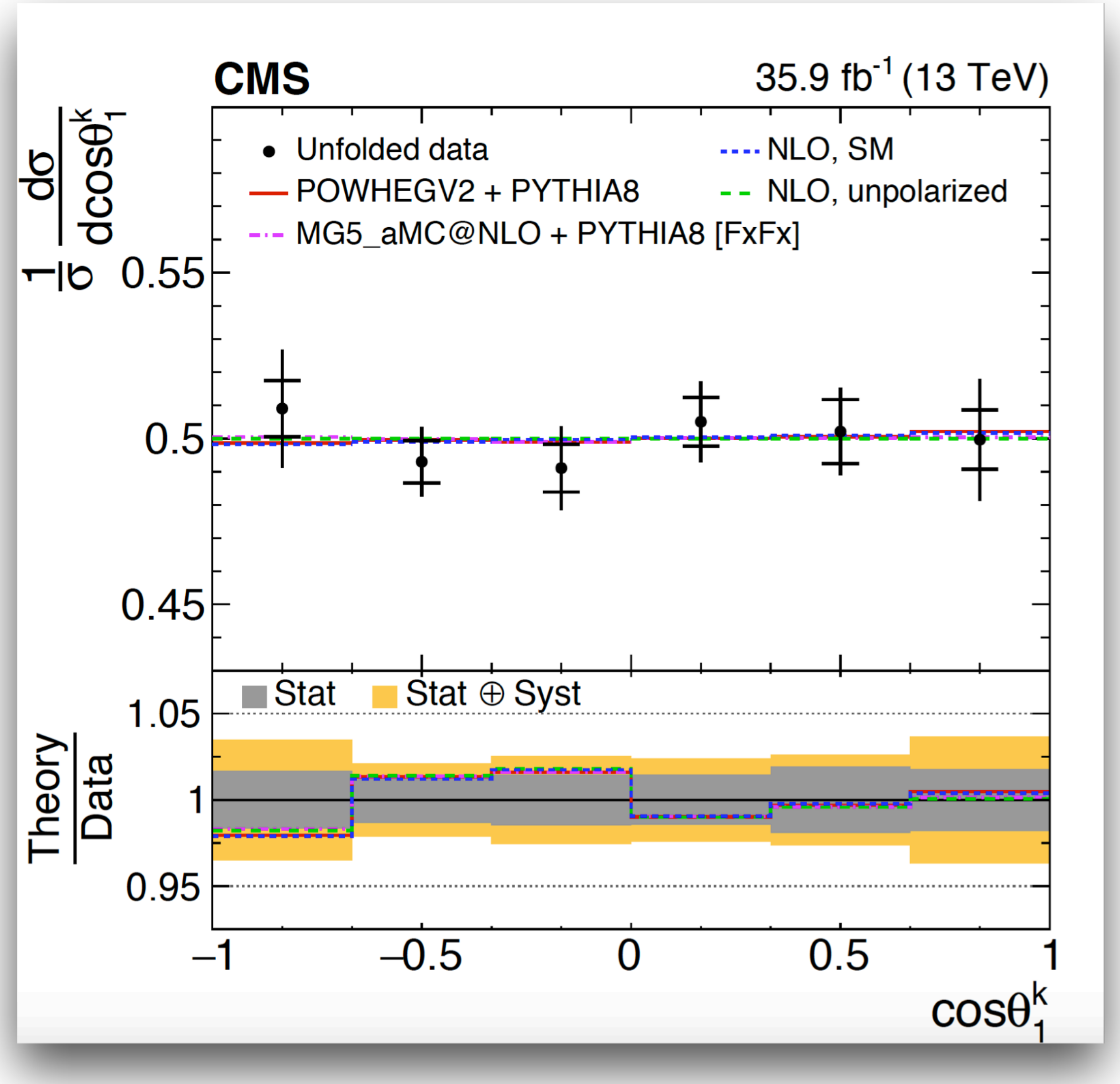
Top spin measurements



- In SM, $t\bar{t}$ production ~ unpolarized
  - $$\frac{1}{\sigma}\frac{d\sigma}{dcos\theta^i_{1/2}} = \frac{1}{2}(1 + B^i_{1/2}cos\theta^i_{1/2})$$
    for polarization coefficients
    → not sensitive to entanglement

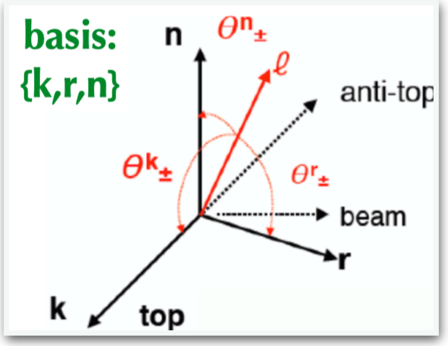
$$B^{+/-} = \begin{pmatrix} x \\ x \\ x \end{pmatrix}$$





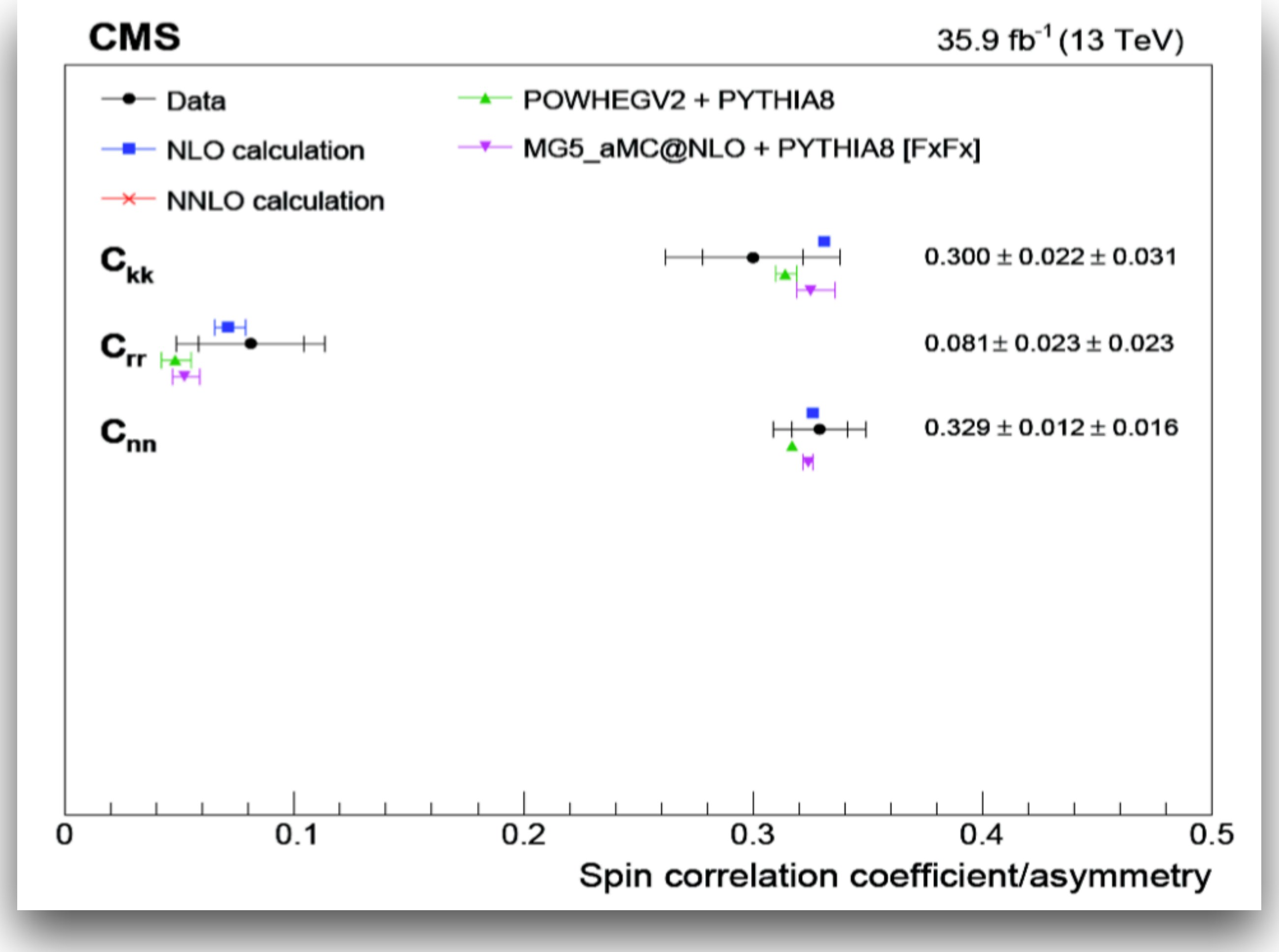
PRD 100 (2019) 072002

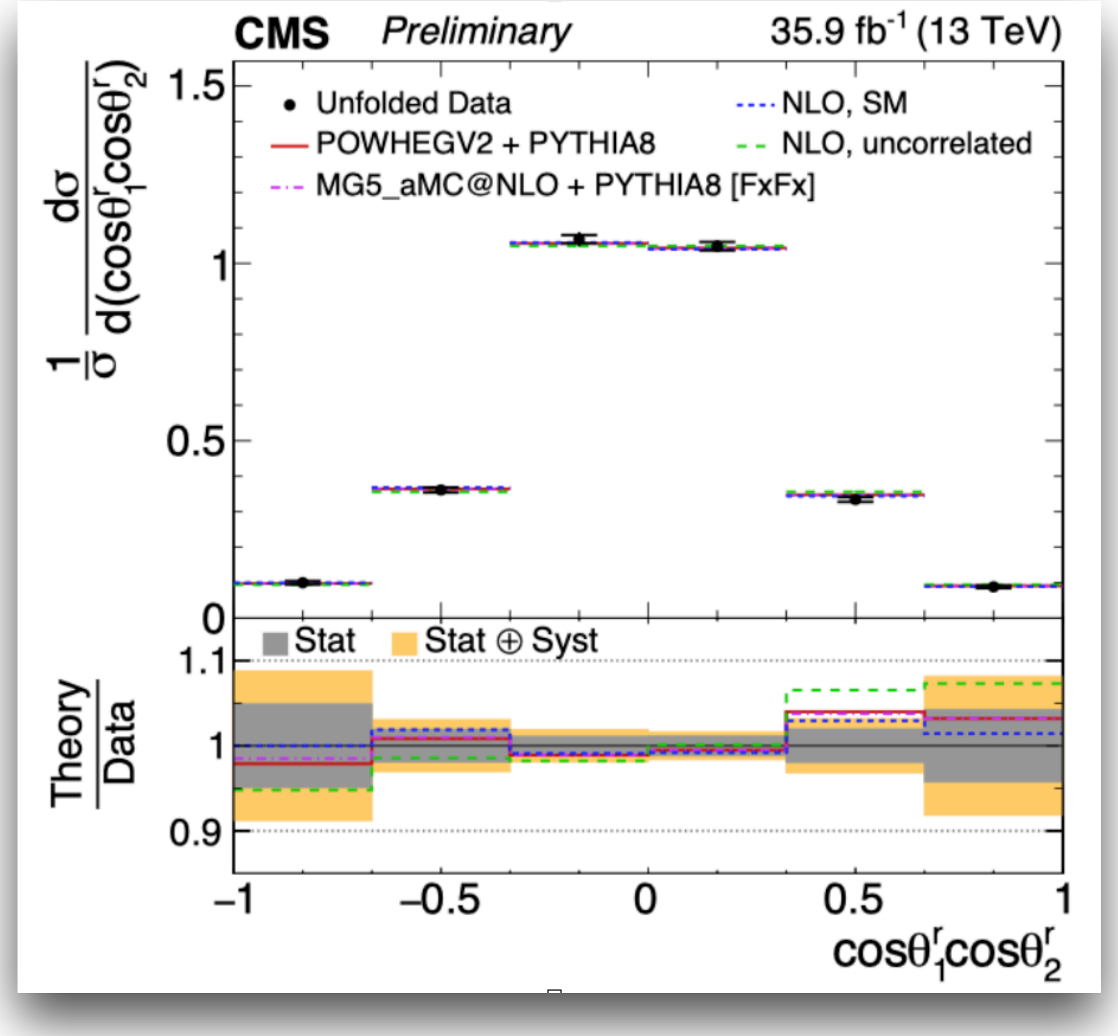
Top spin measurements



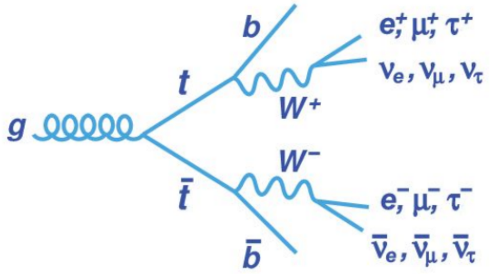
- In SM, $t\bar{t}$ production ~ unpolarized
  but top spins strongly correlated with antitop spins → rich structure of spin correlations
- $$\frac{1}{\sigma}\frac{d\sigma}{d(cos\theta_1^i cos\theta_2^i)} = \frac{1}{2}[1 - C_{ii}(cos\theta_1^i cos\theta_2^i)]\ln\frac{1}{|cos\theta_1^i cos\theta_2^i|}$$
  for diagonal spin correlations coefficients

$$C = \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix}$$

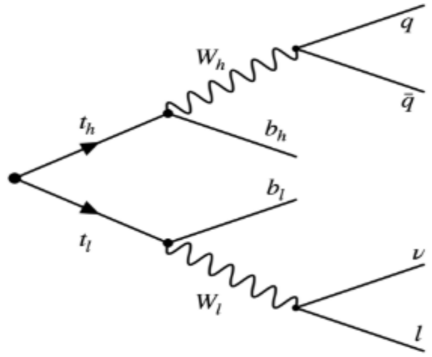




PRD 100 (2019) 072002



Dilepton vs lepton+jets



Dilepton

arXiv:2406.03976
accepted by ROPP

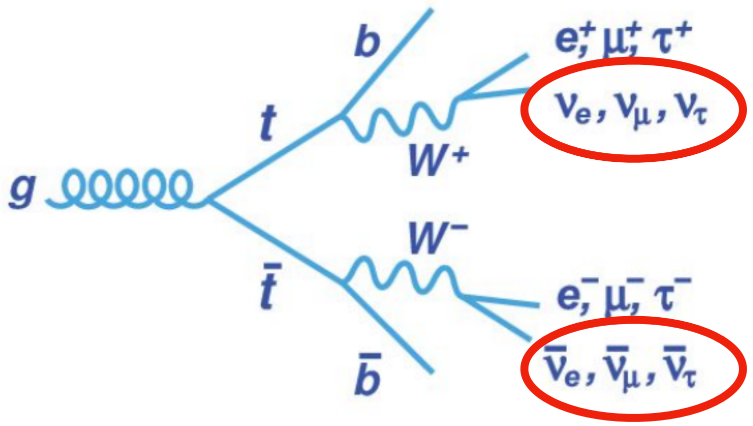
- **36.3 fb^{-1} of 2016 data @13 TeV**
  - based on PRD 100 (2019) 072002
- Lower branching ratio
- top spin info 100 % transmitted to charged leptons → easy to identify
- Lower p_T cuts for leading/subleading lepton (25/20 GeV) → higher efficiency at the threshold
- Worse $m_{t\bar{t}}$ resolution → not ideal for differential measurement
- Best for threshold region
  - high entanglement
  - mostly **time-like separated events**

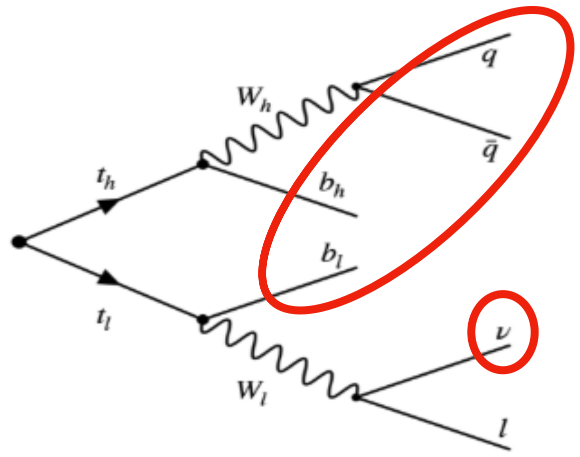
Lepton + jets

arXiv:2409.11067
submitted to PRD

- **138 fb^{-1} of data @13 TeV** collected in **full Run 2**
- Higher branching ratio
- top spin info ~100 % transmitted to down-type quarks → hard to identify
- Higher p_T cut for single lepton (30 GeV) and for 4 jets (30 GeV) → lower efficiency at the threshold but OK for high $m_{t\bar{t}}$
- Better $m_{t\bar{t}}$ resolution → good for differential measurement
- Advantage for high $m_{t\bar{t}}$
  - high entanglement
  - mostly **space-like separated events**

Dilepton vs lepton+jets top quark reconstruction





- **$m_{\ell b}$ weighting method**
  - use algebraic method to solve for neutrino 3-vectors
  - pick solution with smallest $m_{t\bar{t}}$
  - pair lepton and jet according to expected $m_{\ell b}$

- **Artificial NN**
  - goal = correctly identify detector-level objects and up/down jet assignment
  - NN trained on permutations
- For each event:
  - provide all possible permutations of objects as input to NN
  - use permutation resulting in the highest NN score
  - calculate neutrino momentum with W boson mass constraint

$$(p_\nu + p_l)^2 = m_W^2$$

Signal model

Combined signal model: $t\bar{t}$ + toponium (η_t)

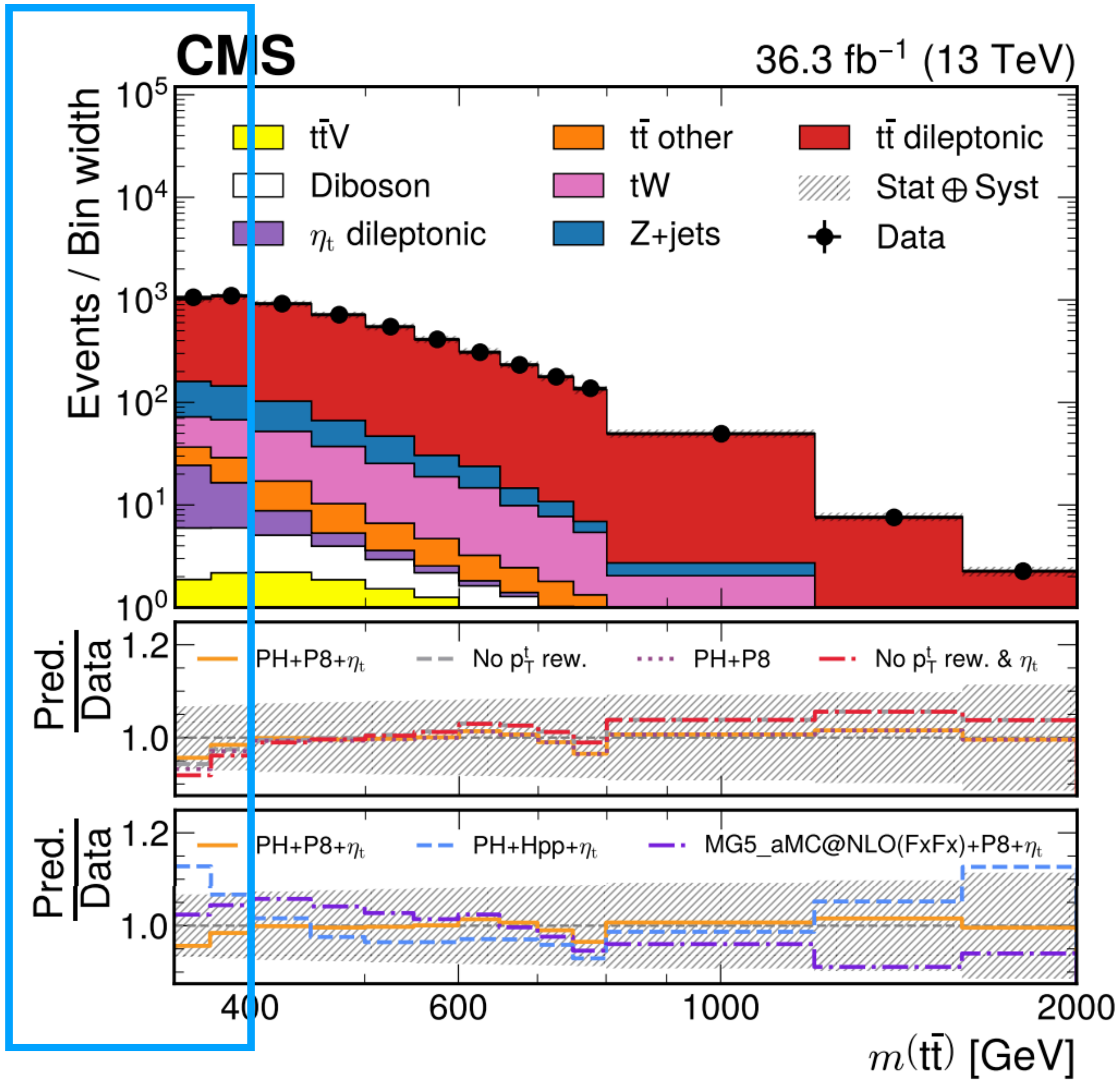
- `PowhegBox+Pythia8` (NLO) as **nominal $t\bar{t}$ sample**

  [Comput. Phys. Commun. 182 (2011) 10](#)

  - inclusion of **EWK corrections at NLO** with `HATHOR`
  - **reweighting to NNLO QCD calculations** [PRL 127 (2021) 062001](#)
    - **dilepton**: p_T reweighting to match the top quark p_T spectrum from a fixed order ME calculation at NNLO
    - **lepton+jets**: NN-based reweighting to NNLO distributions at reco level

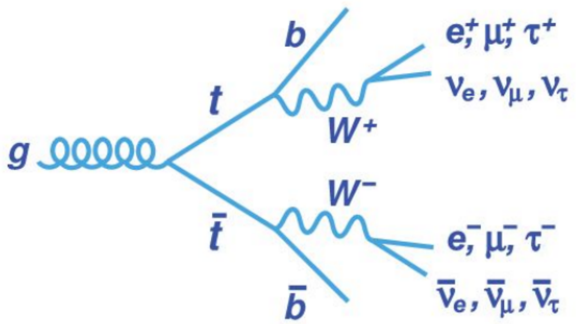
- **toponium model** generated with `MG5 aMC@NLO(LO)+Pythia8` B. Fuks et al. [Phys Rev D 104 034023](#)

  - only pseudoscalar colour singlet and spin-0 η_t state accounted for
  - η_t improves data modeling in the threshold region



Dilepton analysis region

Dilepton: event selection

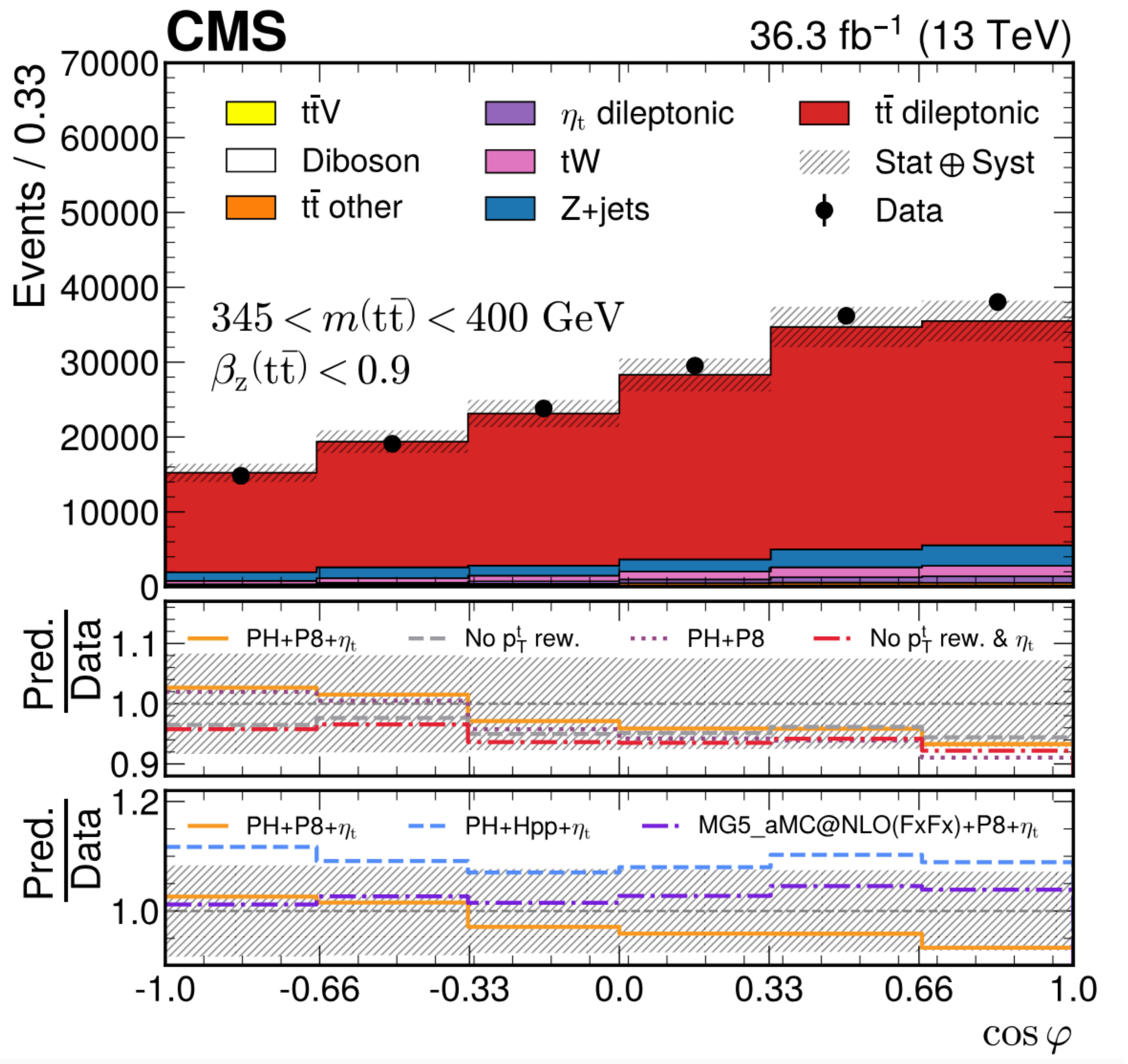


- **= 2 oppositely charged isolated leptons (ee, eμ and μμ)**
  - **including also leptons from tau decays** (different from 2016 analysis)
  - p_T > 25(20) GeV, for leading(trailing) lepton and $|\eta|$ < 2.4
  - reject events with $m_{\ell\bar{\ell}}$ < 20 GeV
  - single lepton + dilepton triggers

- **≥ 2 jets (R=0.4), >=1 b jet**
  - p_T > 30 GeV and $|\eta|$ < 2.4
  - jet cleaning: ΔR(ℓ, jet) > 0.4

- **ee, μμ channels:**
  - E^T_{miss} > 40 GeV
  - Z veto: $|m_Z - m_{\ell\bar{\ell}}|$ > 15 GeV

- **Top quark reconstruction with $m_{\ell b}$ weighting method**
  - take solution with smallest $m_{t\bar{t}}$



Dilepon: top quark reconstruction

- **Use algebraic method to solve for neutrino 3-vectors**
- **Results in quartic equation for neutrino momenta**
- **Pick solution with lowest $m_{t\bar{t}}$**
- **Repeat process 100x for leptons and b jets smeared within resolution**
- **Weight solutions by the $m_{\ell b}$ distribution**

$$0 = \sum_{i=0}^{4} c_i(m_t, p_{\ell^+}, p_{\ell^-}, p_b, p_{\bar{b}})\, p_x(\bar{\nu})^i$$

$$\begin{aligned} \not{E}_x &= p_{\nu_x} + p_{\bar{\nu}_x} \\ \not{E}_y &= p_{\nu_y} + p_{\bar{\nu}_y} \end{aligned}$$

$$\begin{aligned} m_{W^+}^2 &= (E_{\ell^+} + E_\nu)^2 - (p_{\ell_x^+} + p_{\nu_x})^2, \\ &\quad -(p_{\ell_y^+} + p_{\nu_y})^2 - (p_{\ell_z^+} + p_{\nu_z})^2, \\ m_{W^-}^2 &= (E_{\ell^-} + E_{\bar{\nu}})^2 - (p_{\ell_x^-} + p_{\bar{\nu}_x})^2, \\ &\quad -(p_{\ell_y^-} + p_{\bar{\nu}_y})^2 - (p_{\ell_z^-} + p_{\bar{\nu}_z})^2, \\ m_t^2 &= (E_b + E_{\ell^+} + E_\nu)^2 - (p_{b_x} + p_{\ell_x^+} + p_{\nu_x})^2, \\ &\quad -(p_{b_y} + p_{\ell_y^+} + p_{\nu_y})^2 - (p_{b_z} + p_{\ell_z^+} + p_{\nu_z})^2 \\ m_{\bar{t}}^2 &= (E_{\bar{b}} + E_{\ell^-} + E_{\bar{\nu}})^2 - (p_{\bar{b}_x} + p_{\ell_x^-} + p_{\bar{\nu}_x})^2, \\ &\quad -(p_{\bar{b}_y} + p_{\ell_y^-} + p_{\bar{\nu}_y})^2 - (p_{\bar{b}_z} + p_{\ell_z^-} + p_{\bar{\nu}_z})^2 \end{aligned}$$

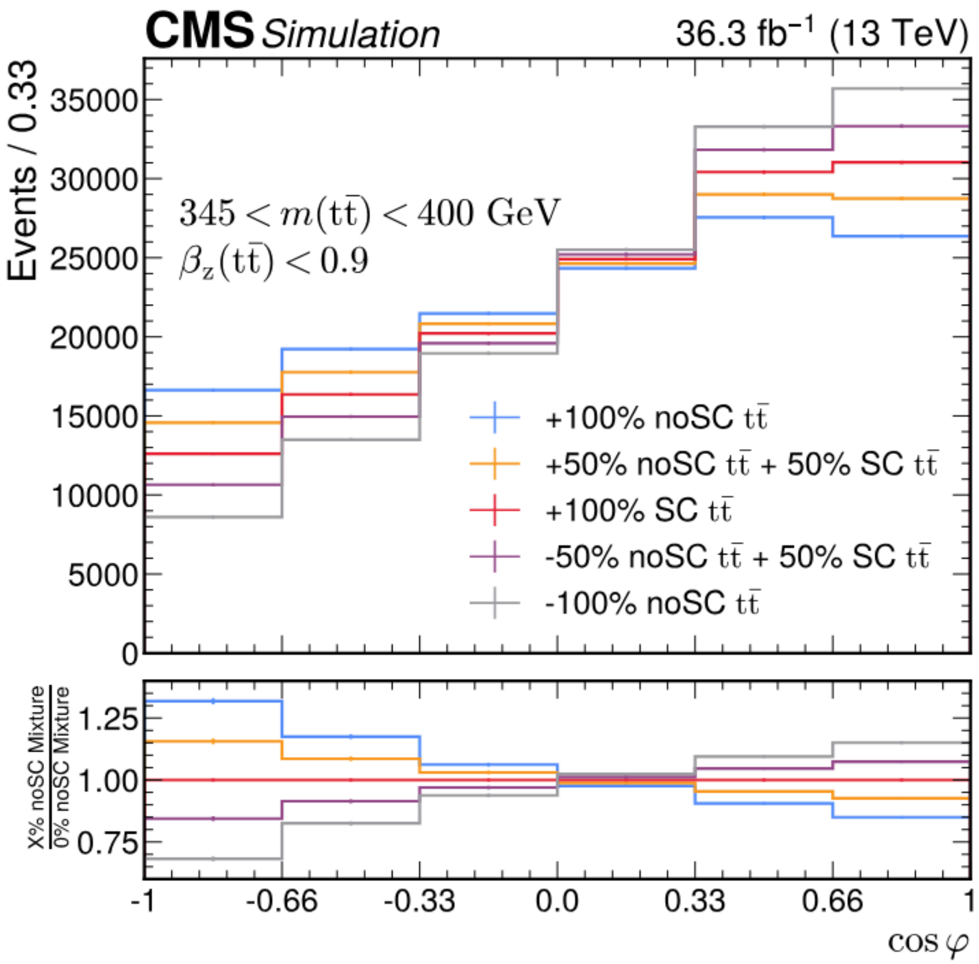
Mixtures of SC and noSC

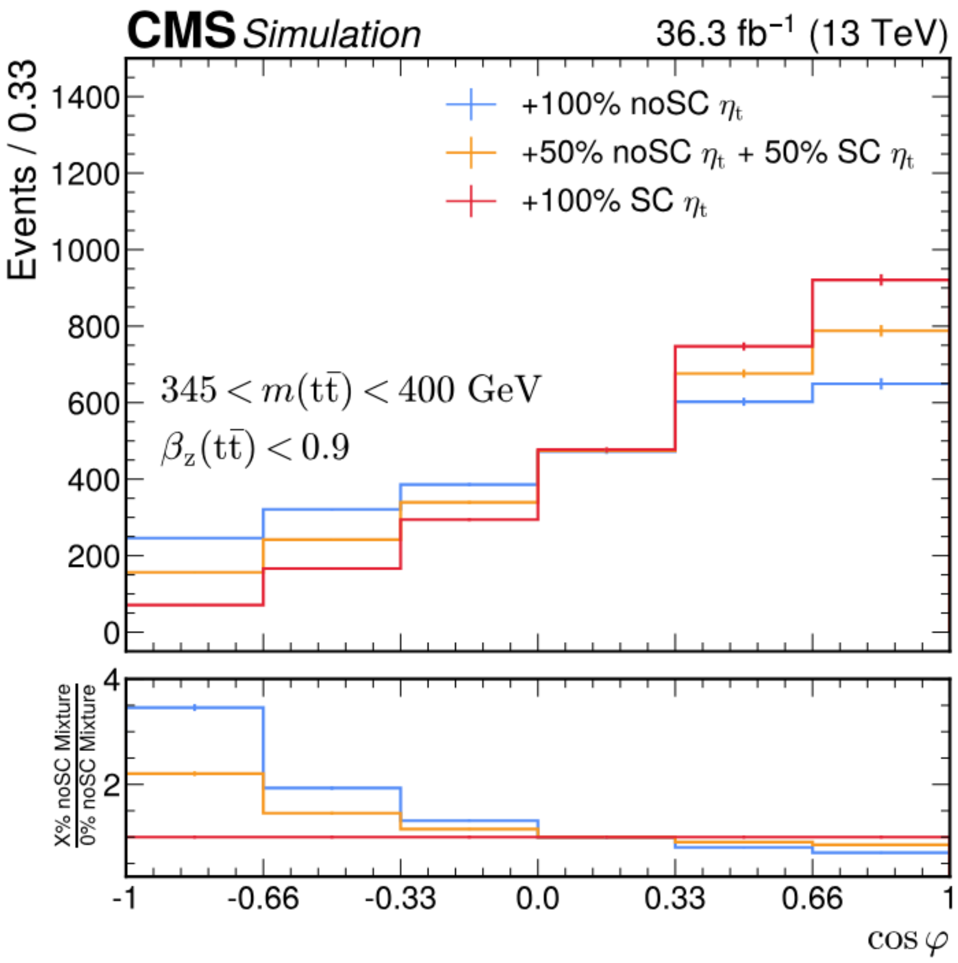
- In order to have templates implementing an alternative value of the entanglement proxy D, we employ the noSC sample and “mix” it in steps ranging from −100% to 100% with the combined signal model SM template
- The negative mixtures are created mirroring the corresponding positive mixtures around the 0% noSC mixture, i.e., the nominal combined signal model

- Any particular mixture of combined SC and noSC signal corresponds to a certain value of D at the parton level by means of calculating a 2-bin asymmetry:

$$A_D = (N(\cos\varphi > 0) - N(\cos\varphi < 0))/(N(\cos\varphi > 0) + N(\cos\varphi < 0))$$

yields D as $-2 \cdot A_D$, with N always being the sum of $\mathrm{t\bar{t}}$ and η_t.





Dilepton results without η_t

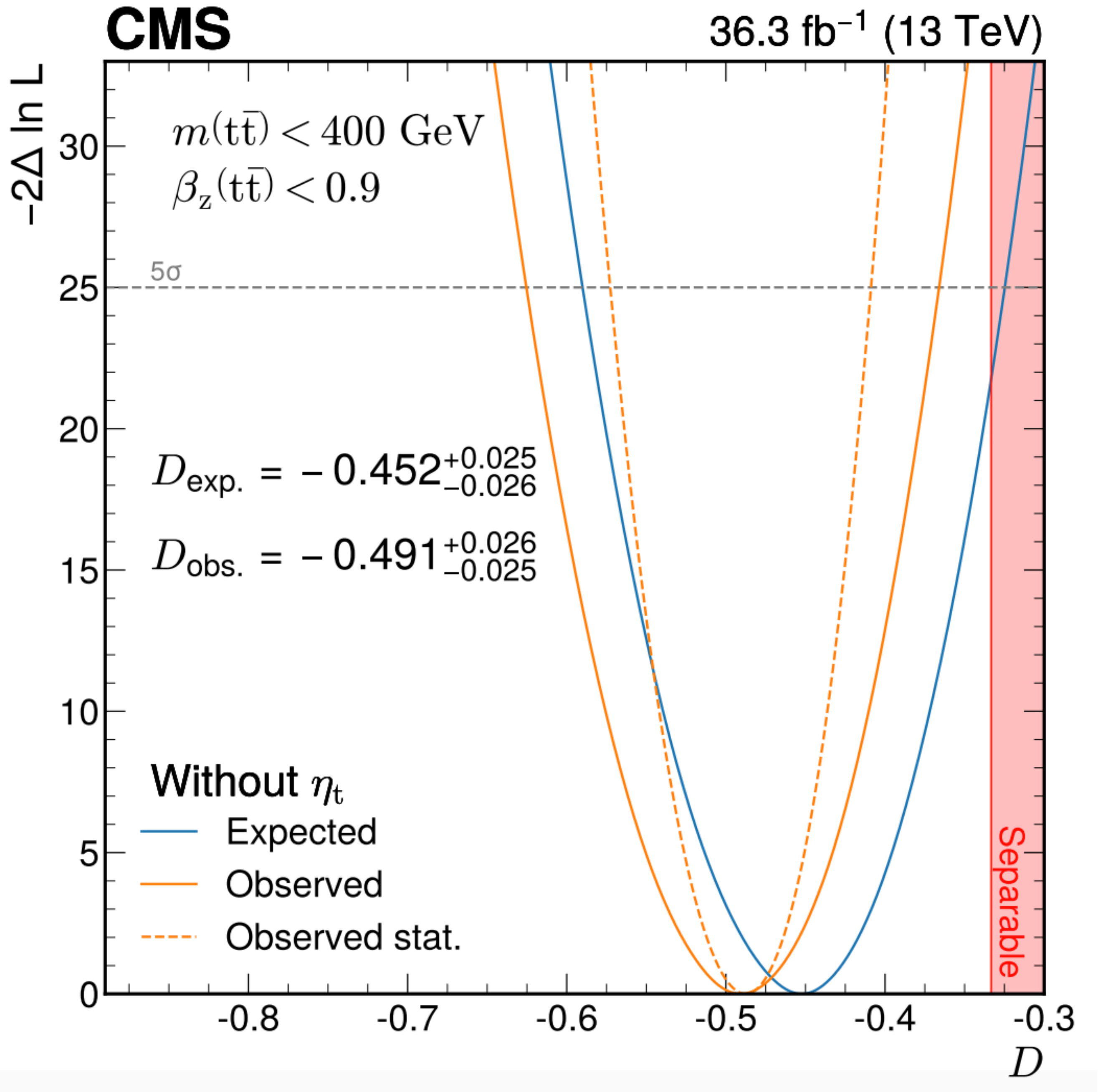
- **Scan of the $-2\Delta lnL$ distribution yields D at parton level, accounting for all detector effects**

$$\boxed{D_{obs} = -0.491^{+0.026}_{-0.025}(\text{tot})}$$

$$D_{exp} = -0.452^{+0.025}_{-0.026}(\text{tot})$$

- **Significance: 6.3σ obs. (4.7σ exp.)**

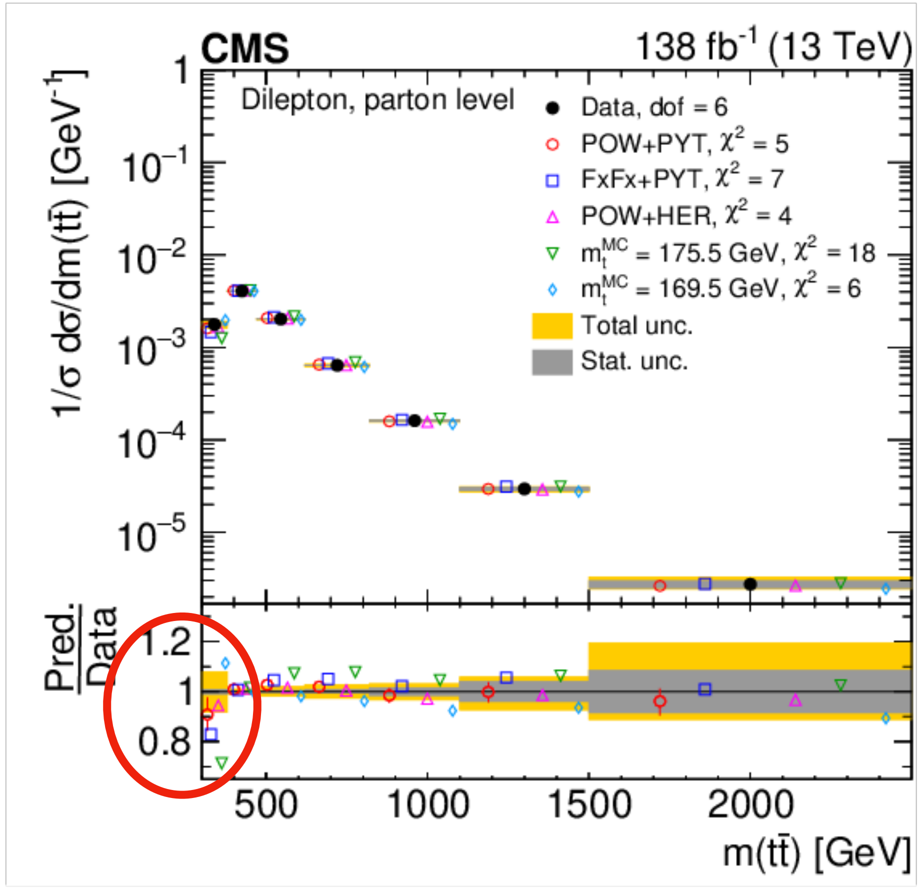
>5 standard deviations observation of top quarks being entangled at $t\bar{t}$ threshold !



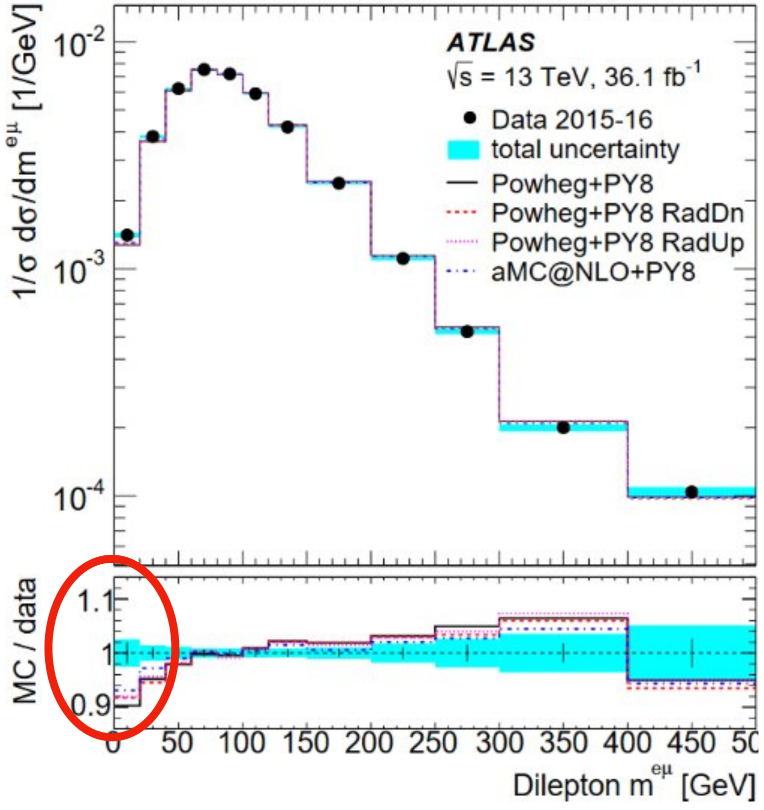
Threshold region

- **Mis-modeling at a level of ~10% seen for $m_{t\bar{t}}$ ~345 GeV ($m_{e\mu}$ < 50 GeV)**
- **Consistent between dilepton and lepton+jets analyses in both CMS and ATLAS**

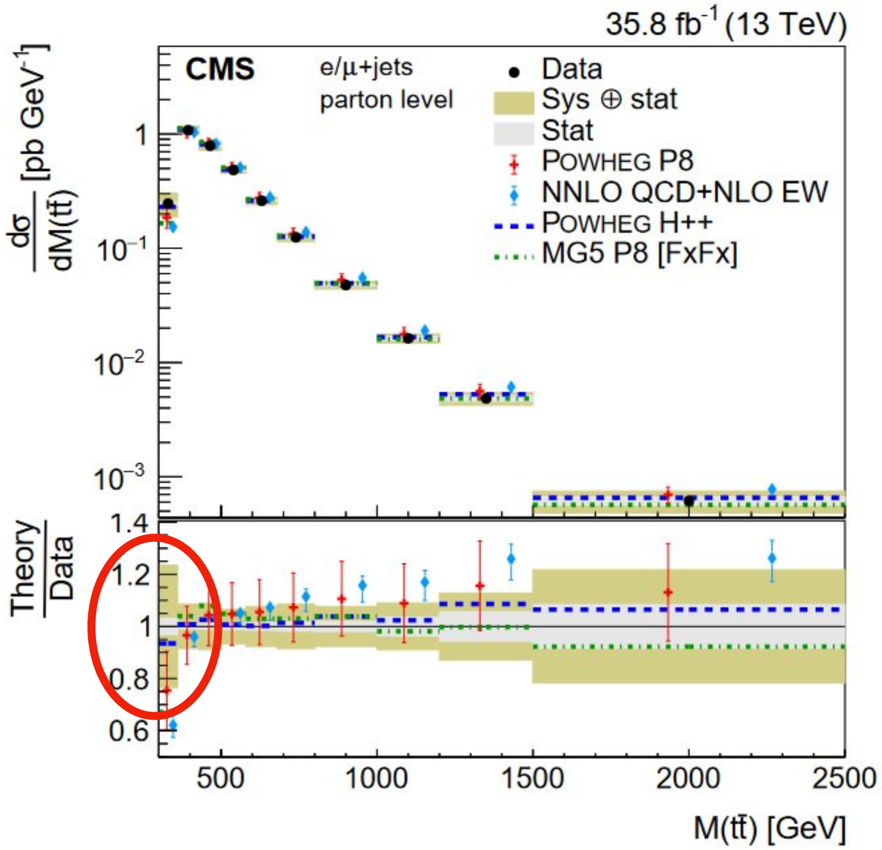
arXiv:2402.08486,
submitted to JHEP



EPJ C 80, 6



Phys. Rev. D 97, 112003



Threshold region

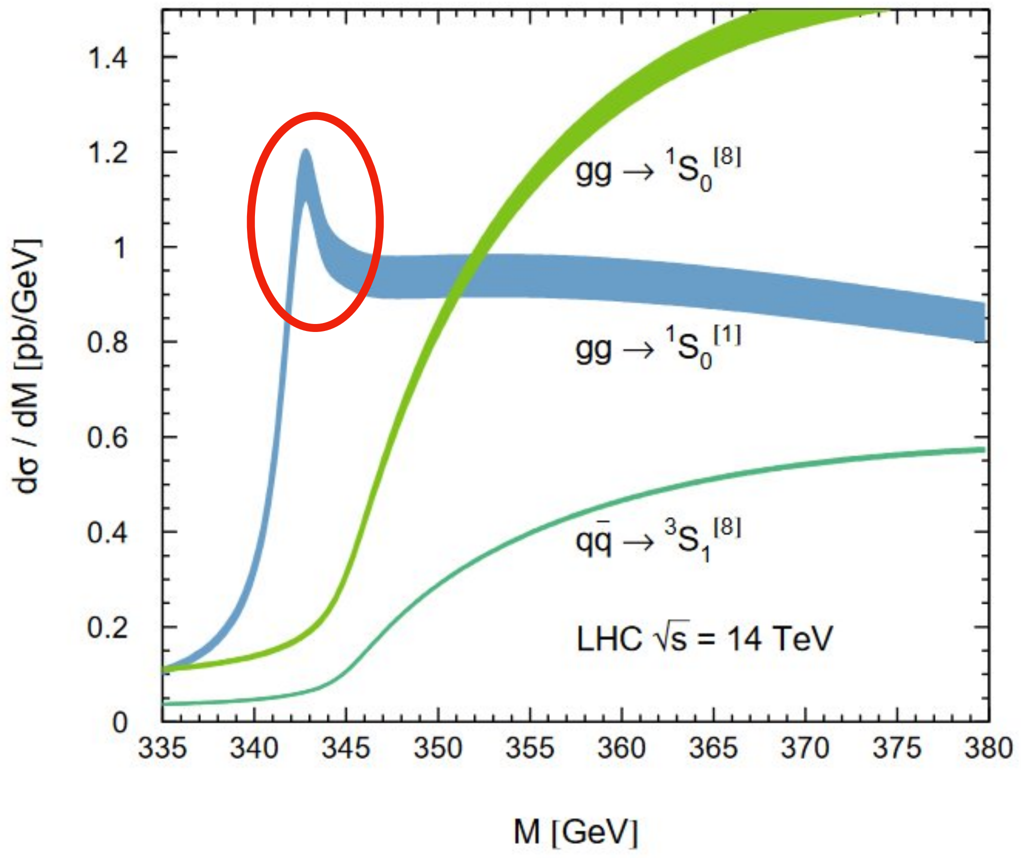
W. Ju, G. Wang, et al.
JHEP 06, 158

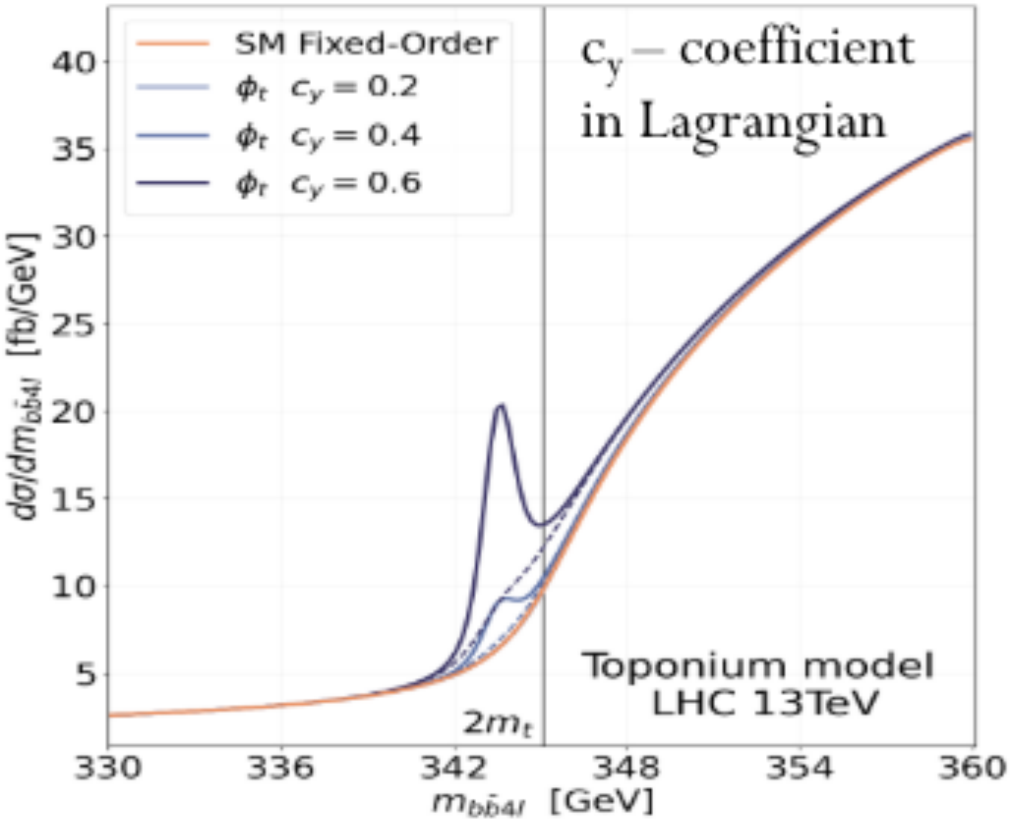
- **NRQCD contributions close to threshold**
  - **toponium:** predicted top quark-antiquark quasi-bound state with a mass of 343 GeV and a width of 7 GeV

- **Excess seen could come from toponium ?**

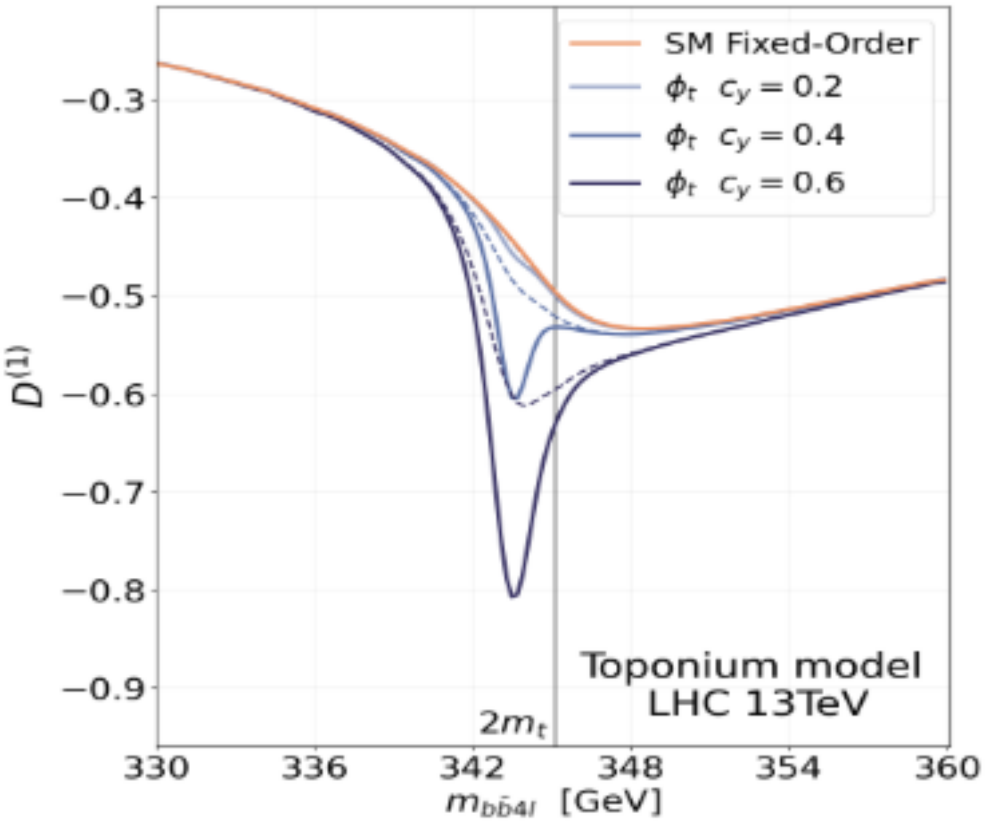
- It affects the invariant mass distribution and the spin correlations at threshold

→ **inclusion of toponium (η_t) contributions in our signal model**





F. Maltoni et al.
JHEP03(2024)099

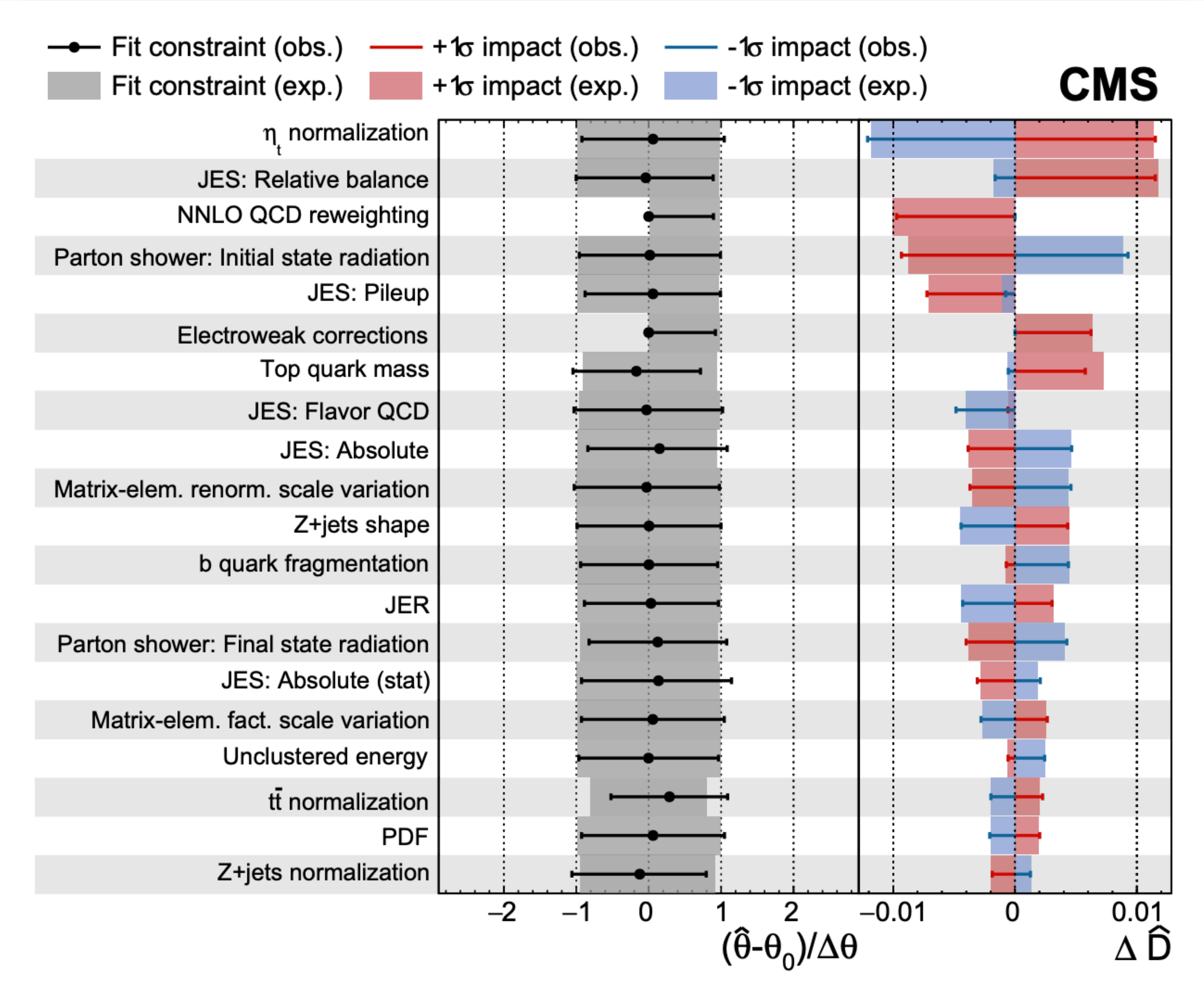


Dilepton: systematic uncertainties

- **Same uncertainties considered in 2016 spin corr analysis + additional ones for toponium:**
  - *toponium cross section varied by 50%* due to missing octet contributions
  - *binding energy uncertainty varied by ±0.5 GeV*

- **Leading theory-based uncertainties:**
  - Toponium normalization
  - NNLO QCD reweighing
  - Parton Shower

- **Leading experimental uncertainties:**
  - Jet energy scale



Comparison with ATLAS

- **Entanglement in top quark observed by both ATLAS and CMS with >5 standard deviations!**
  - despite different analyses…

	ATLAS	**CMS**
Dataset	Full Run 2 (140 fb^{-1})	2016 (35.9 fb^{-1})
$t\bar{t}$ decay	Dilepton: eμ	Dilepton: ee, eμ and μμ
$t\bar{t}$ reconstruction	Ellipse method	Weighting method
Main selections	$340 < m(t\bar{t}) < 380$ GeV	$345 < m(t\bar{t}) < 400$ GeV, beta <0.9
Triggers	Single lepton	Single lepton + dilepton
Corrected to	Particle-level	Parton-level
Fit type	No fit, calibration curve	Profile likelihood template fit
Alternative hypothesis D	Reweighting	Mixing samples with/without spin corr
Threshold effects	Neglected	Considered (toponium contribution)
Nominal MC	PowhegBox+Pythia8	PowhegBox+Pythia8
Alternative MC	PowhegBox+Herwig7, bb4l	PowhegBox+Herwig++, MG5_AMC@NLO
Significance	**>> 5 standard deviations**	**> 5 standard deviations**

$$D_{obs} = -0.547 \pm 0.002(\text{stat}) \pm 0.021(\text{syst})$$
$$D_{exp} = -0.470 \pm 0.002(\text{stat}) \pm 0.018(\text{syst})$$

$$D_{obs} = -0.480^{+0.016}_{-0.017}(\text{stat})^{+0.020}_{-0.023}(\text{syst})$$
$$D_{exp} = -0.467^{+0.016}_{-0.017}(\text{stat})^{+0.021}_{-0.024}(\text{syst})$$

Comparison with ATLAS

- No clear preference for a specific MC prediction

- **Both analyses are dominated by systematic uncertainty**
  - total (stat.) unc. is an order of magnitude larger in the CMS analysis
  - total (syst.) unc. is similar between ATLAS & CMS, but different systematics are considered

$$D_{obs} = -0.547 \pm 0.002(\text{stat}) \pm 0.021(\text{syst})$$
$$D_{exp} = -0.470 \pm 0.002(\text{stat}) \pm 0.018(\text{syst})$$

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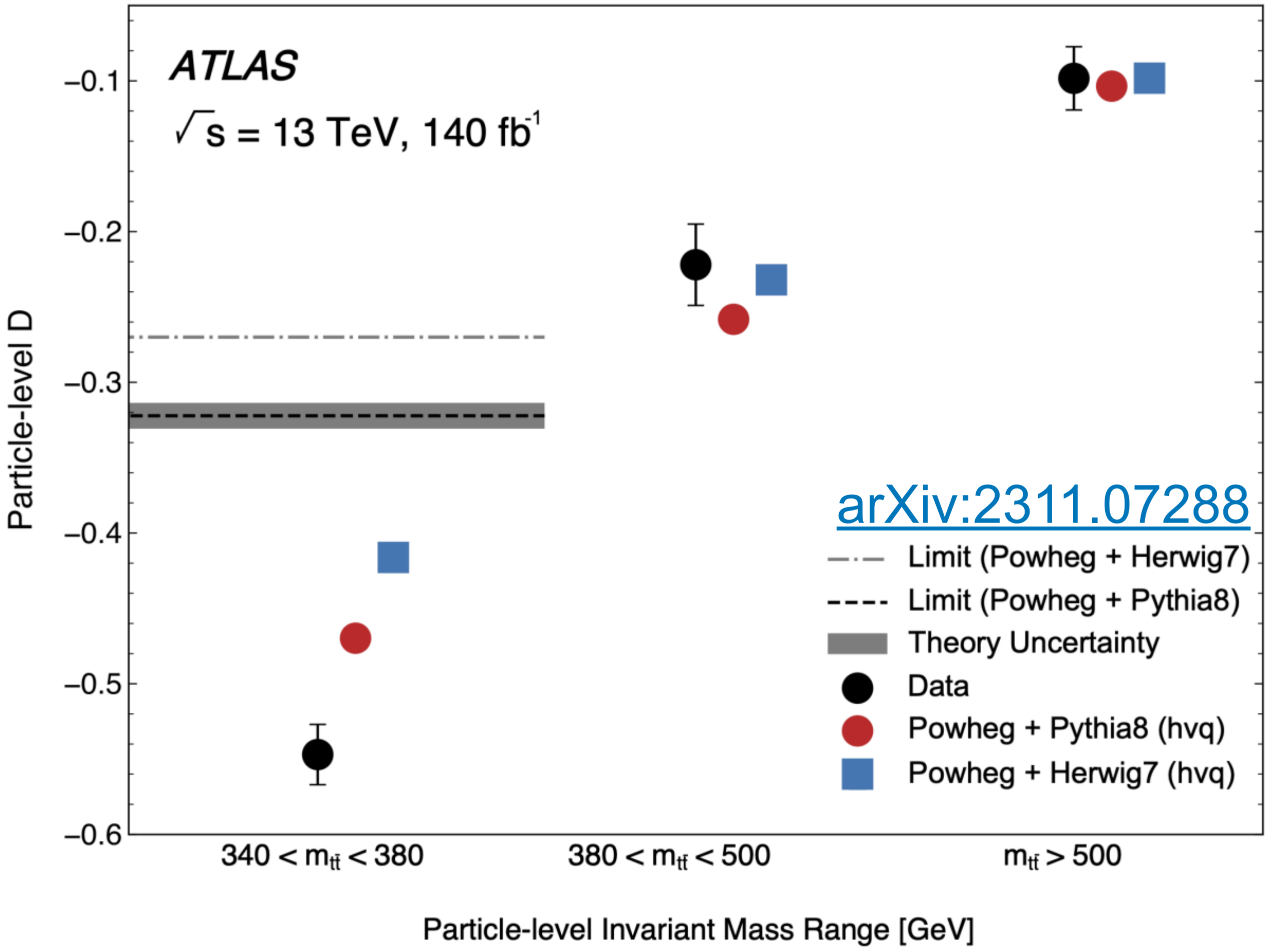
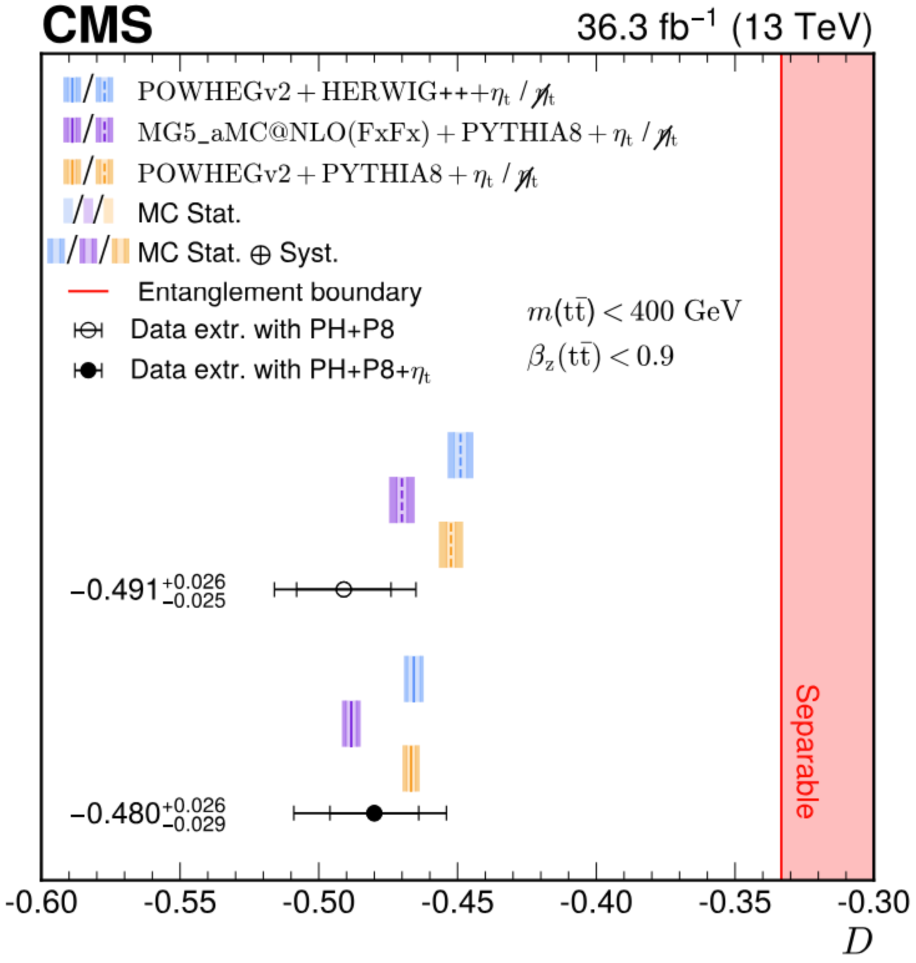


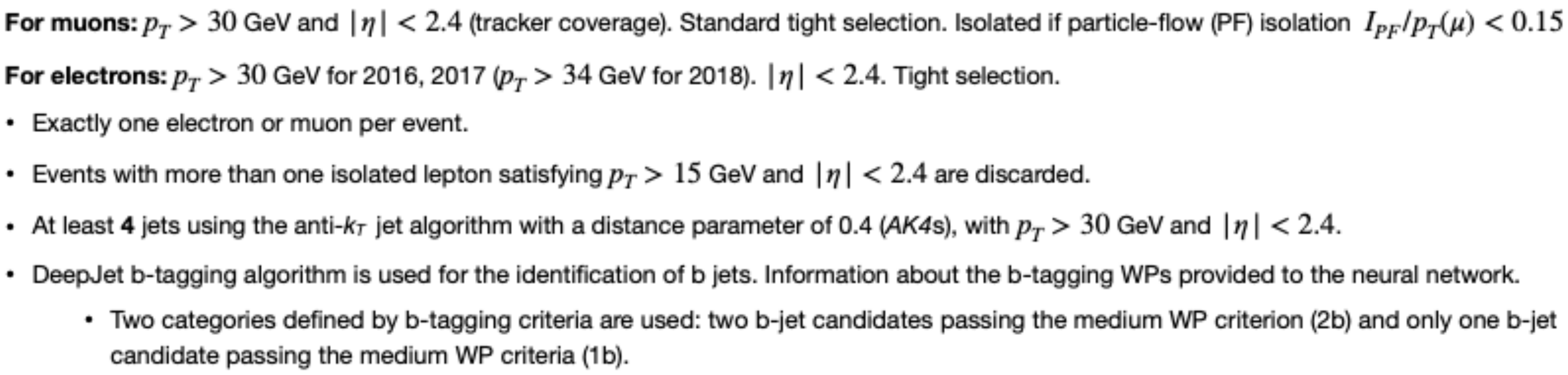
arXiv:2311.07288



ATLAS: limit of D = −1/3 is folded from parton to **particle-level**

CMS: limit of D = −1/3 is shown at **parton-level**

Lepton + jets: event selection



For muons: $p_T > 30$ GeV and $|\eta| < 2.4$ (tracker coverage). Standard tight selection. Isolated if particle-flow (PF) isolation $I_{PF}/p_T(\mu) < 0.15$

For electrons: $p_T > 30$ GeV for 2016, 2017 ($p_T > 34$ GeV for 2018). $|\eta| < 2.4$. Tight selection.

- Exactly one electron or muon per event.
- Events with more than one isolated lepton satisfying $p_T > 15$ GeV and $|\eta| < 2.4$ are discarded.
- At least **4** jets using the anti-k_T jet algorithm with a distance parameter of 0.4 (*AK4s*), with $p_T > 30$ GeV and $|\eta| < 2.4$.
- DeepJet b-tagging algorithm is used for the identification of b jets. Information about the b-tagging WPs provided to the neural network.
  - Two categories defined by b-tagging criteria are used: two b-jet candidates passing the medium WP criterion (2b) and only one b-jet candidate passing the medium WP criteria (1b).

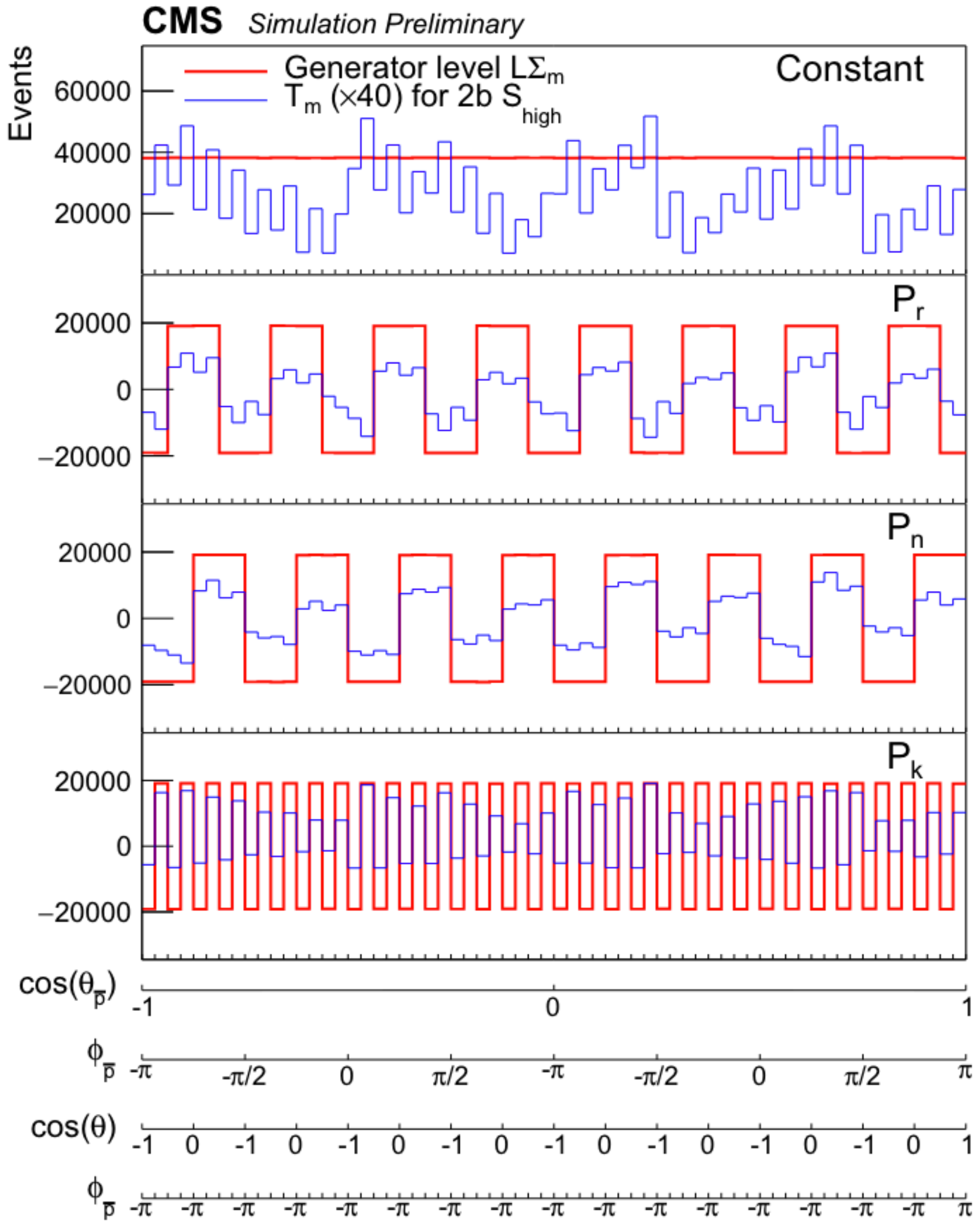
- To enhance fraction of correctly reconstructed events and reduce background contributions
  - $|m(t_l) - 172.5$ GeV$| < 50$ GeV
  - $|m(t_h) - 172.5$ GeV$| < 50$ GeV
  - $|m(W) - 80.4$ GeV$| < 30$ GeV

Lepton + jets: strategy

- At generator level, Σ_m never change because they do not depend on the kinematics of the top quarks
    - calculation of Σ_{tot} uses the average values of Q_m in each bin

- At detector level, T_m change as a function of top quark kinematics due to selection requirements and detector effects
    - if they vary significantly within a fitted bin, the measured Q_m could be biased

- To mitigate this effect, measurements are performed in sufficiently small bins such that either Q_m or T_m are approximately constant within each bin



$L\Sigma_m$ = templates used at gen level

T_m = templates defined at reco level

Lepton+jets: syst. uncertainties

- **Analysis is limited by statistical uncertainties**

- **Leading theoretical uncertainties:**
  - **top quark mass**
  - **renormalization/factorization scale**
  - **NNLO QCD reweighing**
  - **EW corrections**

  - **NB: toponium effect is small for lepton+jets (~5E-04)**

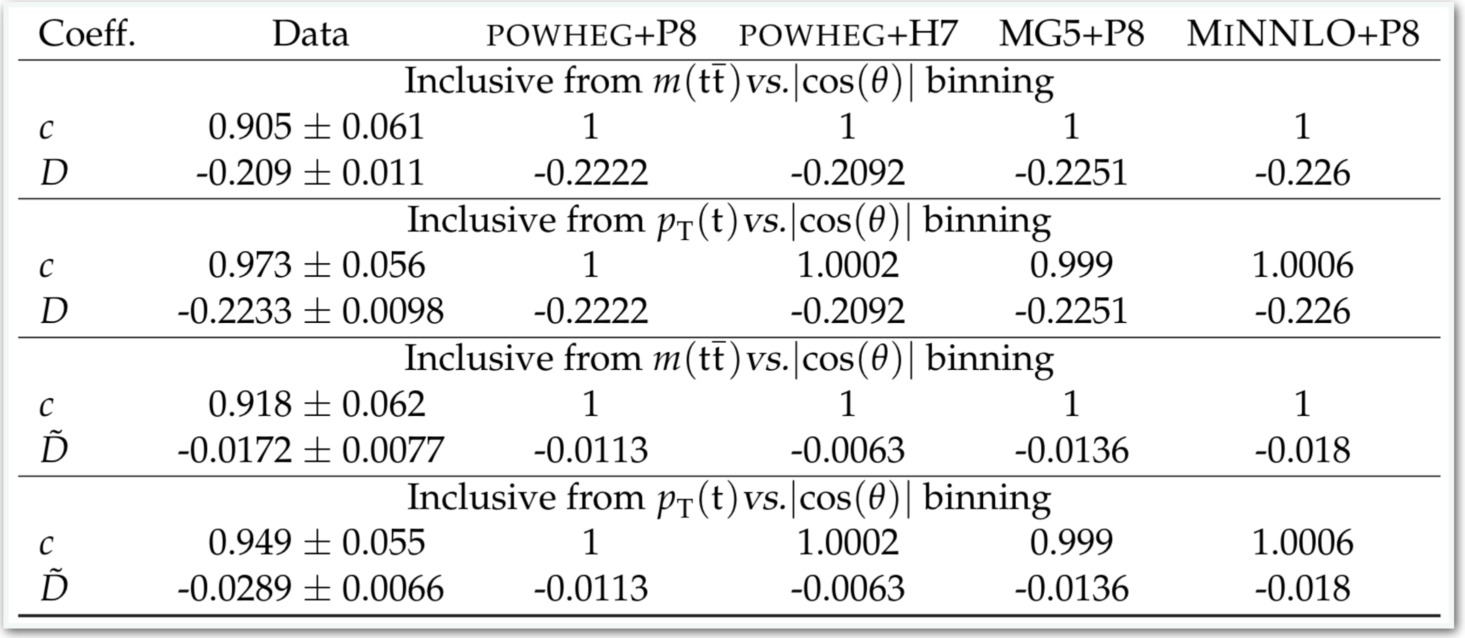
- **Leading experimental uncertainties:**
  - **Jet energy scale**
  - **b-tagging efficiency**

Lepton+jets results: D and $\tilde{D}$

- **Measurement of D, $\tilde{D}$ performed inclusively and differentially** in bins of $m_{t\bar{t}}$ vs $|\cos\theta|$ or p_T(t) vs $|\cos\theta|$

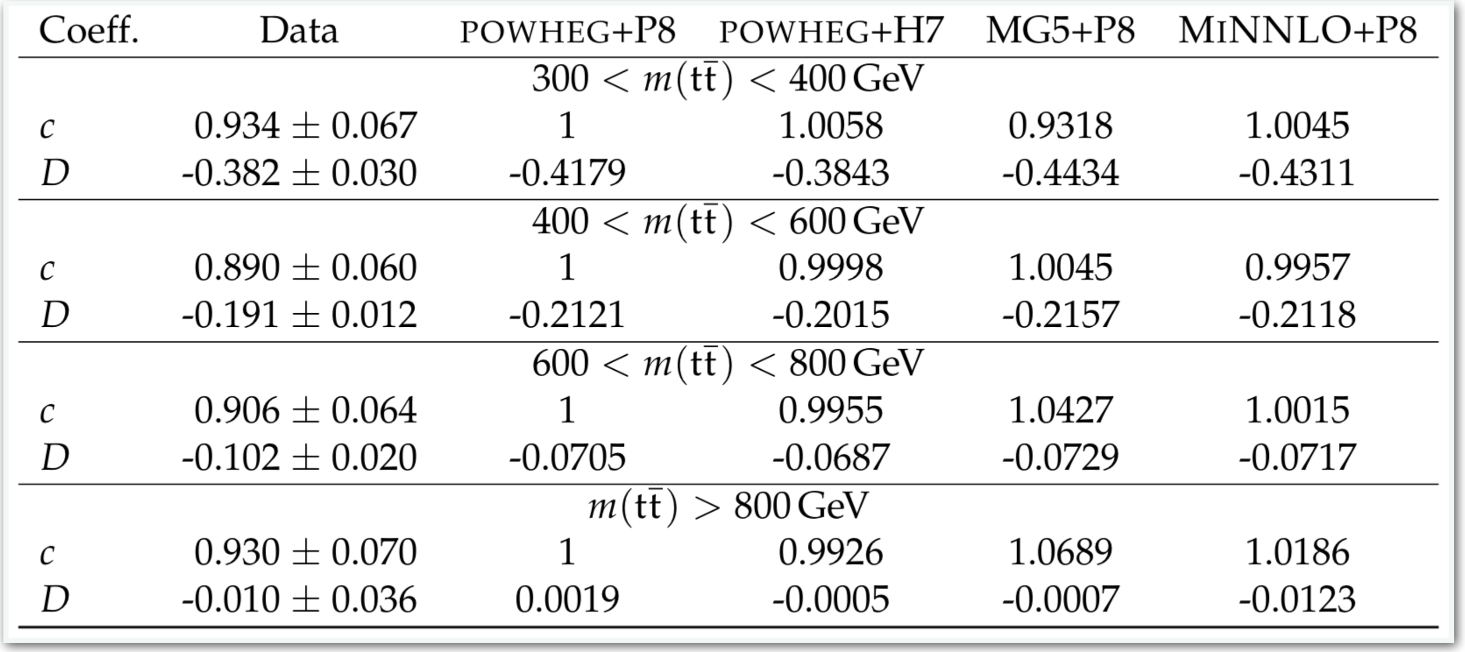
- **Good agreement with SM prediction**

Inclusive D, $\tilde{D}$ measurements



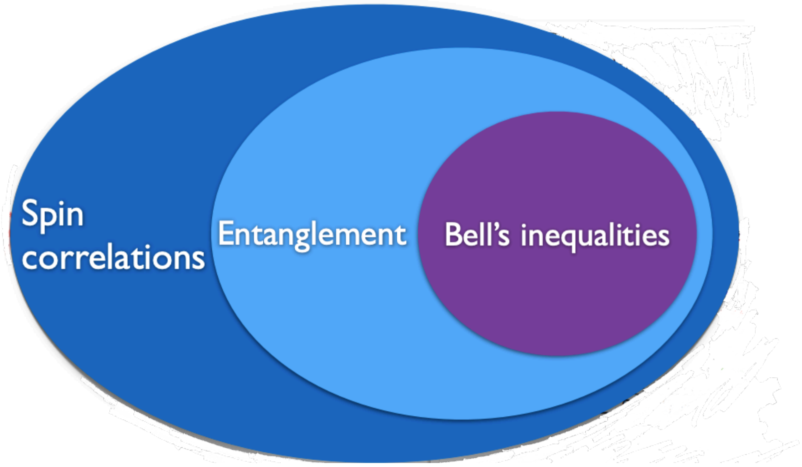
Coeff.	Data	POWHEG+P8	POWHEG+H7	MG5+P8	MiNNLO+P8
Inclusive from $m(\mathrm{t\bar{t}})\,vs.|\cos(\theta)|$ binning					
c	0.905 ± 0.061	1	1	1	1
D	-0.209 ± 0.011	-0.2222	-0.2092	-0.2251	-0.226
Inclusive from $p_{\mathrm{T}}(\mathrm{t})\,vs.|\cos(\theta)|$ binning					
c	0.973 ± 0.056	1	1.0002	0.999	1.0006
D	-0.2233 ± 0.0098	-0.2222	-0.2092	-0.2251	-0.226
Inclusive from $m(\mathrm{t\bar{t}})\,vs.|\cos(\theta)|$ binning					
c	0.918 ± 0.062	1	1	1	1
$\tilde{D}$	-0.0172 ± 0.0077	-0.0113	-0.0063	-0.0136	-0.018
Inclusive from $p_{\mathrm{T}}(\mathrm{t})\,vs.|\cos(\theta)|$ binning					
c	0.949 ± 0.055	1	1.0002	0.999	1.0006
$\tilde{D}$	-0.0289 ± 0.0066	-0.0113	-0.0063	-0.0136	-0.018

D measurements in $m_{t\bar{t}}$ bins



Coeff.	Data	POWHEG+P8	POWHEG+H7	MG5+P8	MiNNLO+P8
$300 < m(\mathrm{t\bar{t}}) < 400\,\mathrm{GeV}$					
c	0.934 ± 0.067	1	1.0058	0.9318	1.0045
D	-0.382 ± 0.030	-0.4179	-0.3843	-0.4434	-0.4311
$400 < m(\mathrm{t\bar{t}}) < 600\,\mathrm{GeV}$					
c	0.890 ± 0.060	1	0.9998	1.0045	0.9957
D	-0.191 ± 0.012	-0.2121	-0.2015	-0.2157	-0.2118
$600 < m(\mathrm{t\bar{t}}) < 800\,\mathrm{GeV}$					
c	0.906 ± 0.064	1	0.9955	1.0427	1.0015
D	-0.102 ± 0.020	-0.0705	-0.0687	-0.0729	-0.0717
$m(\mathrm{t\bar{t}}) > 800\,\mathrm{GeV}$					
c	0.930 ± 0.070	1	0.9926	1.0689	1.0186
D	-0.010 ± 0.036	0.0019	-0.0005	-0.0007	-0.0123

Outlook



- **This is just the beginning…**
- **Threshold region**
  - **potential to probe toponium**
- **High mass region**
  - **potential for observation of Bell's Inequality violation**
- **BI expressed by Clauser, Horne, Simony, Holt (CHSH) inequality** = measurements a, a' and b, b' on subsystems A and B must classically satisfy:

$$\left|\langle ab\rangle - \langle ab'\rangle + \langle a'b\rangle + \langle a'b'\rangle\right| \leq 2$$

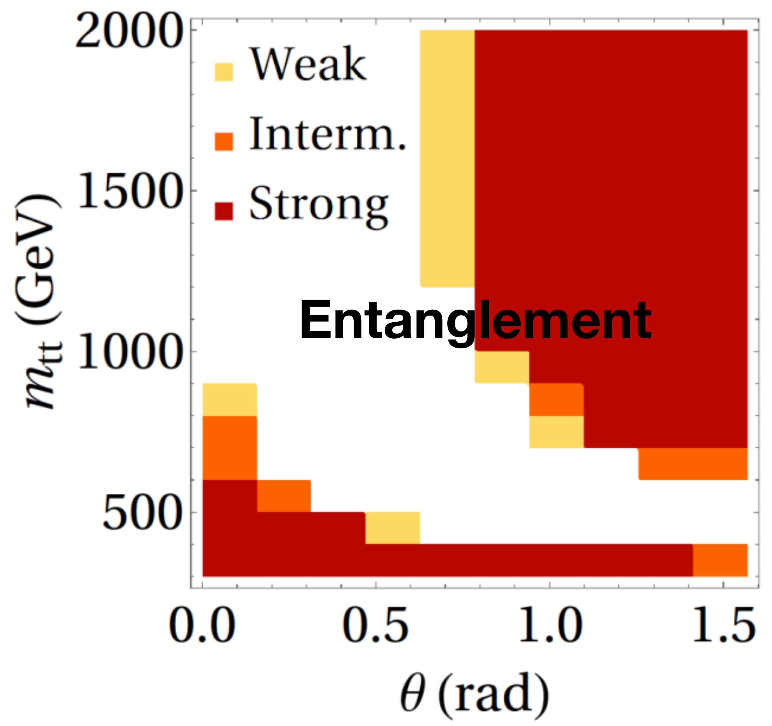
- For $t\bar{t}$ system it can be written in terms of C matrix as

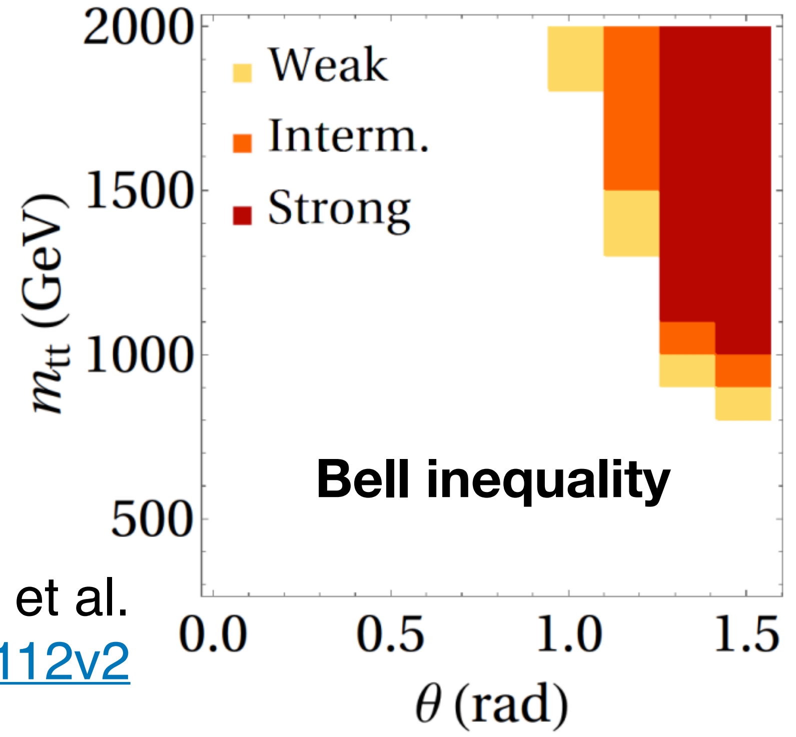
$$|C(a_1,b_1) - C(a_1,b_2) + C(a_2,b_1) + C(a_2,b_2)| \leq 2$$

- or more simply as:

$$\sqrt{2}\left|-C_{rr}+C_{nn}\right| \leq 2$$

- **Expected to happen at even higher $m_{t\bar{t}}$**
  - **more statistics is needed to measure it**





C Severi et al.
arXiv:2110.10112v2