Top spin and quantum entanglement in the ATLAS experiment

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on behalf of ATLAS collaboration

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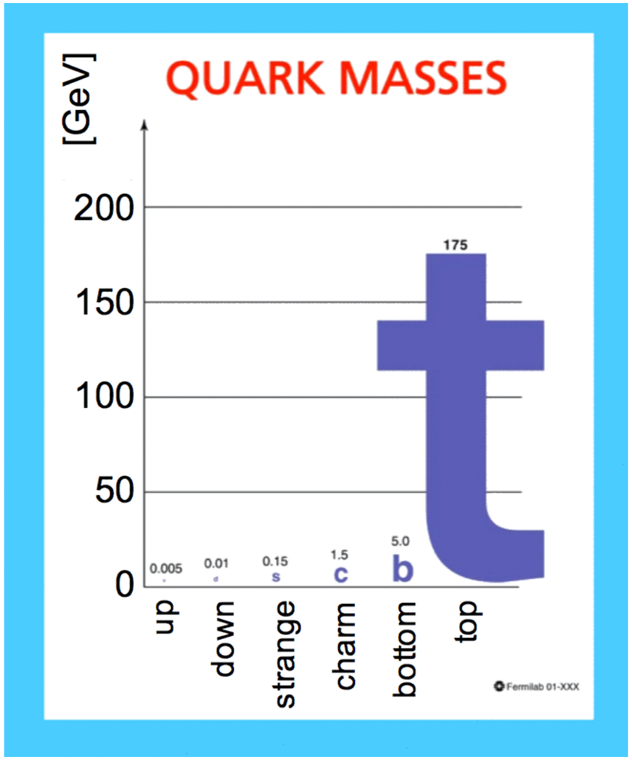
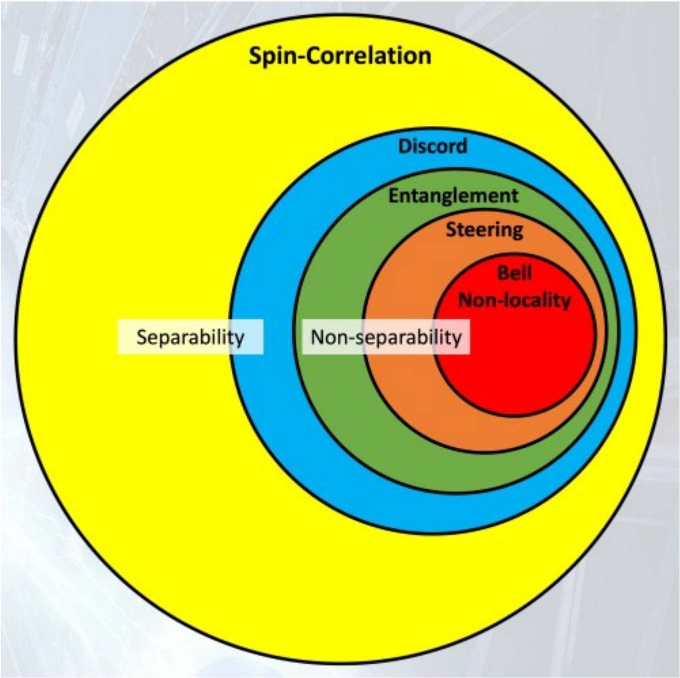
Intro





- Top quark decay products can be used to learn about top spin or top pair spin correlations
  → use these correlations to study the fundamental features of quantum mechanics (entanglement, Bell’s theorem) at the LHC

$t\bar{t}$ spin correlations

$t\bar{t}$ spin correlations

- The angular differential cross-section (for dilepton decay):

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ d\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot \mathbf{C} \cdot \hat{\mathbf{q}}_-}{(4\pi)^2}$$

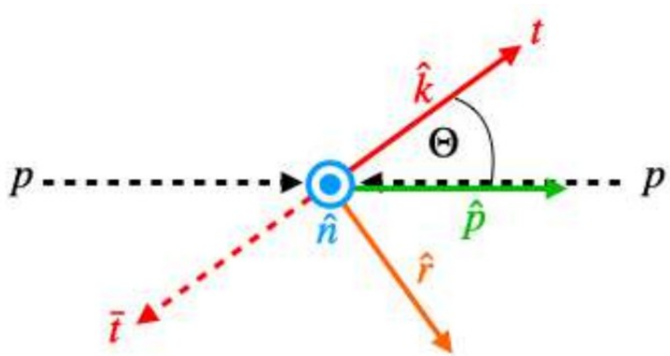
B^{+-} - top/antitop spin polarization vectors (2 x 3 components)

C – spin correlation matrix (3x3)

  – $q_+(q_-)$ - direction of positive (negative) lepton in its parent top (antitop) quark rest frame

→ Overall, **15 coefficients fully characterising the spin information in $t\bar{t}$ production**

- Orthonormal **helicity basis** typically used → axes k,n,r



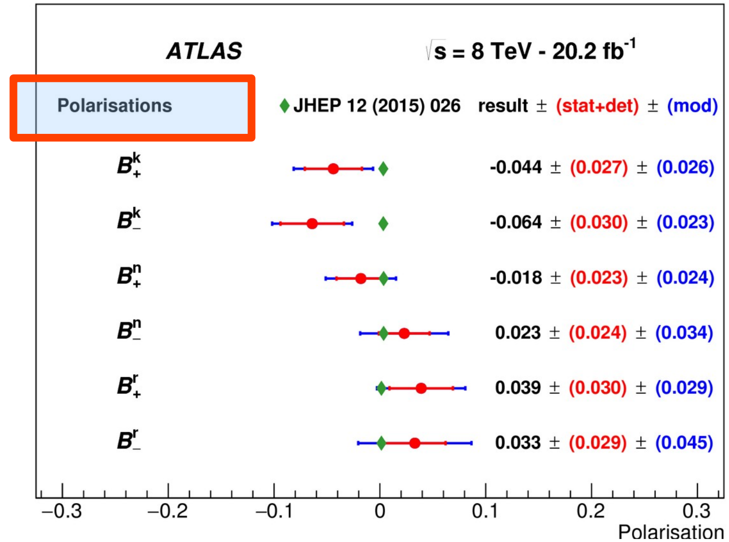
- Coefficients can be determined by measuring **cos(Θ)**$^a_{+/-}$ angles (angle between $q_{+/-}$ and quantized axis ‘a’): $B^a = 3 < \cos\theta^a >$ $C(a,b) = -9 < \cos\theta^a_+ \cos\theta^b_- >$

Measurements of $t\bar{t}$ spin correlations at 8 TeV

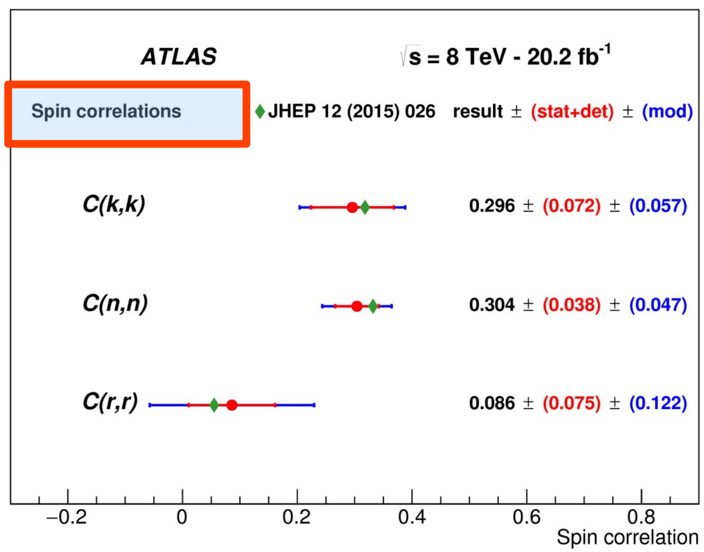
JHEP 03 (2017) 113

Full spin density matrix measured at 8 TeV:

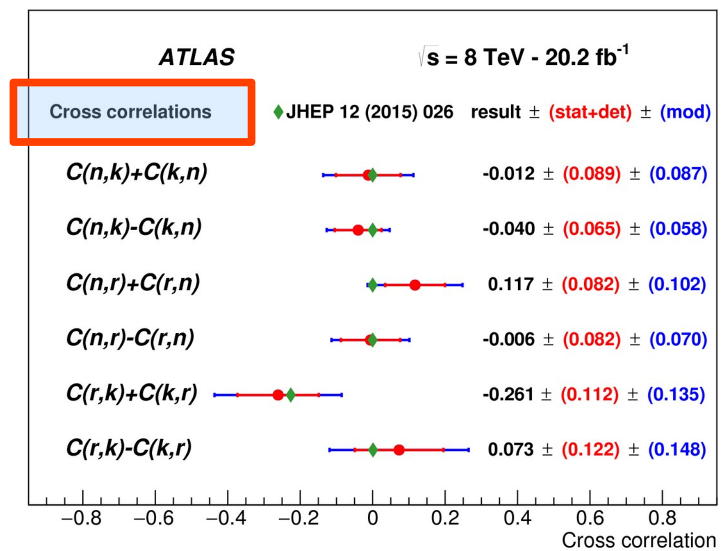
- All 3 dilepton channels
- Neutrino-weighting kinem. reconstruction
- Various unfolded **cos(Θ)**$^{a}_{+/-}$ distributions used to obtain the correlation coefficients
- Results provided for both parton/particle level



3-5%



6-14%

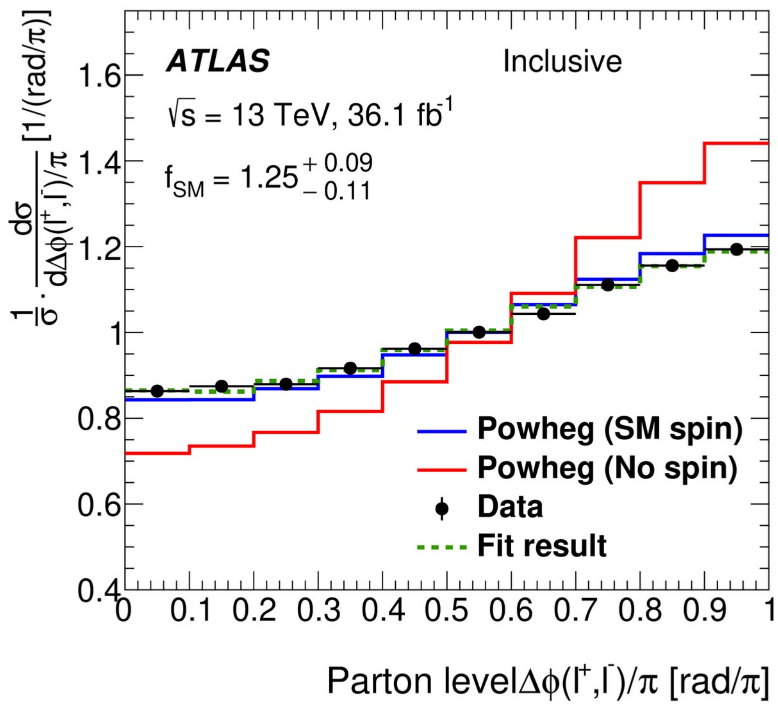


9-19%

Measurements of $t\bar{t}$ spin correlations at 13 TeV

EPJ C 80 (2020) 754

- Just partial Run 2 dataset used
- only $\Delta\phi(l^+,l^-)$ distribution measured
- Template fit to determine amount of correlations compared to SM (SM: $f_{SM} = 1.0$)
  - Also as a function of $m(t\bar{t})$
- A bit higher correlations than expected by Powheg
  - The difference smaller for NNLO prediction



Region	$f_{SM} \pm$ (stat.,syst.,theory)	Significance (excl. theory)
Inclusive	$1.249 \pm 0.024 \pm 0.061 ^{+0.067}_{-0.090}$	2.2 (3.8)
$m_{t\bar{t}} < 450$ GeV	$1.12 \pm 0.04 ^{+0.12}_{-0.13} ^{+0.06}_{-0.07}$	0.78 (0.87)
$450 \leq m_{t\bar{t}} < 550$ GeV	$1.18 \pm 0.08 ^{+0.13}_{-0.14} ^{+0.13}_{-0.15}$	0.84 (1.1)
$550 \leq m_{t\bar{t}} < 800$ GeV	$1.65 \pm 0.19 ^{+0.31}_{-0.41} ^{+0.26}_{-0.33}$	1.2 (1.4)
$m_{t\bar{t}} \geq 800$ GeV	$2.2 \pm 0.9 ^{+2.5}_{-1.7} ^{+1.2}_{-1.5}$	0.49 (0.61)

Quantum entanglement

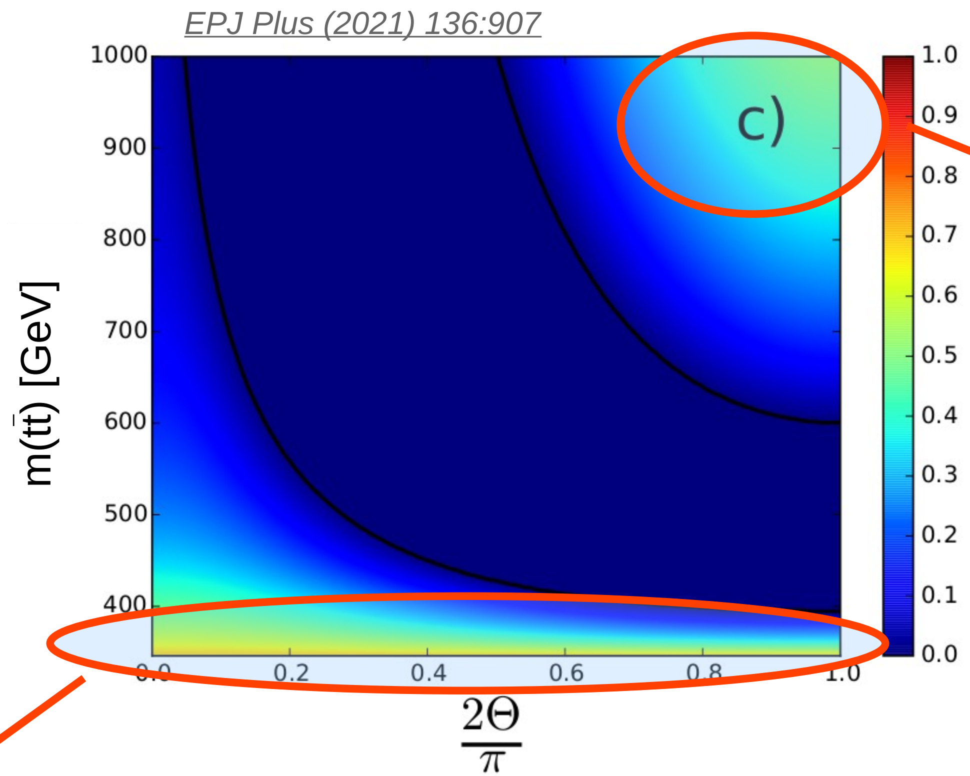
Quantum entanglement in top quark pairs

- Entanglement: quantum state of one particle cannot be described independently from another particle → there are **stronger correlations** than classical system would exhibit
  - A typical example: a system of two fermions in a spin-singlet state

- at the LHC we cannot control the initial state (no pure state) → mixed state
  - described in general by **density matrix** (ρ)
    - e.g. for two top quarks → spin density matrix
      - entanglement can be characterized by their spin correlations magnitude

Quantum entanglement conditions

- A quantitative measure of the entanglement: **concurrence** C[ρ] of the spin density matrix ρ:
  - Sufficient and necessary condition for entanglement: **$C[\rho] > 0$**



high m(tt̄) and $\Theta \sim \pi/2$
→ high p_T

low m(tt̄):

- Dominant contribution: gg fusion with tops in spin singlet state
- The sufficient condition obtained with spin correlation matrix: **$Tr[C] < -1$**

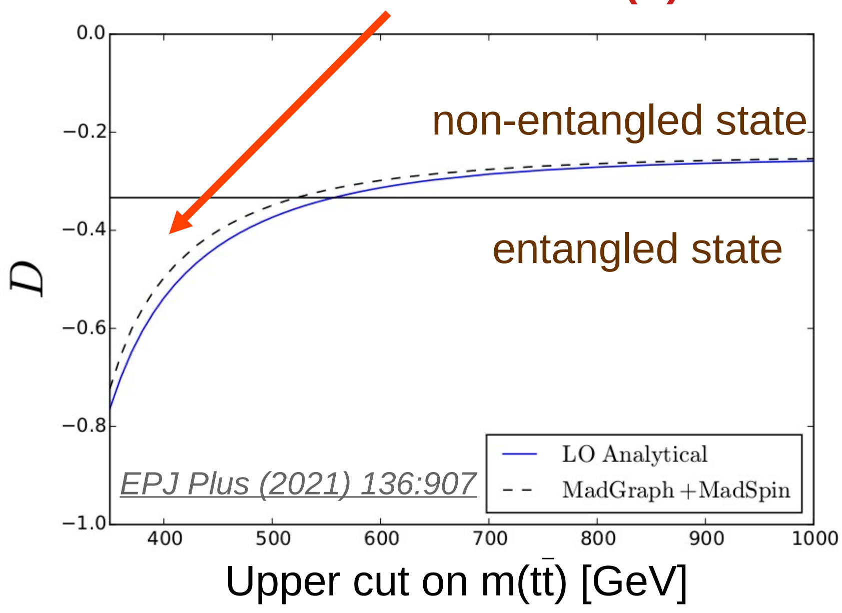
Entanglement observable (D)

- Entanglement can be observed using ***D*** observable
  - can be obtained from the differential distribution:

$$\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{d\sigma_{\ell\bar{\ell}}}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi), \quad D = \frac{\mathrm{tr}[\mathbf{C}]}{3} \qquad D = -3\langle\cos\varphi\rangle$$

φ: angle between the two lepton directions measured in their parent top quark and antiquark rest frames

- Entanglement condition: Tr[C] < -1 → translates to **D < -1/3**
- Crucial to measure ***D*** at **low invariant mass m(tt̄)**



The event selection: dilepton channel

As minimum cuts as possible:

- Require 1 electron and 1 muon: $p_T > 25 – 28$ GeV (|eta| < ~2.5)
- Opposite-sign electric charge of leptons
- No cut on missing transverse momentum
- ≥ 2 jets with $p_T > 25$ GeV
  - ≥ 1 b-tagged jet (85% efficiency)

→ **About 1.1M events after full event selection**

- ~90% purity of top-quark pair signal
- Dominant backgrounds: **single top (60%)**, **fakes (30%)**
  - All but 'fakes' (data-driven) estimated by simulation

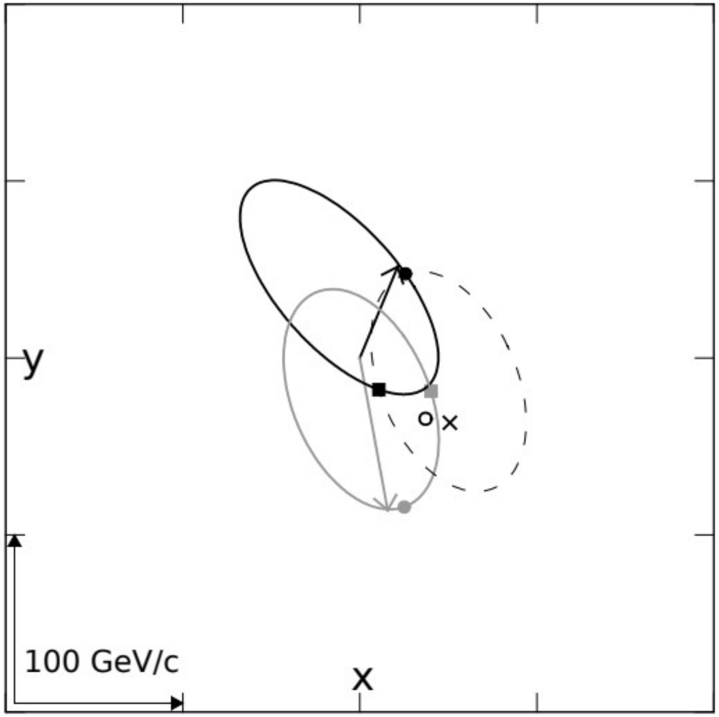
Process	Inclusive			340 – 380 GeV			380 – 500 GeV			> 500 GeV		
$t\bar{t}$	1030000	±	40000	202000	±	8000	408000	±	16000	417000	±	17000
tW	59800	±	1100	10330	±	200	23800	±	500	25700	±	500
Z+jets	9100	±	800	2470	±	240	4000	±	400	2620	±	250
$WW/WZ/ZZ$	5950	±	330	850	±	50	2130	±	120	2960	±	170
$t\bar{t}X$	2959	±	6	437.7	±	2.1	1080.1	±	3.4	1441	±	4
fakes	29000	±	5000	6000	±	1100	11700	±	2100	11700	±	2100
Expectation	1140000	±	40000	220000	±	8100	450000	±	16000	460000	±	17000
Data	1105403			225056			441196			439151		
data/MC	0.97	±	0.03	1.02	±	0.04	0.98	±	0.03	0.95	±	0.04

Top quark pair reconstruction

- Reconstruction of top quarks momenta complicated due to 2 neutrinos
  - Several methods were developed before, using m(top) and m(W) as constraints

NIM A 736 (2014) 169-178

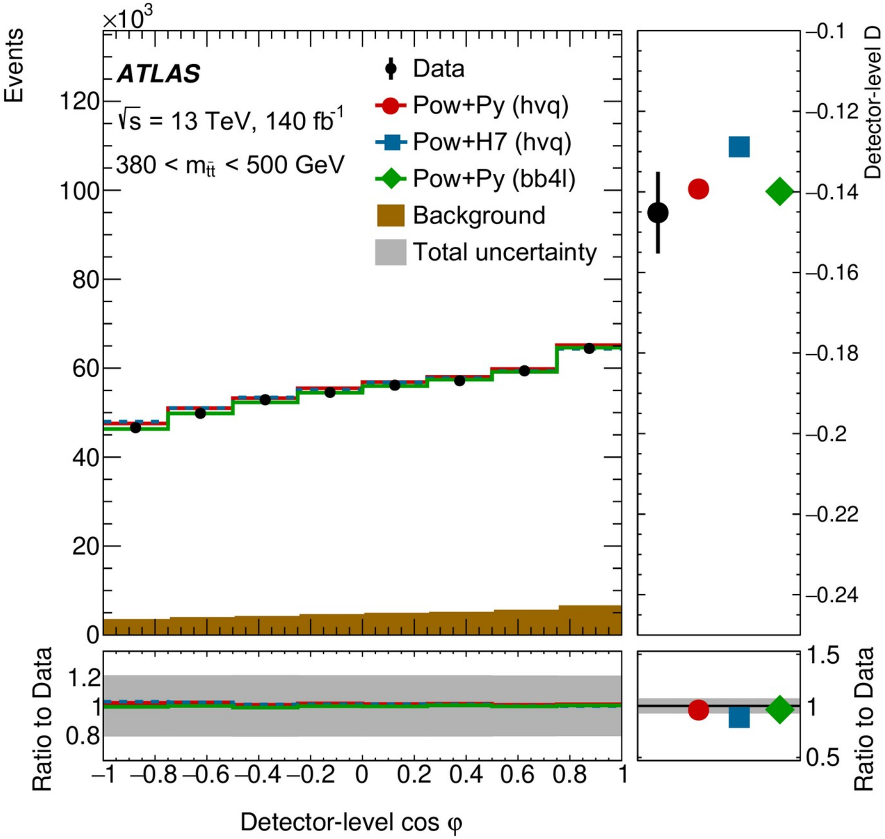
- A combination of various methods used:
  - Main method: **'Ellipse' method** (85% effic.)
    - Analytically calculate two ellipses for $p_T(\nu)$ and find intersections
  - If 'Ellipse' fails → **'Neutrino Weighting'** method (5%)
    - Scans $\eta(\nu)$, $\eta(\bar{\nu})$ phase-space
    - Solutions weighted based on compatibility between p_T of neutrinos and missing p_T
  - If both methods fail: **simple pairing** of leptons with the closest b-jets (10%)
    - Use highest-p_T jet if only 1 b-tagged jet



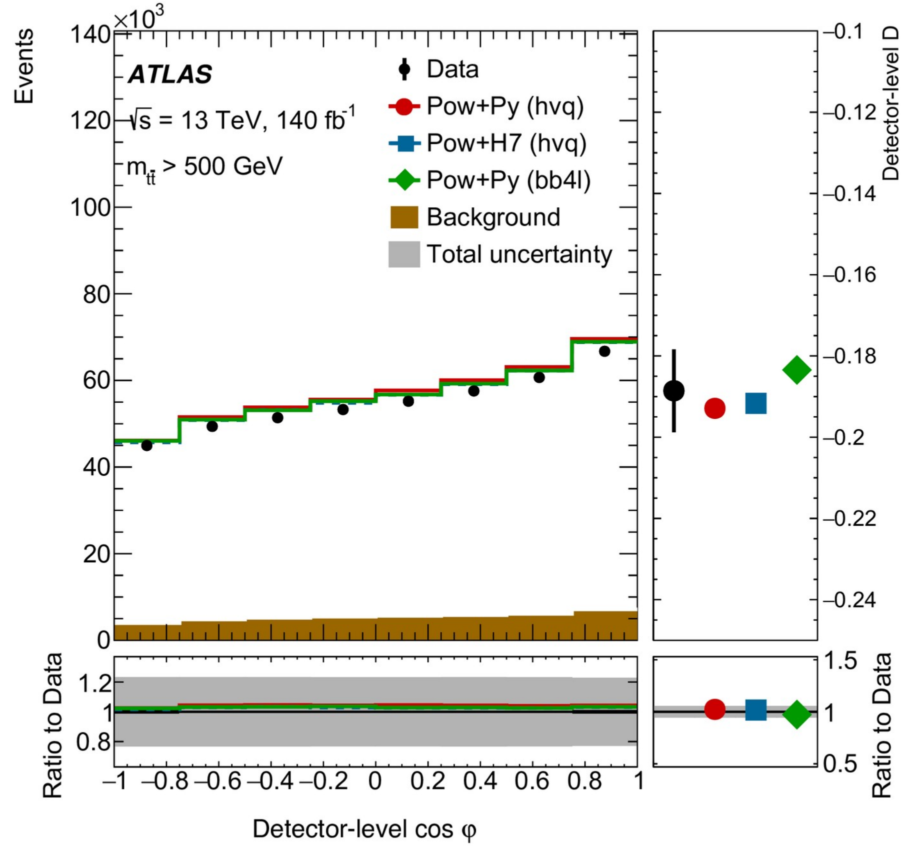
Reconstructed cos φ: validation regions

- Sample divided into 3 regions based on $m(t\bar{t})$:
  - **Signal region: 340 GeV < $m(t\bar{t})$ < 380 GeV**
  - 2 validation regions:

380 < $m(t\bar{t})$ < 500 GeV:



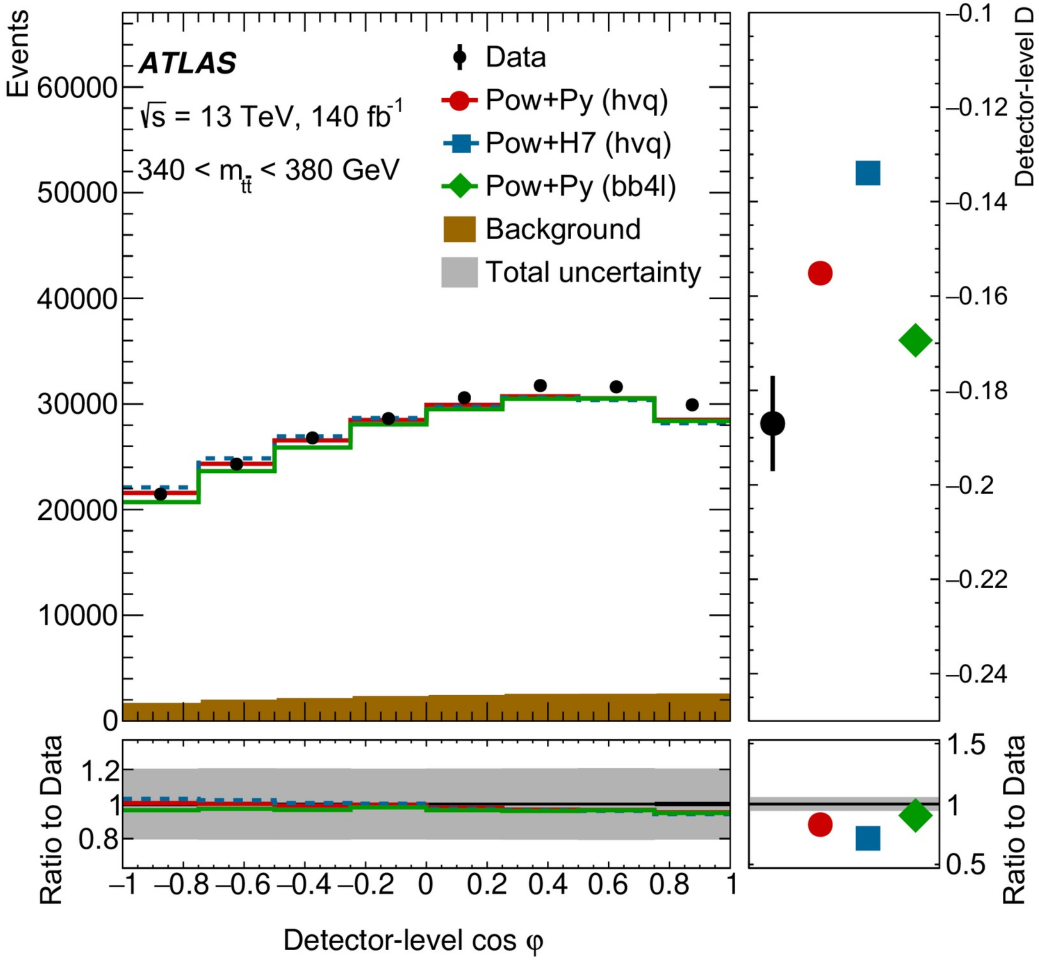
$m(t\bar{t})$ > 500 GeV:



- Data agree with predictions

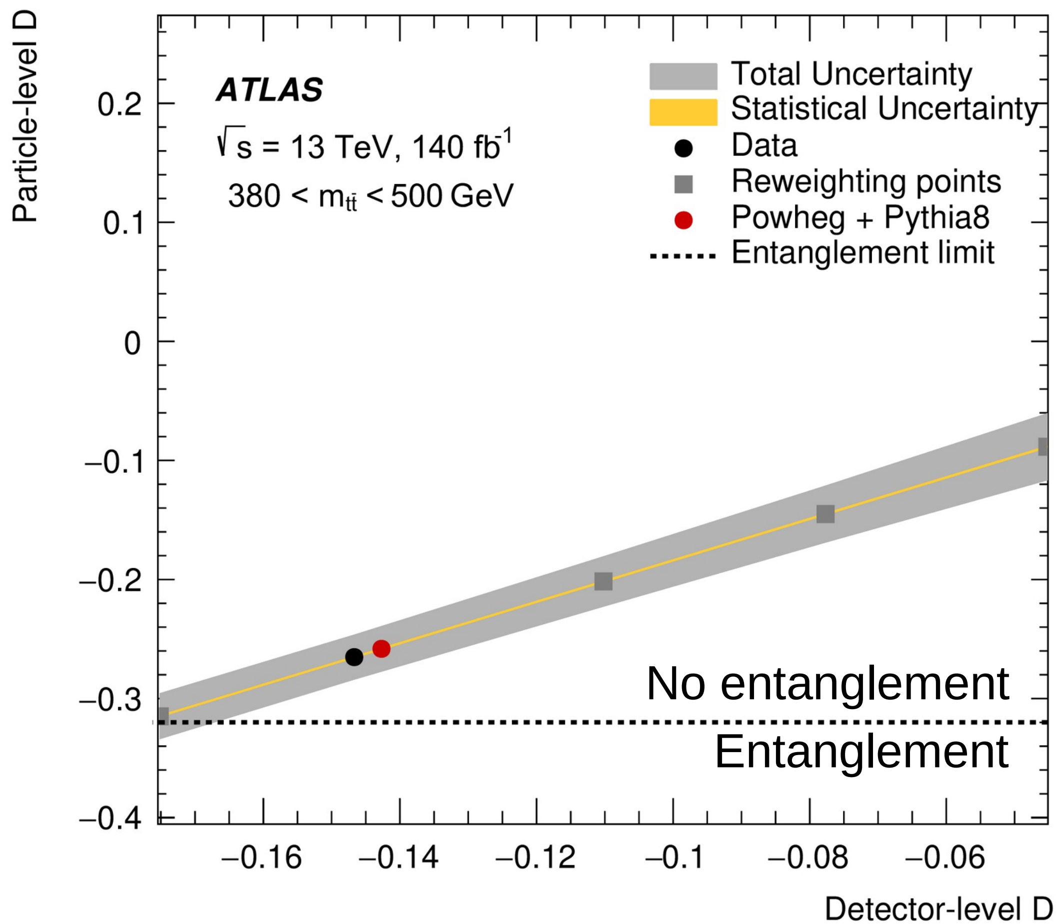
Reconstructed cos φ: signal region

340 < m($t\bar{t}$) < 380 GeV:



- The reconstructed value of observable ***D*** is below predictions
  - More details in backup

Correction to particle level: calibration curve



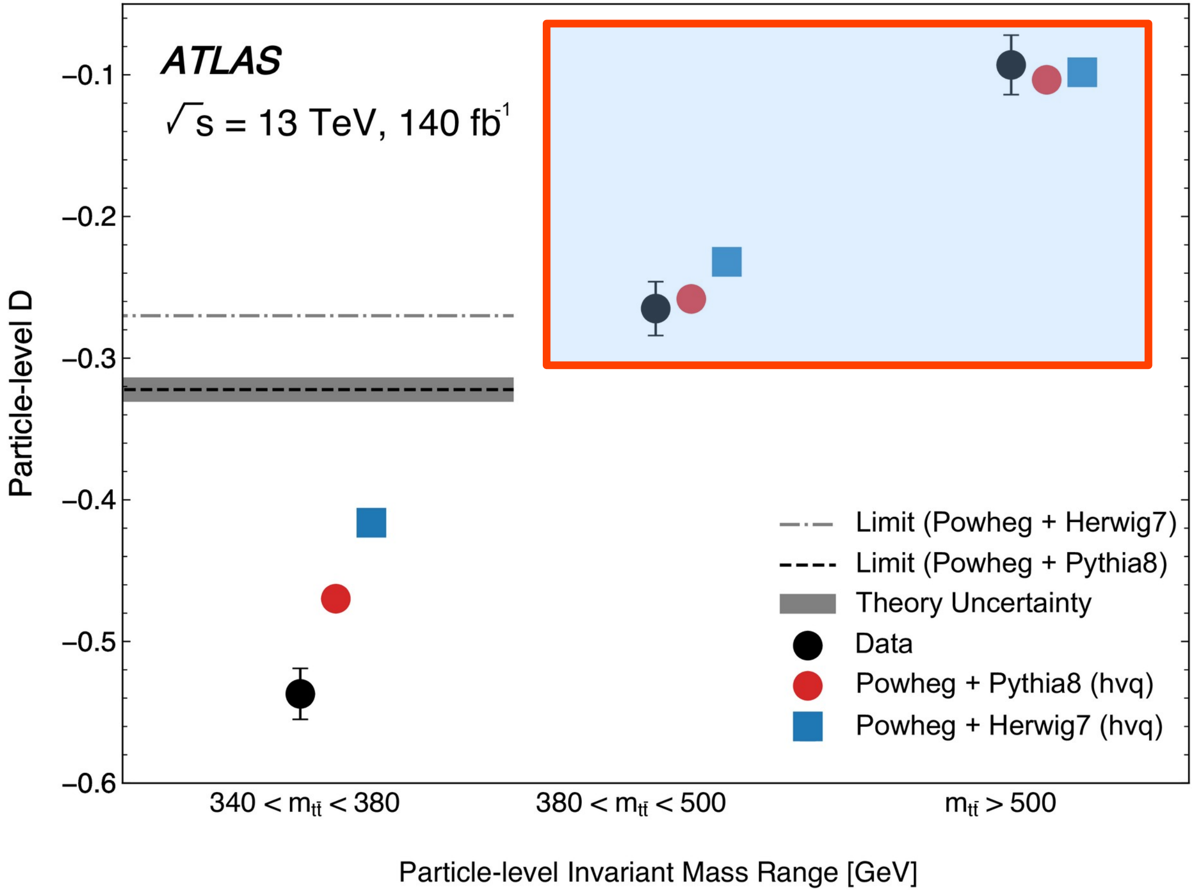
- The measured data (reconstructed level) are corrected to the truth (particle) level
  - The particle level cuts are similar to reconstructed level
- Calibration curve created by reweighting simulation based on truth ***D*** value

Systematic uncertainties

- For each systematic uncertainty: create a new calibration curve
- 3 main categories:
  - Signal modelling → dominant
    - main component: top quark decay (1.6%)
  - Background modelling: $Z \rightarrow \tau\tau$ dominant (0.8%)
  - Object reconstruction

Systematic source	$\Delta D_{\text{expected}}(D = -0.470)$	ΔD [%]
Signal Modelling	0.015	3.2
Electrons	0.002	0.4
Muons	0.001	0.1
Jets	0.004	0.8
b-tagging	0.002	0.4
Pile-up	< 0.001	< 0.1
$E_{\text{T}}^{\text{miss}}$	0.002	0.4
Backgrounds	0.005	1.1
Total Statistical Uncertainty	0.002	0.4
Total Systematic Uncertainty	0.017	3.6
Total Uncertainty	0.017	3.6

Observable ‘D’ in validation regions



- measured ‘D’ in validation regions:

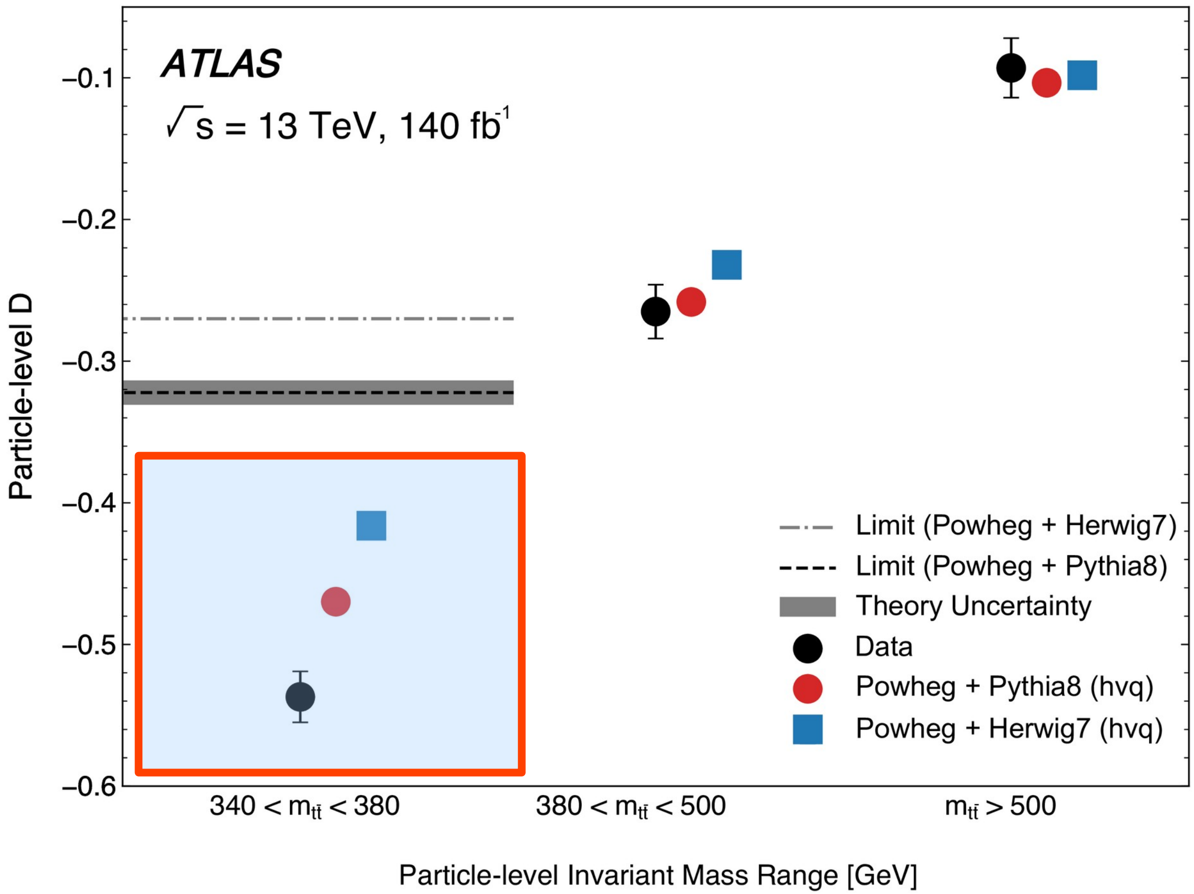
380 < m(tt̄) < 500 GeV:

$$D = -0.265 \pm 0.001\,(\text{stat.}) \pm 0.019\,(\text{syst.})$$

m(tt̄) > 500 GeV:

$$D = -0.093 \pm 0.001\,(\text{stat.}) \pm 0.021\,(\text{syst.})$$

‘D’ in signal region



Nature 633 (2024) 542

- Measured ‘D’ in signal region:

$$D = -0.537 \pm 0.002\,(\mathrm{stat.}) \pm 0.019\,(\mathrm{syst.})$$

- Measured value significantly (>> 5 standard deviations) below entanglement limit (−0.322 ± 0.009 for Powheg+Pythia 8) → **observation of entanglement**

Conclusion

- ATLAS experiment observed quantum entanglement using top-quark pairs
  - $t\bar{t}$ modelling limiting factor in a few areas
  - hopefully such measurements stimulate a progress
- Up to ~20 times more data expected with full LHC program

  → just a beginning of era of quantum information measurements at LHC
    - e.g. can study systems of different particles, test Bell inequalities

Thank you

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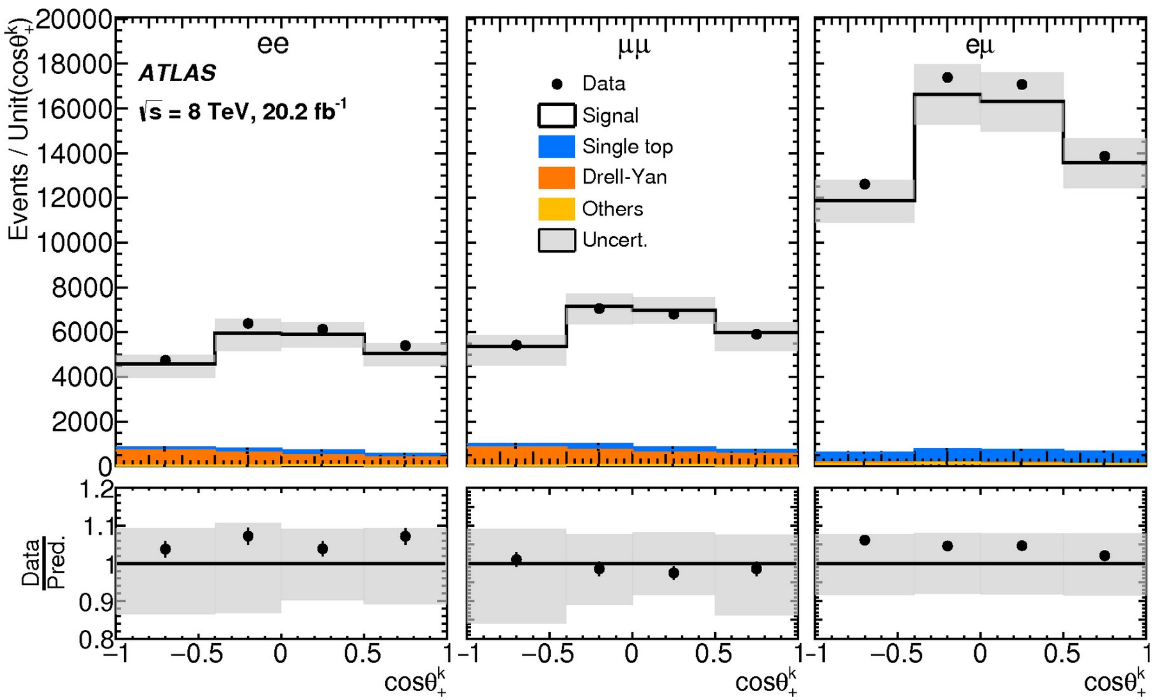
BACKUP

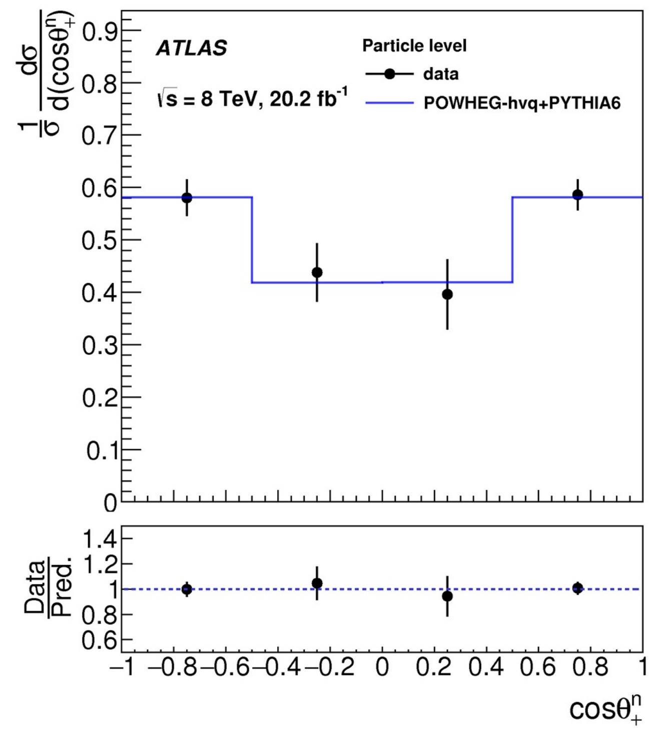
$t\bar{t}$ spin correlations

Spin-correlations at 8 TeV

- Examples of reconstruction level plots and unfolded plot

JHEP 03 (2017) 113





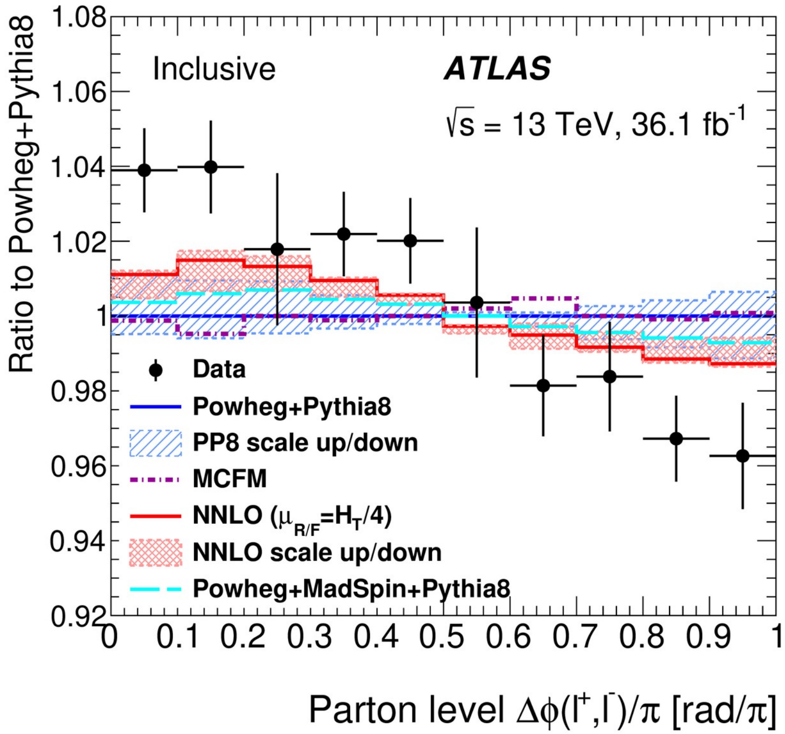
Spin-correlations predictions at 8 TeV

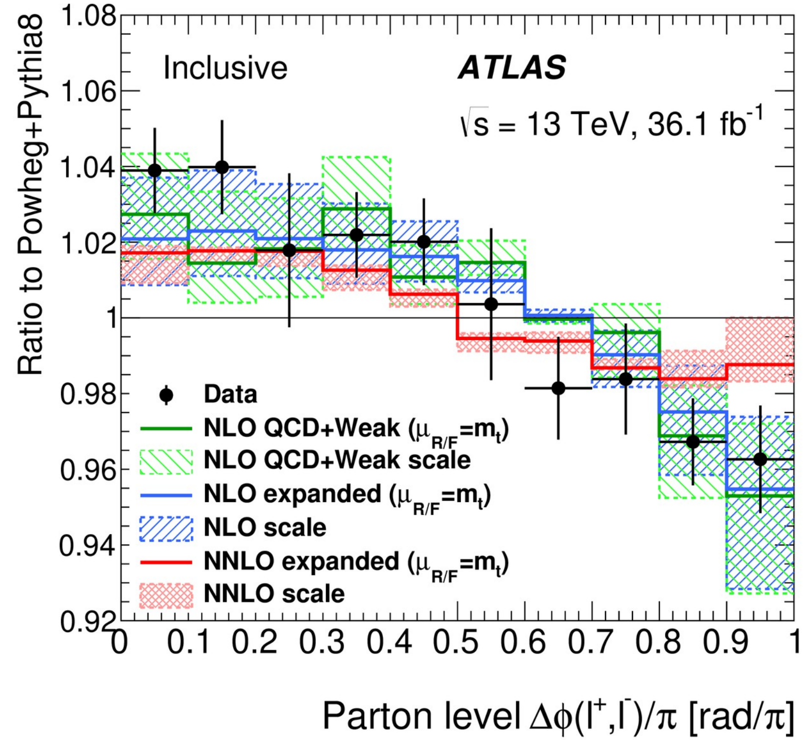
Expectation values	NLO predictions	Observables
B_+^k	0.0030 ± 0.0010	$\cos\theta_+^k$
B_-^k	0.0034 ± 0.0010	$\cos\theta_-^k$
B_+^n	0.0035 ± 0.0004	$\cos\theta_+^n$
B_-^n	0.0035 ± 0.0004	$\cos\theta_-^n$
B_+^r	0.0013 ± 0.0010	$\cos\theta_+^r$
B_-^r	0.0015 ± 0.0010	$\cos\theta_-^r$
$C(k,k)$	0.318 ± 0.003	$\cos\theta_+^k \cos\theta_-^k$
$C(n,n)$	0.332 ± 0.002	$\cos\theta_+^n \cos\theta_-^n$
$C(r,r)$	0.055 ± 0.009	$\cos\theta_+^r \cos\theta_-^r$
$C(n,k) + C(k,n)$	0.0023	$\cos\theta_+^n \cos\theta_-^k + \cos\theta_+^k \cos\theta_-^n$
$C(n,k) - C(k,n)$	0	$\cos\theta_+^n \cos\theta_-^k - \cos\theta_+^k \cos\theta_-^n$
$C(n,r) + C(r,n)$	0.0010	$\cos\theta_+^n \cos\theta_-^r + \cos\theta_+^r \cos\theta_-^n$
$C(n,r) - C(r,n)$	0	$\cos\theta_+^n \cos\theta_-^r - \cos\theta_+^r \cos\theta_-^n$
$C(r,k) + C(k,r)$	-0.226 ± 0.004	$\cos\theta_+^r \cos\theta_-^k + \cos\theta_+^k \cos\theta_-^r$
$C(r,k) - C(k,r)$	0	$\cos\theta_+^r \cos\theta_-^k - \cos\theta_+^k \cos\theta_-^r$

Δφ(l+,l-) at 13 TeV

EPJ C 80 (2020) 754

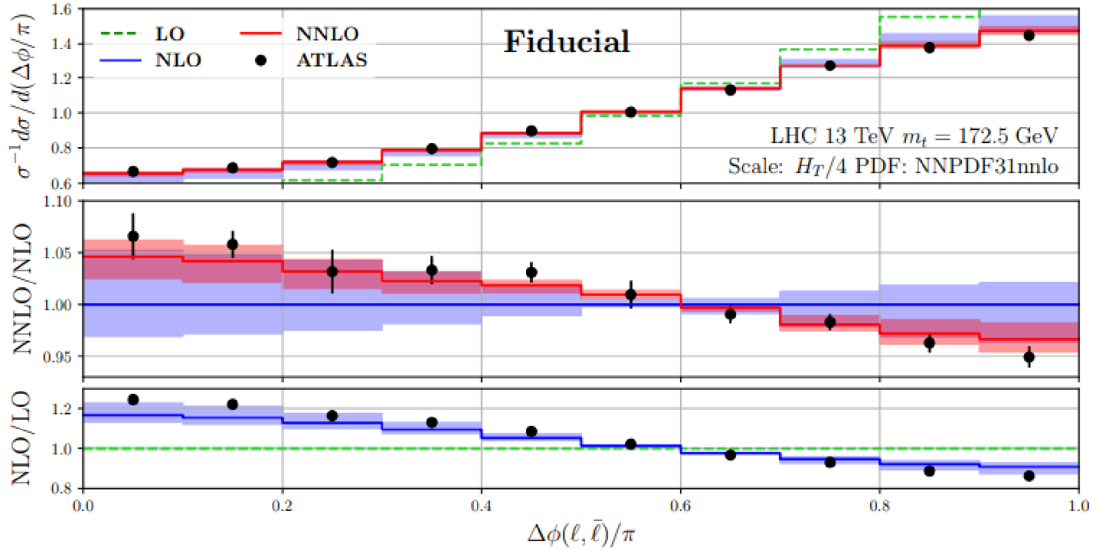
- Comparison of data to various (fixed-order) predictions:

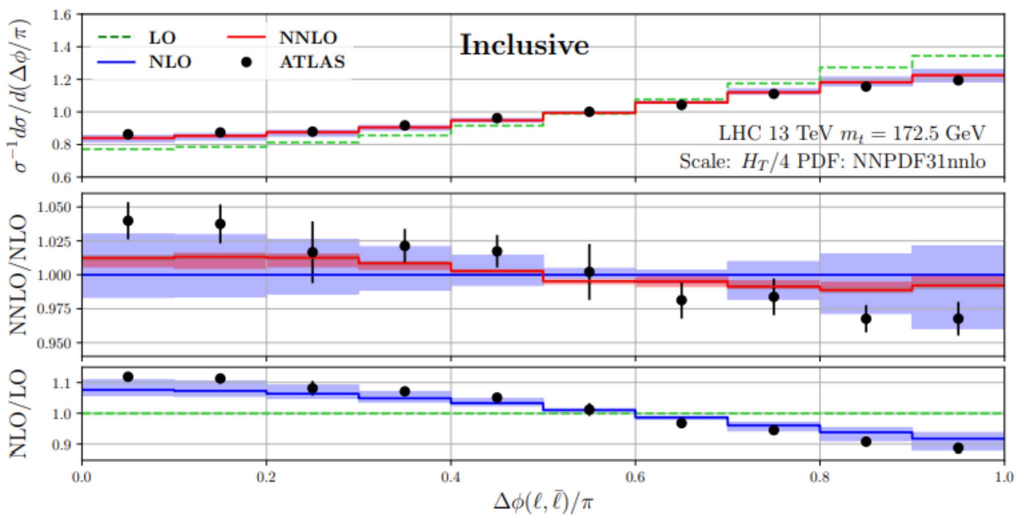




Δϕ(l+,l-): NNLO comparisons

Phys. Rev. Lett. 123, 082001 (2019)





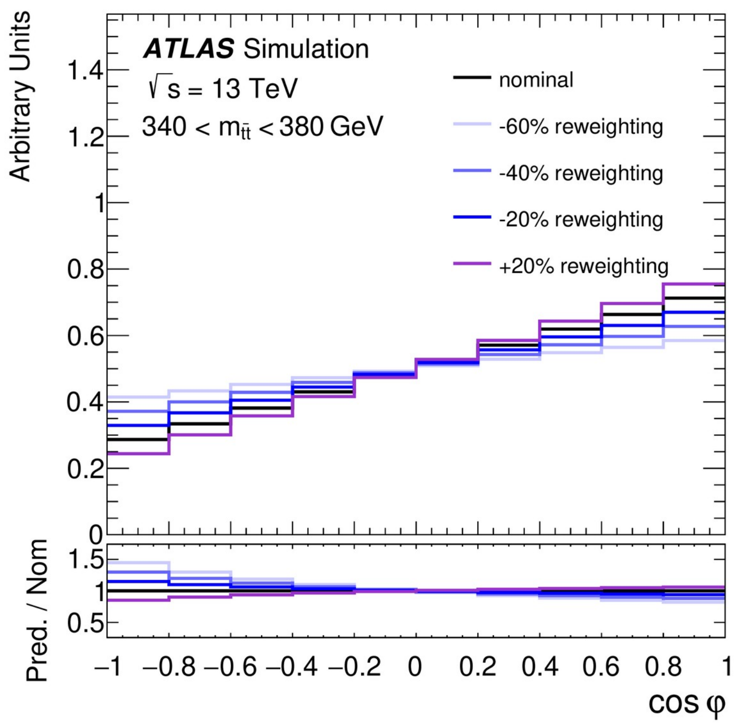
Quantum entanglement

Monte-Carlo simulated samples

- $t\bar{t}$ signal modelling:
  - Nominal sample: PowhegBox (v2,hvq), hdamp=1.5*m(top)
  - Alternative: PowhegBox-RES (bb4l)
    - Includes off-shell and non-resonant effects
  - Alternative parton-shower sample: Powheg(v2) + Herwig 7.21 (default tune)
- Background processes modelling:
  - Single-top quark, tW-channel: PowhegBox(v2); 5FS,DR schemes
  - $t\bar{t}$+X(X=W,Z): MadGraph5_aMC@NLO 2.3.3
  - $t\bar{t}$+H: PowhegBox(v2), 5FS
  - W/Z + jets: Sherpa 2.2.11, including off-shell effects, NNPDF3.0NNLO
    - @NLO in QCD for ≤ 2 additional partons and @LO for ≤ 5 partons
  - VV(V=W,Z): Sherpa 2.2.2, NNPDF3.0NNLO
    - @NLO in QCD for ≤ 1 parton and @LO for ≤ 3 partons
  - Fakes: data driven
- Typically:
  - precision of the modelling: NLO in QCD
  - for ME generations: NNPDF3.0NLO PDF
  - for parton shower: Pythia 8.230, A14 tune, NNPDF2.3LO

Reweighting of cos(phi) distribution

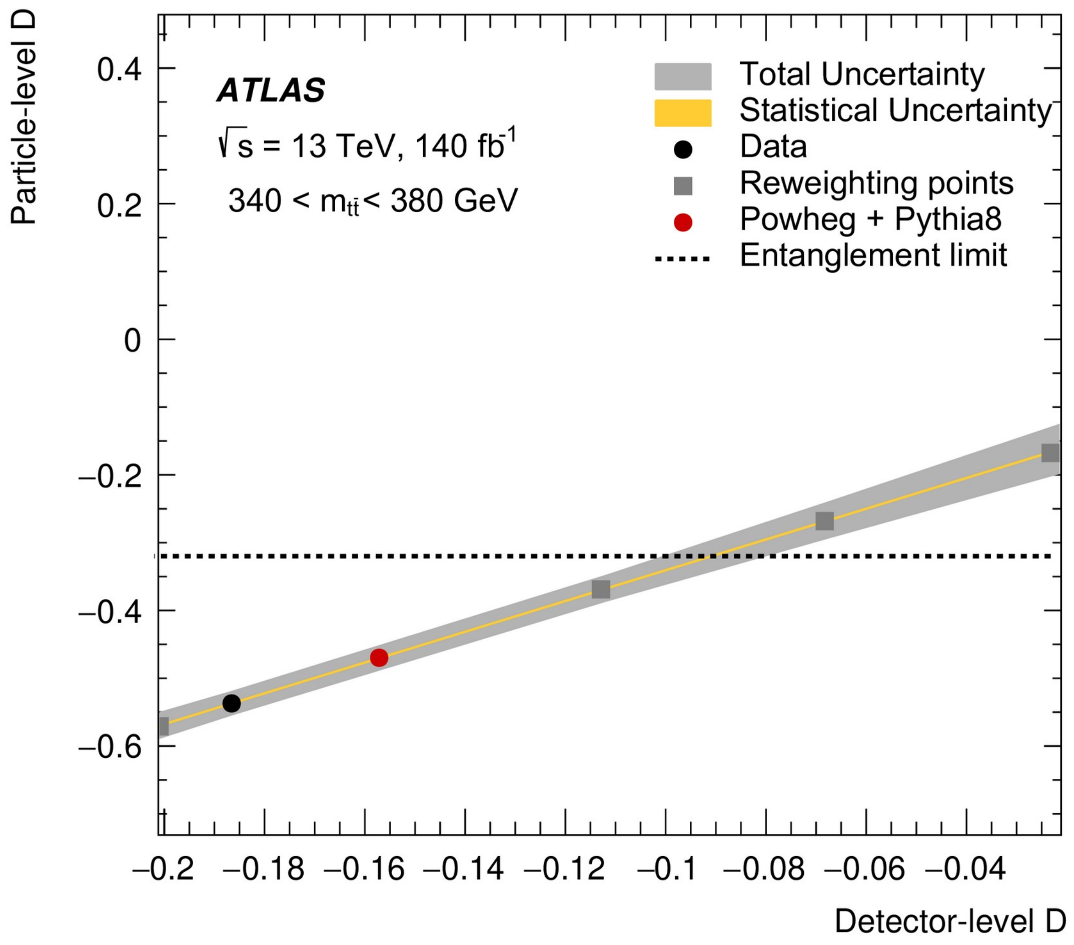
- The effects of quantum entanglement are fundamental to the calculations in the MC generators and cannot be easily changed
- However, the effects of entanglement can be directly accessed via the observable D in the event
- Each events is reweighted according to its parton level value of cos φ in order to change $D = -3\cdot\langle\cos\varphi\rangle$
  - $m(t\bar{t})$ is taken into account to preserve linearity in cos φ



$$w = \frac{1 - D(m_{t\bar{t}})\cdot\chi\cdot\cos\varphi}{1 - D(m_{t\bar{t}})\cdot\cos\varphi}$$

$$\chi = 0.4, 0.6, 0.8, 1.2$$

Calibration curve in signal region



Systematic uncertainties

Source of uncertainty	$\Delta D_{\mathrm{observed}}(D = -0.537)$	ΔD [%]	$\Delta D_{\mathrm{expected}}(D = -0.470)$	ΔD [%]
Signal modeling	0.017	3.2	0.015	3.2
Electrons	0.002	0.4	0.002	0.4
Muons	0.001	0.2	0.001	0.1
Jets	0.004	0.7	0.004	0.8
b-tagging	0.002	0.4	0.002	0.4
Pile-up	< 0.001	< 0.1	< 0.001	< 0.1
$E_{\mathrm{T}}^{\mathrm{miss}}$	0.002	0.4	0.002	0.4
Backgrounds	0.005	0.9	0.005	1.1
Total statistical uncertainty	0.002	0.3	0.002	0.4
Total systematic uncertainty	0.019	3.5	0.017	3.6
Total uncertainty	0.019	3.5	0.017	3.6

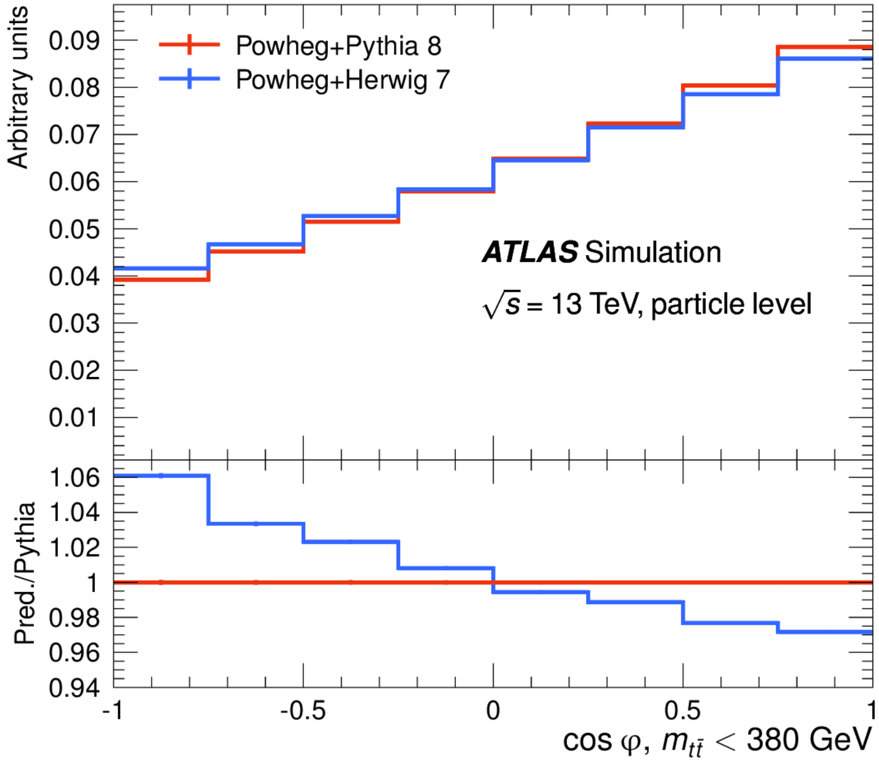
Signal modelling systematics uncertainties

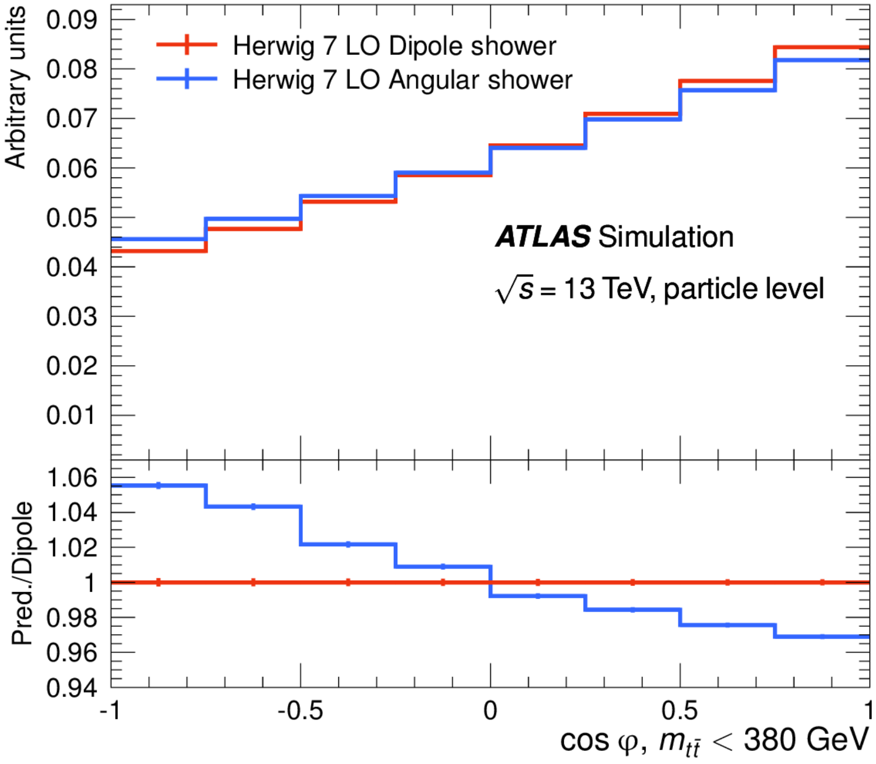
- Top-quark decay: nominal vs. Madspin decay
- Recoil scheme: different schemes where partons recoil against b-quark vs. top-quark
- FSR: change by x2 or ½ the μ_R for emissions from the parton shower
- Scales: change the μ_R and μ_F by factor 2 or ½
- pThard: Powheg parameter which regulates the definition of the region of phase-space that is vetoed in the showering when matched to a parton shower (nominal vs. pThard=1)
- ISR: using the Var3c up/down variants of the A14 tune
- h_{damp}: h_{damp} is a resummation damping factor that controls the matching of ME to PS and thus effectively regulates the high-pT radiation against which the $t\bar{t}$ system recoils (1.5*m(top) vs. 3*m(top))

Systematic uncertainty source	Relative size (for SM D value)
Top-quark decay	1.6%
Parton distribution function	1.2%
Recoil scheme	1.1%
Final-state radiation	1.1%
Scale uncertainties	1.1%
NNLO QCD + NLO EW reweighting	1.1%
pThard setting	0.8%
Top-quark mass	0.7%
Initial-state radiation	0.2%
Parton shower and hadronization	0.2%
h_{damp} setting	0.1%

Parton shower and hadronization effects

- Large difference between Powheg+Pythia 8 and Powheg+Herwig 7 in signal region
  - difference at particle and reco. level, while similar at parton level
  - Two main differences in models:
    - hadronisation model (Lund-string vs. cluster model)
    - shower ordering (pT-ordered vs. angular-ordered shower)
- the majority of the differences seem to originate from the different ordering in the parton shower
- The treatment of spin effects in MC generators combining the ME with PS requires special attention for future higher-precision quantum information studies



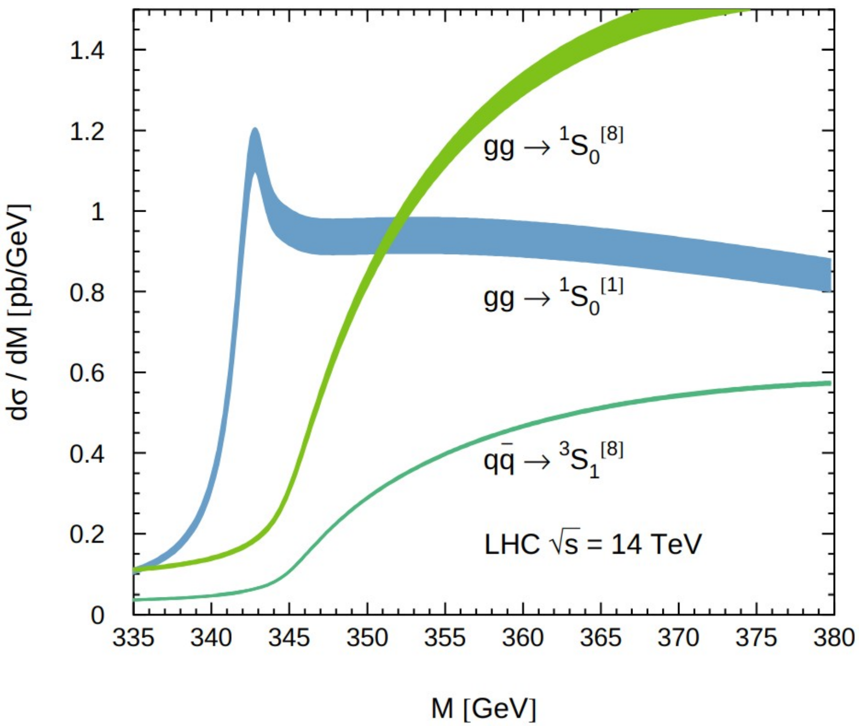


Threshold effects

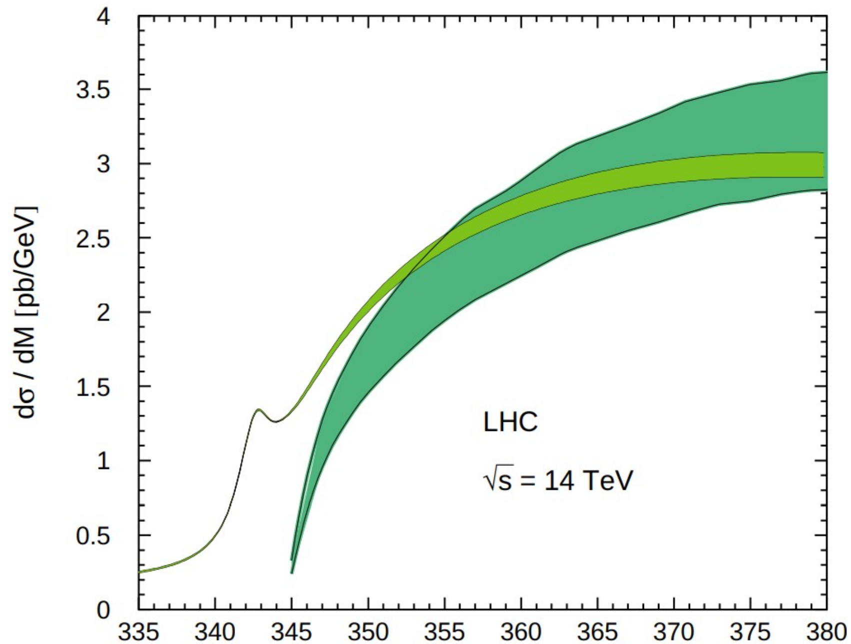
EPJ C 60 375-386,2009

- Non-relativistic threshold (bound state) effects change the $m(t\bar{t})$ dependence of the $t\bar{t}$ cross section
  - Also EWK corrections can have an effect (e.g. virtual Higgs correction)
  - These are not included in MC generators

non-relativ. QCD



non-relativ. QCD +
NLO QCD



- missing effects were tested by introducing them with an ad hoc reweighting of the Monte Carlo based on theoretical predictions → the effect was found to be **0.5%**
  - Main impact: change the line shape of the $m(t\bar{t})$ → similar systematics with larger effect

What is quantum entanglement?

- Phenomenon when quantum state of one particle cannot be described independently from another particle

  → there are **correlations** between observed physical properties of both particles

  → **measurement** of one particle influence other particle entangled with it

observed "here"

affected "over there"

- For a pure quantum state, the general form of the wave function:
  - For the simplest case of two spinless particle moving along the line

$$\psi(x,y) = \sum_i c_i \psi_i(x) \psi_i(y)$$

- The quantum state is separable if wave function can be written as:

$$\psi(x,y) = \psi_1(x) \psi_2(y)$$

- **If the state is not separable → entangled state**

Quantum entanglement for two qubits

- Basic entanglement definition can be generalized also to what are called mixed states
  - Mixed states describe classical statistical mixtures of pure states
    - e.g. at the LHC we cannot control the initial state → mixed state
  - Mixed states described in general by 'density matrix' (ρ)

- A typical example of entanglement is provided by two qubits
  - qubit = a quantum system with two possible states
  - e.g. two particles with ½ spin
    - entanglement is characterized by their spin correlations

- The most general density matrix describing two qubits:

$$\rho = \frac{I_4 + \sum_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

I_4, I_2: unit (4x4), (2x2) matrices
$\sigma_{i,j}$: Pauli matrices

  - The state is described by 15 parameters $B_i^{\pm}$, C_{ij}
  - In case of two particles with ½ spin:
    - $B_i^{\pm}$: individual spin polarizations
    - C_{ij}: spin correlation matrix

A measure of quantum entanglement

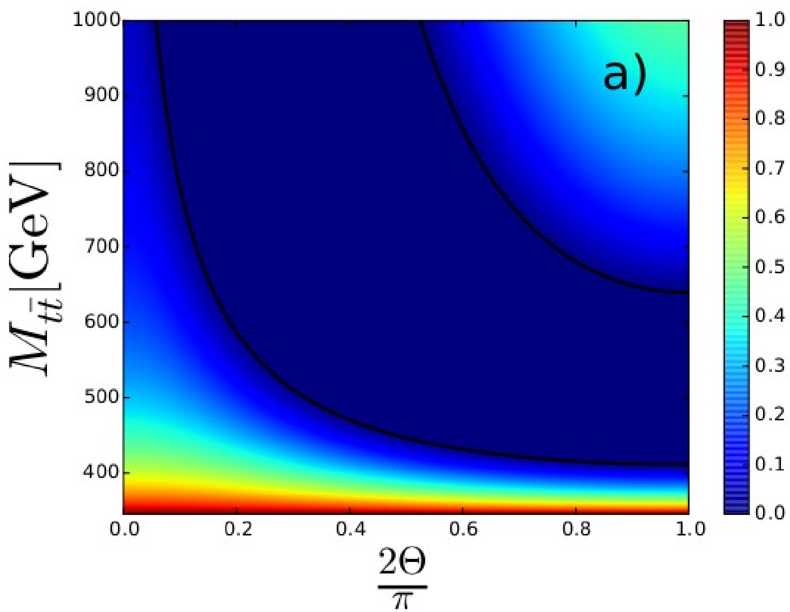
- The Peres-Horodecki criterion is a necessary condition for entanglement in bipartite systems of dimension 2 × 2
- A quantitative measure of the degree of entanglement is obtained by 'concurrence' C[ρ]:

$$C[\rho] \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

  - λ_i: eigenvalues of the matrix $\mathcal{C}(\rho) = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$, with $\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\ \rho^* \ (\sigma_2 \otimes \sigma_2)$
  - 0 ≤ C[ρ] ≤ 1
- Quantum state is entangled if and only if **C[ρ] > 0**

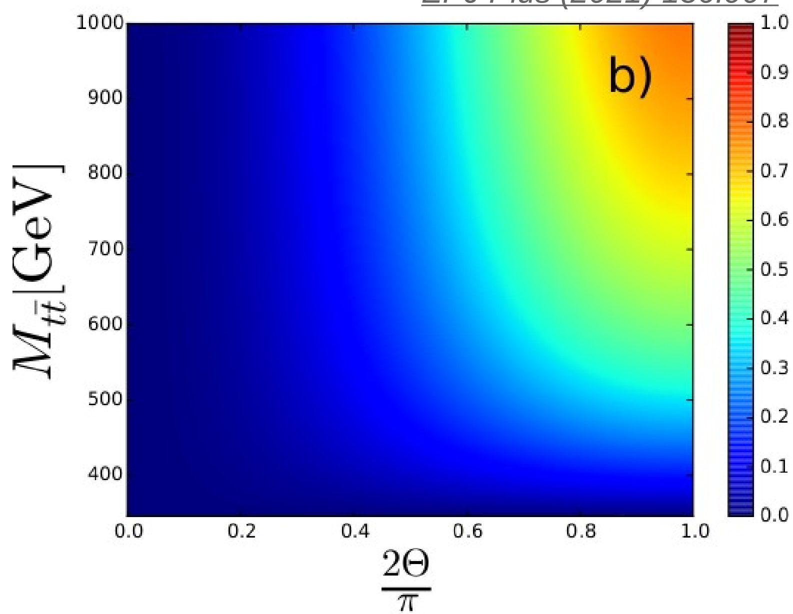
Entanglement for different $t\bar{t}$ initial states

gg → tt



qq → tt

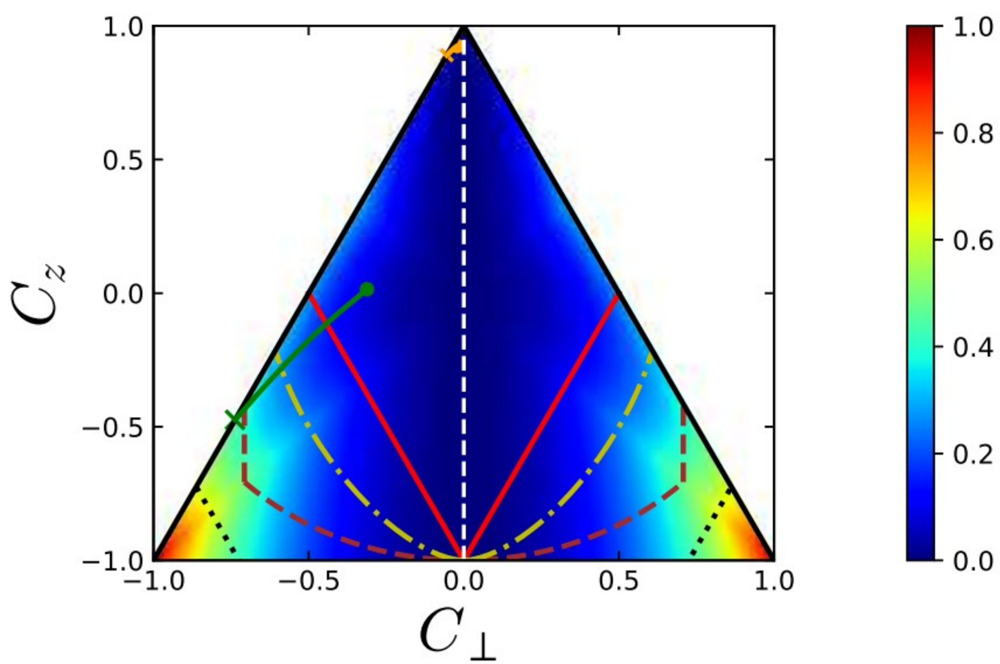
EPJ Plus (2021) 136:907



Quantum discord, steering

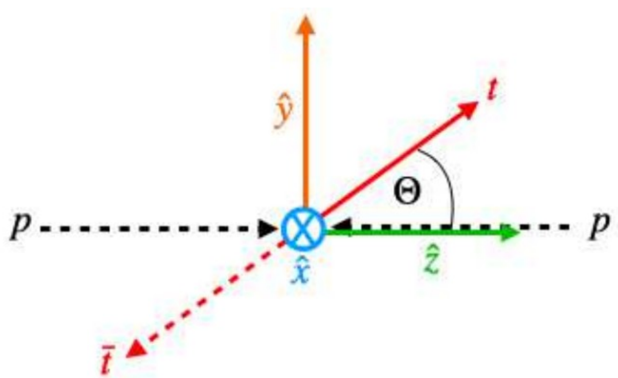
- Quantum discord:
  - Most basic form of quantum correlations
  - more general than entanglement → stronger robustness
- Quantum Steering:
  - measurements on one subsystem can be used to “steer” the other one

Phys. Rev. Lett. 130, 221801 (2023)



Diagonal elements of correlation matrix in the beam basis:

$$C_1 = C_2 = C_\perp \text{ and } C_3 = C_z$$

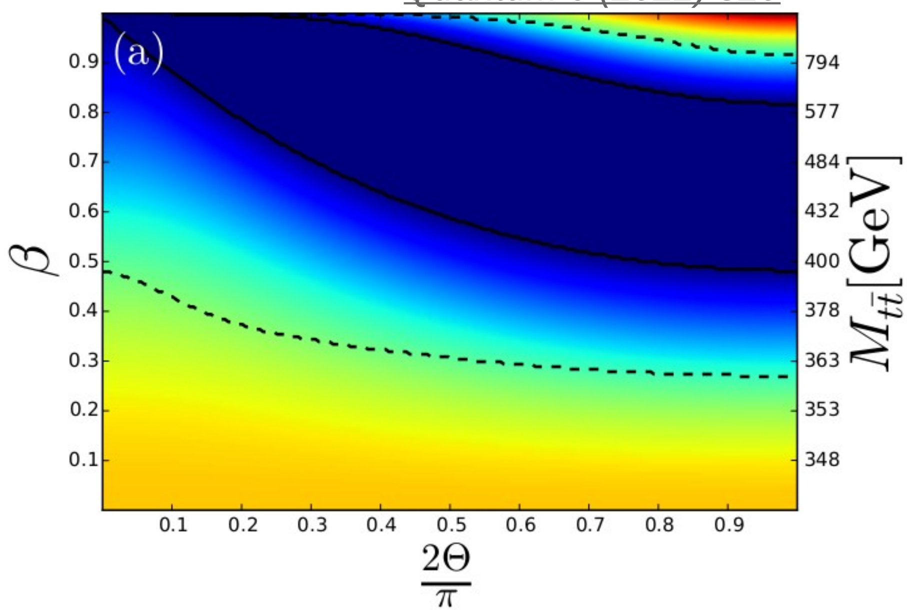


Possible future measurements

- Other measurements of various quantum information concepts at colliders are possible in the future, e.g.
  - test Bell inequalities in $t\bar{t}$ production (*Quantum 6 (2022) 820*)
  - perform tests in other system of particles
    - e.g. test Bell inequalities in Higgs boson decays H → WW (*Phys. Lett. B 825 (2022) 136866*)

tt production, concurrence:

Quantum 6 (2022) 820



CHSH violation limit:

$$|C_\perp| = 1/\sqrt{2}$$