

Atom interferometers as freely falling clocks for time-dilation measurements

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(based on [arXiv:2402.11065](https://arxiv.org/abs/2402.11065))



Motivation

- Applications of atom interferometers based on single-photon transitions:
 - ▶ GW detection in mid-frequency band (100-m prototypes not sensitive enough)
 - ▶ Search for ultralight dark matter (modest exclusion bounds at early stages)

- Are there other interesting measurements (rather than mere null tests) that can be preformed?

Yes, local measurement of *relativistic time dilation* with freely falling atoms.

- Useful *methods* for theoretical *modelling* of such interferometers.

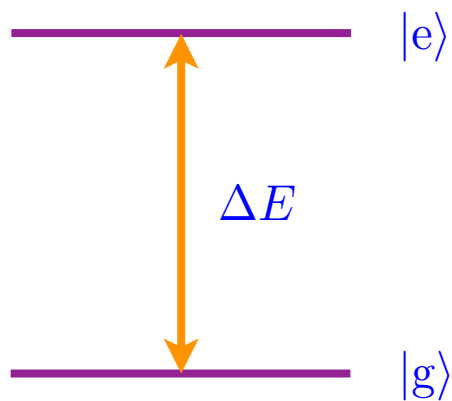
Outline



1. Relativistic effects in freely falling clocks
2. Atom interferometer as a freely falling clock
3. Experimental implementation
4. Equivalence principle violations and external forces
5. Discussion and conclusions

Relativistic effects in freely falling clocks

Quantum clock model



- *Initialization* pulse:

$$|g\rangle \rightarrow |\Phi(0)\rangle = \frac{1}{\sqrt{2}} (|g\rangle + i e^{i\varphi} |e\rangle)$$

- *Evolution*:

$$|\Phi(\tau)\rangle \propto \frac{1}{\sqrt{2}} (|g\rangle + i e^{i\varphi} e^{-i\Delta E \tau / \hbar} |e\rangle)$$

- Theoretical description of the clock:

- ▶ two-level atom (internal state):

$$\hat{H} = \hat{H}_1 \otimes |g\rangle\langle g| + \hat{H}_2 \otimes |e\rangle\langle e|$$

$$m_1 = m_g$$

$$m_2 = m_g + \Delta m$$

$$\Delta m = \Delta E/c^2$$

- ▶ classical action for COM motion:

$$S_n[x^\mu(\lambda)] = -m_n c^2 \int d\tau = -m_n c \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \quad (n = 1, 2)$$

free fall

- Theoretical description of the clock:

- ▶ two-level atom (internal state):

$$\hat{H} = \hat{H}_1 \otimes |g\rangle\langle g| + \hat{H}_2 \otimes |e\rangle\langle e|$$

$$m_1 = m_g$$

$$m_2 = m_g + \Delta m$$

$$\Delta m = \Delta E/c^2$$

- ▶ classical action for COM motion:

$$S_n[x^\mu(\lambda)] = -m_n c^2 \int d\tau \approx \int_{t_0}^t dt' \left(-m_n c^2 + \frac{1}{2} m_n \dot{\mathbf{x}}^2 - m_n U(t', \mathbf{x}) \right)$$

free fall

- Theoretical description of the clock:

- ▶ two-level atom (internal state):

$$\hat{H} = \hat{H}_1 \otimes |g\rangle\langle g| + \hat{H}_2 \otimes |e\rangle\langle e|$$

$$m_1 = m_g$$

$$m_2 = m_g + \Delta m$$

$$\Delta m = \Delta E/c^2$$

- ▶ classical action for COM motion:

$$S_n[x^\mu(\lambda)] = -m_n c^2 \int d\tau - \int d\tau V_n(x^\mu) \quad (n = 1, 2)$$

including external forces

A purple arrow points from the text "including external forces" to the potential energy term $\int d\tau V_n(x^\mu)$ in the equation above.

Propagation of matter-wave packets in curved spacetime (relativistic description)

- Wave-packet evolution in terms of
 - ▶ *central trajectory* (satisfies classical e.o.m.) $X^\mu(\lambda)$
 - ▶ *centered wave packet* $|\psi_c^{(n)}(\tau_c)\rangle$

$$\Delta p/m \ll c$$

$$\Delta x \ll \ell$$

← curvature radius

Comoving frame: $X^\mu(\tau_c) = (c\tau_c, \mathbf{0})$
 (Fermi-Walker) $\hat{H}_n = m_n c^2 + V_n(\tau_c, \mathbf{0}) + \hat{H}_c^{(n)}$

■ Wave-packet evolution: $|\psi^{(n)}(\tau_c)\rangle = e^{i\mathcal{S}_n/\hbar} |\psi_c^{(n)}(\tau_c)\rangle$

▶ *propagation phase*

$$\mathcal{S}_n = - \int_{\tau_1}^{\tau_2} d\tau_c (m_n c^2 + V_n(\tau_c, \mathbf{0}))$$

▶ *centered wave packet*

$$i\hbar \frac{d}{d\tau_c} |\psi_c^{(n)}(\tau_c)\rangle = \hat{H}_c^{(n)} |\psi_c^{(n)}(\tau_c)\rangle$$

$$\hat{H}_c^{(n)} = \frac{1}{2m_n} \hat{\mathbf{p}}^2 + \frac{1}{2} \hat{\mathbf{x}}^T \left(\mathcal{V}^{(n)}(\tau_c) - m_n \Gamma(\tau_c) \right) \hat{\mathbf{x}}$$

$$\mathcal{V}_{ij}^{(n)}(\tau_c) = \partial_i \partial_j V_n(\tau_c, \mathbf{x}) \Big|_{\mathbf{x}=\mathbf{0}}$$

↑ gravity-gradient tensor

For further details:



PHYSICAL REVIEW X **10**, 021014 (2020)

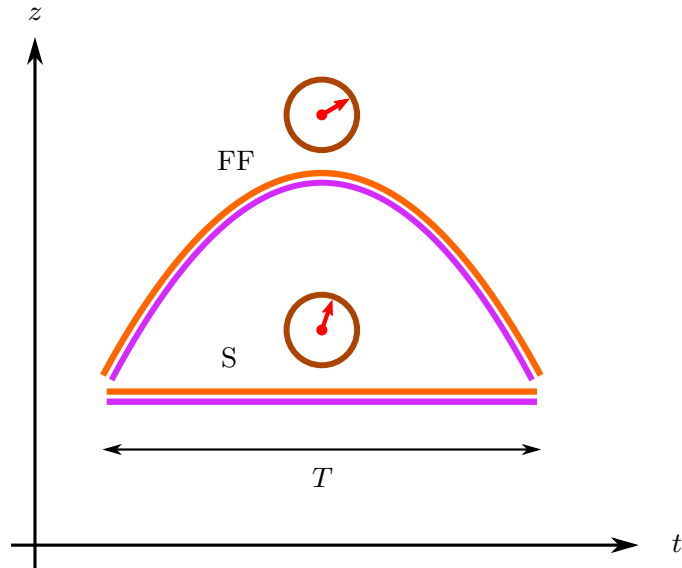
Gravitational Redshift in Quantum-Clock Interferometry

Albert Roura 

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Universität Ulm, Albert-Einstein-Allee 11, 89081 Ulm, Germany*

- *Relativistic* description of atom interferometry in *curved spacetime*.
- Including *external forces* and even *guiding potentials*.
- *Relativistic* interpretation of the *separation phase* in open interferometers.

Relativistic time dilation for a freely falling clock



- Freely falling clock (FF):

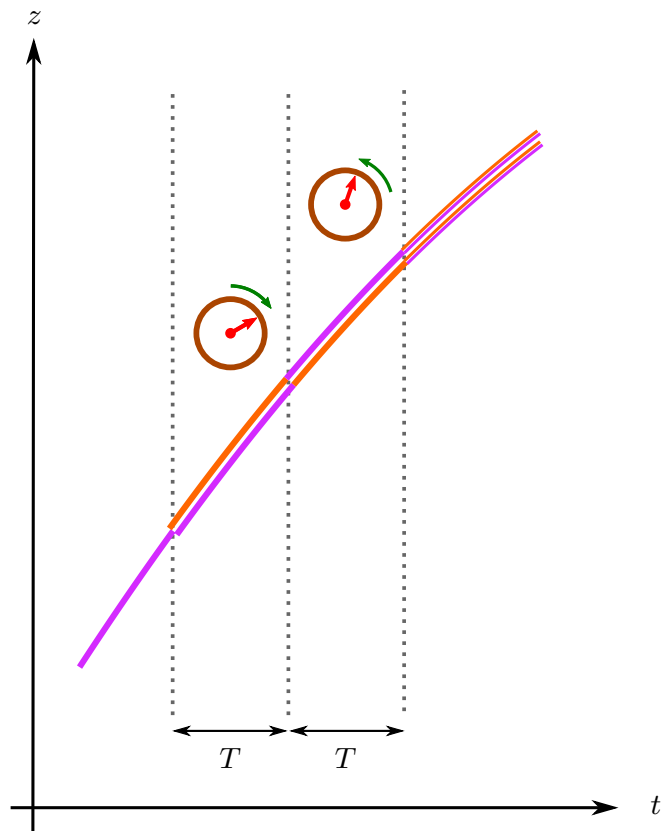
$$\delta\phi = -(\Delta E/\hbar) \left((1 + U_0/c^2) T + \frac{1}{24} \frac{g^2 T^3}{c^2} \right)$$

- Static clock at constant height (S):

$$\delta\phi = -(\Delta E/\hbar) (1 + U_0/c^2) T$$

- Natural implementation: compare atomic fountain clock to optical lattice clock.
- BUT accuracy of best atomic fountain clocks insufficient by more than an order of magnitude.

Freely falling clock with internal-state inversion



- Simultaneity hypersurfaces in the lab frame.
(equal time separation)
- Unbalanced proper times (before and after inversion)
due to relativistic time dilation:

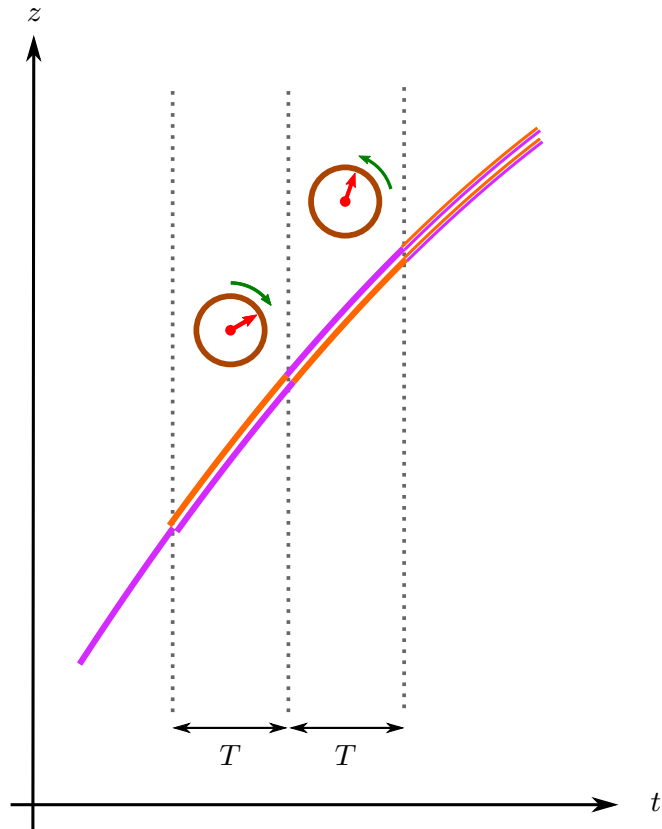
$$\delta\phi = -2 (\Delta E/\hbar) (\mathbf{v}_0 \cdot \mathbf{g} T^2 + g^2 T^3) / c^2$$

$$\frac{d\tau}{dt} = 1 - \frac{1}{2c^2} \left(\frac{d\mathbf{X}}{dt} \right)^2 + \frac{1}{c^2} U(t, \mathbf{X})$$

special relativistic

gravitational redshift

Freely falling clock with internal-state inversion



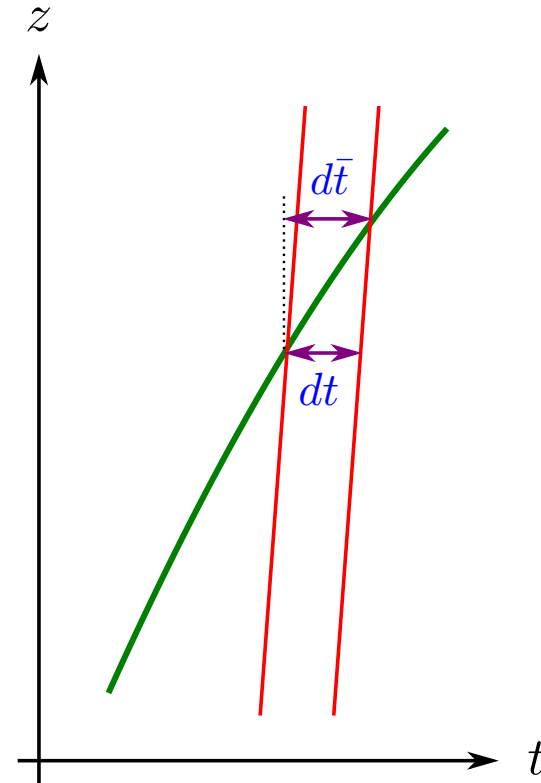
- Possible implementation with *Doppler-free* E2–M1 *two-photon* pulses at $\lambda_2 = 2 \times 698 \text{ nm}$.
- Drawbacks:
 - dedicated high-power laser needed at λ_2
 - residual recoil ($m \Delta \mathbf{v} = -\Delta m \mathbf{v}$)
- Let us consider atom interferometers based on *single-photon* transitions.

Atom interferometer as a freely falling clock

Atom interferometer based on single-photon transitions

- Proper time along a freely falling world line (geodesic) and elapsed between two light rays.
 - ▶ *Retardation* effect due to the finite speed of light:

$$d\bar{t} = dt + (\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}/c) d\bar{t} + O(1/c^3)$$



Atom interferometer based on single-photon transitions

- Proper time along a freely falling world line (geodesic) and elapsed between two light rays.

- ▶ *Retardation* effect due to the finite speed of light: (stationary spacetime)

$$d\bar{t} = dt + (\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}/c) d\bar{t} + O(1/c^3) \quad \longrightarrow \quad \frac{d\bar{t}}{dt} = \frac{1}{1 - \hat{\mathbf{n}} \cdot \bar{\mathbf{v}}/c}$$

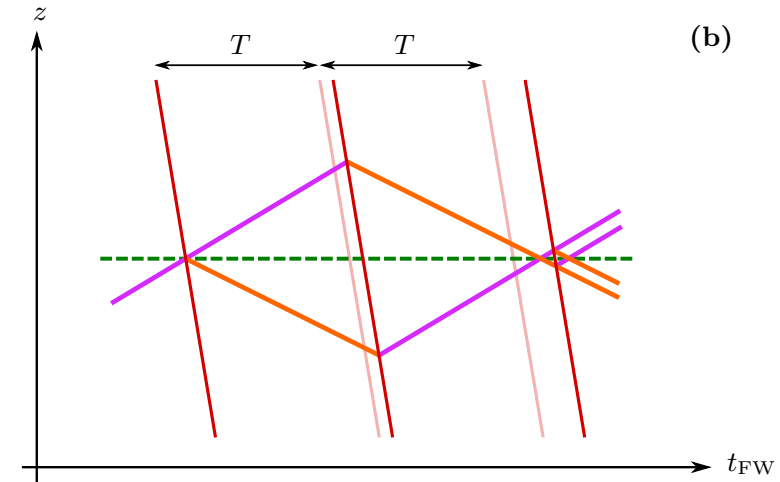
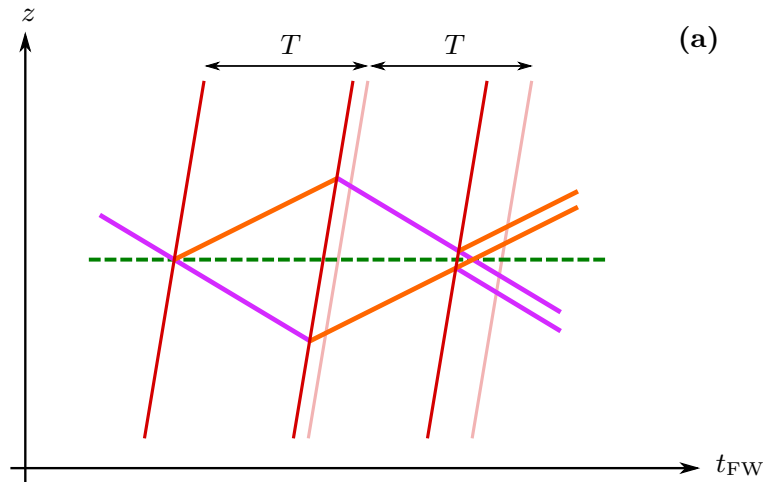
- ▶ Relativistic *time dilation*:

$$\frac{d\bar{\tau}}{d\bar{t}} = 1 - \frac{1}{2c^2} \left(\frac{d\bar{\mathbf{X}}}{d\bar{t}} \right)^2 + \frac{1}{c^2} U(\bar{t}, \bar{\mathbf{X}}) + O(1/c^4)$$

special relativistic

gravitational redshift

Atom interferometer based on single-photon transitions



- *Freely falling frame* comoving with the mid-point world line (Fermi-Walker frame):
 - ▶ light rays (laser wave fronts) have fixed slope,
 - ▶ shifts due to Doppler effect (*opposite sign* in reversed interferometer) and time dilation (*same sign*).

- It is sufficient to calculate the proper times along the *mid-point* world line rather than the actual *arm trajectories* (negligible higher-order corrections to total phase shift).
- Proper time as a function of the phase φ , invariant characterizing each laser wave front:

$$\frac{d\bar{\tau}}{d\varphi} = \frac{d\bar{\tau}}{d\bar{t}} \frac{d\bar{t}}{dt} \left(\frac{dt}{d\varphi} \right) = \frac{d\bar{\tau}}{d\bar{t}} \left(\frac{1}{1 - \hat{\mathbf{n}} \cdot \bar{\mathbf{v}}/c} \right) \left(\frac{dt}{d\varphi} \right)$$

- The Doppler factor can be (partially) compensated through a suitable frequency chirp:

$$\left(\frac{dt}{d\varphi} \right)_{\text{chirp}} = \left(1 - \hat{\mathbf{n}} \cdot \bar{\mathbf{v}}'/c \right) \left(\frac{dt}{d\varphi} \right)_0 \quad (dt/d\varphi)_0 = 1/\omega_0$$

$$\bar{\mathbf{v}}'(t) = \bar{\mathbf{v}}'_0 + \mathbf{g}' (t - t_0)$$

- Phase-shift calculation:

$$\delta\phi = -\frac{\Delta E}{\hbar} \left[\int_0^{\omega_0 T} \left(\frac{d\bar{\tau}}{d\varphi} \right) d\varphi - \int_{\omega_0 T}^{2\omega_0 T} \left(\frac{d\bar{\tau}}{d\varphi} \right) d\varphi \right]$$

- For an approximately uniform gravitational field, $\bar{\mathbf{X}}(\bar{t}) = \bar{\mathbf{v}}_0 + \mathbf{g}(\bar{t} - \bar{t}_0)$ and

$$\delta\phi = -2(\Delta E/\hbar) (\bar{\mathbf{v}}_0 \cdot \mathbf{g} T^2 + g^2 T^3) / c^2 + \delta\phi_{\text{corr}}$$

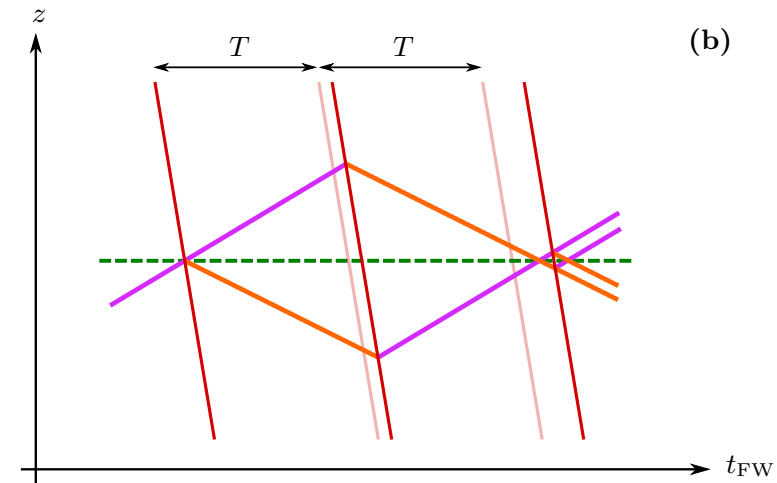
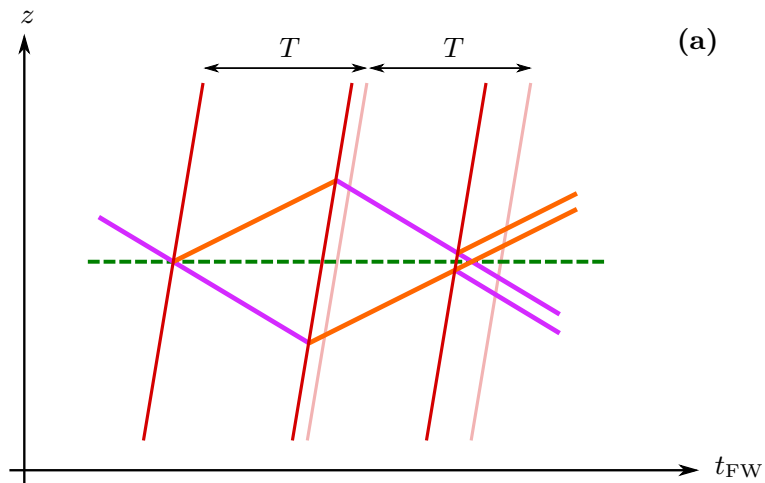
- It agrees with the result for an ideal freely falling clock if $\delta\phi_{\text{corr}}$ can be kept small enough.

- For an imperfect match of the chirped frequency, with $\Delta\mathbf{g} = \mathbf{g} - \mathbf{g}'$ and $\Delta\bar{\mathbf{v}}_0 = \bar{\mathbf{v}}_0 - \bar{\mathbf{v}}'_0$.

$$\delta\phi_{\text{corr}} = \frac{\Delta E}{\hbar} \left[\frac{(\hat{\mathbf{n}} \cdot \Delta\mathbf{g})}{c} T^2 + 2 \frac{(\hat{\mathbf{n}} \cdot \Delta\bar{\mathbf{v}}_0)(\hat{\mathbf{n}} \cdot \mathbf{g})}{c^2} T^2 + \frac{(\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}_0)(\hat{\mathbf{n}} \cdot \Delta\mathbf{g})}{c^2} T^2 + 3 \frac{(\hat{\mathbf{n}} \cdot \mathbf{g})(\hat{\mathbf{n}} \cdot \Delta\mathbf{g})}{c^2} T^3 \right]$$

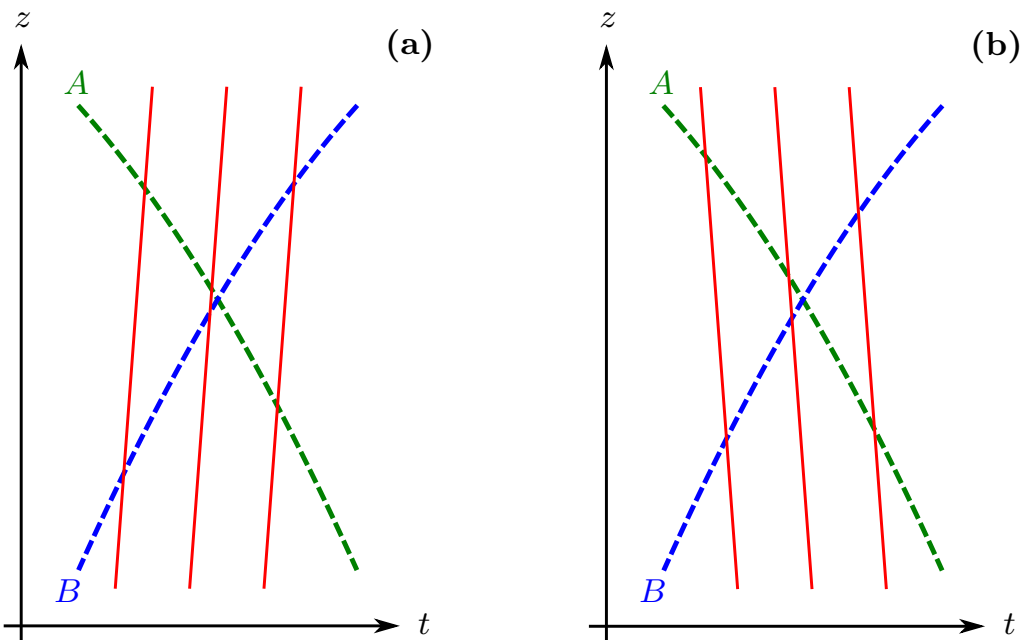
- The dominant term is linear in $\hat{\mathbf{n}}$ and can be suppressed by adding up $\delta\phi$ for two interferometers with opposite $\hat{\mathbf{n}}$.
(reversed interferometers)
- The above result can be straightforwardly generalized to a time dependent $\Delta\mathbf{g}(\bar{t})$.
This can naturally account for *laser phase noise* and *vibrations* of retro-reflection *mirror*.

Reversed interferometers



- Uncompensated Doppler contribution cancels out when adding up their phase shifts.
- Effects of *mirror vibrations* and *laser phase noise* (for reversed interferometers in different shots) do not cancel out → “gradiometric” configuration.

“Gradiometric” configuration



- Differential phase shift between interferometers launched with different velocities (A and B):

$$\delta\phi_A - \delta\phi_B = -2 (\Delta E/\hbar) (\bar{\mathbf{v}}_0^A - \bar{\mathbf{v}}_0^B) \cdot \mathbf{g} T^2/c^2$$

- Similarly for pair of reversed interferometers:
(a) and (b)

- Comparison between two freely falling clocks.
 (no need for time reference in lab frame)

$$\omega_{\text{chirp}}(t) = \left[1 + \frac{(\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}'_0)}{c} \frac{(\hat{\mathbf{n}} \cdot \mathbf{g}')}{c} (t - t_0) + \frac{(\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}'_0)^2}{c^2} + 3 \frac{(\hat{\mathbf{n}} \cdot \bar{\mathbf{v}}'_0) (\hat{\mathbf{n}} \cdot \mathbf{g}')}{c^2} (t - t_0) \right] \omega_0$$

Experimental implementation

- Gradiometric configuration in MAGIS-100 with two simultaneous interferometers launched from the top and bottom atom source.

AOM driven by a stable rf source → second frequency component.

- For $\bar{\mathbf{v}}_0^A = -(20 \text{ m/s}) \hat{\mathbf{z}}$ and $\bar{\mathbf{v}}_0^B = (40 \text{ m/s}) \hat{\mathbf{z}}$ respectively, one gets $\delta\phi^A - \delta\phi^B = 35 \text{ rad}$.
- With $N = 10^5$ detected atoms, a shot-noise-limited sensitivity at the 10^{-5} level can be reached in a hundred shots.
- Stanford's 10-m prototype or AION's 10-m fountain could also measure these time dilation effects with about two orders of magnitude lower sensitivity.

Main systematic effects

- Effects suppressed when adding up the phase shift for *reversed* interferometers:
 - ▶ gravity gradients (co-location at 0.1 mm and 0.1 mm/s level $\rightarrow 10^{-4}$ relative uncertainty)
 - ▶ rotations
 - ▶ wave-front curvature & light shifts
- Pulse timing requirements: $\Delta T \lesssim 0.1 \mu\text{s}$ and $\delta \lesssim 300 \text{ Hz}$ $\rightarrow 10^{-5}$ relative uncertainty
- Magnetic field inhomogeneities: 3 nT / m $\rightarrow 10^{-5}$ relative uncertainty
- Temperature gradients: 2 K / 100 m \rightarrow contribution at 10^{-2} level

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- Temperature gradients: 2 K / 100 m \rightarrow contribution at 10^{-2} level

Equivalence principle violations & external forces

External forces

- The coupling of neutral atoms to magnetic fields and far detuned radiation can be described with *state-dependent* external potentials.

- Replacement in the action: $m_n U(t', \mathbf{X}) \rightarrow m_n U(t', \mathbf{X}) + V_n(t', \mathbf{X})$

- Modified mean acceleration:

$$\bar{\mathbf{a}} = \mathbf{g} - \nabla \bar{V}_n / m_n \qquad \bar{V}_n \equiv \frac{m_n}{2} \left(\frac{V_1}{m_1} + \frac{V_2}{m_2} \right)$$

- Relative acceleration between the two internal states:

$$\delta \mathbf{a} = -\nabla (\delta V_n) / m_n \qquad \delta V_n \equiv m_n \left(\frac{V_2}{m_2} - \frac{V_1}{m_1} \right)$$

- Fermi-Walker frame (mid-point trajectory with acceleration $\bar{\mathbf{a}}$).
- Modified arm trajectories + separation phase \rightarrow no net phase-shift contribution.
- Key contribution to the action evaluated along the mid-point trajectory:

$$V_2(t', \bar{\mathbf{X}}) - V_1(t', \bar{\mathbf{X}}) = \Delta m \frac{\bar{V}_n(t', \bar{\mathbf{X}})}{m_n} + \bar{m} \frac{\delta V_n(t', \bar{\mathbf{X}})}{m_n} \quad \bar{m} = (m_1 + m_2)/2 \approx m$$

- Result for a uniform (state-dependent) force:

$$\delta\phi = -\frac{\Delta E}{\hbar} \left[2 (\bar{\mathbf{v}}_0 \cdot \bar{\mathbf{a}} T^2 + \bar{\mathbf{a}}^2 T^3) / c^2 + \left(\frac{m}{\Delta m} \right) (\delta \mathbf{a} \cdot \bar{\mathbf{v}}_0 T^2 + \delta \mathbf{a} \cdot \bar{\mathbf{a}} T^3) / c^2 \right]$$

Equivalence principle violations

- Consider a *dilaton model* as a consistent parametrization of equivalence principle violations.
- Replacement in the action: $m_n U(t', \mathbf{X}) \rightarrow m_n (1 + \beta_n) U(t', \mathbf{X})$

It can be regarded as a particular case of state-dependent external potential (previous slides).

- The phase-shift result coincides with that for an ideal clock following the mean trajectory:

$$\delta\phi = -2 (\Delta E / \hbar) (1 + \alpha_{e-g}/2) (\bar{\mathbf{v}}_0 \cdot \bar{\mathbf{g}} T^2 + \bar{g}^2 T^3) / c^2$$

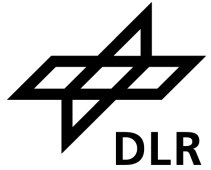
$$\alpha_{e-g} = (\beta_2 - \beta_1) \left(\frac{m}{\Delta m} \right)$$

Test of universality of gravitational redshift (UGR).

Discussion and conclusions

Comparison to quantum-clock interferometry and other proposals

Quantum-clock interferometry



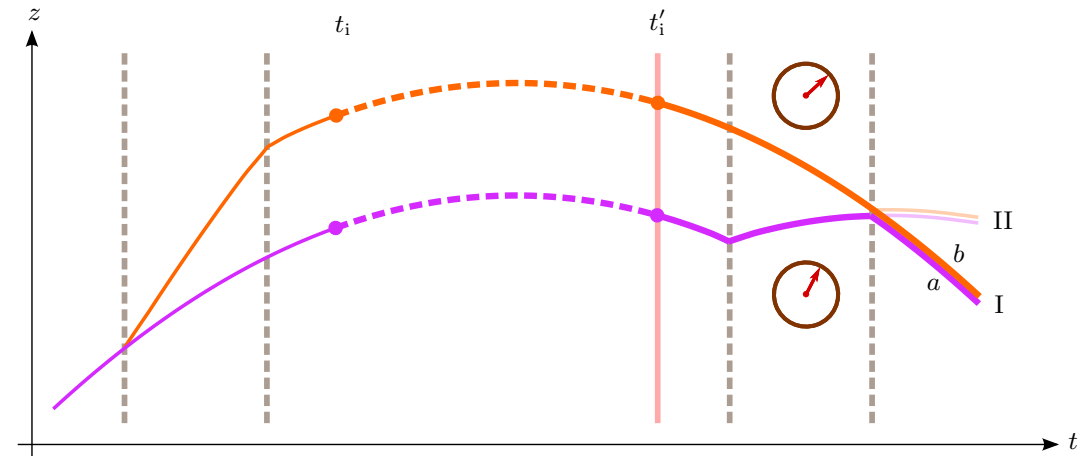
PHYSICAL REVIEW X **10**, 021014 (2020)

Gravitational Redshift in Quantum-Clock Interferometry

Albert Roura

Quantum superposition of a single clock
at two different heights

- Initialization pulse after the spatial superposition has been generated.
- Doubly differential measurement:
 - ▶ state-selective detection
 - ▶ compare different initialization times

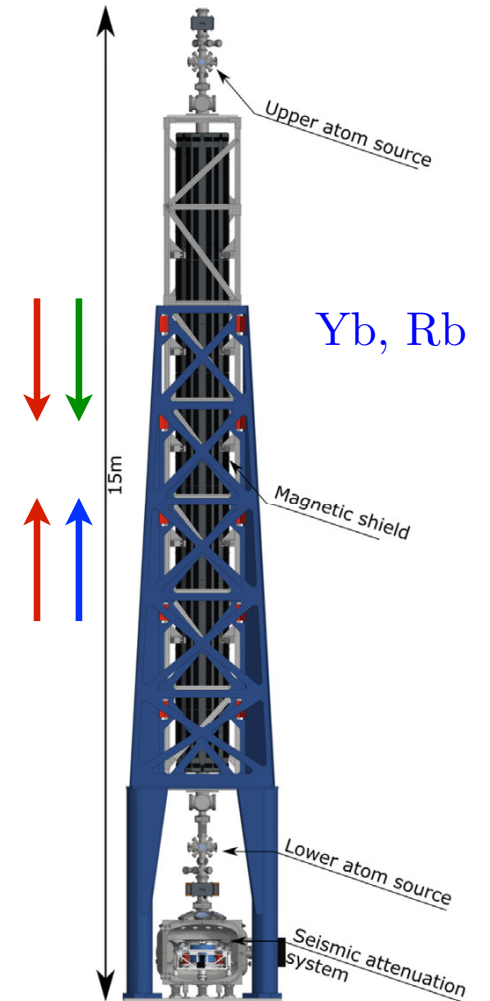
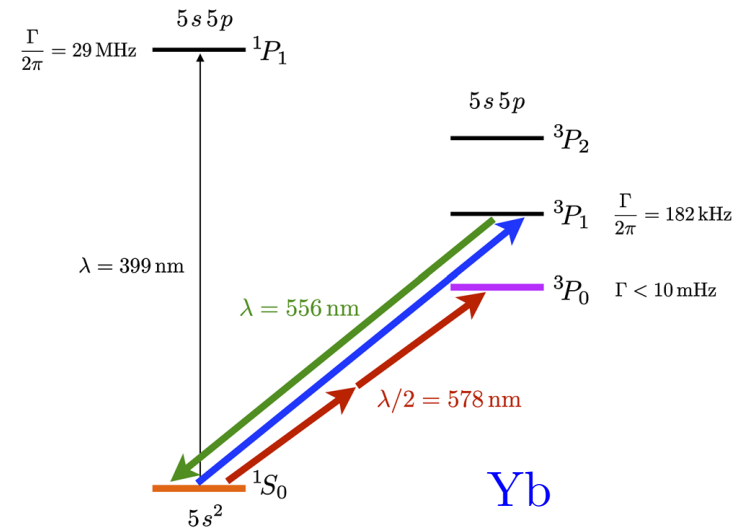
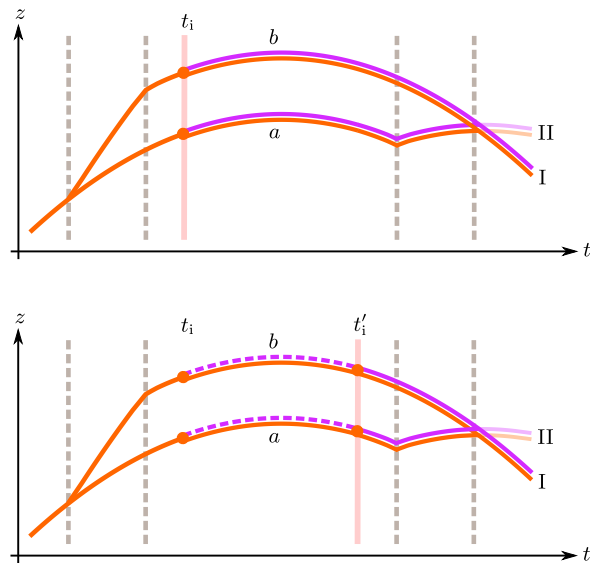


Quantum-clock interferometry

PHYSICAL REVIEW D **104**, 084001 (2021)

Measuring gravitational time dilation with delocalized quantum superpositions

Albert Roura¹, Christian Schubert^{2,3}, Dennis Schlippert² and Ernst M. Rasel²

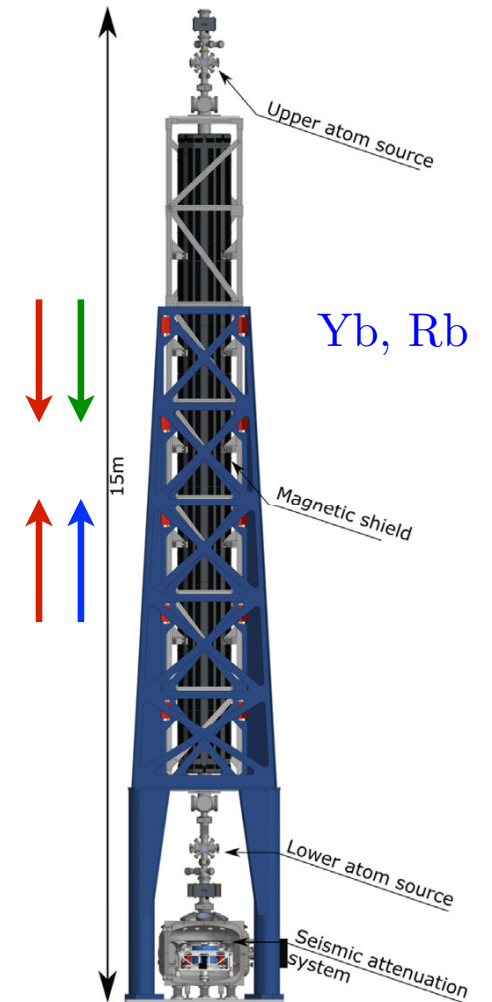
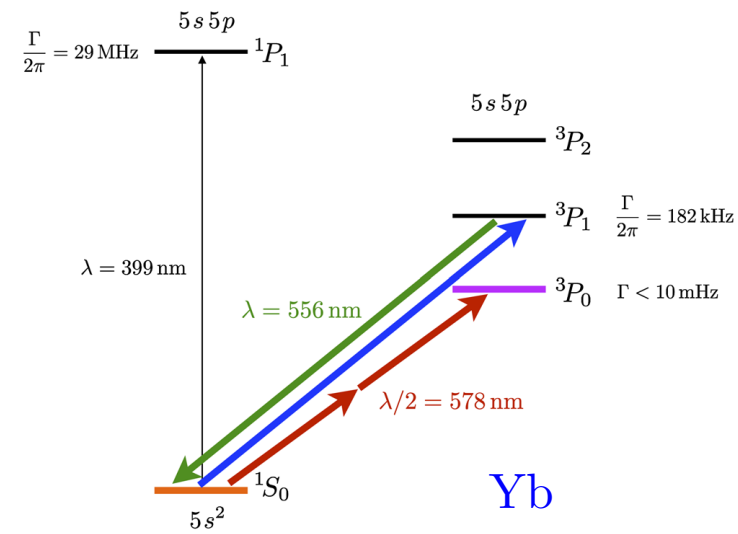
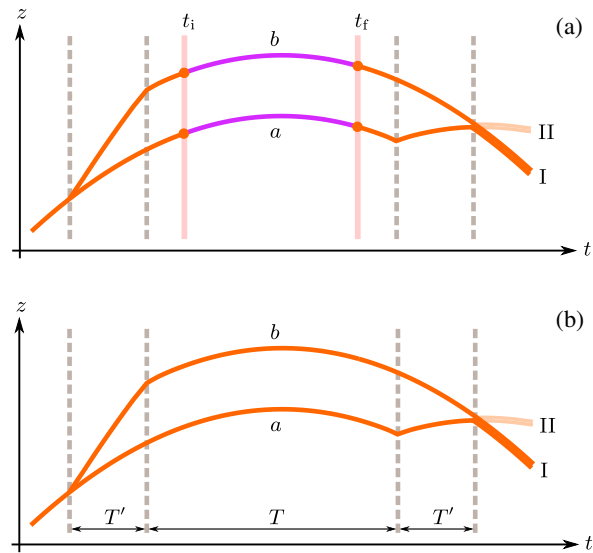


Quantum-clock interferometry

PHYSICAL REVIEW D **104**, 084001 (2021)

Measuring gravitational time dilation with delocalized quantum superpositions

Albert Roura¹, Christian Schubert^{2,3}, Dennis Schlippert² and Ernst M. Rasel²



Comparison with current proposal

- **Quantum-clock interferometry:** single clock in a delocalized quantum superposition of two wave packets experiencing different gravitational time dilation.
- **Current proposal:** each atom interferometer acts as a *freely falling clock*; comparison between two independent clocks in the “gradiometric” configuration.

PHYSICS

Interference of clocks: A quantum twin paradox

Sina Loriani^{1*}, Alexander Friedrich^{2*†}, Christian Ufrecht², Fabio Di Pumpo², Stephan Kleinert², Sven Abend¹, Naceur Gaaloul¹, Christian Meiners¹, Christian Schubert¹, Dorothee Tell¹, Étienne Wodey¹, Magdalena Zych³, Wolfgang Ertmer¹, Albert Roura², Dennis Schlippert¹, Wolfgang P. Schleich^{2,4,5}, Ernst M. Rasel¹, Enno Giese²

Loriani et al., *Sci. Adv.* 2019;5:eaax8966 4 October 2019

Atom-interferometric test of the universality of gravitational redshift and free fall

Christian Ufrecht^{1,*}, Fabio Di Pumpo¹, Alexander Friedrich¹, Albert Roura², Christian Schubert^{3,†}, Dennis Schlippert³, Ernst M. Rasel³, Wolfgang P. Schleich^{1,2,4} and Enno Giese^{1,3}

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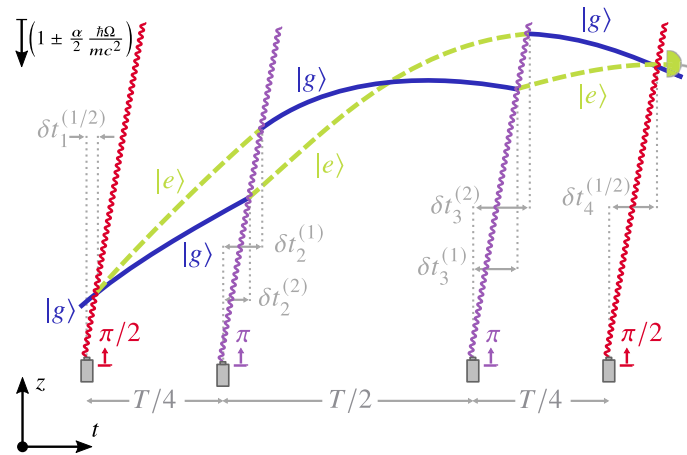
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Proposed UGR test with atom interferometry



F. Di Pumpo, A. Friedrich, C. Ufrecht, E. Giese
Phys. Rev. D **107**, 064007 (2023)

- Null test: non-vanishing result in case of gravitational redshift differences for different isotopes (e.g. ^{87}Sr and ^{88}Sr)
- *Forbidden* clock transition for bosonic isotopes such as ^{88}Sr unless a strong transverse magnetic field is applied \rightarrow not a viable option for precision measurements with VLBAI.
- Little dependence of $\Delta E \propto m_e \alpha^2 c^2$ on the nuclear isotope \rightarrow effects of UGR violations nearly the same for both isotopes.

Conclusions

- Atom interferometers based on single-photon transitions can be used as *freely falling clocks* for time dilation measurements.
- Unprecedented measurement of *relativistic time dilation* in a local measurement with *freely falling* atoms.
- It could be implemented in MAGIS-100 with virtually *no additional requirements*.

A version with limited sensitivity could also be implemented in Stanford's 10-m prototype or AION's 10-m fountain.

- Main challenge for achieving higher sensitivities → temperature gradients.

Further improvement through measurements of temperature profile and post-correction.

For further details:

Atom interferometer as a freely falling clock for time-dilation measurements

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[arXiv:2402.11065](https://arxiv.org/abs/2402.11065)

Other related activities

Q-GRAV Project

Interface of Quantum Mechanics and Gravitation

- Main Topics:
 1. Atom interferometry
 2. Matter-wave lensing for cold atoms
 3. Relativistic quantum information

- Team members:



Nadja Augst



Nico Schwersenz



Albert Roura

ESA-related activities



- ACES Mission (launch in 01/2025)
 - ▶ high-precision measurements with cold atoms in space
 - ▶ tests of general relativity, relativistic geodesy, intercontinental time / frequency distribution

ACES Workshop 2023 organized in Ulm.



- Co-Chair of ESA's *Physical Sciences Working Group* (PSWG).
Member of ESA's *Space Science Advisory Committee* (SSAC).



Thank you for your attention.

