MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Wakefield-Driven Filamentation of Warm Beams in Plasma

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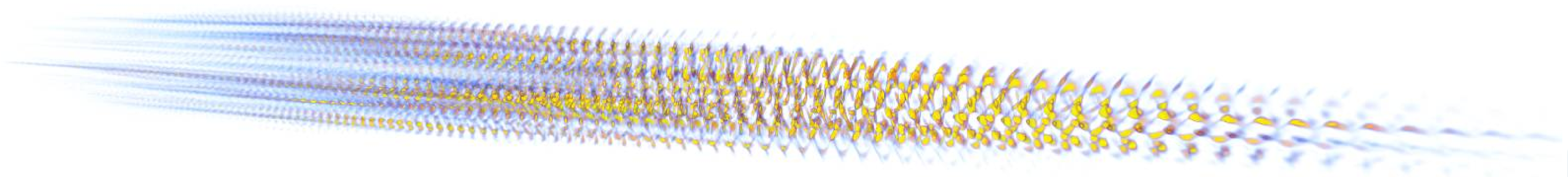


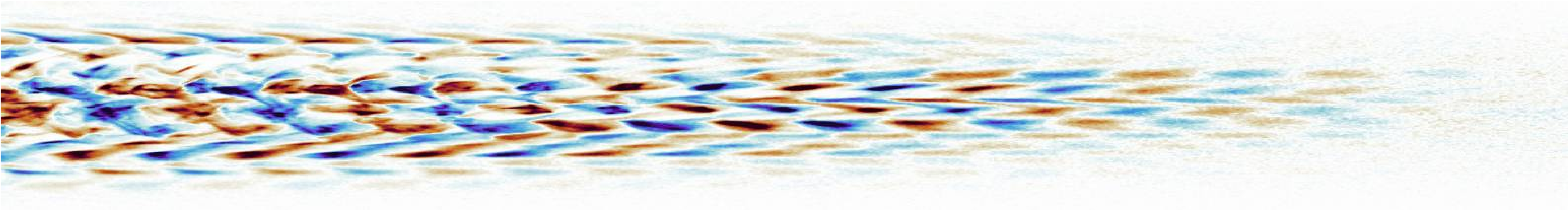
Structure

Filamentation of warm dilute beams

Wakefield-driven two-stream modes

Emittance-driven diffusion

Conclusion



Filamentation of Beams

2 relevant filamentation modes in a relativistic, quasineutral bunch (based on *Shukla, 2018*)

Oblique instability

- Dominant electric field

$$\Gamma_{\mathrm{OTSI}} = \frac{\sqrt{3}}{2^{4/3}}\sqrt[3]{\frac{n_b/n_p}{\gamma_b}}\omega_p t$$

Current filamentation

- EM-fields of similar magnitude

$$\Gamma_{\mathrm{CFI}} = \beta_b\sqrt{\frac{n_b/n_p}{\gamma_b}}\omega_p t$$

Quasineutral bunch

- Neutral: Equal number of positrons (e^+) and electrons (e^-) --> No net charge
- Quasi: Divergence results in small fluctuations: Seed for instability growth

Why quasineutral?

- Enables study of small-scale / higher order instabilities, while avoiding SMI, driven by the bunch shape
- Avoids pinching and hosing for high bunch currents

Filamentation of Beams

2 relevant filamentation modes in a relativistic, quasineutral (e+e-) bunch (based on *Shukla, 2018*)

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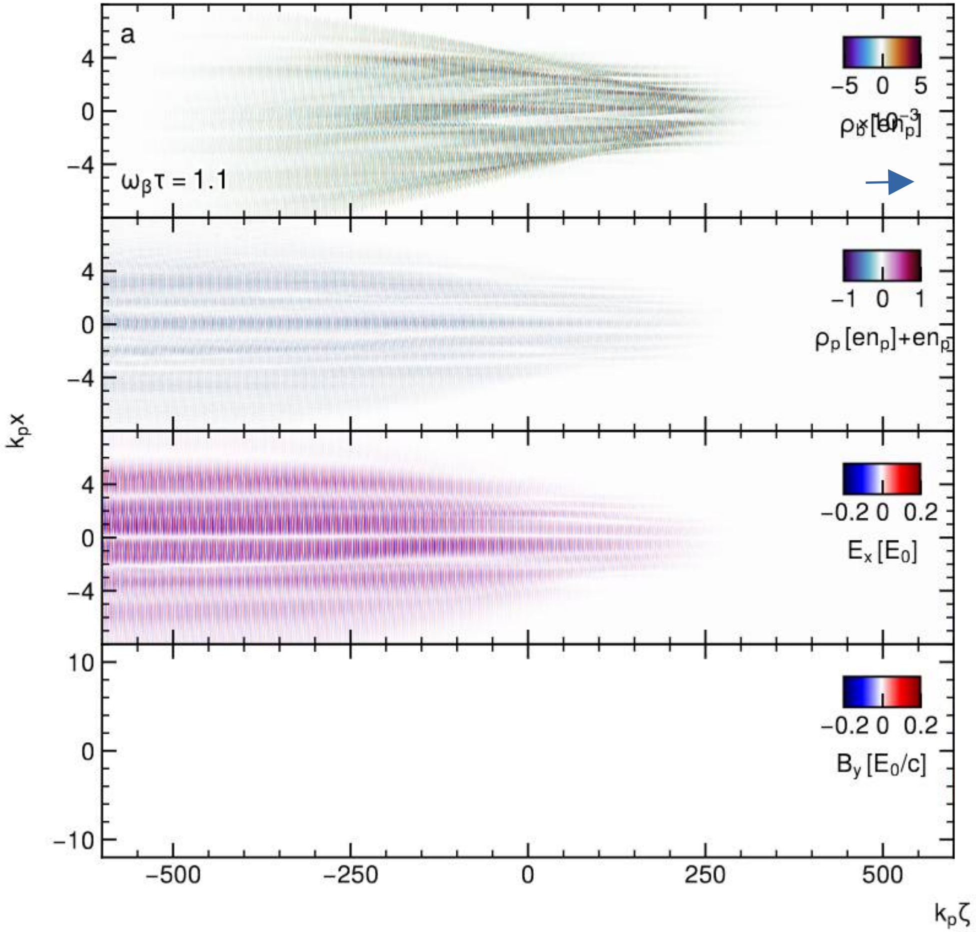

Bunch charge density

Plasma charge density

Electric field

Magnetic field

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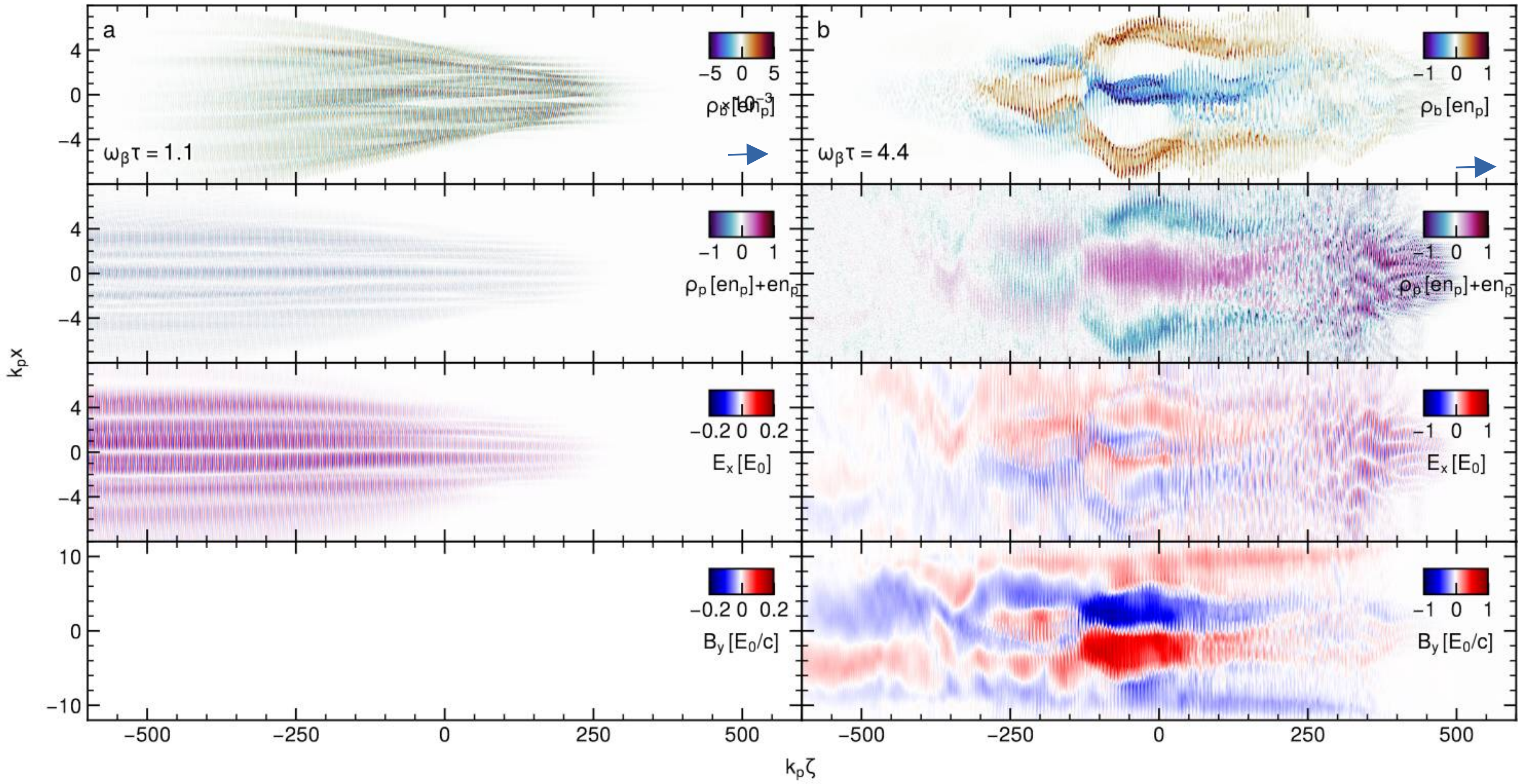

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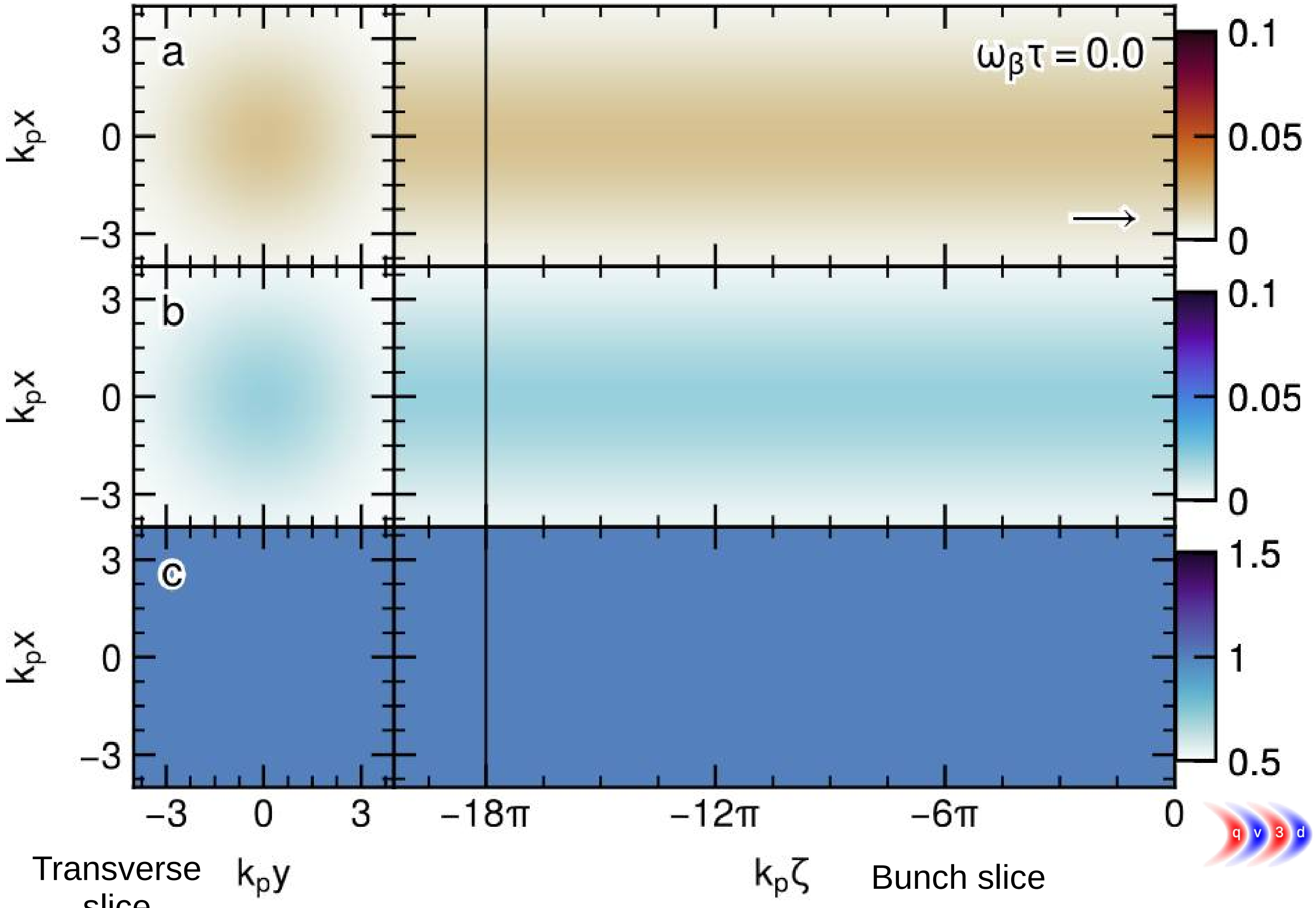
Filamentation of Warm Beams

- Dilute (n_b=0.02 n_p), quasineutral e^+e^- bunch with finite emittance



Positron charge density

Electron charge density

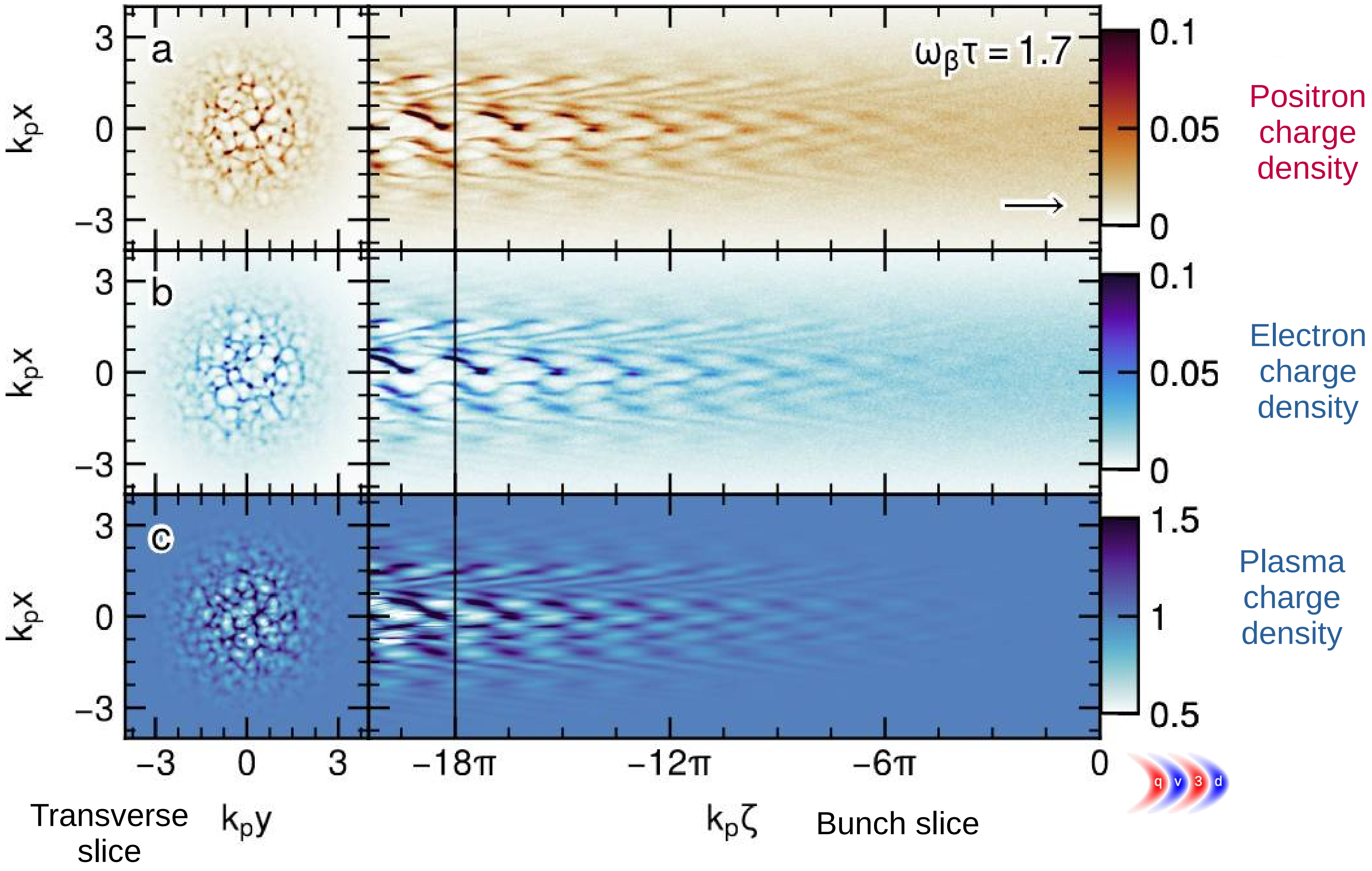
Plasma charge density

Transverse slice

Bunch slice

Filamentation of Warm Beams

- Dilute (n_b=0.02 n_p), quasineutral e^+e^- bunch with finite emittance
- Longitudinal and transvere modulation in bunch and plasma
- Finite, roughly uniform distance between self-modulated filaments



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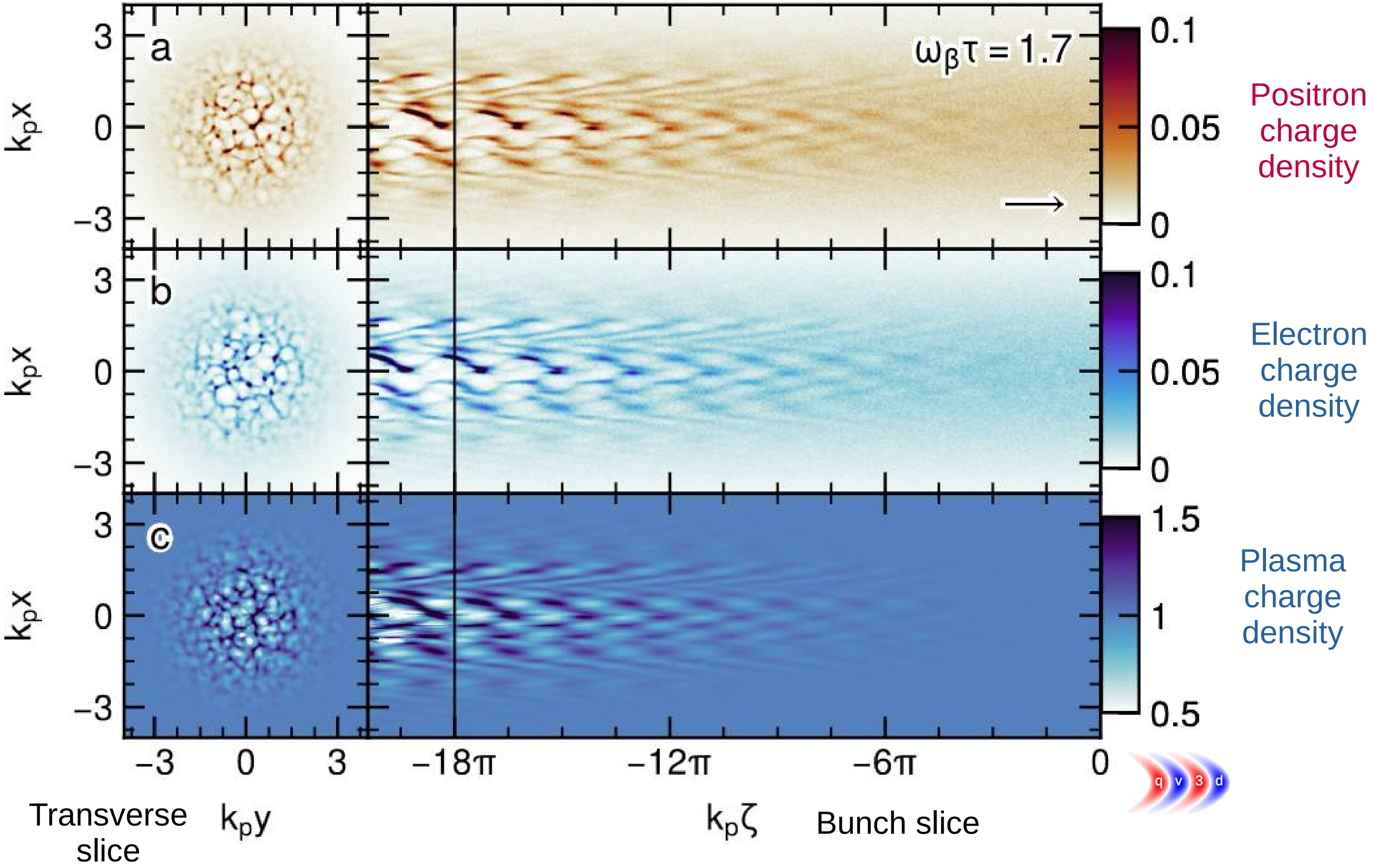
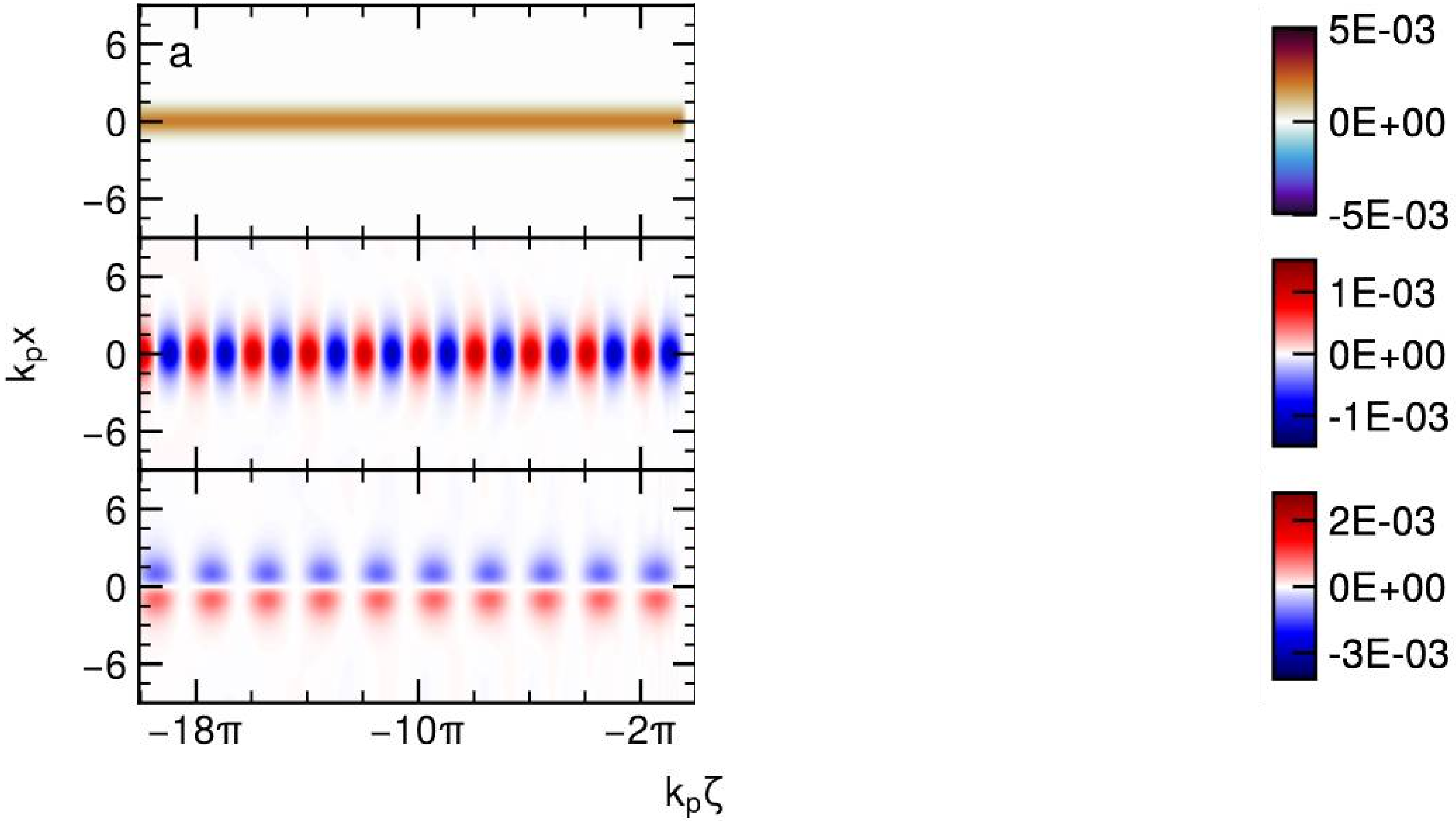
Is filamentation driven by the electrostatic plasma respose?



Wakefield-driven Two-Stream: Seed Fields

SMI: Driven by beam shape

- Gaussian bunch drives focussing and defocussing wakefield



Bunch charge density

Parallel wakefield

Perp. wakefield

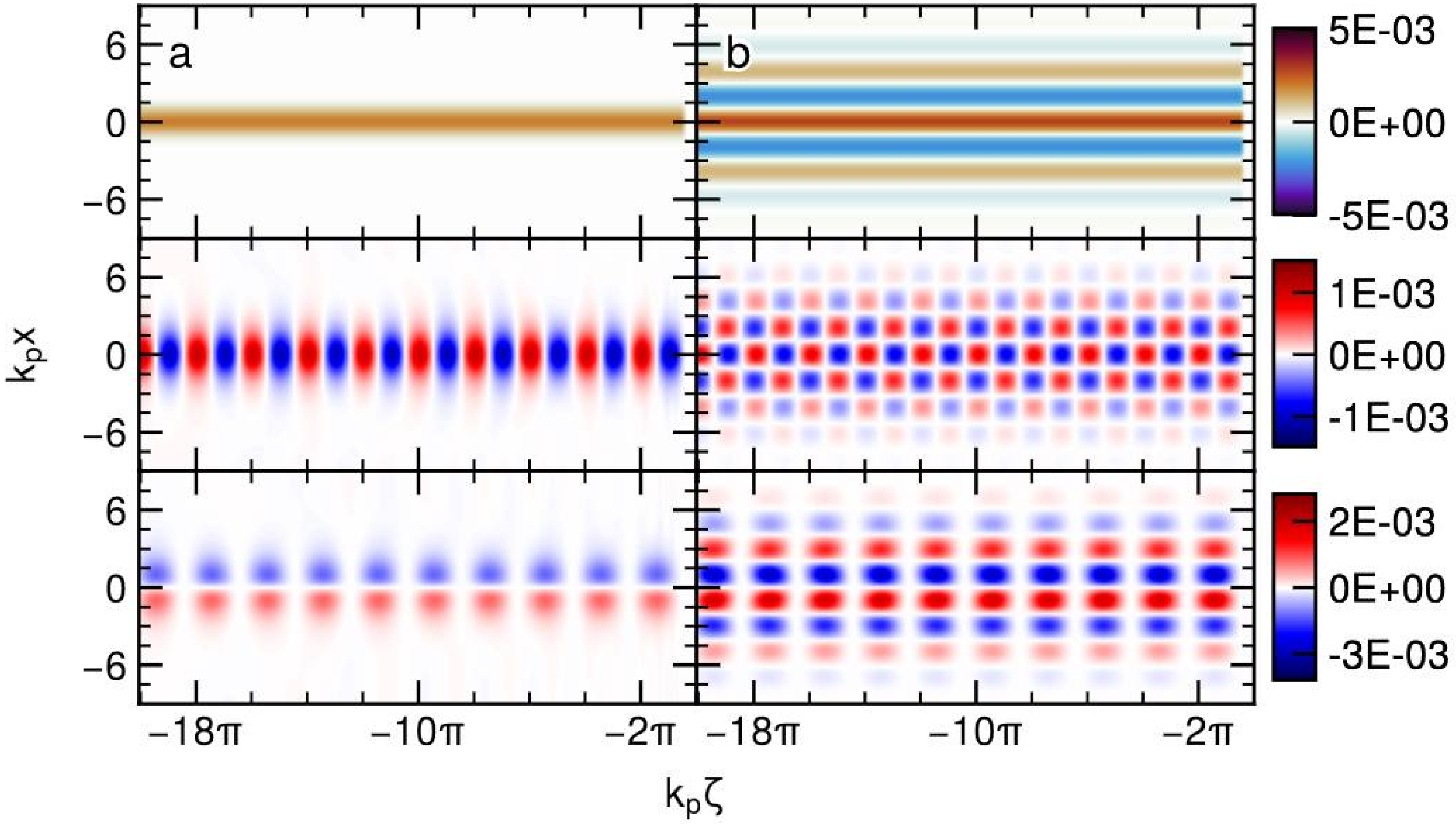
Wakefield-driven Two-Stream: Seed Fields

SMI: Driven by beam shape

- Gaussian bunch drives focussing and defocussing wakefield

Higher-order: Driven by transverse perturbation

- Wakefield alternating focusses and defocusses filaments



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Parallel wakefield

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Wakefield-driven Two-Stream: A Brief Theory

Governing Equations for Wakefields

- Plasma return current neglected

$$
\begin{aligned}
\left(\nabla^2 - \partial_t^2/c^2 - k_p^2\right)\boldsymbol{E} &= \mu_0 \partial_t \boldsymbol{j}_b + \nabla\rho/\epsilon_0 \\
\left(\nabla^2 - \partial_t^2/c^2 - k_p^2\right)\boldsymbol{B} &= -\mu_0 \nabla \times \boldsymbol{j}_b.
\end{aligned}
$$

Initial bunch perturbation

$$\delta n_b = \delta\hat{n}_b f(\zeta) g(x, y) \cos(k_x x)\cos(k_y y)$$

Wakefield-driven Two-Stream: A Brief Theory

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Initial bunch perturbation

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Seed Fields

$$E_z = \frac{\hat{\rho}_b}{\epsilon_0}\frac{k_e}{k_e^2+k_r^2} g(x,y)\cos k_x x \cos k_y y \int_{-\infty}^{\zeta} \mathrm{d}\zeta' f(\zeta') k_e^2 \cos k_e(\zeta-\zeta')$$

$$E_{[x,y]} = \frac{\hat{\rho}_b}{\epsilon_0}\frac{k_{[x,y]}}{k_e^2+k_r^2} g(x,y)\sin k_{[x,y]}[x,y]\cos k_{[y,x]}[y,x]\left[\int_{-\infty}^{\zeta} \mathrm{d}\zeta' f(\zeta') k_e \sin k_e(\zeta-\zeta') + f(\zeta)\right]$$

$$u_b B_{[y,x]} = \pm\beta_b^2 \frac{\hat{\rho}_b}{\epsilon_0}\frac{k_{[x,y]}}{k_p^2+k_r^2} f(\zeta) g(x,y)\sin k_{[x,y]}[x,y]\cos k_{[y,x]}[y,x]$$

$$\begin{aligned} k_e &= k_p/\beta_b \\ k_r &= \sqrt{k_x^2+k_y^2} \end{aligned}$$

Wakefield-driven Two-Stream: Relativistic Factor

Fluid theory

$$\left(\partial_\tau^2 + \frac{\varsigma_r^2}{2m_b\gamma_b^2}\nabla^2\right)\delta n_b = 2\frac{e}{q}\omega_\beta^2\left(\frac{\partial_z E_z}{\gamma_b^2} + \partial_x W_x + \partial_y W_y\right)$$

Wakefield-driven Two-Stream: Relativistic Factor

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Diffusion

TSI: Dominant in the non-relativistic regime

TTS: Dominant in the relativistic limit (γb»1) and $k_z \rightarrow k_p$

Wakefield-driven Two-Stream: Relativistic Factor

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Diffusion

TSI: Dominant in the non-relativistic regime

TTS: Dominant in the relativistic limit ($\gamma b \gg 1$) and $k_z \rightarrow k_p$

Oblique instability: Superposition of TSI and TTS

- TTS is **not** a superposition of SMI and CFI
- SMI is the 0th mode of TTS

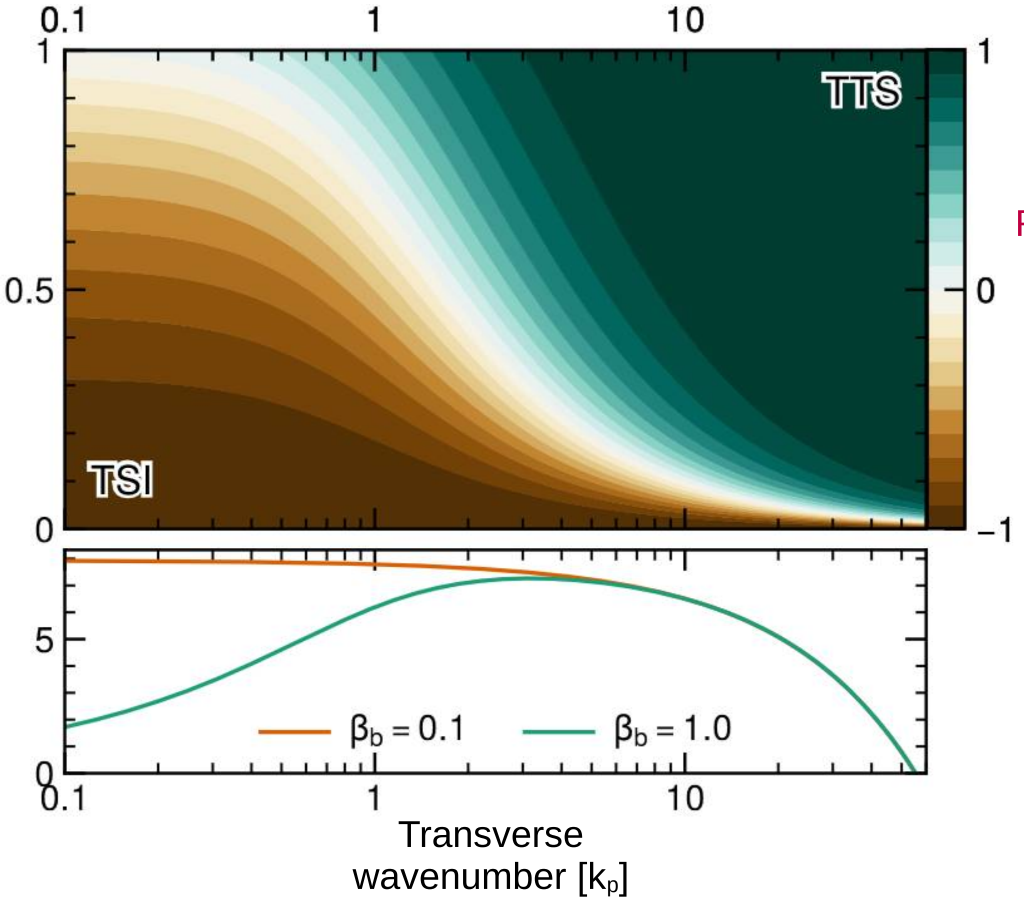
Wakefield-driven Two-Stream: Relativistic Factor

Fluid theory

$$\left(\partial_\tau^2 + \frac{\varsigma_r^2}{2 m_b \gamma_b^2}\nabla^2\right)\delta n_b = 2\frac{e}{q}\omega_\beta^2\left(\frac{\partial_z E_z}{\gamma_b^2} + \partial_x W_x + \partial_y W_y\right)$$

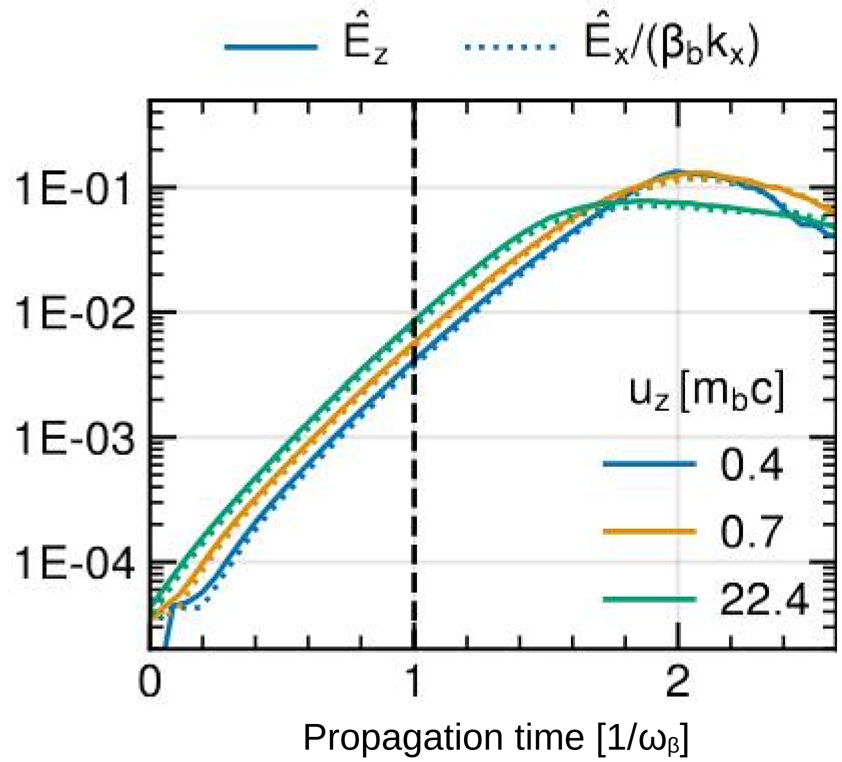
Bunch beta

Two-stream growth



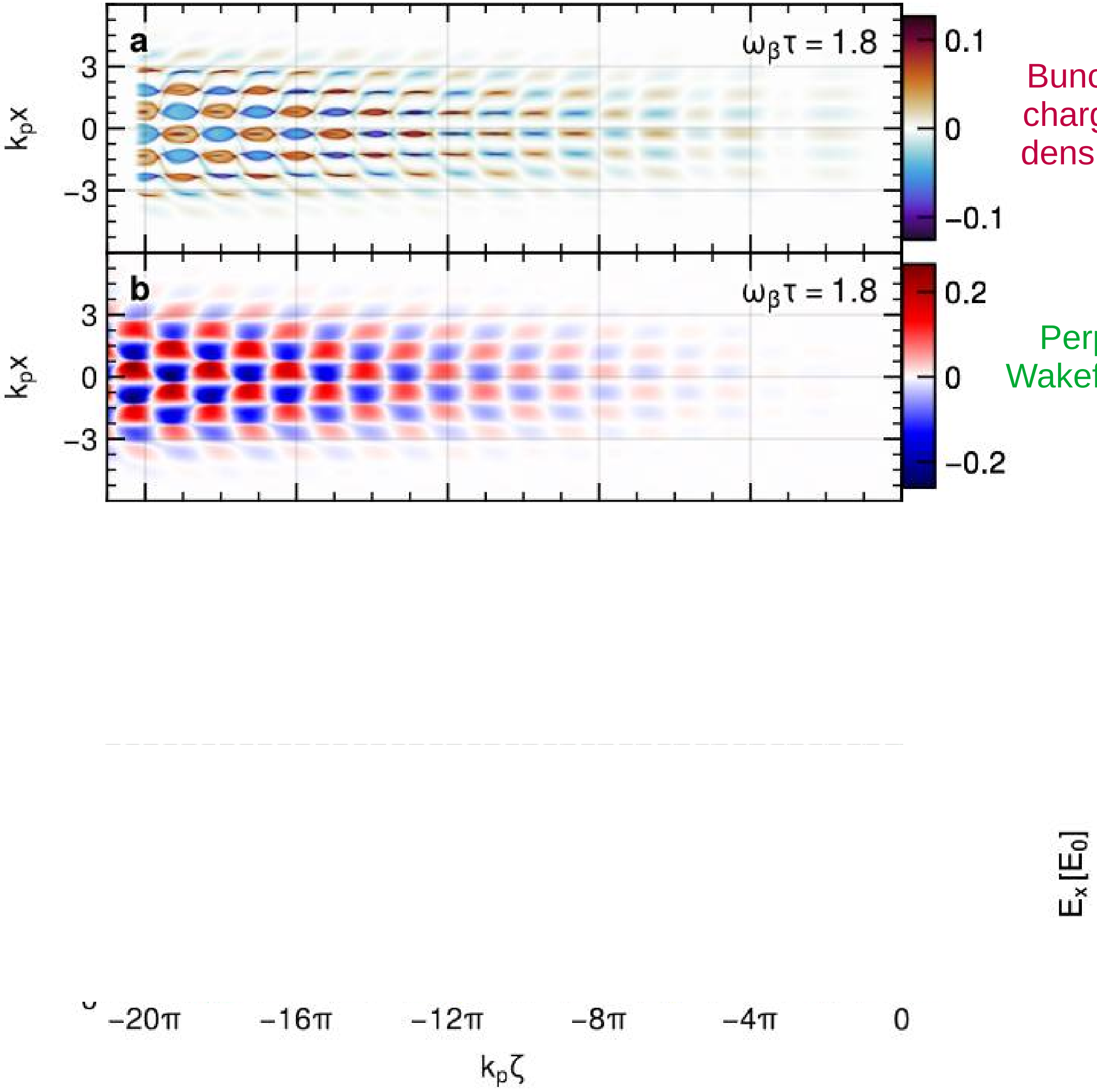
Relative Effect of Wakefield components

Electric field envelope [E_0]



Wakefield-driven Two-Stream: Growth

- Bunch filaments self-modulate due to transverse wakefield
- Saturation due to dephasing



Bunch charge density

Perp. Wakefield

Wakefield-driven Two-Stream: Growth

- Bunch filaments self-modulate due to transverse wakefield
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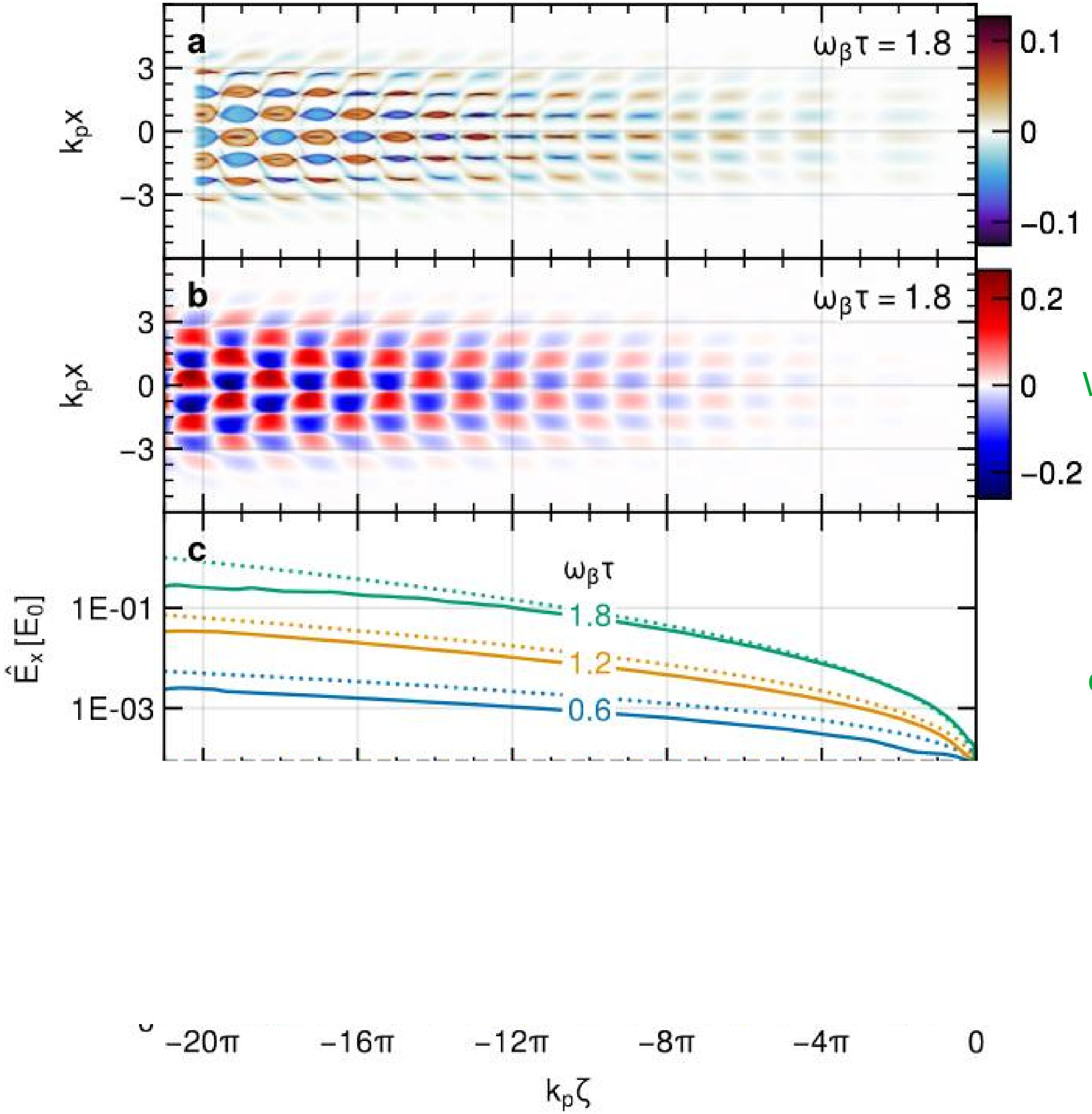
Exponential growth

$$\Gamma_{\mathrm{TS}} = \frac{3^{3/2}}{4^{2/3}}\left[\nu_\beta f(\zeta_0) g(x,y) k_e(\zeta-\zeta_0)\omega_\beta^2\tau^2\right]^{1/3}$$

$$\nu_\beta = \frac{(1-\beta_b^2)k_p^2+\beta_b^2 k_r^2}{k_p^2+\beta_b^2 k_r^2}$$



Bunch charge density

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Electric fields' envelope

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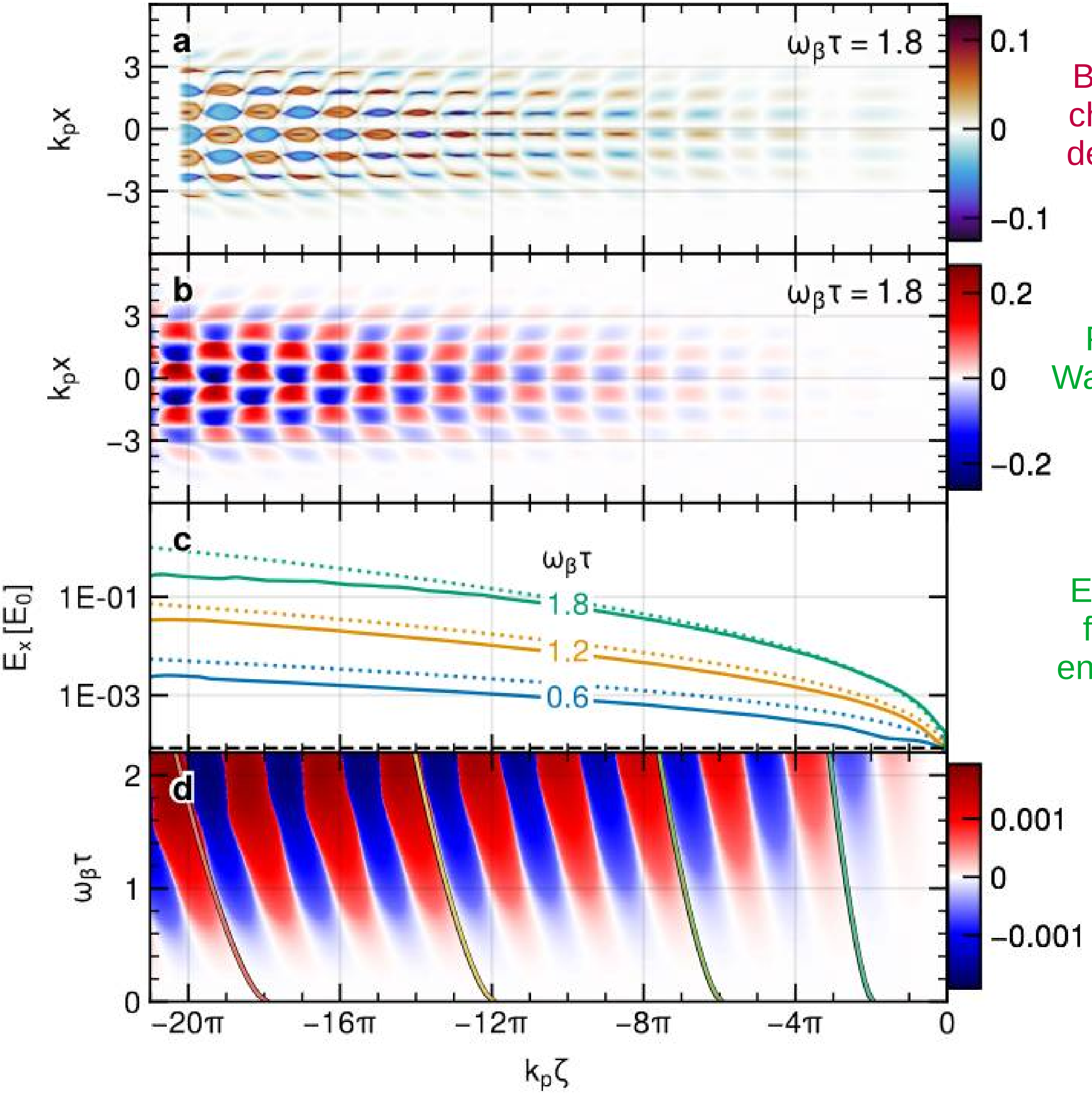
Reduced phase velocity

$$\beta_\psi = \beta_b - \frac{1}{2^{5/3}} \left(\nu_\beta f(\zeta_0) g(x,y) \frac{\omega_\beta^2}{\omega_p^2} \frac{\zeta}{c\tau}\right)^{1/3}$$



Bunch charge density

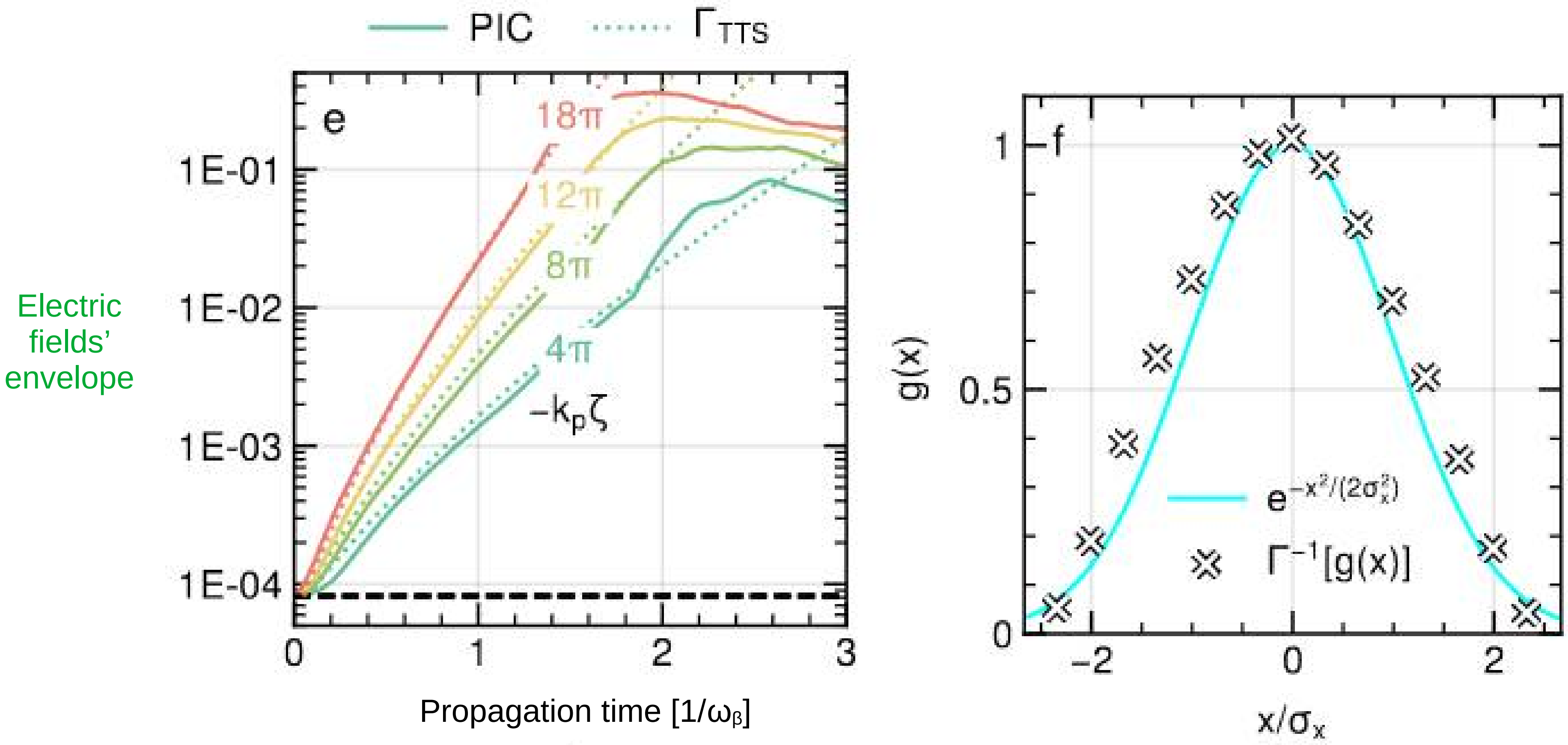
Perp. Wakefield

Electric fields' envelope

Wakefield Phase

Wakefield-driven Two-Stream: Growth

- Simulations and theory in excellent agreement:
  Bunch dominantly driven by electric field
- Two-stream growth proportionally scales with transverse bunch profile



Filamentation of Warm Beams

- Dilute (n_b=0.02 n_p), quasineutral e^+e^- bunch with finite emittance
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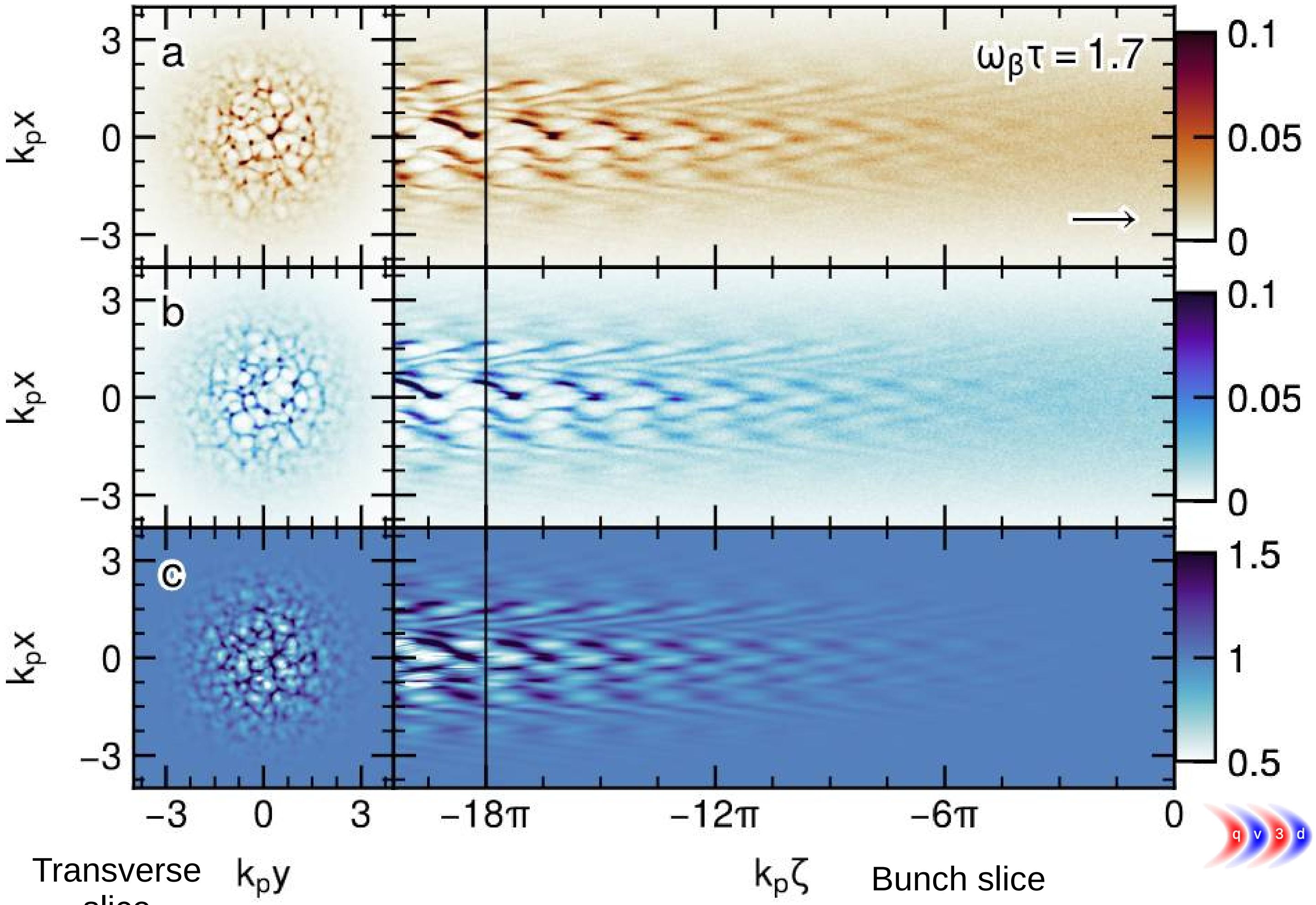
What is the effect of emittance?



Positron charge density

Electron charge density

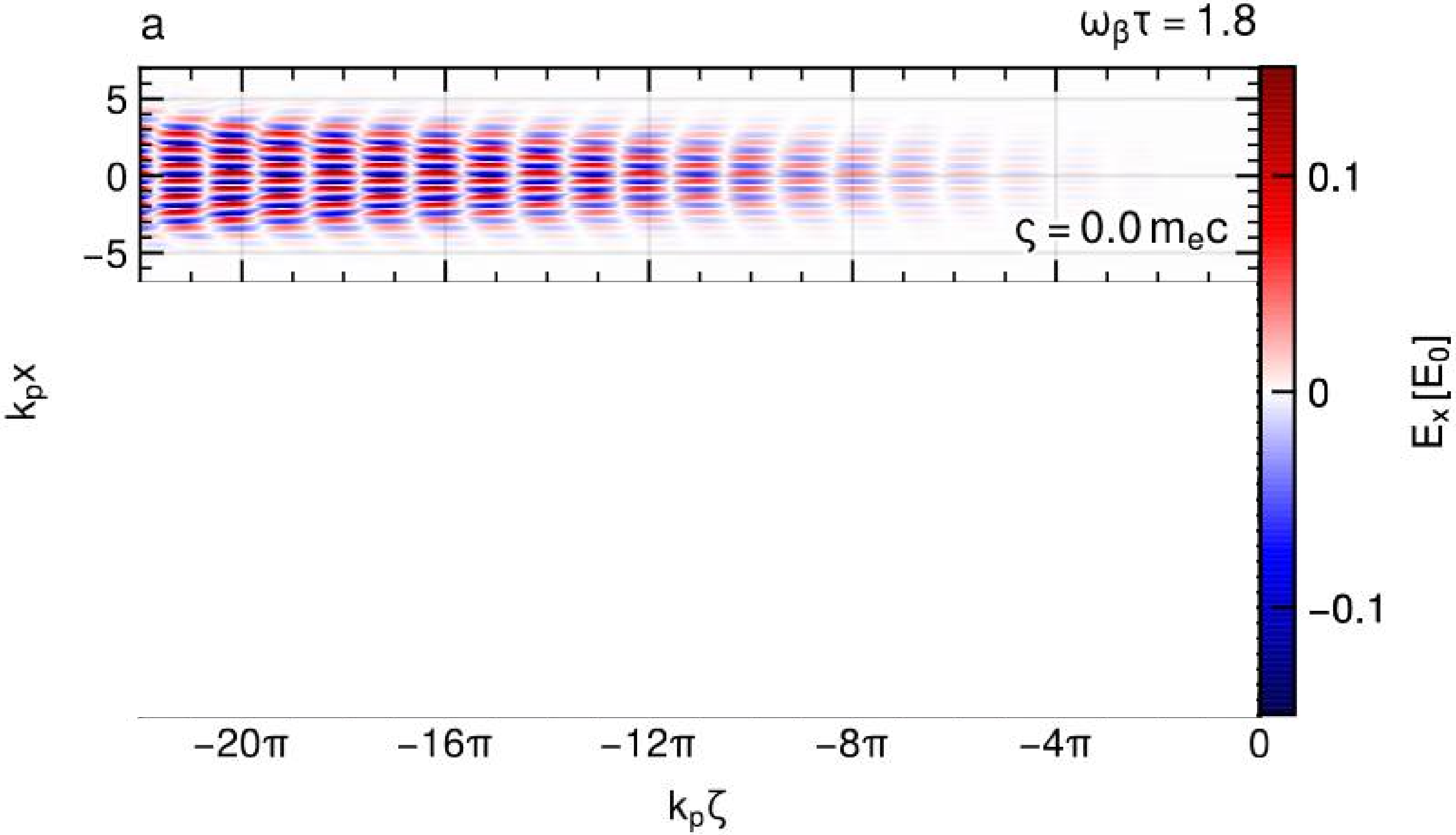
Plasma charge density

Transverse slice

Bunch slice

Emittance-Driven Diffusion

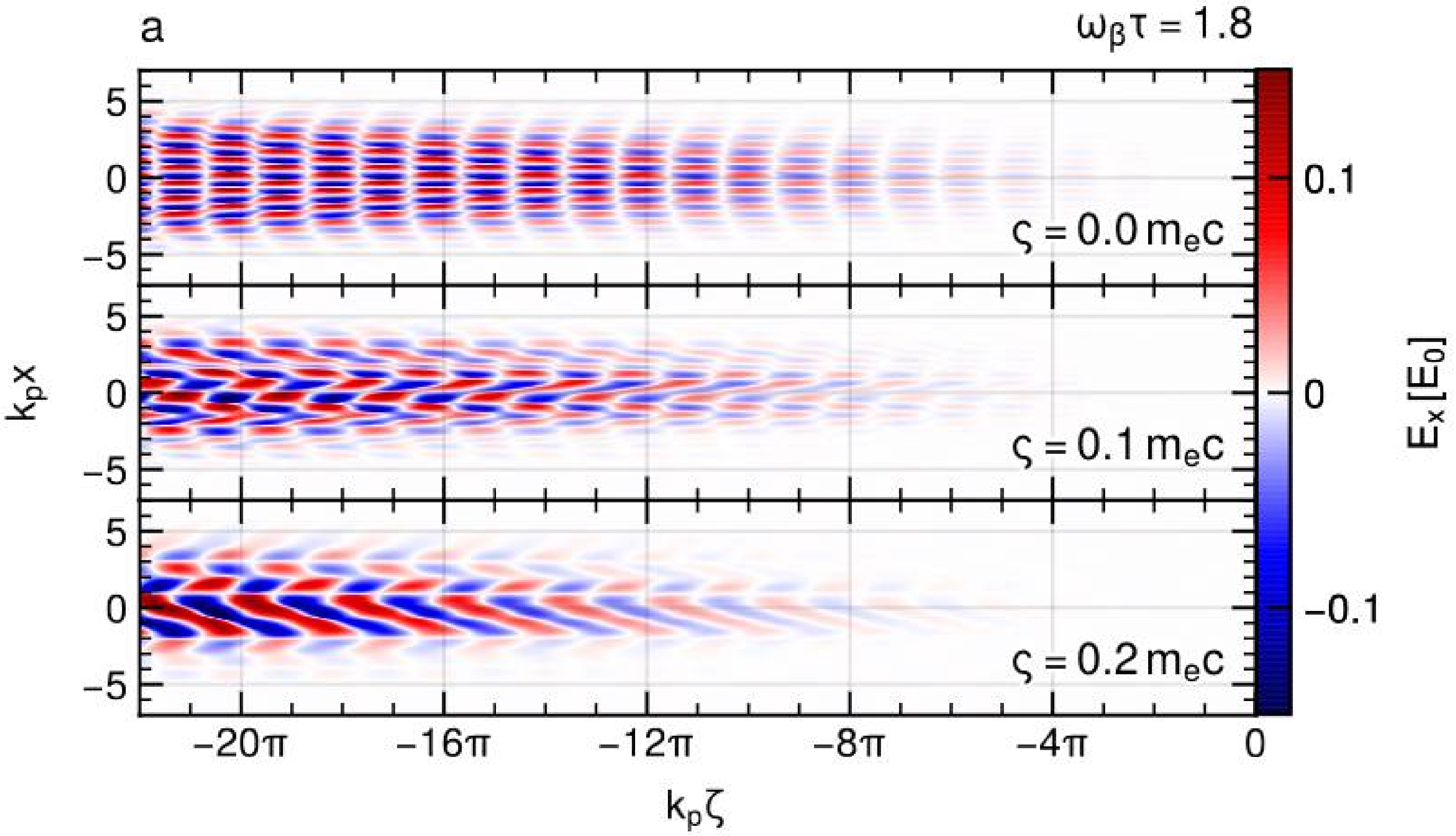
Cold bunch: Seeded wavenumber dominates



Emittance-Driven Diffusion

Cold bunch: Seeded wavenumber dominates

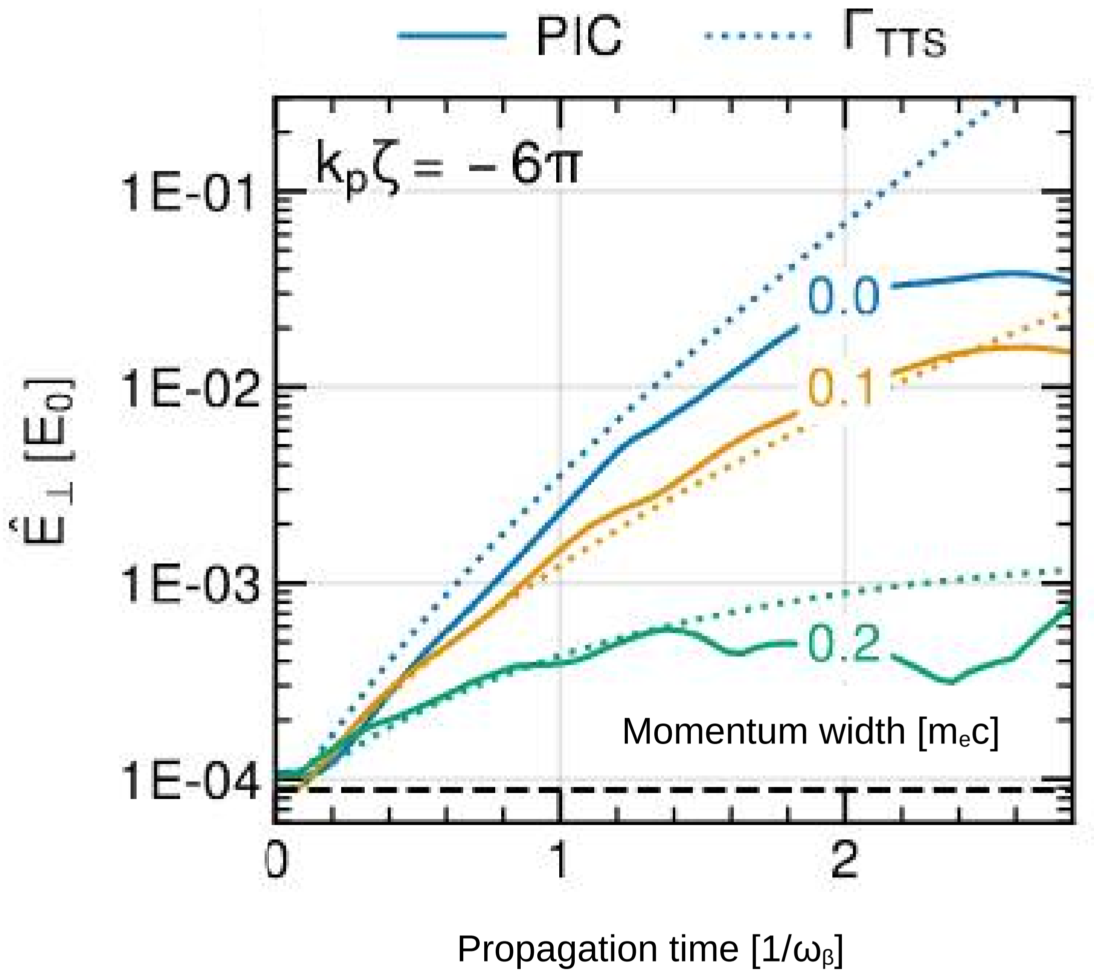
Finite emittance: Small scale phasemixing damps high modes and lower modes, seeded by noise, dominate



Emittance-Driven Diffusion

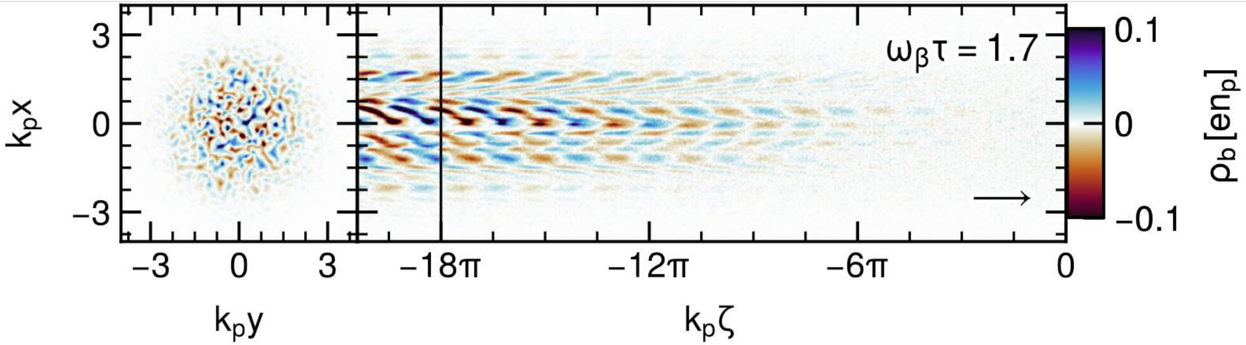
Seed fields result in a purely temporal damping, whose effect is stronger towards the bunch front

Conflict: Damping may be spatiotemporal?



Emittance-Driven Diffusion

Fallback: Self-modulated filaments at different phases.



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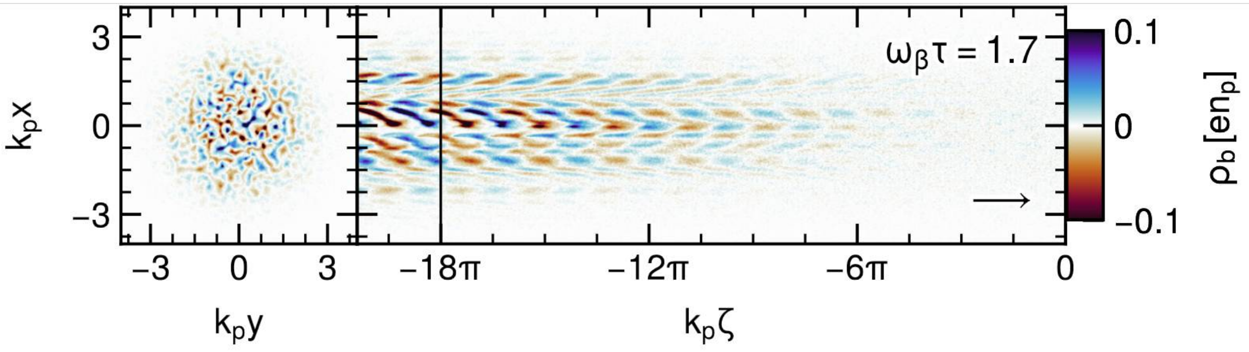
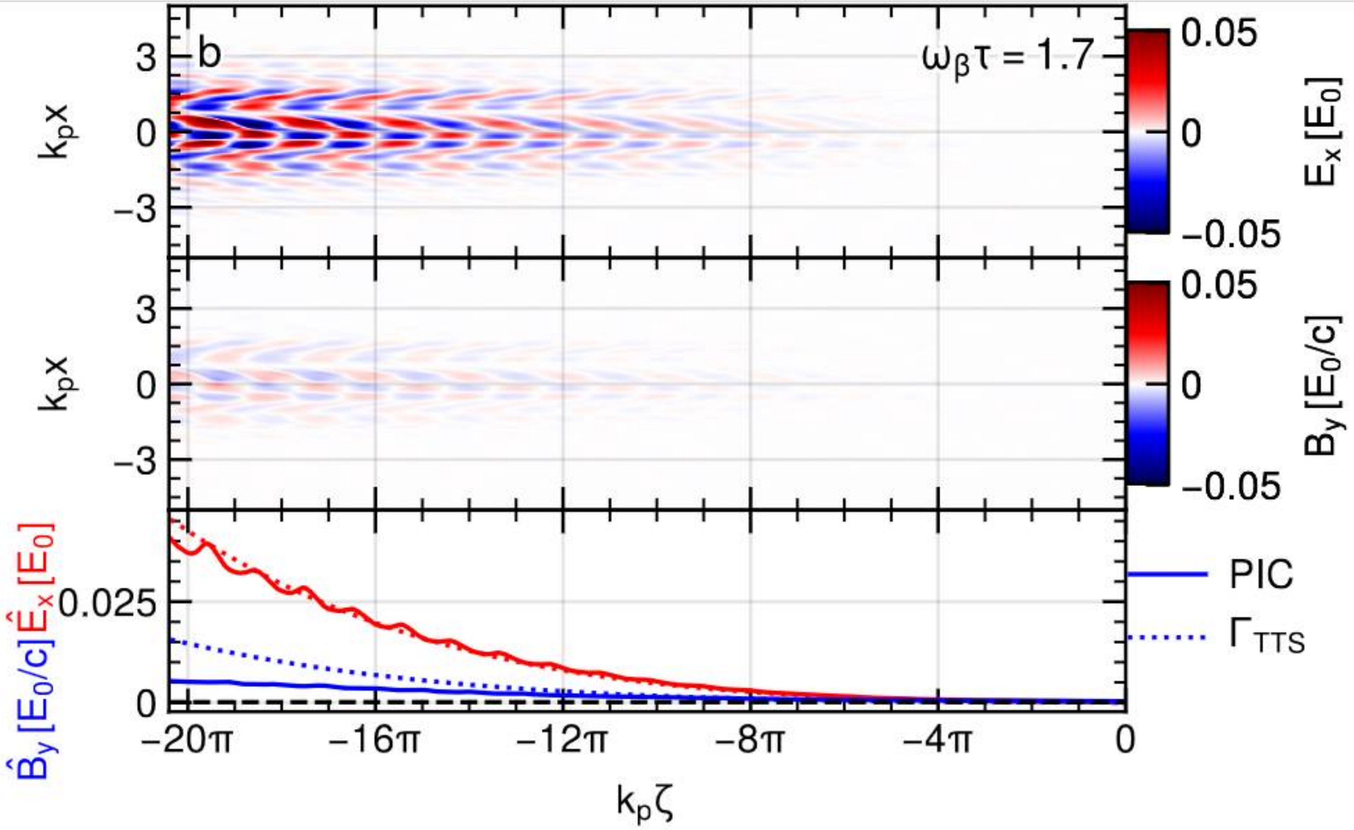
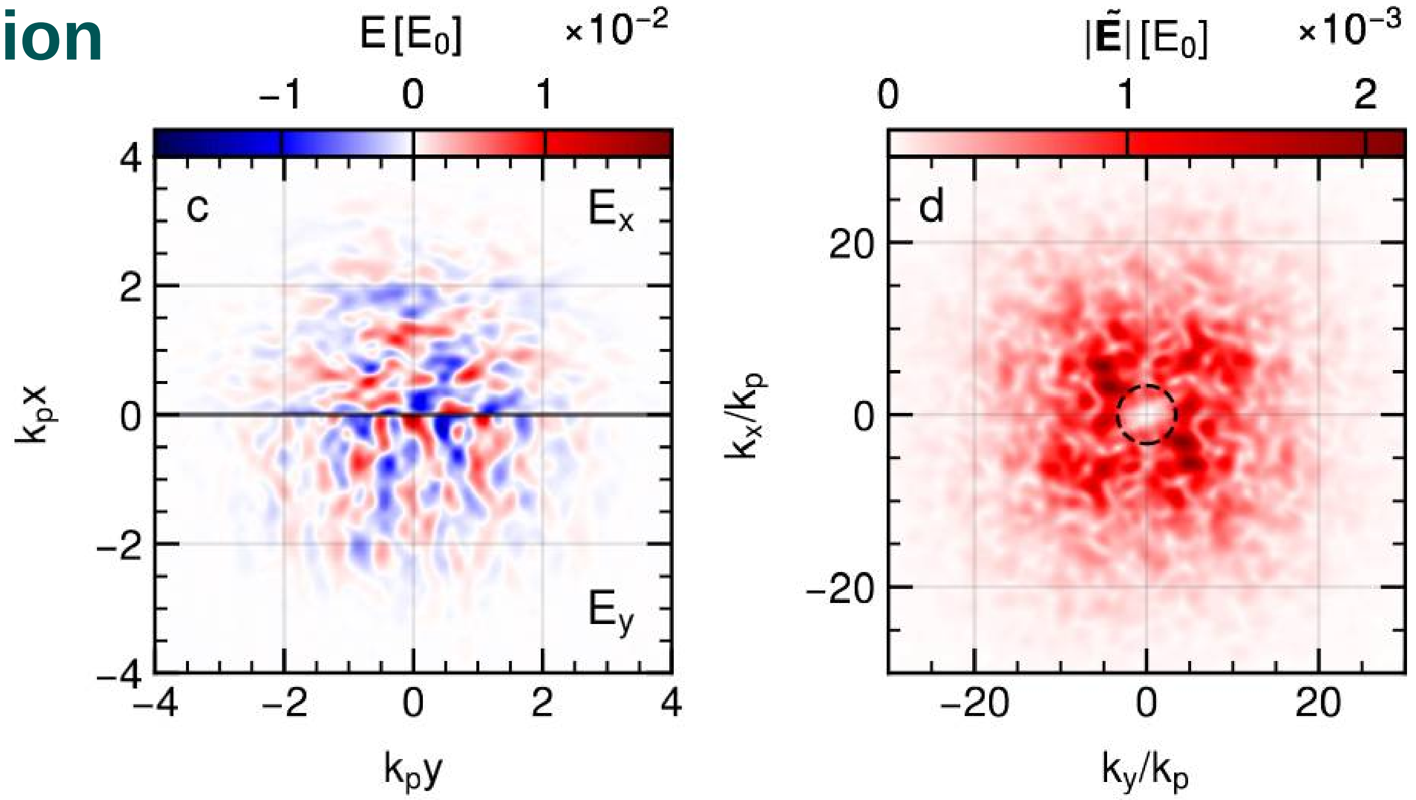
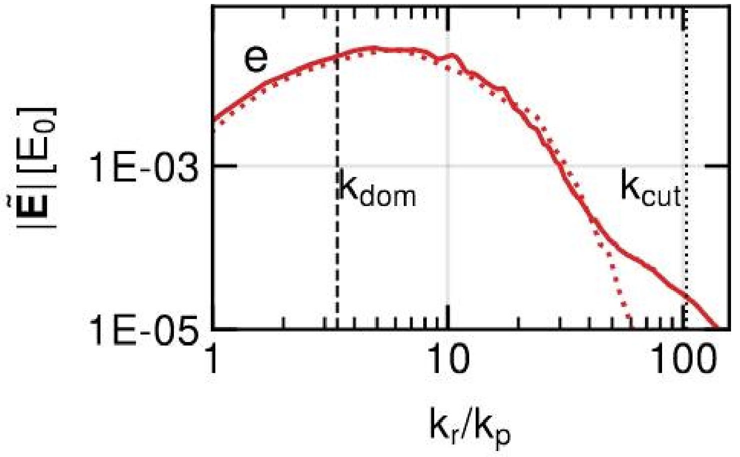


- Theory enables to accuratelly describe the growth of the electric field
- Electric field (plasma response) dominates by order of magnitude as it grows along propagation and along the bunch
- Magnetic field neglible, since it only originates from the local bunch slice



Emittance-Driven Diffusion





- Electric field spectrum spans a ring in (k_x, k_y) space

$$k_r = \sqrt{k_x^2 + k_y^2}$$

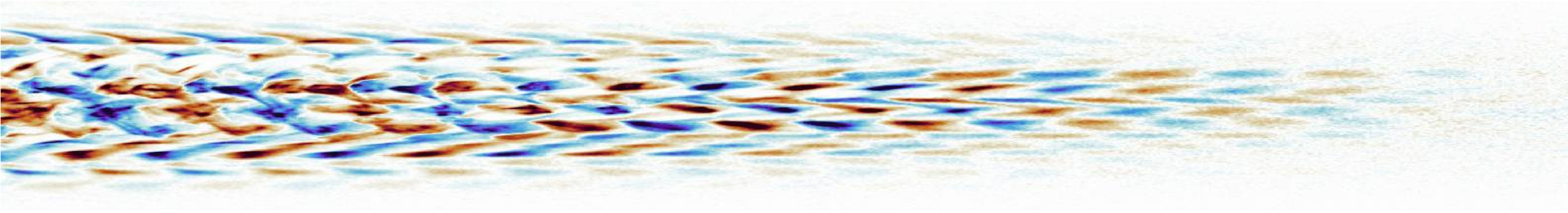
- Spectral distribution agrees well between theory and unseeded simulation, **including** the spectral seed distribution

Summary

Simulations and linear theory confirm in excellent agreement that **higher-order transverse two-stream** result in self-modulated filaments for low-density bunches.

Transverse two-stream is the relativistic limit of the oblique instability („oblique two-stream“).

Emittance-related diffusion damps high modes and results in a finite distance between filaments. Transverse wavevectors are coupled in 3D, which spans a ring of dominant transverse modes.



Thank you for your attention

For questions please contact me: erwin.walter@ipp.mpg.de

Literature

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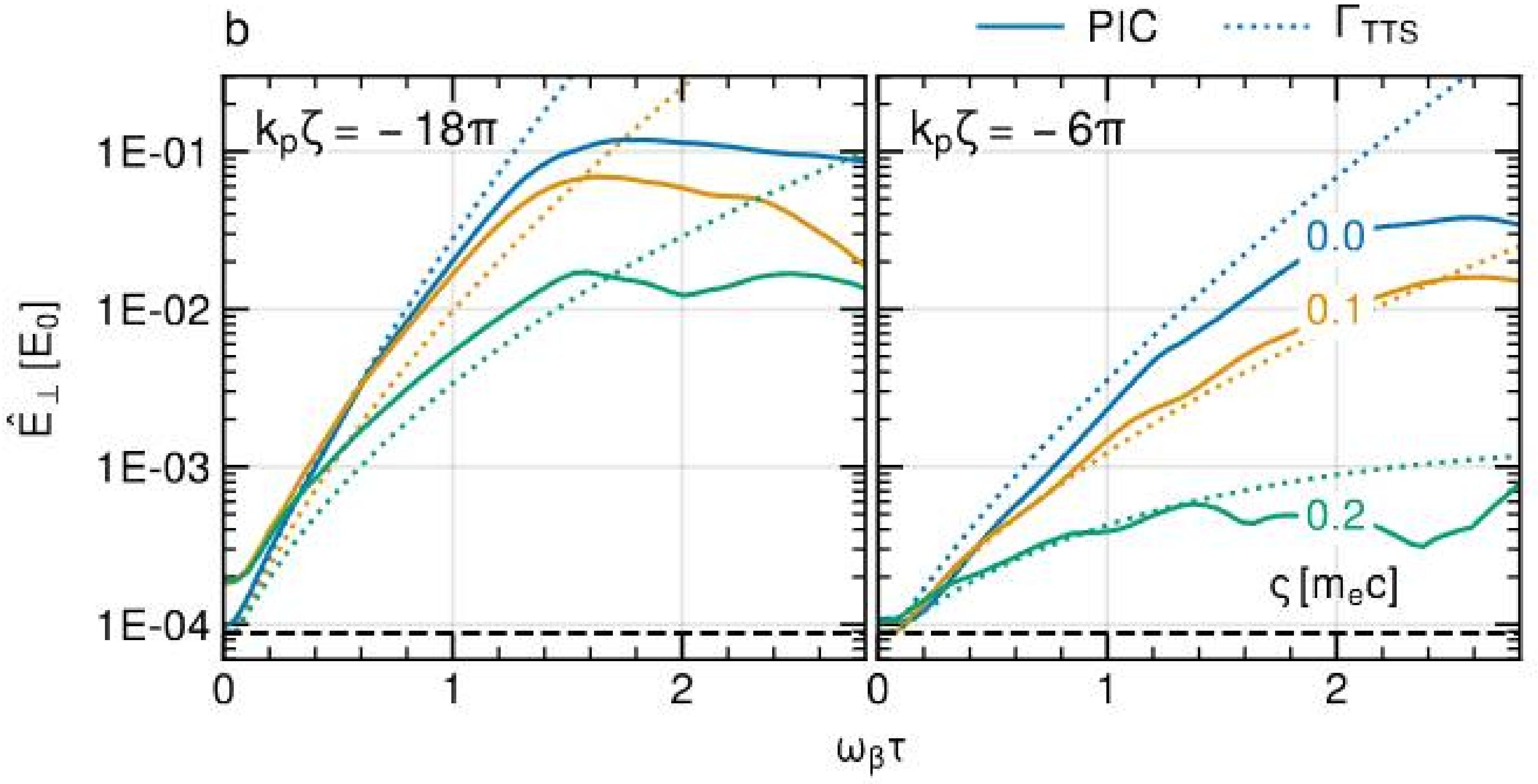

Emittance-Driven Diffusion