Plasma exit beam parameter estimates

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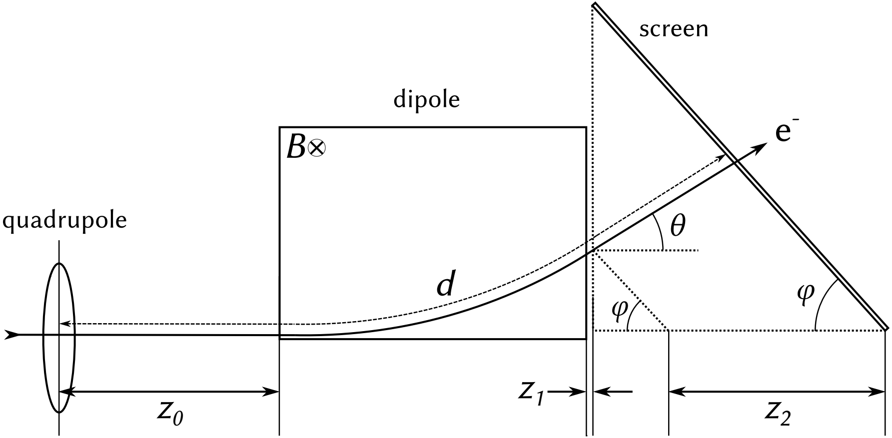
Outline

- Emittance measurement—11/2022
  - Basic premise/various techniques/strengths/weaknesses/assumptions
    - Single shot quadscan
    - Tomographic reconstruction
    - Multishot quadscan
  - Results comparison
  - Validity check from simulation (single shot quadscan only)
  - Run2c prospects

Emittance measurement techniques with the spectrometer

- Beam size method: use measurements of vertical beam size on spectrometer screen and calculated transport matrix to reconstruct upstream Gaussian beam parameters. Assumes beam parameters are invariant with energy and phase space well-described by Gaussian beam.
- Phase space tomography: treat vertical slices of the spectrometer images as different integrated views of the upstream phase space, and use an inverse Radon transform to reconstruct the phase space, acquiring rotation and scale from the calculated transport matrix. Also assumes the phase space is invariant with energy, and further that any energy slice can be considered as equivalent to the sum over the profile horizontally.
- Multishot quadrupole scan: change quad strengths and measure the beam size at the screen, and again reconstruct the Gaussian beam parameters using the transport matrices. Assumes shot-to-shot invariance of beam parameters (but if you get it, can validate the energy invariance assumptions of the other methods).

Spectrometer geometry



Refresher: vertical beam size function

How to determine beam parameters from size measurements? Assuming Gaussian distributions for the beam, a beam matrix is a covariance matrix in position and momentum, in 1D given by,

$$\Sigma = \begin{bmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{bmatrix} \tag{1}$$

thus transformation of beam matrix from one position to another occurs by some (known) transport matrix **R**:

$$\Sigma_{new} = \mathbf{R}\Sigma\mathbf{R}^T \tag{2}$$

so beam size at new position

$$\sigma_{y,new}^2 = R_{11}^2\sigma_y^2 + 2R_{11}R_{12}\sigma_{yy'} + R_{12}^2\sigma_{y'}^2 \tag{3}$$

Refresher: vertical beam size function

We can then calculate **R** matrix elements for each energy corresponding to each column in the image, and find least-squares fit for σ_y^2, $\sigma_{yy'}$, and $\sigma_{y'}^2$. Finally, emittance ϵ is given by:

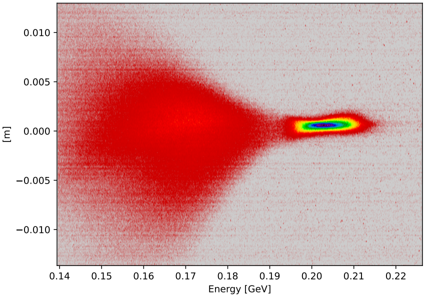
$$\epsilon_y = \sqrt{\sigma_y^2 \sigma_{y'}^2 - \sigma_{yy'}^2} \quad (4)$$

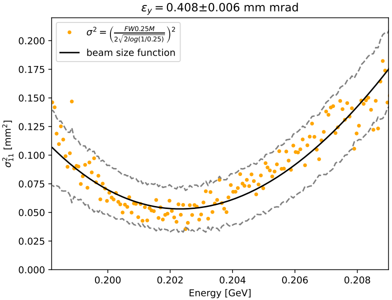
But, the method doesn't include:

- Resolution: must be removed by deconvolution
- Horizontal beam size/divergence: can make a difference if the x emittance is large.

Note: this is identical mathematics to the multi-shot quad scan case, except there we measure at a fixed position and vary **R** by hand.

Example beam size function fit





Fit to region of interest where the bulk of the charge lies; finding a contour in the low-energy tail is hard (automatically, although your eye can see it clearly).

Phase space tomography

Shout-out: **V. Bencini** for introducing me to this: standard computed tomography reconstruction (recreate the 2d shape of an object using many 1d projections), with angle θ calculated from $\mathbf{R}$, and a modification to the scale s calculated from $\mathbf{R}$, since $\mathbf{R}$ is not pure rotation:

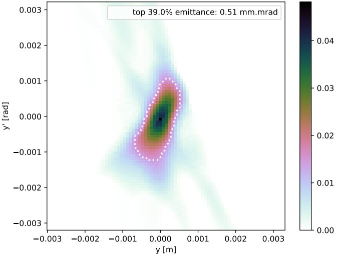
$$\theta = \frac{R_{12}}{R_{11}} \tag{5}$$

$$s = \sqrt{R_{11}^2 + R_{12}^2} \tag{6}$$

So performing these transformations:

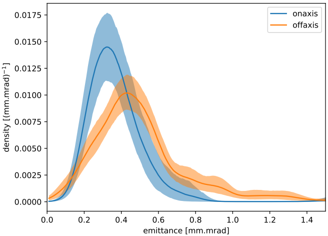
- normalization
- centering
- scaling (s)
- inverse Radon

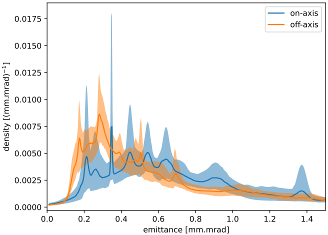
produces a reconstruction of the phase space. Measure emittance as an area of this (I use the top 39%, this is the contour shown here in white):

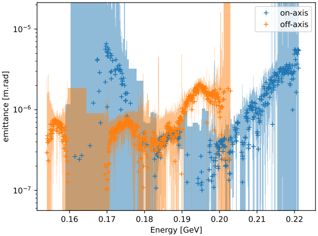


Some artifacts visible from incomplete angle coverage (typically show up as radial spokes).

Comparative results—emittance

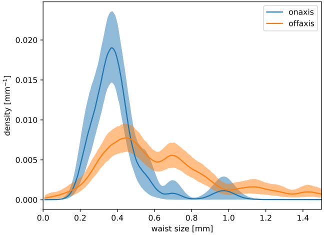


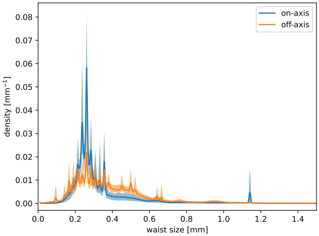


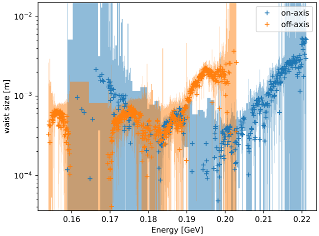


Clockwise from top left: beam size, quad scan, tomography.

Comparative results—waist size

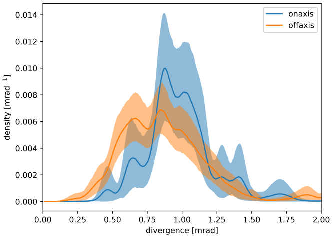


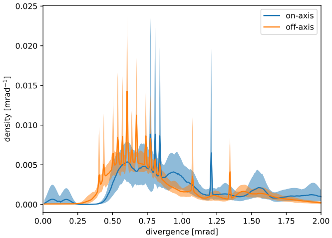


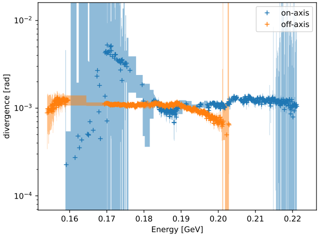


Clockwise from top left: beam size, quad scan, tomography.

Comparative results—divergence

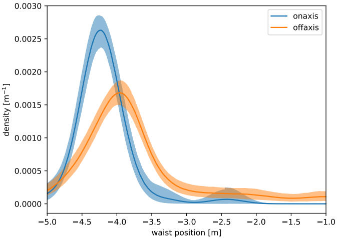


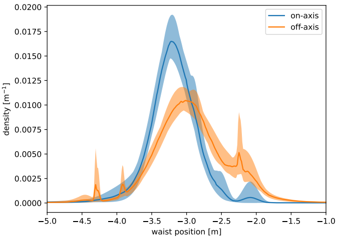


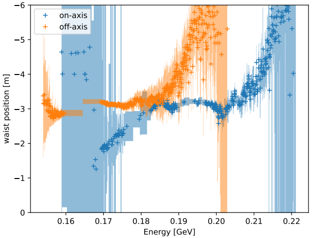


Clockwise from top left: beam size, quad scan, tomography.

Comparative results—waist position







Clockwise from top left: beam size, quad scan, tomography.

Systematic uncertainty from resolution width error

- Currently, the resolution is estimated from simulation and lab measurements of the MTF of the camera/lens system.
- Ongoing measurements at CLEAR for scintillating screen resolution—results might shift if resolution estimate if different.
- Useful to vary the resolution in fitting (numerically differentiate with respect to resolution width)

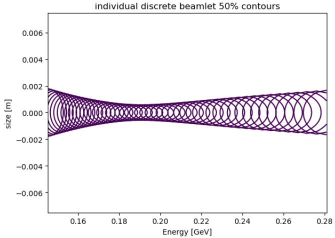
$\delta\sigma_x$	2.4×10^{-3} mm µm^{-1}
$\delta\sigma_{xp}$	1.9×10^{-3} mrad µm^{-1}
$\delta\sigma_w$	3.5 mm µm^{-1}
$\delta\sigma_\epsilon$	2.9×10^{-3} mm mrad µm^{-1}

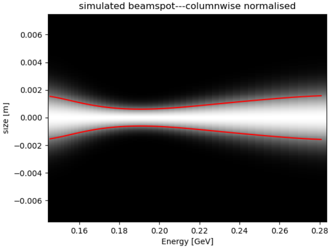
Validity estimate

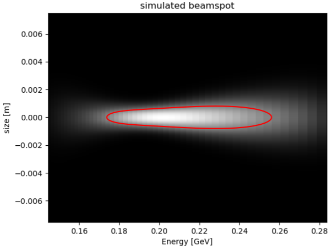
A question naturally arises concerning the smallest emittance that can be measured by this system, and whether or not we measure the true beam parameters this way, or are constrained by the resolution. Since emittance comprises equal contributions from beam size and beam divergence, there is no single answer to ‘smallest measurable emittance’, it depends on the composition of the beam.

So we turn to simulation to check this…

Validity estimate from simulation

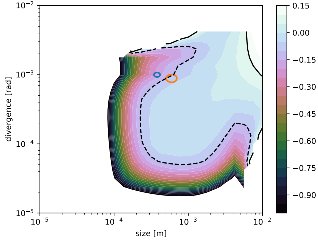


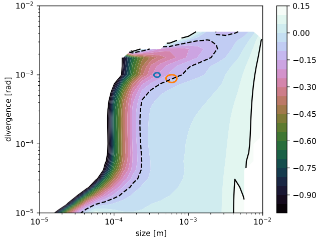


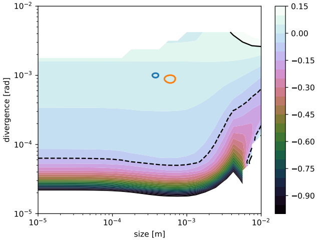


- Shown here with a coarse energy grid for illustrative purposes
- Includes resolution
- Then apply beam size function fit to the simulated image to reconstruct input phase space.

Validity estimate from simulation

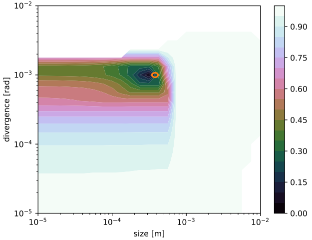


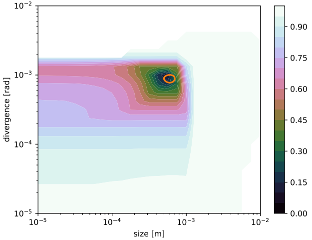




- Maps of relative difference between input and reconstructed emittance, divergence and waist size (clockwise from top left).
- Emittance measurement constrained from all sides, by minimum reconstructable size and divergence, but also by large beam parameters in *x* (and in some cases, the simulation window is too small).
- Measurements shown as ellipses (blue: on-axis, orange: off-axis).

Validity estimate from simulation





- Reformulated emittance maps from previous slide in terms of measurements.
- Relative difference between measured value and reconstructed value of emittance, as a function of simulation input (measurement shown as an ellipse again).

Prospects for emittance measurement in run2c

- Electron optics magnification should help to lower the minimum measurable size and divergence for a given pixel size.
- Target emittance could be $O(10^{-10}) = O(10^{-6})/O(10^{4})$ m, that is, injection normalised emittance is $O(10^{-6})$ m, and $\gamma \sim 10^4$ for multi-GeV electrons. But configuration which has this value in the instrument valid region doesn't seem impossible to achieve.
- Good knowledge of system point-spread function is required (study underway, including measurement campaign at CLEAR in December 2023).
- Field of view is compromised by needing long focal length lenses or macro lens object distances.
- Simulations use a generous energy spread to allow fitting to the size. Low energy spread beams won't give good results (high dynamic range cameras might help a litte); in this case only multishot quadscan method works.
- Only one small area of phase space has been simulated to check validity (just the plane of symmetric beams)—large ϵ_x can spoil the ϵ_y measurement even if ϵ_y is small.