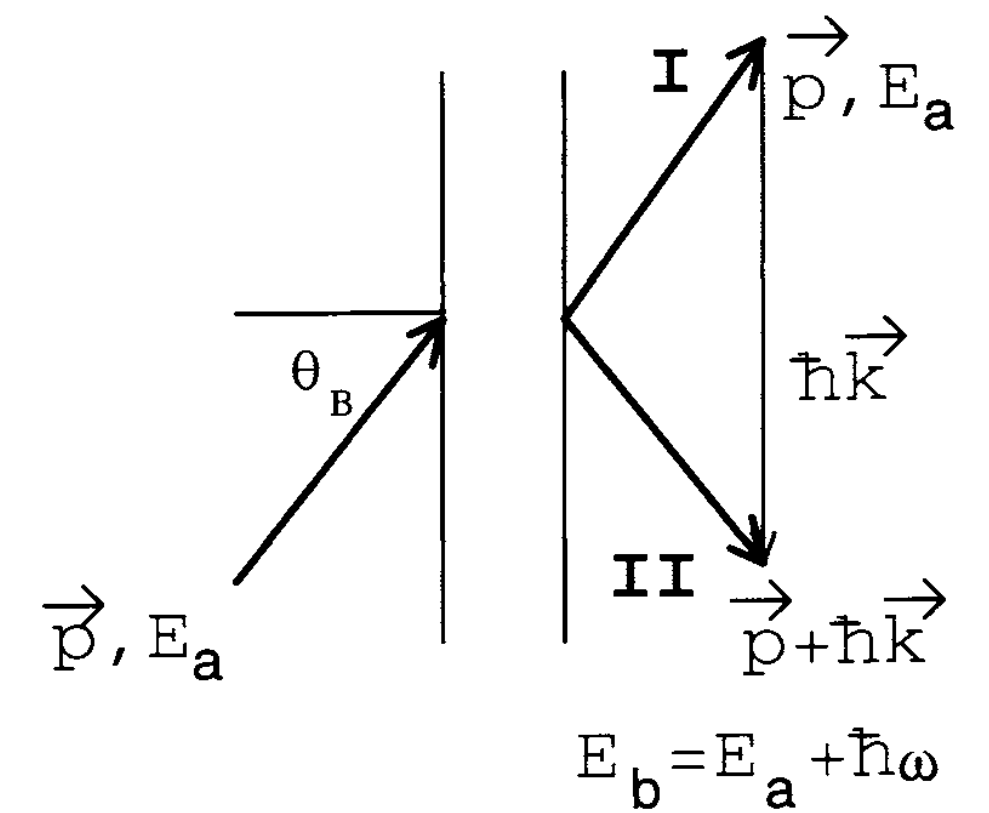
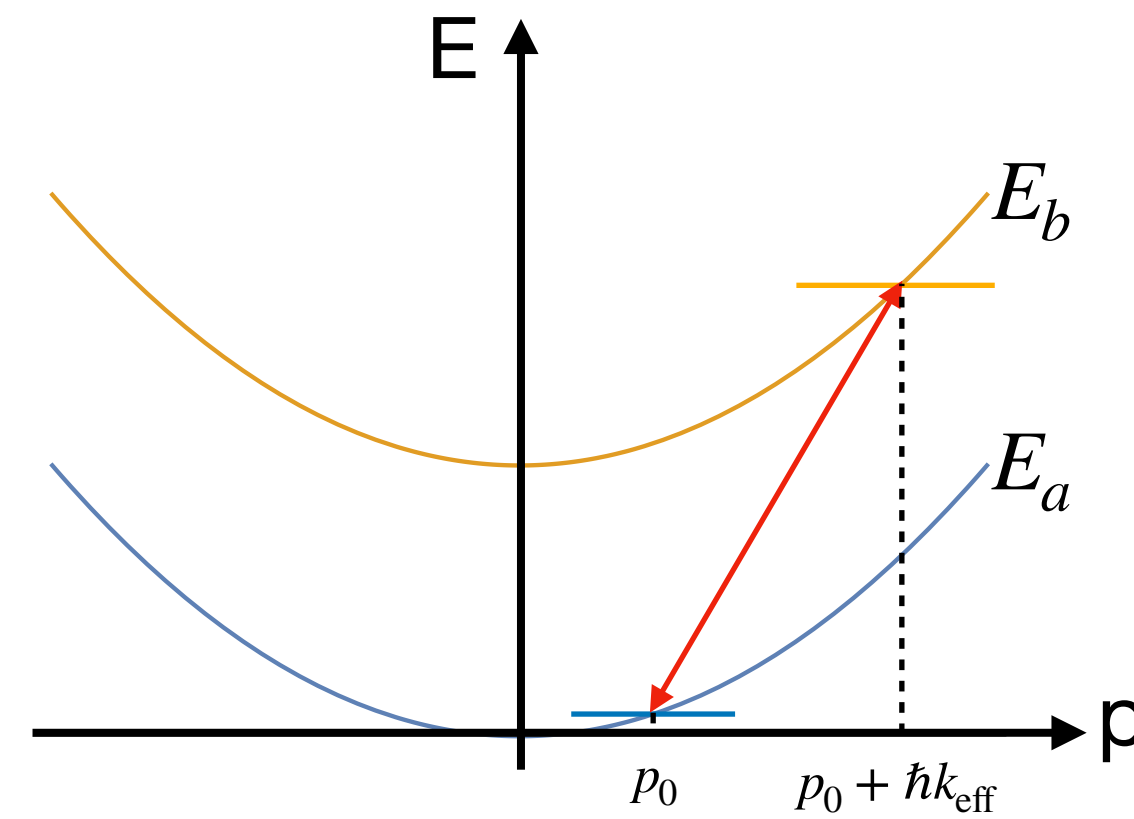
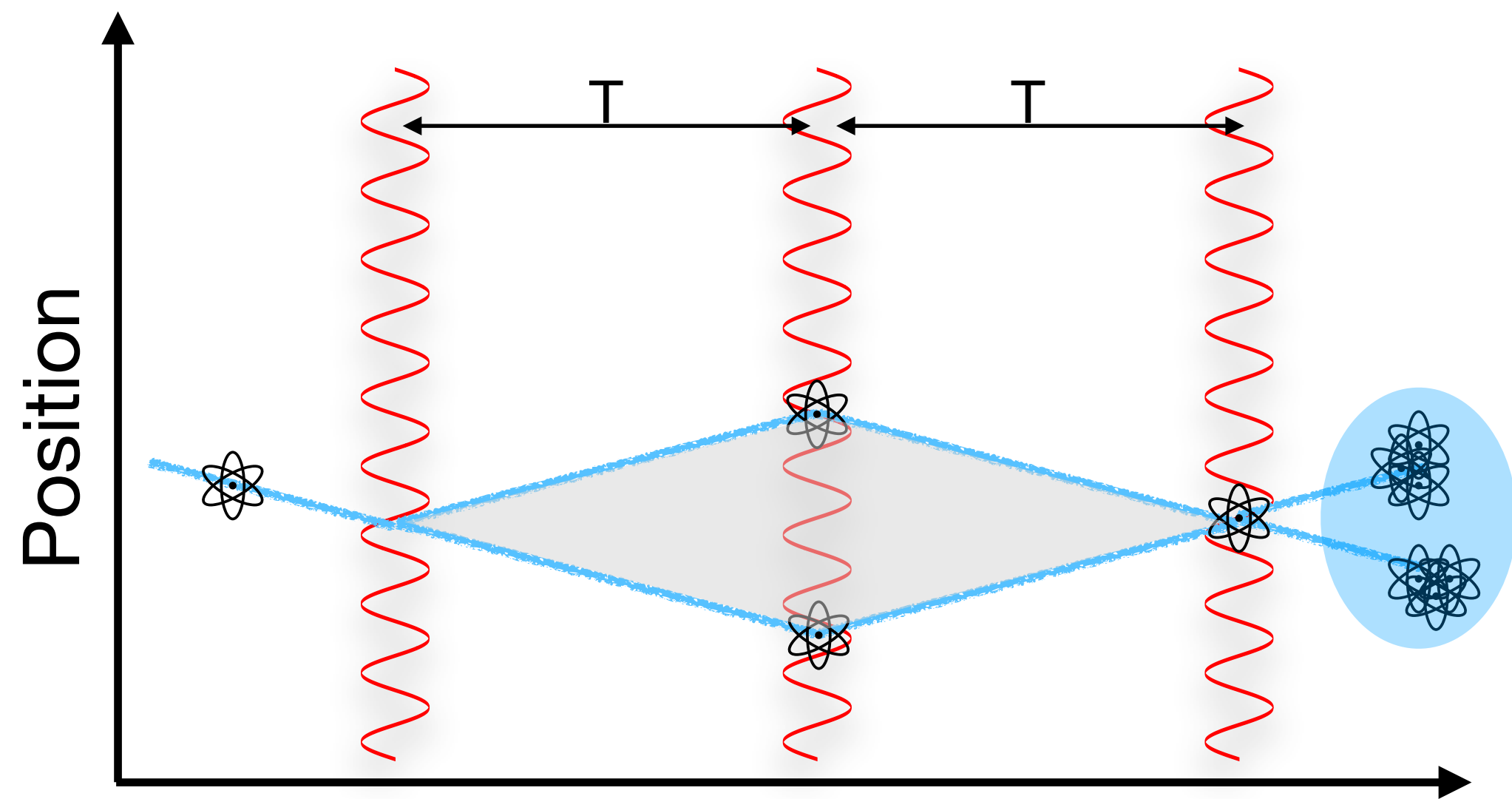


Optimal Floquet Engineering for Large Scale Interferometer

Alexandre Gauguet

Light-pulse Atom Interferometry

Large Momentum Transfer (LMT)

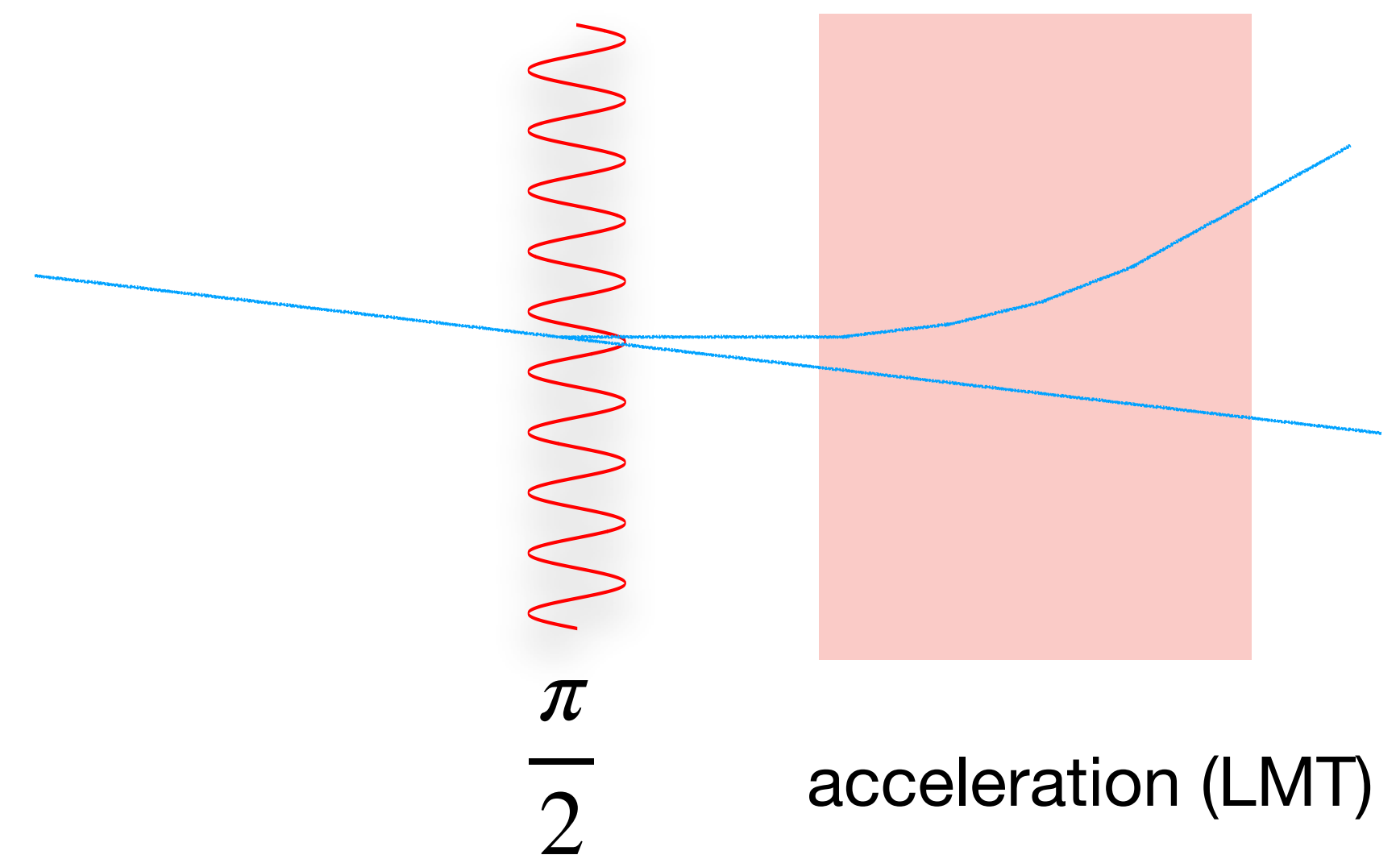
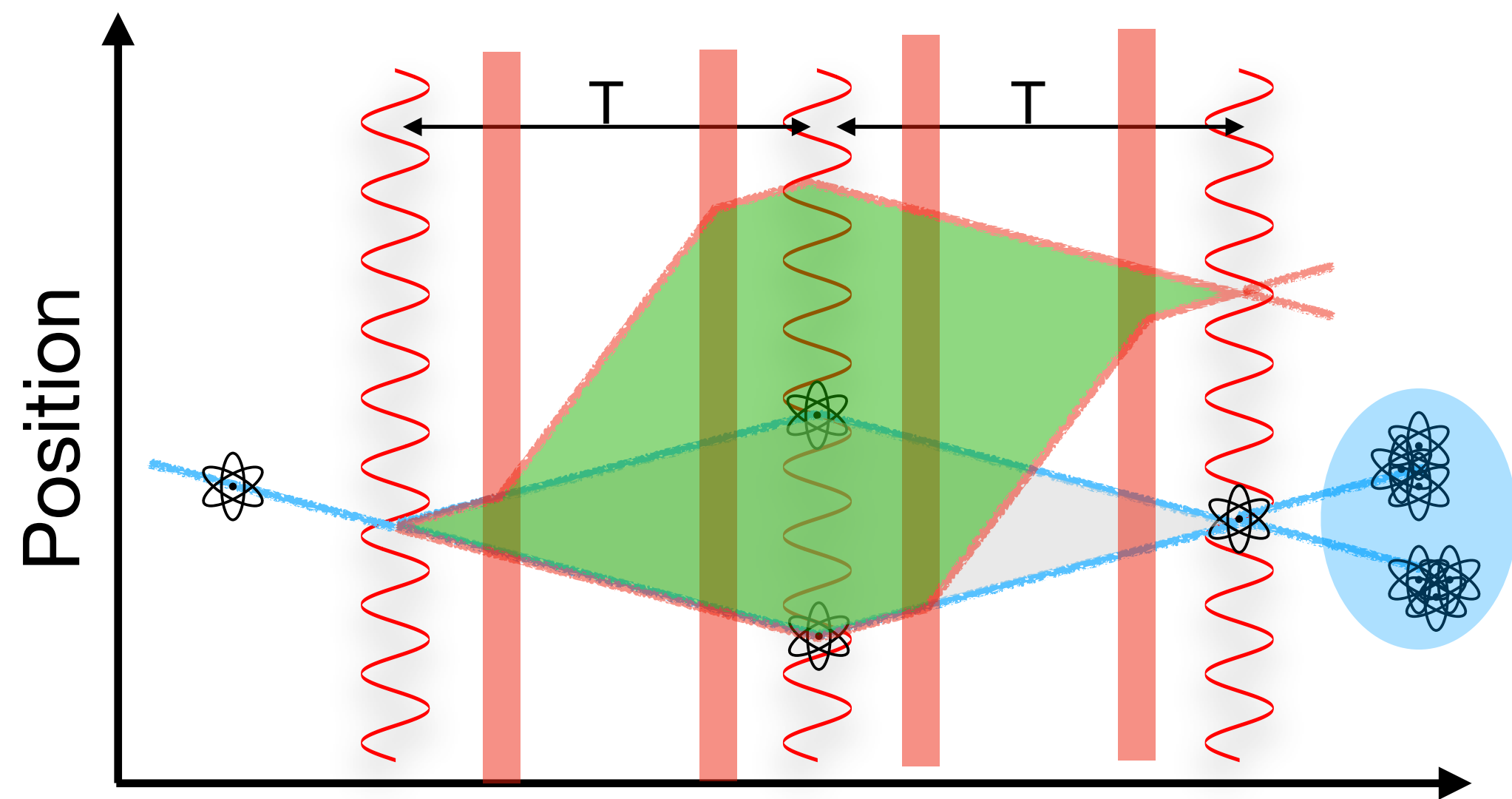


C.J Bordé, *Matter-Wave Interferometers: a synthetic approach*, (1997)

Coherent transfer of momentum from light-fields $\hbar k_{\text{eff}}$ to the atomic wave function.

Light-pulse Atom Interferometry

Large Momentum Transfer (LMT)



TVLBAI - roadmap:

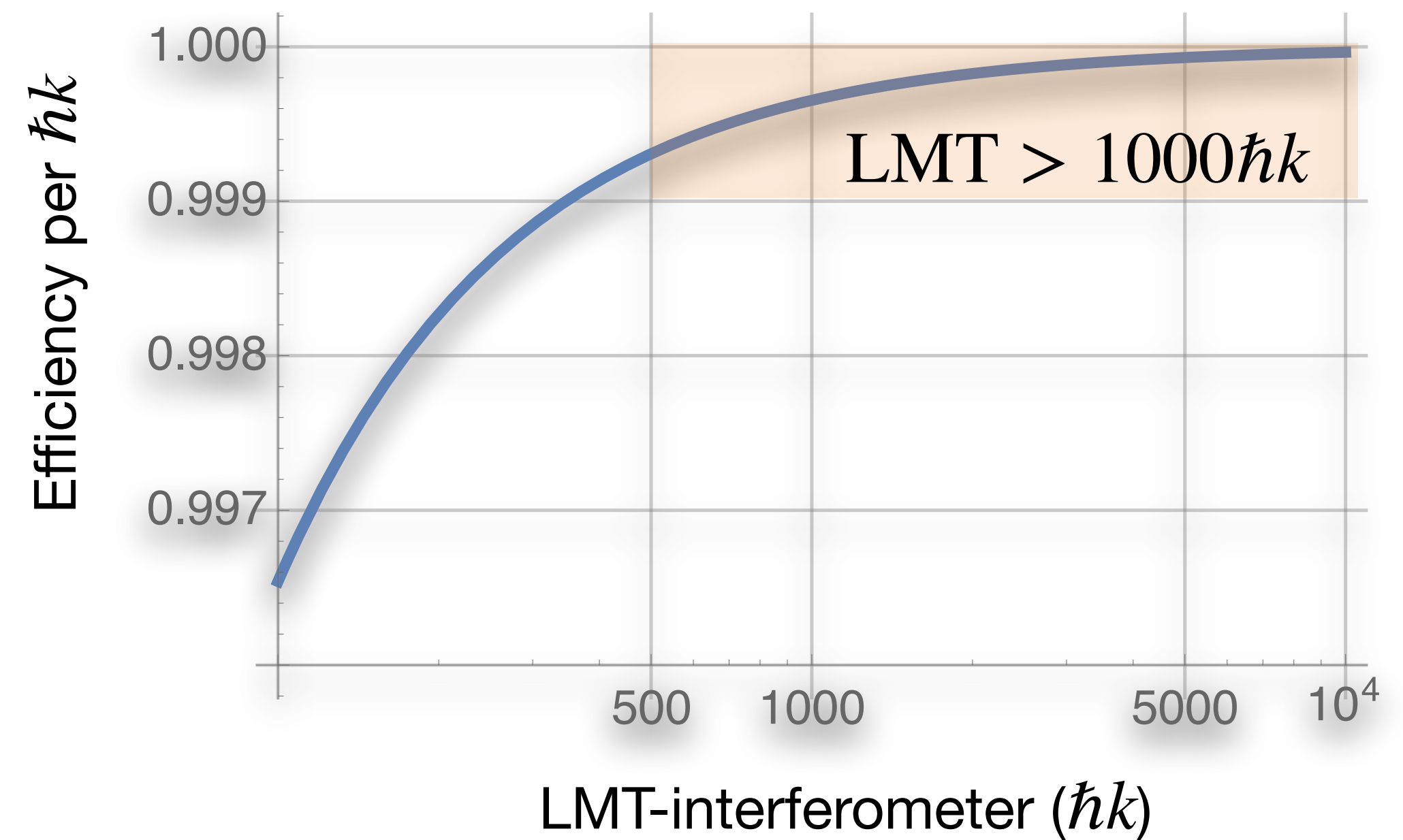
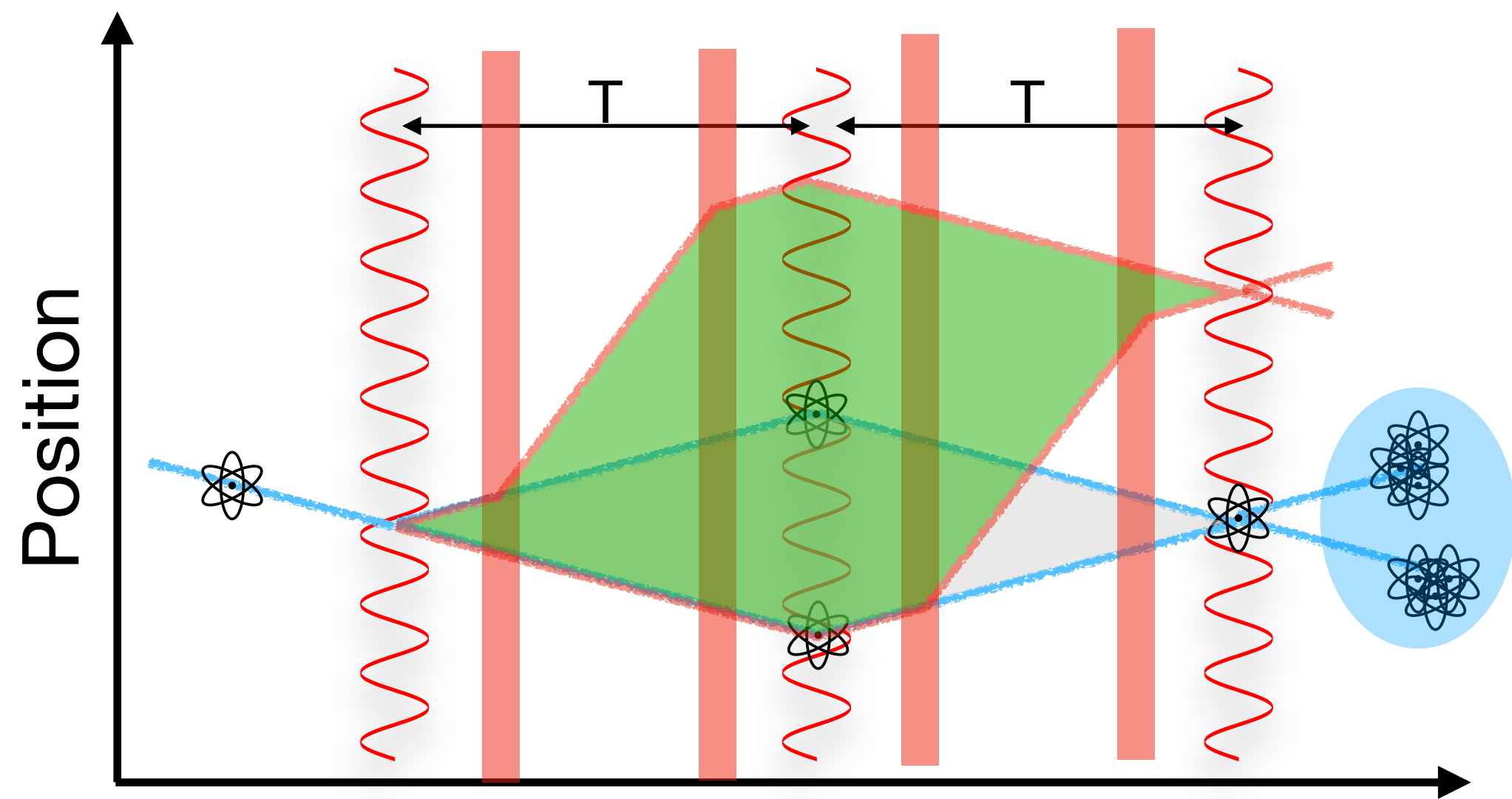
Sensor Technology	State-of-the-art	Target	Enhancement
LMT atom optics	$10^2 \hbar k$	$10^4 \hbar k$	100
Matter-wave lensing	50 pK	5 pK	–
Laser Power	10 W	100 W	–
Spin squeezing	20 dB (Rb), 0 dB (Sr)	20 dB (Sr)	10
Atom flux	10^5 atoms/s (Rb)	10^7 atoms/s (Sr)	10
Baseline length	10 m	1000 m	100

« We believe that there are no serious barriers to realization of momentum transfers greater than $10\hbar k$. »

J.M. McGuirk et al. Large Area Light-Pulse Atom interferometry PRL (2000)

A quick tour of LMT beam splitters

Requirements for Large Scale Interferometers



TVLBAI - roadmap:

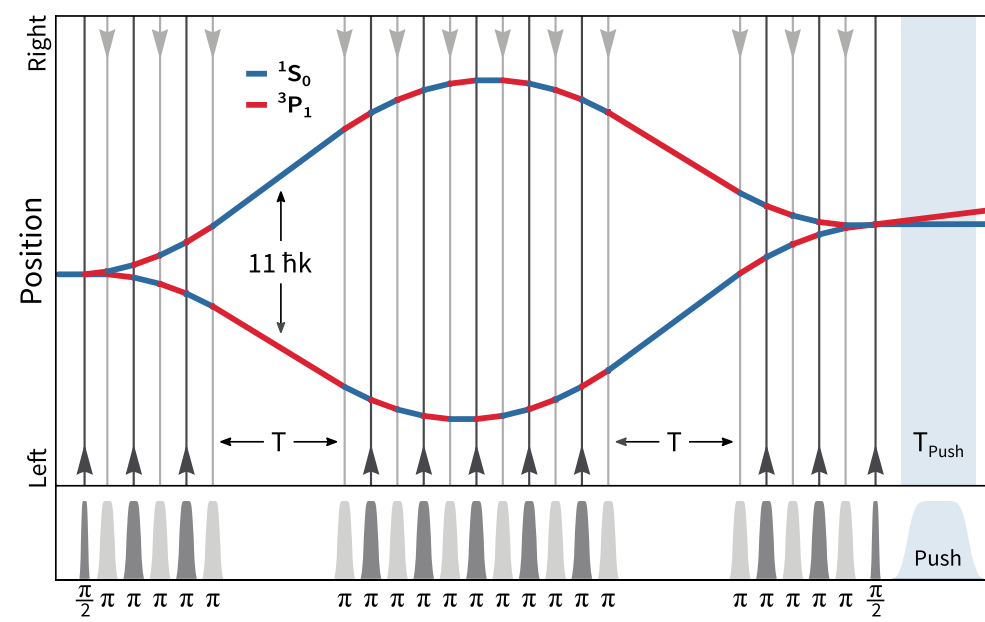
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High efficiency & High rate

Light-pulse Atom Interferometry

Large Momentum Transfer

Single photon transition

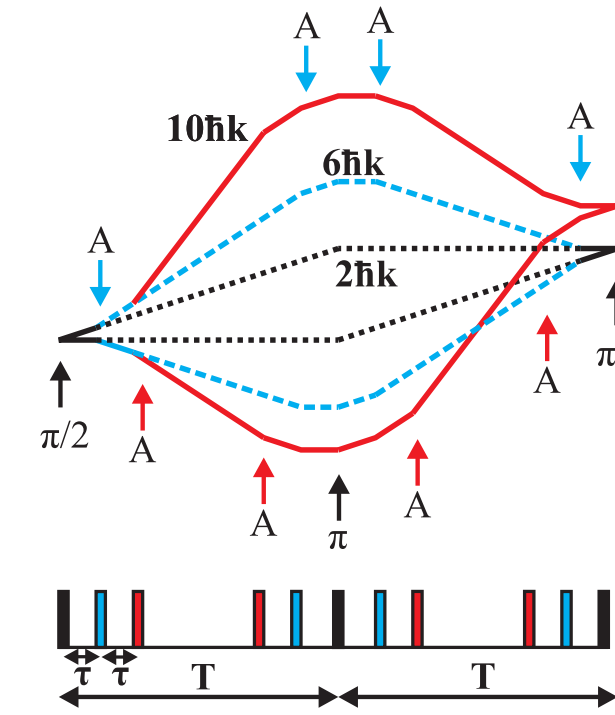


Rudolph et al., Phys. Rev. Lett. **124**, 083604 (2020)

- 2-level system optical transition
- Metastable excited state
- Resonant transition \rightarrow Spont. Emission
- Rabi frequency $\Omega \sim$ MHz
- Demonstrated $400\hbar k$
- Very high rate transfer: $\sim 0.1 \mu\text{s}/\hbar k$

Wilkason et al., Phys. Rev. Lett., **129**, 183202 (2022)

Two-photon Raman transition

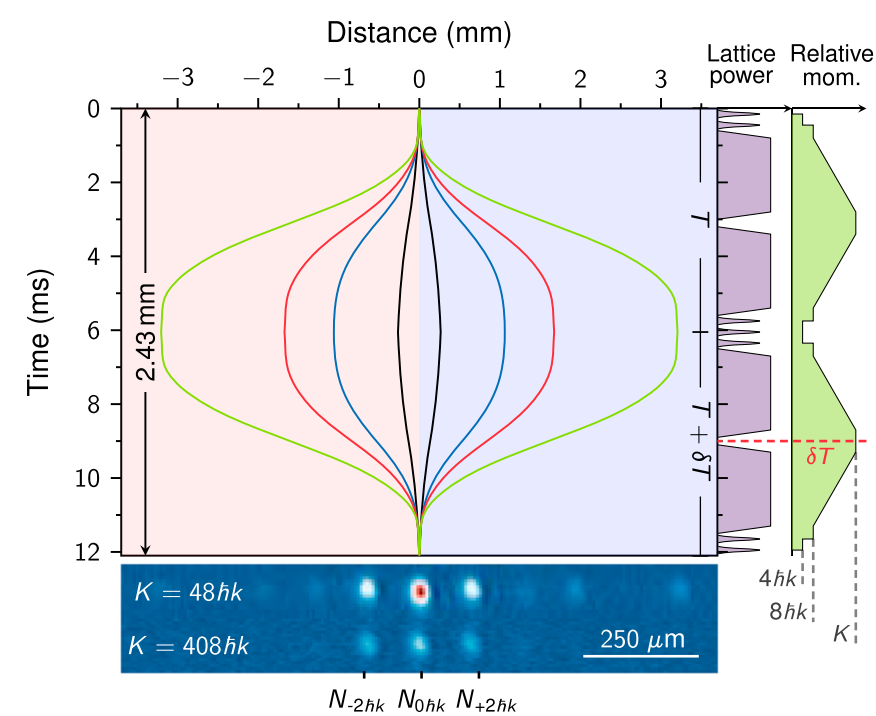


- Effective 2-level system, 2 photons transition
- Ground states
- Rabi frequency $\Omega \sim 100\text{kHz}$
- Demonstrated $30\hbar k$
- High rate transfer: $\sim 2.5 \mu\text{s}/\hbar k$

McGuirk et al., Phys. Rev. Lett., **85**, 4981(2015)

Kotru et al., Phys. Rev. Lett., **115**, 103001(2015)

Optical lattice Bloch-Type acceleration



Gebbe et al., Nat Commun, **12**, 2544 (2021)

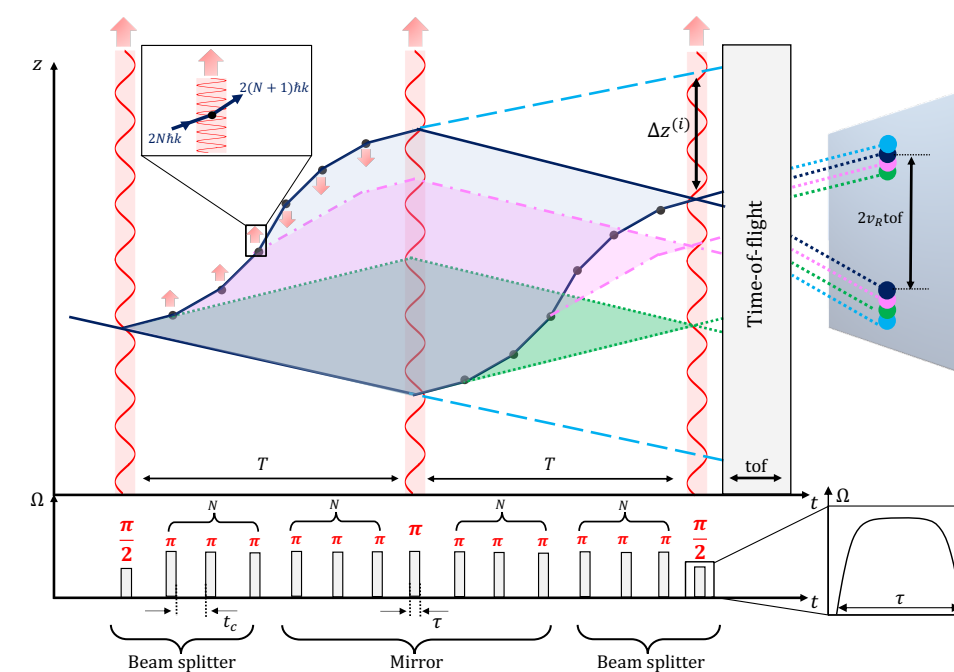
- multi-photon transitions - lattice states
- Ground state
- Rabi frequency $\Omega \sim 10\text{kHz}$
- Demonstrated $> 400\hbar k$
- Rate transfer: $\sim 6 \mu\text{s}/\hbar k$

Cladé et al., Phys. Rev. Lett. **102**, 240402 (2009)

McDonald et al., Phys. Rev. A **88**, 053620 (2013)

Pagel et al., Phys. Rev. A **102** 053312 (2020)

Optical lattice Sequence of Bragg pulses



Béguin et al., Phys. Rev. Lett., **131**, 143401 (2023)

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Chiu et al., Phys. Rev. Lett., **107**, 130403 (2011)

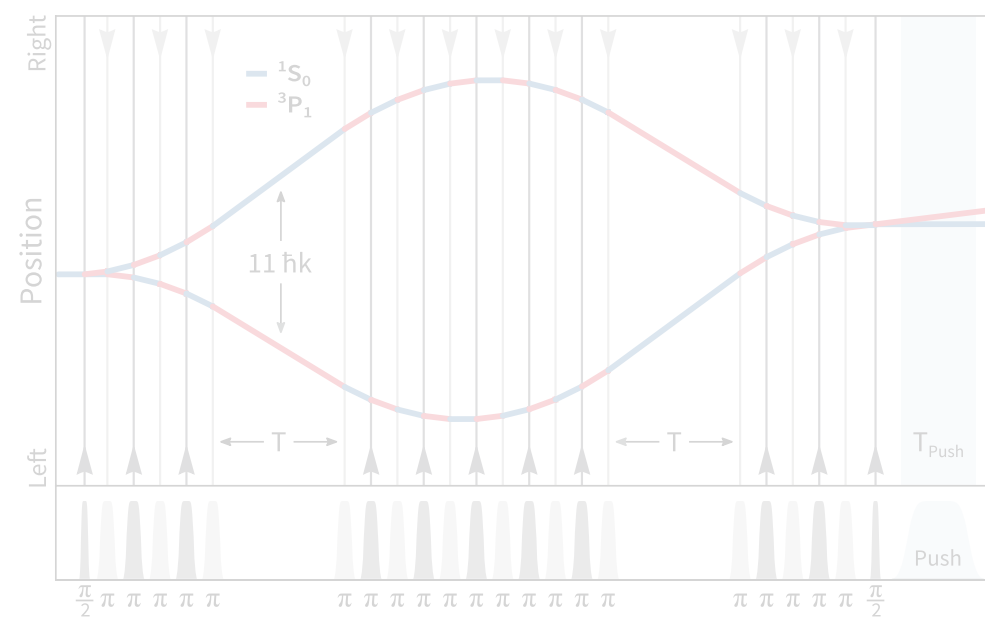
Plotkin-Swing et al., Phys. Rev. Lett., **121**, 133201 (2018)

Rodzinka et al. . arXiv:2403.14337

Light-pulse Atom Interferometry

Large Momentum Transfer

Single photon transition

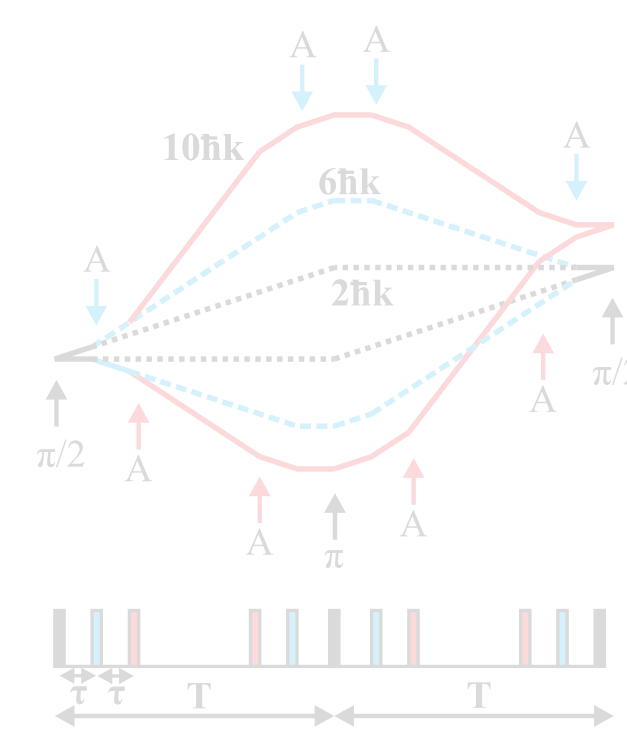


Rudolph et al., Phys. Rev. Lett. **124**, 083604 (2020)

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Wilkason et al., Phys. Rev. Lett., **129**, 183202 (2022)

Two-photon Raman transition

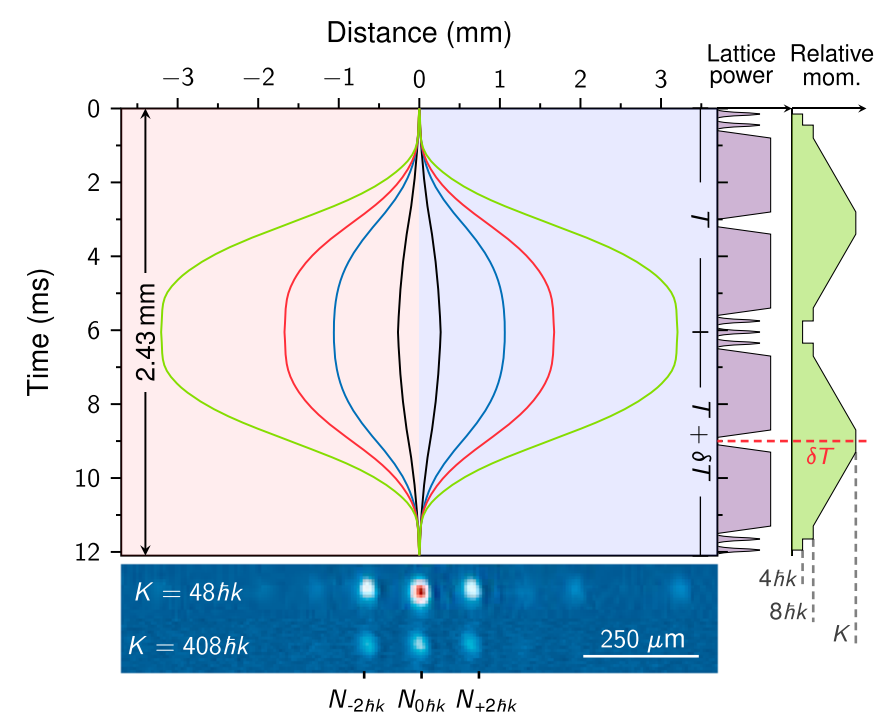


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Gebbe et al., Nat Commun, **12**, 2544 (2021)

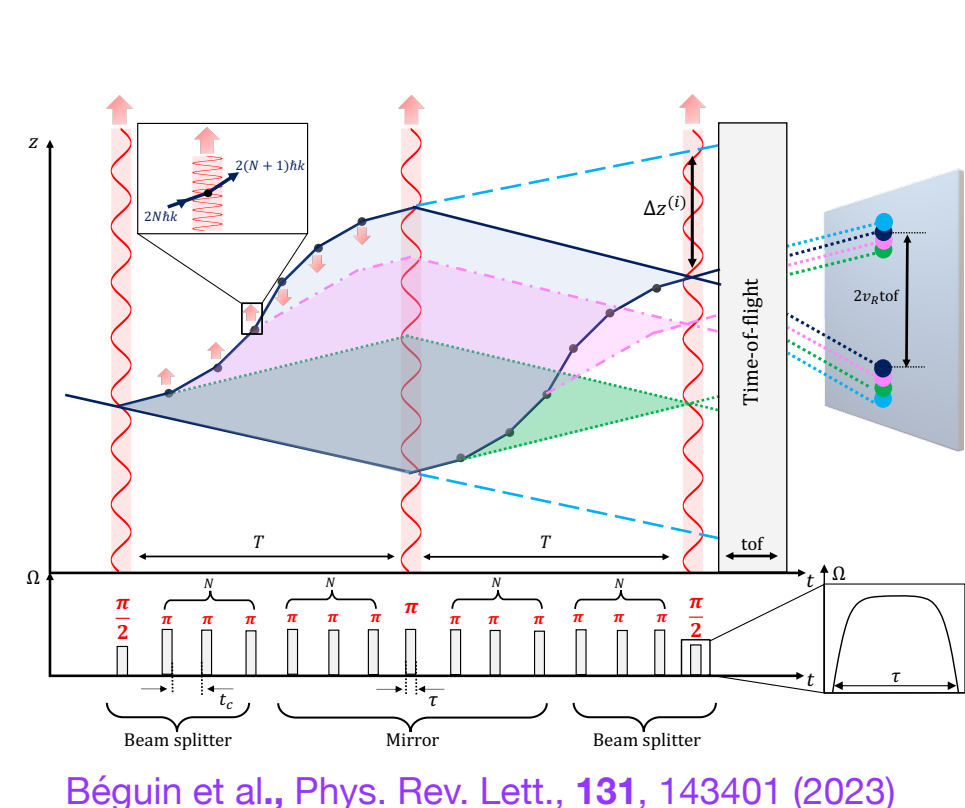
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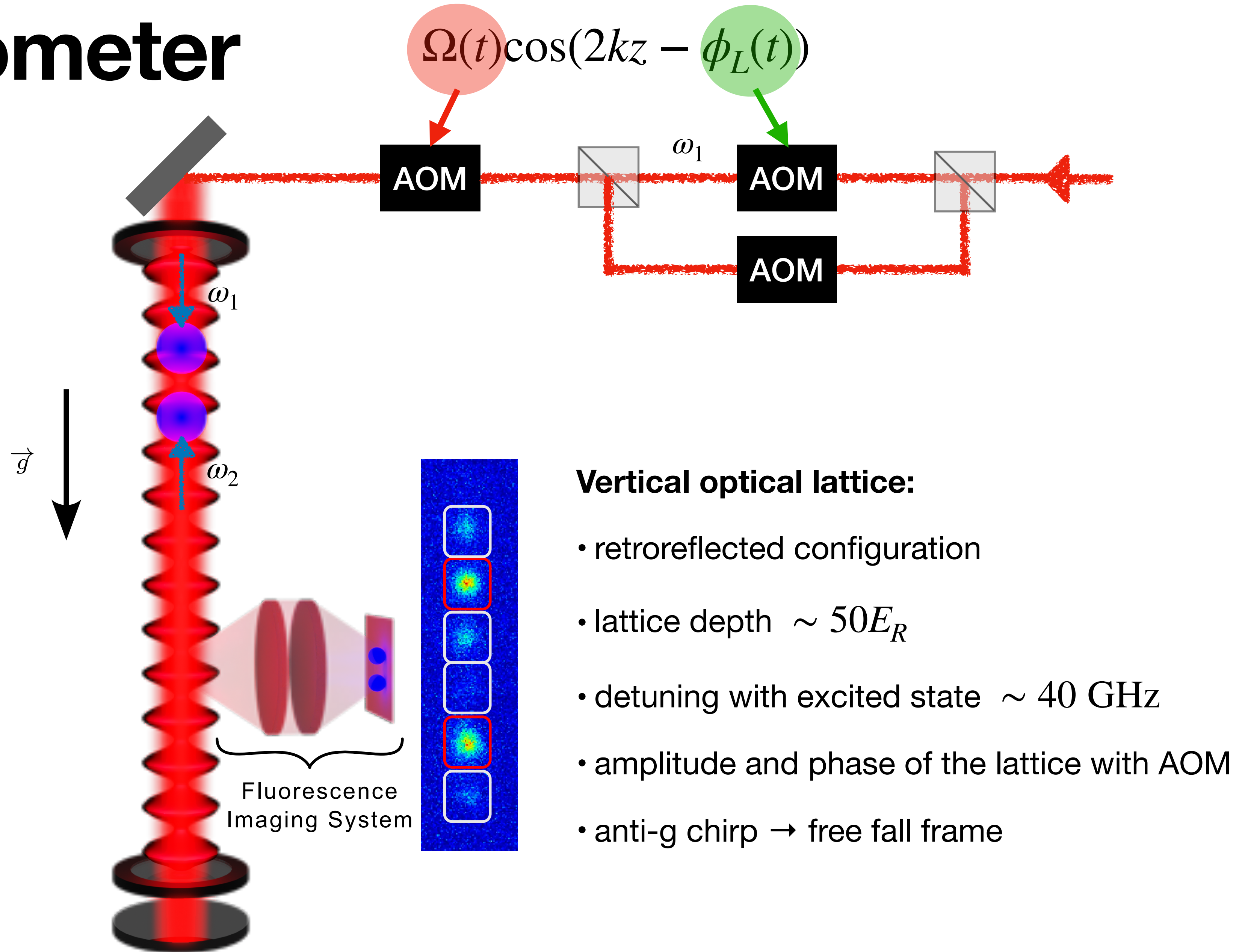
Chiu et al., Phys. Rev. Lett., **107**, 130403 (2011)

Plotkin-Swing et al., Phys. Rev. Lett., **121**, 133201 (2018)

Rodzinka et al. [arXiv:2403.14337](https://arxiv.org/abs/2403.14337)

Atom Interferometer

Experimental setup

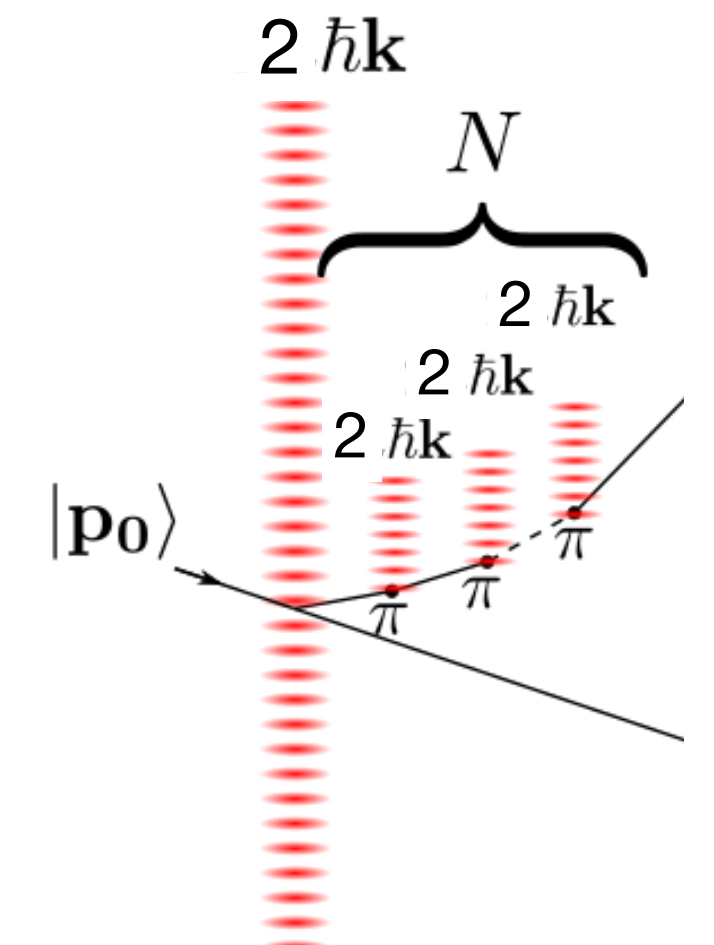
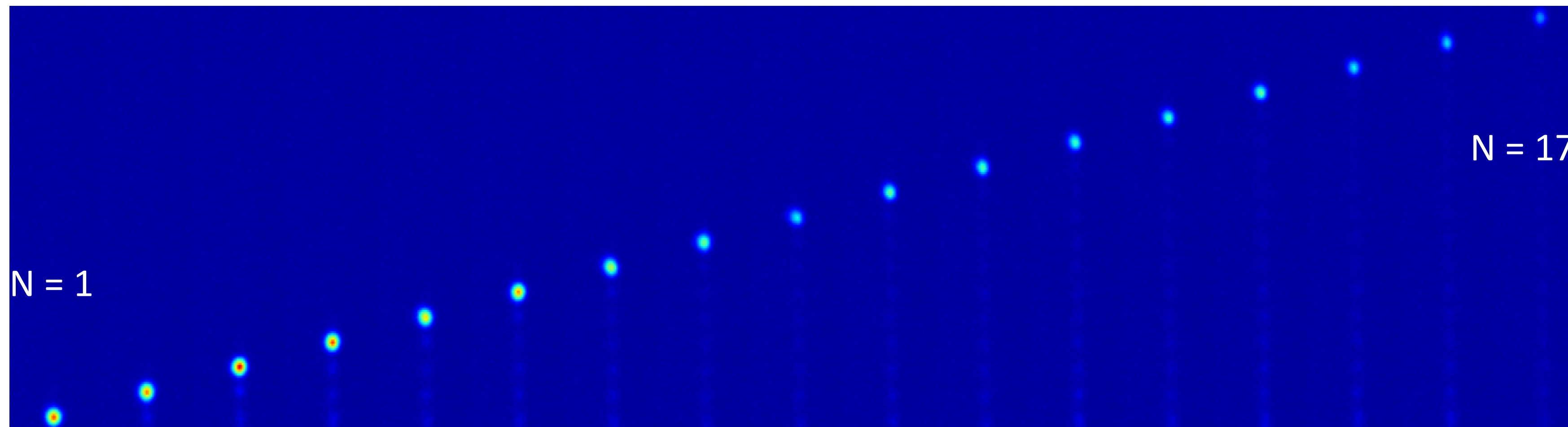
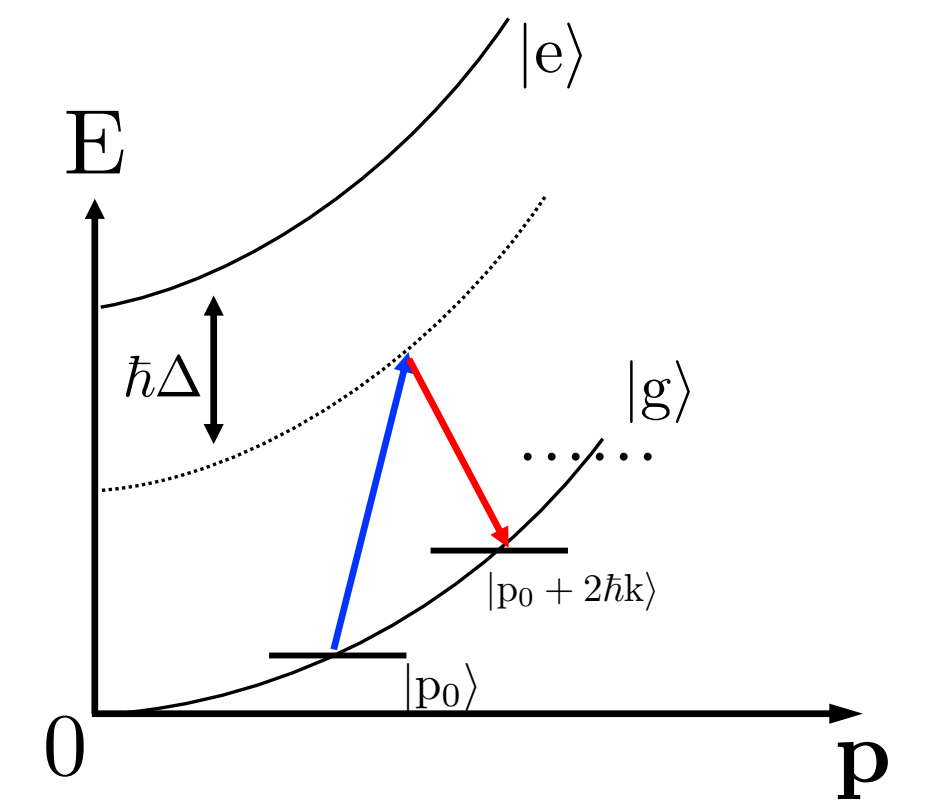


LMT: Bragg pulse sequence

Independent pulses: gaussian pulse

Beam splitter made with multiple $n = 1$ accelerating (mirror) pulses.

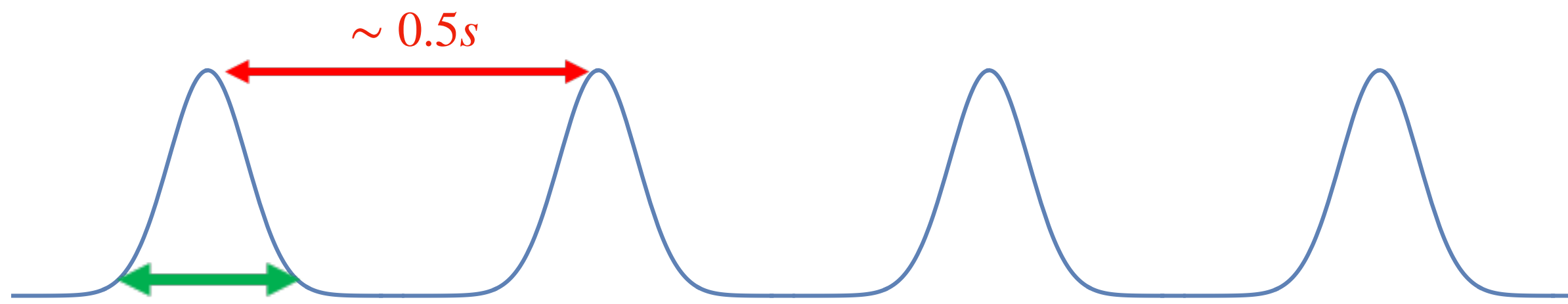
Chiou et al. Phys. Rev. Lett. 107, 130403 (2011)



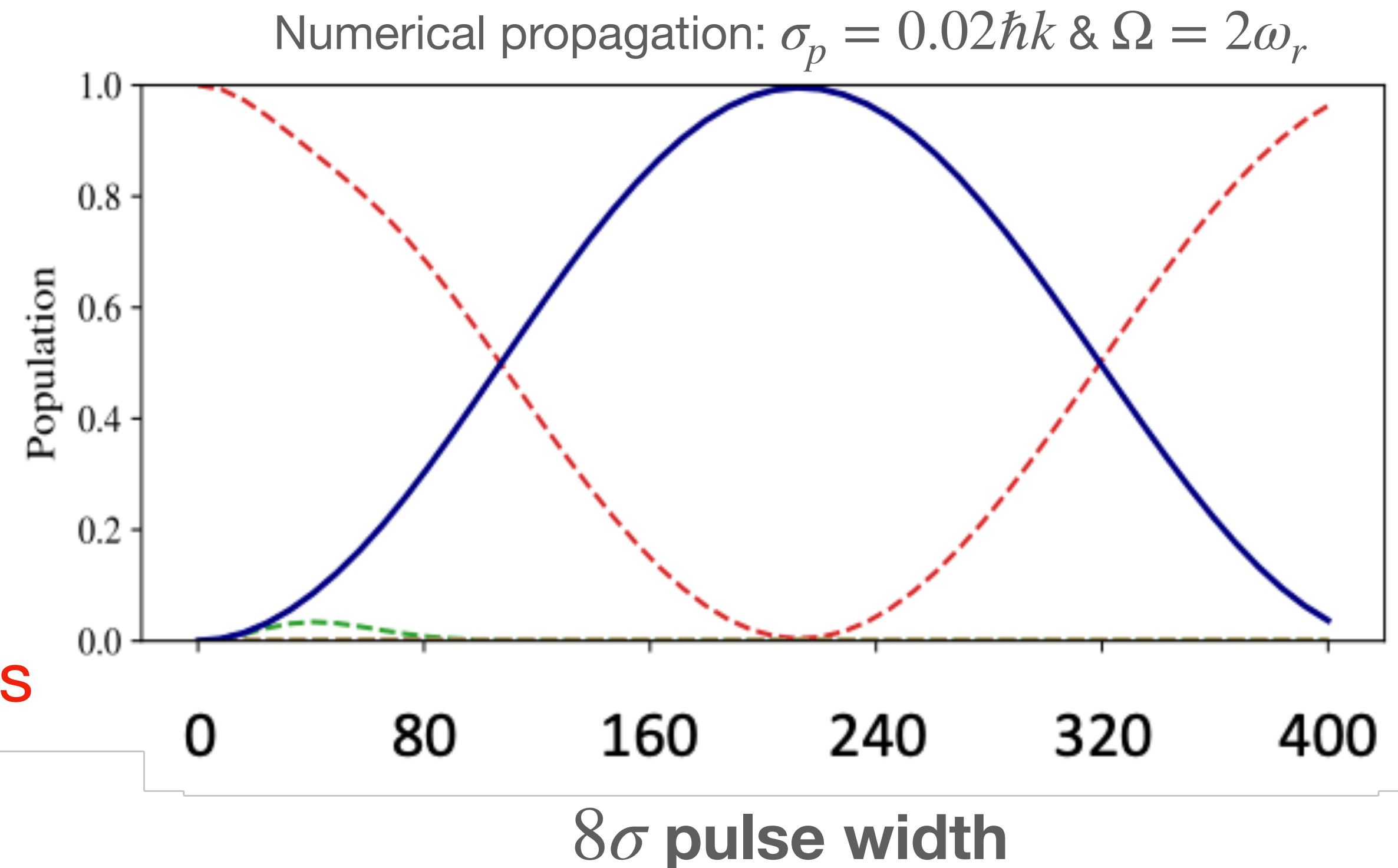
- Better control of non-adiabatic losses
- Less spontaneous emission per $\hbar k$

LMT: Bragg pulse sequence

Independent pulses: gaussian pulse

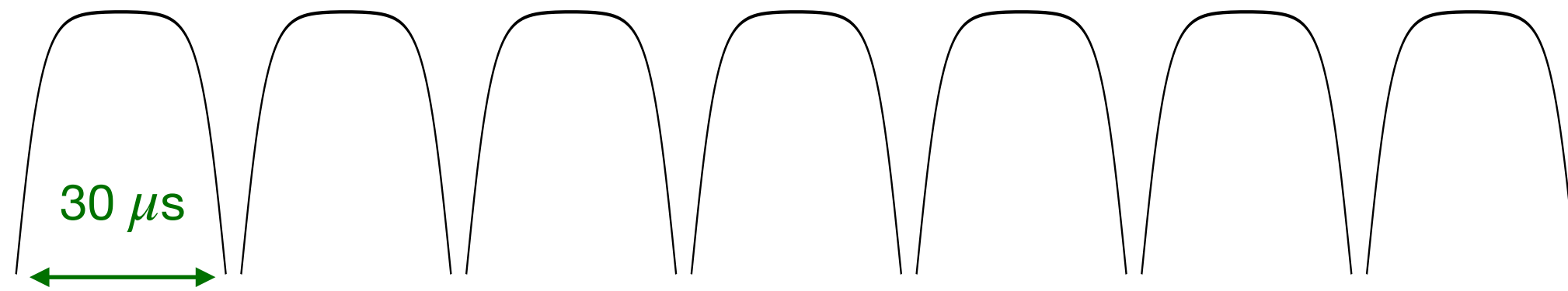


- Adiabatic condition : Long pulses
- Very low temperature < nK
- $> 1000\hbar k$ -interferometer needs few seconds



LMT: Bragg pulse sequence

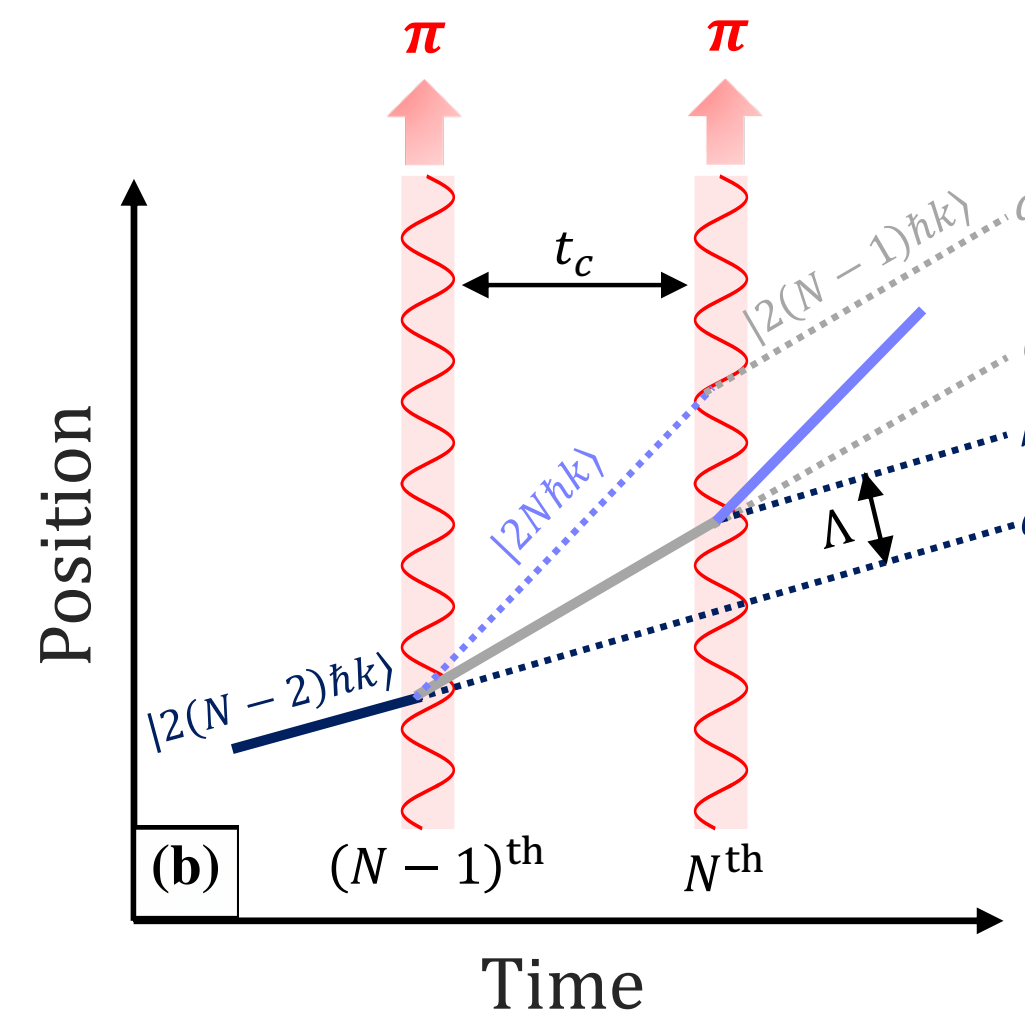
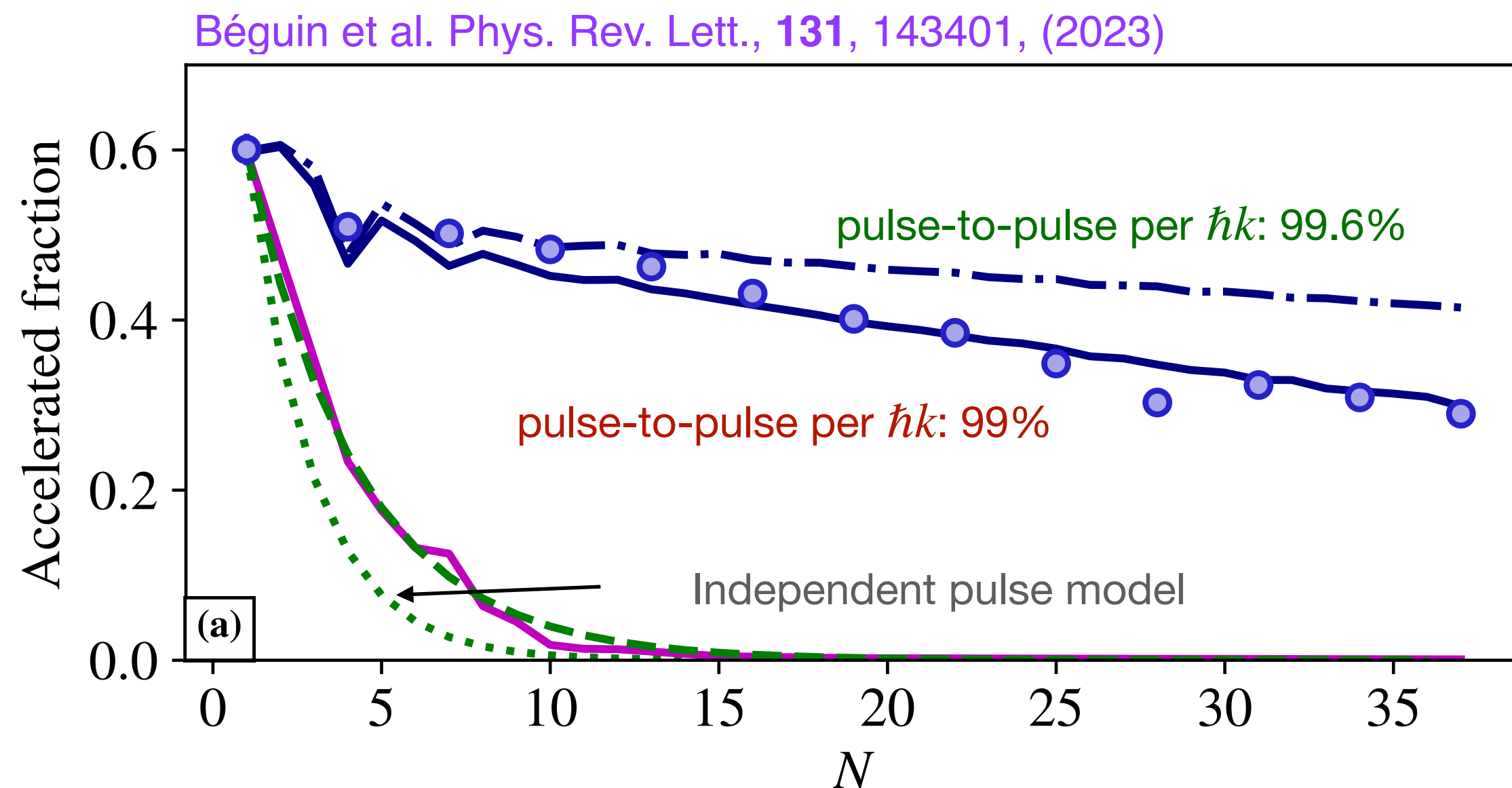
Coherent Enhanced pulses



Short pulses and fast train pulses

Non-adiabatic losses = coherent losses

Small spatial separation $\Lambda = 2v_R t_c \ll \xi = \frac{\hbar}{m\sigma_v}$



Losses interfere destructively:

$$P_{|N-2\rangle} = 2\epsilon^2(1 + \cos(\pi + 4\omega_R t_c))$$

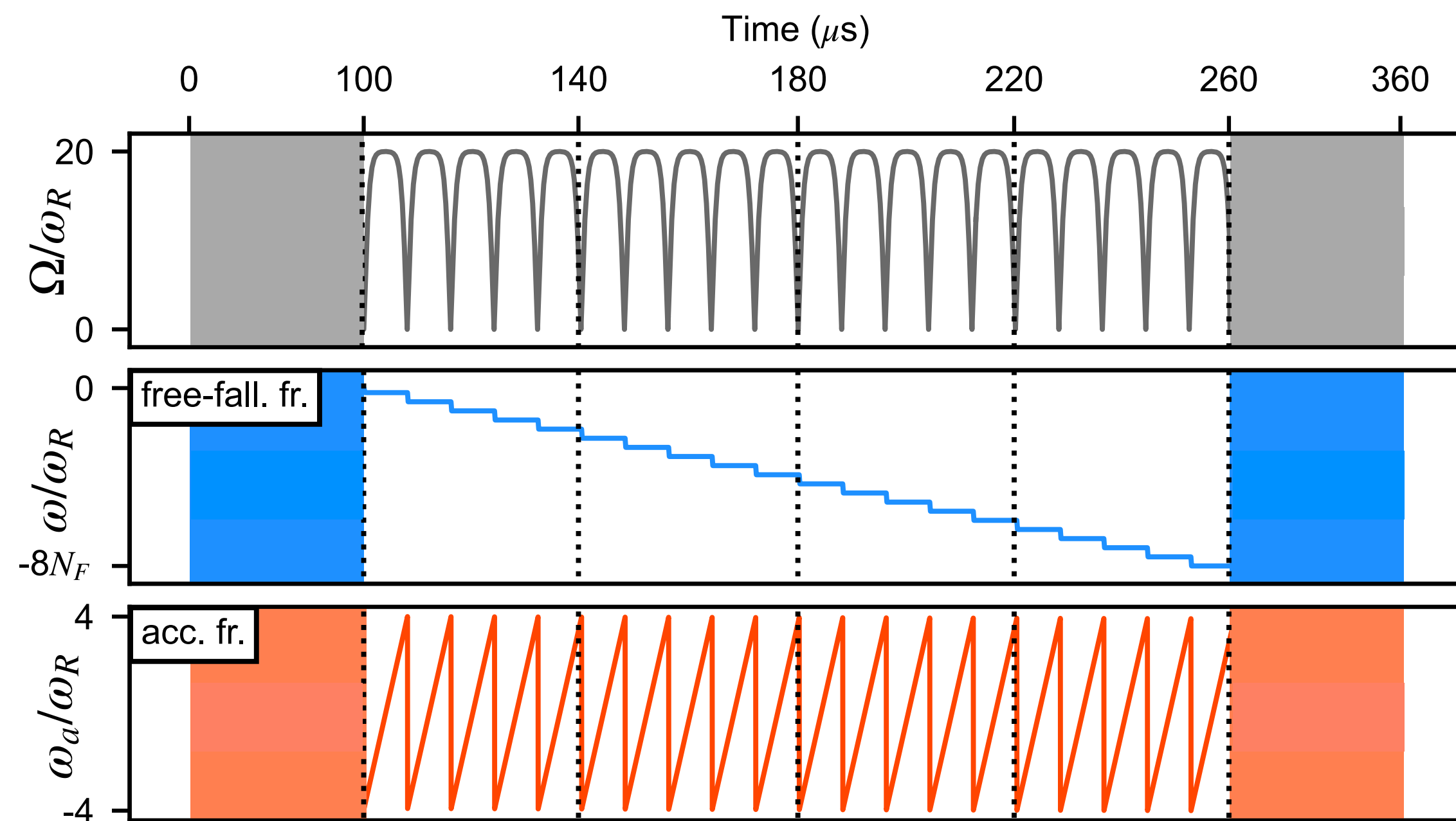
98% pulse-to-pulse efficiency

Higher efficiency ?

Faster transfer ?

Stroboscopic stabilization in the accelerated frame

Optical Lattice with periodic driving



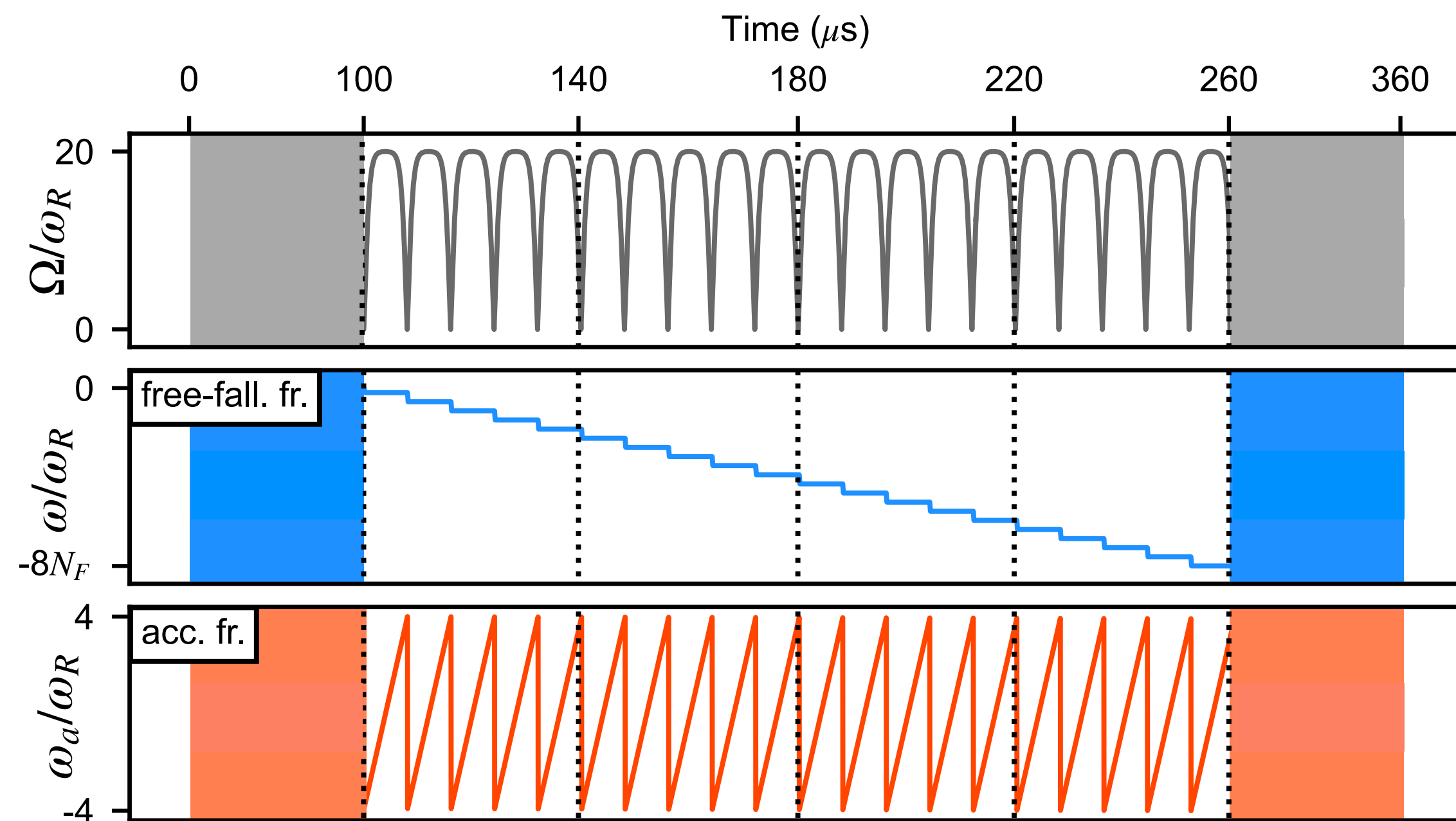
- Periodic driving in the **accelerating frame**
- Periodic hamiltonian $H(t_0) = H(t_0 + \tau)$



Floquet Formalism

Stroboscopic stabilization in the accelerated frame

Floquet's formalism



- Periodic driving in the **accelerating frame**
- Periodic hamiltonian $H(t_0) = H(t_0 + \tau)$



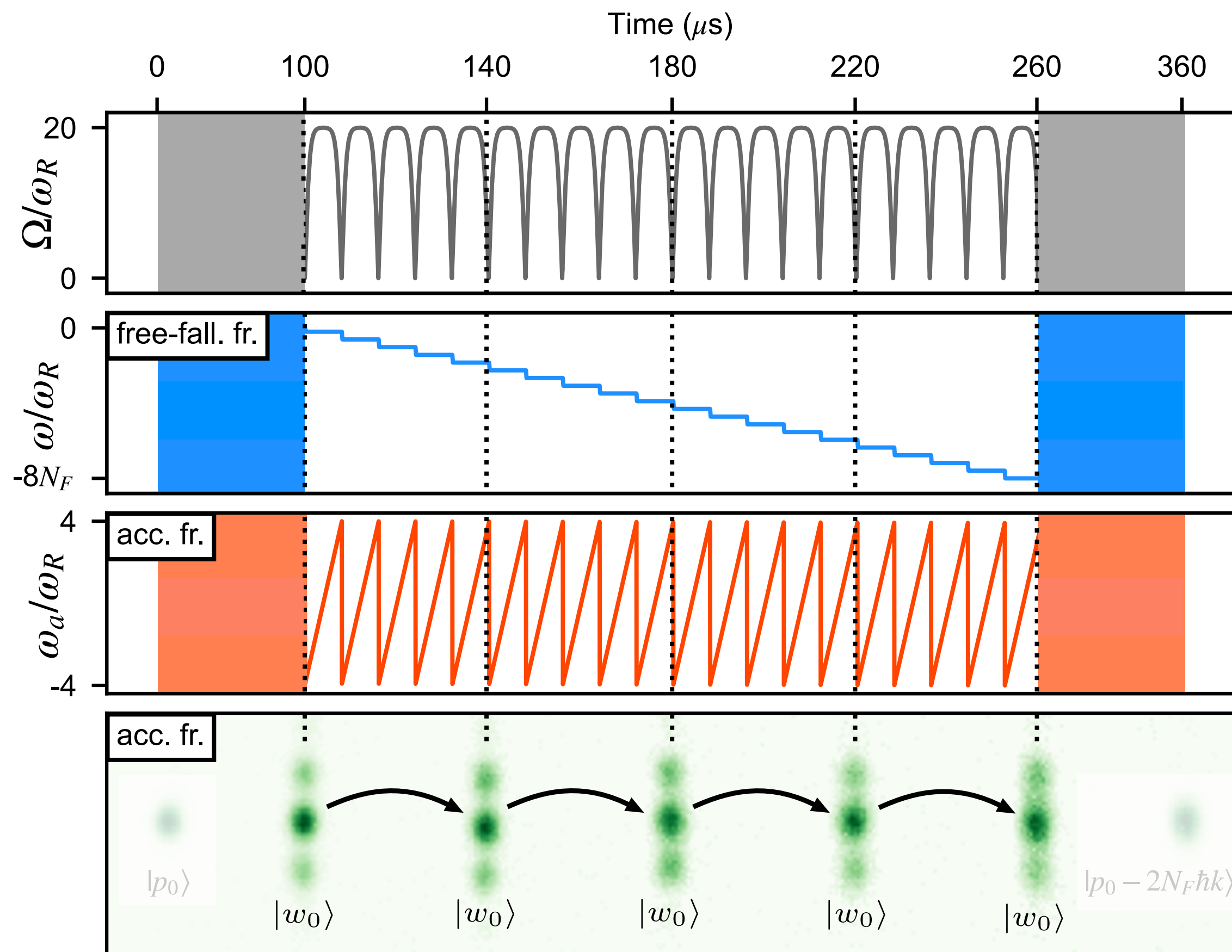
Floquet Formalism

Diagonalization of the one-period propagator = Floquet states $|w_n(t)\rangle = |u_n(t)\rangle e^{i\theta_n}$ with $|u_n(t + \tau)\rangle = |u_n(t)\rangle$

$$\text{Initial state } |\psi(0)\rangle = \sum_n c_n |u_n\rangle \longrightarrow |\psi(t)\rangle = \sum_n c_n |u_n(t)\rangle e^{i\theta_n}$$

Stroboscopic stabilization in the accelerated frame

Stabilization in the accelerated frame



Initial state prepared in a Floquet state

$$|\psi(0)\rangle = |w_k\rangle$$

Stroboscopic stabilization

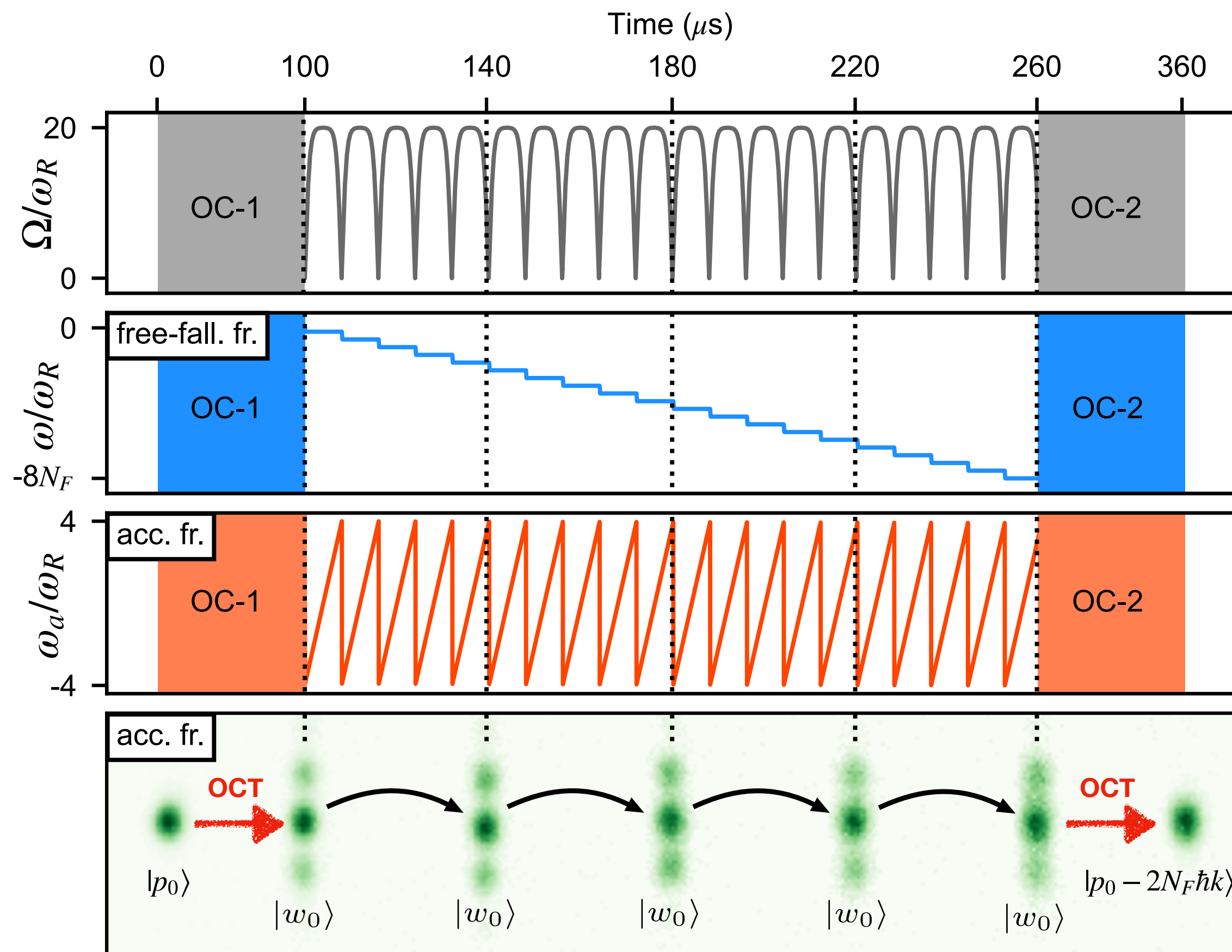
$$|\psi(m\tau)\rangle = |w_k\rangle e^{i\theta_k}$$

The wave function is ideally transported in the accelerated frame.

Floquet states can be defined for any periodic sequence

State preparation $|p_0\rangle \rightarrow |w_0\rangle$

Quantum Optimal Control Theory



We choose the Floquet state $|w_0\rangle$ with the largest projection on $|p_0\rangle$

Hamiltonian with control: amplitude $\Omega(t)$ and frequency $\omega_a(t)$

Find the control fields $\{\Omega(t), \omega_a(t)\}$, maximizing the figure of merit: $|\langle w_0 | \psi(t_f) \rangle|^2$

Optimization procedure with QOCT and implemented with Gradient based method (here GRAPE)

Ansel et al. arXiv: 2403.00532

The complexity of Optimal Control LMT is encapsulated into the Floquet state

Robust preparation

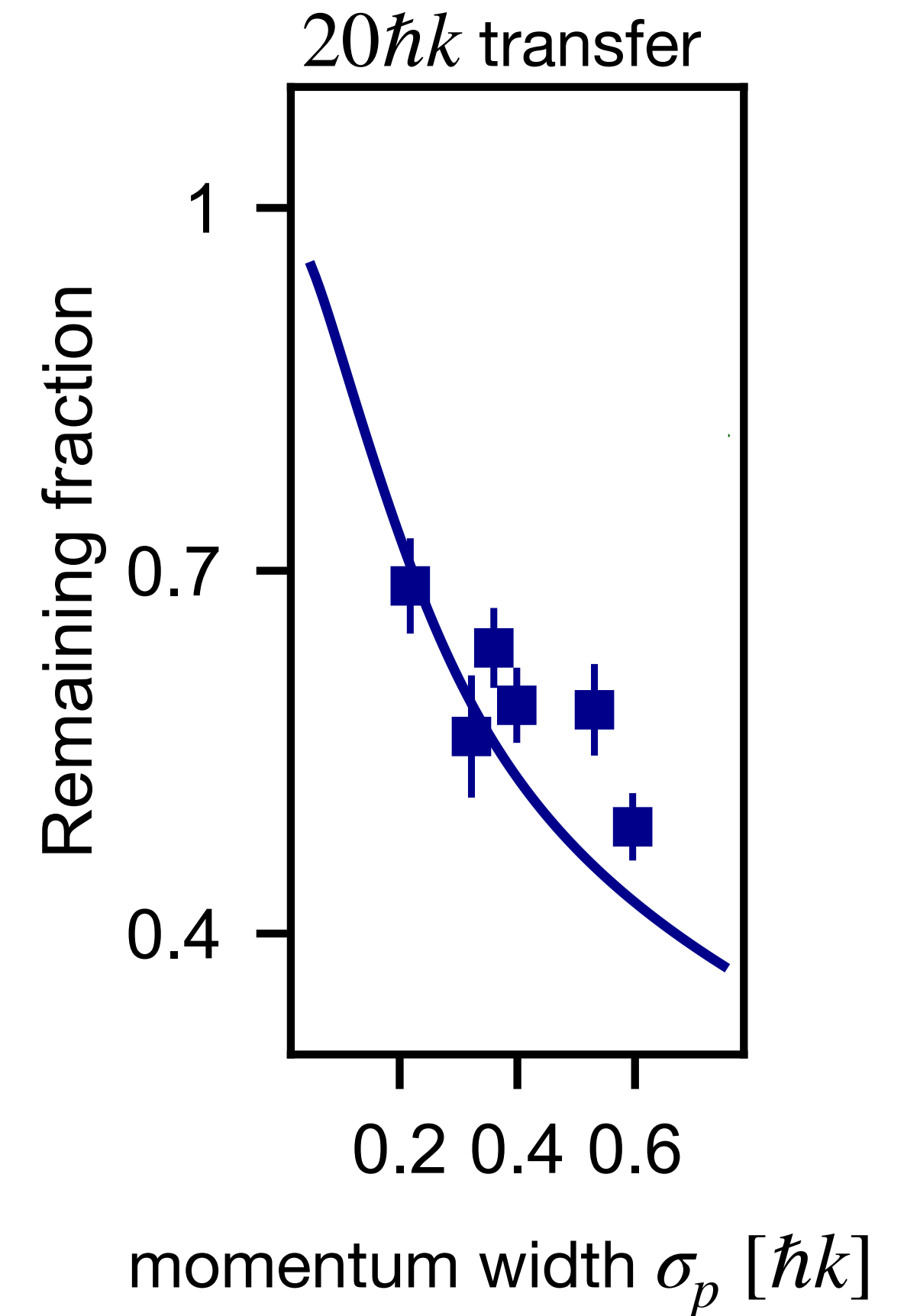
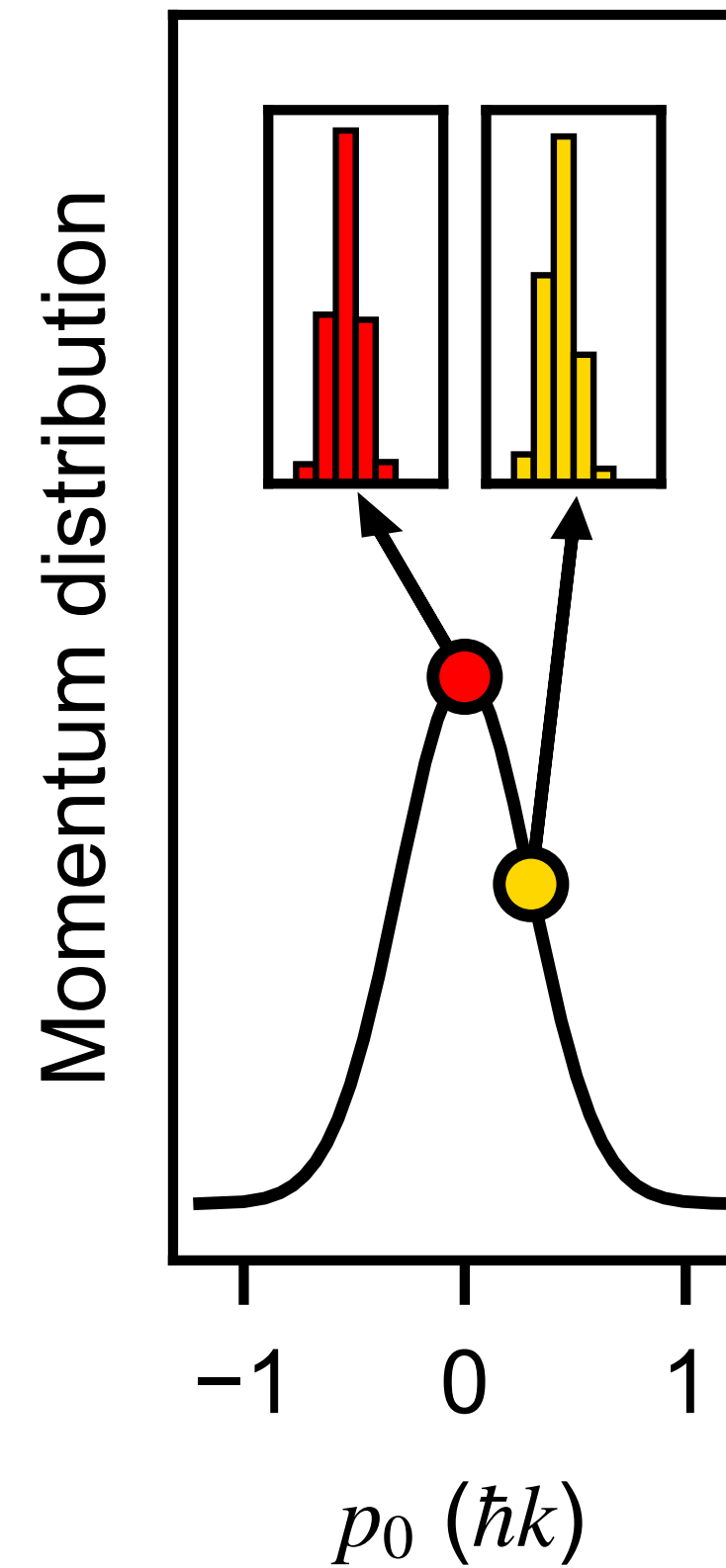
Initial statistical mixture

A floquet state for each momentum of the distribution

$$|p_0\rangle \rightarrow |w_0(p_0)\rangle$$

Simultaneous control for $|w_0(p_0)\rangle = \text{Robust against } p_0$

Rodzinka et al. [arXiv:2403.14337](https://arxiv.org/abs/2403.14337)



Robust preparation

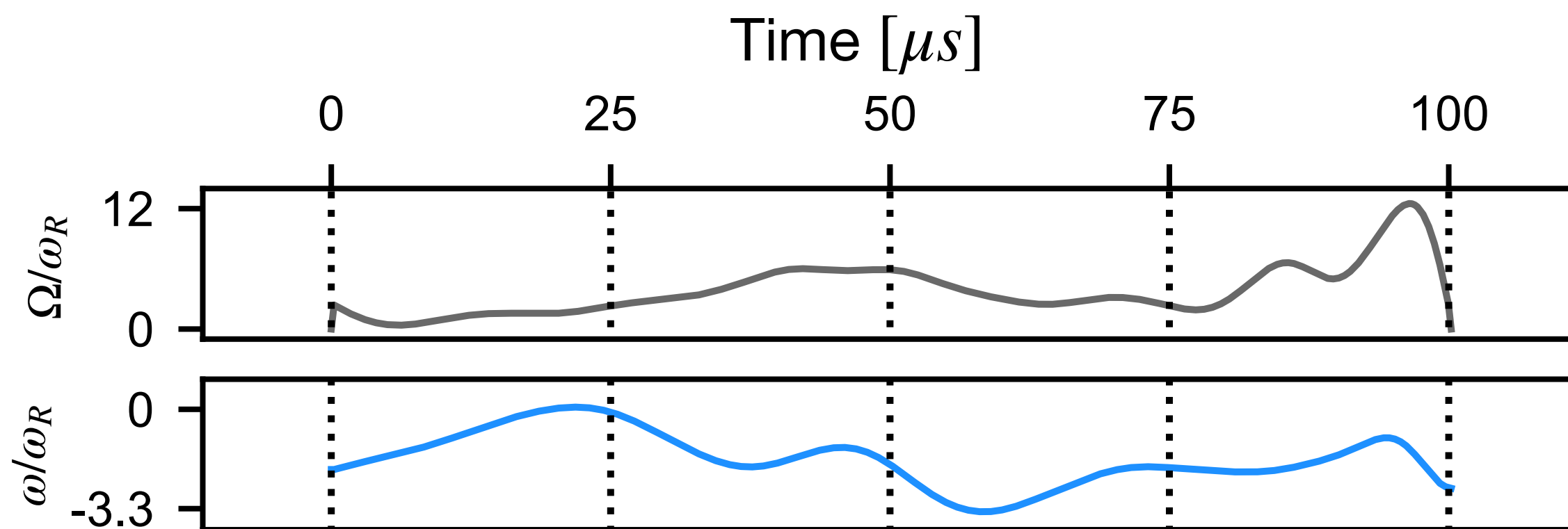
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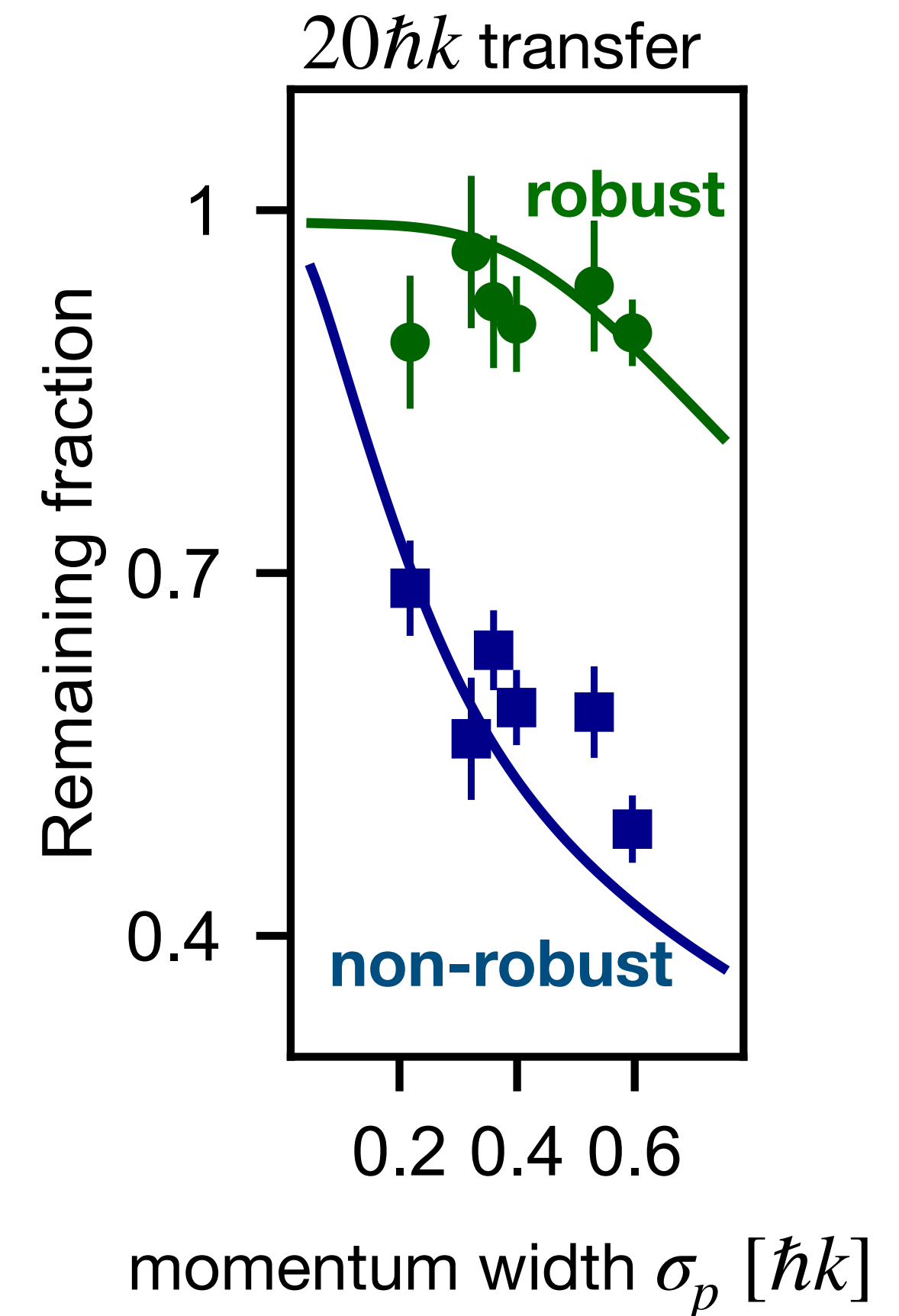
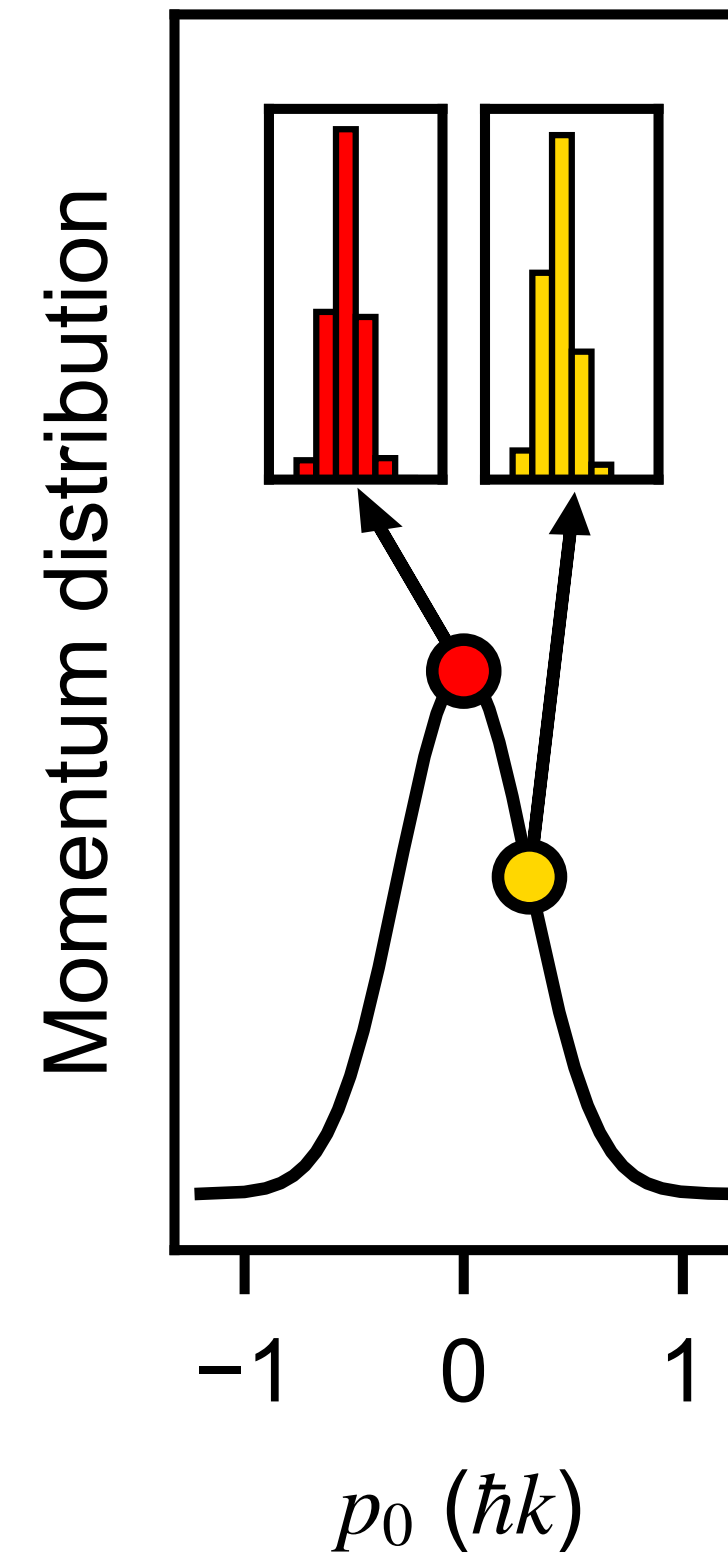
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Simultaneous control for $|w_0(p_0)\rangle = \text{Robust against } p_0$

Figure of merit $F_1 = \int_{-\infty}^{+\infty} |\langle \psi(\tau_c) | w_0(p_0) \rangle|^2 f(p_0) dp_0$



Rodzinka et al. [arXiv:2403.14337](https://arxiv.org/abs/2403.14337)

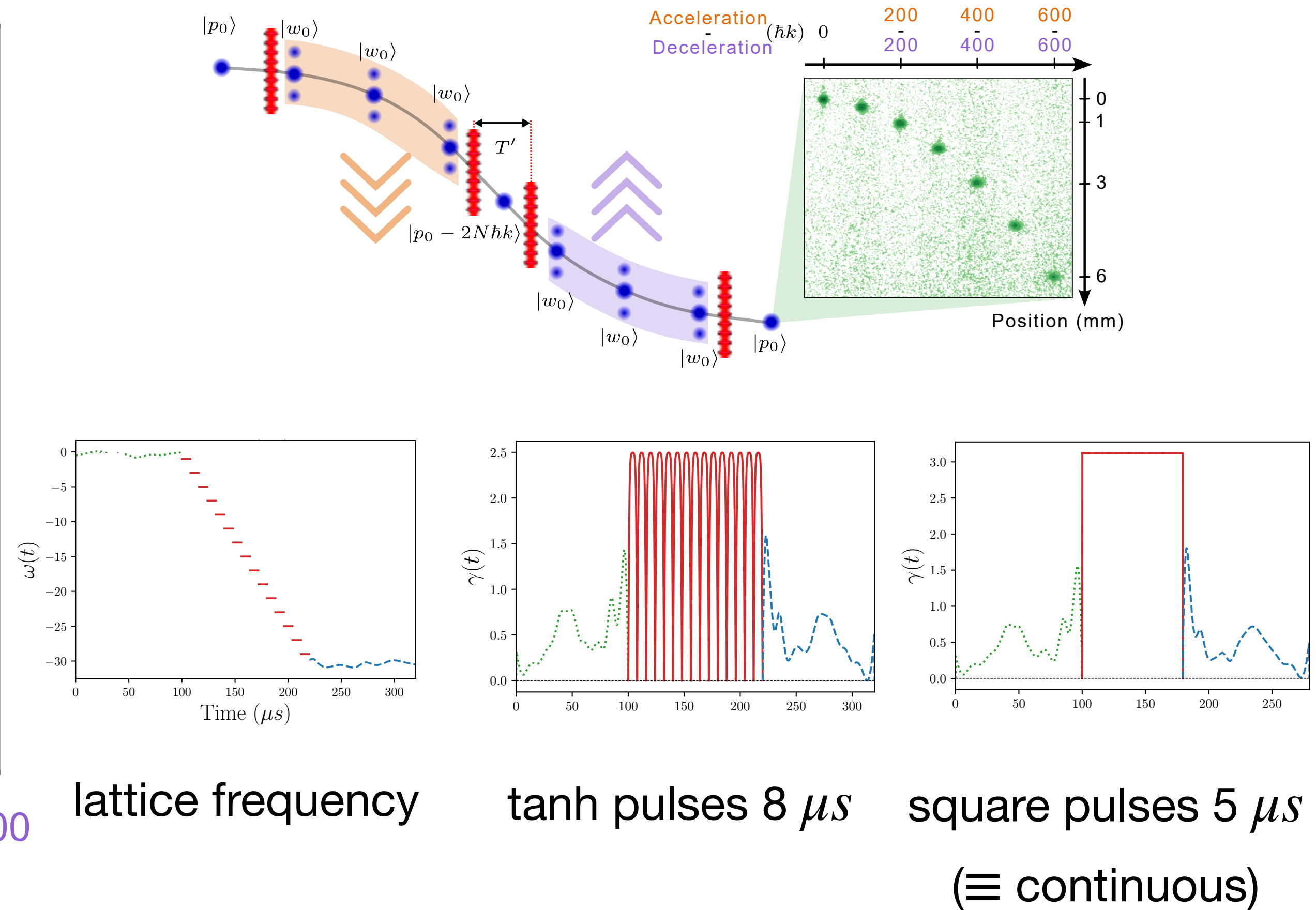
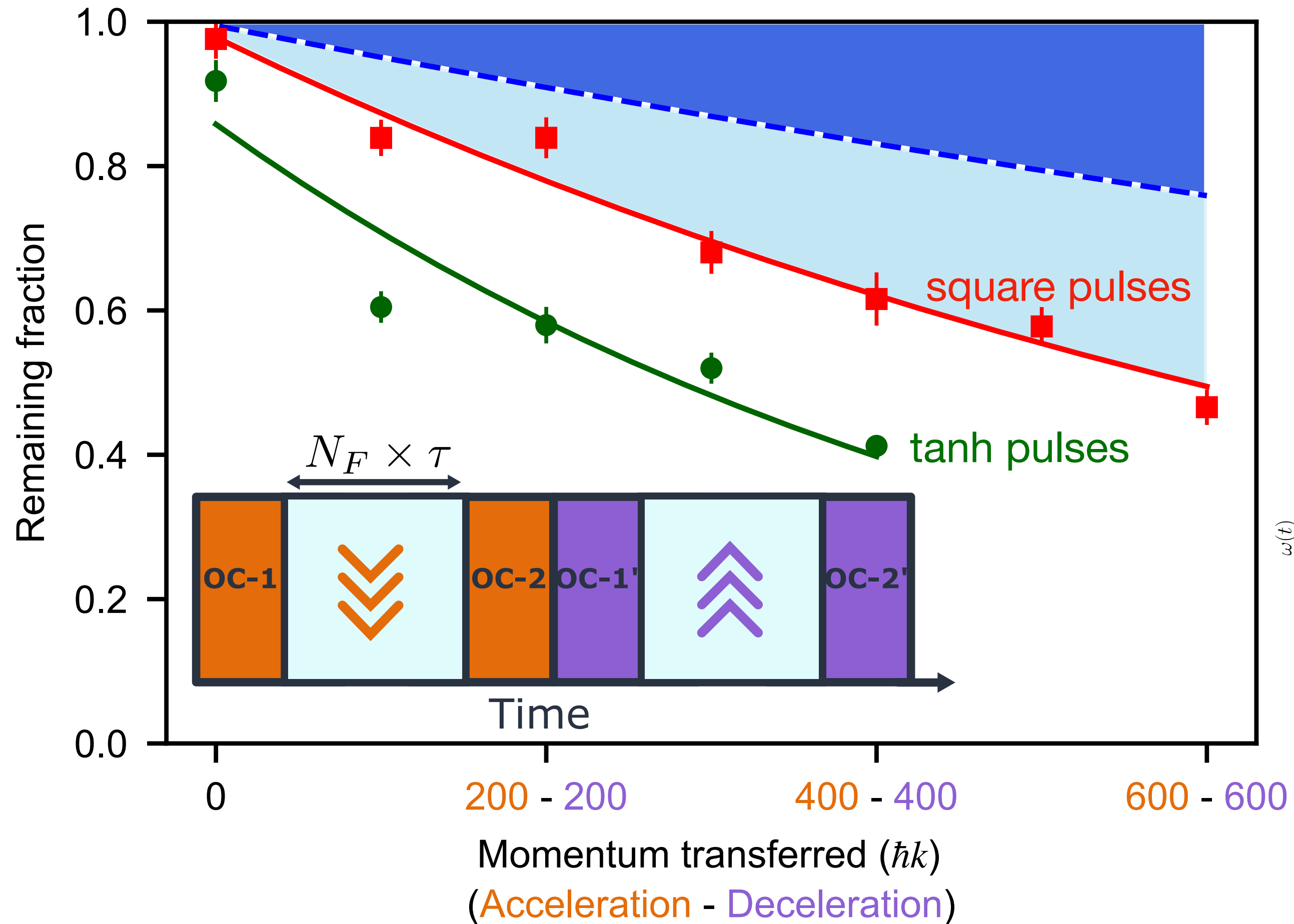


Robust control up to $0.35\hbar k$

Floquet acceleration

Normalized atom number in the fully accelerated state

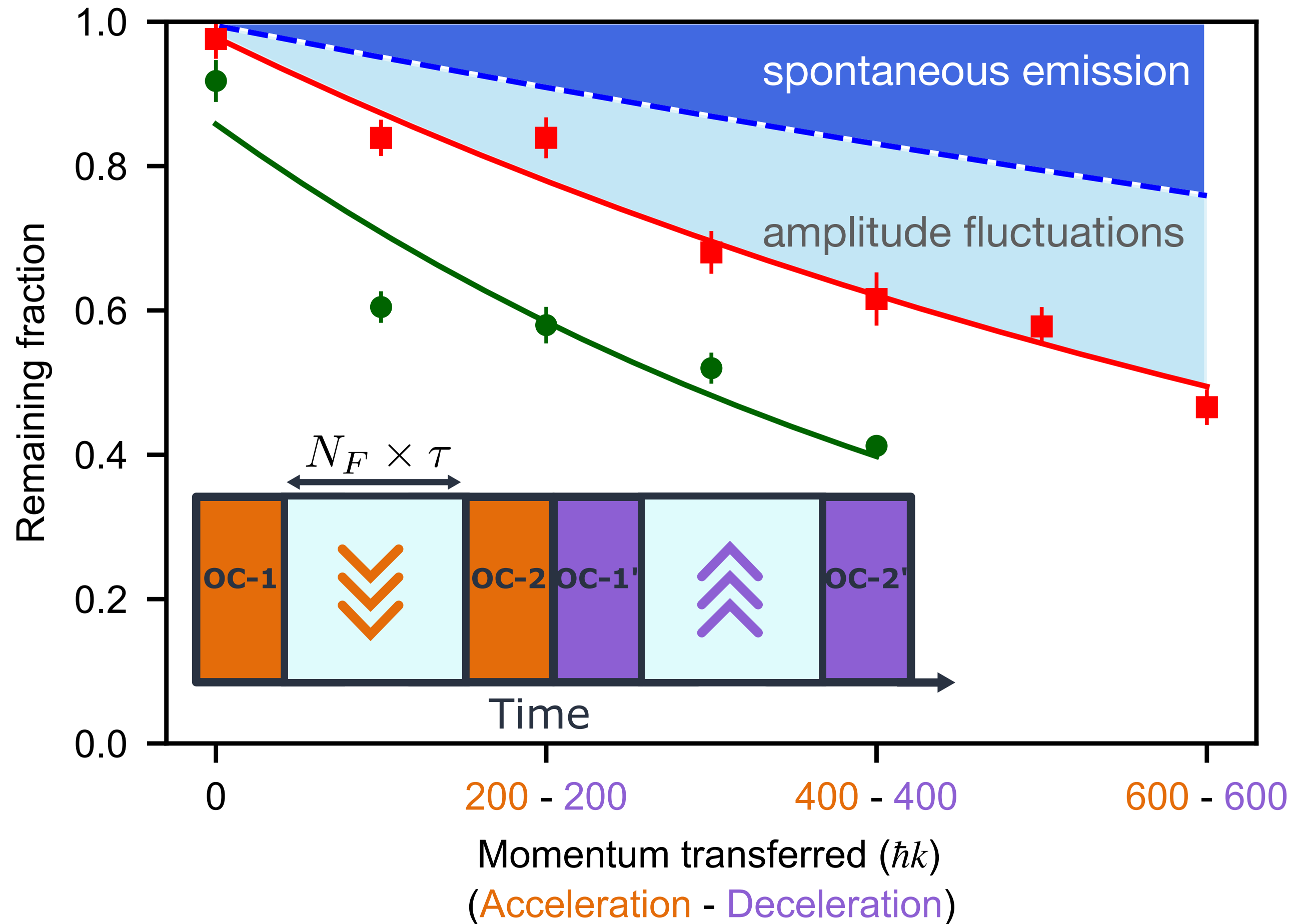
Rodzinka et al. [arXiv:2403.14337](https://arxiv.org/abs/2403.14337)



Floquet acceleration

Normalized atom number in the fully accelerated state

Rodzinka et al. [arXiv:2403.14337](https://arxiv.org/abs/2403.14337)



Fast LMT peak momentum transfer :

$$2.5 \mu\text{s}/\hbar k$$

Efficiency per $\hbar k$: 0.99945(5)

Limitations:

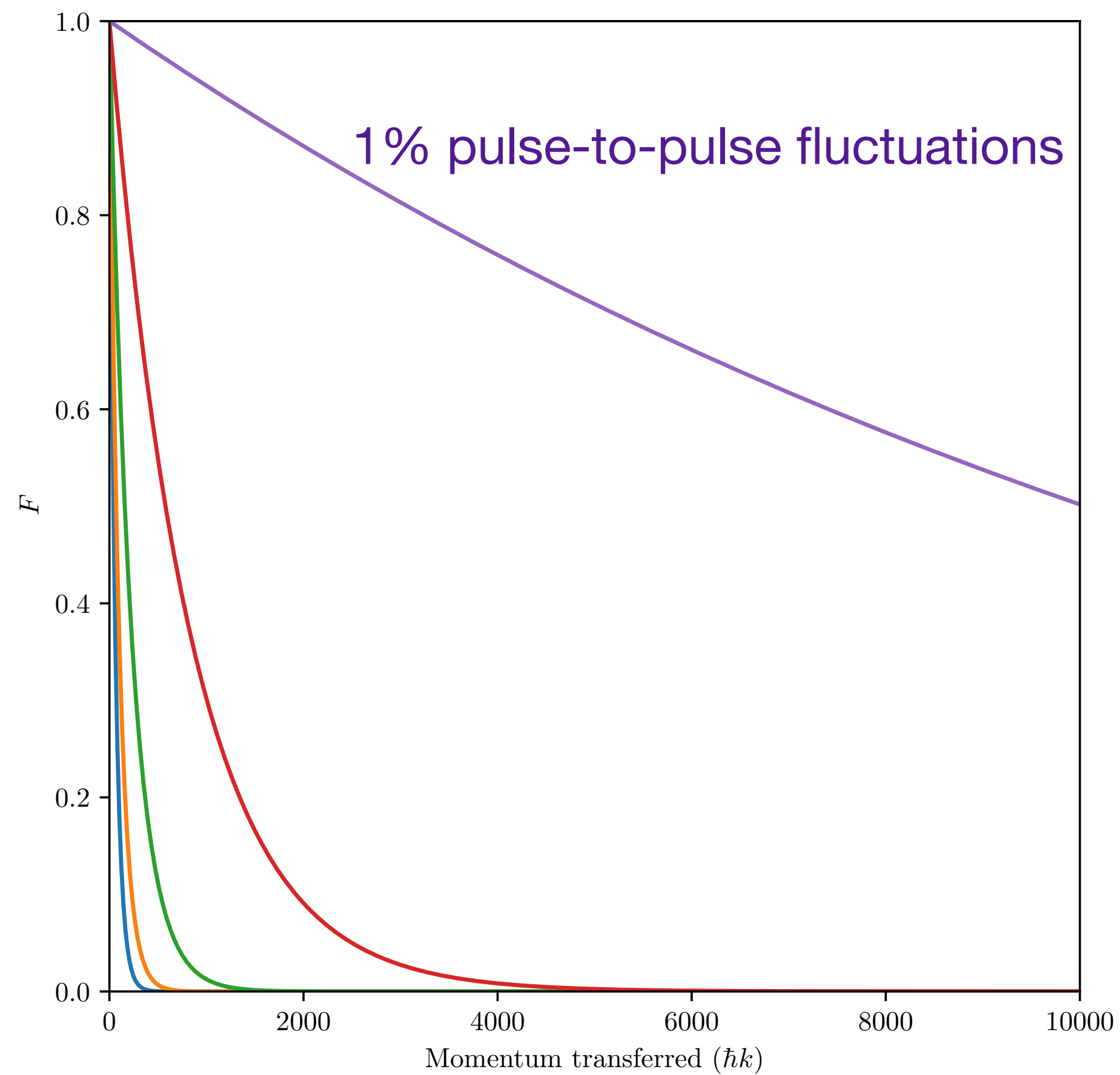
spontaneous emission: $\Delta = 40$ GHz

pulse-to-pulse fluctuations: 4.5 %

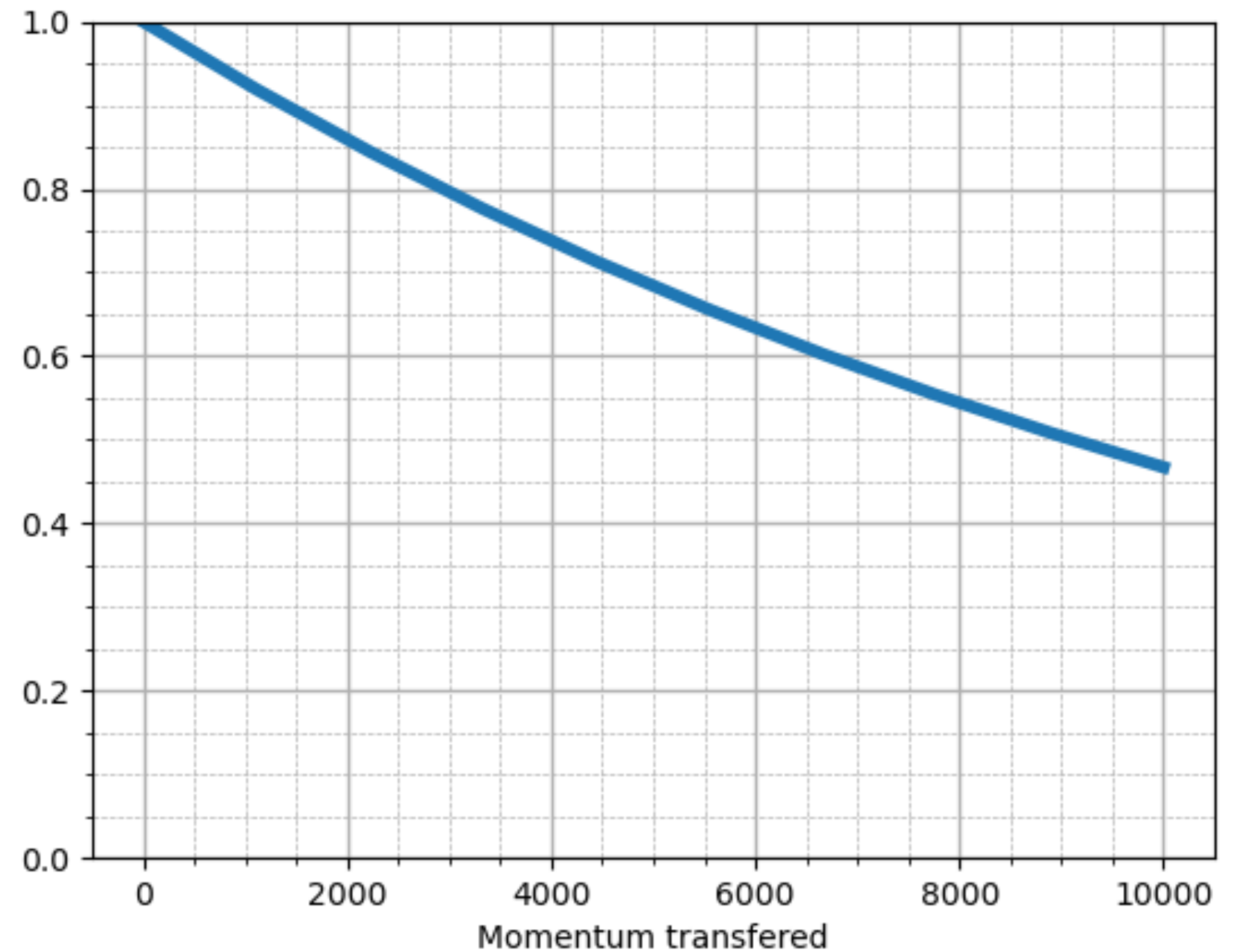
Floquet acceleration

Simulation to reach 10000 $\hbar k$

Pulse-to-pulse fluctuations

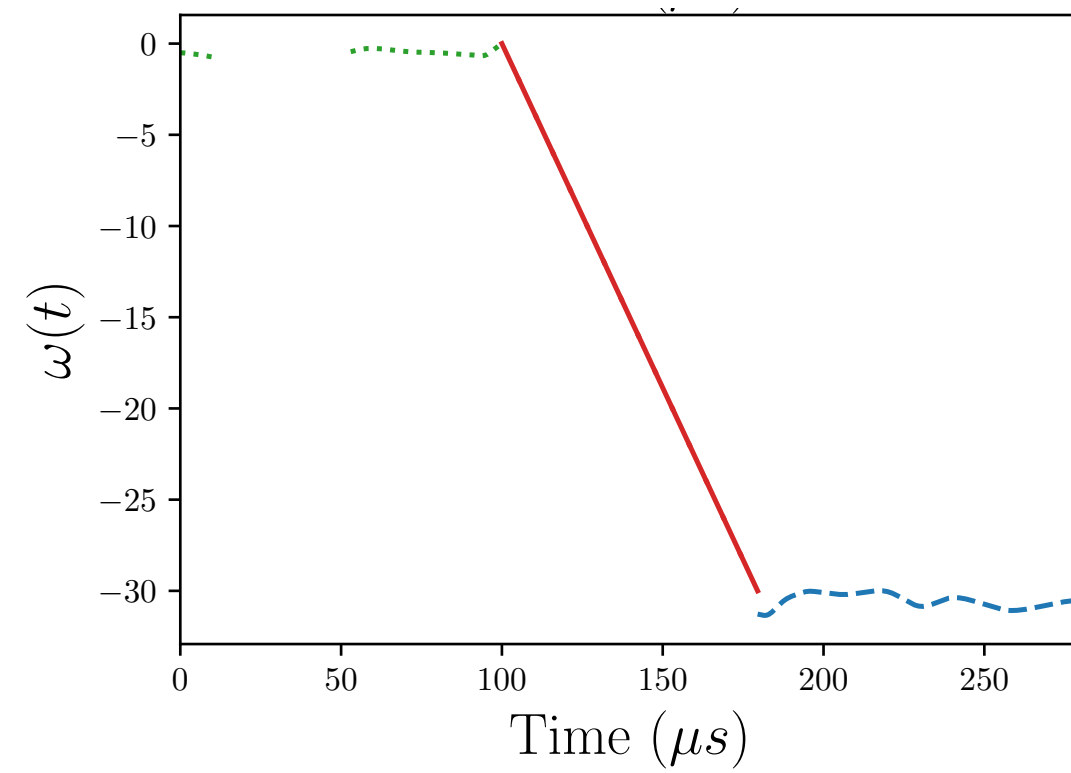
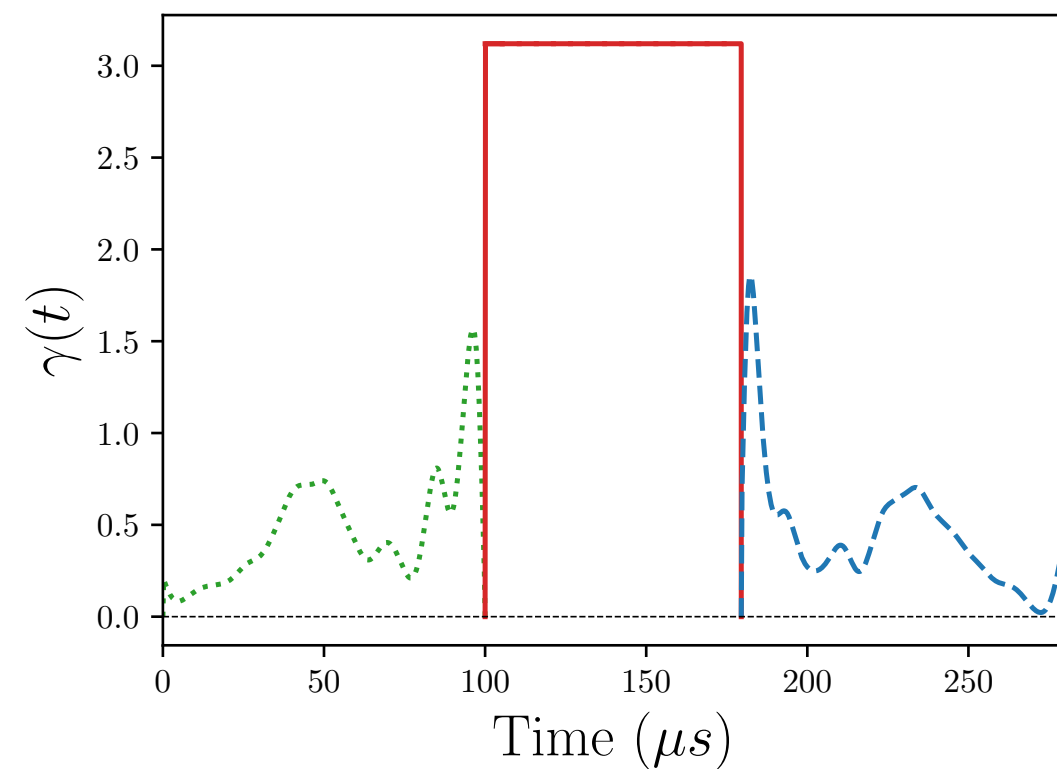


Spontaneous emission 250 GHz

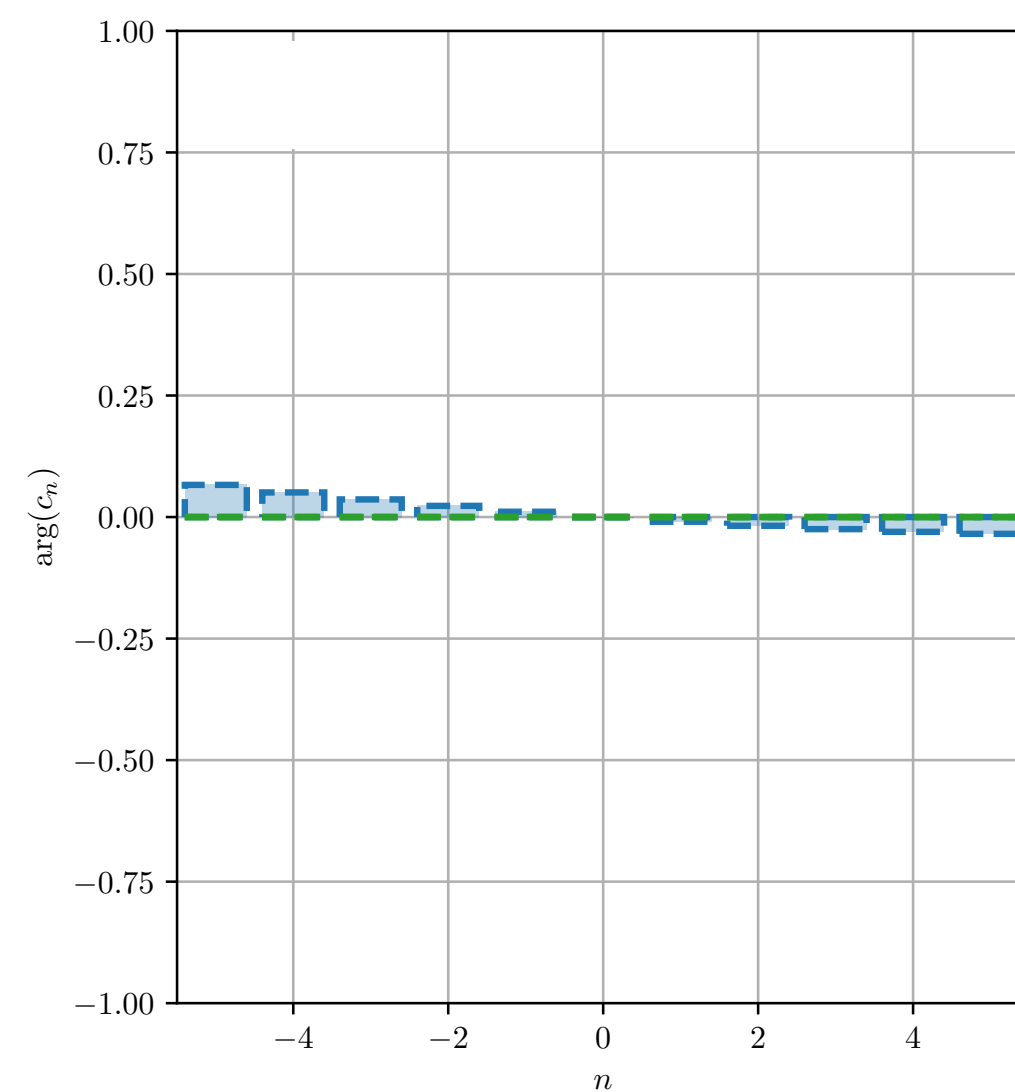
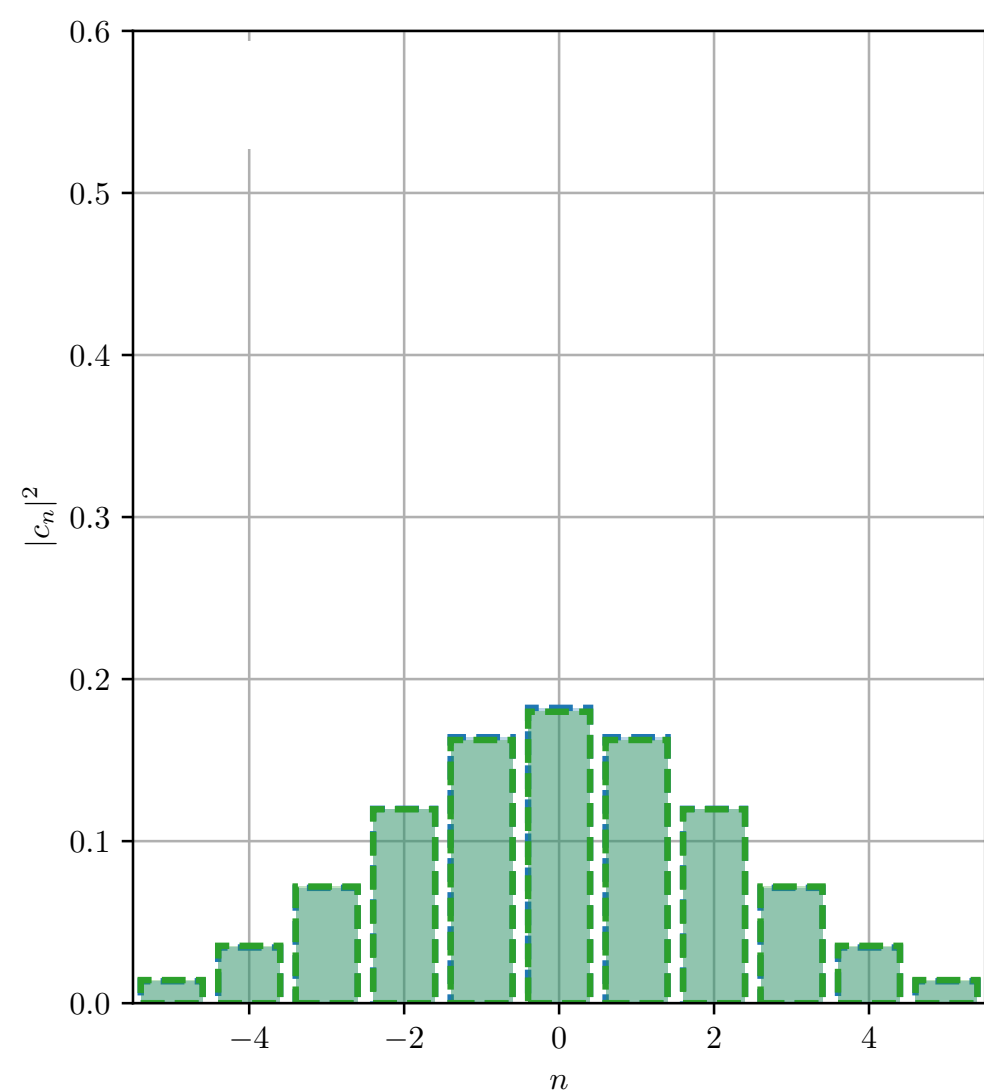


Bloch-type acceleration

Constant amplitude and frequency in accelerated frame



Optimal solution results in similar efficiency



For infinitely deep lattices, the Floquet state converges to the lattice ground state.

Comparison with :

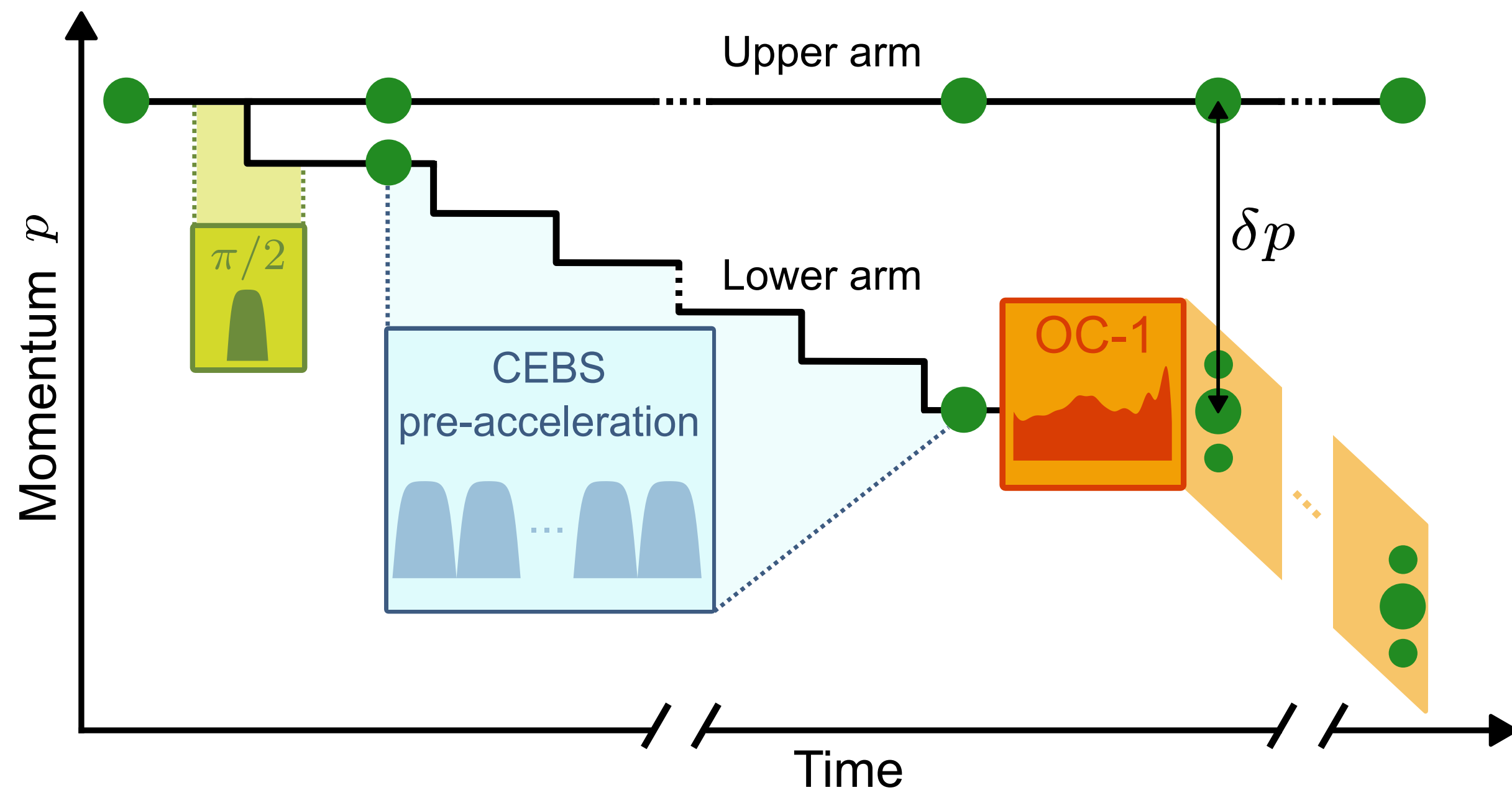
[Rahman et al. arXiv:2308.04134](#)

[Fitzek et al. arXiv:2306.09399](#)

LMT-Beam splitters

Pre-acceleration

Floquet states potentially have a large momentum expansion. This can interfere with the other arm during acceleration. Need for a pre-acceleration step.



$\pi/2$ - pulse quasi-Bragg regime

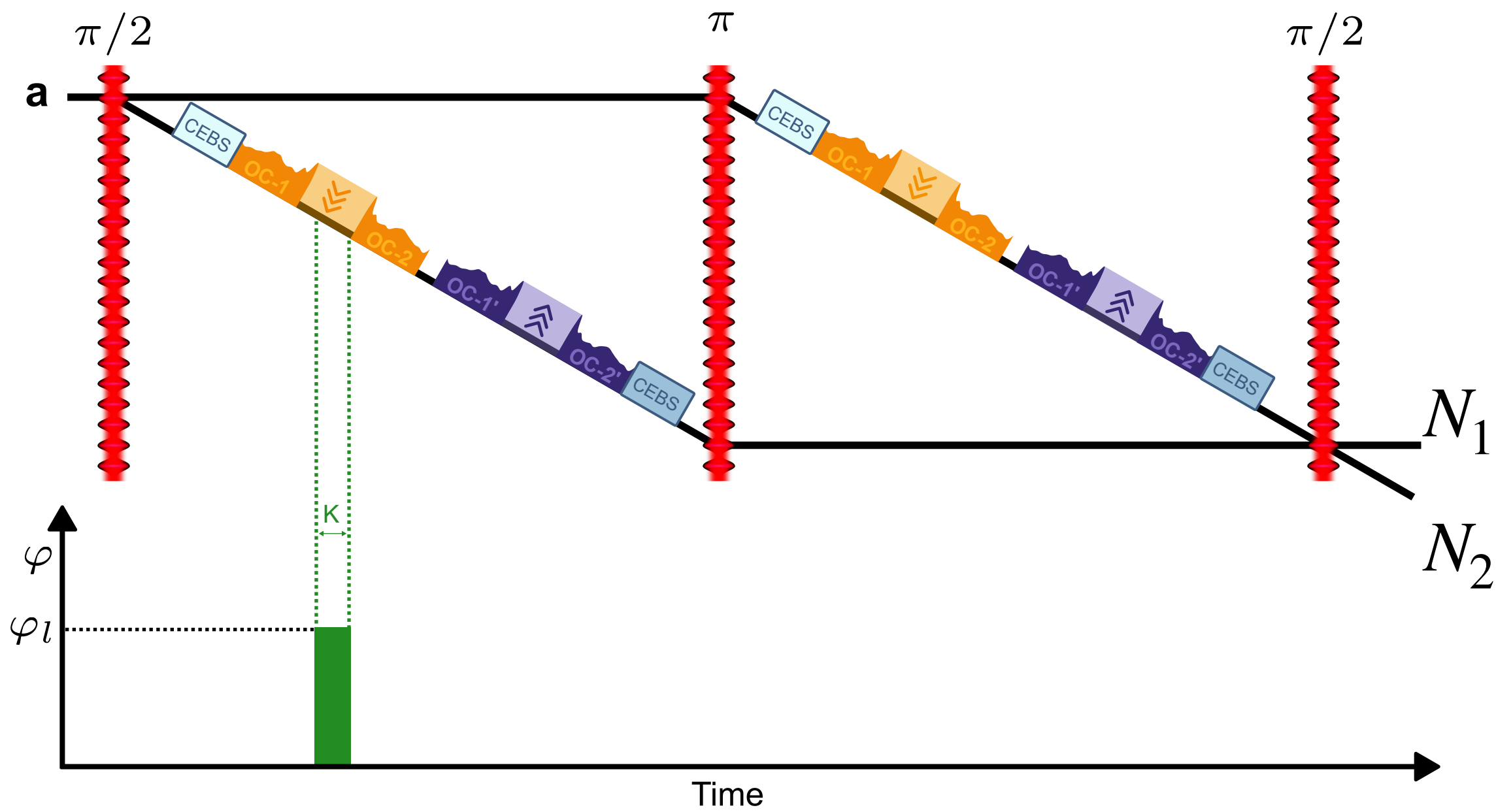
11 CEBS pulses ($40 \mu s$)

Floquet acceleration: N pulses

$$\text{Total Momentum separation} = (1 + 11 + N) \times 2\hbar k$$

LMT - Interferometer

$600\hbar k$ - interferometer



Interferometer signal $\frac{N_1}{N_1 + N_2} = A \left(1 + V \sin(\Delta\phi) \right)$

Lattice phase imprinted on the atom at each momentum transfer

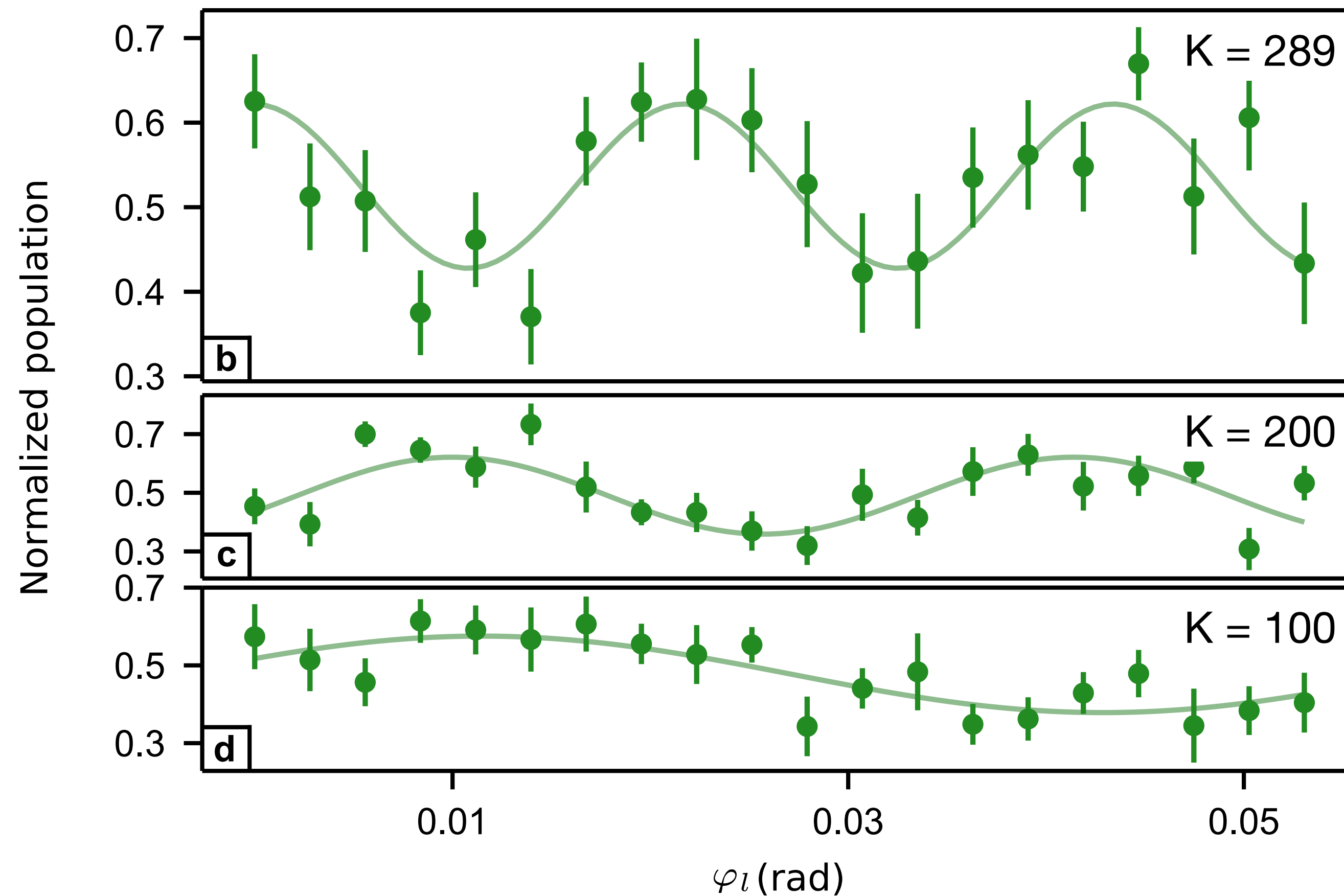
Phase-shift scaling with lattice phase $\Delta\phi = K \times \varphi_l$

Scan the fringes by incrementing φ_l

LMT - Interferometer

$600\hbar k$ - interferometer

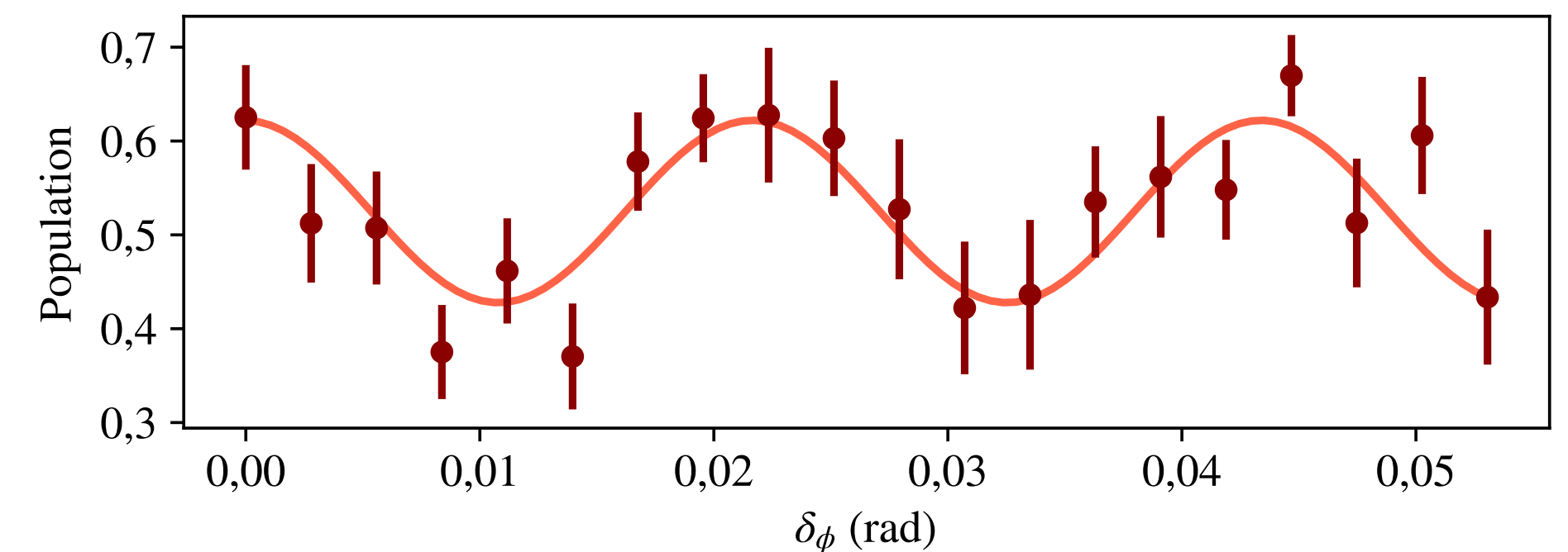
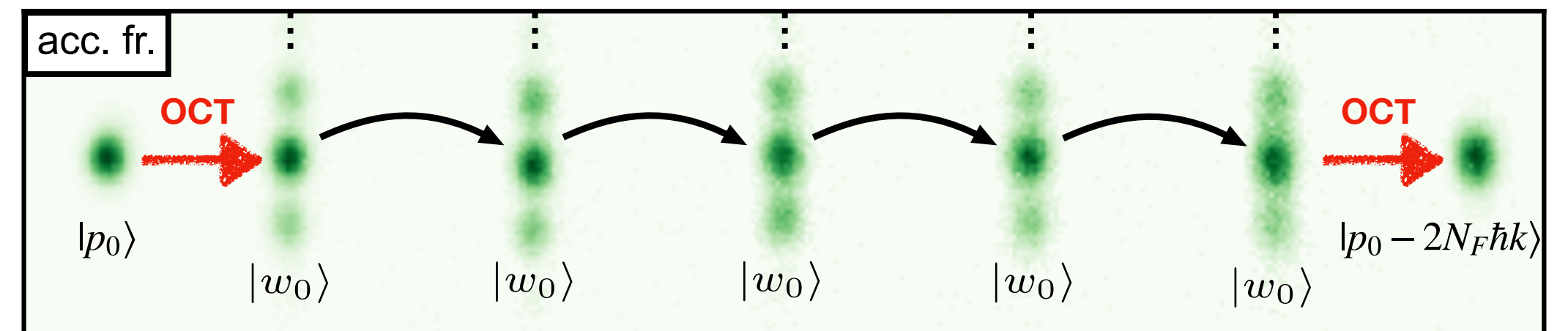
Rodzinka et al. [arXiv:2403.14337](https://arxiv.org/abs/2403.14337)



- LMT - Interferometer $600\hbar k$
 - ▶ limit = detection volume
- Visibility: $18\% \pm 4\%$
 - ▶ limit = spontaneous emission & pre-acceleration efficiency

Conclusion & Discussion

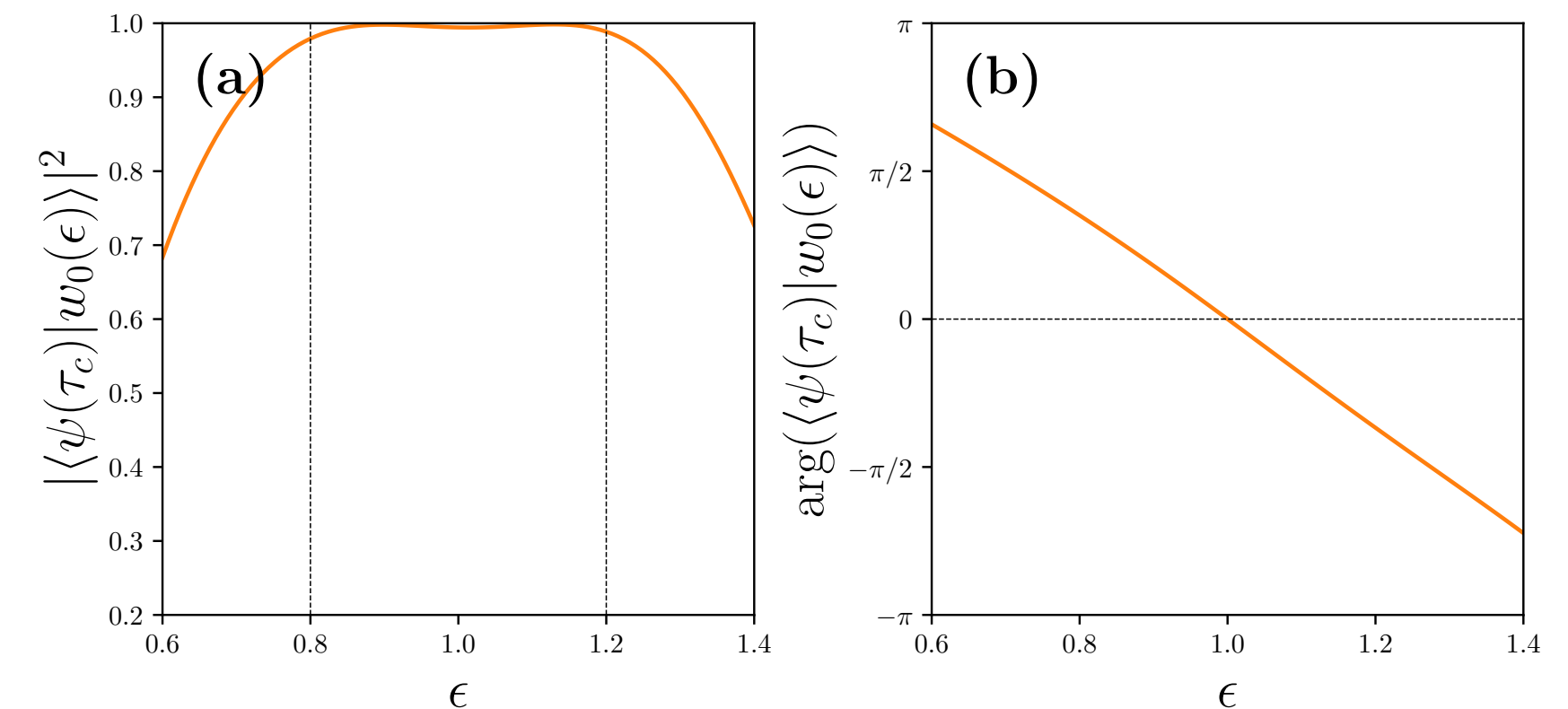
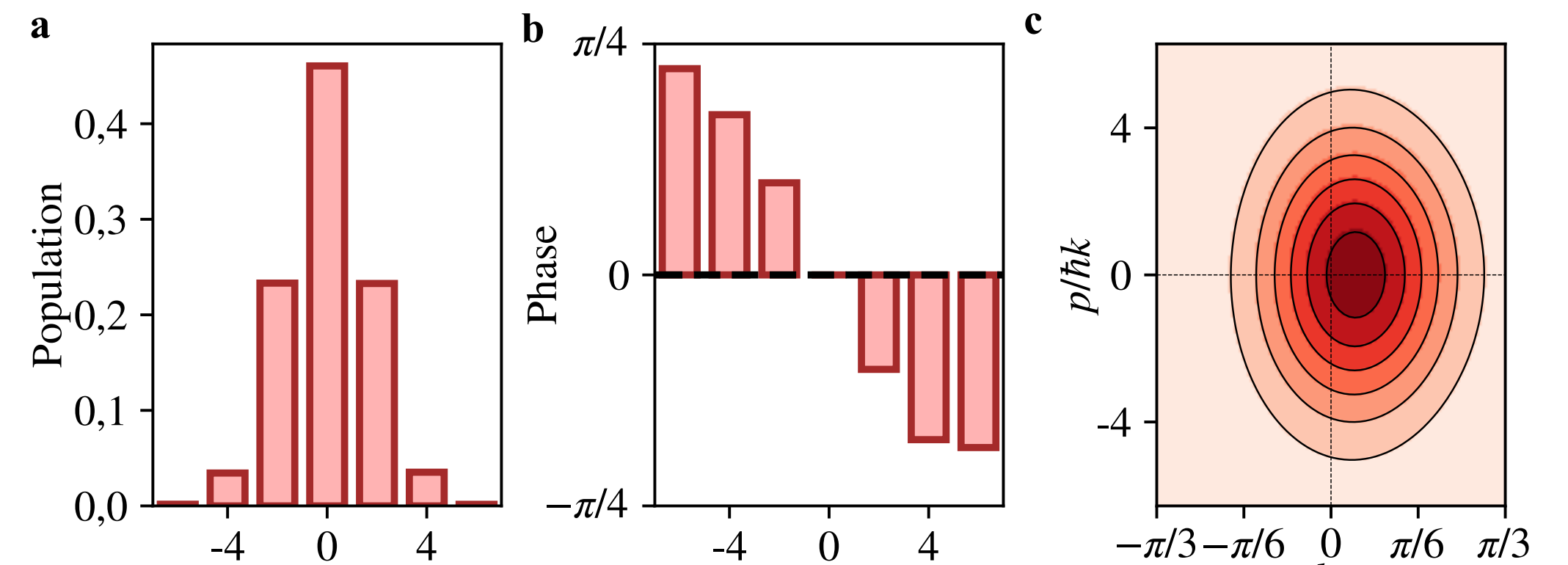
- Floquet approach for sequential and continuous acceleration.
- New QOCT implementation for navigating large Hilbert spaces.
- Fast and very efficient LMT.
- Demonstrates 600 $\hbar k$ atom interferometer.



We believe that there are no serious barriers to realization of momentum transfers greater than $1000\hbar k$.

Conclusion & Discussion

- Improving the beam splitting and pre-acc
- Robust against lattice depth fluctuations
 - Pulse Sequence Engineering
 - More powerful and stable laser
- Metrology of LMT interferometer
 - QOCT for Phase shifts robustness
 - Phase shift measurements





T. Rodzinka
(PhD)

S. Beldjoudi
(PhD)

L. Calmels
(PhD)

Ashley Béguin
(PhD)

Baptiste Allard
(Ass. Prof.)



D. Guéry-Odelin
(Prof.)



E. Dionis
(PhD)



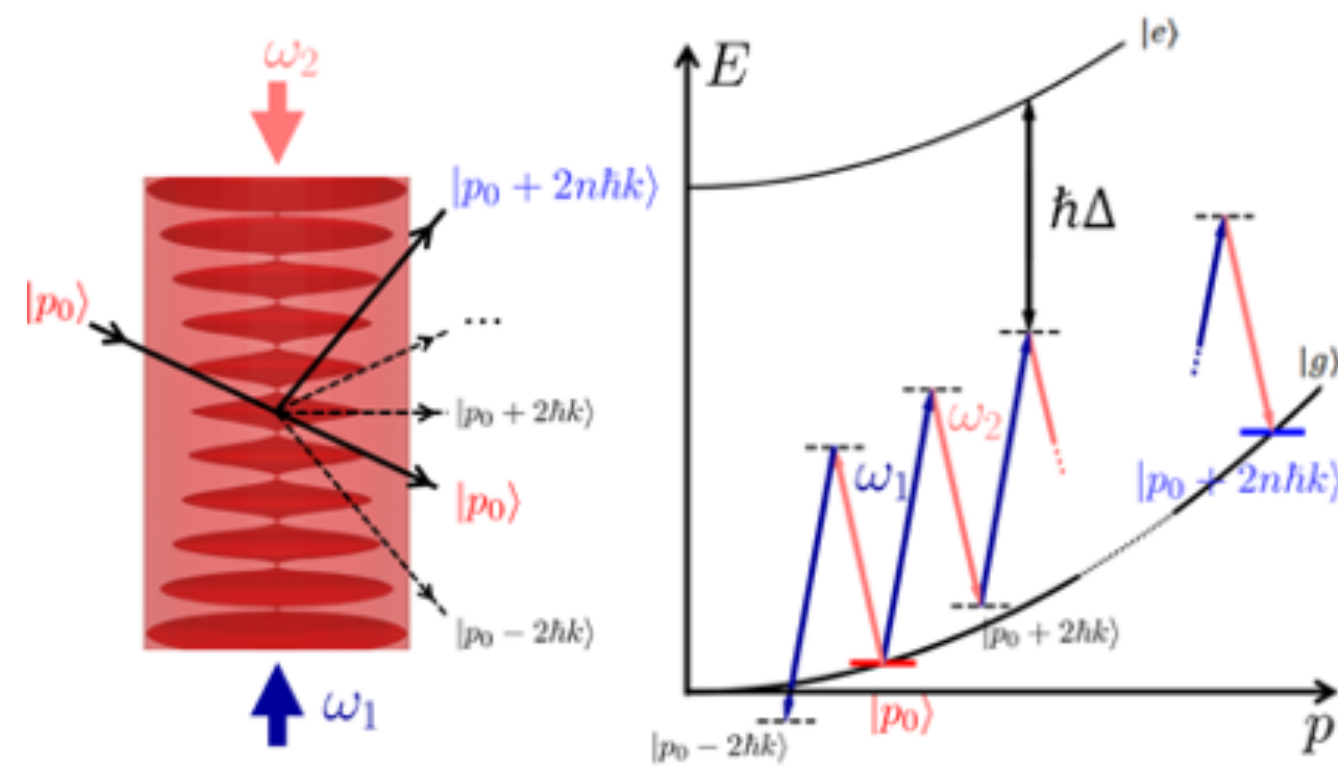
D. Sugny
(Prof.)



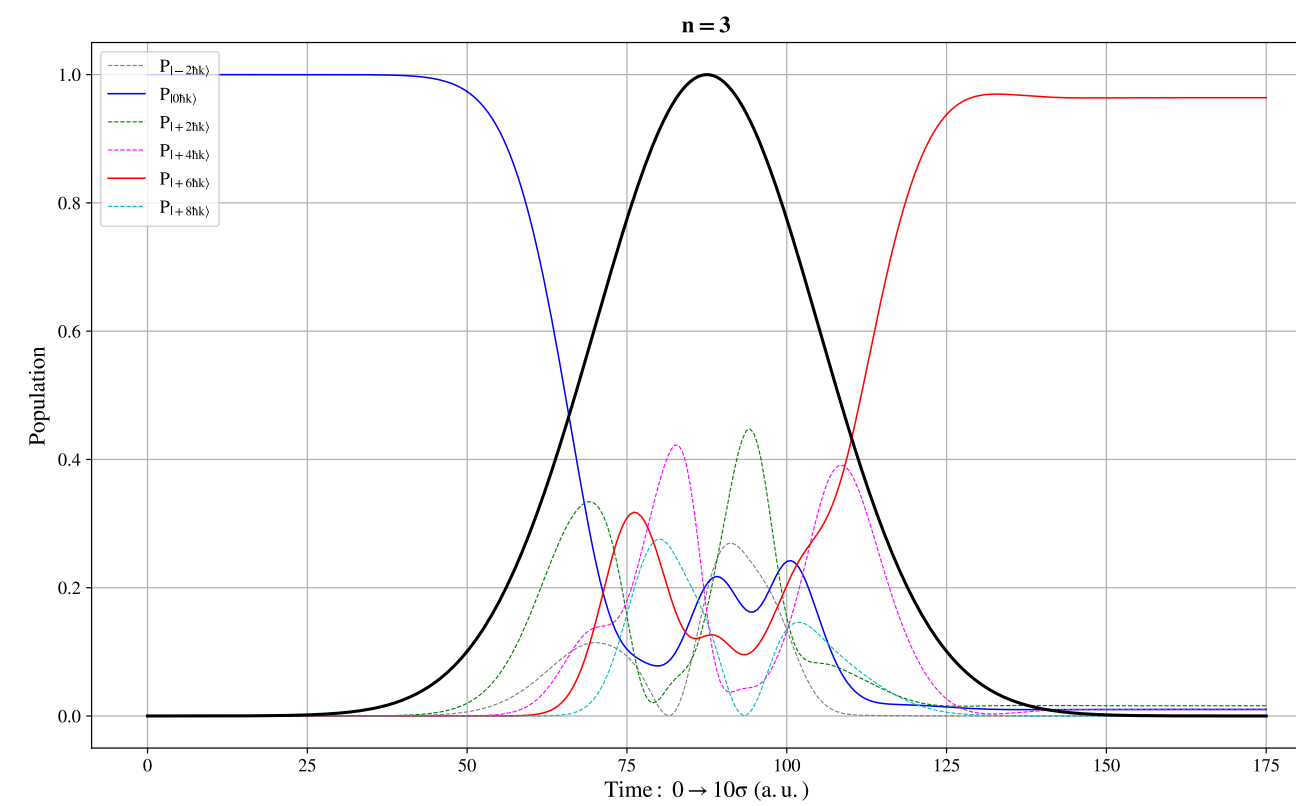
The Machine

Quasi-Bragg diffraction

High-order diffraction and Brute-Force Optimal Control

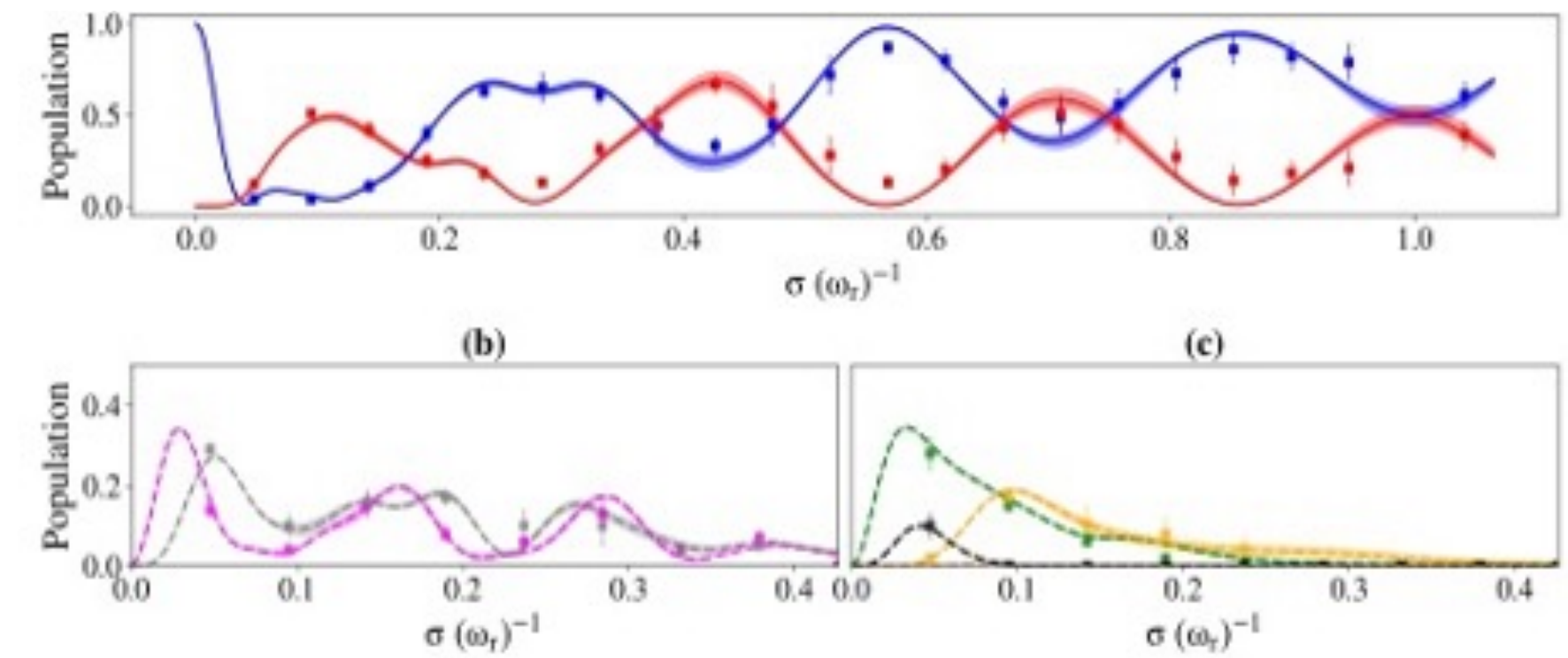


Multiple loss path



Gaussian pulse

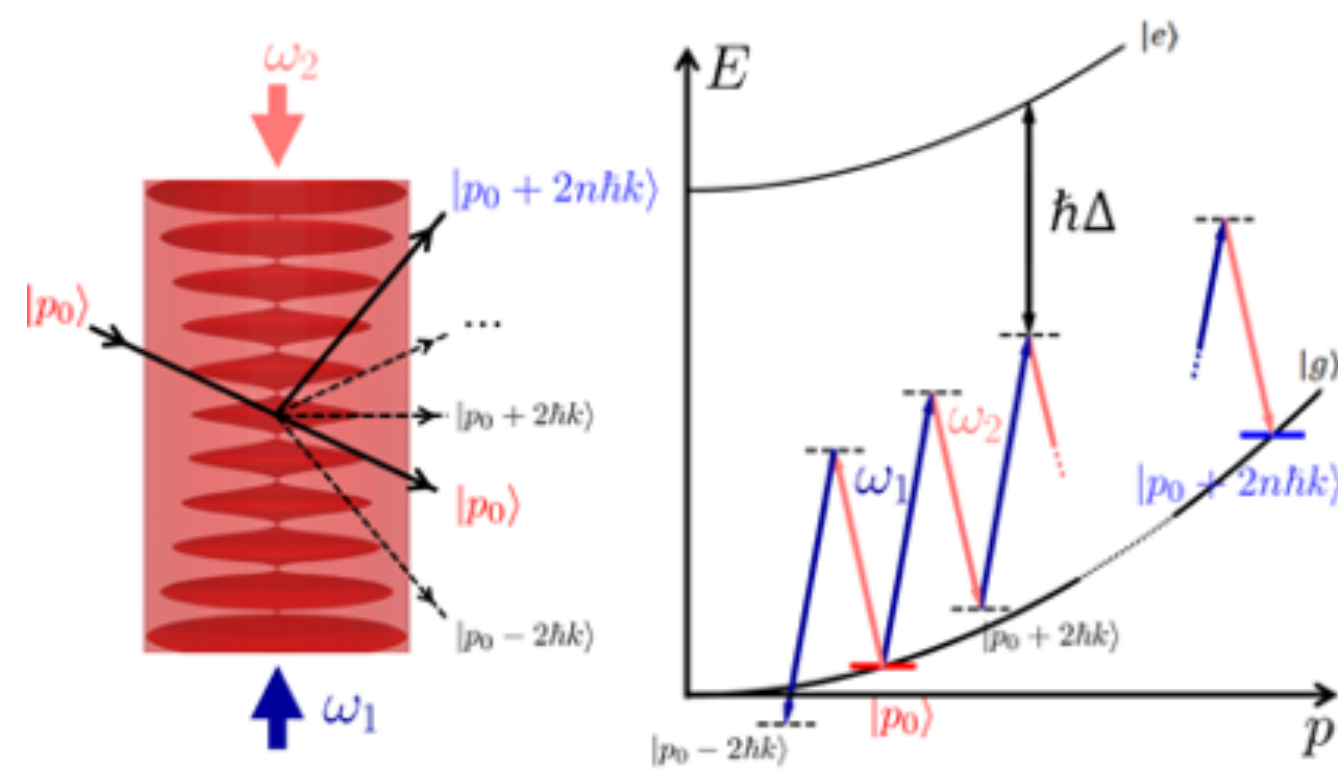
Béguin et al., Phys. Rev. A, **105**, 03302 (2022)



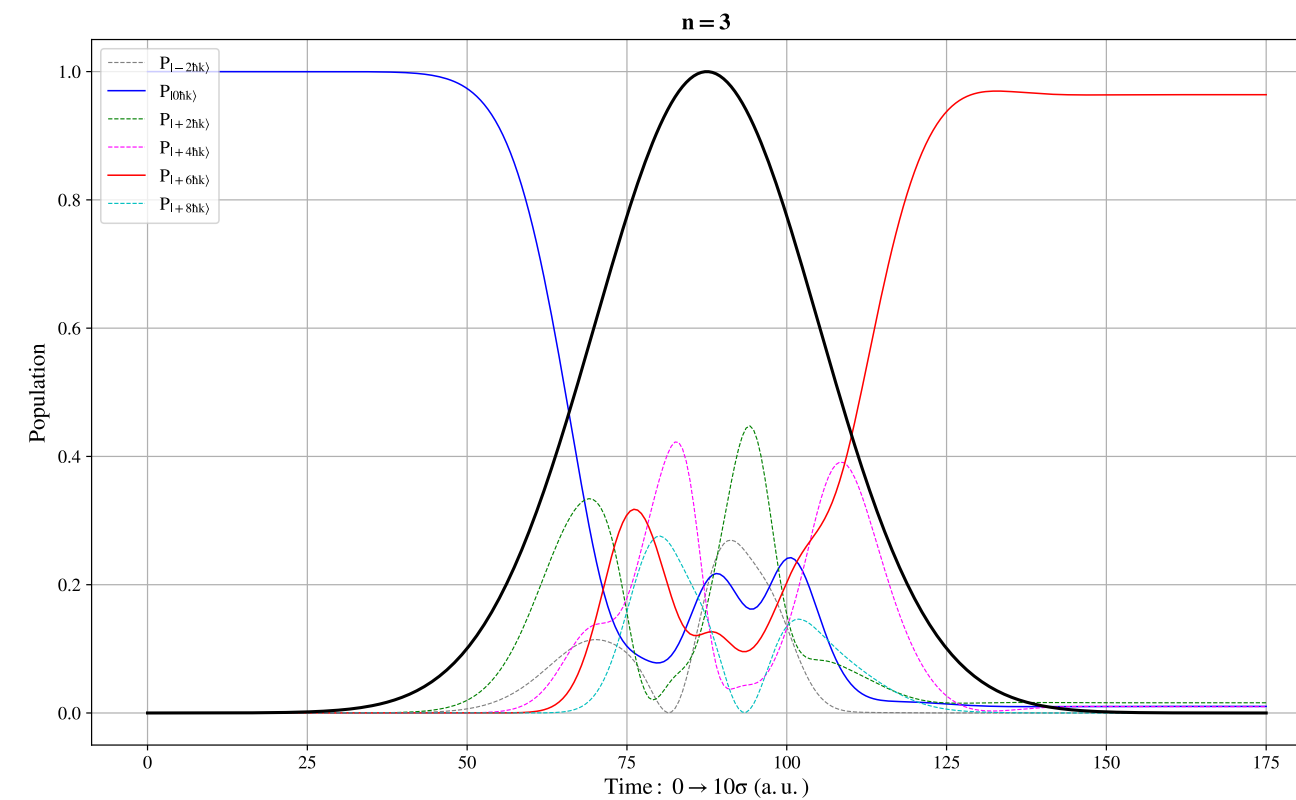
non-adiabatic vs velocity selection

Quasi-Bragg diffraction

High-order diffraction and Brute-Force Optimal Control

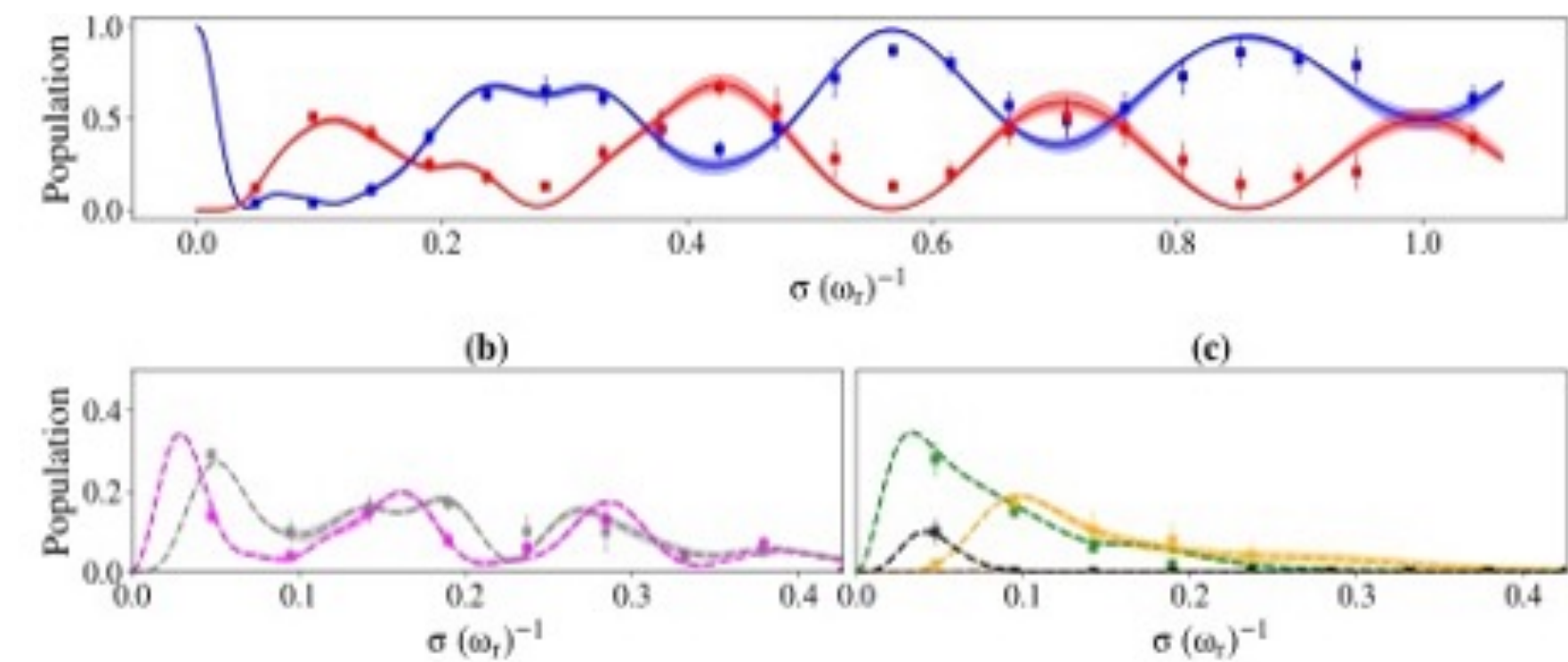


Multiple loss path

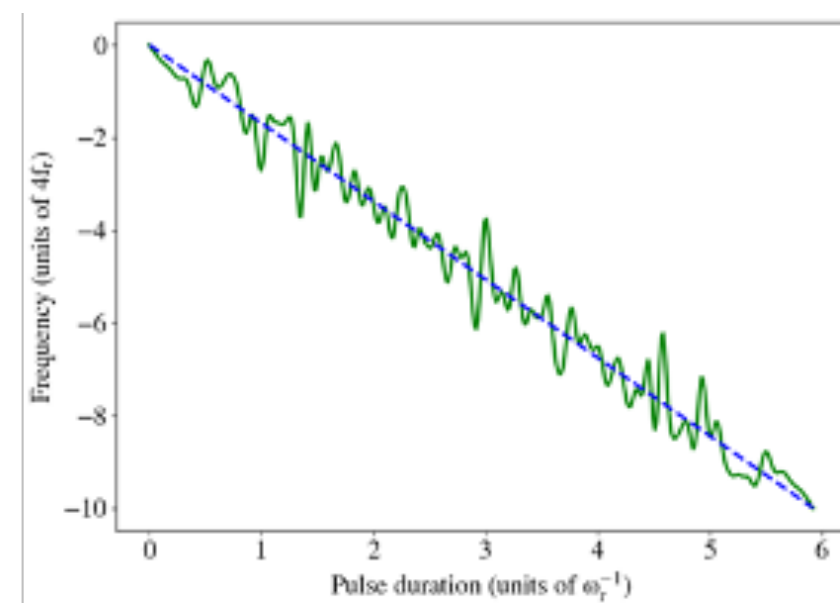
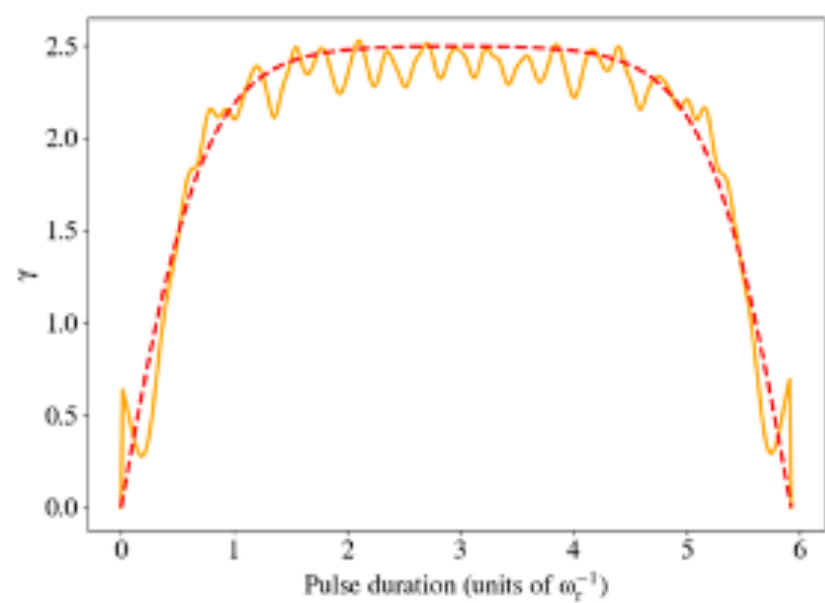


Gaussian pulse

Béguin et al., Phys. Rev. A, 105, (2022)



non-adiabatic vs velocity selection

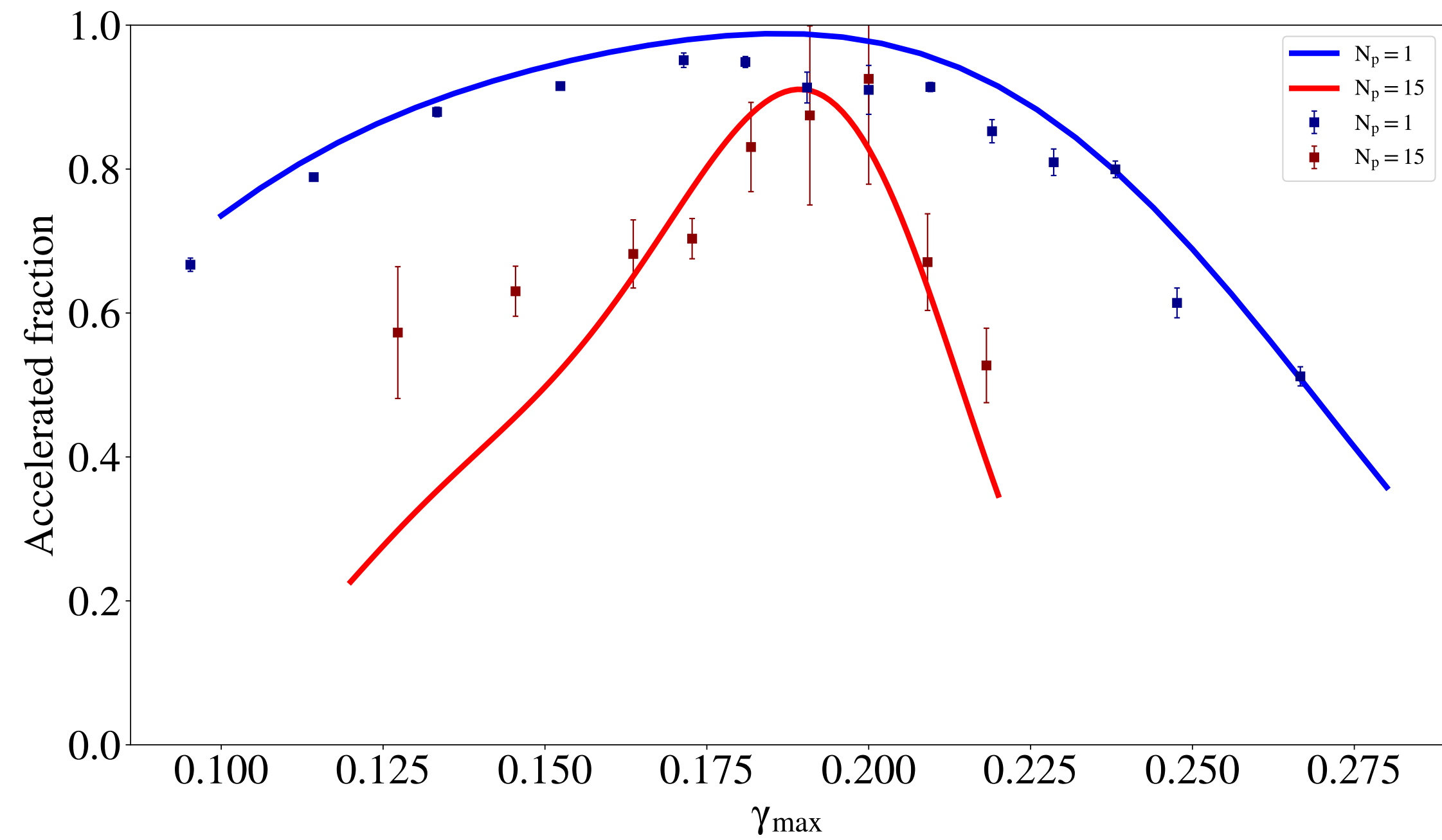
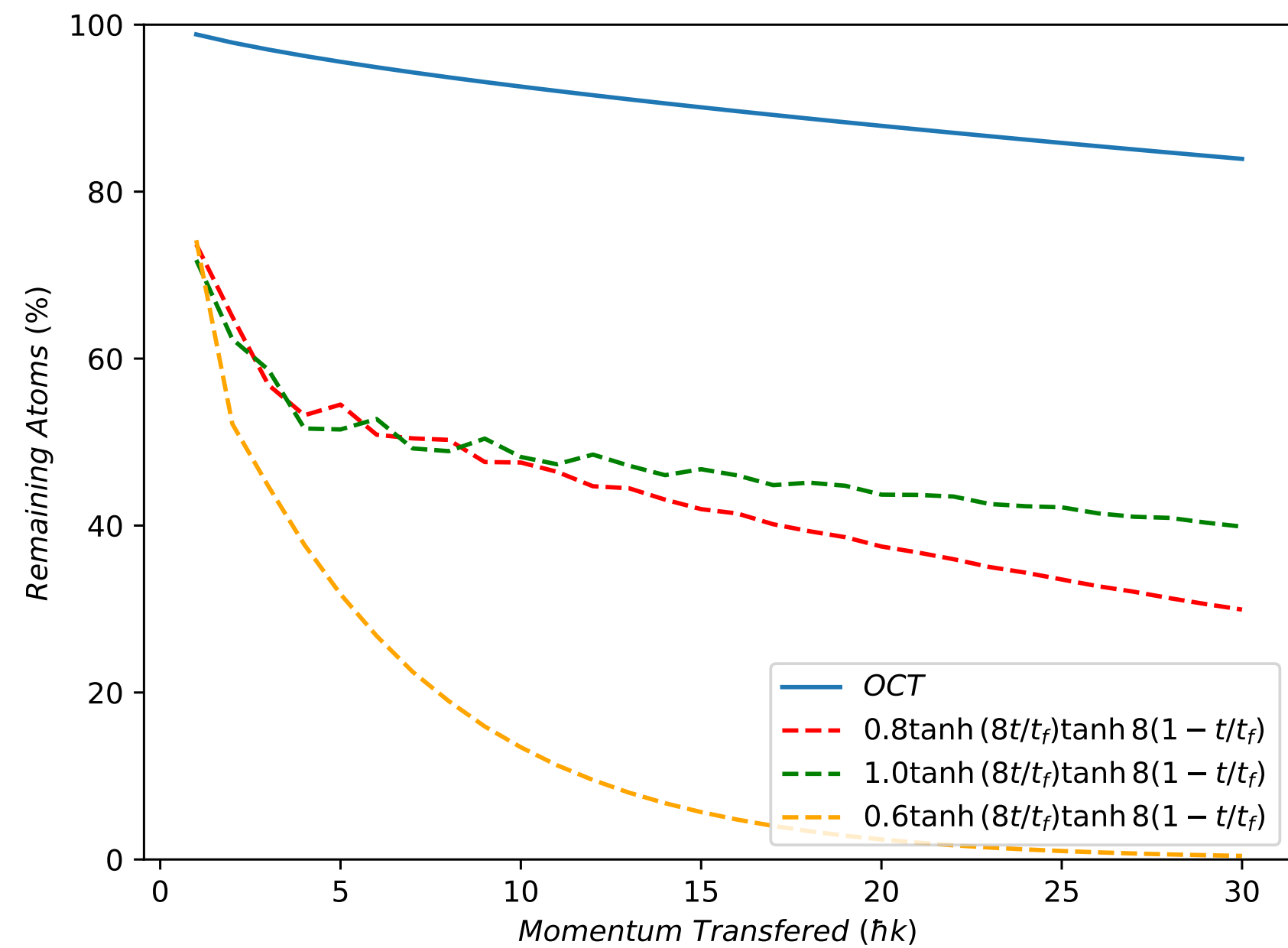
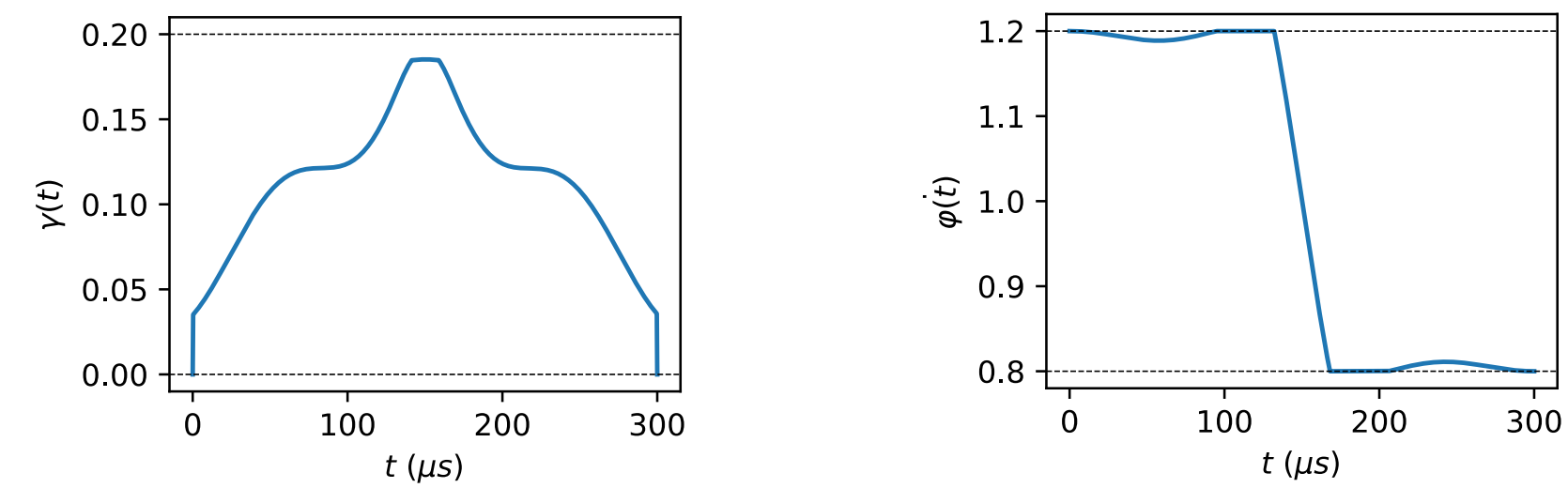


Pulse shape based on Optimal Control Theory

- Can we improve gaussian pulse with OCT ?
- Enhanced robustness to Doppler detuning, lattice depth, etc.
- Very hard to go beyond a few tens of $\hbar k$ with a brute force numerical approach.

LMT: Bragg pulse sequence

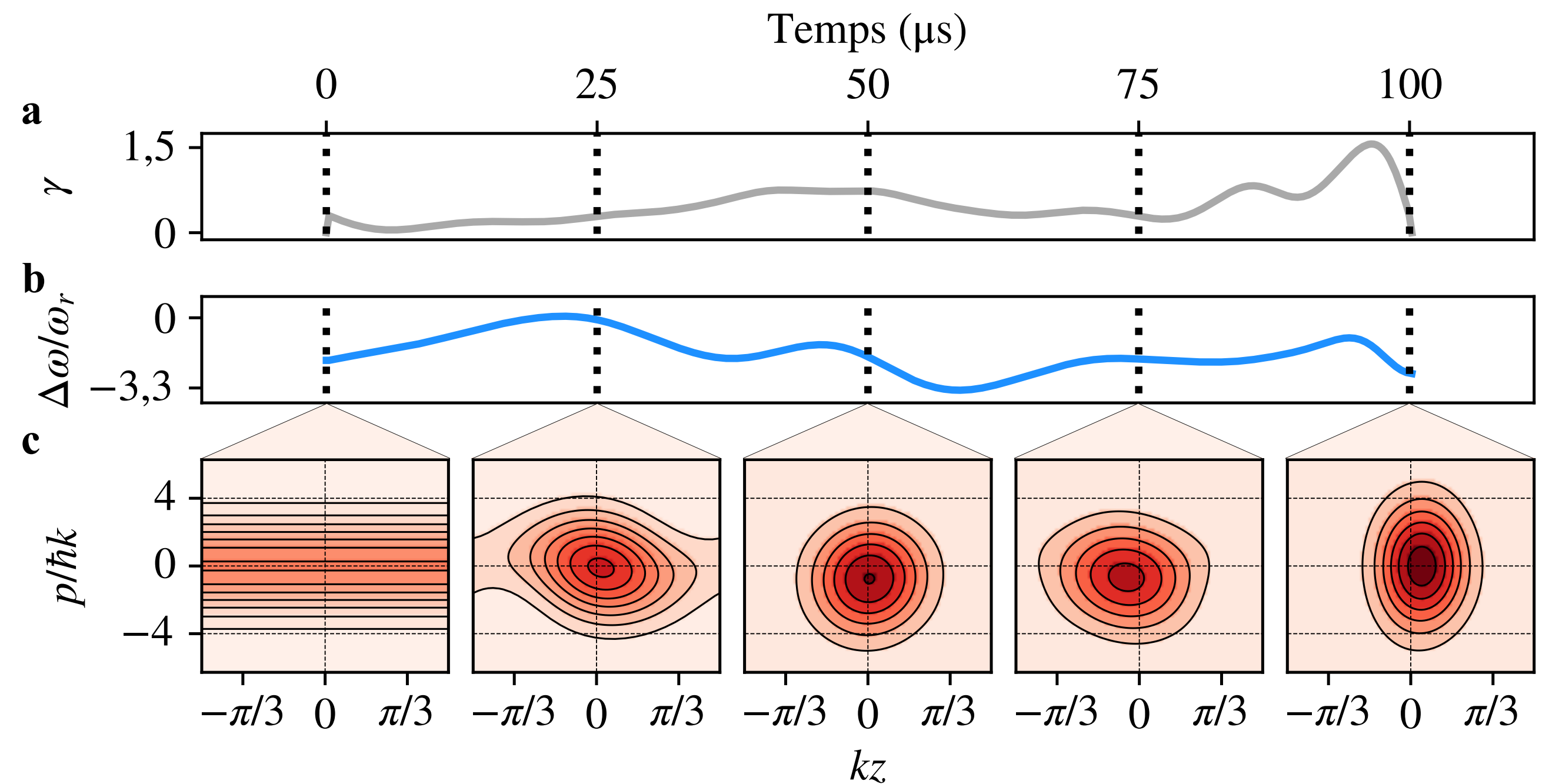
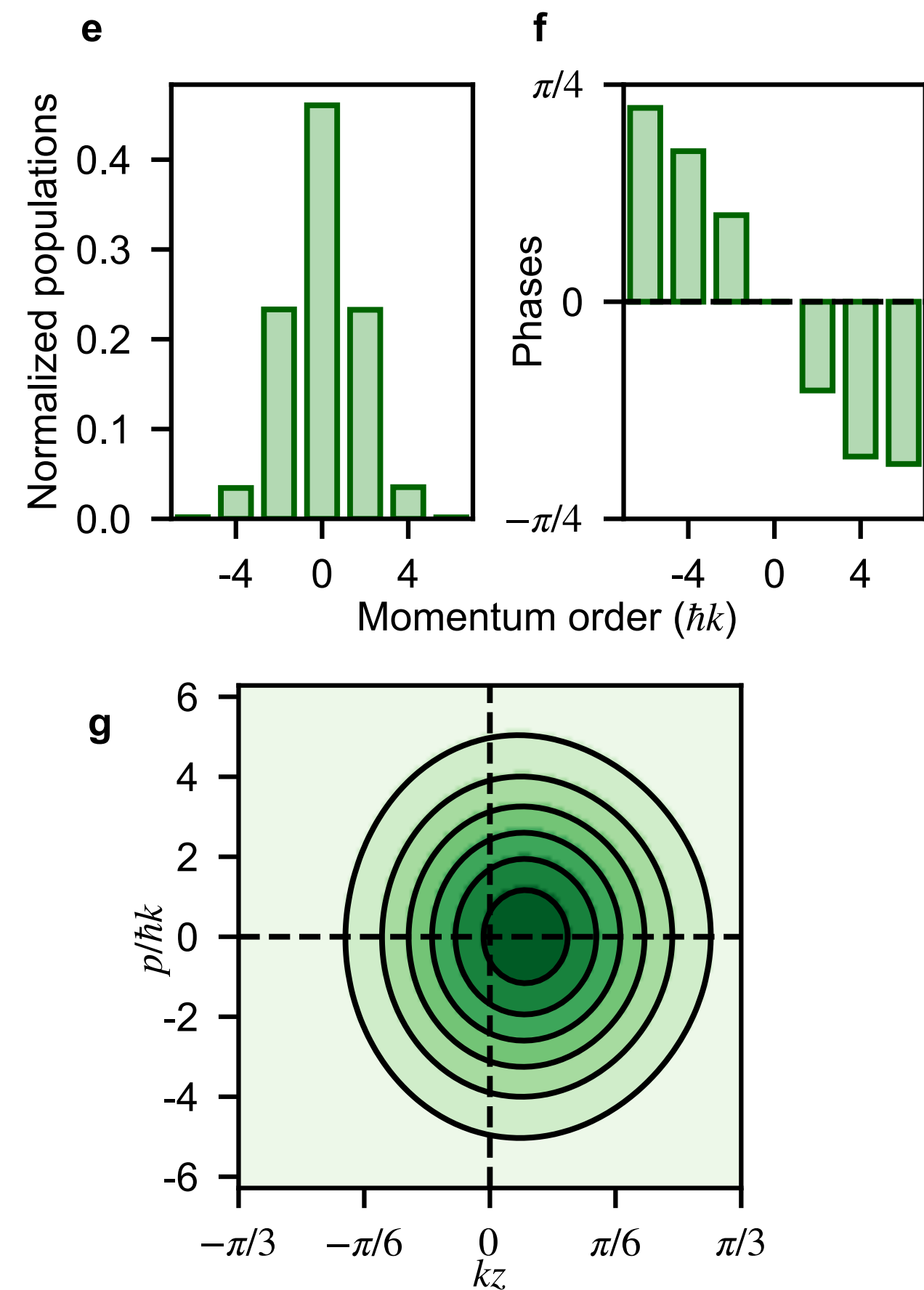
Independent pulses: OCT pulses



- Experimental efficiency limited by detection
- **Testing robustness ?**
- Comparison with gaussian pulse $8\sigma = 300 \mu s$

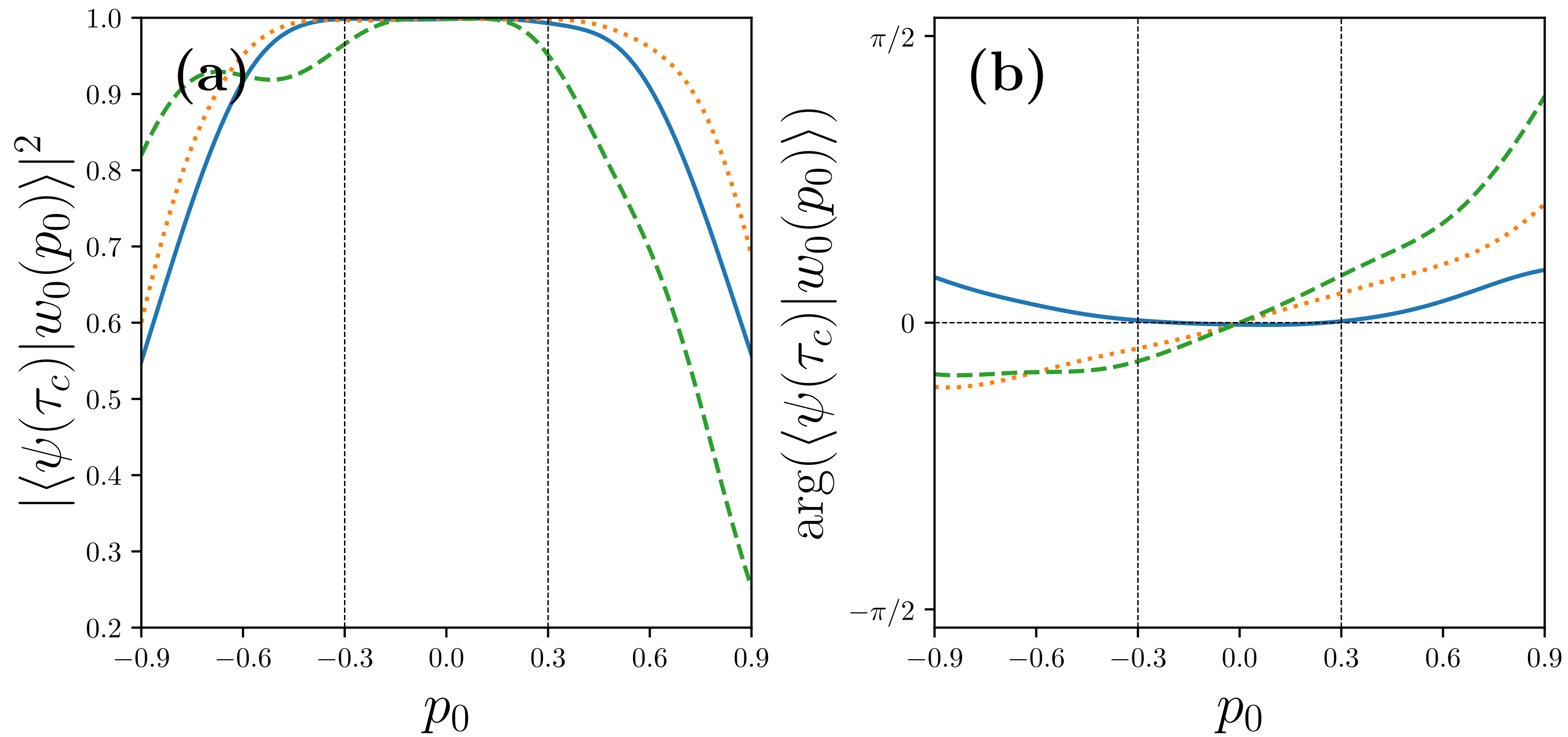
Floquet state in phase space

A simple parameterization of $|w_0\rangle$



Phase dispersion

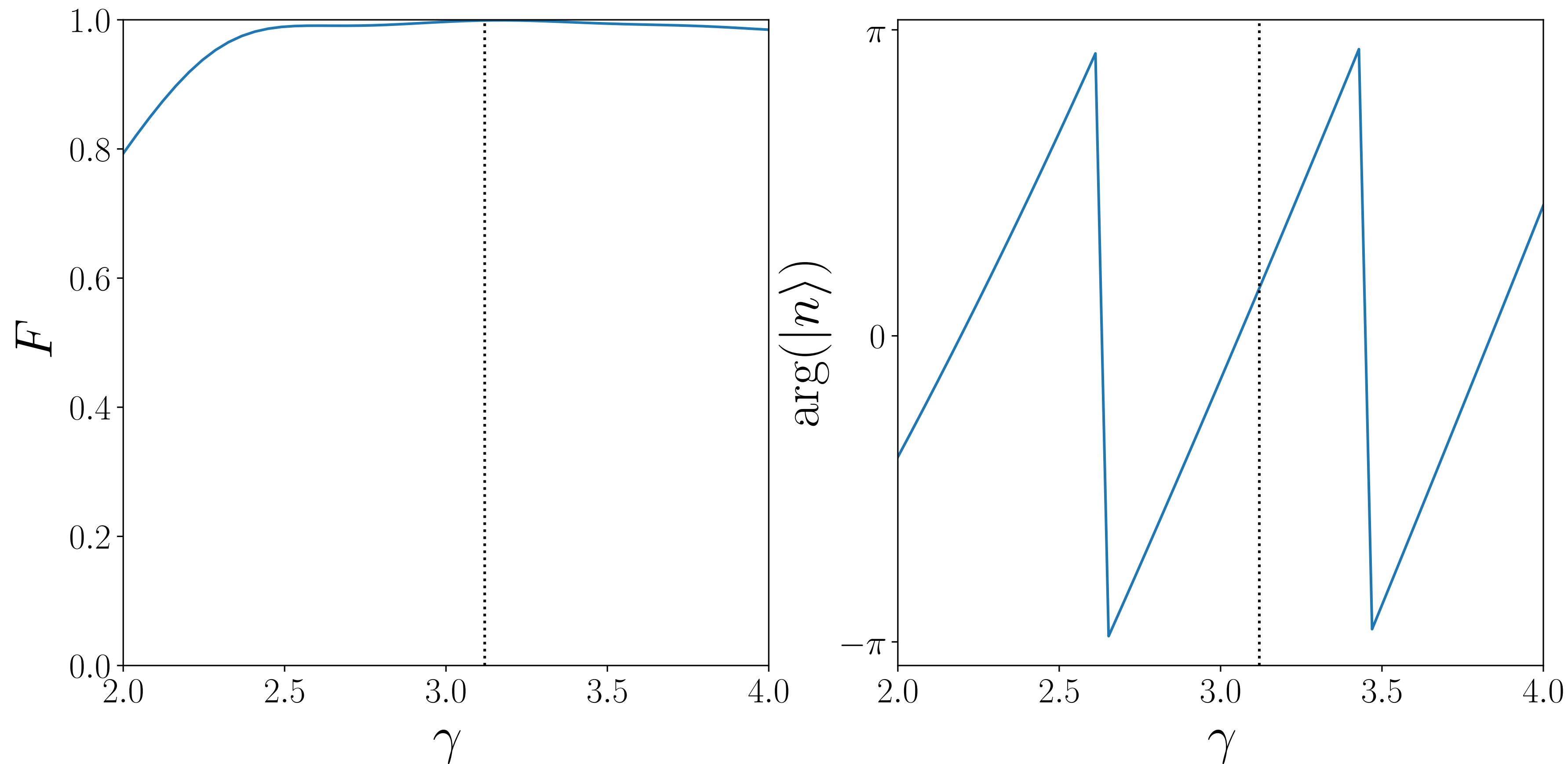
QOCT allows non-dispersive phase



Phase shift vs Lattice depth

Fidelity and phase of the accelerated state

$20\hbar k$ momentum transfer:



$$\frac{\Delta\gamma}{\gamma} = 10^{-6} \rightarrow \sim 1 \text{ mrad}/1000\hbar k$$

Can be improved with
sequence engineering