Single-photon large-momentum-transfer atom interferometry with application to determining the fine-structure constant

Outline

- Why measure the fine-structure constant, α ?
- Proposal: measurement by (single-photon) atom interferometry with Sr (method being developed in MAGIS-100 and AION projects)
- Conclusions
- Preprint: http://arxiv.org/abs/2403.10225

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Fine-structure constant (α)

- Coupling strength for electromagnetic interaction

$$\alpha^2 = \frac{2R_\infty h}{m_e c} = \frac{2R_\infty}{c} \times \frac{A_r(X)}{A_r(e)} \times \frac{h}{m_X}$$

$\Leftarrow$ 1 Hartree = $2hcR_\infty = \alpha^2 m_e c^2$

Rydberg constant spectroscopy of H

Cyclotron & spin-precession frequency ratios

Atom interferometry

- Atomic recoil velocity $= \hbar k/m$; $m \equiv m_X$

$$\Delta\phi \sim \frac{\Delta S}{\hbar} \sim \frac{m}{2\hbar}\int \Delta(v^2)dt \sim \frac{\hbar k^2 T}{m} \sim \omega_{\mathrm{rec}} T$$

- Recoil frequency: $\omega_{\mathrm{rec}} = \dfrac{\hbar k^2}{2m_X}$
- Time = T

m_X = mass of atomic species, e.g. m_{Sr}

Fine-structure constant (α) – current experiments

- Coupling strength for electromagnetic interaction

$$\alpha^2 = \frac{2R_\infty h}{m_e c} = \frac{2R_\infty}{c} \times \frac{A_r(X)}{A_r(\mathrm{e})} \times \frac{h}{m_X}$$

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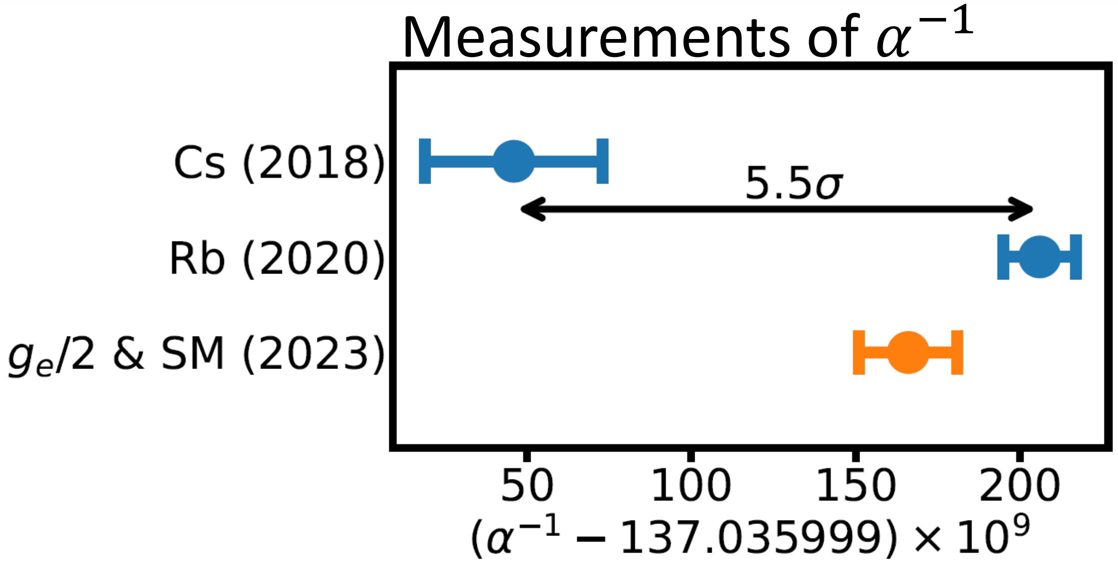


R. Parker et al., Science **360**, 191 (2018)
L. Morel et al., Nature **588**, 61 (2020)
X. Fan,... G. Gabrielse. PRL **130**, 071801 (2023)

Testing the Standard Model of Particle Physics

- Electron magnetic moment. X. Fan,… G. Gabrielse. PRL **130**, 071801 (2023)

$$-\frac{\mu_e}{\mu_B} = \frac{g_e}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 +$$

$$\ldots + a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}}$$

- Standard Model comparison limited by > 5σ discrepancy in α
- Improvement of $g_e - 2$ would make it competitive with muon *g*-2 experiment’s sensitivity to beyond SM effects.

- Other precision measurements can also be used α to test the SM, e.g. G. Adkins et al., Phys. Rep. **975**, 1 (2022)

Atom interferometry: uncertainties in determining α

$$\frac{\Delta\alpha}{\alpha} \approx \frac{1}{2}\sqrt{\left(\frac{\Delta R_\infty}{R_\infty}\right)^2 + \left(\frac{\Delta A_r(\mathrm{e})}{A_r(\mathrm{e})}\right)^2 + \left(\frac{\Delta A_r(\mathrm{Sr})}{A_r(\mathrm{Sr})}\right)^2 + \left(\frac{\Delta m(\mathrm{Sr})}{m(\mathrm{Sr})}\right)^2}$$

$$\frac{\Delta R_\infty}{R_\infty} = 2 \times 10^{-12}$$

- negligible for now

$$\frac{\Delta A_r(\mathrm{e})}{A_r(\mathrm{e})} = 3 \times 10^{-11} \qquad \frac{\Delta A_r(\mathrm{Sr})}{A_r(\mathrm{Sr})} = 7 \times 10^{-11}$$

- assume that mass ratios can be improved

$$\frac{\Delta m(\mathrm{Rb})}{m(\mathrm{Rb})} = 15 \times 10^{-11}$$

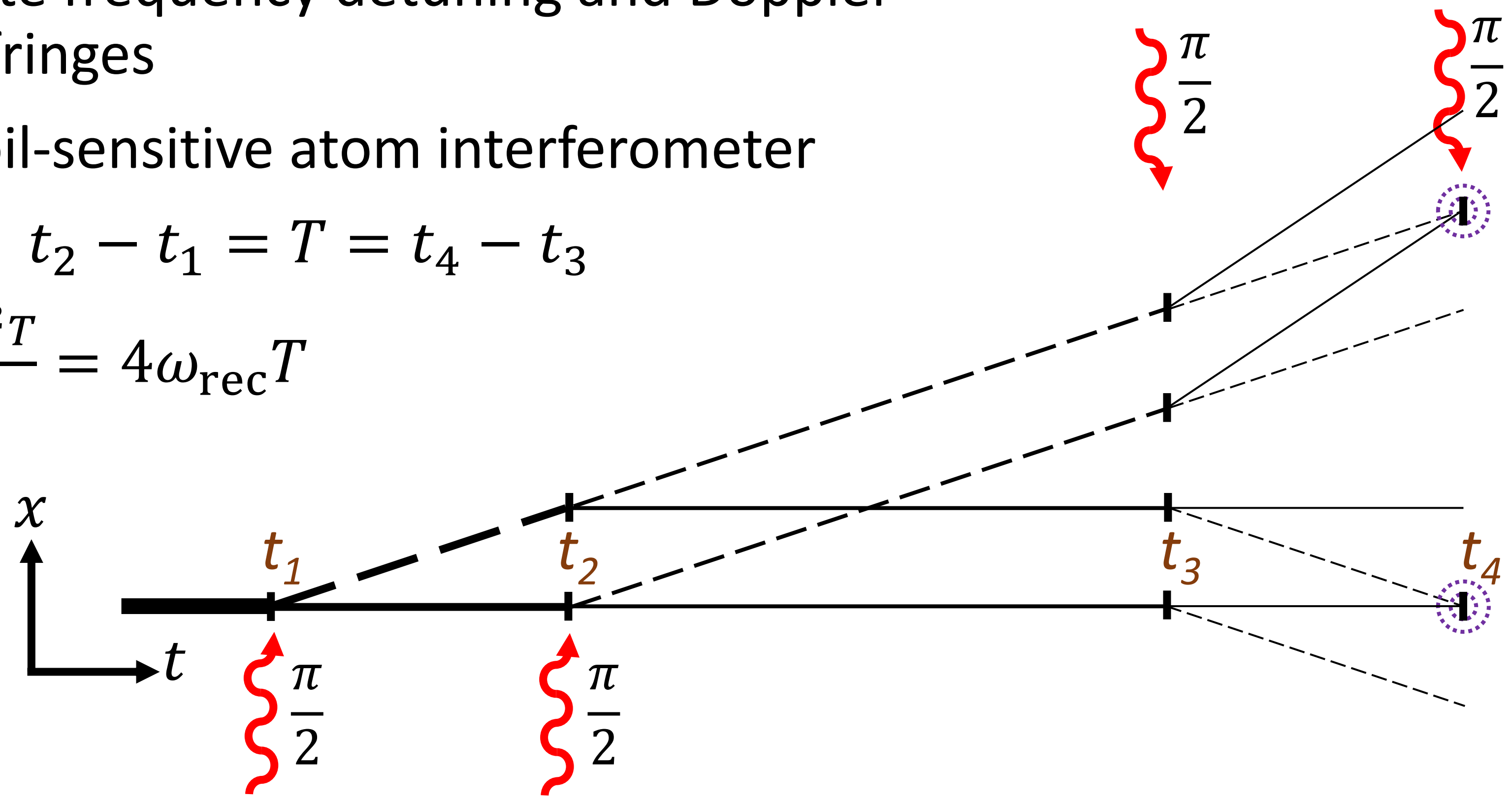
- Current experiment

- $\frac{\Delta\alpha}{\alpha} = 8 \times 10^{-11}$ is the uncertainty currently (LKB, Paris 2020)
- $\frac{\Delta\alpha}{\alpha} \approx 1 \times 10^{-11}$ would require $\frac{\Delta m(\mathrm{Sr})}{m(\mathrm{Sr})} \lesssim 2 \times 10^{-11}$, and improved mass ratios.
- N.B. Rest mass difference between ground and excited states: $\frac{m(Sr\ ^3P_0)}{m(Sr\ ^1S_0)} - 1 \approx 2 \times 10^{-11}$

E. Tiesinga..., Rev. Mod. Phys. **93**, 025010 (2021); R. Rana..., PRA **86**, 050502 (2012); M. Wang..., CPC **45**, 030003 (2021)

Ramsey-Bordé interferometer(s)

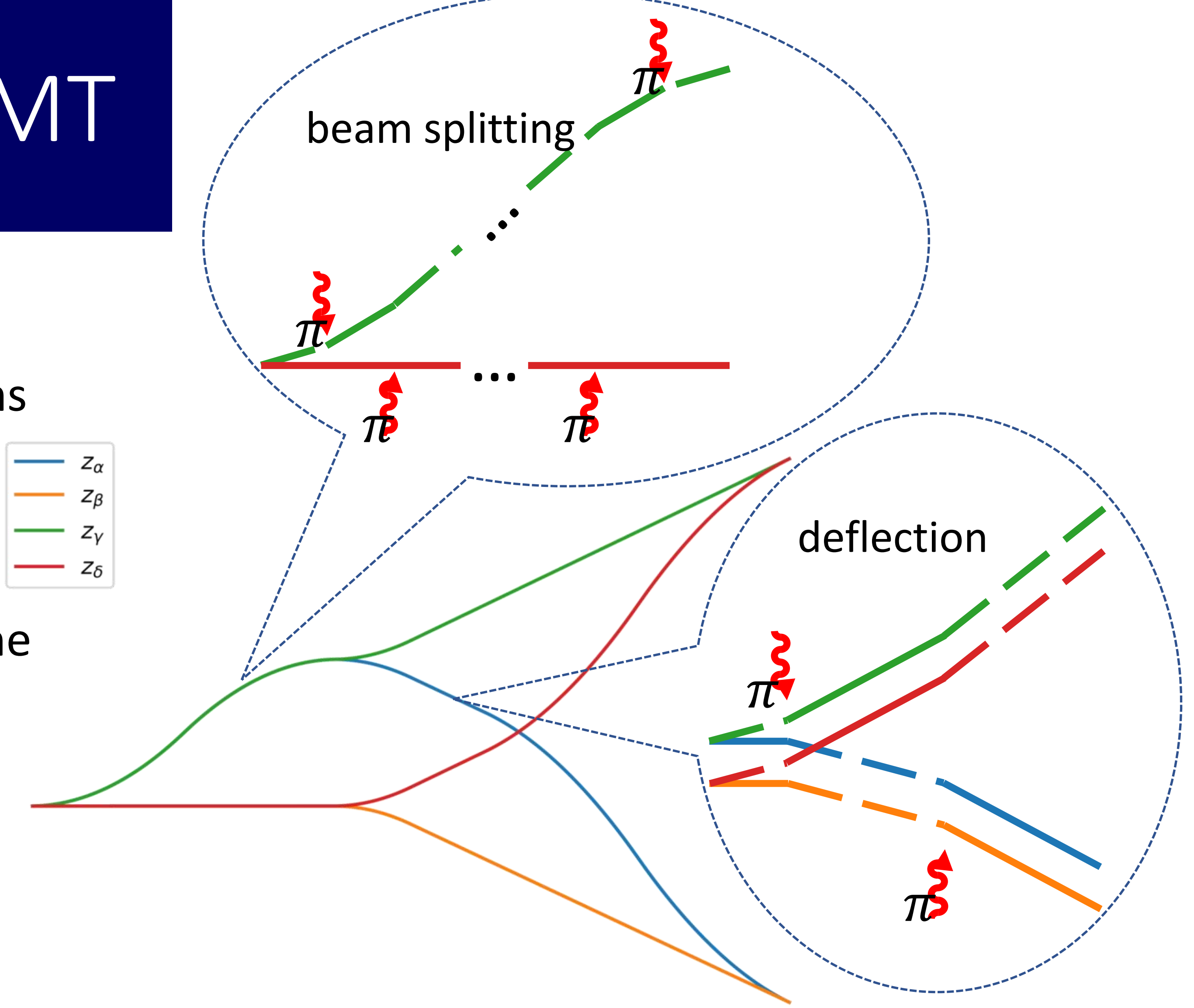
- Four $\pi/2$ pulses = two oppositely directed pairs
- Originally developed to separate frequency detuning and Doppler dephasing for optical Ramsey fringes
- Bordé realised its use as a recoil-sensitive atom interferometer
- Two interferometers close for: $t_2 - t_1 = T = t_4 - t_3$
- Differential phase: $\Delta\Phi = \frac{2\hbar k^2 T}{m} = 4\omega_{\text{rec}}T$



C. Bordé et al., PRA **30**, 1836 (1984); C.Bordé, Phys. Lett. A **140**, 10 (1989); J.Bergquist et al., PRL **38**, 159 (1977).

Increasing sensitivity – LMT

- Large momentum transfer (LMT) by a sequence of π pulses in alternating directions to increase momentum separation.
- LMT beam-splitter from pulses between initial/final pairs of $\pi/2$ pulses. For recoil frequency measurement we address only one of the arms
  - $\Delta\phi$ scales as recoil-energy difference
  - $E_{\rm rec} = \hbar\omega_{\rm rec} = m v_{\rm rec}^2/2$
- The two interferometers can be deflected (apart) using LMT in the middle region



Recoil measurement scheme

Use Large Momentum Transfer (LMT) pulses to:

1. Increase recoil-energy separation, then cancel it

   (otherwise two interferometers do not both close)

2. Deflect the interferometers in opposite directions
3. Close the interferometers

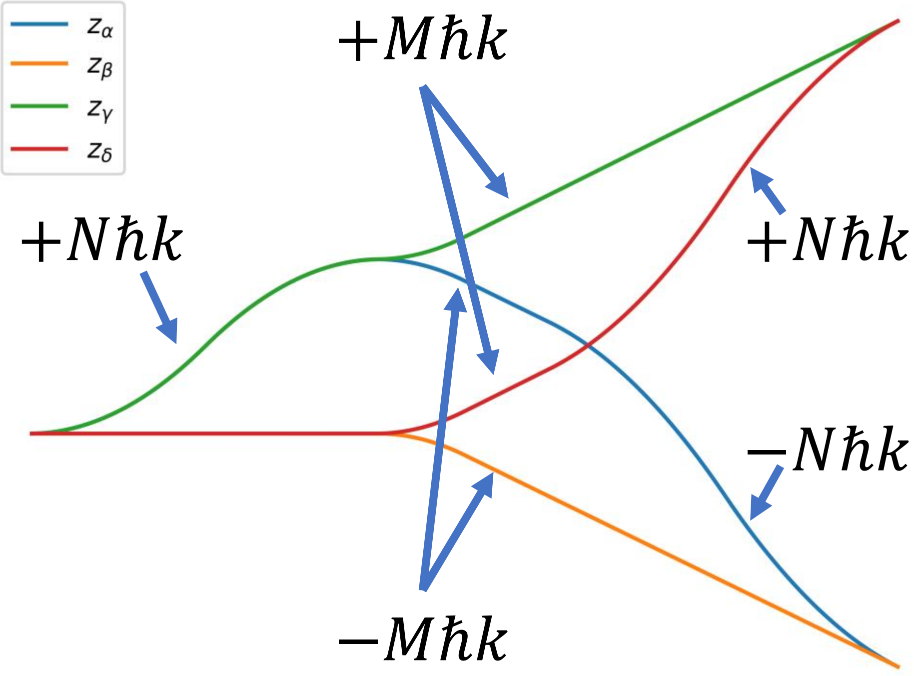
- Differential phase:

$$\Delta\Phi = \frac{(N+1)(N+2M+2)\hbar k^2(T-N\Delta t)}{m} - \frac{N(N+1)(N+2)\hbar k^2 \Delta t}{3m}$$

$$\geq (N+1)\left(2M(N+1) + \frac{(N+2)(2N+3)}{3}\right)\frac{\hbar k^2 \Delta t}{m}$$

Δt – repetition time of pulses

$$t_2 - t_1 = T = t_4 - t_3$$

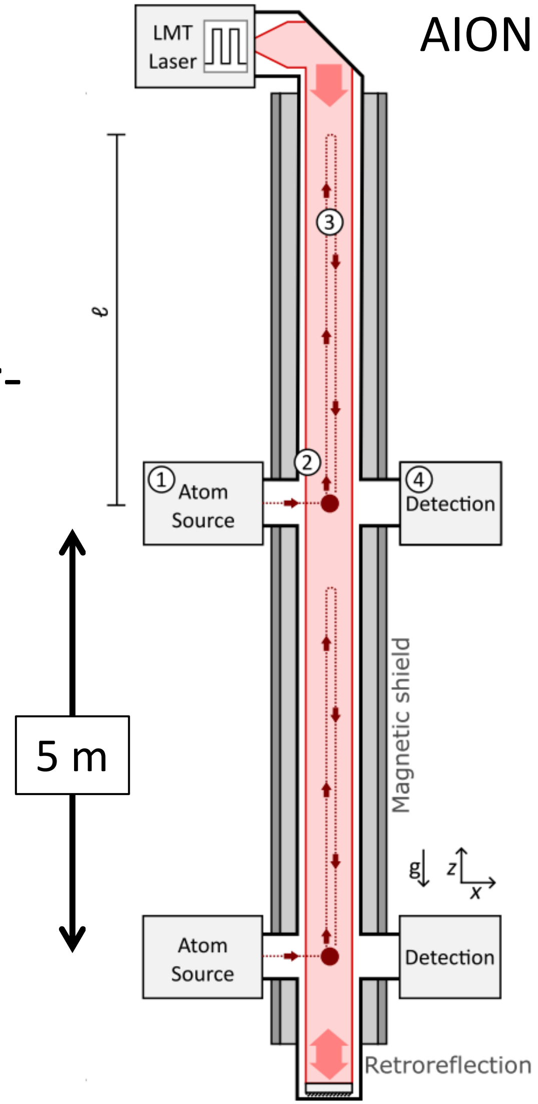


$(2N+1)\Delta t \leq T$ due to pulse timing constraints

Intermediate-scale atom interferometers

- Long coherence times of the optical clock transitions
  - $\tau \sim 2$ min for ^{87}Sr
- Single-photon transitions avoids some issues for counter-propagating pulses for Raman transitions/Bragg pulses
- Intermediate scale (~ 10 m) prototypes under development for future VLBAI (see other talks).

AION Prototype



Gravity gradient mitigation – asymmetric launch

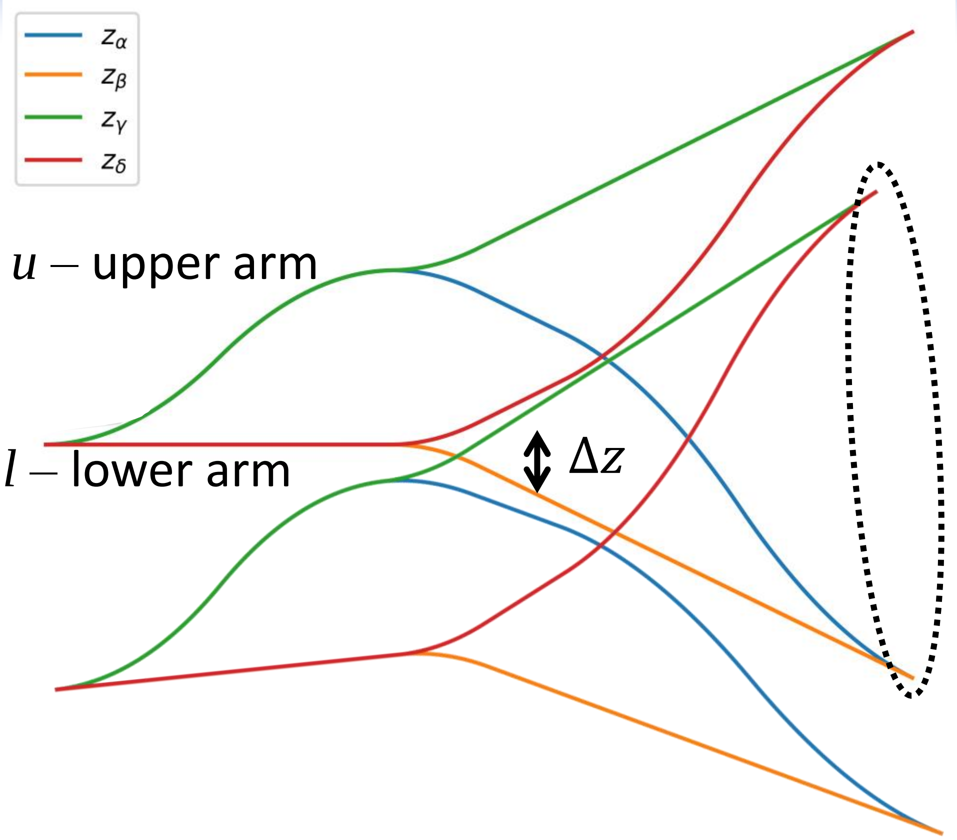
- Gravity gradient phase is a large systematic when scaling up

$$\Delta\phi_\gamma = \frac{m\gamma A}{\hbar}\Delta z$$

$$\mathrm{A} = \int (z_l(t) - z_u(t))dt$$

where A is the space-time area enclosed (between two arms)

- $\gamma = G_{zz} = \frac{dg}{dz}$, the gradient of the gravitational acceleration, or any (spatial) gradient of acceleration.
- Strategy as in Zhong et al., (Berkeley):
  - Offset two Ramsey-Bordé interferometers
  - Compare top interferometer of (initially) lower set with bottom one of upper set.
  - Acceleration gradients along vertical cancel.



Offset simultaneous conjugate atom interferometers. W. Zhong, … H. Müller, Phys. Rev. A **101**, 053622 (2020).

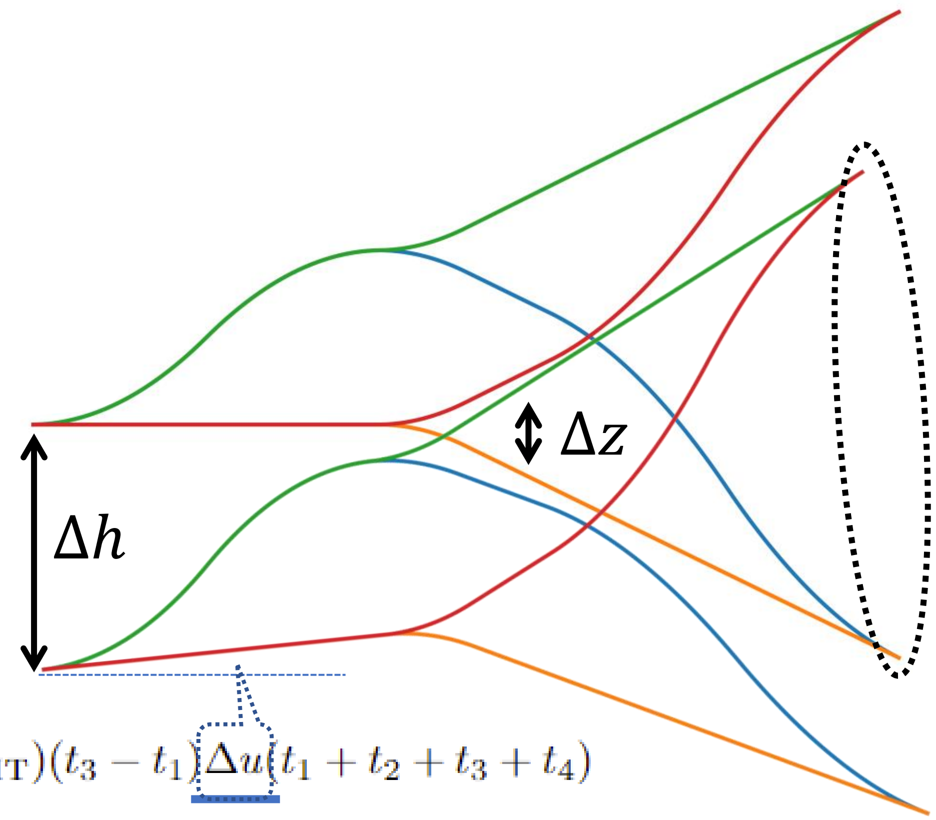
Gravity gradient mitigation – asymmetric launch

- Gravity gradient phase → large systematic

$$\Delta\phi_\gamma = \frac{m\gamma A}{\hbar}\Delta z$$

- Compare top interferometer of (initially) lower set with bottom one of upper set
- Differential phase (for our scheme):

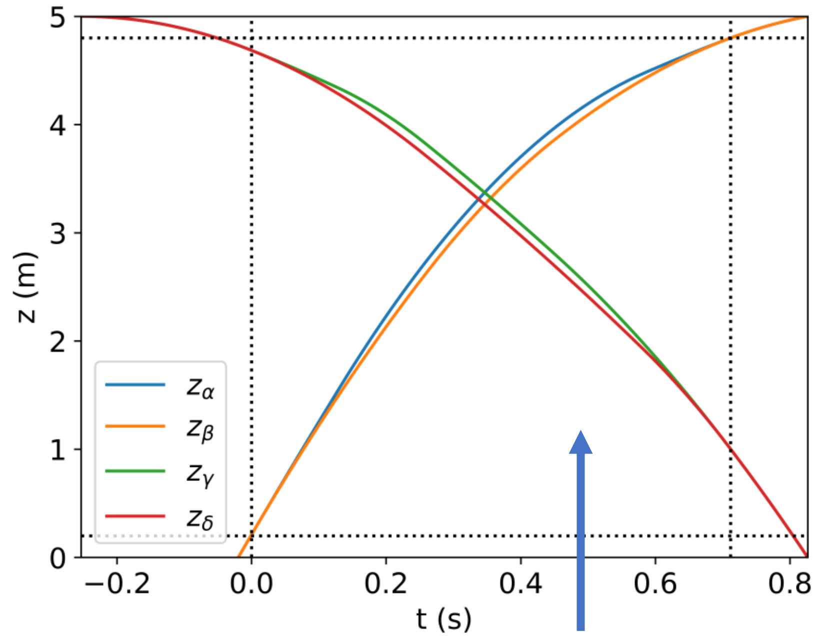
Set to 0 and solve for asymmetric launch parameters



$$\Delta\Phi_\gamma \approx -\frac{(N+1)k\gamma}{180}\Big(180(T-N\Delta t_{\mathrm{LMT}})(t_3-t_1)\Delta h + 45(T-N\Delta t_{\mathrm{LMT}})(t_3-t_1)\Delta u(t_1+t_2+t_3+t_4)$$
$$+\frac{15\hbar k(T-N\Delta t_{\mathrm{LMT}})}{m}\big[2(N+2M+2)T^2 - T(4N+2M+5)N\Delta t_{\mathrm{LMT}} + 6(t_3-t_2)(t_3-t_1)$$
$$+6M(t_3-t_m)(t_4-(t_m+(M-1)\Delta\tau_{\mathrm{LMT}})) + 6M(t_4-t_m)(t_3-(t_m+(M-1)\Delta\tau_{\mathrm{LMT}}))$$
$$+(3N^2+2NM+6N+2M+3)N\Delta t_{\mathrm{LMT}}^2 + 2M(M-1)(2M-1)\Delta\tau_{\mathrm{LMT}}^2\big]$$
$$-\frac{N(N+2)(3N^2+6N+1)\hbar k\Delta t_{\mathrm{LMT}}^3}{m}\Big)$$

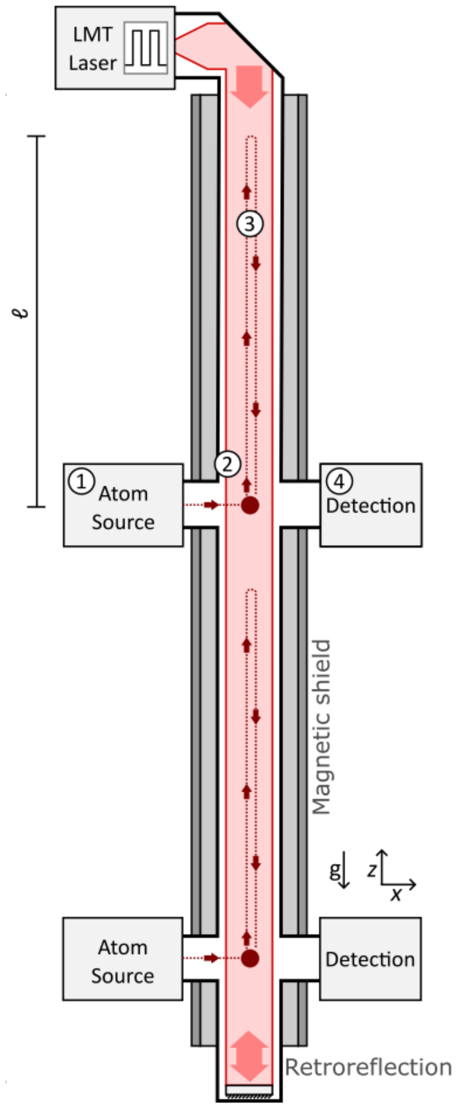
Prospects for intermediate-scale atom interferometers

‘X’ configuration

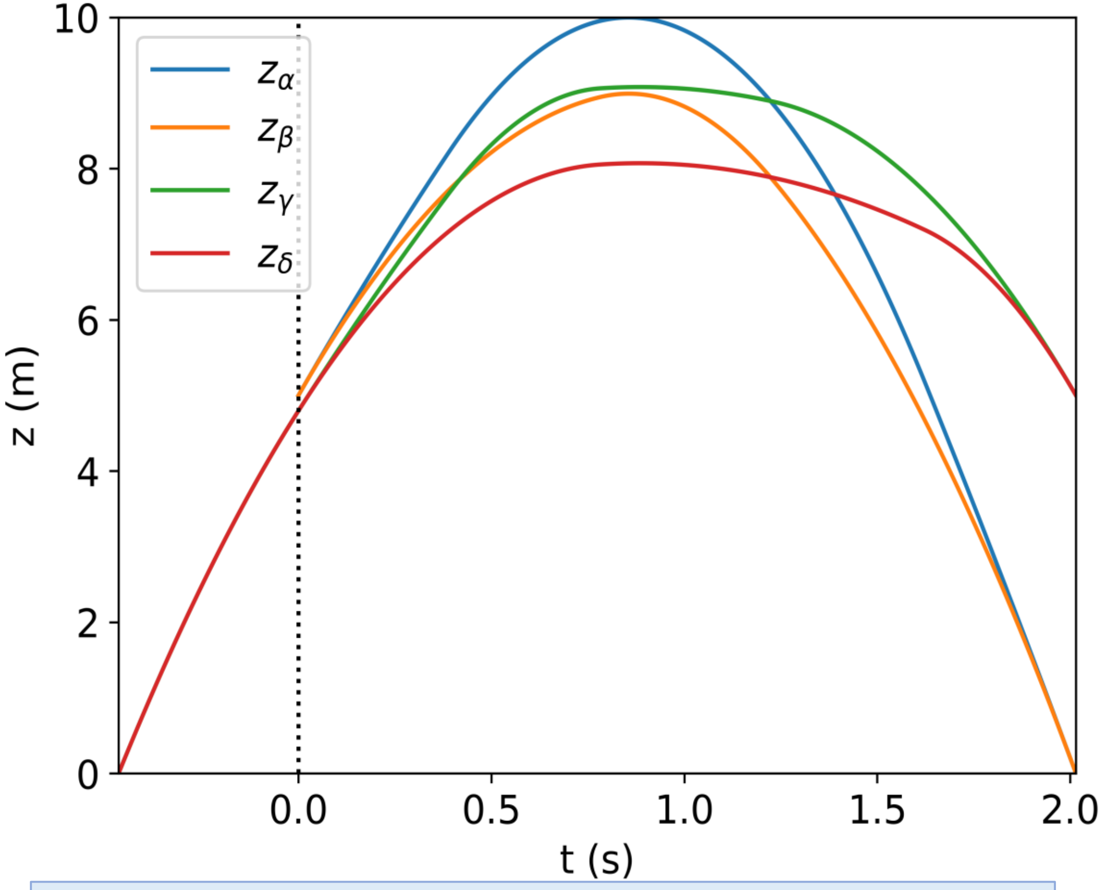


Restricted interferometry region

AION Prototype



Fountain configuration



Fountain ⇒ crossing the source region (questionable magnetic shielding.

Prospects for intermediate-scale atom interferometers

L = 3m, $\Delta\Phi$ = 1.4 × 10⁸ rad $\Rightarrow \dfrac{\Delta m(\mathrm{Sr})}{m(\mathrm{Sr})} < 1 \times 10^{-11}$ (at 1 mrad resolution) $(\Delta t = 1\ \mathrm{ms})$

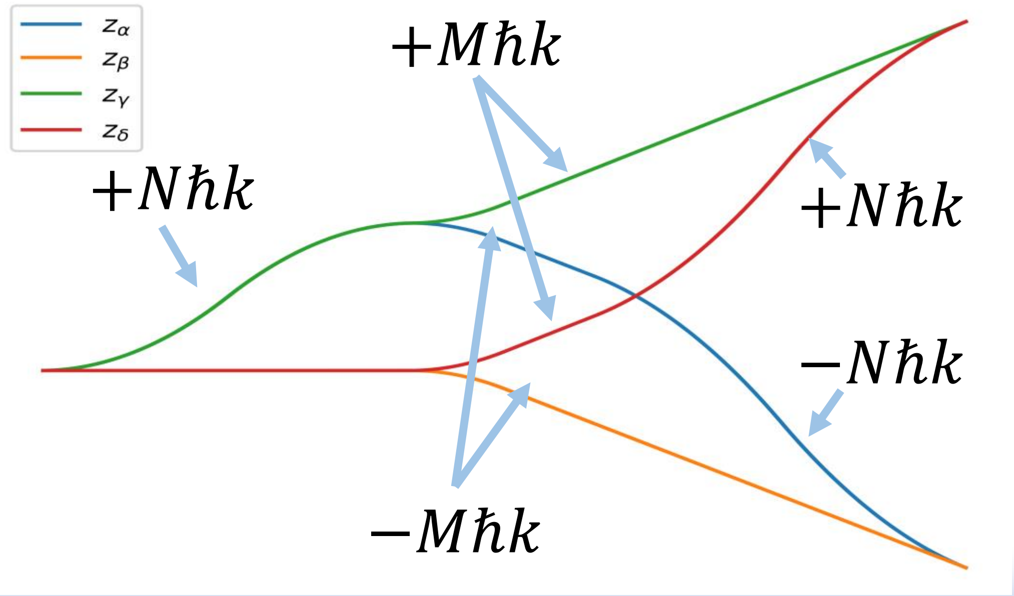
TABLE IV. Optimal parameters for X configuration with Sr at different values of L

L (m)	N	M	$h - 0.2\,\mathrm{m}$ (mm)	u ($\mathrm{m\,s^{-1}}$)	z_0 (m)	$\Delta\Phi$ (rad)
1	24	54	5.403	4.79312	0.7355[a]	4.6×10^6
1.5	42	78	41.107	5.71647	1.2165[a]	2.0×10^7
2	56	108	12.499	6.72874	1.6995[a]	4.9×10^7
2.5	72	118	51.929	7.39303	2.198[a]	9.0×10^7
3	84	140	22.604	8.18404	2.6915[a]	1.4×10^8

L = 3m, N = 84 pulses, M = 140 pulses

Total number of pulses = 4(N+1)+M = 480

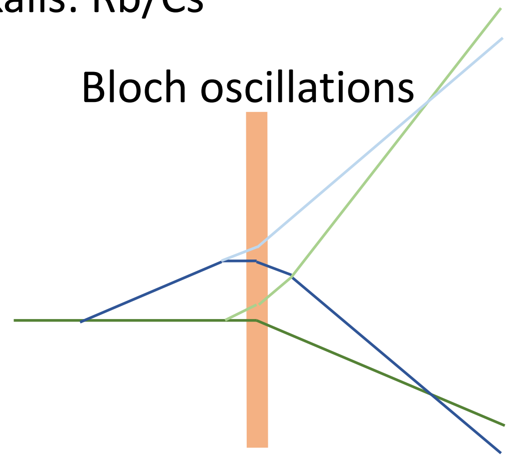
N=M=0 is the standard Ramsey-Borde interferometer

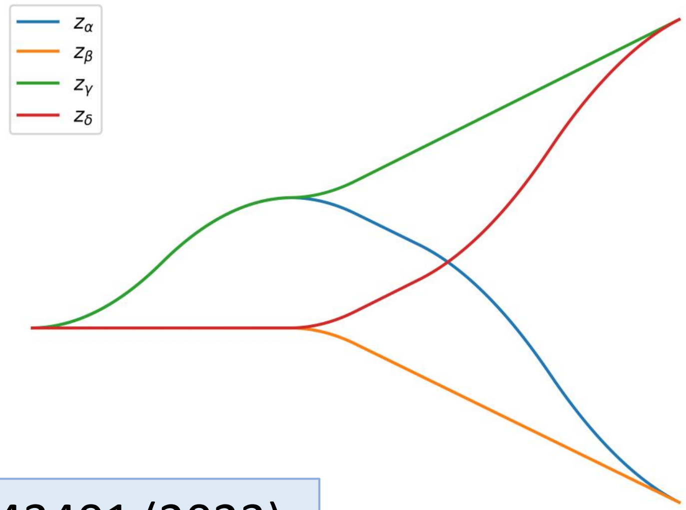


Possible advantages of Sr scheme over Rb or Cs ?

- Long lifetime of the excited state
  - Reduced spontaneous emission loss
- Single photon vs Raman/Bragg pulses
  - Single propagation direction scales better to larger instruments
  - Near-resonant pulses vs far-detuned pulses (laser power requirements)
  - Other systematics – to be investigated?
  - See also other talks in this session: Pierre Clade, Subhadeep Gupta, …
- LMT beam splitters ⇒ quadratic scaling with LMT order:
  - Sr (2022): $400\hbar k$
  - Rb (2023): $200\hbar k$
- To be discussed at this workshop?

Scheme for alkalis: Rb/Cs



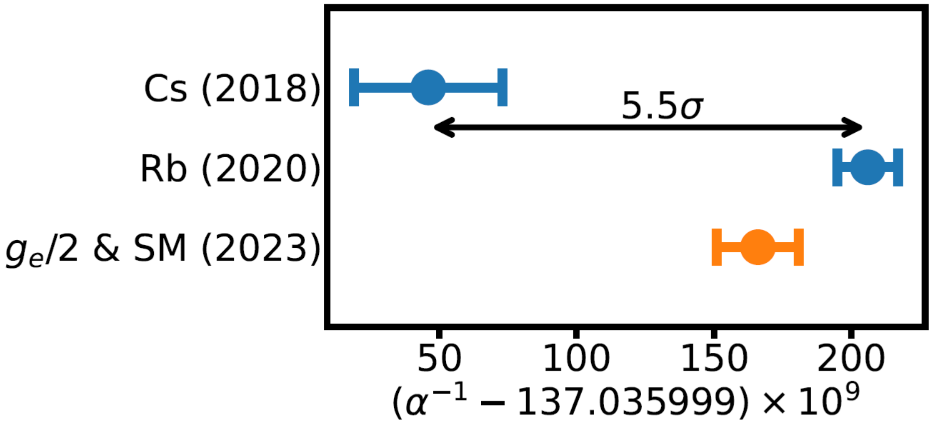


T. Wilkason et al., PRL **129**, 183202 (2022); A. Béguin et al., PRL **131**, 143401 (2023)

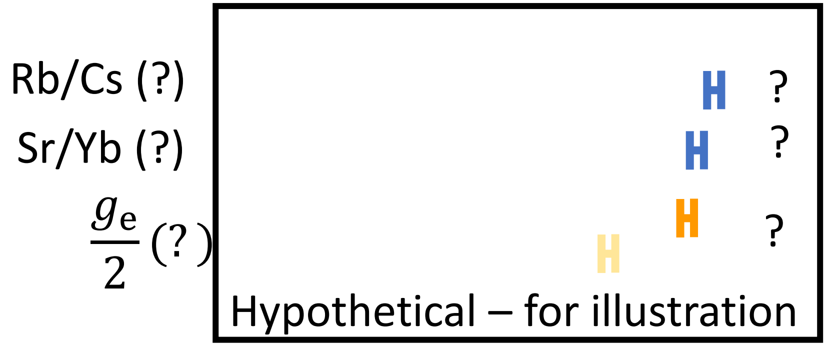
Conclusions – atom interferometry with application to determining α

- Sr interferometry could provide another determination of α
  - 3m high atom interferometer would improve α to level for testing the Standard Model (with other measurements)
  - Limited by current knowledge of relative mass $A_r(\mathrm{Sr})$ [which can be improved?]
  - Same scheme works for other neutral atoms with optical clock transitions, e.g. Yb
- Systematic uncertainties ?
  - All atom interferometry techniques require high quality laser beams: shifts from beam curvature, Gouy phase etc.
  - Single-photon interferometry avoids light shifts in Raman transitions and reduces sensitivity to vibrations.
  - Single-photon interferometry may have other systematics?
  - Atom Interferometry: Metrology & Systematics (tomorrow)
- Open question: Implementation of velocity selective pulses resonant at two different velocities? (to be modelled)

Current measurements of α^{-1}



Possible future measurements of α^{-1}



Strontium Laser Lab, University of Oxford, AION project.

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Other members of AION @Oxford:

- Ian Shipsey
- Adam Lowe
- Daniela Bortoletto
- John March-Russell
- Dan Weatherill
- Dan Wood
- Charu Mishra

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Suppl. notes: How M enters phase shift

- Use LMT to:

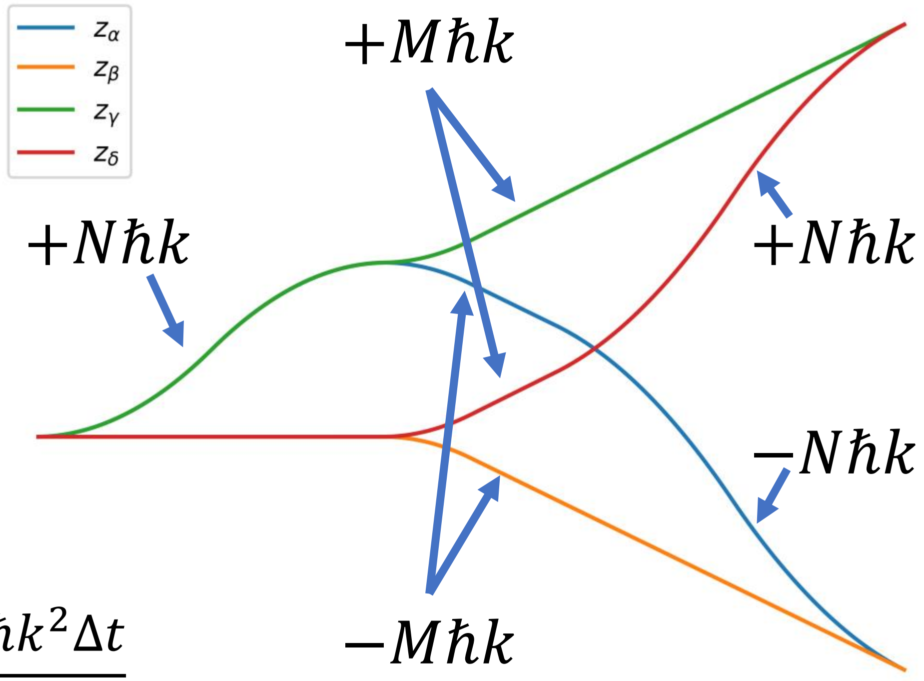
1. Increase recoil-energy separation, then cancel it

   (otherwise two interferometers do not both close)

2. Deflect the interferometers in opposite directions
3. Close the interferometers

- Differential phase is

$$\Delta\Phi = \frac{(N+1)(N+2M+2)\hbar k^2(T-N\Delta t)}{m} - \frac{N(N+1)(N+2)\hbar k^2 \Delta t}{3m}$$



(N+1)(N+2M+2) scaling from:
The **maximum difference in recoil frequencies** (between the final π/2-pulses):
1. For the upward interferometer, is proportional to **$(M+1)^2 - (N+M+2)^2 = -(N+1)(N+2M+3)$**
2. For the downward interferometer, is proportional to **$(-N-M-1)^2 - (-M)^2 = (N+1)(N+2M+1)$**
3. The difference is, thus, proportional to **2(N+1)(N+2M+2)**

- The terms that scale like Δt come from the momentum transfer spread over time.

Supplementary notes: muon *g*-2

- The muon *g*-2 tension is the $\sim 4\sigma$ discrepancy between the experimental value and the theoretical value according to some determinations of the lowest-order hadronic vacuum polarisation contribution (see the latest review by the Particle Data Group on the topic for further details).
- A factor of 2.2 improvement for the electron *g*-2 theoretical value would probe (beyond the) Standard Model contributions at the level of the muon *g*-2 discrepancy.
- This assumes whatever new physics might be proposed affects all leptons (equally) and the larger rest mass of the muon enhances its sensitivity (since higher energy leads to higher probability of virtual particle creation) – see, e.g., Testing new physics with the electron *g* − 2 | Journal of High Energy Physics (springer.com), pp 7-8 .