Two-loop QCD corrections to $pp \to t\bar{t}j$

Colomba Brancaccio
University of Turin

In collaboration with: Simon Badger, Matteo Becchetti, Heribertus Bayu Hartanto, Simone Zoia [arXiv:24xx.xxxxx, arXiv:2404.12325, arXiv:2201.12188]
(Thanks also to Gaia Fontana for the wonderful drawings)

17th International Workshop on Top Quark Physics

Towards Two-loop QCD corrections to $pp \to t\bar{t}j$

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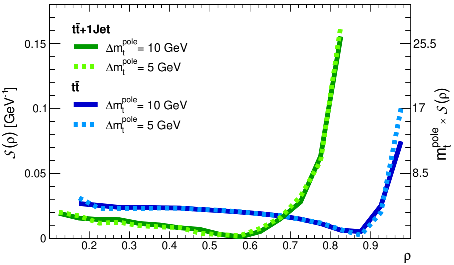
WEIGHTY motivations for $t\bar{t}j$ production

Why is $t\bar{t}j$ important?



[Alioli, Fernandez, Fuster, Irles, Moch, Uwer '13]

- 50% of $t\bar{t}$ events produced at LHC are associated with a jet
- $t\bar{t}j$ normalised differential cross-section **highly sensitive to m_t** [see also Olaf's talk]

What do we know about $t\bar{t}j$?

- NLO QCD corrections [Dittmaier, Uwer, Weinzierl, '07]
- Full off-shell decays and interfaces with parton shower
  [Alioli, Bevilacqua, Czakon, Hartanto, Kraus, Melnikov, Moch, Schulze, Uwer, Worek '10-'16]
- Mixed QCD and EW corrections [Gütschow, Lindert, Schönherr '18]
- **NNLO QCD corrections needed**

Current frontier: 2 → 3 two-loop scattering amplitudes

Massless external particles:

- $pp \to \gamma\gamma\gamma$
  [Abreu, Page, Pascual, Sotnikov (2020)]
  [Chawdhry, Czakon, Mitov, Poncelet (2021)]
  [Abreu, De Laurentis, Ita, Klinkert, Page, Sotnikov (2023)]
- $pp \to \gamma\gamma j$
  [Agarwal, Buccioni, von Manteuffel, Tancredi (2021)]
  [Chawdhry, Czakon, Mitov, Poncelet (2021)]
  [Badger, Brönnum-Hansen, Chicherin, Gehrmann, Hartanto, Henn, Marcoli, Moodie, Peraro, Zoia (2021)]
- $pp \to \gamma jj$
  [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia (2023)]
- $pp \to jjj$
  [Abreu, Febres Cordero, Ita, Page, Sotnikov (2021)]
  [De Laurentis, Ita, Klinkert, Sotnikov (2023)]
  [Agarwal, Buccioni, Devoto, Gambuti, von Manteuffel, Tancredi (2023)]
  [De Laurentis, Ita, Sotnikov (2023)]

One massive external particle: (full colour missing)

- $pp \to Wbb$
  [Badger, Hartanto, Zoia (2021)]
  [Hartanto, Poncelet, Popescu, Zoia (2022)]
- $pp \to Wjj$
  [Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov (2022)]
- $pp \to Hbb$
  [Badger, Hartanto, Krys, Zoia (2021)]
- $pp \to W\gamma j$
  [Badger, Hartanto, Krys, Zoia (2022)]
- $pp \to W/Z + bb$
  [Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini (2022)]
  [Mazzitelli, Sotnikov, Wiesemann (2024)]
- $pp \to W\gamma\gamma^*$
  [Badger, Hartanto, Wu, Zhang, Zoia (2024)]

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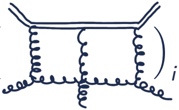
*subleading contribution numerically available

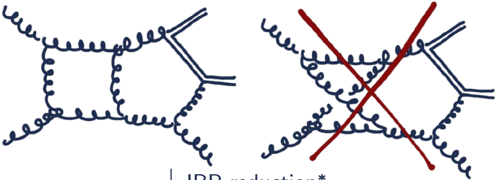

INTERNAL MASSES

Amplitude workflow: a MASSIVE effort

$$A^{2L}(\epsilon) = \sum_i \Big(\quad \Big)_i$$



$\downarrow$ colour decomposition



$\downarrow$ IBP reduction*

$$A^{hel,2L}_{\mathrm{LC}}(\epsilon) = \sum_i \boxed{d_i(\epsilon)}\ \boxed{\mathrm{MI}_i(\epsilon)}$$

$\longrightarrow$ Elliptic sector

[Badger, Becchetti, Giraudo, Zoia '24]
[Badger, Becchetti, Chaubey, Marzucca '23]

$\downarrow$ ϵ expansion

$$A^{hel,2L}_{\mathrm{LC}}(\epsilon) = \sum_i \sum_{k=-4}^{0} \epsilon^k\ \boxed{r_{ki}}\ \boxed{\mathrm{F}_i}$$

$\longrightarrow$ Special functions

$A(hel; n_t, n_{\bar{t}})$ helicity amplitudes encode spin correlations in the narrow width approximation

*Optimised IBP relations from NeatIBP [Wu, Boehm, Ma, Xu, Zhang '23]

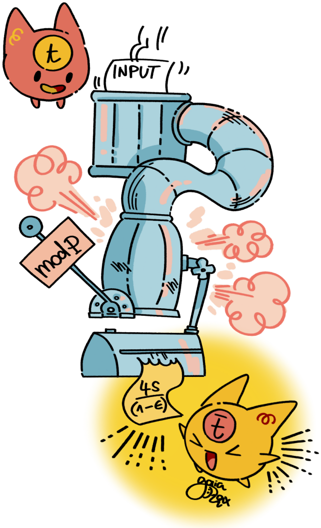
A MASSIVE effort: Algebraic complexity

- Intermediate steps in scattering amplitude computations can produce very **large expressions**
- To manage complexity, use numerical methods and then restore analytic dependence
- Replace symbolic operations with numerical evaluations in a **finite field** (integers mod prime P)

  [von Manteuffel, Schabinger '14] [Peraro '16]

- Numerical framework: **FiniteFlow**

  [Peraro '19]



A MASSIVE effort: Analytic complexity

- Expanding $\mathsf{MI}_i(\epsilon)$ around $\epsilon = 0$:

$$\mathsf{MI}_i(\epsilon) = \sum_{k=-4}^{0} \epsilon^k \underbrace{\mathbf{MI_i^k}}$$

  - **Polylogarithmic functions**, fast+stable numerics
  - **Elliptic functions**, need further investigation

- $$\cancel{\mathrm{d}\mathsf{MI}_i(\epsilon) = \epsilon\, \mathrm{d}A_{ij}\mathsf{MI}_j(\epsilon)} \quad \longrightarrow \quad \text{elliptic sector}$$

  $\rightarrow$ DEs arranged so that elliptic sectors only appear at $\mathrm{O}(\epsilon^0)$

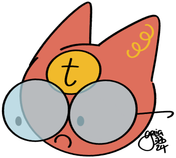
- A basis of special functions has been identified such that: **UV/IR poles can be identified analytically and the finite remainder computed directly**

TOP findings

- **First working code** to evaluate the finite reminder of $t\bar{t}j$ two-loop helicity amplitudes ✓
- **Ward identity check** (gauge invariant results) ✓
- **Pole check** (analytically checked) ✓
- **Special functions evaluated** via their DEs using GSE ✓ [**DiffExp**, Hidding '20][Moriello '18]
- **Evaluation** of two-loop helicity amplitudes (gg channel) at leading colour for benchmark physical phase-space points:

Phase-Space	$A^{2L}_{LC}(+++++; n_t n_{\bar{t}})$ [GeV^{-2}]
point #1	$19.028262 - 3.1078961\, i$
point #2	$0.07061470 - 0.00649655\, i$
point #3	$-29.219122 - 27.542150\, i$
point #4	$-0.97280521 + 0.86357506\, i$
point #5	$-0.40407926 - 0.53165671\, i$

Preliminary



TOPline summary

What?

- **Two-loop scattering amplitude for $\mathbf{pp \to t\bar{t}j}$** → towards two-loop QCD corrections

Why?

- Bottleneck computation for $t\bar{t}j$ **precise theoretical predictions** → high-priority process at the LHC

4 key questions

How?

- **Finite fields** framework
- **Special function** basis → direct determination of the finite reminder**!**

What's next?

- Deliver **phenomenological viable** results
- Explore analytical **reconstruction** viability

What?

- **Two-loop scattering amplitude for pp $\rightarrow$ t$\bar{\text{t}}$j**
  $\rightarrow$ towards two-loop QCD corrections

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Thank you!!!

How?

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- **Special function** basis
  $\rightarrow$ direct determination of the finite reminder**!**

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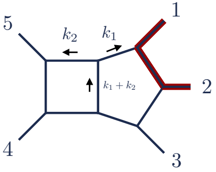
Backup

Integral basis for leading colour $t\bar{t}j$

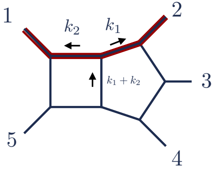
PB$_A$



[Badger, Becchetti, Chaubey, Marzucca '23]

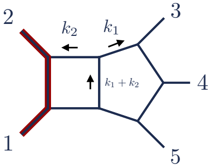
- 88 MIs
- $\mathrm{d}\log$-form

PB$_B$



- 121 MIs
- Elliptic sector + Nested square roots

PB$_C$



[Badger, Becchetti, Giraudo, Zoia '24]

- 109 MIs
- $\mathrm{d}\log$-form

Numerical evaluation of the helicity amplitudes

- Two-loop helicity amplitudes for the process $\boxed{4_g 5_g \to 1_t 2_{\bar{t}} 3_g}$
- Amplitude computed at leading colour
- Decay direction fixed in a non-physical direction $(n_t = n_{\bar{t}} = p_3)$
- Amplitude computed in 't Hooft-Veltman scheme
- Finite reminder directly computed

Preliminary

Phase-Space points	$A^{2L}_{LC}(+++++;n_t n_{\bar{t}})[\mathrm{GeV}^{-2}]$
$\mathrm{d}_{12} \to 0.1074, \mathrm{d}_{23} \to 0.2719, \mathrm{d}_{34} \to -0.1563,$ $\mathrm{d}_{45} \to 0.5001, \mathrm{d}_{15} \to -0.03196, \mathrm{mt}^2 \to 0.02502$	$19.028262 - 3.1078961\, i$
$\mathrm{d}_{12} \to 0.3915, \mathrm{d}_{23} \to 0.06997, \mathrm{d}_{34} \to -0.06034,$ $\mathrm{d}_{45} \to 0.5002, \mathrm{d}_{15} \to -0.1293, \mathrm{mt}^2 \to 0.02499$	$0.07061470 - 0.00649655\, i$
$\mathrm{d}_{12} \to 0.2167, \mathrm{d}_{23} \to 0.02186, \mathrm{d}_{34} \to -0.01149,$ $\mathrm{d}_{45} \to 0.5007, \mathrm{d}_{15} \to -0.04709, \mathrm{mt}^2 \to 0.02502$	$-29.219122 - 27.542150\, i$
$\mathrm{d}_{12} \to 0.2986, \mathrm{d}_{23} \to 0.1599, \mathrm{d}_{34} \to -0.05978,$ $\mathrm{d}_{45} \to 0.4998, \mathrm{d}_{15} \to -0.2899, \mathrm{mt}^2 \to 0.02500$	$-0.97280521 + 0.86357506\, i$
$\mathrm{d}_{12} \to 0.2882, \mathrm{d}_{23} \to 0.04770, \mathrm{d}_{34} \to -0.1080,$ $\mathrm{d}_{45} \to 0.5000, \mathrm{d}_{15} \to -0.1583, \mathrm{mt}^2 \to 0.02502$	$-0.40407926 - 0.53165671\, i$

with $d_{ij} = p_i \cdot p_j$, normalised here w.r.t $2\, p_4 \cdot p_5$

Helicity amplitudes for massive fermions

We make use of the following basis for spin structures:

$$\boxed{\begin{aligned} A^{2L}(1_t^+, 2_{\bar{t}}^+, 3^{h_3}, 4^{h_4}, 5^{h_5}; n_t, n_{\bar{t}}) &= m_t \Phi(3^{h_3}, 4^{h_4}, 5^{h_5}) \\ &\sum_{i=1}^{4} \Theta_i(1, 2; n_t, n_{\bar{t}}) A^{2L,[i]}(1_t^+, 2_{\bar{t}}^+, 3^{h_3}, 4^{h_4}, 5^{h_5}) \end{aligned}}$$

- The phase factor Φ account for the massless parton helicities
- Four basis functions Θ_i for the spin dependence of the top-quark pair and the associated subamplitudes $A^{2L,[i]}$

For more details see [arXiv:2201.12188, arXiv:2102.13450]