Review of analytic reconstruction methods of $t\bar{t}$ dileptonic decay in proton-proton collisions at the CMS Experiment

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ABSTRACT: Reconstruction of the $t\bar{t}$ system is crucial for many top quark studies at the Large Hadron Collider (LHC). The dileptonic decay of $t\bar{t}$ which produces two neutrinos that escape detection, presents significant challenges for reconstruction algorithms. We review the two analytic reconstruction methods, ***m_{lb}*-weighting** and ***Ellipse***. Both methods are based on the constraints of W mass, top mass and measured missing transverse energy. The first method is widely used for $t\bar{t}$ dileptonic decay reconstruction in the CMS Collaboration. It uses smearing and permutation to increase the chance of finding a solution to the quartic equation. Overall, 94% of $t\bar{t}$ dileptonic decays can be reconstructed with this method. We also review the second method since it visualizes the problem in a geometric way and can apply to semi-leptonic decay as well.

Constraints and Quartic Equation[1]

[Phys.Rev.D73:054015,2006]

$$m_t^2 = (E_{l^+} + E_\nu + E_b)^2 - (p_{x,l^+} + \boxed{p_{x,\nu}} + p_{x,b})^2 - (p_{y,l^+} + \boxed{p_{y,\nu}} + p_{y,b})^2 - (p_{z,l^+} + \boxed{p_{z,\nu}} + p_{z,b})^2$$

$$m_{\bar{t}}^2 = (E_{l^-} + E_{\bar{\nu}} + E_{\bar{b}})^2 - (p_{x,l^-} + \boxed{p_{x,\bar{\nu}}} + p_{x,\bar{b}})^2 - (p_{y,l^-} + \boxed{p_{y,\bar{\nu}}} + p_{y,\bar{b}})^2 - (p_{z,l^-} + \boxed{p_{z,\bar{\nu}}} + p_{z,\bar{b}})^2$$

$$m_{W^+}^2 = (E_{l^+} + E_\nu)^2 - (p_{x,l^+} + p_{x,\nu})^2 - (p_{y,l^+} + p_{y,\nu})^2 - (p_{z,l^+} + p_{z,\nu})^2$$

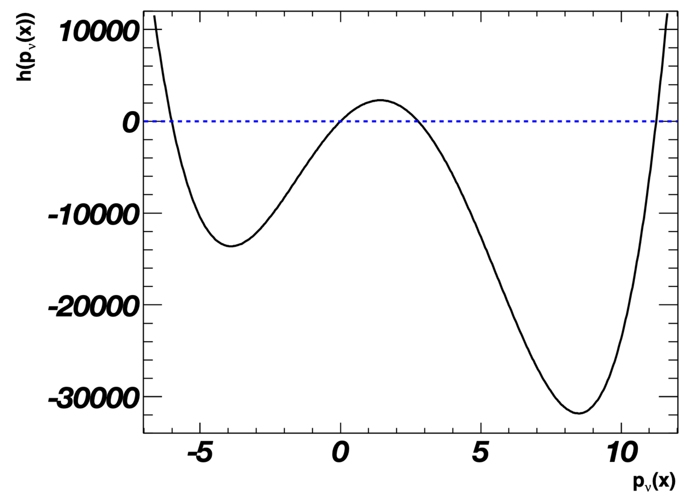
$$m_{W^-}^2 = (E_{l^-} + E_{\bar{\nu}})^2 - (p_{x,l^-} + p_{x,\bar{\nu}})^2 - (p_{y,l^-} + p_{y,\bar{\nu}})^2 - (p_{z,l^-} + p_{z,\bar{\nu}})^2$$

$$E_x^{MET} = p_{x,\nu} + p_{x,\bar{\nu}}$$

$$E_y^{MET} = p_{y,\nu} + p_{y,\bar{\nu}}$$

- Solve for the 3-vectors of the two neutrinos using top quark mass, W boson mass and missing transverse momentum (MET) constraints.
- Neutrino mass is negligible compared to its energy in the current LHC. Therefore, $E^2 = \boldsymbol{p}^2$.
- Ideally without any radiation, it results in a quartic equation of $p_{x,\bar{\nu}}$ with 0, 2 or 4 solutions, where each d_i is a complicated expression of leptons, b quarks' four momentum and values of the constraints. The solution of $p_{x,\bar{\nu}}$ can determine both neutrinos' momenta.

$$0 = \sum_{i=0}^{4} d_i\, p_{x,\bar{\nu}}{}^i$$



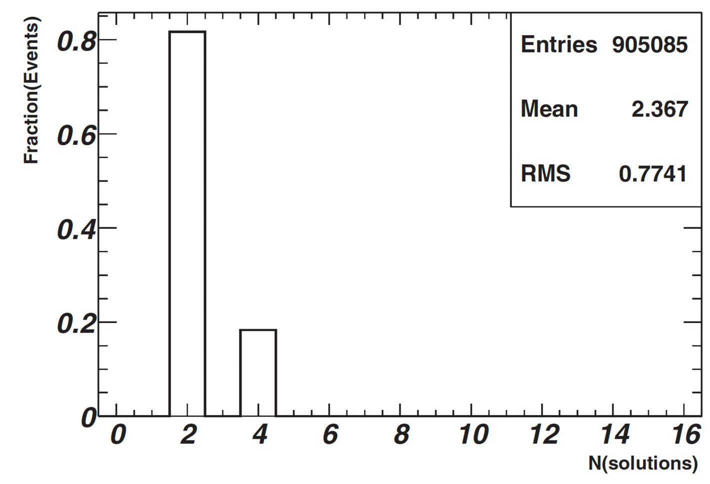


Figure 1. (Left) An example of a quartic equation whose real roots in $p_{x,\bar{\nu}}$ are solutions of the set of equations for $t\bar{t}$ dileptonic decay. (Right) Distribution of number of analytic solutions when all the information including leptons' four momentum, b quarks' four momentum and transverse missing energy are known exactly.

[1] Sonnenschein, L., (2006). *Analytical solution of ttbar dilepton equations.* Phys.Rev.D73:054015,2006

Reconstruction on Data and Monte-Carlo

[Eur. Phys. J. C 75 (2015) 542]

- The selected events on top of data contains not only $t\bar{t}$ dileptonic decays, but also other processes partially due to their similar final signature as $t\bar{t}$ dileptonic decays and partially due to object mis-identification in reconstruction.
- Figure 2 proves that there is little bias on Data vs MC with the method.

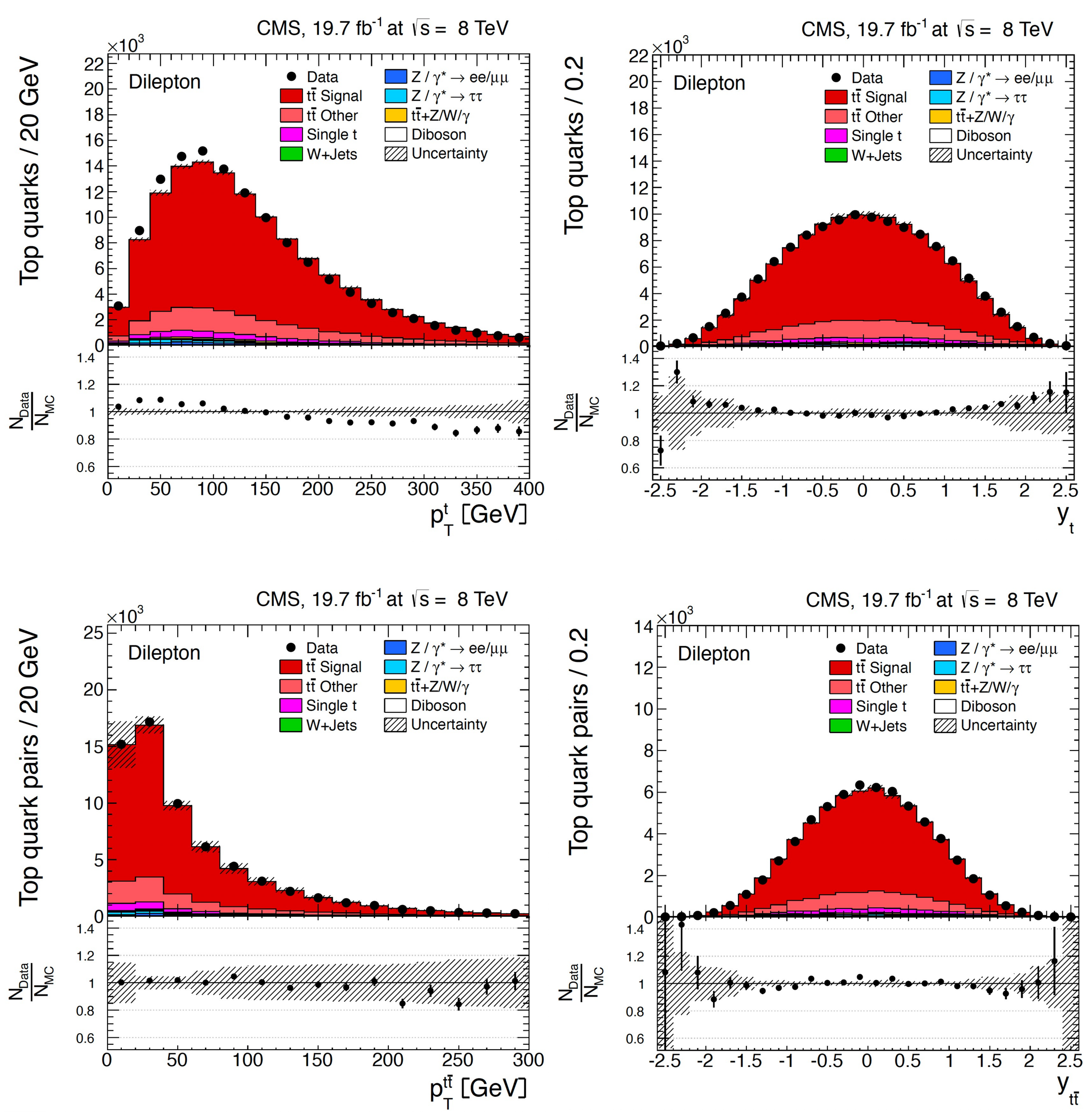
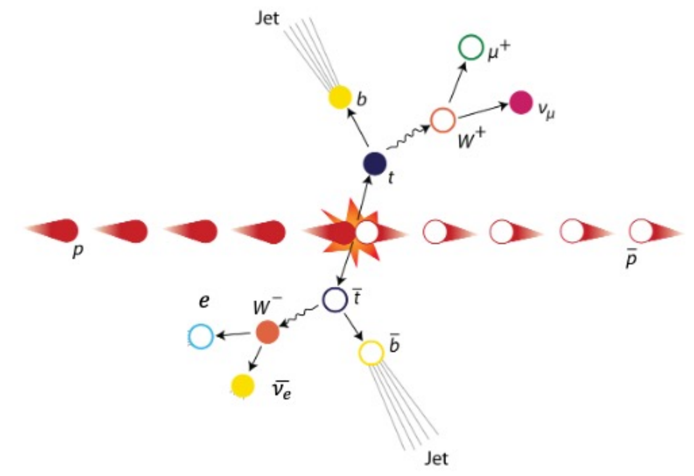


Figure 2. Distribution of top quark and antiquark (left) and $t\bar{t}$ (right) quantities as observed from the m_{lb} weighting method in dilepton channels. The top row shows the transverse momentum, and the bottom row shows the rapidity. The QCD multijet background is negligible and not shown. The Z/γ^* +jets background determined from data. The hatched regions correspond to the shape uncertainties for Monte-Carlos.

Event Topology and Selections in CMS

W^+ DECAY MODES		Fraction (Γ_i/Γ)
$\ell^+\nu$	[c]	(10.86 ± 0.09) %
$e^+\nu$		(10.71 ± 0.16) %
$\mu^+\nu$		(10.63 ± 0.15) %
$\tau^+\nu$		(11.38 ± 0.21) %
hadrons		(67.41 ± 0.27) %

τ^- DECAY MODES		Fraction (Γ_i/Γ)
$e^-\bar{\nu}_e\nu_\tau$	[g]	(17.82 ± 0.04) %
$\mu^-\bar{\nu}_\mu\nu_\tau$	[g]	(17.39 ± 0.04) %



- A τ lepton's lifetime is short and it instantly decays after being generated. Its leptonic decay generates 2 neutrinos and a muon (μ) or an electron (e).
- These events are also reconstructed with the analytic method, treating the two neutrinos from a tau as one.

Event selections and object reconstruction:

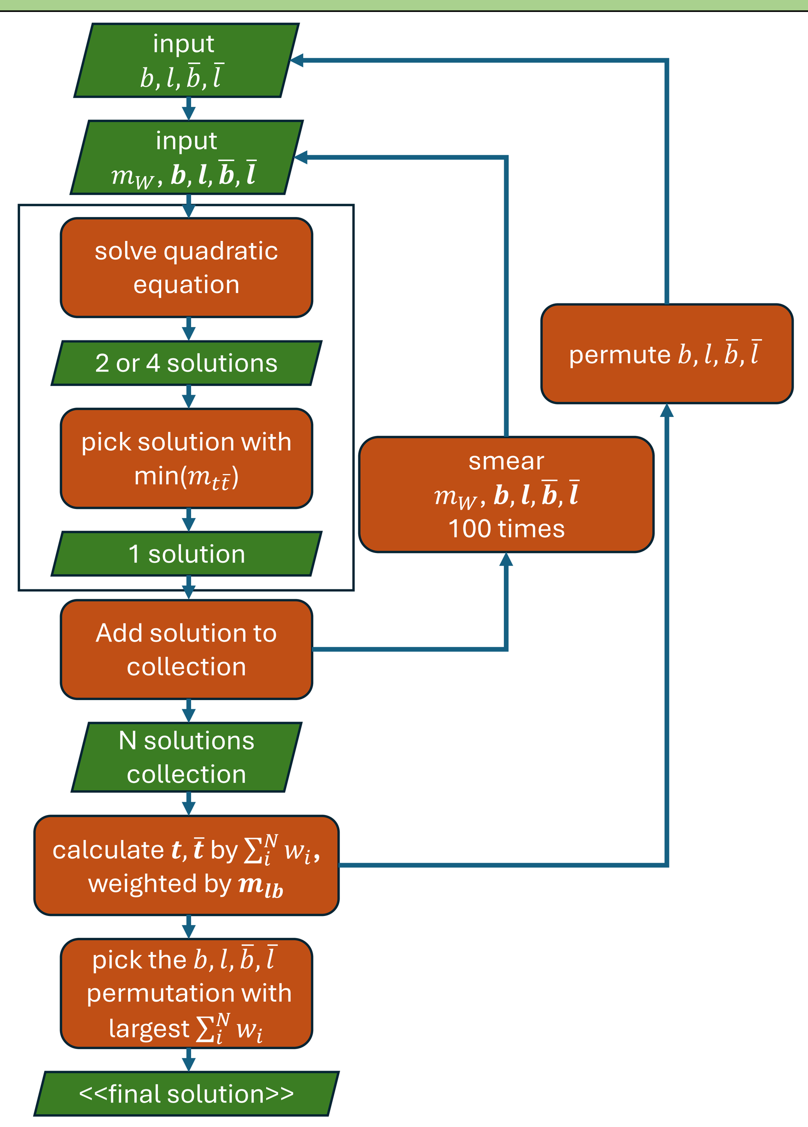
- The two leptons' p_T> 25(20) GeV and $|\eta|$ < 2.4, isolation required.
- The event has at least 2 jets with p_T > 30 GeV & $|\eta|$ < 2.4, and at least 1 jet is tagged as from a b quark.
- MET > 40 GeV in events with ee or $\mu\mu$.
- $M_{l\bar{l}}$> 20 GeV to suppress heavy-flavor resonance and Drell-Yan.
- $|M_{l\bar{l}} - M_Z|$> 15GeV in events with ee or $\mu\mu$ to suppress Z boson.

Solutions, Smearing and m_{lb} weighting

- Quartic equation solutions:
  - pick the solution with minimal $m_{t\bar{t}}$.
- Smearing:
  - Repeat process 100 times per event smearing lepton and b-jets' energy resolution and direction. W mass is smeared with Breit-Wigner distribution.
  - Calculate per-smear weight based on the m_{lb} distribution.
  - Top quark momentum is weighted average.
- Permutation:
  - The detector cannot tell b-quark from anti-b-quark's shower. It can also mis-identify the charge of leptons sometimes.
  - Pick the permutation with the largest weighted average

*In case of zero solution in solution collection, solutions are computed again including 1 b-tagged jet and 1 untagged jet.

Through picking the solution with minimal $m_{t\bar{t}}$, smearing and permutation, 94% of the selected events find a solution.

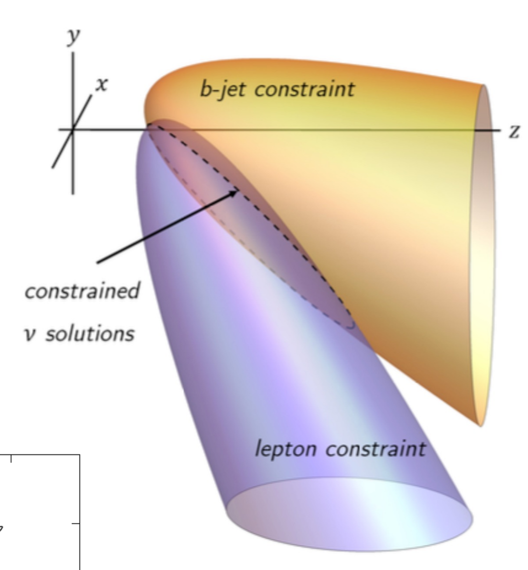


Another analytic approach: Ellipse[2]

[arXiv:1305.1878]

For a single top quark decay, constraints from the t mass and W mass each forms an ellipsoidal surface for $\boldsymbol{p_W}$ and $\boldsymbol{p_\nu}$.

- Since $\boldsymbol{p_W} = \boldsymbol{p_\mu} + \boldsymbol{p_\nu}$, and $\boldsymbol{p_\mu}$ is well measured, the first ellipsoidal surface also becomes a $\boldsymbol{p_\nu}$ ellipsoidal.
- The solution set of $\boldsymbol{p_\nu}$ is the intersection of two ellipsoidal surfaces.



Constraint by MET for $t\bar{t}$ system dileptonic decay:

- It looks for inter-section of the constraint with both sets of neutrino solutions.
- It provides the same solution as in Figure 1, with 0, 2 or 4 solutions.

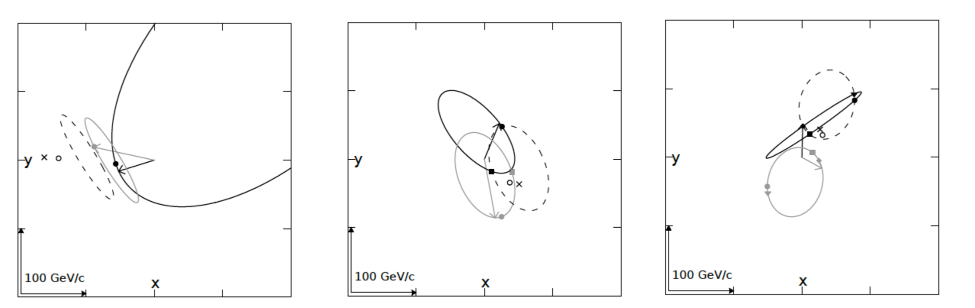


Figure 3: Visualization of two sets of neutrino momenta solutions. The arrows represent the true momentum of neutrinos. Neutrino solution is on black solid ellipse and anti-neutrino is on grey solid ellipse. The dashed ellipse is from the momentum imbalance constraint (o) which is the sum transverse momentum of the neutrinos. MET is represented by (x). The intersections of dashed line with each solid ellipse represents momentum solutions for each neutrino. From left to right presents 0, 2, 4 solutions.

An application of the ellipse method is used to study spin correlation and polarization sensitivity at the HL-LHC phase [CMS-PAS-FTR-18-034] by **Purdue University**.

[2] Betchart, B. A.,Demina, R., and Harel, A., (2013), *Analytic solutions for neutrino momenta in decay of top quarks*, arXiv:1305.1878.

Past and Future: The ***m_{lb}*-weighting** method evolves from D0 experiment's neutrino-weighting method for measuring top quark mass [Phys.Rev.Lett.80:2063-2068,1998]. The m_{lb}-weighting method was published in 2014 with the measurement of differential cross section for $t\bar{t}$ in pp collision at $\sqrt{s}$ = 8TeV. It has been used in many top quark related analyses by the CMS experiment for 10 years. The ***Ellipse*** method was published in 2013, and some physics results were published with it as well, including measurement of $t\bar{t}$ differential cross section in the lepton+jets final state at $\sqrt{s}$ = 13TeV[Phys.Rev.D.95,092001]. Both methods present decent mathematical interpretations of the kinematic system, but ambiguities in the solutions are unavoidable. They both fail to reconstruct some events due to limits in detector resolution and object reconstruction. Improving $t\bar{t}$ kinematic reconstruction will benefit the frontier of top quark studies in the CMS experiment. We plan to explore other approaches including machine learning methods to investigate the probability to improve the accuracy of top quark momentum reconstruction.