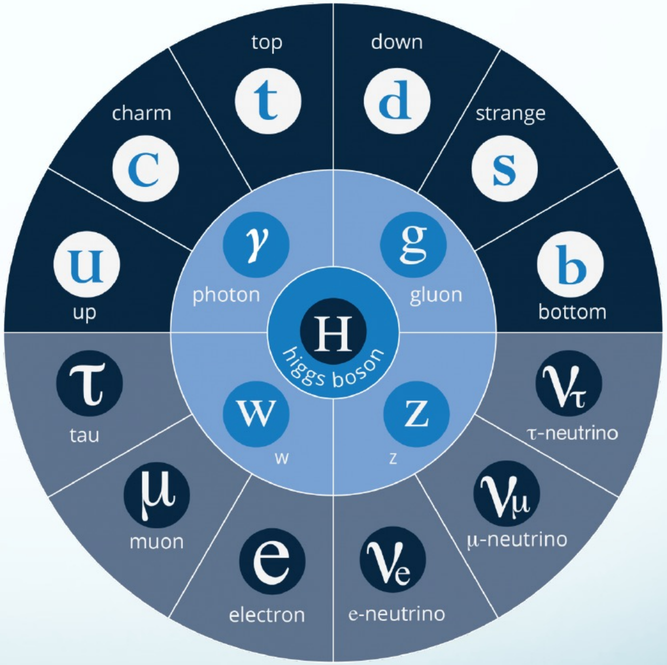
The Standard-Model

Ulla Blumenschein, Queen Mary University of London

- Introduction Standard model
- Precision tests of electroweak physics at the LHC



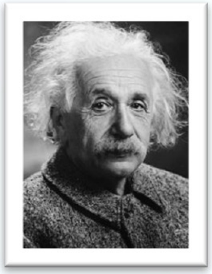
From time immemorial, man has desired to comprehend the complexity of nature in terms of as few elementary concepts as possible.

Abdus Salam (Nobel Price in physics 1979)

Relativistic notation:

Space-time 4-vector: $x^\mu = (ct, x, y, z)$ $x_\mu = (ct, -x, -y, -z)$

Minkowski metric:



$$a \cdot b = a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

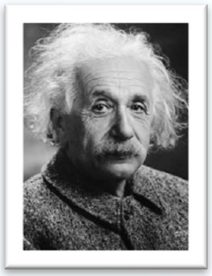
Albert Einstein

Herbert Minkowski, Goettingen 1902-09

Relativistic notation:

Space-time 4-vector: $x^\mu = (ct, x, y, z)$ $\quad x_\mu = (ct, -x, -y, -z)$

Minkowski metric:



Albert Einstein

$$a \cdot b = a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

Herbert Minkowski, Goettingen 1902-09

HEP: Energy-momentum 4-vector:

$$p^\mu = m\gamma \frac{dx^\mu}{dt} = \left(\frac{E}{c}, \mathbf{p}\right)$$

$E = \gamma m c^2$ $\quad \mathbf{p} = \gamma m \mathbf{v}$

Lorentz-factor $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$

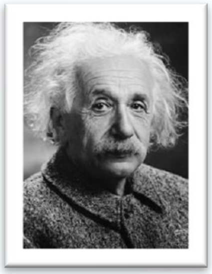
In most slides, we use natural units: h = c = 1

Relativistic notation:

Space-time 4-vector:

$$x^\mu = (ct, x, y, z) \qquad x_\mu = (ct, -x, -y, -z)$$

Minkowski metric:



Albert Einstein

$$a \cdot b = a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

Herbert Minkowski, Goettingen 1902-09

HEP: Energy-momentum 4-vector:

$$p^\mu = m\gamma \frac{dx^\mu}{dt} = \left(\frac{E}{c}, \mathbf{p}\right)$$

$$E = \gamma m c^2 \qquad \mathbf{p} = \gamma m \mathbf{v}$$

The norm of an Energy-momentum 4-vector is its (invariant) mass

$$p^2 = \frac{E^2}{c^2} - |\mathbf{p}|^2 = m^2$$

Lorentz-factor

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

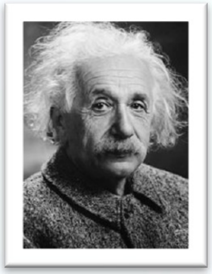
In most slides, we use natural units: h = c = 1

Relativistic notation:

Space-time 4-vector:

$$x^\mu = (ct, x, y, z) \qquad x_\mu = (ct, -x, -y, -z)$$

Minkowski metric:



Albert Einstein

$$a \cdot b = a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

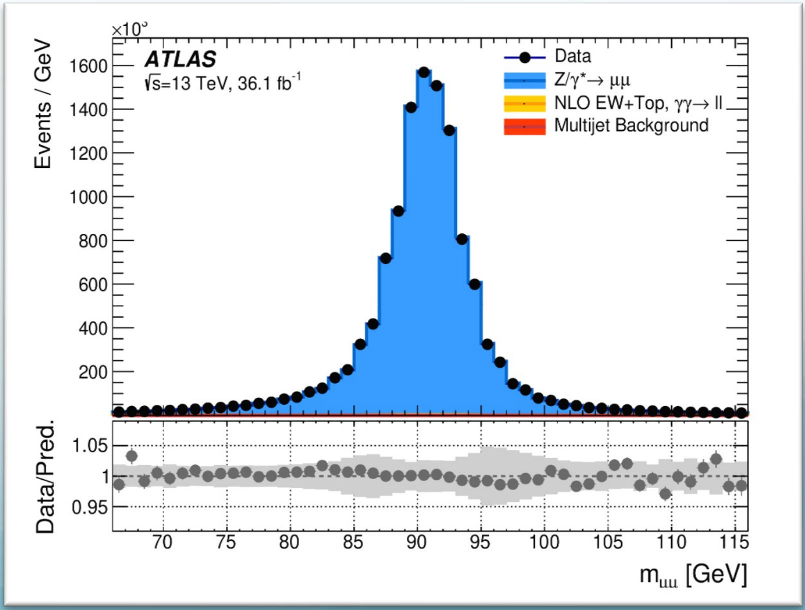
Herbert Minkowski, Goettingen 1902-09

HEP: Energy-momentum 4-vector:

$$p^\mu = m\gamma \frac{dx^\mu}{dt} = \left(\frac{E}{c}, \mathbf{p}\right)$$

The norm of an Energy-momentum 4-vector is its (invariant) mass

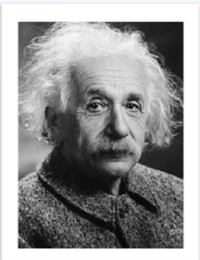
$$p^2 = \frac{E^2}{c^2} - |\mathbf{p}|^2 = m^2c^2$$



Relativistic notation:

Space-time 4-vector: $x^\mu = (ct, x, y, z)$ $x_\mu = (ct, -x, -y, -z)$

Minkowski metric:



$$a \cdot b = a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

Albert Einstein

Herbert Minkowski, Goettingen 1902-09

HEP: Energy-momentum 4-vector:

$$p^\mu = m\gamma \frac{dx^\mu}{dt} = \left(\frac{E}{c}, \mathbf{p}\right)$$

Relativistic energy-momentum relation

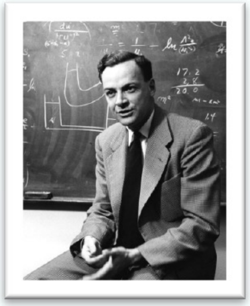
$$E^2 = |p|^2 c^2 + m^2 c^4$$

The norm of an Energy-momentum 4-vector is its (invariant) mass

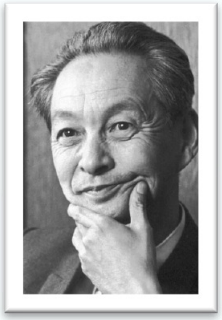
$$p^2 = \frac{E^2}{c^2} - |\mathbf{p}|^2 = m^2 c^2$$

In most slides, we use natural units: h = c = 1

(1) Towards QED



Richard Feynman



Shin'Ichiro Tomonaga

Julian Schwinger

"I would rather have questions that can't be answered than answers that can't be questioned."

Richard Feynman (Nobel Price in physics 1965)

QM description of Fermions

Applying quantum substitution $\vec{p} \rightarrow i\hbar\vec{\nabla}$ $E \rightarrow i\hbar\frac{\partial}{\partial t}$

E. Schroedinger

(1) to **classic energy-momentum relation** $E = p^2/2m + V$

QM description of Fermions

Applying quantum substitution $\vec{p} \rightarrow i\hbar\vec{\nabla}$ $E \rightarrow i\hbar\frac{\partial}{\partial t}$


E. Schroedinger

(1) to **classic energy-momentum relation** $E = p^2/2m + V$

→ Schroedinger equation: $$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + V(\vec{r},t)\psi(\vec{r},t)$$

QM description of Fermions

Applying quantum substitution $\vec{p} \to i\hbar\vec{\nabla}$ $E \to i\hbar\frac{\partial}{\partial t}$

E. Schroedinger

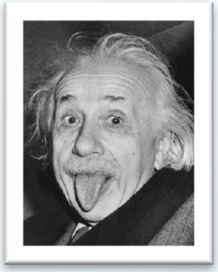
(1) to **classic energy-momentum relation** $E = p^2/2m + V$

→ Schroedinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + V(\vec{r},t)\psi(\vec{r},t)$$

Applying quantum substitution $p_\mu \to i\hbar\partial_\mu$ $\partial_\mu = \frac{\partial}{\partial X^\mu}$

(2) to **relativistic energy-momentum relation:** $E^2 = p^2c^2 + m^2c^4$



A. Einstein

QM description of Fermions

Applying quantum substitution $\vec{p} \rightarrow i\hbar\vec{\nabla}$ $\quad E \rightarrow i\hbar\frac{\partial}{\partial t}$

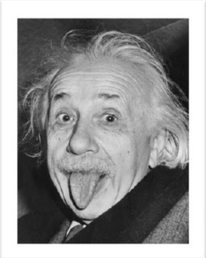
E. Schroedinger

(1) to **classic energy-momentum relation** $E = p^2/2m + V$

→ Schroedinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + V(\vec{r},t)\psi(\vec{r},t)$$

Applying quantum substitution $p_\mu \rightarrow i\hbar\partial_\mu$ $\quad \partial_\mu = \frac{\partial}{\partial X^\mu}$



A. Einstein

(2) to **relativistic energy-momentum relation:** $E^2 = p^2c^2 + m^2c^4$

→ Klein-Gordon equation:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0.$$

QM description of Fermions

Applying quantum substitution $\vec{p} \to i\hbar\vec{\nabla}$ $E \to i\hbar\frac{\partial}{\partial t}$

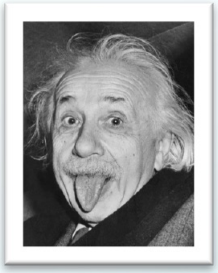
E. Schroedinger

(1) to **classic energy-momentum relation** $E = p^2/2m + V$

→ Schroedinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + V(\vec{r},t)\psi(\vec{r},t)$$

Applying quantum substitution $p_\mu \to i\hbar\partial_\mu$ $\partial_\mu = \frac{\partial}{\partial X^\mu}$



A. Einstein

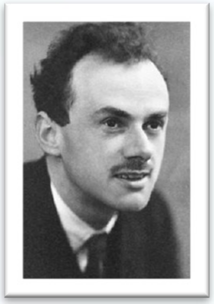
(2) to **relativistic energy-momentum relation:** $E^2 = p^2c^2 + m^2c^4$

→ Klein-Gordon equation:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0.$$

describes scalar particles but not fermions. → need equation first order in time

The Dirac equation



Paul Dirac

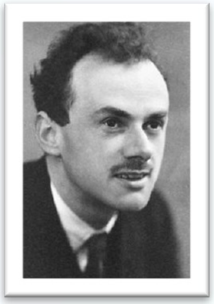
→ 4D matrices: γ-matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Dirac equation



Paul Dirac

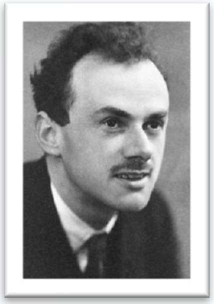
→ 4D matrices: γ-matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Fermion becomes a Dirac spinor:

$$\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$$

The Dirac equation



Paul Dirac

→ 4D matrices: γ-matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Fermion becomes a Dirac spinor:

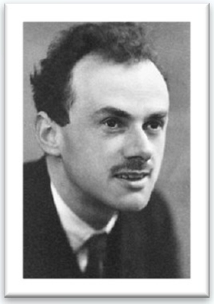
$$\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$$

→ Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\partial_\mu = \frac{\partial}{\partial X^\mu}$$

The Dirac equation



Paul Dirac

→ 4D matrices: γ-matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Fermion becomes a Dirac spinor:

$$\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$$

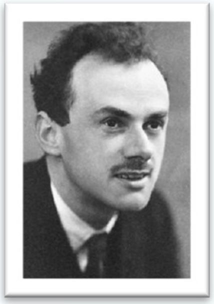
→ Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\partial_\mu = \frac{\partial}{\partial X^\mu}$$

Implicit sum convention: Sum over all indices: μ = 0,1,2,3

The Dirac equation



Paul Dirac

→ 4D matrices: γ-matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Fermion becomes a Dirac spinor:

$$\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$$

→ Dirac equation:

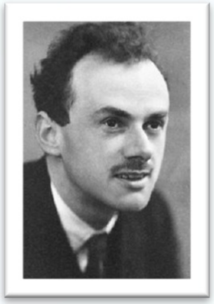
$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Solutions:

$$\psi^{(i)} = u^{(i)}(E, \mathbf{p}) e^{-\frac{1}{\hbar}(Et - \mathbf{px})}$$

spinor wave

The Dirac equation



Paul Dirac

→ 4D matrices: γ-matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Fermion becomes a Dirac spinor:

$$\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$$

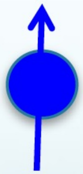
→ Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Solutions:

$$\psi^{(i)} = u^{(i)}(E, \mathbf{p}) e^{-\frac{1}{\hbar}(Et - \mathbf{px})}$$

Electrons:



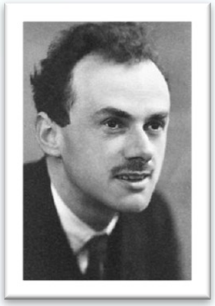
$$u_1 = \begin{bmatrix} 1 \\ 0 \\ p_z/(E+m) \\ (p_x + ip_y)/(E+m) \end{bmatrix} \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ (p_x - ip_y)/(E+m) \\ -p_z/(E+m) \end{bmatrix}$$

Positrons

spin component in direction of motion

$$v_1 = \begin{bmatrix} (p_x - ip_y)/(E+m) \\ -p_z/(E+m) \\ 0 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} p_z/(E+m) \\ (p_x + ip_y)/(E+m) \\ 1 \\ 0 \end{bmatrix}$$

The Dirac equation



Paul Dirac

→ 4D matrices: γ-matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Fermion becomes a Dirac spinor:

$$\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$$

→ Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Solutions:

$$\psi^{(i)} = u^{(i)}(E, \mathbf{p}) e^{-\frac{i}{\hbar}(Et - \mathbf{px})}$$

probability density/current:

$$j^\mu = \overline{\psi}\gamma^\mu\psi = (\overline{\psi}\gamma_0\psi + \overline{\psi}\vec{\gamma}\psi) = (\rho, \vec{j})$$

Electrons:

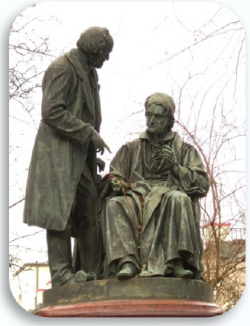
$$u_1 = \begin{bmatrix} 1 \\ 0 \\ p_z/(E+m) \\ (p_x + ip_y)/(E+m) \end{bmatrix} \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ (p_x - ip_y)/(E+m) \\ -p_z/(E+m) \end{bmatrix}$$

Positrons

spin component in direction of motion

$$v_1 = \begin{bmatrix} (p_x - ip_y)/(E+m) \\ -p_z/(E+m) \\ 0 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} p_z/(E+m) \\ (p_x + ip_y)/(E+m) \\ 1 \\ 0 \end{bmatrix}$$

The EM force

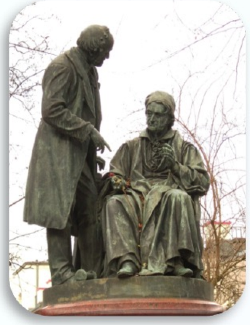


Classic Maxwell equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0 \, .$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \, .$$

The EM force



Classic Maxwell equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

→ **Homogeneous MWE:** E and B as derivative/curl of potentials V and $\vec{A}$

$$\vec{B}(\vec{x},t) = \nabla \times \vec{A}(\vec{x},t) \qquad \vec{E}(\vec{x},t) = -\nabla V(\vec{x},t) - \frac{\partial \vec{A}(\vec{x},t)}{\partial t}$$

The EM force

Classic Maxwell equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

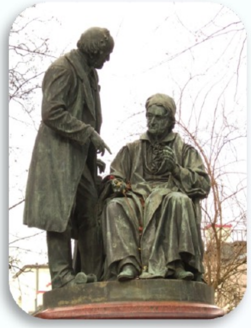
→ **Homogeneous MWE:** E and B as derivative/curl of potentials V and $\vec{A}$

$$\vec{B}(\vec{x},t) = \nabla \times \vec{A}(\vec{x},t) \qquad \vec{E}(\vec{x},t) = -\nabla V(\vec{x},t) - \frac{\partial \vec{A}(\vec{x},t)}{\partial t}$$

→ relativistic description: 4-potential $A^\mu = (V, \vec{A})$

4-current $J^\mu = (\rho, \vec{J})$

The EM force



Classic Maxwell equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

→ **Homogeneous MWE:** E and B as derivative/curl of potentials V and $\vec{A}$

$$\vec{B}(\vec{x},t) = \nabla \times \vec{A}(\vec{x},t) \qquad \vec{E}(\vec{x},t) = -\nabla V(\vec{x},t) - \frac{\partial \vec{A}(\vec{x},t)}{\partial t}$$

→ relativistic description: 4-potential $A^\mu = (V, \vec{A})$

4-current $J^\mu = (\rho, \vec{J})$

Field strength tensor:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}$$

The EM force

Classic Maxwell equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0 .$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}.$$

→ **Homogeneous MWE:** E and B as derivative/curl of potentials V and $\vec{A}$

$$\vec{B}(\vec{x},t) = \nabla \times \vec{A}(\vec{x},t) \qquad \vec{E}(\vec{x},t) = -\nabla V(\vec{x},t) - \frac{\partial \vec{A}(\vec{x},t)}{\partial t}$$

→ relativistic description: 4-potential $A^\mu = (V, \vec{A})$

4-current $J^\mu = (\rho, \vec{J})$

Field strength tensor:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}$$

→ **Inhomogeneous MWE:**

$$\partial_\mu F^{\mu\nu} = J^\nu$$

The Lagrangian density

Classical mechanics: Lagrange function: $L = T - V$

Fundamental law of motion: Euler-Lagrange equation: $$\frac{\partial L}{\partial q_j} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_j} = 0$$

The Lagrangian density

Classical mechanics: Lagrange function: $L = T - V$

Fundamental law of motion: Euler-Lagrange equation: $\frac{\partial L}{\partial q_j} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_j} = 0$

Field theory: Lagrangian is a function of fields and their 4-derivatives $\mathcal{L}(\phi, \partial_\mu \phi)$

→ Euler-Lagrange equation: $\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} = \frac{\delta \mathcal{L}}{\delta \phi}$

The Lagrangian density

Classical mechanics: Lagrangefunction: $L = T - V$

Fundamental law of motion: Euler-Lagrange equation: $\frac{\partial L}{\partial q_j} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_j} = 0$

Field theory: Lagrangian is a function of fields and their 4-derivatives

$$\mathcal{L}(\phi, \partial_\mu \phi)$$

→ Euler-Lagrange equation: $\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} = \frac{\delta \mathcal{L}}{\delta \phi}$

Example **Dirac Lagrangian:** $e^\pm$

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

→ Euler Lagrange equation
= Dirac equation $(i\gamma^\mu \partial_\mu - m)\psi = 0$

The Lagrangian density

Classical mechanics: Lagrangefunction: $L = T - V$

Fundamental law of motion: Euler-Lagrange equation: $\frac{\partial L}{\partial q_j} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_j} = 0$

Field theory: Lagrangian is a function of fields and their 4-derivatives

$$\mathcal{L}(\phi, \partial_\mu \phi)$$

→ Euler-Lagrange equation: $\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} = \frac{\delta \mathcal{L}}{\delta \phi}$

Example **Dirac Lagrangian:** $e^{\pm}$

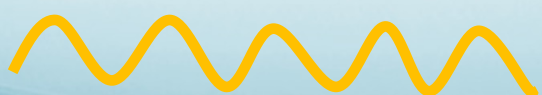
$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

→ Euler Lagrange equation = Dirac equation $(i\gamma^\mu \partial_\mu - m)\psi = 0$

Example: **Lagrangian of free photon:**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(B^2 - E^2)$$

$\bar{\psi} \equiv \psi^\dagger \gamma^0$



Gauge symmetries

Noether theorem:
Continuous symmetries → corresponding conserved quantities

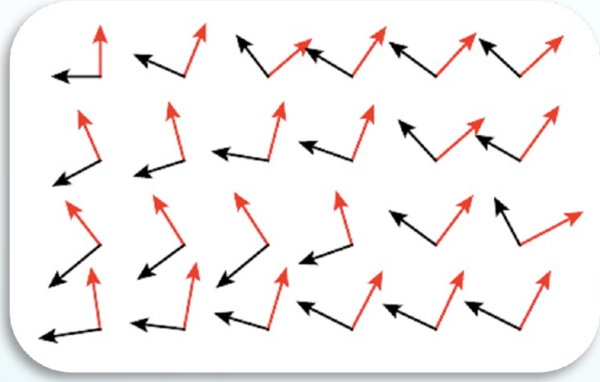
Emmy Noether,
Goettingen 1915-33

Gauge symmetries

Noether theorem:
Continuous symmetries → corresponding conserved quantities

Dirac Lagrangian is invariant
under **global phase translation**

Emmy Noether,
Goettingen 1915-33

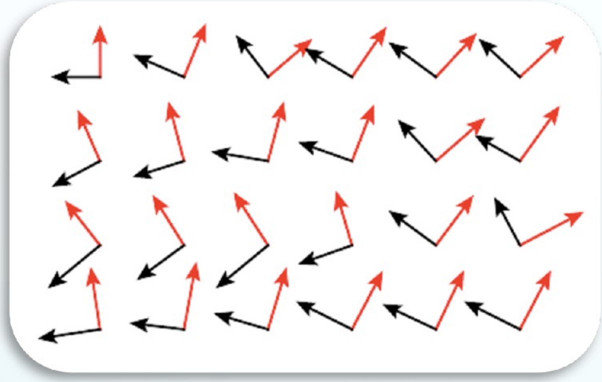


Gauge symmetries

Noether theorem:
Continuous symmetries → corresponding conserved quantities

Emmy Noether, Goettingen 1915-33

Dirac Lagrangian is invariant under **global phase translation**



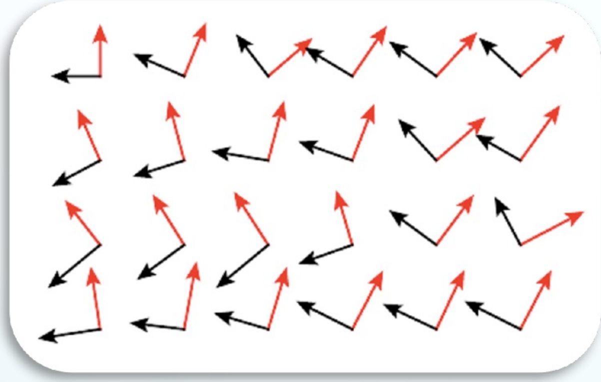
$$\psi \rightarrow \psi' = e^{-iq\alpha}\psi$$

Gauge symmetries

Noether theorem:
Continuous symmetries → corresponding conserved quantities

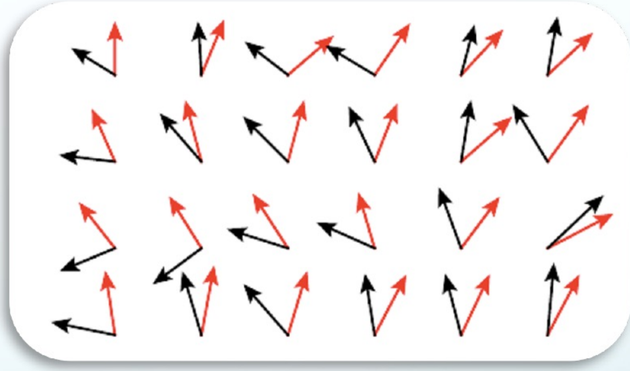
Emmy Noether,
Goettingen 1915-33

Dirac Lagrangian is invariant under **global phase translation**



$$\psi \rightarrow \psi' = e^{-iq\alpha}\psi$$

Now require also invariance under **local phase transitions**



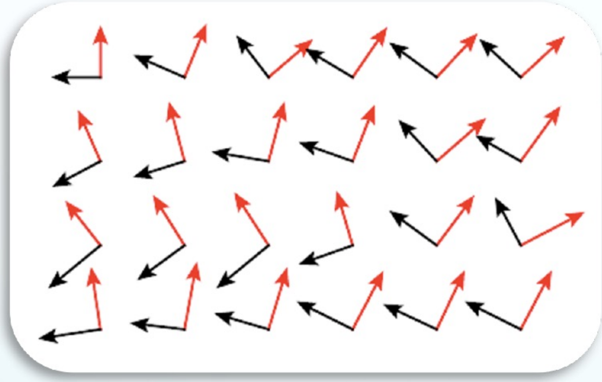
Objects are only influenced by their immediate surroundings
(principle of locality)

Gauge symmetries

Noether theorem:
Continuous symmetries → corresponding conserved quantities

Emmy Noether, Goettingen 1915-33

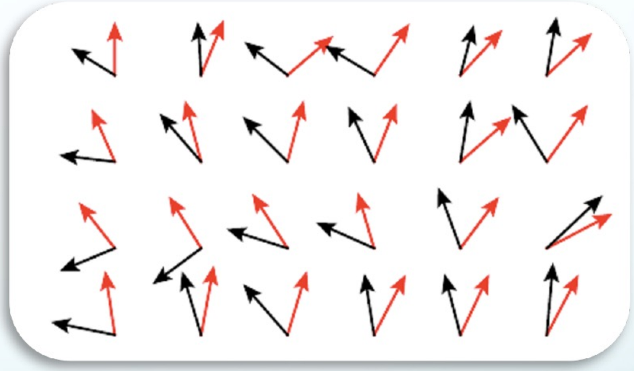
Dirac Lagrangian is invariant under **global phase translation**



$$\psi \rightarrow \psi' = e^{-iq\alpha}\psi$$

$e^{\pm}$

Now require also invariance under **local phase transitions**



$$\psi \rightarrow \psi' = e^{-iq\alpha(x_\mu)}\psi$$

Gauge invariance

When calculating the QED Lagrangian with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$e^{\pm}$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_{\mu})}\psi$$

Gauge invariance

When calculating the QED Lagrangian with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$e^{\pm}$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_{\mu})}\psi$$

We obtain a spurious inner derivative $\partial_{\mu}\alpha(x)$ destroying the gauge invariance

Gauge invariance

When calculating the QED Lagrangian with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$e^{\pm}$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_{\mu})}\psi$$

We obtain a spurious inner derivative $\partial_{\mu}\alpha(x)$ destroying the gauge invariance

But what about the principle of locality

?

Gauge invariance

When calculating the QED Lagrangian with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

$e^{\pm}$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_\mu)}\psi$$

We obtain a spurious inner derivative $\partial_\mu\alpha(x)$ destroying the gauge invariance

→ Replace the 4-derivative by the covariant derivative D_μ by adding interaction with photon field A_μ

$$D_\mu = \partial_\mu + iqA_\mu$$

Gauge invariance

When calculating the QED Lagrangian with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$e^{\pm}$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_{\mu})}\psi$$

We obtain a spurious inner derivative $\partial_{\mu}\alpha(x)$ destroying the gauge invariance

→ Replace the 4-derivative by the **Covariant Derivative D_{μ}** by adding interaction with photon field A_{μ}

with the photon field transforming as:

$$D_{\mu} = \partial_{\mu} + iqA_{\mu}$$

$$A_{\mu} \rightarrow A'_{\mu} = A_{\mu} + \partial_{\mu}\alpha(x)$$

Gauge invariance

When calculating the QED Lagrangian with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$e^{\pm}$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_{\mu})}\psi$$

We obtain a spurious inner derivative $\partial_{\mu}\alpha(x)$ destroying the gauge invariance

→ Replace the 4-derivative by the **Covariant Derivative D_{μ}** by adding interaction with photon field A_{μ}

with the photon field transforming as:

$$D_{\mu} = \partial_{\mu} + iqA_{\mu}$$

$$A_{\mu} \rightarrow A'_{\mu} = A_{\mu} + \partial_{\mu}\alpha(x)$$

Gauge invariance

When calculating the QED Lagrangian $e^{\pm}$ with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_{\mu})}\psi$$

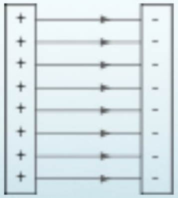
We obtain a spurious inner derivative $\partial_{\mu}\alpha(x)$ destroying the gauge invariance

→ Replace the 4-derivative by the **Covariant Derivative D_{μ}** by adding interaction with photon field A_{μ}

with the photon field transforming as:

$$D_{\mu} = \partial_{\mu} + iqA_{\mu}$$

$$A_{\mu} \rightarrow A'_{\mu} = A_{\mu} + \partial_{\mu}\alpha(x)$$



using the gauge freedom of the photon field

Gauge invariance

When calculating the QED Lagrangian with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$e^{\pm}$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_{\mu})}\psi$$

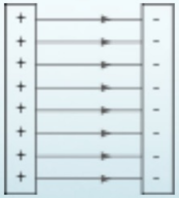
We obtain a spurious inner derivative $\partial_{\mu}\alpha(x)$ destroying the gauge invariance

→ Replace the 4-derivative by the **Covariant Derivative D_{μ}** by adding interaction with photon field A_{μ}

with the photon field transforming as:

$$D_{\mu} = \partial_{\mu} + iqA_{\mu}$$

$$A_{\mu} \rightarrow A'_{\mu} = A_{\mu} + \partial_{\mu}\alpha(x)$$



using the gauge freedom of the photon field

→ inner derivative is cancelled → gauge invariance restored

Gauge invariance

→ New Lagrangian:

$$\mathcal{L}_{QED} = \quad i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - m\bar{\psi}\psi$$

$$D_{\mu} = \partial_{\mu} + iqA_{\mu}$$

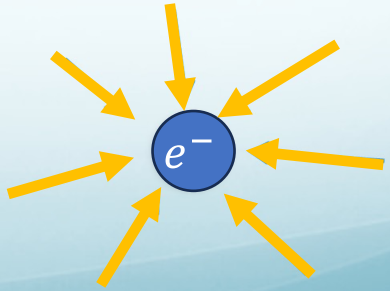
Adding kinematic term for free photon:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - m\bar{\psi}\psi$$

.. Yields the Maxwell-equations for current density $j^{\mu}_{EM} = e\bar{\psi}\gamma^{\mu}\psi$

Needed to introduce photon field by requiring local U(1) gauge invariance of the Lagrangian

Charged particles are always accompanied by EM field



Perturbation theory

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi - e\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

$\longrightarrow$ Scattering matrix

$$V = e\int d^3x\, \bar{\psi}\gamma^{\mu}\psi A_{\mu}$$

Perturbation Hamiltonian
in the Interaction picture

$$S = T\exp\left(-i\int_{-\infty}^{\infty} dt V(t)\right)$$

$$S_{\alpha\beta} = \langle \alpha_{in}|S|\beta_{out}\rangle$$

Perturbation theory

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - e\bar{\psi}\gamma^\mu A_\mu\psi$$

→ Scattering matrix

$$S_{\alpha\beta} = \langle\alpha_{in}|S|\beta_{out}\rangle$$

$$S = T\exp\left(-i\int_{-\infty}^{\infty} dt V(t)\right)$$

$$V = e\int d^3x\,\bar{\psi}\gamma^\mu\psi A_\mu$$

Perturbation Hamiltonian in the Interaction picture

→ Perturbation expansion: terms can be graphically represented as Feynman diagrams

Perturbation theory

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi - e\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

→ Scattering matrix

$$S_{\alpha\beta} = \langle \alpha_{in} | S | \beta_{out} \rangle$$

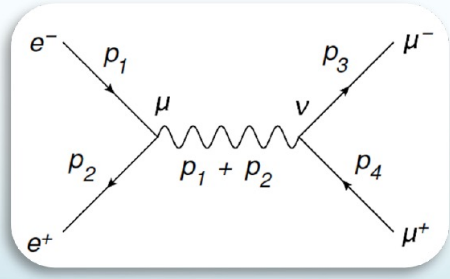
$$S = T\exp\left(-i\int_{-\infty}^{\infty} dt V(t)\right)$$

$$V = e\int d^3x\,\bar{\psi}\gamma^{\mu}\psi A_{\mu}$$

Perturbation Hamiltonian in the Interaction picture

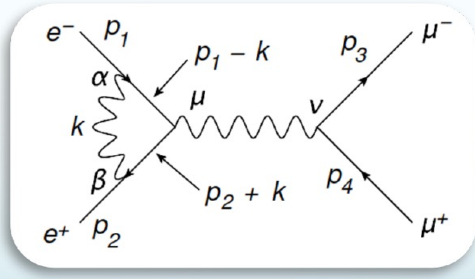
→ Perturbation expansion: terms can be graphically represented as Feynman diagrams

example: ee → μμ



LO

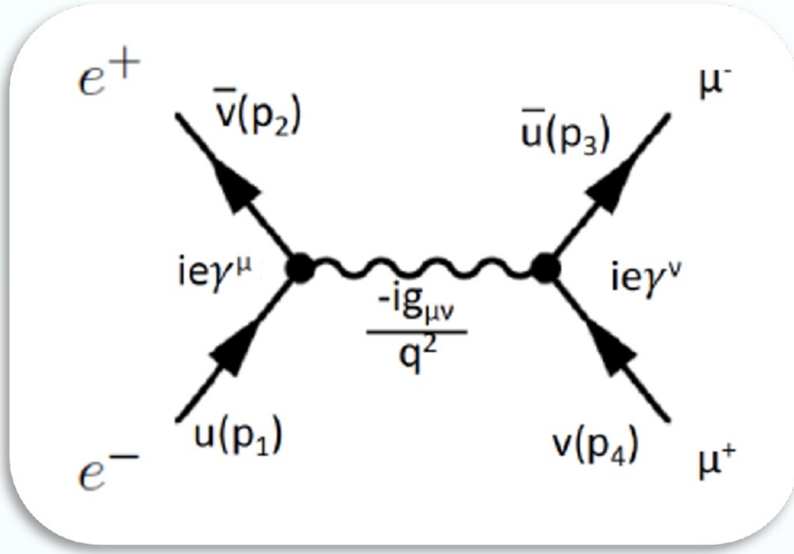
+



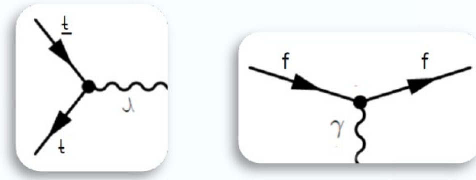
NLO

+ ....

Feynman diagrams



EM vertex:



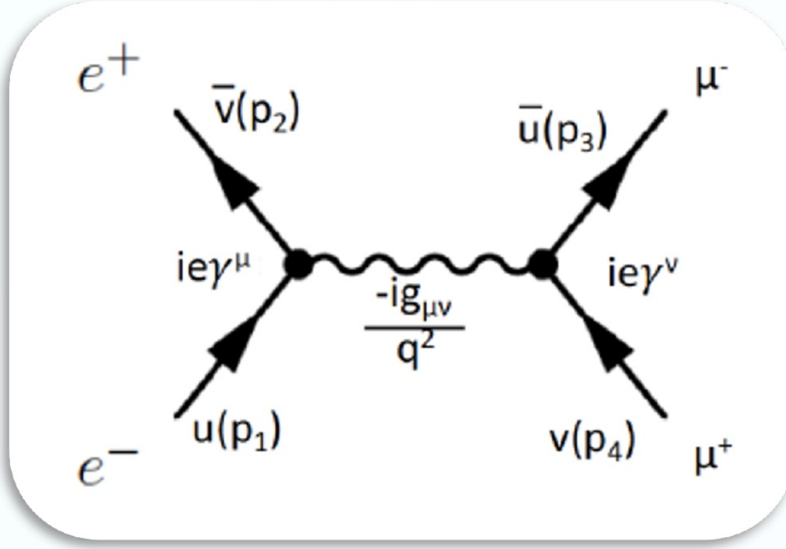
Each line and each vertex represents a mathematical term

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

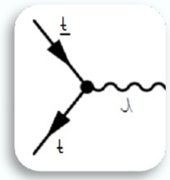
$$\mathbf{g} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

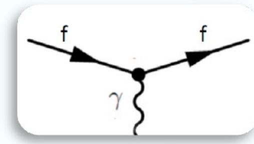
A set of Feynman Rules allows to translate any diagram to an amplitude without explicitly carrying out the perturbative expansion of the S-matrix

Feynman diagrams



EM vertex:





Each line and each vertex represents a mathematical term

→ Resulting matrix element:

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

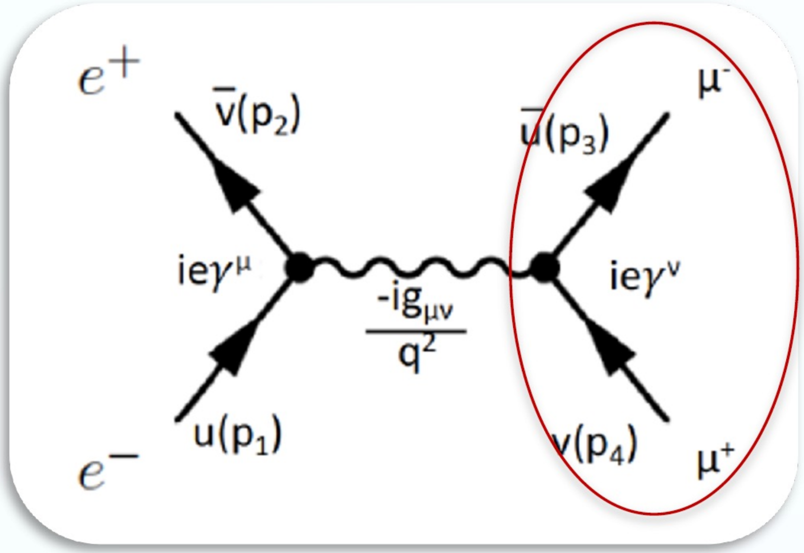
$$\mathcal{M} = \frac{-e^2}{(p_1+p_2)^2}[\bar{u}(p_3)\gamma^\mu v(p_4)][\bar{v}(p_2)\gamma_\mu u(p_1)]$$

$$\mathbf{g} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

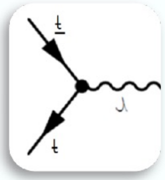
QED fermion current: $j^\mu_{EM} = e\overline{\psi}\gamma^\mu\psi$

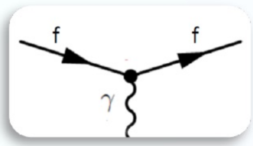
A set of Feynman Rules allows to translate any diagram to an amplitude without explicitly carrying out the perturbative expansion of the S-matrix

Feynman diagrams



EM vertex:





Each line and each vertex represents a mathematical term

→ Resulting matrix element:

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

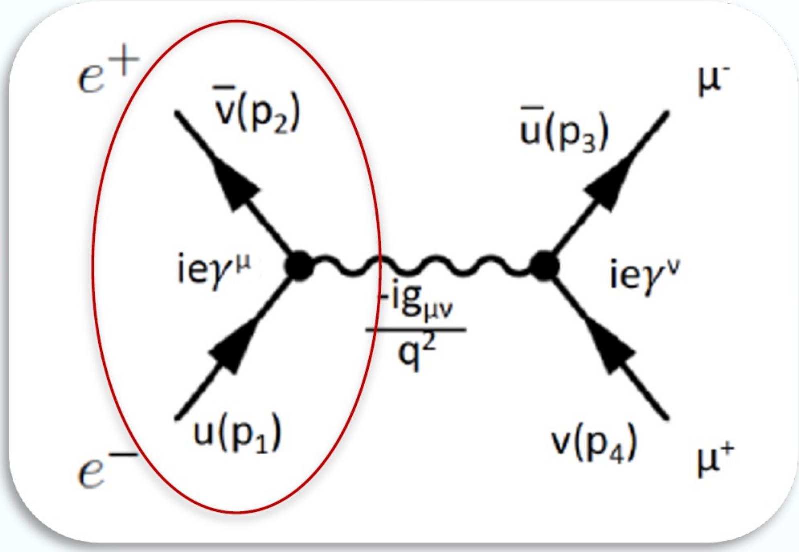
$$\mathbf{g} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\mathcal{M} = \frac{-e^2}{(p_1 + p_2)^2}[\bar{u}(p_3)\gamma^\mu v(p_4)][\bar{v}(p_2)\gamma_\mu u(p_1)]$$

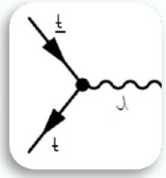
QED fermion current: $j^\mu_{EM} = e\bar{\psi}\gamma^\mu\psi$

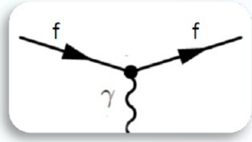
A set of Feynman Rules allows to translate any diagram to an amplitude without explicitly carrying out the perturbative expansion of the S-matrix

Feynman diagrams



EM vertex:





Each line and each vertex represents a mathematical term

→ Resulting matrix element:

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

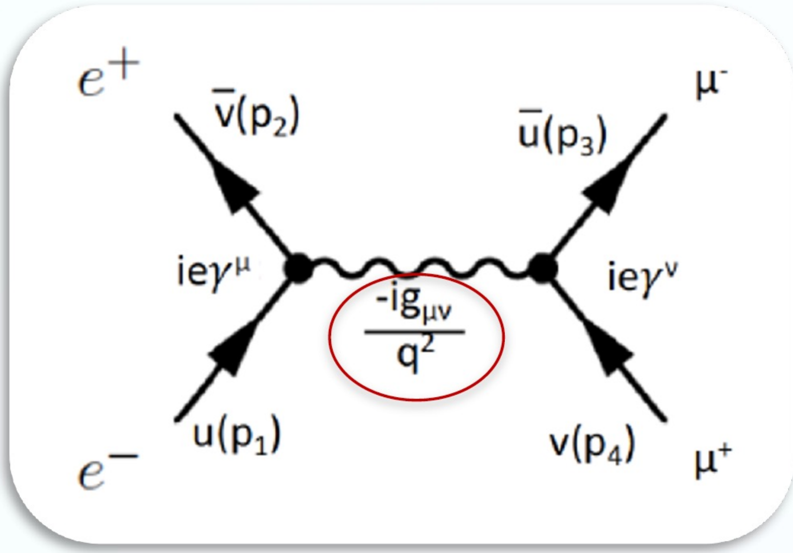
$$\mathcal{M} = \frac{-e^2}{(p_1+p_2)^2}[\bar{u}(p_3)\gamma^\mu v(p_4)][\bar{v}(p_2)\gamma_\mu u(p_1)]$$

$$\mathbf{g} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

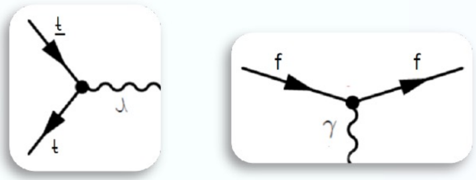
QED fermion current: $j^\mu_{EM} = e\overline{\psi}\gamma^\mu\psi$

A set of Feynman Rules allows to translate any diagram to an amplitude without explicitly carrying out the perturbative expansion of the S-matrix

Feynman diagrams



EM vertex:



Each line and each vertex represents a mathematical term

→ Resulting matrix element:

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

$$\mathbf{g} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\mathcal{M} = \frac{-e^2}{(p_1 + p_2)^2}[\bar{u}(p_3)\gamma^\mu v(p_4)][\bar{v}(p_2)\gamma_\mu u(p_1)]$$

QED fermion current: $j^\mu_{EM} = e\overline{\psi}\gamma^\mu\psi$

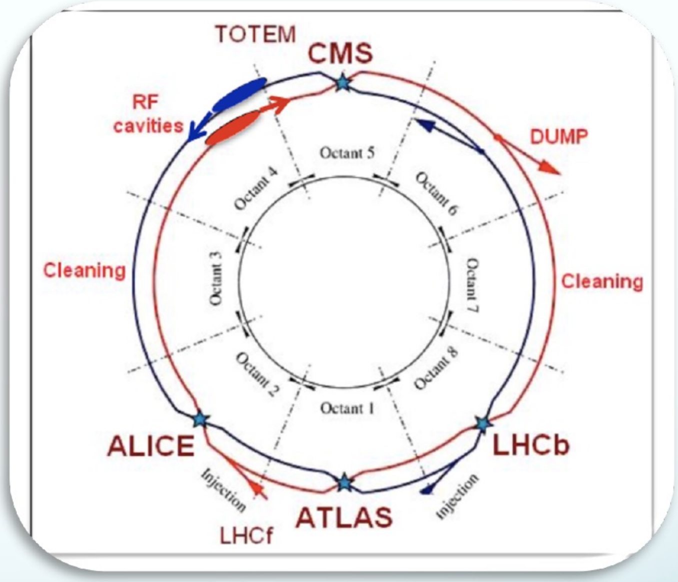
A set of Feynman Rules allows to translate any diagram to an amplitude without explicitly carrying out the perturbative expansion of the S-matrix

From amplitudes to cross sections

Particle Collider: Interaction rate depends on luminosity and cross section

$$N_{\text{events}} = \sigma(e^+e^- \to \mu^+\mu^-) \cdot \mathcal{L}dt$$

Luminosity $\mathcal{L}$ depends on number of particles per bunch, bunch frequency and beam profile: "flux"



$s = (E_1+E_2)^2$

From amplitudes to cross sections

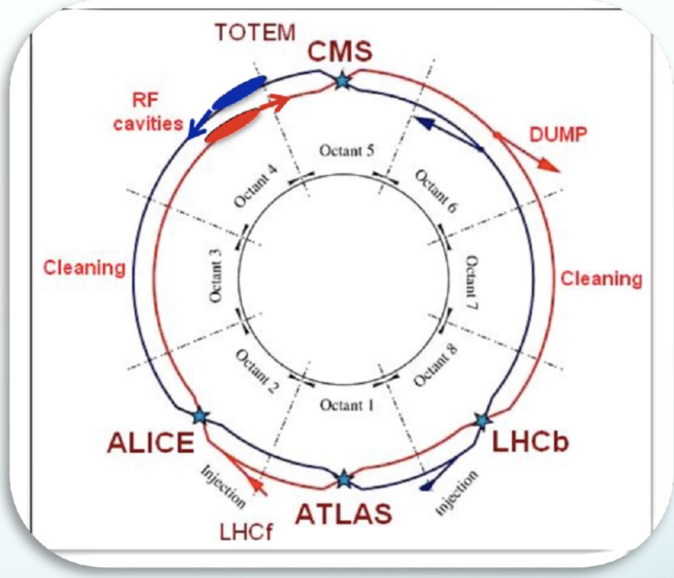
Particle Collider: Interaction rate depends on luminosity and cross section

$$N_{\text{events}} = \sigma(e^+e^- \to \mu^+\mu^-) \cdot \mathcal{L}dt$$

Luminosity $\mathcal{L}$ depends on number of particles per bunch, bunch frequency and beam profile: "flux"

Differential cross section:
depends on squared matrix element

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|}$$



$s = (E_1+E_2)^2$

From amplitudes to cross sections

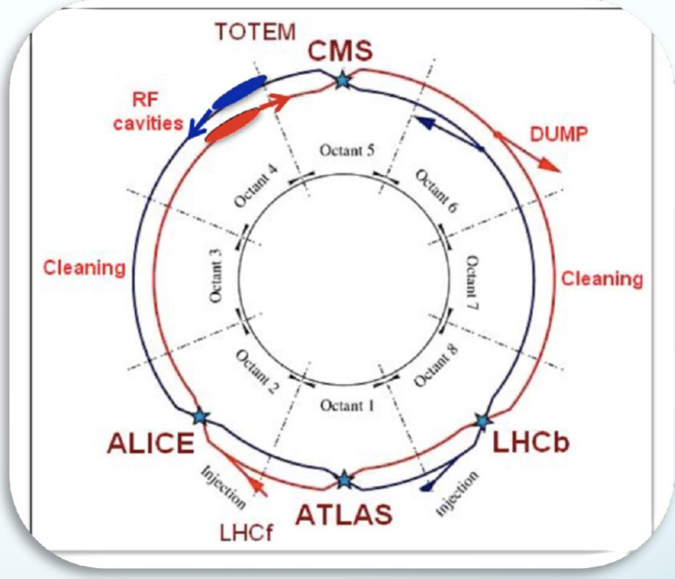
Particle Collider: Interaction rate depends on luminosity and cross section

$$N_{\text{events}} = \sigma(e^+e^- \to \mu^+\mu^-) \cdot \mathcal{L}dt$$

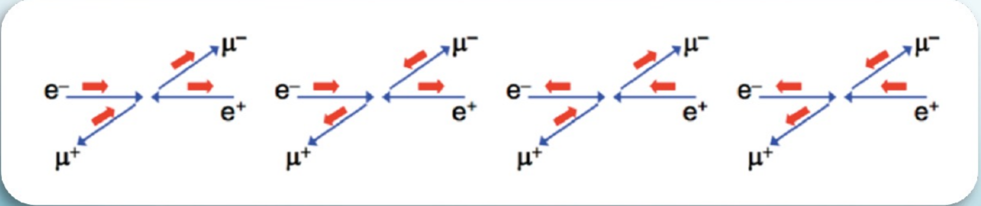
Luminosity $\mathcal{L}$ depends on number of particles per bunch, bunch frequency and beam profile: “flux”

Differential cross section:
depends on squared matrix element

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|}$$



Add matrix elements for the various spin combinations, example: $e^+e^- \to \mu^+\mu^-$



$s = (E_1+E_2)^2$

From amplitudes to cross sections

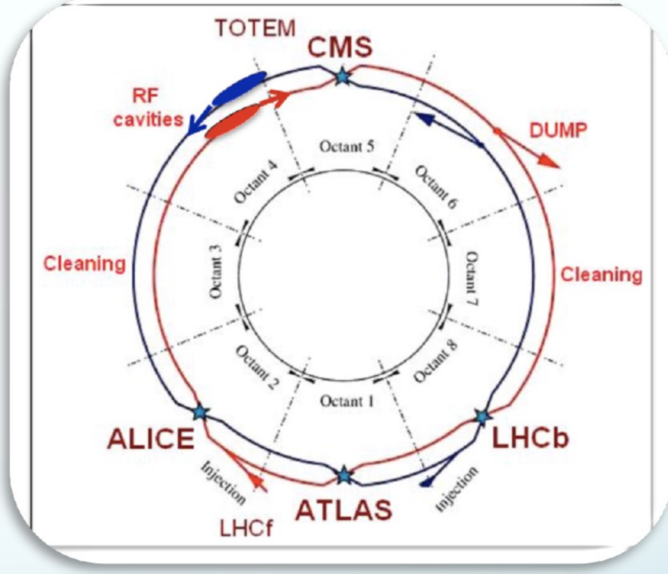
Particle Collider: Interaction rate depends on luminosity and cross section

$$N_{\text{events}} = \sigma(e^+e^- \to \mu^+\mu^-) \cdot \mathcal{L}dt$$

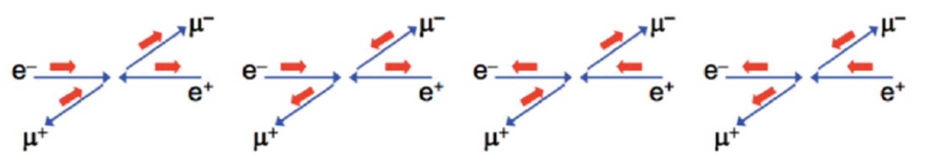
Luminosity $\mathcal{L}$ depends on number of particles per bunch, bunch frequency and beam profile: "flux"

Differential cross section:
depends on squared matrix element

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|}$$



Add matrix elements for the various spin combinations, example: $e^+e^- \to \mu^+\mu^-$



Average over initial-state spins, sum over final-state spins

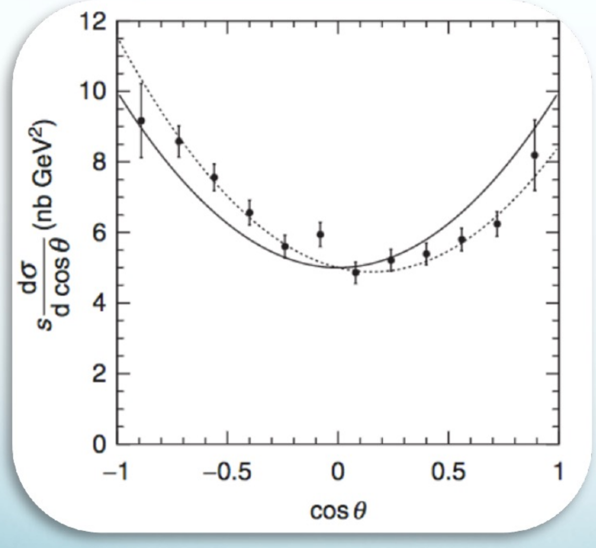
$s = (E_1+E_2)^2$

From amplitudes to cross sections

Integration over phase space → total cross section

Example: $e^+e^- \to \mu^+\mu^-$

$$\frac{d\sigma}{\sin\theta d\theta d\phi} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$



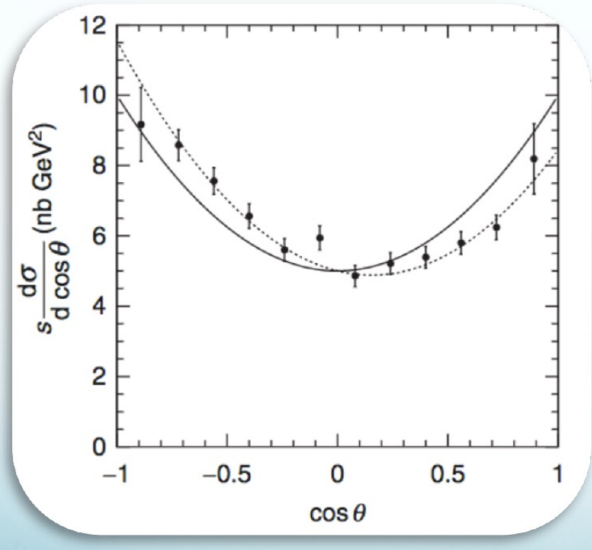
$s = (E_1+E_2)^2$

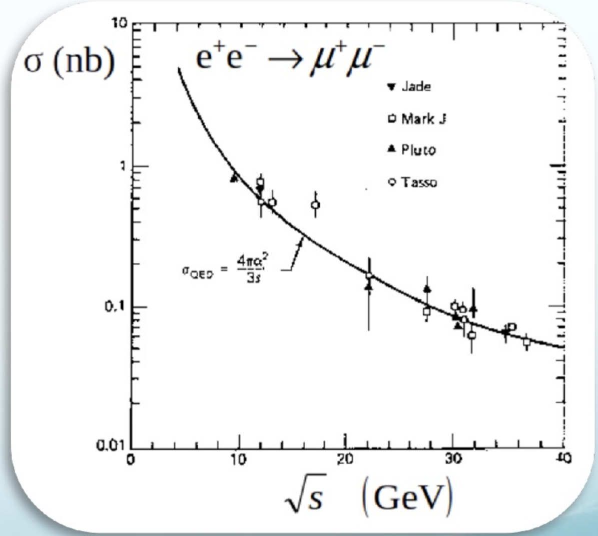
From amplitudes to cross sections

Integration over phase space → total cross section

Example: $e^+ e^- \to \mu^+ \mu^-$

$$\frac{d\sigma}{\sin\theta d\theta d\phi} = \frac{\alpha^2}{4s}(1+\cos^2\theta) \quad \rightarrow \quad \sigma = \frac{4\pi\alpha^2}{3s}$$





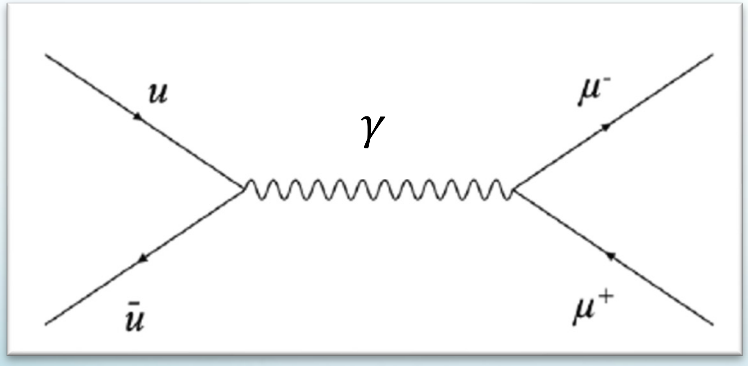
$s = (E_1+E_2)^2$

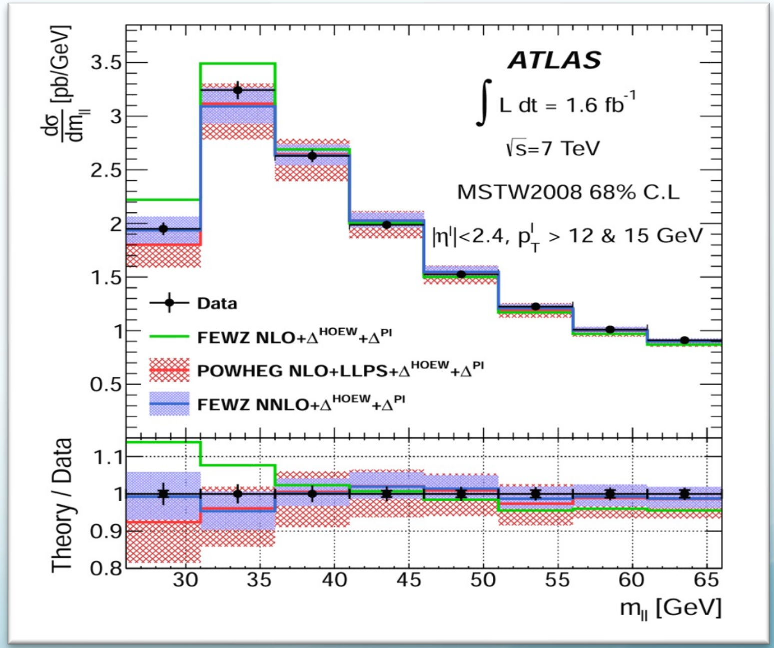
From amplitudes to cross sections

Integration over phase space → total cross section

Example: $e^+e^- \rightarrow \mu^+\mu^-$

$$\frac{d\sigma}{\sin\theta d\theta d\phi} = \frac{\alpha^2}{4s}(1+\cos^2\theta) \rightarrow \sigma = \frac{4\pi\alpha^2}{3s}\frac{1}{3}Q_q^2$$





(2) Weak interactions and Electroweak unification

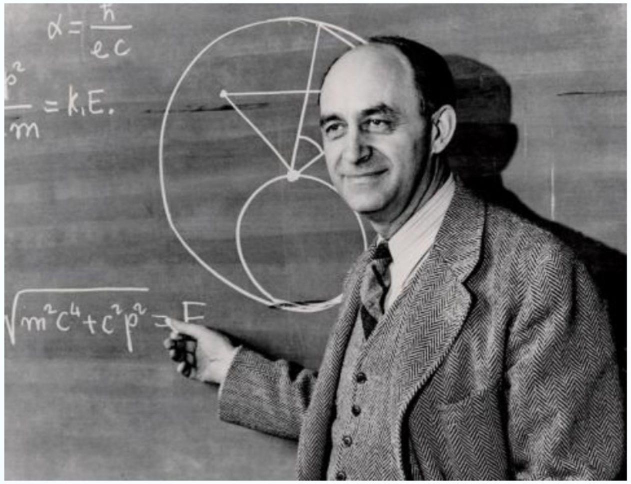
Chien-Shiun Wu

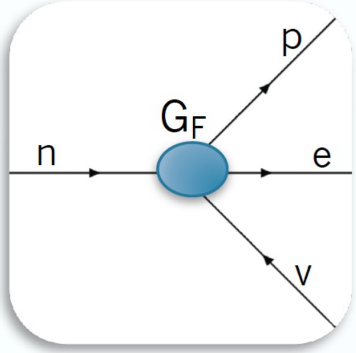
... it was unthinkable that anyone would question the validity of symmetries under space inversion, charge conjugation and time reversal. It would have been almost sacrilegious to do experiments to test such unholy thoughts

Chien-Shiun Wu: discovered parity violation in weak interactions

Fermi Theory

Fermi theory of weak interaction, 1933: Effective Field Theory





$G_F = 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}}[\bar{u}(p)\gamma^\mu u(n)][\bar{u}(\nu)\gamma_\mu u(e)]$$

Effective coupling parameter G_F, related to new physics at a scale m_W which was not accessible in 1933

Cross section diverges at high energies and does not explain observed parity violation

Today's relation at low energies:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

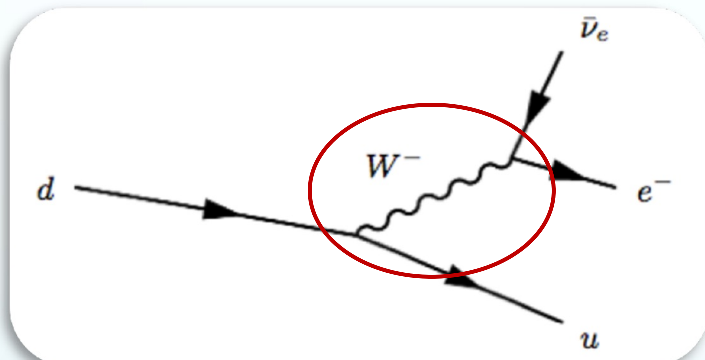
Contemporary theory

Contemporary Matrix element (low energy limit): **W boson**

$$\mathcal{M} = [\frac{g_w}{\sqrt{2}}\bar{u}(u)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(d)]\frac{1}{M_W^2-q^2}[\frac{g_w}{\sqrt{2}}\bar{u}(\nu_e)\gamma_\mu\frac{1}{2}(1-\gamma^5)u(e)]$$

Projection to
left-handed fermions

4-momentum
transfer q



Chien-Shiun Wu

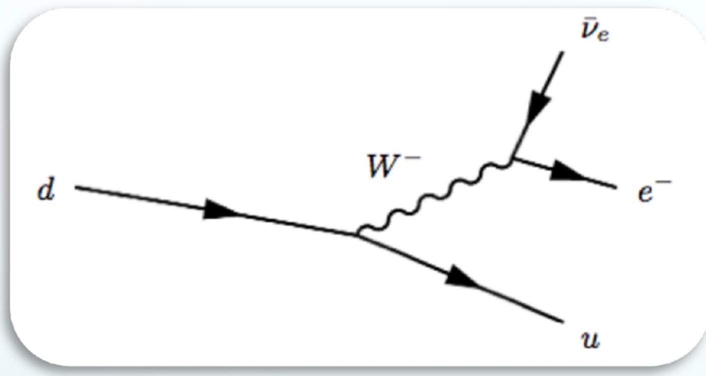
Contemporary theory

Contemporary Matrix element (low energy limit): **W boson**

$$\mathcal{M} = \left[\frac{g_w}{\sqrt{2}}\bar{u}(u)\gamma^{\mu}\frac{1}{2}(1-\gamma^5)u(d)\right]\frac{1}{M_W^2-q^2}\left[\frac{g_w}{\sqrt{2}}\bar{u}(\nu_e)\gamma_{\mu}\frac{1}{2}(1-\gamma^5)u(e)\right]$$

Projection to
left-handed fermions

Chien-Shiun Wu



$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Contemporary theory

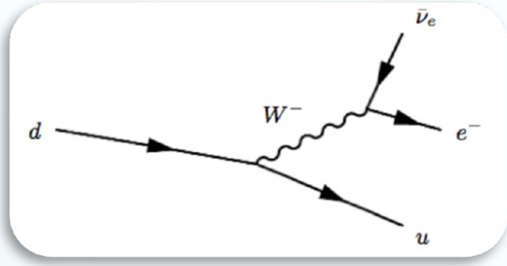
Contemporary Matrix element (low energy limit): **W boson**

$$\mathcal{M} = [\tfrac{g_w}{\sqrt{2}}\bar{u}(u)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(d)]\frac{1}{M_W^2 - q^2}[\tfrac{g_w}{\sqrt{2}}\bar{u}(\nu_e)\gamma_\mu \tfrac{1}{2}(1-\gamma^5)u(e)]$$

Projection to left-handed fermions

M_W ~80 GeV

4-momentum transfer q



The W couples with **pairs of left-handed fermions**

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L \begin{pmatrix}u\\ d'\end{pmatrix}_L \begin{pmatrix}c\\ s'\end{pmatrix}_L \begin{pmatrix}t\\ b'\end{pmatrix}_L$$

Chien-Shiun Wu

Contemporary theory

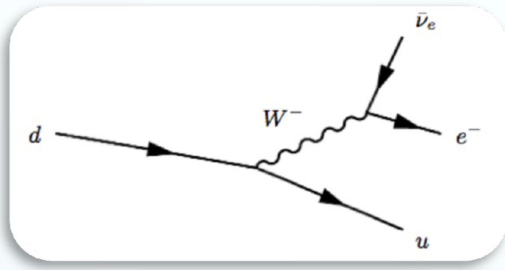
Contemporary Matrix element (low energy limit): **W boson**

$$\mathcal{M} = [\tfrac{g_w}{\sqrt{2}}\bar{u}(u)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(d)]\frac{1}{M_W^2 - q^2}[\tfrac{g_w}{\sqrt{2}}\bar{u}(\nu_e)\gamma_\mu \tfrac{1}{2}(1-\gamma^5)u(e)]$$

Projection to left-handed fermions

M_W ~80 GeV

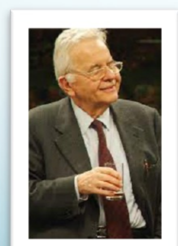
4-momentum transfer q



The W couples with **pairs of left-handed fermions**

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L \begin{pmatrix}u\\ d'\end{pmatrix}_L \begin{pmatrix}c\\ s'\end{pmatrix}_L \begin{pmatrix}t\\ b'\end{pmatrix}_L$$

Chien-Shiun Wu



Nicola Cabibbo

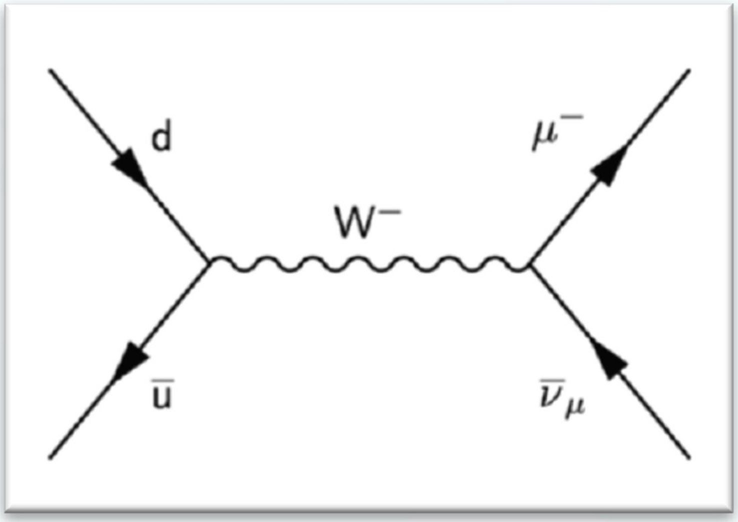
CKM matrix: connection between weak and mass eigen states

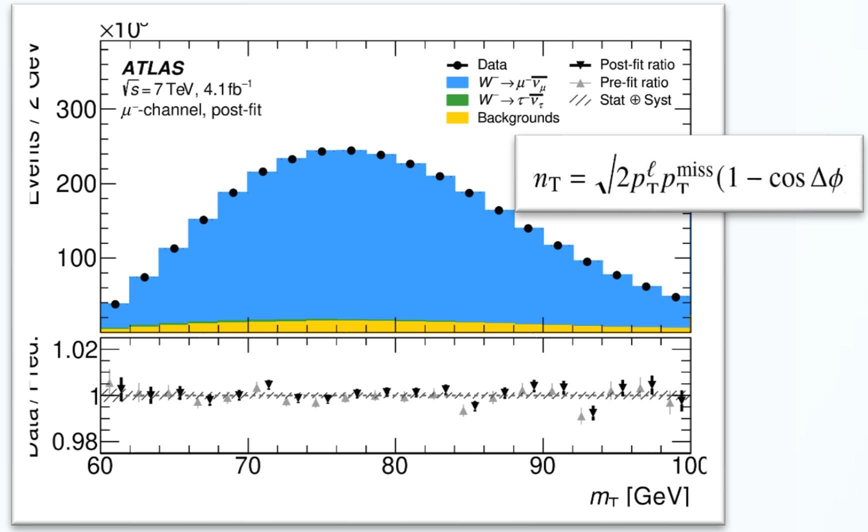
$$\begin{bmatrix}d'\\ s'\\ b'\end{bmatrix} = \begin{bmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}\begin{bmatrix}d\\ s\\ b\end{bmatrix}$$

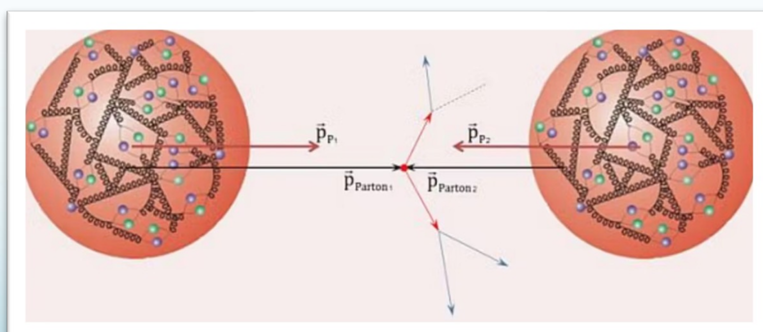
Toshide Maskawa

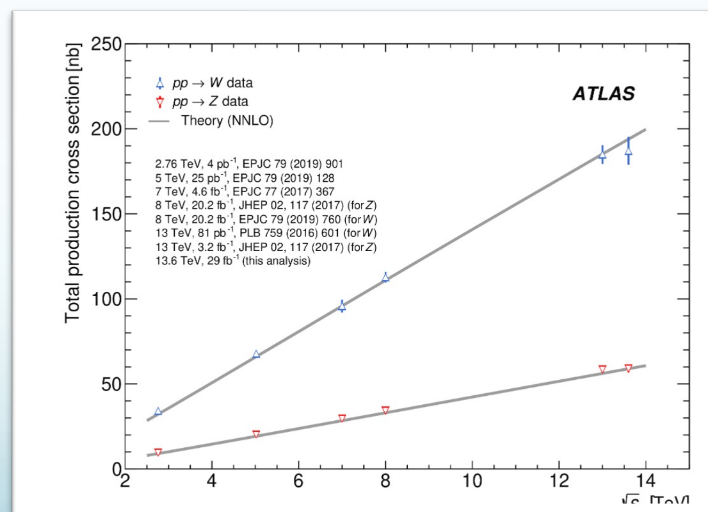
Makoto Kobayashi

W boson production at the LHC









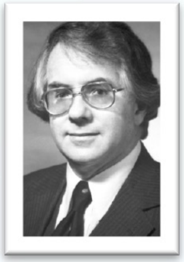
Electroweak unification

Require the Dirac Lagrangian to be invariant under a local $SU(2)_L \times U(1)_Y$ transformation.

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

Steven Weinberg

Abdus Salam



Sheldon. Glashow

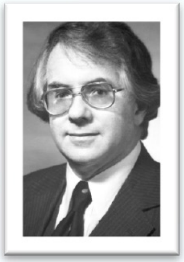
Electroweak unification

Require the Dirac Lagrangian to be invariant under a local $SU(2)_L \times U(1)_Y$ transformation.

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

Steven Weinberg

Abdus Salam



Sheldon. Glashow

$U(1)_Y$: generator is the weak hypercharge $Y = 2(Q - T_3)$, $T_3 = \pm \frac{1}{2}$ for left-handed fermions, $T_3 = 0$ for right-handed fermions

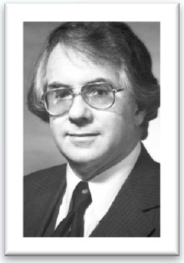
Electroweak unification

Require the Dirac Lagrangian to be invariant under a local $SU(2)_L \times U(1)_Y$ transformation.

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

Steven Weinberg

Abdus Salam



Sheldon. Glashow

$U(1)_Y$: generator is the weak hypercharge $Y = 2(Q - T_3)$, $T_3 = \pm \frac{1}{2}$ for left-handed fermions, $T_3 = 0$ for right-handed fermions

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L \begin{pmatrix}u\\ d'\end{pmatrix}_L \begin{pmatrix}c\\ s'\end{pmatrix}_L \begin{pmatrix}t\\ b'\end{pmatrix}_L$$

$SU(2)_L$: only transforms left-handed fermion-doublets generators of SU(2) are the Pauli matrices:

$$T_i = \frac{1}{2}\sigma_i \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$SU(2) \ni U = \exp(i\alpha_i \sigma_i)$

Electroweak unification

In QED (local U(1) symmetry): new covariant derivative

$$D_\mu = \partial_\mu + iqA_\mu$$

Now: more advanced covariant derivative with one gauge field per generator:

For U(1): B_μ. For SU(2): W^1_μ, W^2_μ, W^3_μ (non physical fields)

$$\begin{aligned} L: \quad & D_\mu = \partial_\mu + ig_w \frac{\sigma_i}{2} W^i_\mu + ig\frac{Y}{2}B_\mu \\ \psi_R,\ \psi'_R: \quad & D_\mu = \partial_\mu + ig\frac{Y}{2}B_\mu \end{aligned}$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \begin{pmatrix} u \\ d' \end{pmatrix}_L \begin{pmatrix} c \\ s' \end{pmatrix}_L \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

Electroweak unification

In QED (local U(1) symmetry): new covariant derivative

$$D_\mu = \partial_\mu + iqA_\mu$$

Now: more advanced covariant derivative with one gauge field per generator:

For U(1): B_μ. For SU(2): $W^1{}_\mu$, $W^2{}_\mu$, $W^3{}_\mu$ (non physical fields)

$$\begin{aligned} L: \quad & D_\mu = \partial_\mu + ig_w \frac{\sigma_i}{2} W^i_\mu + ig\frac{Y}{2} B_\mu \\ \psi_R,\ \psi'_R: \quad & D_\mu = \partial_\mu + ig\frac{Y}{2} B_\mu \end{aligned}$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \begin{pmatrix} u \\ d' \end{pmatrix}_L \begin{pmatrix} c \\ s' \end{pmatrix}_L \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

Electroweak unification

In QED (local U(1) symmetry): new covariant derivative

$$D_\mu = \partial_\mu + iqA_\mu$$

Now: more advanced covariant derivative with one gauge field per generator:

For U(1): B_μ. For SU(2): W^1_μ, W^2_μ, W^3_μ

$$\begin{aligned} L: \quad & D_\mu = \partial_\mu + ig_w \tfrac{\sigma_i}{2} W^i_\mu + ig \tfrac{Y}{2} B_\mu \\ \psi_R,\ \psi'_R: \quad & D_\mu = \partial_\mu + ig \tfrac{Y}{2} B_\mu \end{aligned}$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

Physical fields are linear combinations of B_μ, W^1_μ,, W^2_μ, and W^3_μ

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left[W^1_\mu \mp iW^2_\mu\right]$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}$$

with weak mixing angle θ_W

Electroweak unification

In QED (local U(1) symmetry): new covariant derivative $$D_\mu = \partial_\mu + iqA_\mu$$

Now: more advanced covariant derivative with one gauge field per generator:
For U(1): B_μ. For SU(2): W^1_μ, W^2_μ, W^3_μ

$$\begin{aligned} L: \quad & D_\mu = \partial_\mu + ig_w \tfrac{\sigma_i}{2} W^i_\mu + ig\tfrac{Y}{2} B_\mu \\ \psi_R,\ \psi'_R: \quad & D_\mu = \partial_\mu + ig\tfrac{Y}{2} B_\mu \end{aligned}$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

Physical fields are linear combinations of B_μ, W^1_μ,, W^2_μ, and W^3_μ

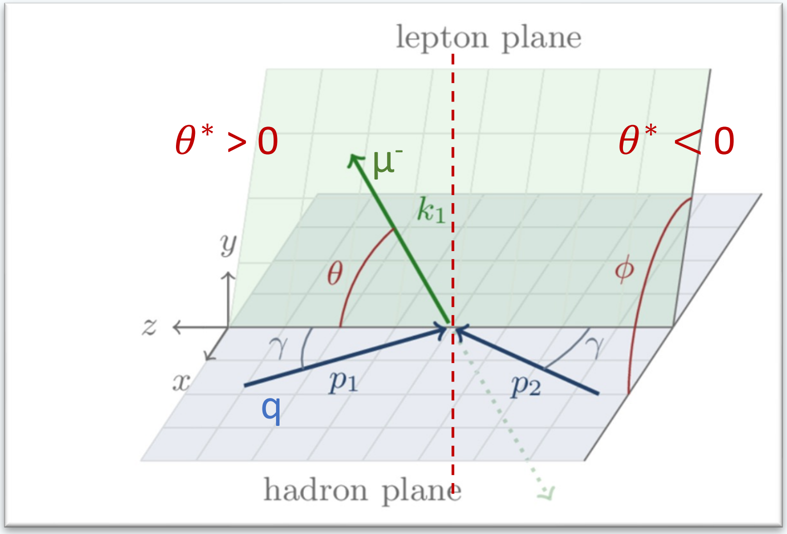
$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left[W^1_\mu \mp iW^2_\mu\right]$$

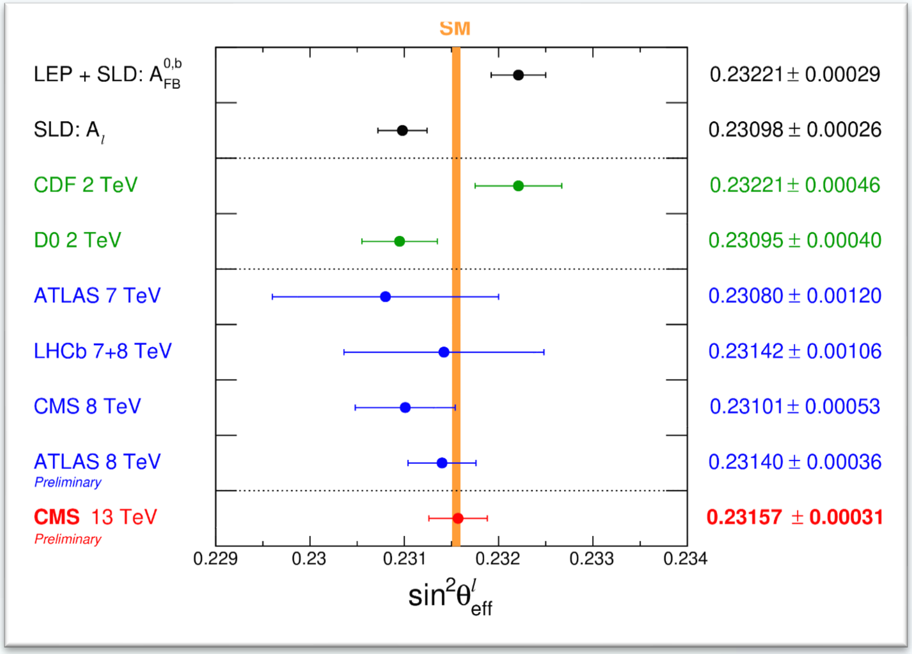
$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}$$

For consistency with QED,
The weak mixing angle θ_W
needs to be :

$$\cos\theta_w = \frac{g_w}{\sqrt{g_w^2 + g^2}} \qquad \sin\theta_w = \frac{g}{\sqrt{g_w^2 + g^2}}$$

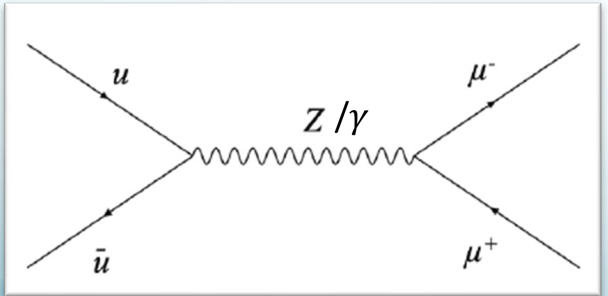
The weak mixing angle





Extracted via the forward-backward asymmetry A_{FB} of the polar angle θ^* in the leptons rest frame (Colin-Soper frame)

LHC expects to resolve the tensions between previous measurements.



$$A_{FB} = \frac{N(\cos\theta^* > 0) - N(\cos\theta^* < 0)}{N(\cos\theta^* > 0) + N(\cos\theta^* < 0)}$$

Electroweak unification

→ plug D_μ into the Dirac Lagrangian:

$$\mathcal{L}^{EW} = i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi$$

→ Lagrangian in terms of the physical fields W^+_μ, W^-_μ, A_μ, Z_μ: see appendix.

→ Here: Z_μ part of Lagrangian: in terms of vector (c_V) and axial vector (c_A) couplings for up and down components of the weak doublet:

$$\mathcal{L}^{EW}_{int\ Z^0} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \frac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu \frac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

Electroweak unification

→ plug D_μ into the Dirac Lagrangian:

$$\mathcal{L}^{EW} = i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi$$

→ Lagrangian in terms of the physical fields W^+_μ, W^-_μ, A_μ, Z_μ: see appendix.

→ Here: Z_μ part of Lagrangian: in terms of vector (c_V) and axial vector (c_A) couplings for up and down components of the weak doublet:

$$\mathcal{L}^{EW}_{int\ Z^0} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu\tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

c_V and c_A are functions of the weak mixing angle and the electric charge Q:

$$\begin{array}{ll} c_V^f = \frac{1}{2} - 2\sin^2\theta_w Q & c_A^f = \frac{1}{2} \\ c_V^{f'} = -\frac{1}{2} - 2\sin^2\theta_w Q & c_A^{f'} = -\frac{1}{2} \end{array}$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

Electroweak unification

→ plug D_μ into the Dirac Lagrangian:

$$\mathcal{L}^{EW} = i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$$

→ Lagrangian in terms of the physical fields W^+_μ, W^-_μ, A_μ, Z_μ: see appendix.

→ Here: Z_μ part of Lagrangian: in terms of vector (c_V) and axial vector (c_A) couplings for up and down components of the weak doublet:

$$\mathcal{L}^{EW}_{int\ Z^0} = -\sum_f \frac{g_w}{\cos\theta_w} \left[\bar{\psi}\gamma^\mu \frac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \bar{\psi'}\gamma^\mu \frac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi' \right] Z_\mu$$

c_V and c_A are functions of the weak mixing angle and the electric charge Q:

$$\begin{array}{ll} c_V^f = \frac{1}{2} - 2\sin^2\theta_w Q & c_A^f = \frac{1}{2} \\ c_V^{f'} = -\frac{1}{2} - 2\sin^2\theta_w Q & c_A^{f'} = -\frac{1}{2} \end{array}$$

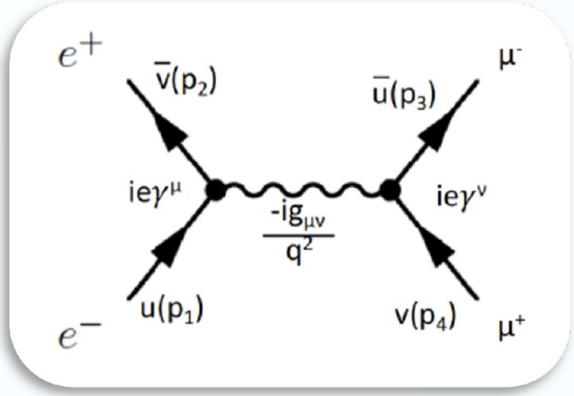
$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

example: $c_V(e,\mu,\tau) = -0.04$ and $c_A(e,\mu,\tau) = -0.5$

also in terms of right- and left-handed couplings: $c_V = c_L + c_R$ $c_A = c_L - c_R$

Electroweak unification

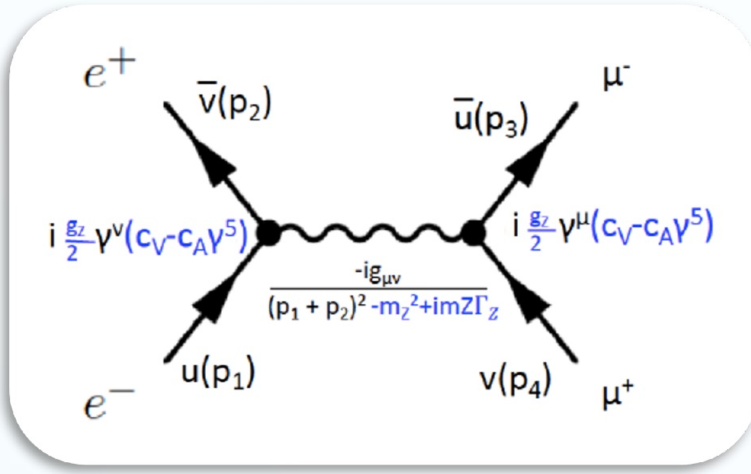
reminder: $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$:



$$\mathcal{M} = \frac{-e^2}{(p_1+p_2)^2}[\bar{u}(p_3)\gamma^\mu v(p_4)][\bar{v}(p_2)\gamma_\mu u(p_1)]$$

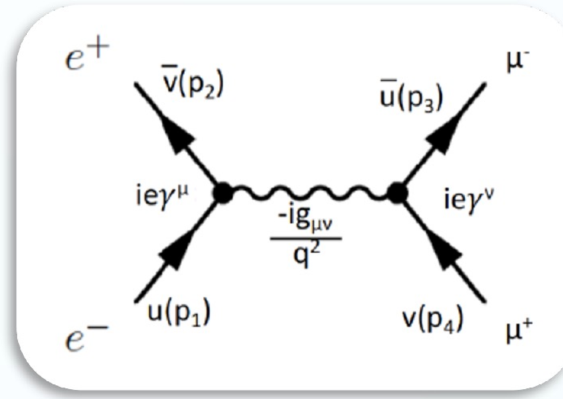
Electroweak unification

example: $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$:



$c_V(e,\mu,\tau) = -0.04$ and $c_A(e,\mu,\tau) = -0.5$

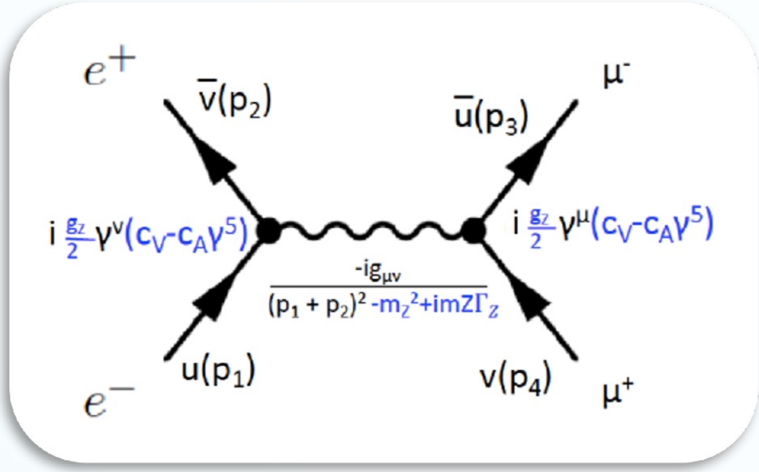
reminder: $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$:



$$\mathcal{M} = \frac{-e^2}{(p_1+p_2)^2}[\bar{u}(p_3)\gamma^\mu v(p_4)][\bar{v}(p_2)\gamma_\mu u(p_1)]$$

Electroweak unification

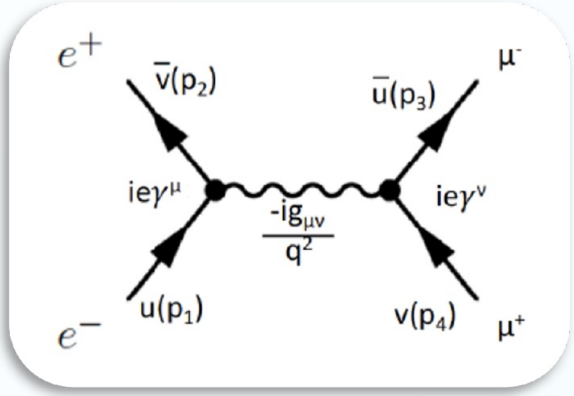
example: $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$:



$c_V(e,\mu,\tau) = -0.04$ and $c_A(e,\mu,\tau) = -0.5$

$$\mathcal{M} = -\frac{g_Z^2}{4((p_1+p_2)^2 - m_Z^2 + im_Z\Gamma_Z)}[\bar{u}(3)\gamma^\mu(0.5\gamma^5 - 0.04)v(4)]$$
$$[\bar{v}(2)\gamma_\mu(0.5\gamma^5 - 0.04)u(1)]$$

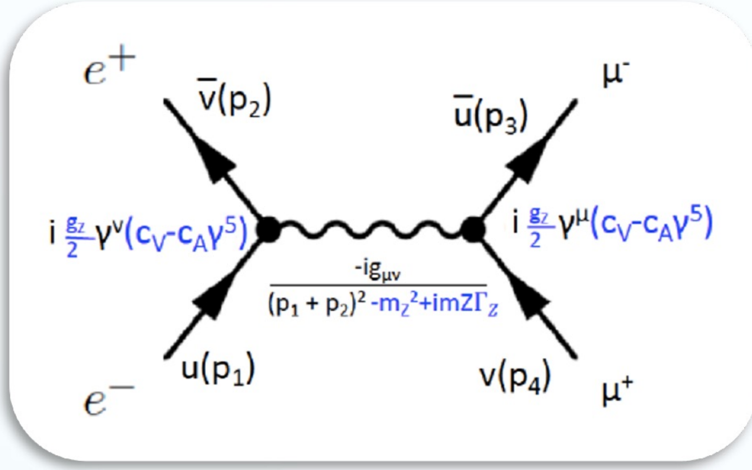
reminder: $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$:



$$\mathcal{M} = \frac{-e^2}{(p_1+p_2)^2}[\bar{u}(p_3)\gamma^\mu v(p_4)][\bar{v}(p_2)\gamma_\mu u(p_1)]$$

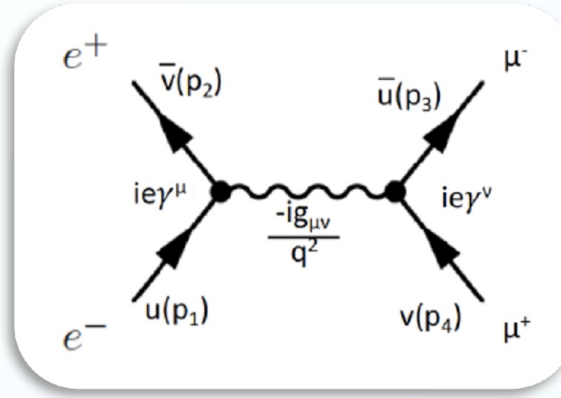
Electroweak unification

example: $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$:



$c_V(e,\mu,\tau) = -0.04$ and $c_A(e,\mu,\tau) = -0.5$

reminder: $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$:

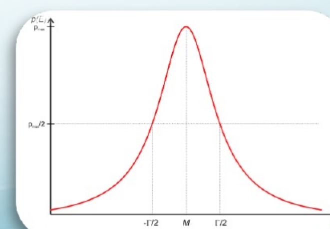


$$\mathcal{M} = \frac{-e^2}{(p_1+p_2)^2}[\bar{u}(p_3)\gamma^\mu v(p_4)][\bar{v}(p_2)\gamma_\mu u(p_1)]$$

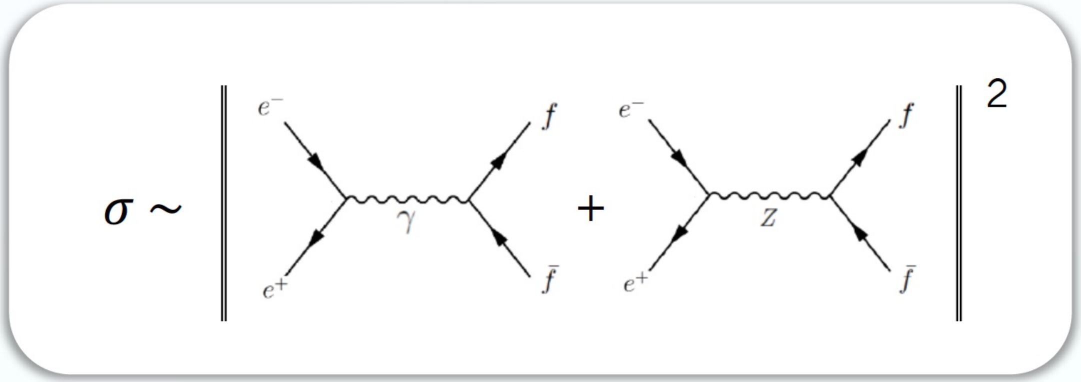
$$\mathcal{M} = -\frac{g_Z^2}{4((p_1+p_2)^2 - m_Z^2 + im_Z\Gamma_Z)}[\bar{u}(3)\gamma^\mu(0.5\gamma^5 - 0.04)v(4)]$$
$$[\bar{v}(2)\gamma_\mu(0.5\gamma^5 - 0.04)u(1)]$$

Breit-Wigner Resonanz

$$\rightarrow \quad \sigma(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-) = \frac{12\pi s}{m_Z^2}\frac{\Gamma_{e^+e^-}\Gamma_{\mu^+\mu^-}}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

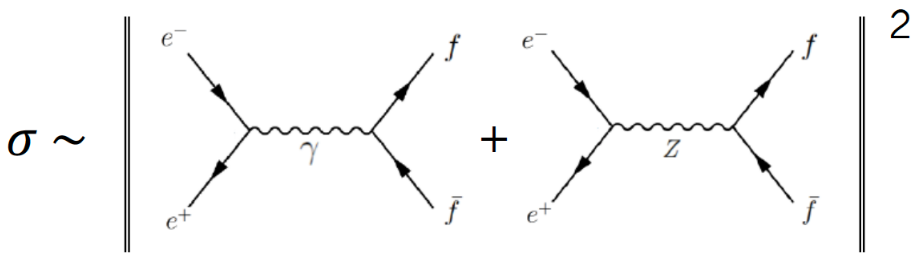


Electroweak unification



Z-γ interference

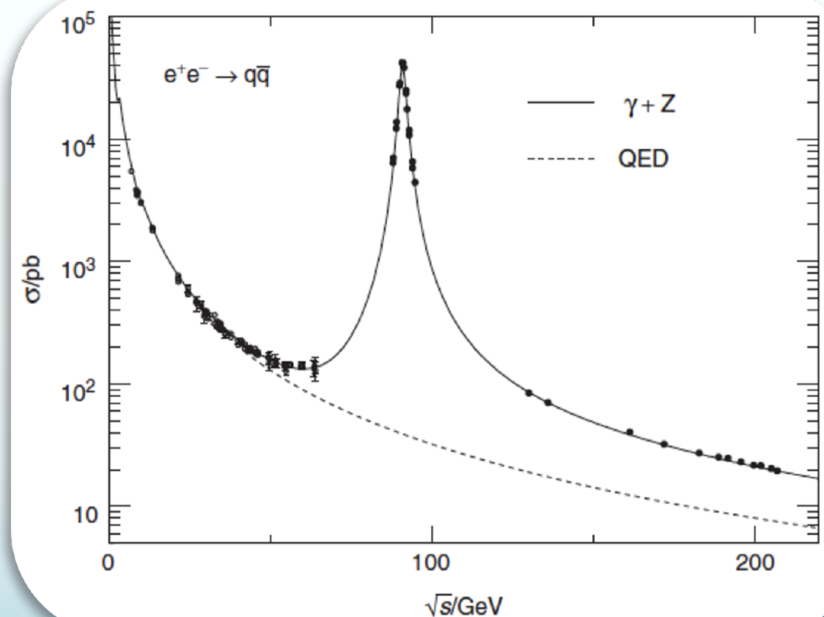
Electroweak unification



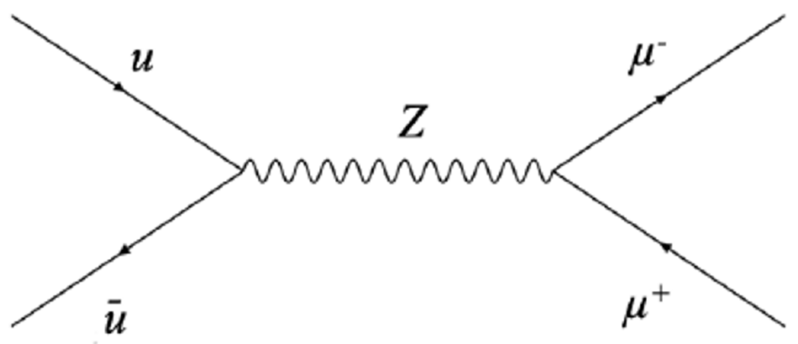
Z-γ interference

For small energies ($\sqrt{s} < 50$ GeV), the photon (QED) contribution dominates.

Around the Z mass ($\sqrt{s} \sim 91$ GeV), the Z contribution dominates.

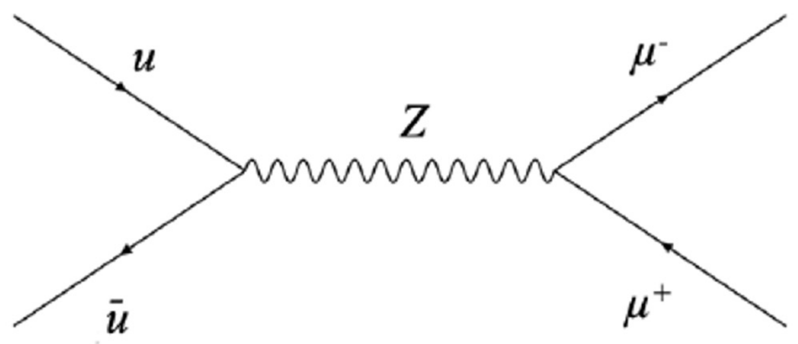


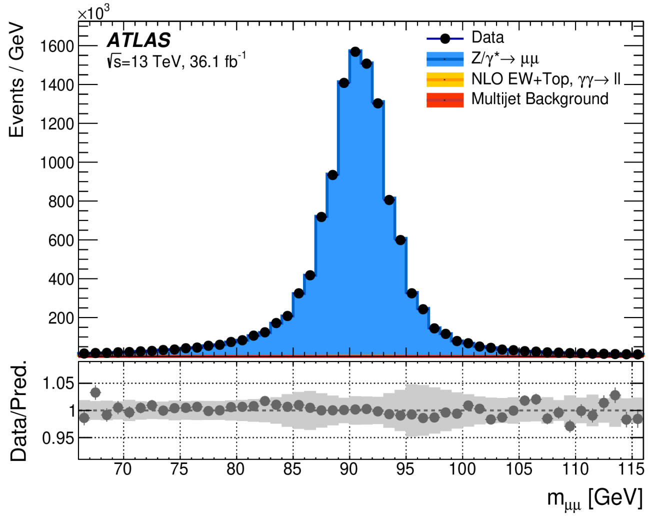
Z production at hadron colliders



- At hadron colliders the Z boson can be produced via the Drell-Yan Process, e.g. $u\bar{u} \to Z \to \mu^+\mu^-$

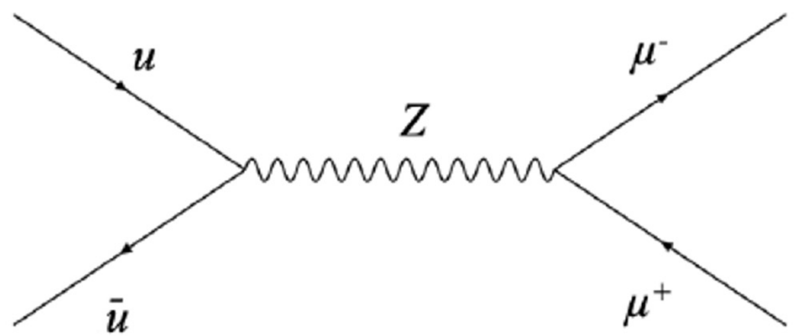
Z production at hadron colliders

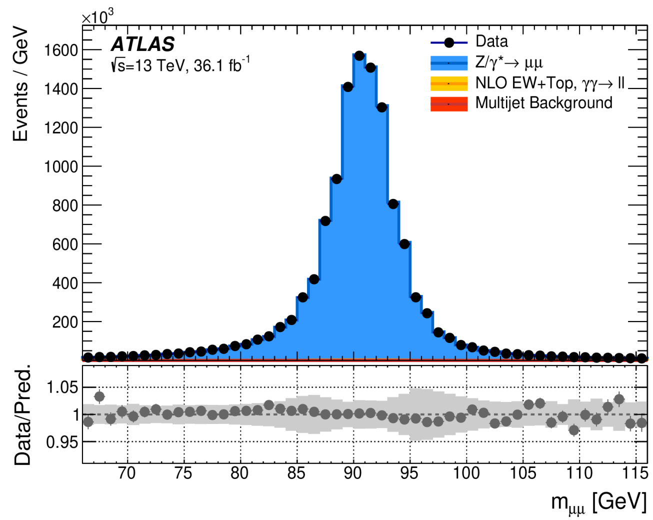




- At hadron colliders the Z boson can be produced via the Drell-Yan Process, e.g. $u\bar{u} \rightarrow Z \rightarrow \mu^+\mu^-$

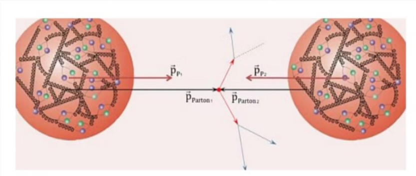
Z production at hadron colliders



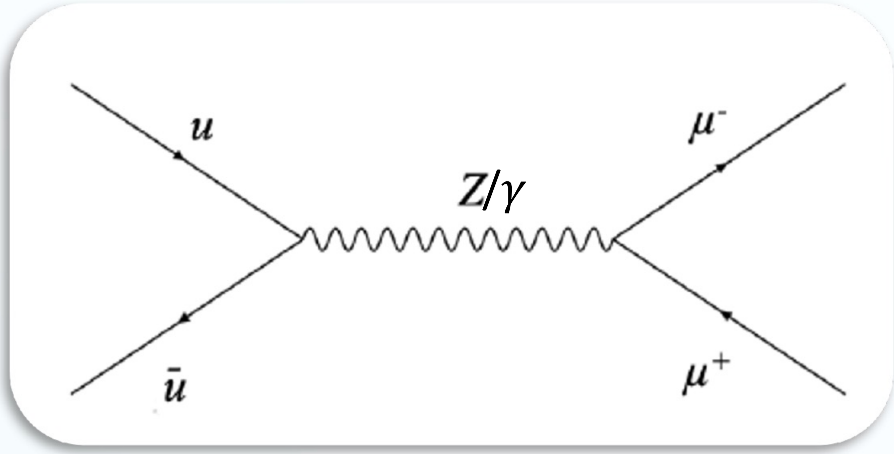


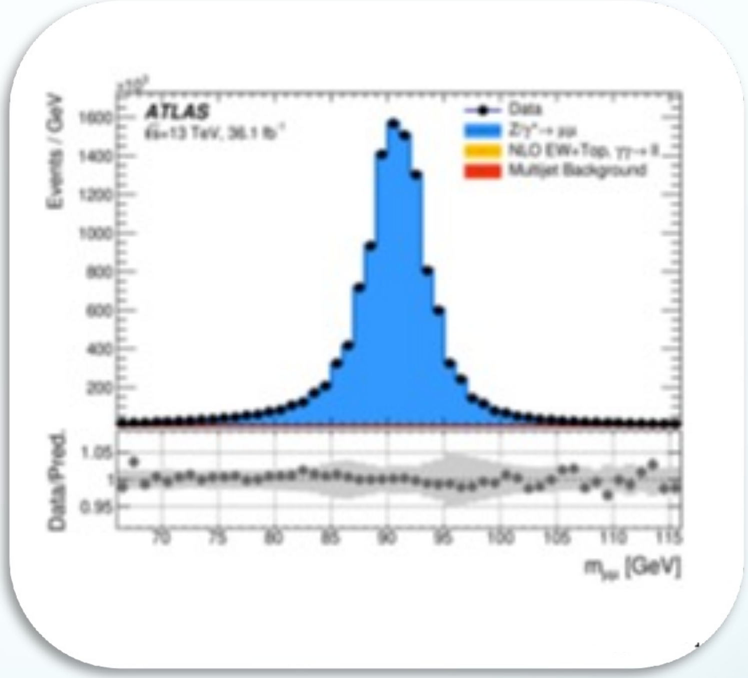
- At hadron colliders the Z boson can be produced via the Drell-Yan Process, e.g. $u\bar{u} \rightarrow Z \rightarrow \mu^+\mu^-$

- As the quarks can carry a wide range of the proton energy fraction, we automatically scan the mass range



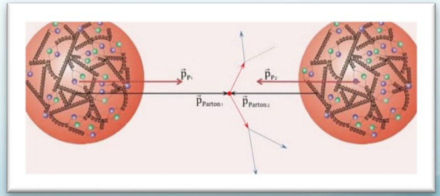
Z production at hadron colliders





- At hadron colliders the Z boson can be produced via the Drell-Yan Process, e.g. $u\bar{u} \to Z \to \mu^+\mu^-$

- As the quarks can carry a wide range of the proton energy fraction, we automatically scan the mass range



- Similar to LEP, the cross section results from an interference between photon and Z

[Higgs mechanism]

Peter Higgs

Francois Englert

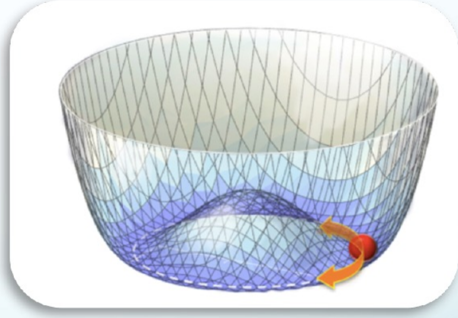
Robert Brout

So far no mass terms for the gauge bosons W,Z as they would destroy the local gauge invariance

→ Mass terms introduced by interaction with a scalar field through the covariant derivative in the kinetic term

$$\mathcal{L}_\phi^{EW} = (D_\mu\phi)^\dagger D^\mu\phi - \left(\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2\right) \qquad (\mu^2 < 0,\ \lambda > 0)$$

after assuming a non-zero vacuum expectation value (electroweak symmetry breaking)



$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \qquad \langle 0|\phi^\dagger\phi|0\rangle = \frac{v^2}{2} \simeq (174\ \mathrm{GeV})^2$$

the kinematic term creates mass terms for the W and Z

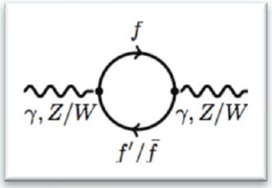
$$m_{W+} = \frac{g_w v}{2} \qquad m_{W-} = \frac{g_w v}{2} \qquad m_Z = \frac{g_w v}{2\cos\theta_w} = \frac{m_W}{\cos\theta_w}$$

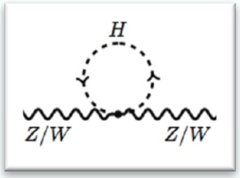
(3) The electroweak fit and the W mass

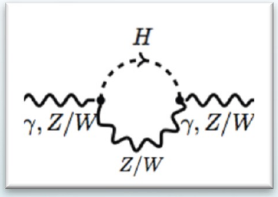
The electroweak fit

- Since LEP/SLD, EWK precision data used together with accurate SM calculations to predict parameters of the theory
- **All changed in 2012:** Higgs discovery, mH → EWK SM sector over constrained → Check **consistency of SM** by comparing fitted with measured values

Example:
Higgs, W, Z, and top masses are connected via radiative corrections



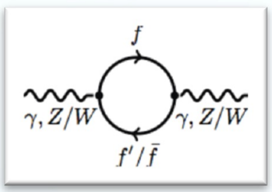


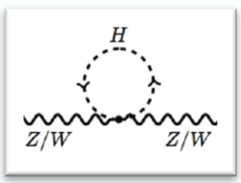


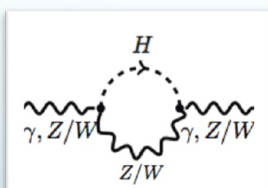
The electroweak fit

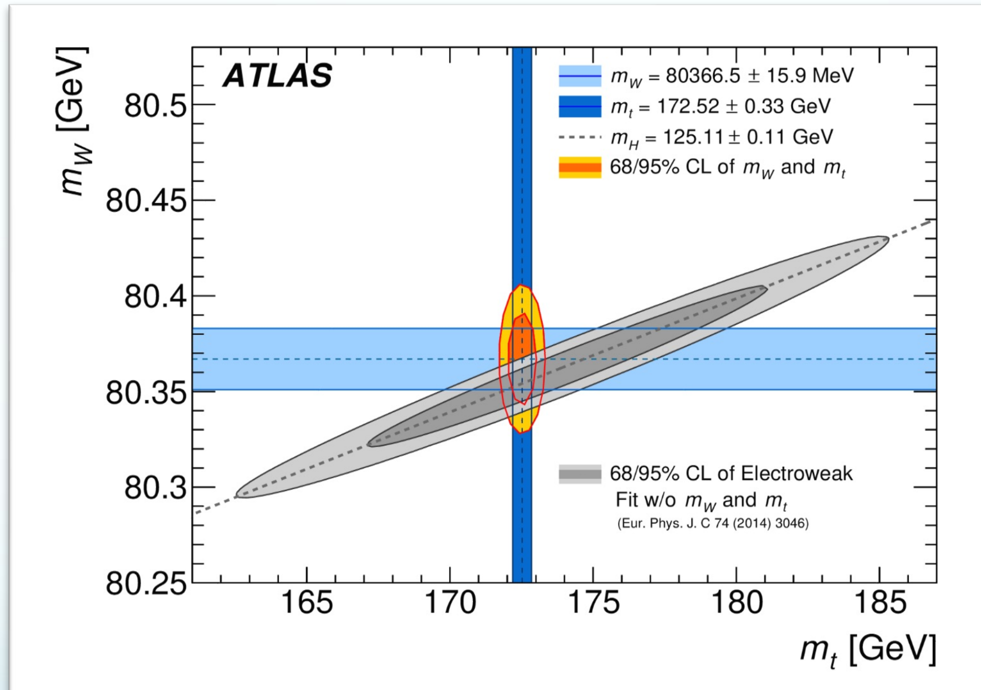
- Since LEP/SLD, EWK precision data used together with accurate SM calculations to predict parameters of the theory
- **All changed in 2012:** Higgs discovery, mH → EWK SM sector over constrained → Check **consistency of SM** by comparing fitted with measured values

Example:
Higgs, W, Z, and top masses are connected via radiative corrections





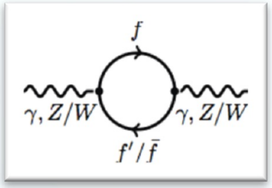


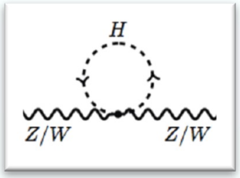


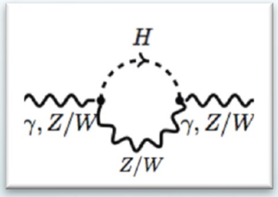
The electroweak fit

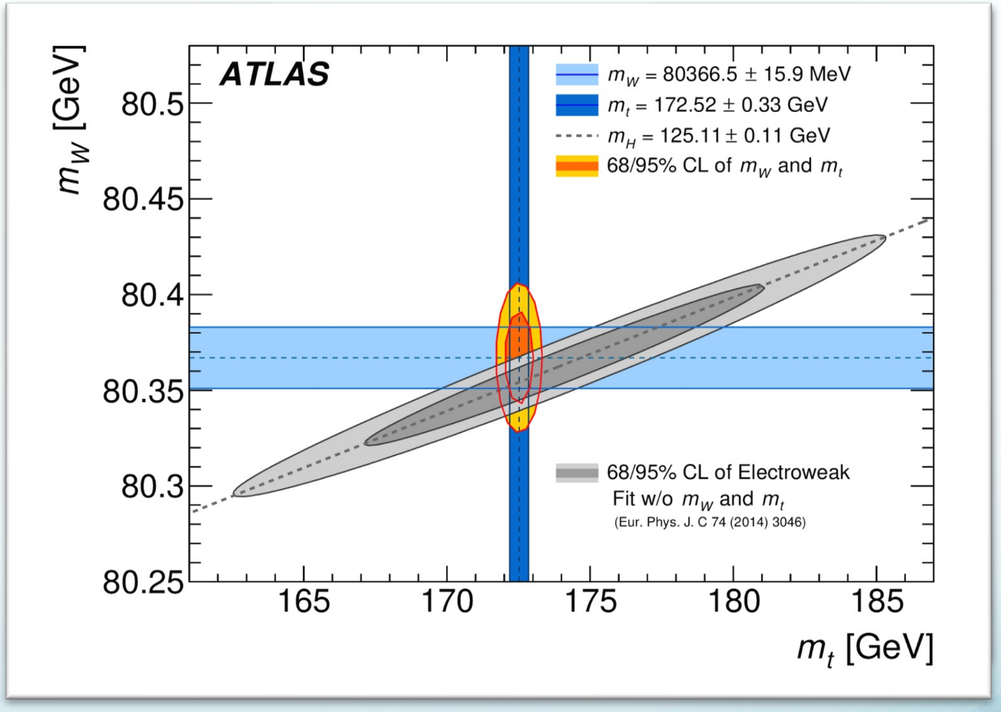
- Since LEP/SLD, EWK precision data used together with accurate SM calculations to predict parameters of the theory
- **All changed in 2012:** Higgs discovery, mH → EWK SM sector over constrained → Check **consistency of SM** by comparing fitted with measured values

Example:
Higgs, W, Z, and top masses are connected via radiative corrections







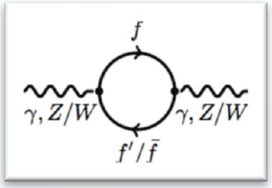


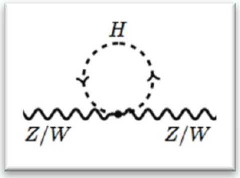
Direct measurements of top quark and W mass

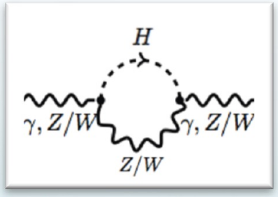
The electroweak fit

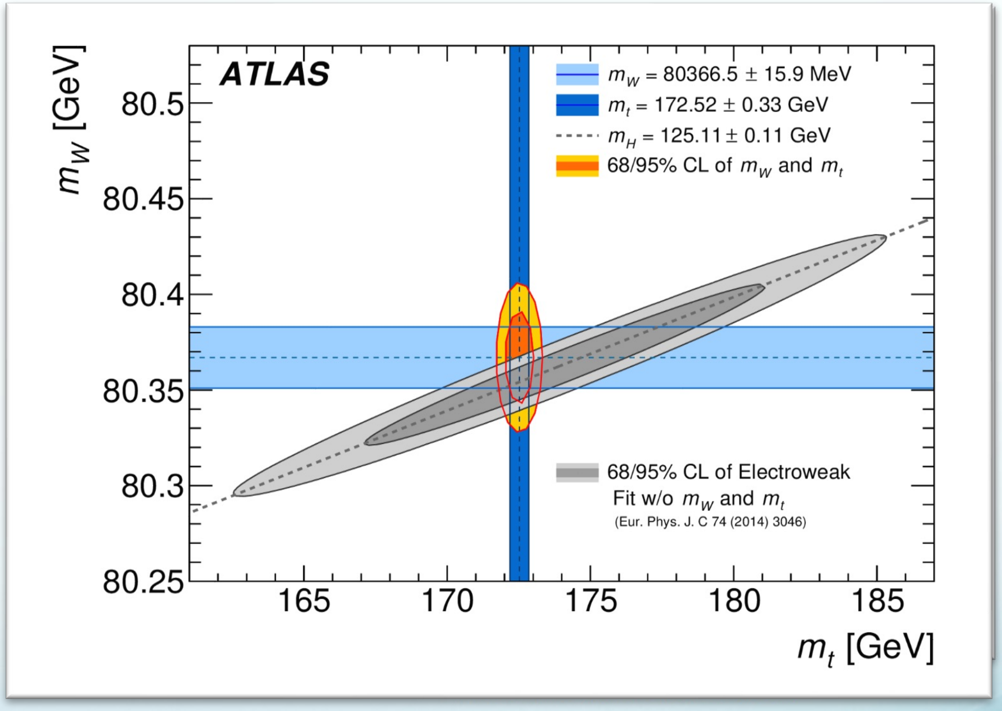
- Since LEP/SLD, EWK precision data used together with accurate SM calculations to predict parameters of the theory
- **All changed in 2012:** Higgs discovery, mH → EWK SM sector over constrained → Check **consistency of SM** by comparing fitted with measured values

Example:
Higgs, W, Z, and top masses are connected via radiative corrections







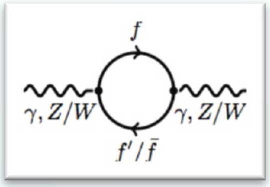


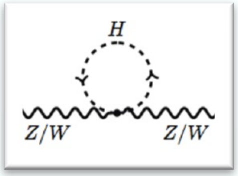
EW fit without using top and W mass measurements

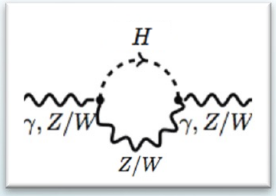
The electroweak fit

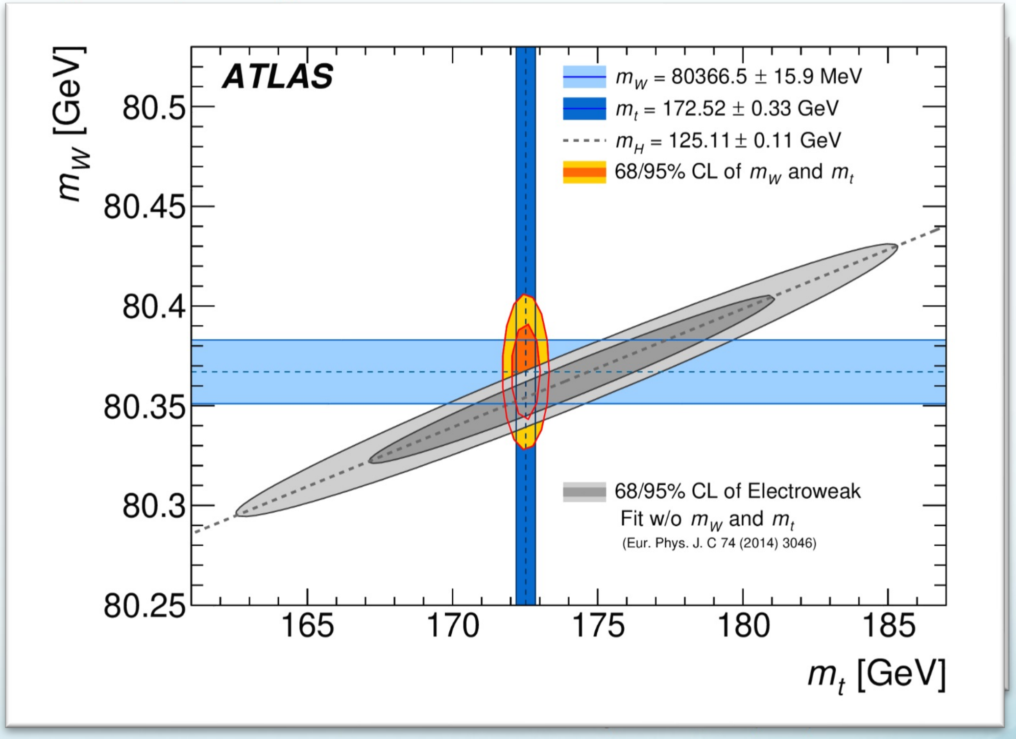
- Since LEP/SLD, EWK precision data used together with accurate SM calculations to predict parameters of the theory
- **All changed in 2012:** Higgs discovery, mH → EWK SM sector over constrained → Check **consistency of SM** by comparing fitted with measured values

Example:
Higgs, W, Z, and top masses are connected via radiative corrections







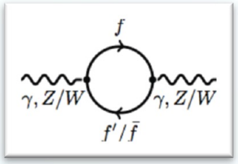


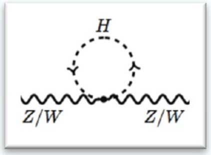
- Direct and indirect W, top and Higgs mass measurements in agreement.

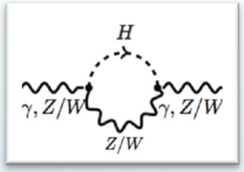
The electroweak fit: all observables

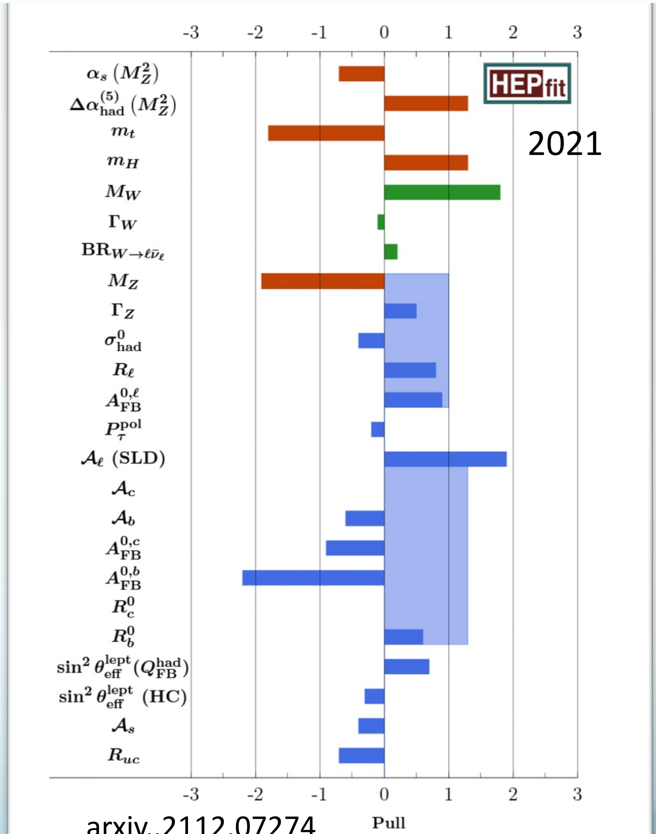
- Since LEP/SLD, EWK precision data used together with accurate SM calculations to predict parameters of the theory
- **All changed in 2012:** Higgs discovery, mH → EWK SM sector over constrained → Check **consistency of SM** by comparing fitted with measured values

Example:
Higgs, W, Z, and top masses are connected via radiative corrections





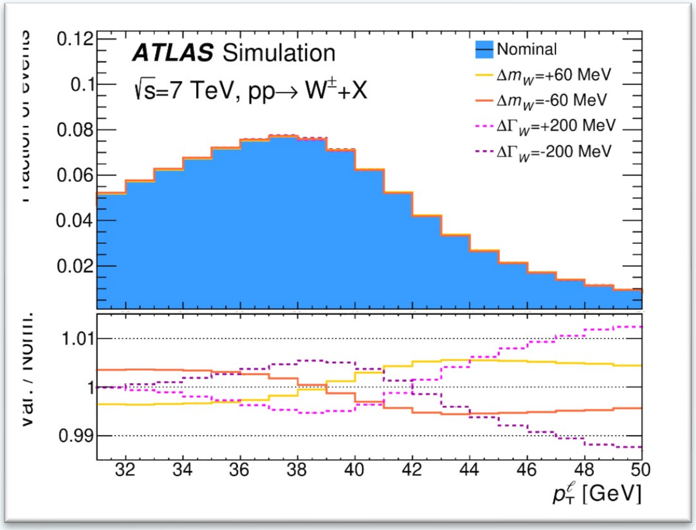




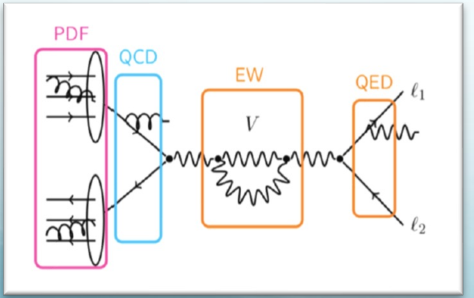
arxiv..2112.07274

W mass precision measurement

Typical: fit of predictions with different masses
to the data in kinematic variables: pT(e/μ), mT, …

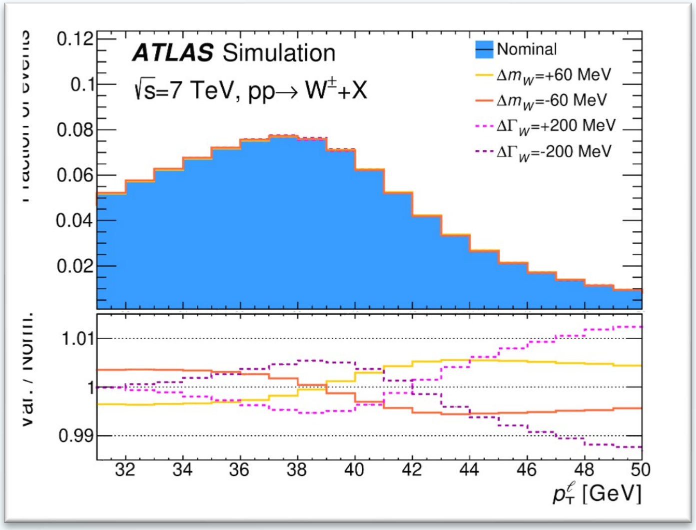


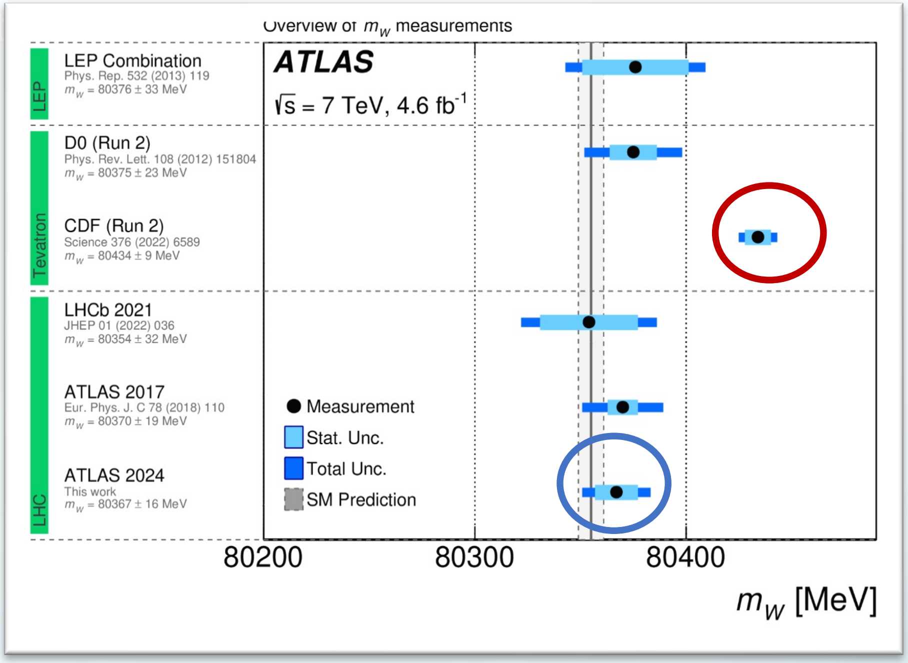
Need precise predictions:



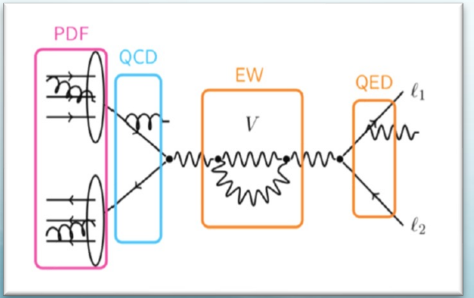
W mass precision measurement

Typical: fit of predictions with different masses
to the data in kinematic variables: pT(e/μ), mT, …





Need precise predictions:



Tension between EW fit/LHC measurements and CDF measurement
→ hopefully resolved this year by first CMS measurement

(4) Boson scattering

Electroweak gauge fields

→ Lagrangian of free gauge fields

$$\mathcal{L}_{gauge}^{EW} = -\frac{1}{4}F_{\mu\nu}^{a}F_{a}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

with:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Electroweak gauge fields

→ Lagrangian of free gauge fields

$$\mathcal{L}_{gauge}^{EW} = -\frac{1}{4}F_{\mu\nu}^{a}F_{a}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

with:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$F_{\mu\nu}^{a} = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g_w \epsilon_{abc} W_\mu^b W_\nu^c$$

$$\left[\frac{\sigma_a}{2}, \frac{\sigma_b}{2}\right] = i\epsilon_{abc}\frac{\sigma_c}{2}$$

W/Z bosons carry EM and/or weak charges

Electroweak gauge fields

→ Lagrangian of free gauge fields

$$\mathcal{L}^{EW}_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

with:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_w \epsilon_{abc} W^b_\mu W^c_\nu$$

$$\left[\frac{\sigma_a}{2}, \frac{\sigma_b}{2}\right] = i\epsilon_{abc}\frac{\sigma_c}{2}$$

W/Z bosons carry EM and/or weak charges

Leads to triple and quartic gauge boson couplings:

$$\begin{aligned}\mathcal{L}^{EW}_{gauge} = \; & -\tfrac{1}{4}\left[(W^-_{\mu\nu})^\dagger W^{-\mu\nu} + (W^+_{\mu\nu})^\dagger W^{+\mu\nu} + Z_{\mu\nu}Z^{\mu\nu} + A_{\mu\nu}A^{\mu\nu}\right] \\ & -ig_w\left[(\cos\theta_w Z^\mu + \sin\theta_w A^\mu)(W^-_{\mu\nu}W^{+\nu} - W^+_{\mu\nu}W^{-\nu})\right. \\ & \quad \left.+(\cos\theta_w Z_{\mu\nu} + \sin\theta_w A_{\mu\nu})W^{+\mu}W^{-\nu}\right] \\ & -\tfrac{g_w^2}{2}\left[2\cos^2\theta_w(W^+_\mu W^{-\mu}Z_\nu Z^\nu - W^+_\mu W^{-\nu}Z_\nu Z^\mu)\right. \\ & \quad +2\sin^2\theta_w(W^+_\mu W^{-\mu}A_\nu A^\nu - W^+_\mu W^{-\nu}A_\nu A^\mu) \\ & \quad +2\cos\theta_w\sin\theta_w(2W^+_\mu W^{-\mu}Z_\nu A^\nu - W^+_\mu W^{-\nu}Z_\nu A^\mu - W^+_\mu W^{-\nu}A_\nu Z^\mu) \\ & \quad \left.-W^+_\mu W^{+\mu}W^-_\nu W^{-\nu} + W^+_\mu W^{-\mu}W^-_\nu W^{+\nu}\right]\end{aligned}$$

Electroweak gauge fields

→ Lagrangian of free gauge fields

$$\mathcal{L}^{EW}_{gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

with:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_w \epsilon_{abc} W^b_\mu W^c_\nu$$

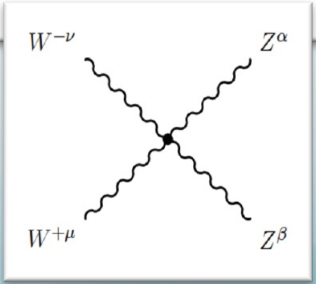
$$\left[\frac{\sigma_a}{2}, \frac{\sigma_b}{2}\right] = i\epsilon_{abc}\frac{\sigma_c}{2}$$

Leads to triple and quartic gauge boson couplings:

$$\begin{aligned}
\mathcal{L}^{EW}_{gauge} = \; & -\tfrac{1}{4}\left[(W^-_{\mu\nu})^\dagger W^{-\mu\nu} + (W^+_{\mu\nu})^\dagger W^{+\mu\nu} + Z_{\mu\nu}Z^{\mu\nu} + A_{\mu\nu}A^{\mu\nu}\right] \\
& -ig_w \left[(\cos\theta_w Z^\mu + \sin\theta_w A^\mu)(W^-_{\mu\nu}W^{+\nu} - W^+_{\mu\nu}W^{-\nu}) \right. \\
& \quad \left. +(\cos\theta_w Z_{\mu\nu} + \sin\theta_w A_{\mu\nu})W^{+\mu}W^{-\nu}\right] \\
& -\tfrac{g_w^2}{2}\left[2\cos^2\theta_w (W^+_\mu W^{-\mu} Z_\nu Z^\nu - W^+_\mu W^{-\nu} Z_\nu Z^\mu) \right. \\
& \quad +2\sin^2\theta_w (W^+_\mu W^{-\mu} A_\nu A^\nu - W^+_\mu W^{-\nu} A_\nu A^\mu) \\
& \quad +2\cos\theta_w \sin\theta_w (2W^+_\mu W^{-\mu} Z_\nu A^\nu - W^+_\mu W^{-\nu} Z_\nu A^\mu - W^+_\mu W^{-\nu} A_\nu Z^\mu) \\
& \quad \left. -W^+_\mu W^{+\mu} W^-_\nu W^{-\nu} + W^+_\mu W^{-\mu} W^-_\nu W^{+\nu}\right]
\end{aligned}$$

W/Z bosons carry EM and/or weak charges

quartic gauge boson coupling



Electroweak gauge fields

→ Lagrangian of free gauge fields

$$\mathcal{L}^{EW}_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

with:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_w \epsilon_{abc} W^b_\mu W^c_\nu$$

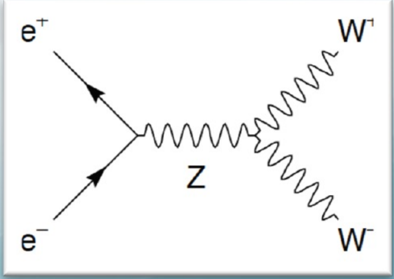
$$\left[\frac{\sigma_a}{2}, \frac{\sigma_b}{2}\right] = i\epsilon_{abc}\frac{\sigma_c}{2}$$

Leads to triple and quartic gauge boson couplings:

$$\begin{aligned}\mathcal{L}^{EW}_{gauge} = \; & -\tfrac{1}{4}\left[(W^-_{\mu\nu})^\dagger W^{-\mu\nu} + (W^+_{\mu\nu})^\dagger W^{+\mu\nu} + Z_{\mu\nu}Z^{\mu\nu} + A_{\mu\nu}A^{\mu\nu}\right] \\ & -ig_w\left[(\cos\theta_w Z^\mu + \sin\theta_w A^\mu)(W^-_{\mu\nu}W^{+\nu} - W^+_{\mu\nu}W^{-\nu})\right. \\ & \qquad \left. +(\cos\theta_w Z_{\mu\nu} + \sin\theta_w A_{\mu\nu})W^{+\mu}W^{-\nu}\right] \\ & -\tfrac{g_w^2}{2}\left[2\cos^2\theta_w(W^+_\mu W^{-\mu}Z_\nu Z^\nu - W^+_\mu W^{-\nu}Z_\nu Z^\mu)\right. \\ & \qquad +2\sin^2\theta_w(W^+_\mu W^{-\mu}A_\nu A^\nu - W^+_\mu W^{-\nu}A_\nu A^\mu) \\ & \qquad +2\cos\theta_w\sin\theta_w(2W^+_\mu W^{-\mu}Z_\nu A^\nu - W^+_\mu W^{-\nu}Z_\nu A^\mu - W^+_\mu W^{-\nu}A_\nu Z^\mu) \\ & \qquad \left. -W^+_\mu W^{+\mu}W^-_\nu W^{-\nu} + W^+_\mu W^{-\mu}W^-_\nu W^{+\nu}\right]\end{aligned}$$

W/Z bosons carry EM and/or weak charges

triple gauge boson coupling



Electroweak gauge fields

→ Lagrangian of free gauge fields

$$\mathcal{L}^{EW}_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

with:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_w \epsilon_{abc} W^b_\mu W^c_\nu$$

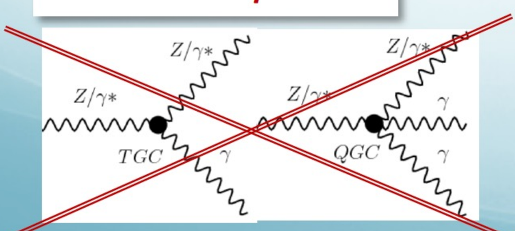
$$\left[\frac{\sigma_a}{2}, \frac{\sigma_b}{2}\right] = i\epsilon_{abc}\frac{\sigma_c}{2}$$

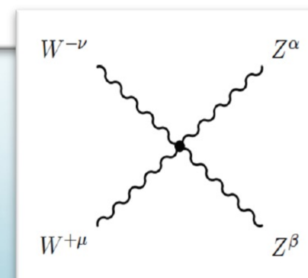
Leads to triple and quartic gauge boson couplings:

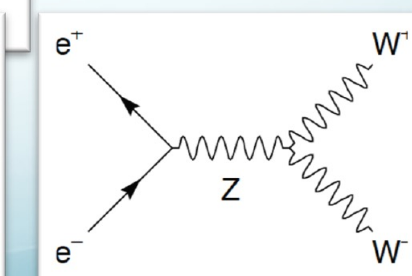
$$\begin{aligned}\mathcal{L}^{EW}_{gauge} = \; & -\tfrac{1}{4}\left[(W^-_{\mu\nu})^\dagger W^{-\mu\nu} + (W^+_{\mu\nu})^\dagger W^{+\mu\nu} + Z_{\mu\nu}Z^{\mu\nu} + A_{\mu\nu}A^{\mu\nu}\right] \\ & -ig_w\left[(\cos\theta_w Z^\mu + \sin\theta_w A^\mu)(W^-_{\mu\nu}W^{+\nu} - W^+_{\mu\nu}W^{-\nu})\right. \\ & \qquad \left.+(\cos\theta_w Z_{\mu\nu} + \sin\theta_w A_{\mu\nu})W^{+\mu}W^{-\nu}\right] \\ & -\tfrac{g_w^2}{2}\left[2\cos^2\theta_w(W^+_\mu W^{-\mu}Z_\nu Z^\nu - W^+_\mu W^{-\nu}Z_\nu Z^\mu)\right. \\ & \qquad +2\sin^2\theta_w(W^+_\mu W^{-\mu}A_\nu A^\nu - W^+_\mu W^{-\nu}A_\nu A^\mu) \\ & \qquad +2\cos\theta_w\sin\theta_w(2W^+_\mu W^{-\mu}Z_\nu A^\nu - W^+_\mu W^{-\nu}Z_\nu A^\mu - W^+_\mu W^{-\nu}A_\nu Z^\mu) \\ & \qquad \left.-W^+_\mu W^{+\mu}W^-_\nu W^{-\nu} + W^+_\mu W^{-\mu}W^-_\nu W^{+\nu}\right]\end{aligned}$$

W/Z bosons carry EM and/or weak charges

No vertex with 3 or 4 Z/γ



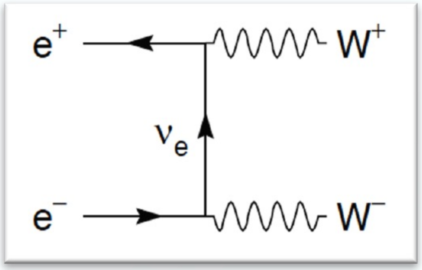


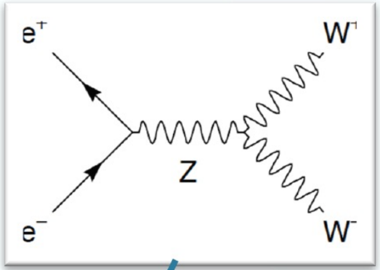


SM precision at LEPII

LEP2 (1996-2000): WW, ZZ, γγ

- W leptonic and hadronic BF
- W mass and width
- Triple gauge couplings:WWZ, WWγ & anomalous TGC: ZZZ, ZZγ*

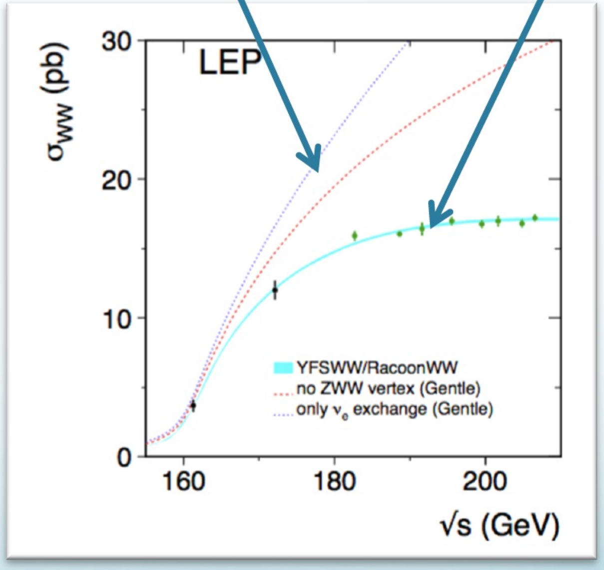




$$
\begin{aligned}
m_W &= 80.376 \pm 0.033\ \mathrm{GeV} \\
\Gamma_W &= 2.195 \pm 0.083\ \mathrm{GeV} \\
B(W \to \mathrm{had}) &= 67.41 \pm 0.27\ \% \\
g_1^Z &= 0.984^{+0.018}_{-0.020} \\
\kappa_\gamma &= 0.982 \pm 0.042 \\
\lambda_\gamma &= -0.022 \pm 0.019\,.
\end{aligned}
$$

Phys.Rept.532,119-224, 2013

WW cross section



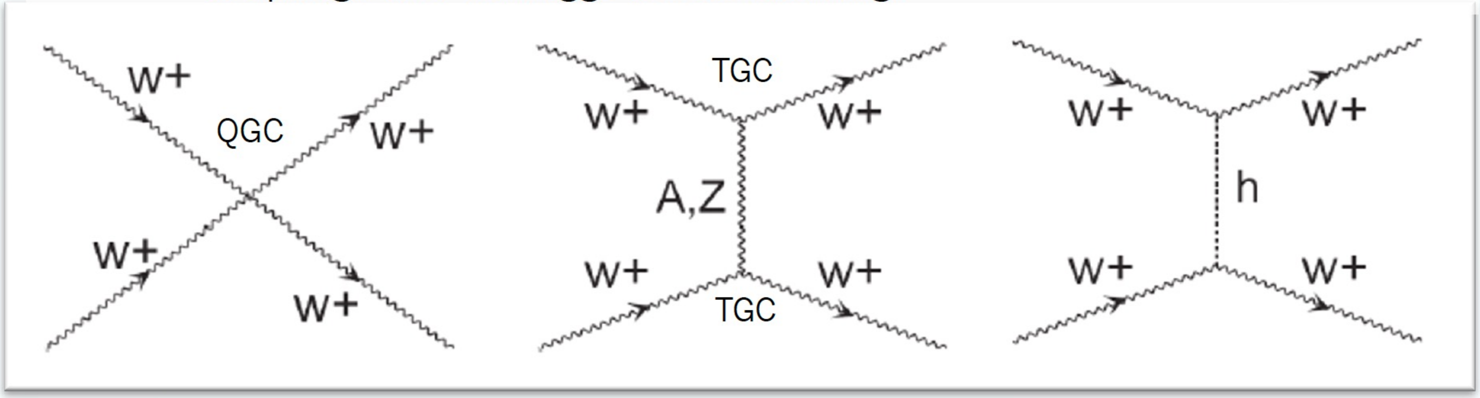
→ Precise measurement of W properties

→ TGC as predicted in SM

→ Quartic Gauge Couplings at the LHC

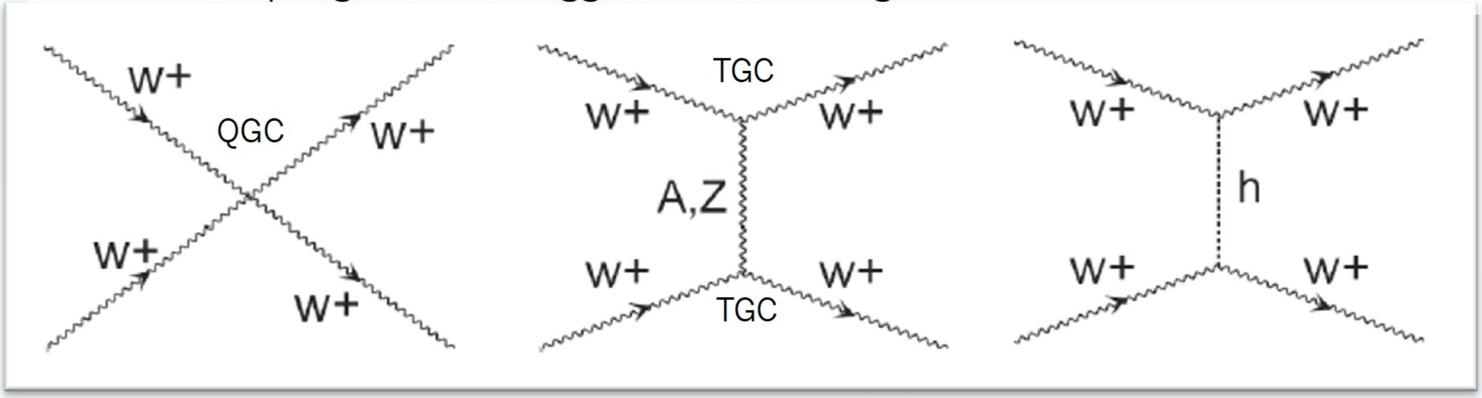
Boson scattering

- The SM predicts gauge boson scattering via Triple and Quartic Gauge couplings and via Higgs boson exchange



Boson scattering

- The SM predicts gauge boson scattering via Triple and Quartic Gauge couplings and via Higgs boson exchange



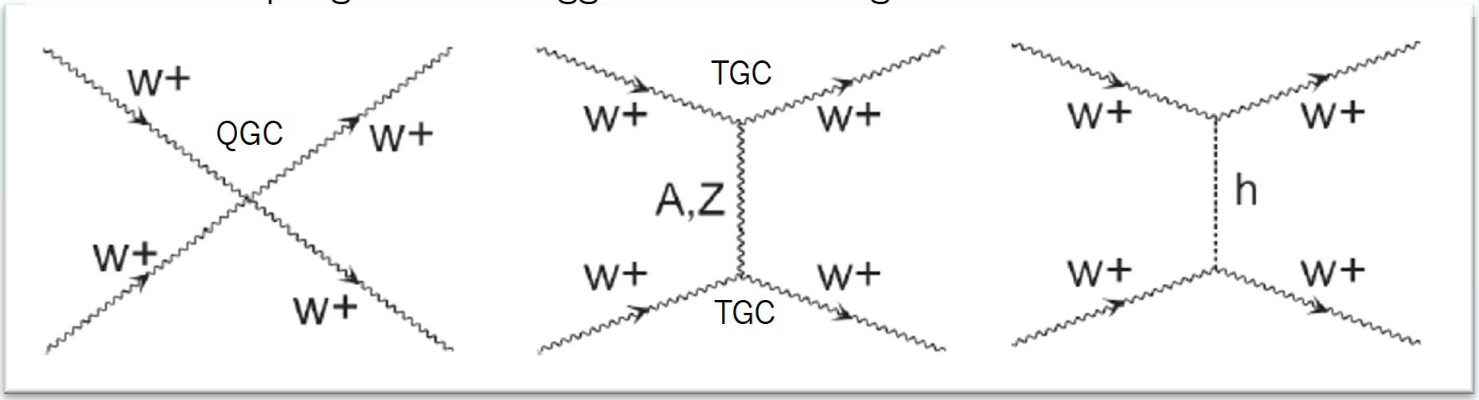
$$\mathcal{M}_{Gauge} = -g^2 \frac{s}{4M_W^2} + \mathcal{O}(s^0)$$

- Without the Higgs contribution the cross section diverges for high energies

s = squared center-of-mass energy of the interacting bosons

Boson scattering

- The SM predicts gauge boson scattering via Triple and Quartic Gauge couplings and via Higgs boson exchange



$$\mathcal{M}_{Gauge} = -g^2 \frac{s}{4M_W^2} + \mathcal{O}(s^0)$$

$$\mathcal{M}_H = g_{HWW}^2 \frac{s}{M_W^4} + \mathcal{O}(s^0)$$

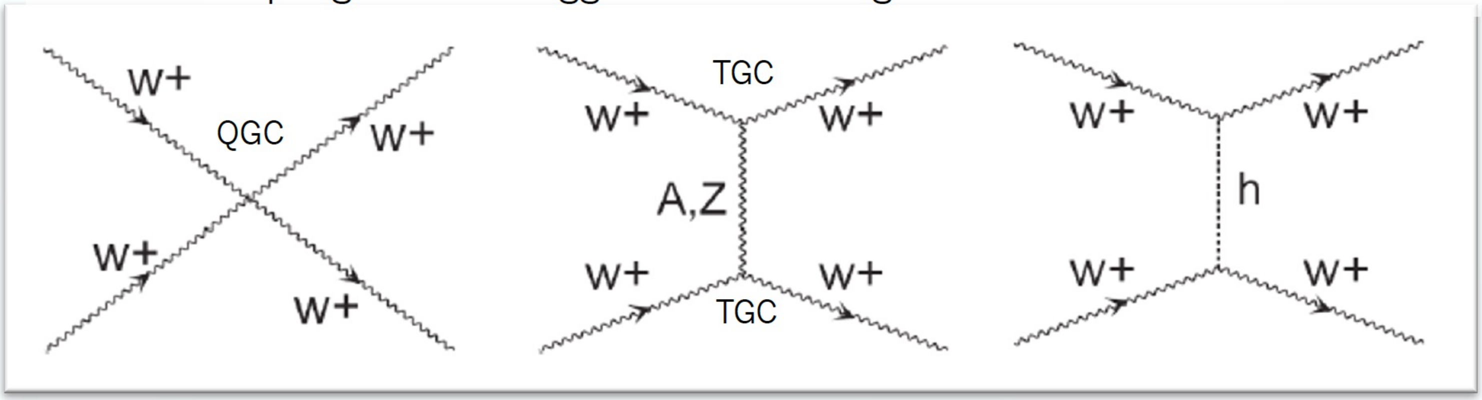
s = squared center-of-mass energy of the interacting bosons

- Without the Higgs contribution the cross section diverges for high energies

- Higgs contribution cancels divergence if Higgs couples proportional to W mass: $g_{HWW} = gM_W$ ... as expected from SM Higgs

Boson scattering

- The SM predicts gauge boson scattering via Triple and Quartic Gauge couplings and via Higgs boson exchange



$$\mathcal{M}_{Gauge} = -g^2 \frac{s}{4M_W^2} + \mathcal{O}(s^0)$$

$$\mathcal{M}_H = g_{HWW}^2 \frac{s}{M_W^4} + \mathcal{O}(s^0)$$

s = squared center-of-mass energy of the interacting bosons

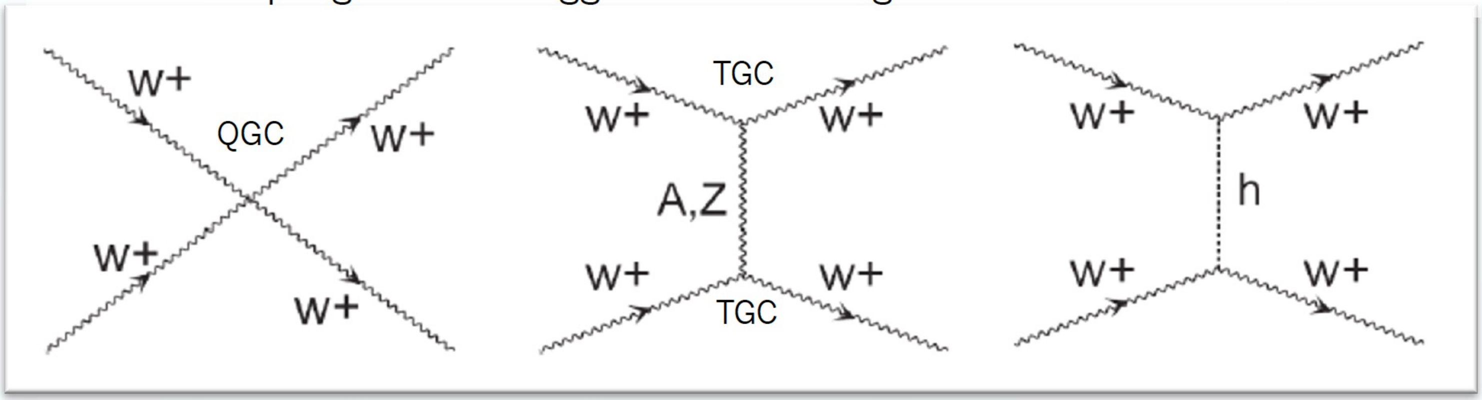
- Without the Higgs contribution the cross section diverges for high energies

- Higgs contribution cancels divergence if Higgs couples proportional to W mass: $g_{HWW} = gM_W$ ... as expected from SM Higgs

$$\mathcal{M}_{Gauge} + \mathcal{M}_H = g^2 \frac{M_H^2}{4M_W^2}$$

Boson scattering

- The SM predicts gauge boson scattering via Triple and Quartic Gauge couplings and via Higgs boson exchange



$$\mathcal{M}_{Gauge} = -g^2 \frac{s}{4M_W^2} + \mathcal{O}(s^0)$$

$$\mathcal{M}_H = g_{HWW}^2 \frac{s}{M_W^4} + \mathcal{O}(s^0)$$

s = squared center-of-mass energy of the interacting bosons

- Without the Higgs contribution the cross section diverges for high energies

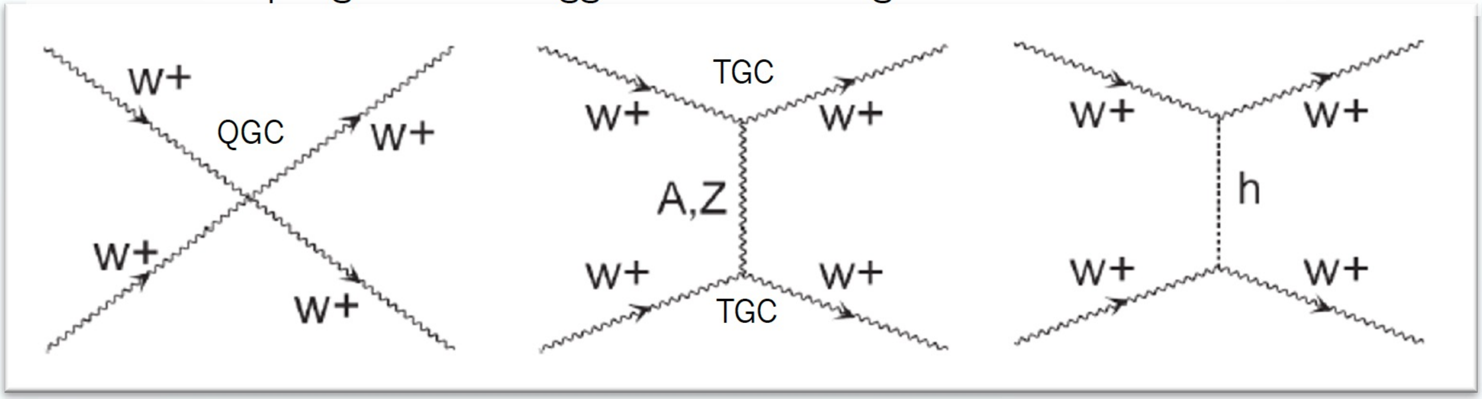
- Higgs contribution cancels divergence if Higgs couples proportional to W mass: $g_{HWW} = gM_W$ ... as expected from SM Higgs

$$\mathcal{M}_{Gauge} + \mathcal{M}_H = g^2 \frac{M_H^2}{4M_W^2}$$

... similar for W+W- scattering, WZ scattering, etc

Boson scattering

- The SM predicts gauge boson scattering via Triple and Quartic Gauge couplings and via Higgs boson exchange



$$\mathcal{M}_{Gauge} = -g^2 \frac{s}{4M_W^2} + \mathcal{O}(s^0)$$

$$\mathcal{M}_H = g_{HWW}^2 \frac{s}{M_W^4} + \mathcal{O}(s^0)$$

s = squared center-of-mass energy of the interacting bosons

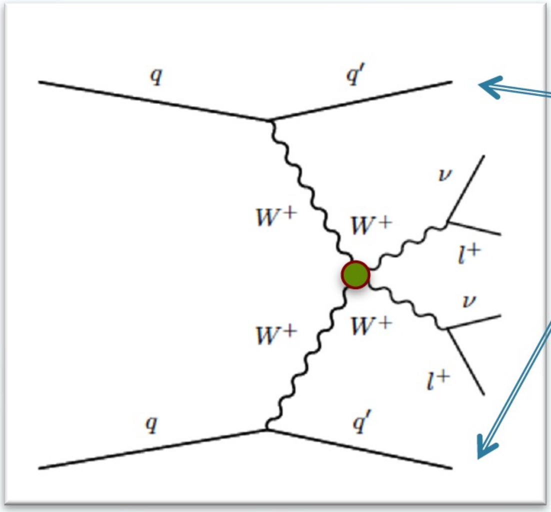
- Without the Higgs contribution the cross section diverges for high energies

- Higgs contribution cancels divergence if Higgs couples proportional to W mass: $g_{HWW} = gM_W$ ... as expected from SM Higgs

$$\mathcal{M}_{Gauge} + \mathcal{M}_H = g^2 \frac{M_H^2}{4M_W^2}$$

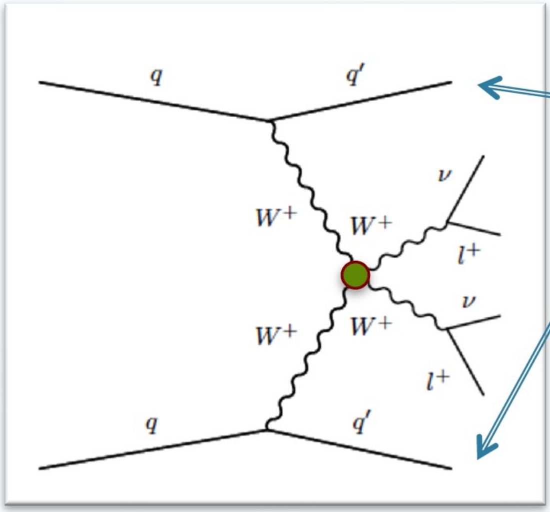
Small BSM effects in either side can lead to larger cross section deviations

WW scattering and QGC

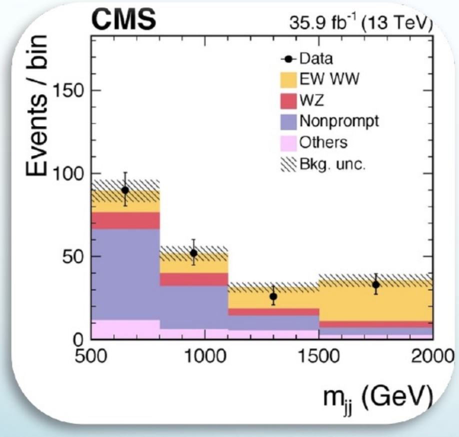


- Characteristic for EWK VV production: presence of two forward jets
- EWK VV events accumulate at large m(jj) and large rapidity gaps between jets $|\Delta\eta(jj)|$

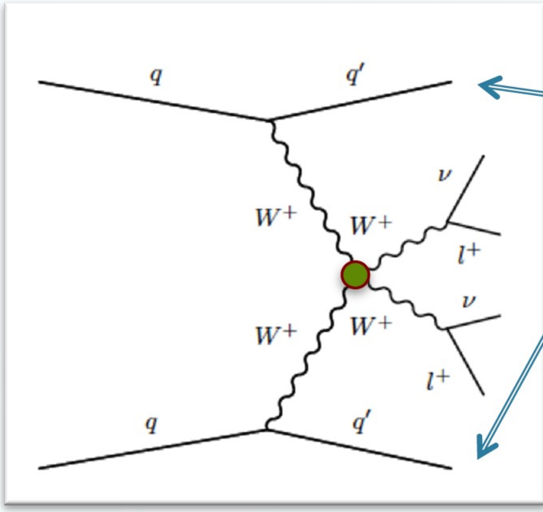
WW scattering and QGC



- Characteristic for EWK VV production: presence of two forward jets
- EWK VV events accumulate at large m(jj) and large rapidity gaps between jets |Δη(jj)|

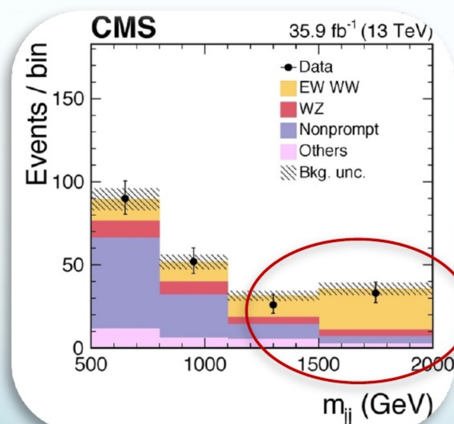


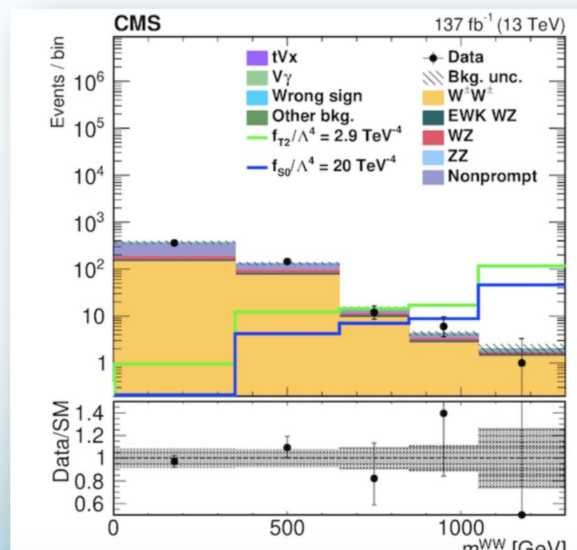
WW scattering and QGC



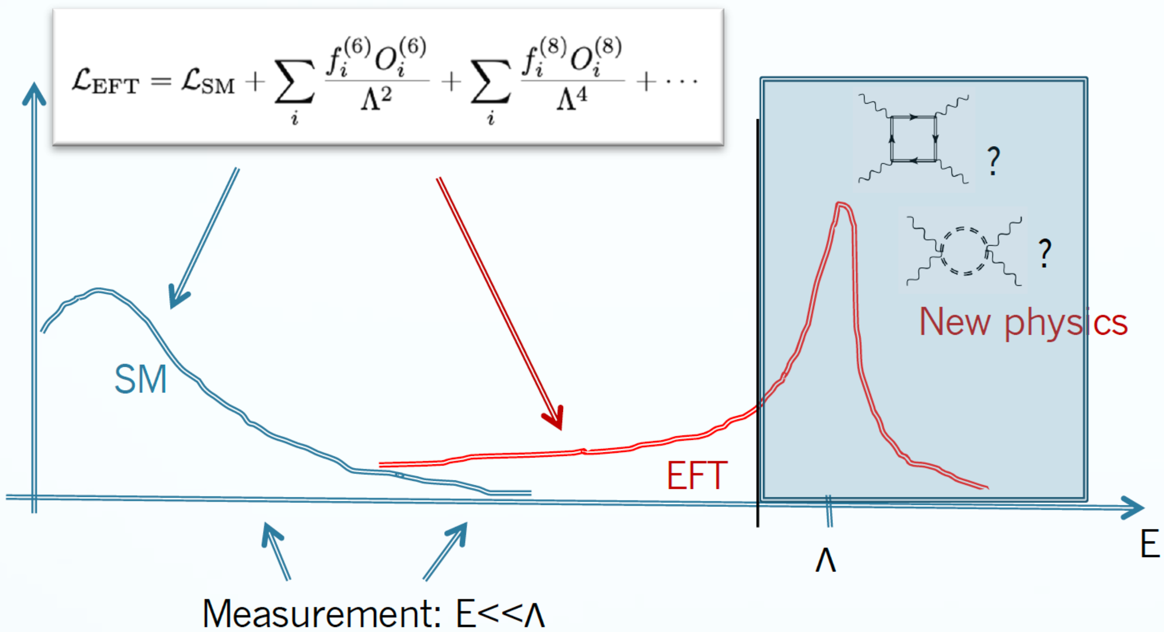
- Characteristic for EWK VV production: presence of two forward jets
- EWK VV events accumulate at large m(jj) and large rapidity gaps between jets |Δη(jj)|

- Searching for new physics $m_T(WW)$

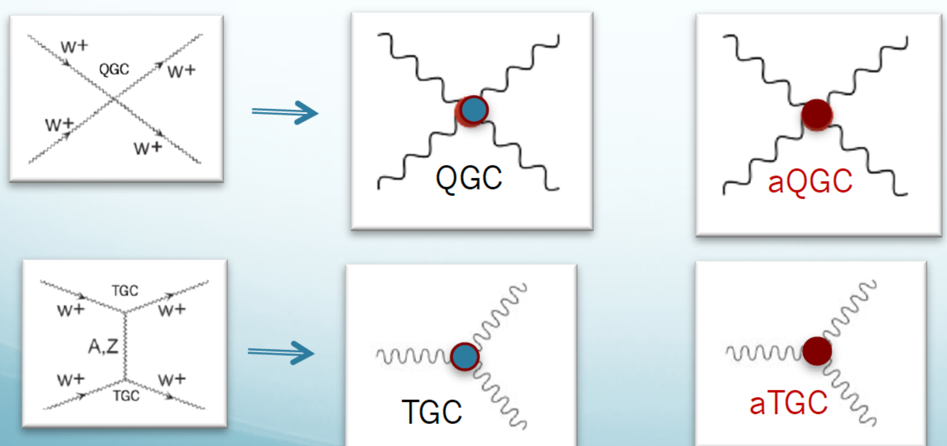




Parametrising new physics

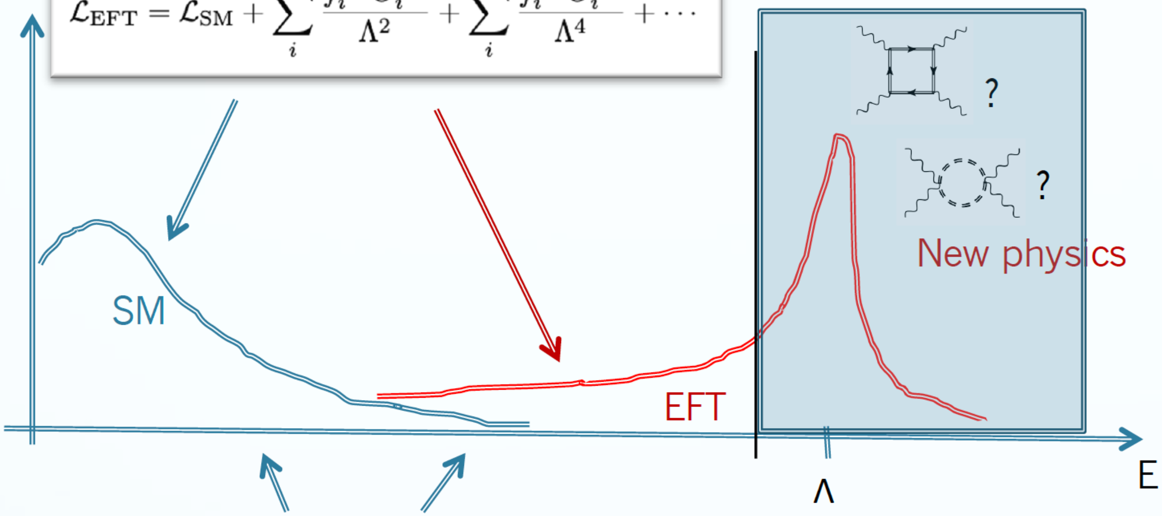


- Parametrize unknown new physics at energy scale Λ with effective Lagrangian at scale $E << \Lambda$
- EFT approach only valid for $E << \Lambda$
- Higher-dimensional terms suppressed by Λ^2



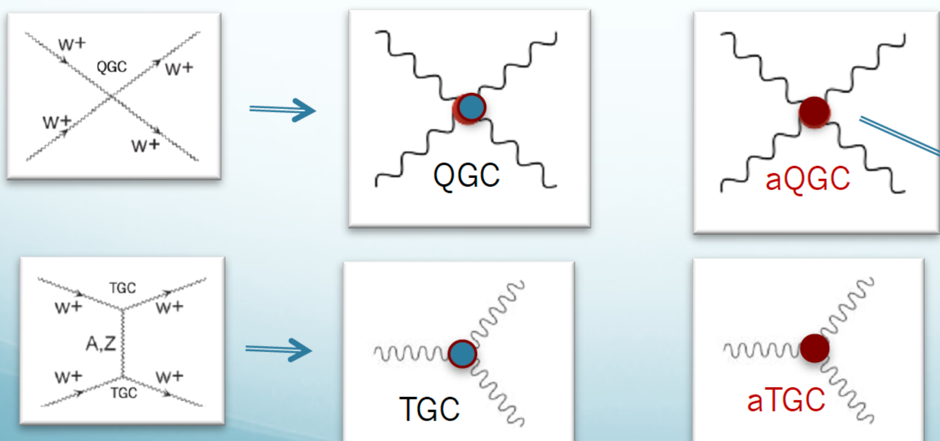
Parametrising new physics

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{f_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{f_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$



Measurement: E<<Λ

- Parametrize unknown new physics at energy scale Λ with effective Lagrangian at scale E << Λ
- EFT approach only valid for E << Λ
- Higher-dimensional terms suppressed by Λ^2

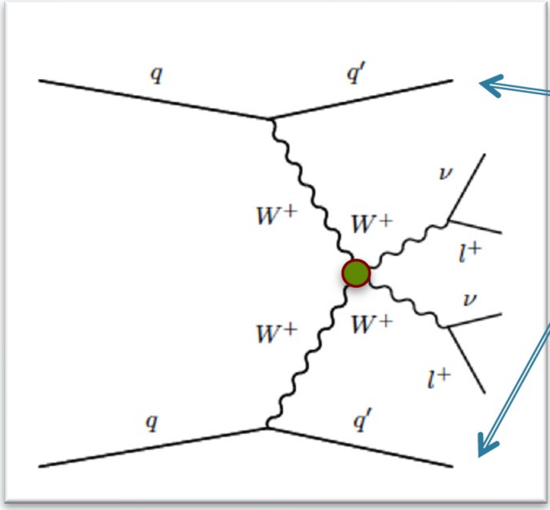


Example: Dim-8 EFT operators (aQGC)

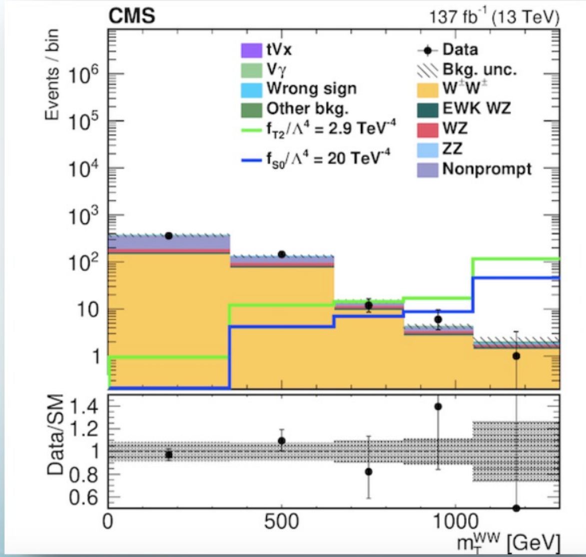
$$O_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right]$$
$$O_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right]$$
$$O_{S,2} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\nu\Phi)^\dagger D^\mu\Phi\right]$$
$$O_{M,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$O_{M,1} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$O_{M,2} = \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$O_{M,3} = \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$O_{M,4} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\mu\Phi\right] \times \hat{B}^{\beta\nu}$$
$$O_{M,5} = \tfrac{1}{2}\left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\nu\Phi\right] \times \hat{B}^{\beta\mu} + h.c.$$
$$O_{M,7} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right]$$

$$O_{T,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right]$$
$$O_{T,1} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right]$$
$$O_{T,2} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right]$$
$$O_{T,5} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta}$$
$$O_{T,6} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$
$$O_{T,7} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$
$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta}$$
$$O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha},$$

WW scattering and QGC



- Characteristic for EWK VV production: presence of two forward jets
- EWK VV events accumulate at large m(jj) and large rapidity gaps between jets |Δη(jj)|



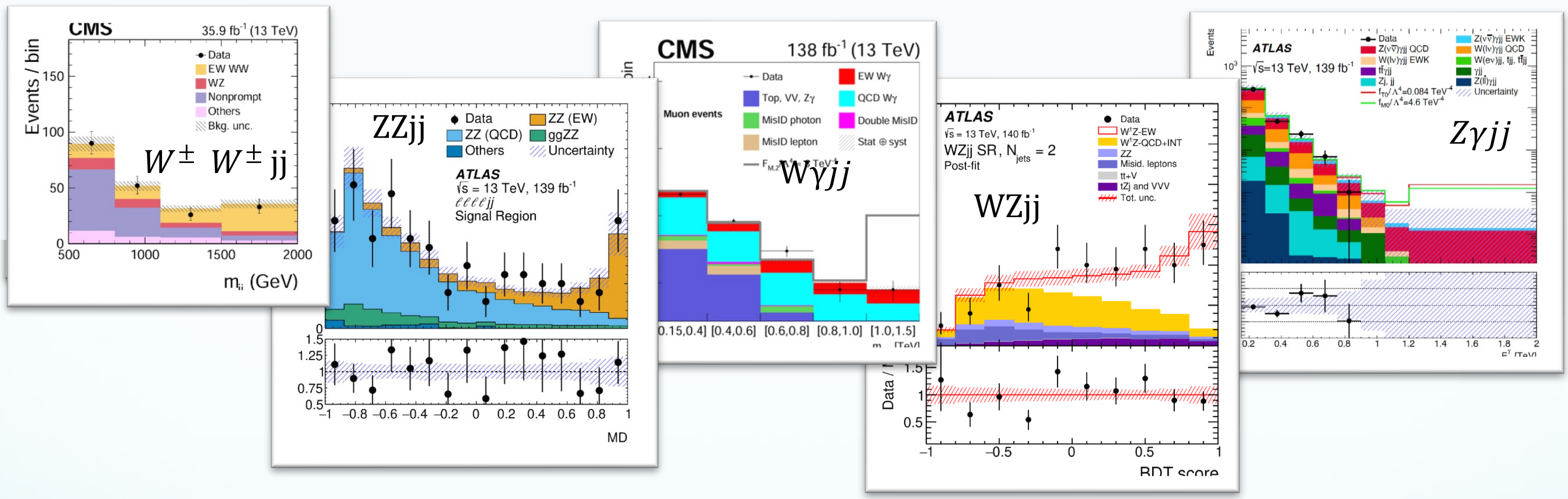
- Extract constraints on EFT parameters

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{f_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{f_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

	Observed ($W^\pm W^\pm$) (TeV^{-4})	Expected ($W^\pm W^\pm$) (TeV^{-4})
f_{T0}/Λ^4	[-0.28, 0.31]	[-0.36, 0.39]
f_{T1}/Λ^4	[-0.12, 0.15]	[-0.16, 0.19]
f_{T2}/Λ^4	[-0.38, 0.50]	[-0.50, 0.63]
f_{M0}/Λ^4	[-3.0, 3.2]	[-3.7, 3.8]
f_{M1}/Λ^4	[-4.7, 4.7]	[-5.4, 5.8]
f_{M6}/Λ^4	[-6.0, 6.5]	[-7.5, 7.6]
f_{M7}/Λ^4	[-6.7, 7.0]	[-8.3, 8.1]
f_{S0}/Λ^4	[-6.0, 6.4]	[-6.0, 6.2]
f_{S1}/Λ^4	[-18, 19]	[-18, 19]

Other Boson scattering processes

LHC Run~2: Golden VBS Era:
observed $W^{\pm}\ W^{\pm}$jj, $W^{+}W^{-}$jj, ZZjj, 4ljj, WZjj, Wγjj, $Z\gamma jj$



With more data, we can improve the precision and reach in EFT-sensitive regimes.

Summary Standard Model

- **Relativistic Quantum Field Theory**
- **Symmetry requirements**
- **Renormalizability**

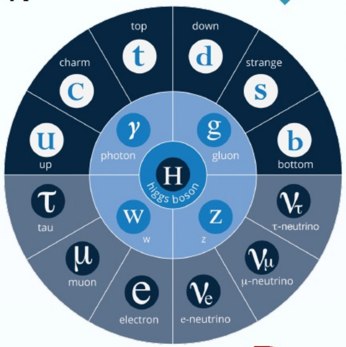
Symmetries and fields:

- **Lorentz boosts/rotations, translations**
  matter particles, spin1/2: ψ
  described by Dirac formalism:

$$\mathcal{L}_{SM} = i\bar{\Psi}_i \gamma^\mu \partial_\mu \Psi_i$$

2x6 leptons: e, ν_e, μ, ν_μ, τ, ν_τ

2x6 quarks: u, d, c, s, b, t



- **Fermions and bosons masses**
  ➔ scalar Higgs field φ
  EW Symmetry breaking in the ground state (vacuum)

- **Local gauge symmetries:**
  ➔ force fields: spin 1: **V**

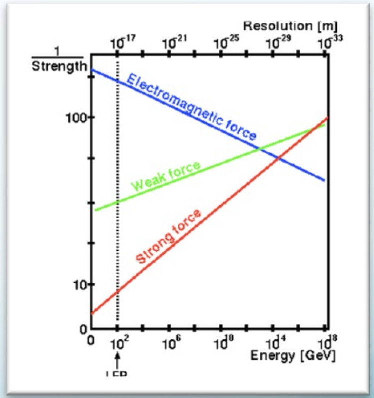
SU(2) x U(1) ➔ γ, Z, $W^\pm$: EW force

SU(3) ➔ 8 gluons: strong force

$$\mathbf{D}_\mu = \partial_\mu + \mathrm{i}\mathrm{g}_\mathrm{k} \mathbf{V}^\mathrm{k}_\mu$$

Renormalizability

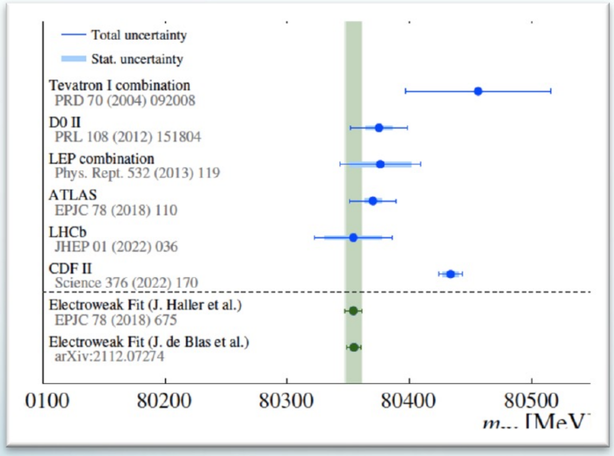
Effective parameters can be adjusted in all orders of perturbation expansion such that theory keeps finite

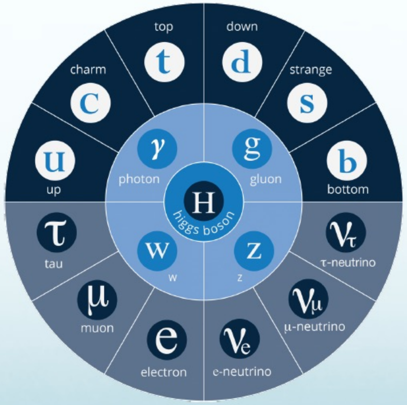


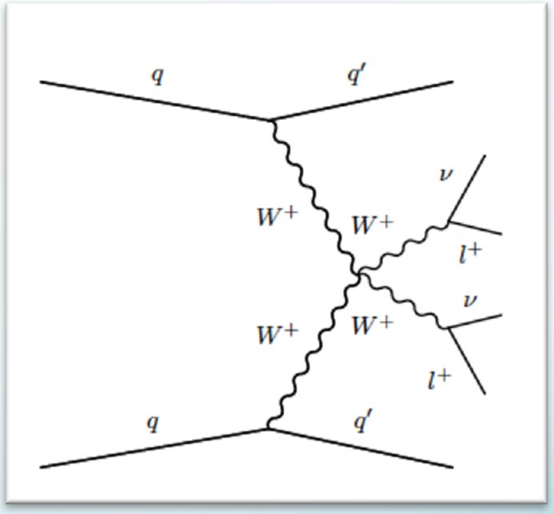
Summary

The Standard-Model has been probed at high precision at e+e- colliders.

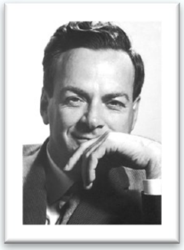
LHC has reached a high precision in EWK physics measurements and
Is probing important SM parameters at a competitive precision.





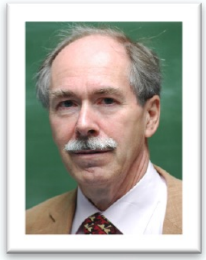


SM: an international development



Richard Feynman
(US), QED

Abdus Salam
(Pakistan)
EW theory



Gerard 't Hooft
Netherlands,
renormalisation

Chien-Shiun Wu
(China/US)
Parity violation

Shinichiro Tomonaga
(Japan) QED

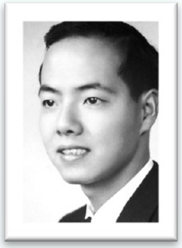
Yang Chen-Ning
(China) Gauge theories

Frank Wilczek
(US) QCD

Emmy Noether
(Germany):
Symmetries and
conservation

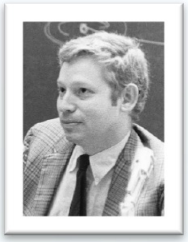
Makoto Kobayashi
(Japan): Quark mixing

Murray Gell-Man
(US) Quarjks



Tsung-Dao Lee
(China) parity violation

Peter Higgs
(UK) EWSB



Steven Weinberg
(US) EW theory

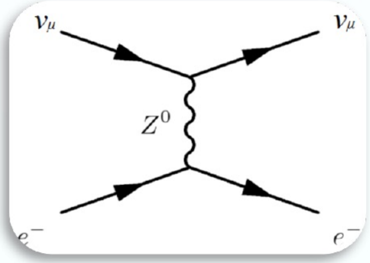
"Scientific thought and its creation is the common and shared heritage of mankind."

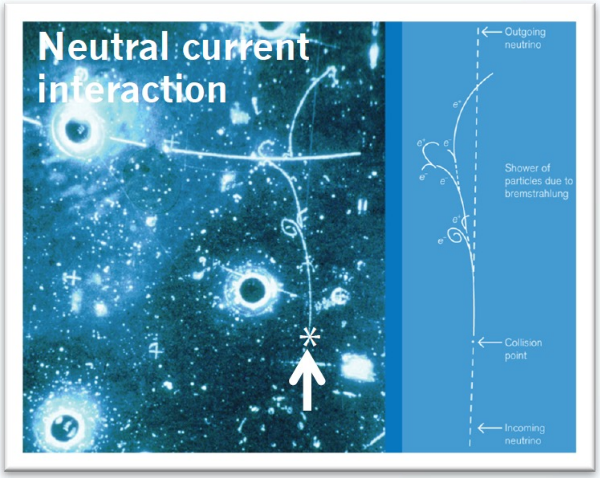
Abdus Salam

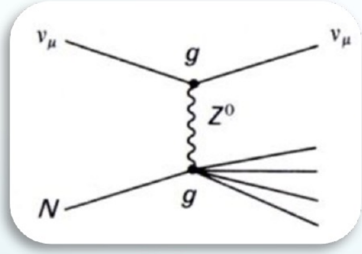
Appendix

Experimental milestones

- **1972: Gargamelle: discovery of neutral currents**



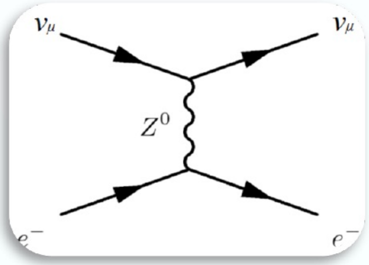


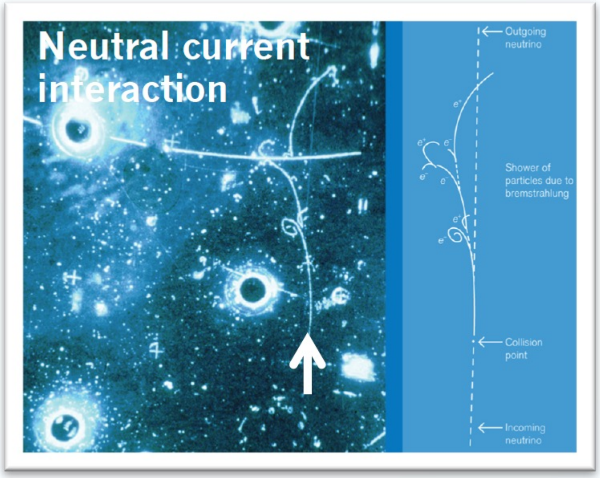


The Neutral Current was not part of Gargamelle's core program
… but they reacted fast to new challenge

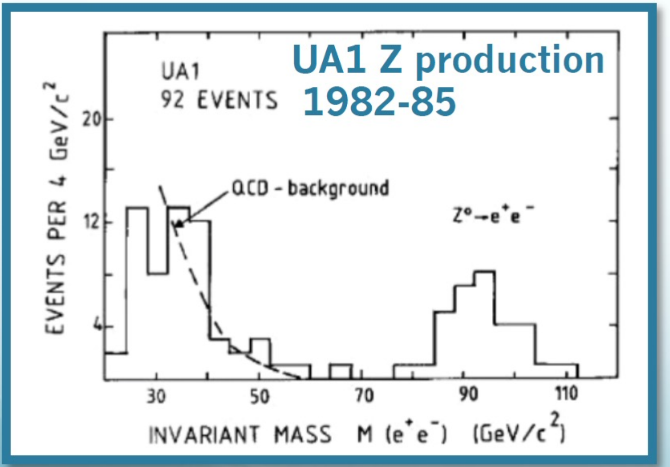
Experimental milestones

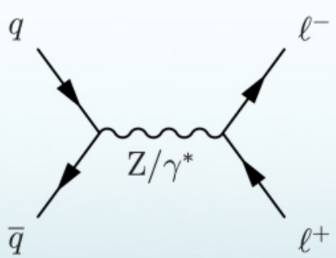
- **1972: Gargamelle: discovery of neutral currents**

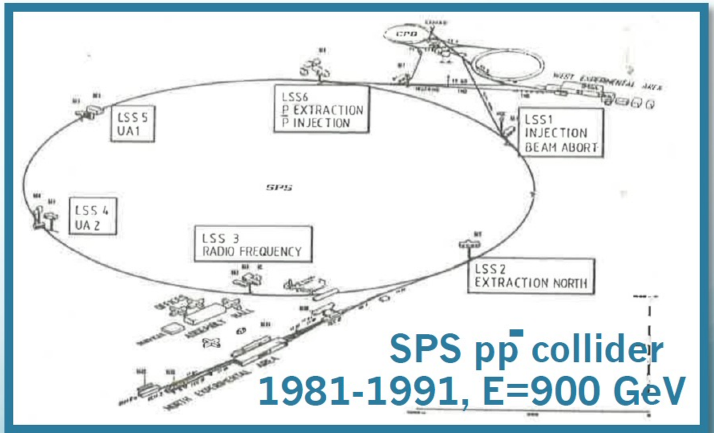




- **1984: UA1/UA2: observation of W and Z bosons**

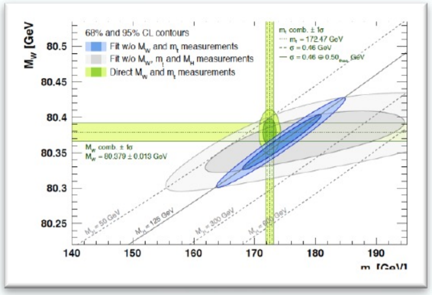


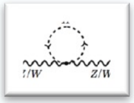


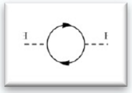


Electroweak fit: status 2021

Example: *W* *top and Higgs mass measurements*

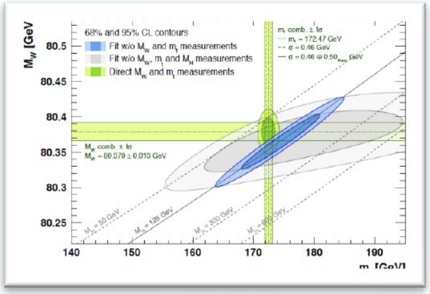


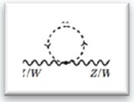


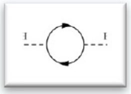


Electroweak fit: status 2021

Example: W top and Higgs mass measurements

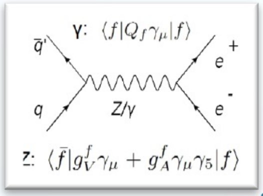






Example: Z pole precision measurements

- *Forward/backward asymmetries*
- *Left/Right asymmetries*
- *Branching fractions*



$$\boxed{\frac{g_{\mathrm{Vf}}}{g_{\mathrm{Af}}} = 1 - 4|Q_{\mathrm{f}}|\sin^2\theta_{\mathrm{eff}}^{\mathrm{f}}}$$

$$A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$$

$$R_\ell^0 \equiv \Gamma_{\mathrm{had}}/\Gamma_\ell$$

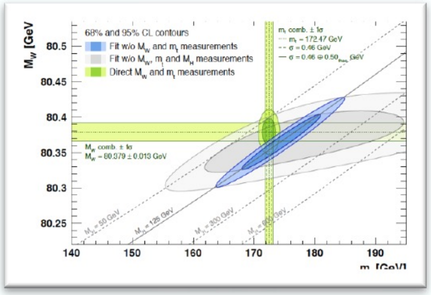
$$R_q^0 \equiv \Gamma_{q\bar{q}}/\Gamma_{\mathrm{had}}$$

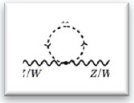
$$A_{\mathrm{f}} = \frac{g_{\mathrm{Lf}}^2 - g_{\mathrm{Rf}}^2}{g_{\mathrm{Lf}}^2 + g_{\mathrm{Rf}}^2}$$

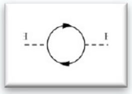
$$= 2\frac{g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1 + (g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2}$$

Electroweak fit: status 2021

Example: *W* *top and Higgs mass measurements*

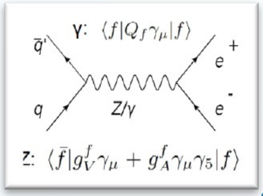






Example: Z pole precision measurements

- *Forward/backward asymmetries*
- *Left/Right asymmetries*
- *Branching fractions*



$$\frac{g_{\mathrm{Vf}}}{g_{\mathrm{Af}}} = 1 - 4|Q_{\mathrm{f}}|\sin^2\theta_{\mathrm{eff}}^{\mathrm{f}}$$

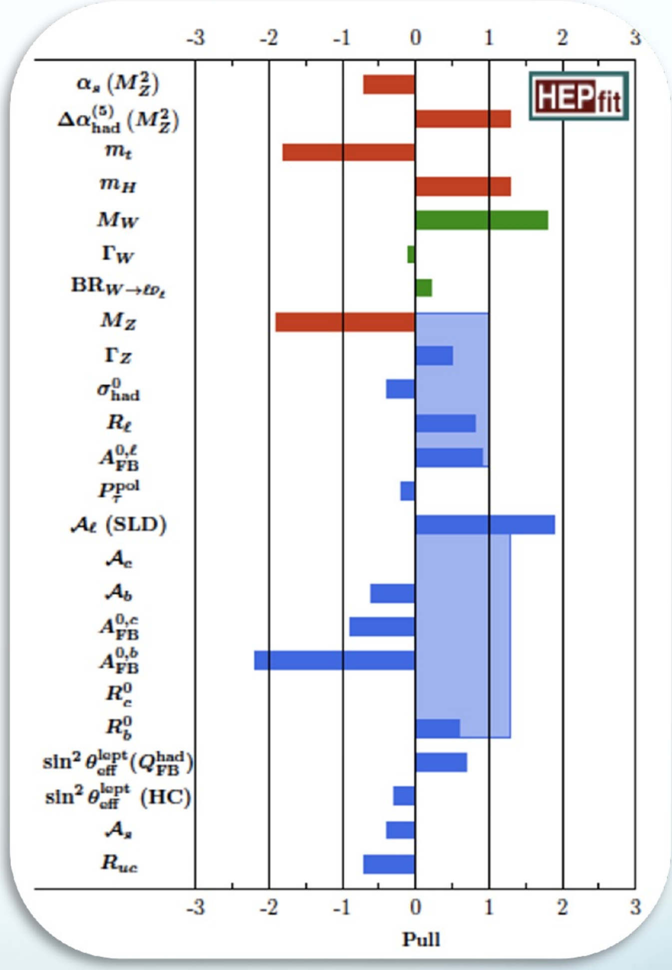
$$A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$$

$$R_\ell^0 \equiv \Gamma_{\mathrm{had}}/\Gamma_\ell$$

$$R_q^0 \equiv \Gamma_{q\bar{q}}/\Gamma_{\mathrm{had}}$$

$$A_{\mathrm{f}} = \frac{g_{\mathrm{Lf}}^2 - g_{\mathrm{Rf}}^2}{g_{\mathrm{Lf}}^2 + g_{\mathrm{Rf}}^2}$$

$$= 2\frac{g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1 + (g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2}$$

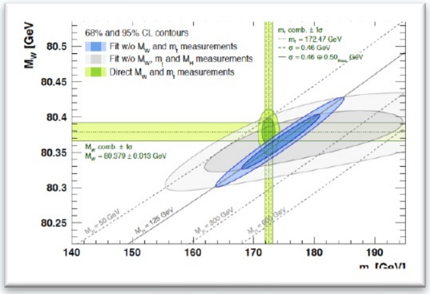


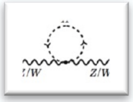
arXiv:2112.07274

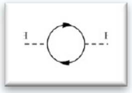
Pulls between direct determination and SM fit (with direct measurement excluded)

Electroweak fit: status 2021

Example: W top and Higgs mass measurements

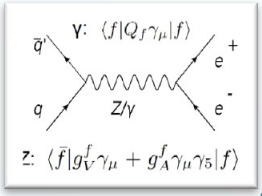






Example: Z pole precision measurements

- *Forward/backward asymmetries*
- *Left/Right asymmetries*
- *Branching fractions*



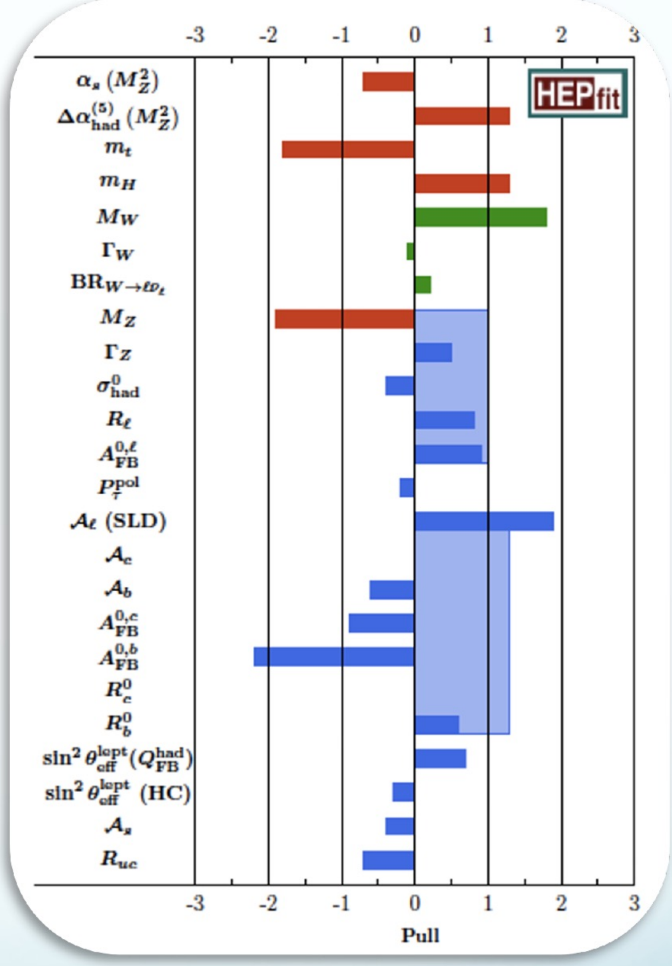
$$\frac{g_{Vf}}{g_{Af}} = 1 - 4|Q_f|\sin^2\theta_{eff}^{f}$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$R_\ell^0 \equiv \Gamma_{had}/\Gamma_\ell$$

$$R_q^0 \equiv \Gamma_{q\bar{q}}/\Gamma_{ha}$$

$$A_f = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = 2\frac{g_{Vf}/g_{Af}}{1+(g_{Vf}/g_{Af})^2}$$



arXiv:2112.07274

Pulls between direct determination and SM fit (with direct measurement excluded)

- Direct measurements in general in agreement with EW fit,
- Some tensions: A_l (SLD) and $A_{FB}^{(0,b)}$, ... → Potential for new precision measurements

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

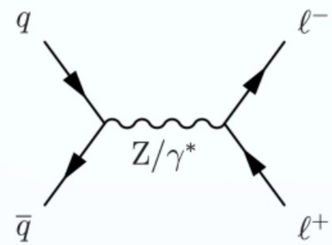
$$\mathcal{L}_{int}^{EW} = -\sum_f \quad \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_+ L \boxed{W_\mu^+} + \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_- L \boxed{W_\mu^-} + \\ e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w \left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu$$

$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\begin{aligned}\mathcal{L}_{int}^{EW} = -\sum_f \quad & \frac{g_w}{\sqrt{2}}\overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}}\overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ & e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\boxed{A_\mu} + \\ & \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3 L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right]\boxed{Z_\mu}\end{aligned}$$

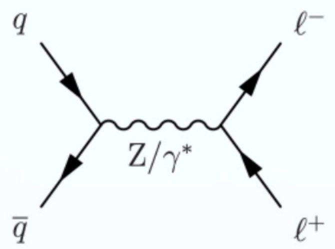


$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\mathcal{L}_{int}^{EW} = -\sum_f \quad \frac{g_w}{\sqrt{2}}\overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}}\overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu$$



$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

→ Rewriting the Z interaction in terms of axial (c_A) and vector (c_V) coupling for the two components of the lefthanded doublet:

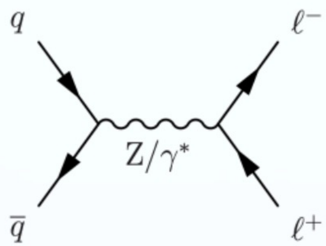
$$\mathcal{L}_{int\ Z^0}^{EW} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu \tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\begin{aligned}\mathcal{L}_{int}^{EW} = -\sum_f \quad & \frac{g_w}{\sqrt{2}}\overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}}\overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ & e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ & \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu\end{aligned}$$



$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

→ Rewriting the Z interaction in terms of axial (c_A) and vector (c_V) coupling for the two components of the lefthanded doublet:

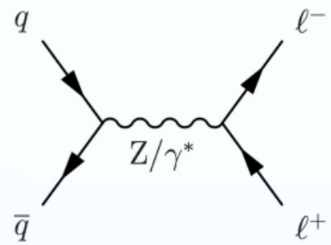
$$\mathcal{L}_{int\ Z^0}^{EW} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu \tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\mathcal{L}_{int}^{EW} = -\sum_f \quad \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu$$



$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

→ Rewriting the Z interaction in terms of axial (c_A) and vector (c_V) coupling for
→ the two components of the lefthanded doublet:

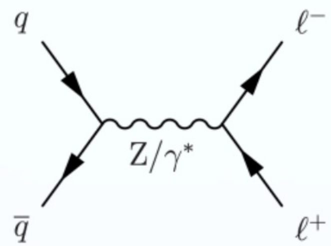
$$\mathcal{L}_{int\ Z^0}^{EW} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu \tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\begin{aligned}\mathcal{L}_{int}^{EW} = -\sum_f \quad & \frac{g_w}{\sqrt{2}}\overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}}\overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ & e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ & \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu\end{aligned}$$



$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

→ Rewriting the Z interaction in terms of axial (c_A) and vector (c_V) coupling for

→ the two components of the lefthanded doublet:

$$\mathcal{L}_{int\ Z^0}^{EW} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu\tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

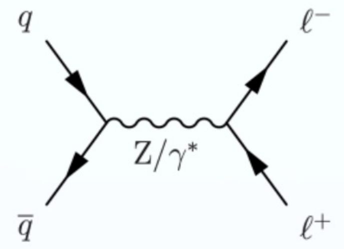
$$\begin{aligned} c_V^f &= \tfrac{1}{2} - 2\sin^2\theta_w Q & c_A^f &= \tfrac{1}{2} \\ c_V^{f'} &= -\tfrac{1}{2} - 2\sin^2\theta_w Q & c_A^{f'} &= -\tfrac{1}{2}\end{aligned}$$

$$L = \begin{pmatrix}\psi_L \\ \psi'_L\end{pmatrix}$$

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\begin{aligned}\mathcal{L}_{int}^{EW} = -\sum_f \quad & \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ & e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ & \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu\end{aligned}$$



$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

→ Rewriting the Z interaction in terms of axial (c_A) and vector (c_V) coupling for
→ the two components of the lefthanded doublet:

$$\mathcal{L}_{int\ Z^0}^{EW} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu \tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

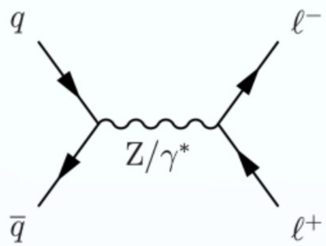
$$\begin{aligned} c_V^f &= \tfrac{1}{2} - 2\sin^2\theta_w Q & c_A^f &= \tfrac{1}{2} \\ c_V^{f'} &= -\tfrac{1}{2} - 2\sin^2\theta_w Q & c_A^{f'} &= -\tfrac{1}{2}\end{aligned}$$

$$L = \begin{pmatrix} \psi_L \\ \psi'_L \end{pmatrix}$$

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\mathcal{L}_{int}^{EW} = -\sum_f \quad \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu$$



$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

→ Rewriting the Z interaction in terms of axial (c_A) and vector (c_V) coupling for
→ the two components of the lefthanded doublet:

$$\mathcal{L}_{int\ Z^0}^{EW} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu \tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

$$c_V^f = \tfrac{1}{2} - 2\sin^2\theta_w Q \qquad c_A^f = \tfrac{1}{2}$$
$$c_V^{f'} = -\tfrac{1}{2} - 2\sin^2\theta_w Q \qquad c_A^{f'} = -\tfrac{1}{2}$$

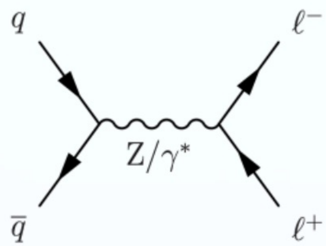
example:

$c_V(e, \mu, \tau) = -0.04$ and $c_A(e, \mu, \tau) = -0.5$

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\mathcal{L}_{int}^{EW} = -\sum_f \quad \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu$$



$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

→ Rewriting the Z interaction in terms of axial (c_A) and vector (c_V) coupling for
→ the two components of the lefthanded doublet:

$$\mathcal{L}_{int\ Z^0}^{EW} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu \tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

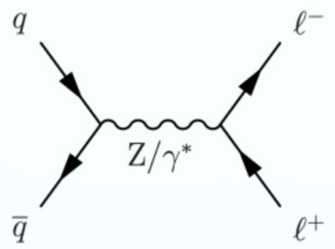
$$c_V^f = \tfrac{1}{2} - 2\sin^2\theta_w Q \qquad c_A^f = \tfrac{1}{2}$$
$$c_V^{f'} = -\tfrac{1}{2} - 2\sin^2\theta_w Q \qquad c_A^{f'} = -\tfrac{1}{2}$$

Couplings are averages for right and left handed fermions

Electroweak unification

→ Interaction Lagrangian (in terms of physical fields A, Z):

$$\mathcal{L}_{int}^{EW} = -\sum_f \quad \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_+ L\, W_\mu^+ + \frac{g_w}{\sqrt{2}} \overline{L}\gamma^\mu T_- L\, W_\mu^- + \\ e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right) A_\mu + \\ \frac{g_w}{\cos\theta_w}\left[\overline{L}\gamma^\mu T_3\, L - \sin^2\theta_w\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)\right] Z_\mu$$



$$T_+ = \frac{\sigma_+}{2} = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_- = \frac{\sigma_-}{2} = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

→ Rewriting the Z interaction in terms of axial (c_A) and vector (c_V) coupling for
→ the two components of the lefthanded doublet:

$$\mathcal{L}_{int\ Z^0}^{EW} = -\sum_f \frac{g_w}{\cos\theta_w}\left[\ \overline{\psi}\gamma^\mu \tfrac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu \tfrac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right] Z_\mu$$

$$c_V^f = \tfrac{1}{2} - 2\sin^2\theta_w Q \qquad c_A^f = \tfrac{1}{2}$$
$$c_V^{f'} = -\tfrac{1}{2} - 2\sin^2\theta_w Q \qquad c_A^{f'} = -\tfrac{1}{2}$$

Different couplings for right/left handed fermions

$c_V = c_L + c_R$ $\quad$ $c_A = c_L - c_R$

[Higgs mechanism]

So far no mass terms for the gauge bosons W,Z as they would destroy the local gauge invariance

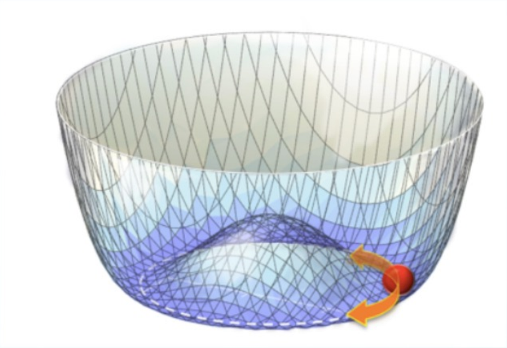
→ Mass terms introduced by interaction with a scalar field through the covariant derivative in the kinetic term

$$\mathcal{L}_{\phi}^{EW} = (D_\mu \phi)^\dagger D^\mu \phi - \left(\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2\right) \qquad (\mu^2 < 0,\ \lambda > 0)$$

Peter Higgs

Francois Englert

Robert Brout



[Higgs mechanism]

Peter Higgs

Francois Englert

Robert Brout

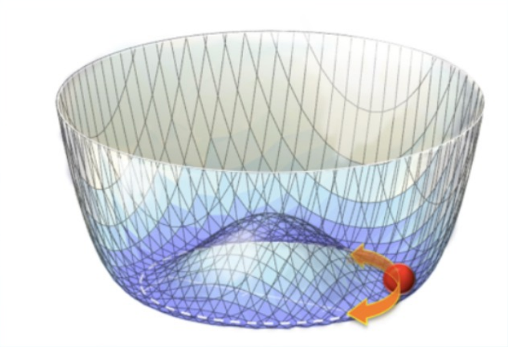
So far no mass terms for the gauge bosons W,Z as they would destroy the local gauge invariance

→ Mass terms introduced by interaction with a scalar field through the covariant derivative in the kinetic term

$$\mathcal{L}_{\phi}^{EW} = (D_{\mu}\phi)^{\dagger} D^{\mu}\phi - \left(\mu^2\phi^{\dagger}\phi + \lambda(\phi^{\dagger}\phi)^2\right) \qquad (\mu^2 < 0,\ \lambda > 0)$$

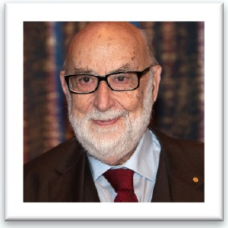
reminder:

$$D_{\mu} = \partial_{\mu} + ig_w \frac{\sigma_i}{2} W_{\mu}^{i} + ig\frac{Y}{2} B_{\mu}$$



[Higgs mechanism]

Peter Higgs



Francois Englert

Robert Brout

So far no mass terms for the gauge bosons W,Z as they would destroy the local gauge invariance

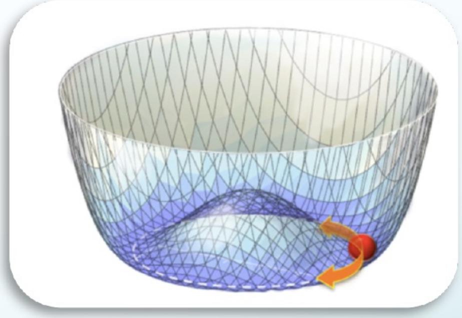
→ Mass terms introduced by interaction with a scalar field through the covariant derivative in the kinetic term

$$\mathcal{L}_\phi^{EW} = (D_\mu\phi)^\dagger D^\mu\phi - \left(\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2\right) \qquad (\mu^2 < 0,\ \lambda > 0)$$

after assuming a non-zero vacuum expectation value (electroweak symmetry breaking)

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\langle 0|\phi^\dagger\phi|0\rangle = \frac{v^2}{2} \simeq (174\ \text{GeV})^2$$



[Higgs mechanism]

Peter Higgs

Francois Englert

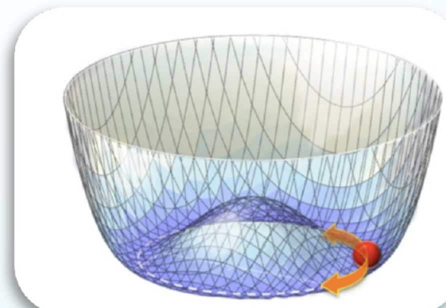
Robert Brout

So far no mass terms for the gauge bosons W,Z as they would destroy the local gauge invariance

→ Mass terms introduced by interaction with a scalar field through the covariant derivative in the kinetic term

$$\mathcal{L}_{\phi}^{EW} = (D_\mu \phi)^\dagger D^\mu \phi - \left(\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2\right) \qquad (\mu^2 < 0,\ \lambda > 0)$$

after assuming a non-zero vacuum expectation value (electroweak symmetry breaking)



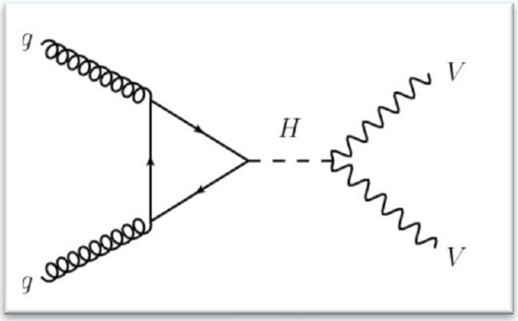
$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\langle 0 | \phi^\dagger \phi | 0 \rangle = \frac{v^2}{2} \simeq (174 \text{ GeV})^2$$

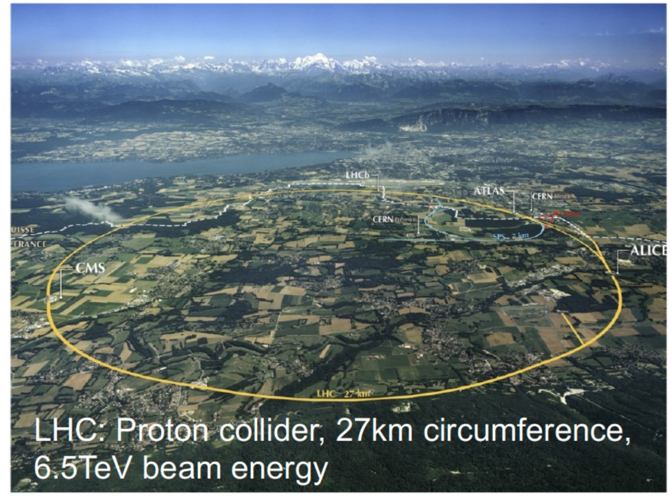
the kinematic term creates mass terms for the W and Z

$$m_{W+} = \frac{g_w v}{2} \qquad m_{W-} = \frac{g_w v}{2} \qquad m_Z = \frac{g_w v}{2 \cos\theta_w} = \frac{m_W}{\cos\theta_w}$$

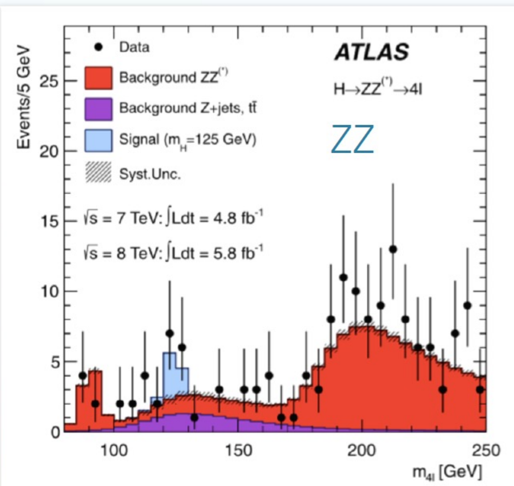
[LHC: the Higgs boson]

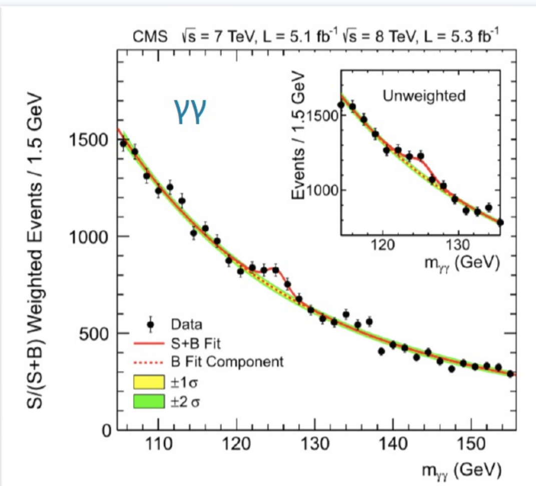


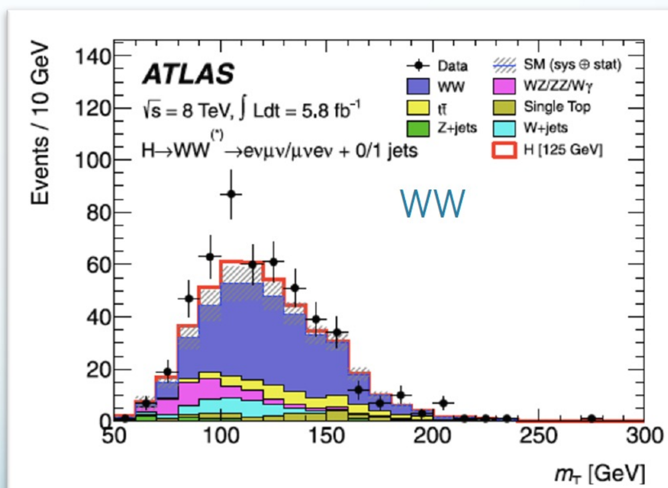
Production:
- gg fusion
- VBF
- bbH, ttH
- ZH,WH



2012: Higgs boson discovery:
ATLAS & CMS experiments, mH =125.1±0.2 GeV







Phys. Lett. B 716 (2012)