Review

International Journal of Modern Physics A
Vol. 31, No. 16 (2016) 1630022 (12 pages)

DOI: 10.1142/S0217751X16300222

Imagining the future, or how the Standard Model may survive the attacks*

Gerard 't Hooft

Institute for Theoretical Physics, EMMEΦ,
Centre for Extreme Matter and Emergent Phenomena, Science Faculty,
Utrecht University, P.O. Box 80.195, 3808TD, Utrecht, the Netherlands
g.thooft@uu.nl
http://www.staff.science.uu.nl/~hooft101/

Published 9 June 2016

After the last missing piece, the Higgs particle, has probably been identified, the Standard Model of the subatomic particles appears to be a quite robust structure, that can survive on its own for a long time to come. Most researchers expect considerable modifications and improvements to come in the near future, but it could also be that the Model will stay essentially as it is. This, however, would also require a change in our thinking, and the question remains whether and how it can be reconciled with our desire for our theories to be "natural".

Keywords: Standard Model; Higgs particle; Lagrangian; gravitation; naturalness; scale invariance.

1. Memories of Abdus Salam

I first met Abdus Salam in the 1971 Amsterdam Conference on Elementary Particle Physics, where I was given the opportunity to briefly present my graduation work on the renormalization of Yang–Mills theories. When I met him again in London, 1974, he invited me in his office, and besides his personality, I was impressed by the fact that he had three telephones on his desk; he was continuously on one of these phones, but the three cords were in one gigantic knot, so I was wondering whether he actually knew which phone he was using. "You see, my country is at war", he explained to me. At that moment, he was top level science advisor to the Government of Pakistan.

Salam was one of those deep thinkers of theoretical physics. In 1959 he had the idea of two-component neutrinos,[1] which would later play an important role in his

*Based on talk given at the Memorial Meeting for Nobel Laureate Prof. Abdus Salam's 90th Birthday, Singapore, 25–28 January 2016.

ideas with John C. Ward on unified electroweak theories. In their earlier work[2,3] in 1964, he and Ward realized, intuitively, how important gauge theories had to be for the renormalization of these theories, even if they did not yet have the group structure quite right.

Later, with J. Pati, he had imaginative ideas about Grand Unification, regarding lepton number as the 4th color, still today believed to be quite likely correct. He also had wilder ideas that were bound to fail, such as what he called the "superpropagator" for gravity, an idea that was rooted in work of Lehmann and Pohlmeyer in 1971.

Salam also expressed his suspicion that the Nobel Committee must have been influenced by his relentless efforts to bring top science to the developing countries; in 1964, Salam had founded the International Centre for Theoretical Physics (ICTP), Trieste, in the North-East of Italy. He served as its director until 1993.

2. The Standard Model

The history of the numerous developments, consisting of experimental and theoretical searches, successes and failures, and some occasional triumphs, has been covered at many occasions.[4] There is no universal agreement as to how to define the Standard Model. For some, it is just the Lagrangian consisting of the three Yang–Mills terms for the Lie groups $SU(3)$, $SU(2)$, and $U(1)$, and the kinetic terms for the quark and lepton representations, in the form of a triple repetition of the representations $(3)\otimes(2)_{\text{left}}\oplus(3)\otimes((1)+(1))_{\text{right}}$ for the spin-1/2 quarks, the representations $(2)_{\text{left}}\oplus((1)+(1))_{\text{right}}$ for the leptons, plus the antiparticles, and the Higgs scalar as a (2) representation, each with their specific $U(1)$ charges.

It is true that it is not quite understood where the numerous Yukawa coupling parameters come from, but most theoreticians would also include these Yukawa couplings to be part of the Standard Model. We do not know any sound theoretical principle that explains the origin and the values of all associated coupling parameters, and therefore we include all these numbers into what we consider the Standard Model to be, resting with the fact that the actual values of neither the gauge couplings, nor the Higgs and the Yukawa couplings are predicted.

The model owes its success to the fact that in total, there are only about 25 such numbers, most of them fairly well-known, while there are thousands of interesting experimental observations and measurements that are explained by them.

It is by now quite well understood how to do detailed calculations, where some effects are known with extreme precision, while others, notably the masses and interaction properties of the hadrons, are more difficult to pin down with margins of error less than a percent or so.

The construction of the Standard Model was finalized with the discovery of a particle that in all respects appeared to be identifiable as the Standard Model Higgs particle, in the ATLAS[5] and CMS[6] experiments at CERN's Large Hadron Collider, July 2012. In all respects this indicates that the world of the subatomic particles, up to the TeV domain, is accurately described by a Lagrangian, which, in a telegraphic notation, takes the form

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \bar{\psi}(\gamma D + \mu\phi)\psi - D\phi^\dagger D\phi - V(\phi)\,, \tag{1}$$

where each of the terms stands for a relatively simple algebraic expression in terms of the quantized fields, which are the dynamical variables of the system: the first term is the Yang–Mills action for the vector fields, leading to a straightforward generalization of the Maxwell equations for the spin-1 particles. The second term describes the fermionic fields, each having spin-1/2, and finally we have the kinetic part and the self-interaction part of the necessary scalar field, the Higgs sector of the theory, yielding at least one spin-0 particle.

When written in full detail, all interaction parameters mentioned above enter, determining the strengths of the various interactions, and the masses of the numerous elementary particles it describes. Each of these parameters have to be determined by experimental observations and measurements; none of them can be calculated from first principles.

At first sight, these equations may seem to be complicated, but we have to remember that there are hundreds of particle types, and having a theory that does not require more than a handful of interaction parameters to match precisely with all these observations, is quite a feat.

Thus, what took us by surprise, is the relative simplicity of this system. We had expected much more to come; maybe there still is much more, but in that case, for some reason, all conceivable additions seem to be hiding in energy domains that have been difficult to attain as of today.

The model we have, our beloved Standard Model, cannot be all there is. There are at least three reasons to expect that there must be more. The first reason to think that there is more, comes from various arguments originating from astronomy. The most important one is the observation of mismatches in the gravitational interactions between stars and galaxies. A substance called 'dark matter' produces strong additions to the gravitational fields of stars and galaxies, while the particles that this substance must be made of, cannot be found within our Standard Model.[a]

A related question concerns the role played by 'dark energy', an effect that leads to unexpected gravitational interactions at cosmic scales. It is not so easy to link this effect to elementary particles in the cosmos, because of the very strong negative pressure that is associated to dark energy, which is difficult to ascribe to particles. Dark energy is more likely to be regarded as a new feature in Einstein's gravity equations. Its effect in Einstein's theory is well understood, going back to Einstein himself. Dark energy is then assumed to be the consequence of an extra term, the cosmological constant, but its raison d'être is somewhat mysterious: what can the

[a] Alternatively, there are various theories describing modifications of Einstein's gravity theory to account for the observed anomalies. These would represent more fundamental, and more enigmatic, departures from the standard picture, which are strongly being debated.

origin be of an extra force in the Standard Model that is more than 120 orders of magnitude smaller than the natural units one would have to use to express it in?

Actually, an explanation of the cosmological constant will probably have to wait until we have a better understanding of the quantum features of gravity itself, and this brings us to the second shortcoming of the Standard Model: the gravitational force will need to be subject to the rules of quantum mechanics before we can add it to this model. In spite of five decades of intense study, we still do not understand how to do this properly.

An other aspect of imperfection is sometimes added to this: the hierarchy problem, which according to some researchers can be fixed by using supersymmetry. We shall discuss below what the hierarchy problem entails, and it will be apparent that relatively simple notions such as supersymmetry will not be powerful enough to cure that purported shortcoming.

From a mathematical point of view, the hierarchy problem alone would not invalidate the Standard Model, but we do have a more profound way to formulate a third reason for suspecting that the Standard Model cannot be the ultimate theory, which is often totally ignored in the reviews. In spite of its internal robustness and apparent beauty, the *mathematics* of the system is not completely clean.[7] The equations can be handled in a completely unambiguous way only if we perform perturbation expansions with respect to the various small coupling parameters of the theory. This is now done routinely, but, regardless of how many terms we include in our calculations, the results will always be approximations, and it is basically impossible to prove that any desired accuracy can be reached this way. To the contrary, it is believed that many expressions will continue to contain small margins of error that we cannot improve on; the theory simply is not suitable for more accuracy. We, theoreticians, can never be content with such an incomplete theory, even if, in many instances, the fundamental margins of theoretical error may be much tinier than the uncertainties in the experimental measurements.

3. A Scenario

Thus, we do expect modifications, fundamental departures from the Standard Model, to arrive sooner or later. What will the first discoveries be? Will there be

– super symmetry?
– extra dimensions?
– new, smaller, building blocks ("technicolor")?
– *nothing?*

We think that there is one clue, perhaps, provided by the latest result from the Large Hadron Collider:[5,6] the mass, 125 GeV, found for the Higgs particle, is a very special value. The Higgs mass has been the one unknown parameter in the Standard Model, but, since the vacuum value of the Higgs field is precisely known from the W and Z mass and Fermi's interaction constant for the weak force, having the Higgs mass now also yields the Higgs self-interaction parameter, λ_H. The value of λ_H, following from the given value of the Higgs mass, appears to be very special. One can compute how λ_H changes as we look at different energy scales. It is a running coupling parameter. If $M_H \approx 125$ GeV, the Higgs self-coupling parameter runs almost to zero at very high energies.[8]

This means that the renormalization group β function rapidly runs to zero at high energies, a property it will have in common with the other β functions that describe how the gauge field forces run to zero. Apparently, our field theory of the subatomic particles, at sufficiently high energies, becomes *scale-invariant*.

Why is this? What do we have to infer from that?

If there is scale invariance at higher energies, the masses of heavy particles would be at odds with this, and this could explain why we have not seen them, and also the massive superpartners of the SM particles, expected by many investigators, would be at odds with this symmetry. Is this why we see no heavy particles at all? Then, what about the preferred explanation of 'dark matter'? It was always assumed to consist of WIMPs, 'Weakly Interacting Massive Particles'. WIMPs would also have to be forbidden. So, perhaps LHC gave us a hint, but the hint is difficult for us to understand.

It is more likely that the role played by scaling transformations will be a more subtle one,[9] and that it will not forbid the occurrence of heavier masses in the system, *if the particles associated with these heavy masses, interact sufficiently weakly*, as, indeed, is likely to be the case with the dark matter particles.

It is also the case with gravity.[10–14] In units where $\hbar = c = 1$, Newton's constant G_N has dimension length-squared, or inverse mass-squared. The associated length, called Planck length, is very small, and accordingly, the associated mass, the Planck mass, is very large:

$$\hbar = 1.0546 \times 10^{-34} \text{ kg m}^2 \text{ sec}^{-1}\,, \tag{2}$$

$$c = 2.99792458 \times 10^{8} \text{ m/sec}\,, \tag{3}$$

$$G_N = 6.672 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}\,, \tag{4}$$

which yields the Planck units:

$$L_{\rm Pl} = \sqrt{\frac{\hbar G_N}{c^3}} = 1.616 \times 10^{-33} \text{ cm}\,, \tag{5}$$

$$M_{\rm Pl} = \sqrt{\frac{\hbar c}{G_N}} = 21.8\ \mu\text{g}\,. \tag{6}$$

This length unit is very tiny, even at the scale of subatomic particles, and the mass is very large compared to subatomic particles. Nevertheless, one can elegantly restore exact scale invariance in perturbative quantum gravity. This, we do by observing that only the truly constant quantities in a theory, such as interaction constants, must be dimensionless if we want scale invariance. In contrast, the metric tensor $g_{\mu\nu}$, which determines the distance scales between neighboring points in space–time in terms of metres, is actually a dynamical variable. Like many other dynamical variables of a theory, it may have nontrivial dimensions even if the theory itself is scale invariant.

Writing

$$g_{\mu\nu}(\vec{x}, t) \equiv \omega^2(\vec{x}, t)\hat{g}_{\mu\nu}(\vec{x}, t)\,, \tag{7}$$

we can take the field $\omega(\vec{x}, t)$ to have dimension of a length — just like all other fields in the Standard Model — while all components of the tensor $\hat{g}_{\mu\nu}(\vec{x}, t)$ are kept dimensionless.

In perturbative quantum gravity, we now keep $\hat{g}_{\mu\nu}$ close to the identity matrix $\eta_{\mu\nu} \equiv \mathrm{diag}(-1, 1, 1, 1)$. In that case, however, we must postulate that ω stays close to one *in the geometrically flat vacuum*, while in general it may fluctuate. Or, we say:

$$\langle\emptyset|\omega(\vec{x}, t)|\emptyset\rangle = 1\,. \tag{8}$$

This means that scale invariance is a gauge symmetry that is spontaneously broken, a situation that one often encounters in quantum field systems.

Actually, in gravity, we also have general coordinate transformations, so we can also say that the theory has *local conformal invariance*, which is spontaneously broken. In that case, Eq. (8) expresses the fact that we have a BEH mechanism here. We can choose the gauge such that $\omega(\vec{x}, t) = 1$, or we can fix the gauge constraint in some other way. In either case, we have a new local gauge symmetry, and this is a very important observation shedding a different light on quantum gravity.

One can even argue that gravity herewith becomes a renormalizable theory,[15] but before arriving at such a conclusion, one would have to add kinetic terms for the $\hat{g}_{\mu\nu}$ field.[10–12] Such terms can be written down (the Weyl action), but that ruins positivity: the Weyl action adds a negative metric massive spin-2 particle to the system, something that will be difficult to accept, as this is believed to make our theory internally inconsistent.

The Weyl action, however, seems to be such a fundamental interaction, that some of us suspect it can be used anyway, inviting us to think again about stability of theories and the exact role of indefinite metric particles[b] in a local conformally invariant theory.

4. Black Holes

Glancing at the history of science, one cannot help noticing how new ideas and insights were first achieved. More often than not, this happened when something in

[b]To older generation physicists, this suggestion may sound familiar; we had such suggestions when Niels Bohr puzzled about an apparent energy loss in weak interactions, and again when weak interaction theories were proposed where renormalizability seemed to require contributions from negative metric ghost fields. In all these cases, the more conventional no-go theorems turned out to be right, and all more exotic suggestions were admitted to be dead alleys.

the existing theories did not seem to be quite right. While a majority of scientists were busy ignoring the problem, or dismissing it while hiding the fact that they did not quite understand what was going on, a few mavericks addressed the difficulty and searched for natural sounding solutions, until, unexpectedly, this lead to a breakthrough. This happened when J. C. Maxwell noted an inconsistency in the equations governing electric and magnetic fields, and it happened again when Max Planck pondered about the laws of radiation that failed to obey the rules of thermodynamics.

Do we have such puzzles in today's science? Are there situations where, applying the known laws of physics, appears to give blatantly wrong results?

The answer is yes, and again, as if history is repeating itself, too many investigators try to hide the fact that they do not really understand what is going on. The laws of nature that should apply are known, and yet the answers make no sense: the quantum properties of black holes.

The major discovery by S. W. Hawking[16,17] was that one can apply standard quantum field theory in the coordinate frame of a black hole to see how its quantum variables evolve as seen by a distant observer. His result was that the distant observer will detect particles emerging from the black hole, while a local observer cannot see anything but a vacuum. What seemed to be only mildly puzzling at first was that Hawking saw his particles coming out in a mixed quantum state. Physically, this seemed to be a quite reasonable result that must be right. Black holes must have a finite temperature, as had been suggested earlier by Bekenstein,[18] so, apparently, they glow like a light bulb. Furthermore, we must assume that the radiation energy must be provided by the black hole itself, so it looses energy, and with that, also mass.

While the mass decreases, the total power of the radiation increases, and the whole process must come to a violent end when all mass has been used up. So far, this scenario does not look unreasonable at all.

However, during the last phases, this black hole must be smaller than an elementary particle; describing it as a mixed quantum state is fine, but what are the pure quantum states like? The fact that they are mixed just seems to indicate that we are applying statistics to describe its quantum modes. Why can't we describe its pure quantum states more succinctly?

Hawking originally thought that one cannot describe a black hole at all in terms of pure states, but he was overruled by string theorists, who claimed that black holes are well described by string theory,[19–21] even though, what they really described were only black holes in a particular extreme limiting case, not the generic black hole. Also, the answers given by string theory itself were poorly understood as well.

As is typical for modern science, researchers then turn on their fantasy, assuming far-fetched ideas and models. These ideas and models are not manifestly incorrect, but they are also not manifestly right, and therefore, they are probably wrong as well.

One problem was the question "what happens to the quantum information carried by particles that enter the black hole, soon to be absorbed by the central singularity? Do they wiggle their way out again, or at least the quantum information carried along with them? When you take the existing theory and equations literally, there is no chance that they can wiggle out. Should we ignore this?

First of all, we claim that this problem is a much more elementary and important one than what the literature suggests, and that most of the suggested cures are not well enough thought through. What should be done, is to stretch as much as is possible what one can do using standard, mainstream physics. Then, finally, impose the demand that black holes should behave, as much as possible, as ordinary forms of matter. When doing this properly, this leads to amazing observations.

Only few people noted that the application of existing physical knowledge can bring us much further than any wild speculation. Only recently,[c] we discovered a way to calculate things that could have been applied decades earlier. Our observation was[22] that we can use a partial wave expansion to describe energy and momentum entering the black hole, and that, applying Einstein's equations, the information carried in by each partial wave is carried out again, in a partial wave with the same values of the quantum numbers ℓ and m.

This calculation seems to be as elementary as the calculation of the spectrum of a hydrogen atom using quantum mechanics. As in the hydrogen atom, the boundary conditions are of crucial importance, but in a black hole, these boundary conditions are quite counter intuitive.

What this calculation does, is to expose our problem more clearly than ever before, and now we can see what the answers must be. The general coordinate transformation relating the "inside" of a black hole with what an outside observer sees, must be a topologically nontrivial one.[23–25] Only then do the equations make sense.

This does remarkable things with the fabric of space and time itself, not noticed before. For one thing, space and time must be discrete. This was already suspected long ago, leading people to fantasize about noncommutative space and time. However, noncommutative geometry itself does not give the right equations, one has to do this more precisely. This should now be possible.

Yet the deepest mysteries are still there. The fact that black holes emit radiation at a finite temperature seems to be a strong indication that degrees of freedom near the horizon are discrete, so that space and time themselves will be discrete. We do not know, let alone understand, what this does to local Lorentz invariance. The question asked can be summarized as:

> If space and time are discrete, how does one distribute fields on space–time? How can we reconcile this with Lorentz invariance? In short: *what is Nature's book keeping system?*[26]

[c] Some of the remarks reported here in the proceedings are to be considered as *notes added in proof*; these concepts were not yet completely clear when the presentation was given in the conference.

It seems that, at the Planck scale, all we really need to specify is: how is the information processed? At that scale, our world seems to be nothing more than an information processing machine. Many investigators now assume that this machine will be processing *quantum information*, or qubits, by some, continuous or discrete, version of Schrödinger's equation. This lecturer is suspecting a bolder scenario, which is that the information at the Planck scale will be no more than just classical information.[27] This information is apparently being processed in such a complex manner that, if we go to larger scales in space and time, only effectively quantum mechanical equations can tell us how the bulk of the information is processed at much larger scales of space and time.

Superstring theory does suggest to us what some of the answers to such questions will be like: a particle cannot be completely localized to one point in space. Even the black hole horizon will be 'fuzzy'. My point here is, that such answers are not precise enough. Fuzzy answers are often confused with sloppy answers. The precise equations are much more interesting. The early developments in quantum physics constitute important lessons here. The notion that a particle can be both a particle and a wave, seems sloppy at first, but once we had the Schrödinger equation, all sloppiness disappeared. We have to work out these problems precisely.

5. The Hierarchy Problem

As stated, the value found for the Higgs mass, around 125 GeV, is remarkably special, making the inner world just barely stable against scale transformations. Exactly at this mass value, the Higgs self-interactions stay practically constant as we go to very much tinier scales. This could indicate that there will be remarkably little structure in the desert between the TeV scale and the Planck scale, see Fig. 1.

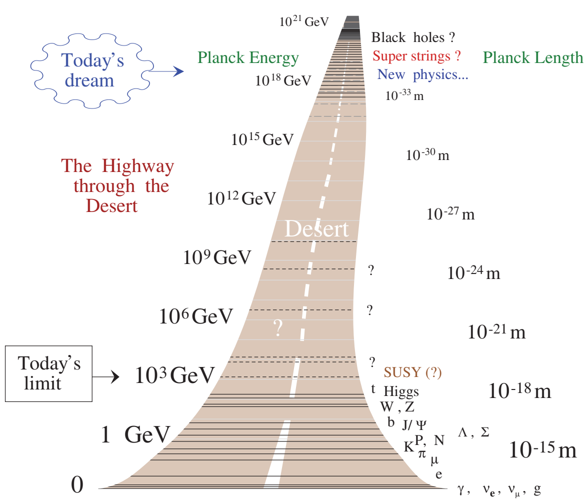

Fig. 1. The highway through the desert. Horizontal lines: known particles. Milestones: left, the energies per particle, in factors of 1000; right, the length scales, in factors 1000.

But then we are led to another problem: if the fundamental equations governing our universe are relatively simple, then

Why is the universe so complex?

The problem begins by noting the enormously different scales of things in our universe:

$$\begin{aligned} \text{Size of universe itself:}\quad & 10^{27}\ \text{m}\,,\\ \text{Size of stars and planets:}\quad & 10^{6}\text{–}10^{12}\ \text{m}\,,\\ \text{Size of humans:}\quad & 1\ \text{m}\,,\\ \text{Size of atoms:}\quad & 10^{-10}\ \text{m}\,,\\ \text{Subnuclear particles}\quad & 10^{-18}\text{–}10^{-15}\ \text{m}\,,\\ \text{Planck size:}\quad & 10^{-35}\ \text{m}\,. \end{aligned} \tag{9}$$

One may suspect that these scale differences are related to scale differences in the fundamental constants,

$$\begin{aligned} \text{finestructure constant:}\quad & \alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137.036}\,,\\ \frac{\text{proton mass}}{\text{electron mass}}:\quad & \frac{m_p}{m_e} = 1836.1527\,,\\ \frac{\text{proton mass}}{\text{Planck units}}:\quad & \kappa = m_p\sqrt{\frac{G}{\hbar c}} = 7.685\times 10^{-20}\,,\\ \frac{\text{cosmological constant}}{\text{Planck units}}:\quad & \frac{\Lambda\hbar G}{c^5} = 3\times 10^{-122}\,. \end{aligned} \tag{10}$$

Some very rough estimates then suggest where the scales of the universe come from. Admittedly, we used some hand waving[d] to guess the following size distributions:

$$\begin{aligned} \text{Size atomic nucleus:}\quad & \frac{\hbar}{m_\pi c} \approx 1.5\times 10^{-15}\ \text{m}\,,\\ \text{Size of atoms:}\quad & \frac{\hbar}{\alpha m_e c} \approx 0.5\times 10^{-10}\ \text{m}\,,\\ \text{Density of rock:}\quad & m_p\left(\frac{\alpha m_e c}{\hbar}\right)^3 \approx 11\ \text{g/cm}^3\,,\\ \text{Mass of planet (chemical):}\quad & \sqrt{\frac{\alpha^3 c^3\hbar^3}{m_p^4 G^3}} \approx 385 M_{\text{Earth}} \approx 1.21 M_{\text{Jupiter}}\,,\\ \text{Mass of star (nuclear)}:\quad & \frac{M_{\text{Planck}}^3}{m_p^2} \approx 1.85 M_\odot\,,\\ \frac{\text{“Size of universe”}}{\text{Planck length}}:\quad & \sqrt{\frac{c^5}{\Lambda\hbar G}} \approx 10^{61}\,. \end{aligned} \tag{11}$$

[d] For instance, one could refine these numbers by taking the numbers of protons and neutrons (A and Z) in an average nucleus into account, but their distributions seem to be due to some more spurious accidents in the mass ratios and coupling strengths of the nucleons.

These numbers[e] suggest that, if we can explain the variations in Nature's fundamental constants, Eq. (10), then we can also understand the big scale differences (9). Obviously, any theory that generates such numbers or ratios of numbers must possess some minimal amount of inherent complexity. There are two fundamental possibilities:

(i) a relatively simple theory can be found that requires mathematical manipulations that, all by themselves, generate these delicate, large number ratios, or

(ii) a very large 'landscape' of theories exists, that houses our universe together with countless others.

String theories have been quite unable to accommodate for possibility (i), so many theoreticians now speculate about (ii), that there should be a landscape. However, from our considerations in the subatomic domain, sparked, perhaps, by the LHC observations, one might conclude that the naturalness argument must be refined. Possibility (i) is not as far-fetched as one might think. Relatively simple mathematical constructions can easily generate excessively large numbers. The discussion is not over.

References

1. A. Salam, On parity conservation and neutrino mass, *Nuovo Cimento Ser. X* **5**, 299–301 (1957).
2. A. Salam and J. C. Ward, Electromagnetic and weak interactions, *Phys. Lett.* **13**, 168–171 (1964).
3. A. Salam and J. C. Ward, Gauge theory of elementary interactions, *Phys. Rev. B* **136**, 763–768 (1964).
4. L. Maiani and G. Rolandi (eds.), *The Standard Theory of Particle Physics — Essays to Celebrate CERN's 60th Anniversary* (World Scientific, 2016).
5. ATLAS Collab., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, *Phys. Lett. B* **716**, 1–29 (2012), arXiv:1207.7214.
6. CMS Collab., Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, *Phys. Lett. B* **716**, 30 (2012), arXiv:1207.7235 [hep-ex].
7. G. 't Hooft, The conceptual basis of quantum field theory, in *Handbook of the Philosophy of Science*, Philosophy of Physics, Part A, eds. J. Butterfield and J. Earman (Elsevier), pp. 661–729, ISBN-13 978-0-444-53001-1/10 0-444-53001-0.
8. K. G. Chetyrkin and M. F. Zoller, Three-loop β-functions for top-Yukawa and the Higgs self-interaction in the Standard Model, arXiv:1205.2892.
9. G. 't Hooft, Local conformal symmetry: the missing symmetry component for space and time, *Int. J. Mod. Phys. D* **24**, 1543001 (2015), arXiv:1410.6675v2 [gr-qc].
10. P. D. Mannheim and D. Kazanas, Exact vacuum solution to conformal Weyl gravity and galactic rotation curves, *Astrophys. J.* **342**, 635 (1989).

[e] We define a planet as a body where the gravitational potential for a proton reaches the scale of chemical reactions, while in a star this approaches the scale of nuclear interactions.

11. D. Kazanas and P. D. Mannheim, General structure of the gravitational equations of motion in conformal Weyl gravity, *Astrophys. J. Suppl.* **76**, 431 (1991).
12. P. D. Mannheim, Solution to the ghost problem in fourth order derivative theories, *Found. Phys.* **37**, 532 (2006), arXiv:hep-th/0608154.
13. G. 't Hooft, Probing the small distance structure of canonical quantum gravity using the conformal group, arXiv:1009.0669 [gr-qc].
14. G. 't Hooft, The conformal constraint in canonical quantum gravity, arXiv:1011.0061 [gr-qc].
15. G. 't Hooft, Singularities, horizons, firewalls, and local conformal symmetry, Karl Schwarzschild Memorial Lecture, arXiv:1511.04427.
16. S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys.* **43**, 199 (1975).
17. S. W. Hawking, Breakdown of predictability in gravitational collapse, *Phys. Rev. D* **14**, 2460 (1976).
18. J. D. Bekenstein, Black holes and entropy, *Phys. Rev. D* **7**, 2333–2346 (1973).
19. J. Dai, R. Leigh and J. Polchinski, *Mod. Phys. Lett. A* **4**, 2073 (1989).
20. J. Polchinski, *Phys. Rev. Lett.* **75**, 4724 (1995), arXiv:hep-th/9510017.
21. J. M. Maldacena, Black holes in string theory, Ph.D. Thesis, Princeton University, June 1996, arXiv:hep-th/9607235.
22. G. 't Hooft, Diagonalizing the black hole information retrieval process, arXiv:1509.01695.
23. N. Sanchez, Semiclassical quantum gravity in two and four dimensions, in *Gravitation in Astrophysics*: *Cargèse 1986*, Nato Science Series B, eds. B. Carter and J. B. Hartle.
24. N. Sanchez and B. F. Whiting, Quantum field theory and the antipodal identification of black holes, *Nucl. Phys. B* **283**, 605–623 (1987).
25. G. 't Hooft, Black hole unitarity and antipodal entanglement, *Found. Phys.* (2016), doi: 10.1007/s10701-016-0014-y, arXiv:1601.03447 [gr-qc].
26. G. 't Hooft, Nature's book keeping system, A contribution to "*Space, Time, and Frontiers of Human Understanding*," a collection of essays compiled by S. Iyengar (Springer), arXiv:1605.00027.
27. G. 't Hooft, The cellular automaton interpretation of quantum mechanics, to be published by Springer, arXiv:1405.1548.