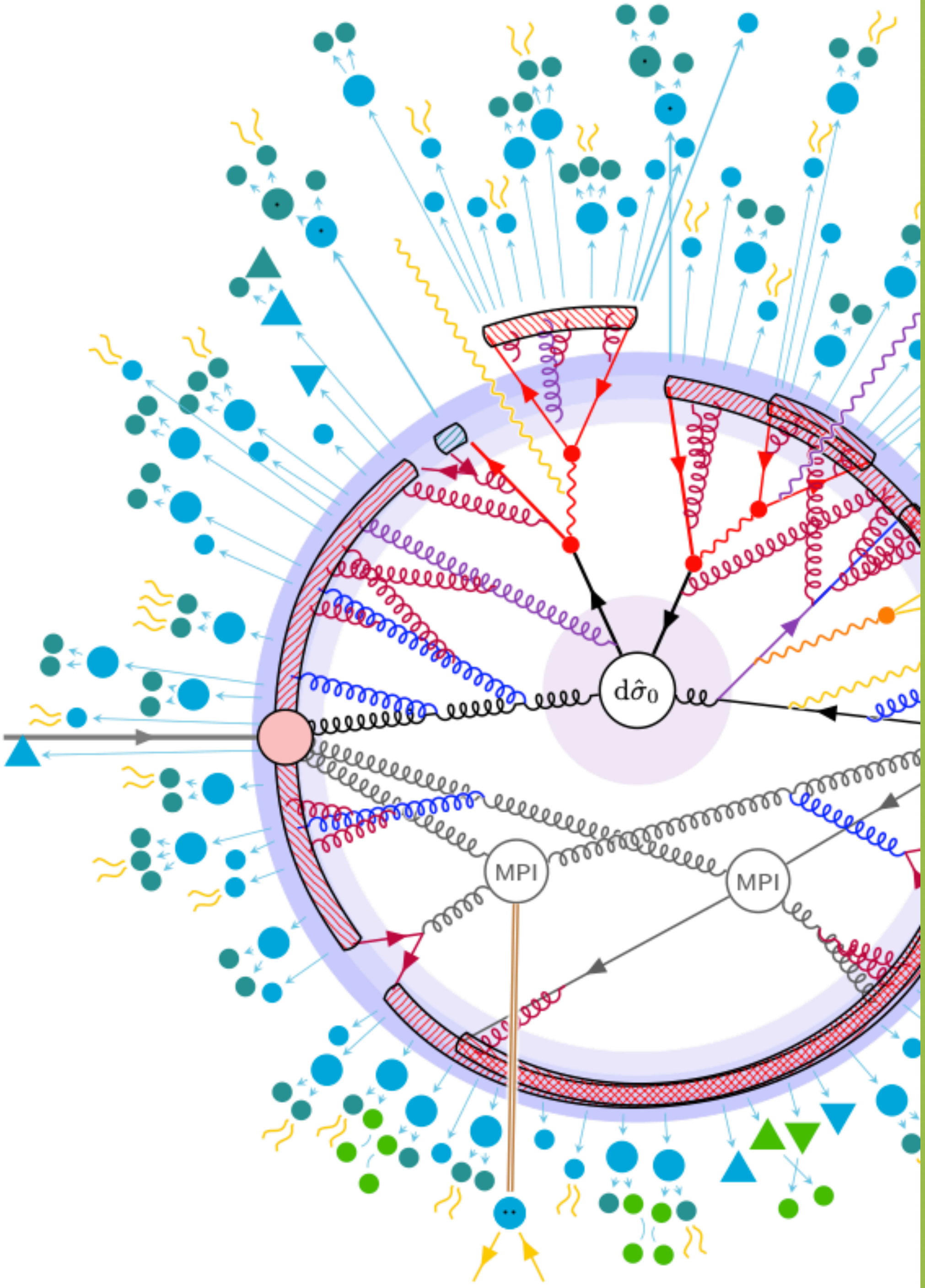


Introduction to QCD

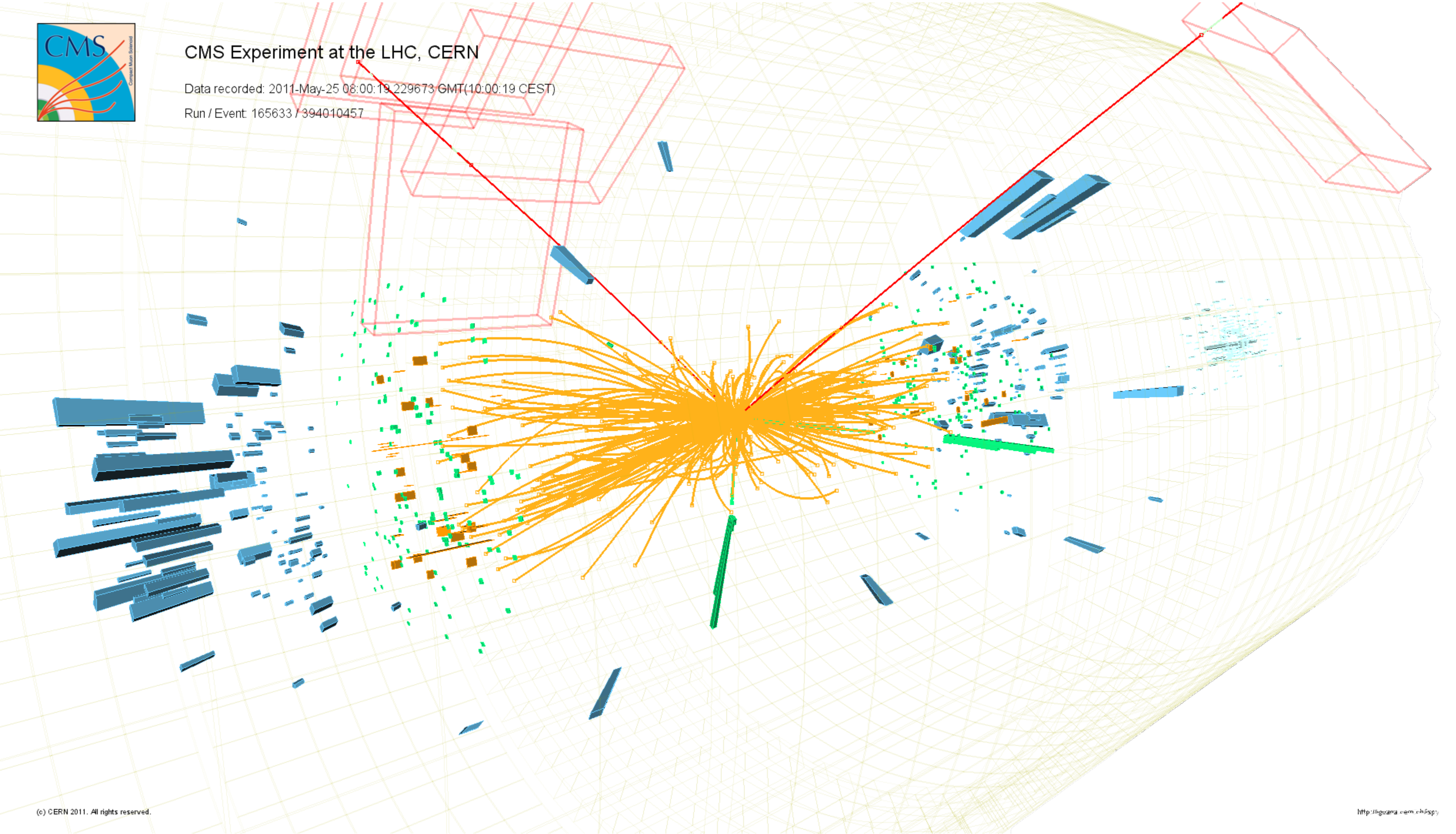
Lecture 1

HASCO Summer School 2024
Göttingen

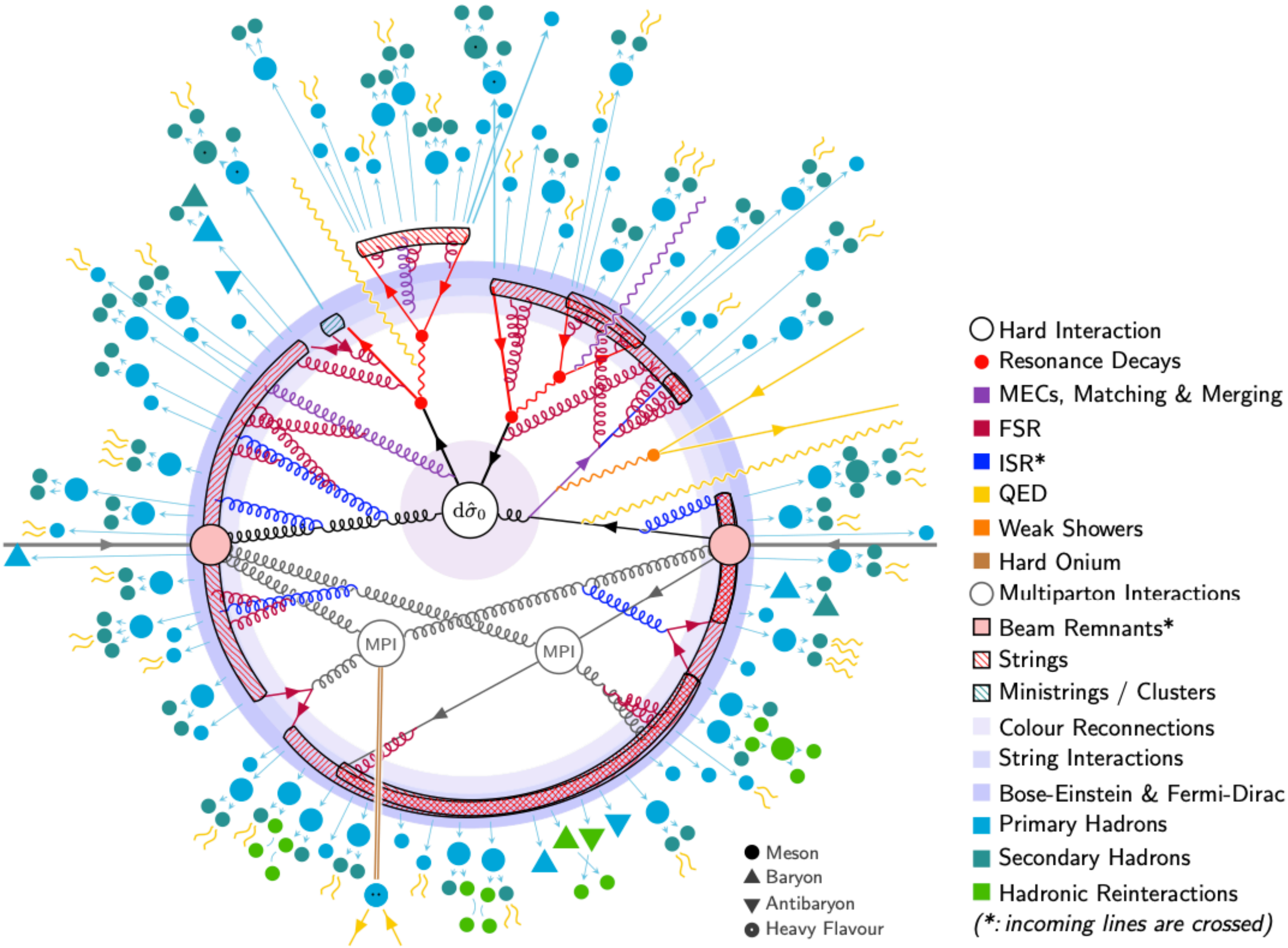
Christian T Preuss
(University of Wuppertal)

BERGISCHE
UNIVERSITÄT
WUPPERTAL

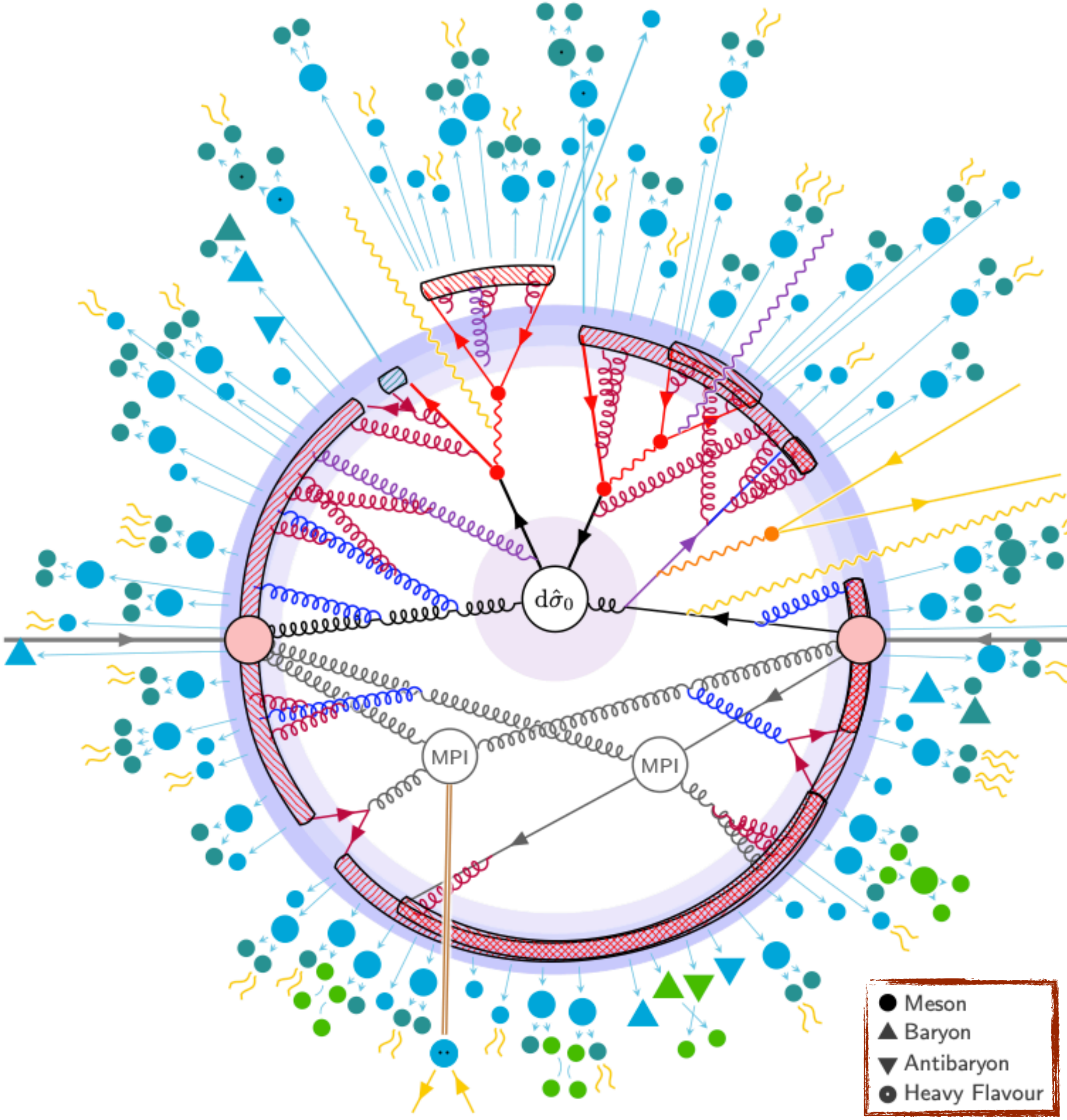
Hadron collisions in experiments



Hadron collisions in theory



Hadron collisions in theory



QCD

- ○ Hard Interaction
- ● Resonance Decays
- 🟪 MECs, Matching & Merging
- 🟥 FSR
- 🟦 ISR*
- 🟨 QED
- 🟧 Weak Showers
- 🟫 Hard Onium
- ○ Multiparton Interactions
- ☐ Beam Remnants*
- ▨ Strings
- ▨ Ministrings / Clusters
- ☐ Colour Reconnections
- ☐ String Interactions
- ☐ Bose-Einstein & Fermi-Dirac
- 🟦 Primary Hadrons
- 🟩 Secondary Hadrons
- 🟩 Hadronic Reinteractions

(: incoming lines are crossed)*

Overview

Lecture 1 (today):

- Hadrons, partons, and all that
- Colour charges
- QCD Lagrangian and Feynman rules
- QCD at test
- QCD-improved parton model

Lecture 2 (tomorrow):

- Fixed-order calculations and jets
- QCD radiation
- Running coupling and confinement
- QCD in event generators

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I'm standing on the shoulders of giants…

The content of these lectures is compiled from many different sources.

Previous HASCO lectures:

- Enrico Bothmann 2021 & 2022
- Daniel Reichelt 2023

"Standard" text books:

- Griffiths — Introduction to Elementary Particles
- Halzen, Martin — Quarks & Leptons
- Ellis, Stirling, Webber — QCD and Collider Physics
- Thomson — Modern Particle Physics

A tour of QCD
at hadron colliders.
concepts and phenomenology

Enrico Bothmann
Institut für Theoretische Physik,
Universität Göttingen

HASCO 2021

Quantum Chromodynamics
HASCO Summer School, Göttingen,
17 July - 25 July 2023

Daniel Reichelt

PHYSICS TEXTBOOK
David Griffiths
WILEY-VCH
Introduction to Elementary Particles
Second, Revised Edition

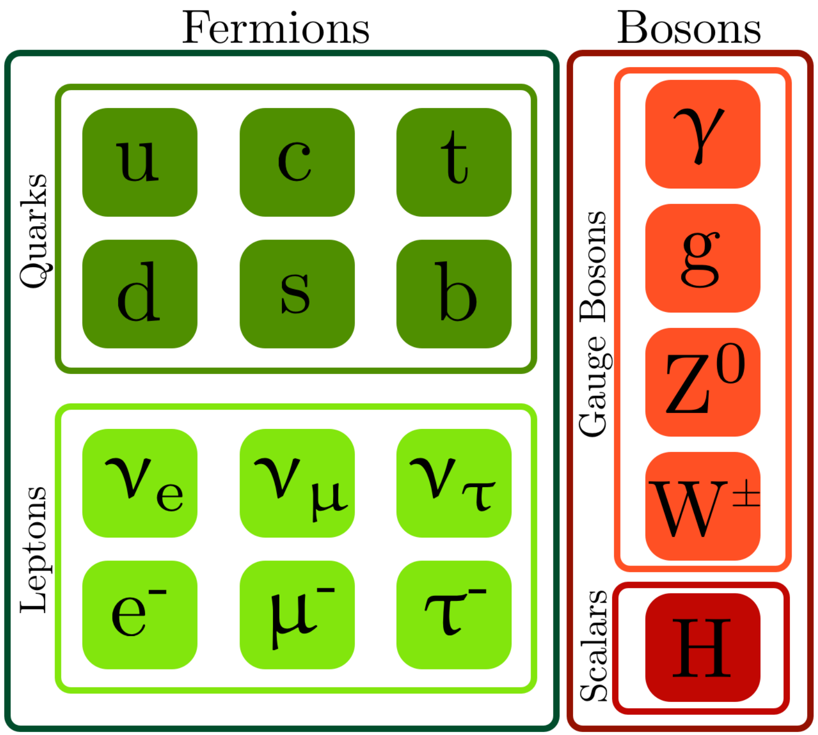
QCD and
Collider Physics
.K. ELLIS, W. J. S
AND B.R. WEB
CAMBRIDGE MONO
RTICLE PHYSICS, NU
AND COSMOLO
8

QUARKS & LEPTONS:
An Introductory Course in Modern Particle Physics
Francis Halzen
Alan D. Martin

MODERN PARTICLE PHYSICS
MARK THOMSON

Quantum Chromodynamics — overview

QCD describes **strong interaction** in the Standard Model of particle physics



Ingredients:

- quarks/antiquarks:
    - basic constituents of matter
    - 3 “families”, 6 “flavours” (up, down, charm, strange, top, bottom)
    - 3 “colours”
- gluons:
    - “mediators” of the strong force
    - 8 “colours”
- strong coupling $\alpha_s \approx 0.1$

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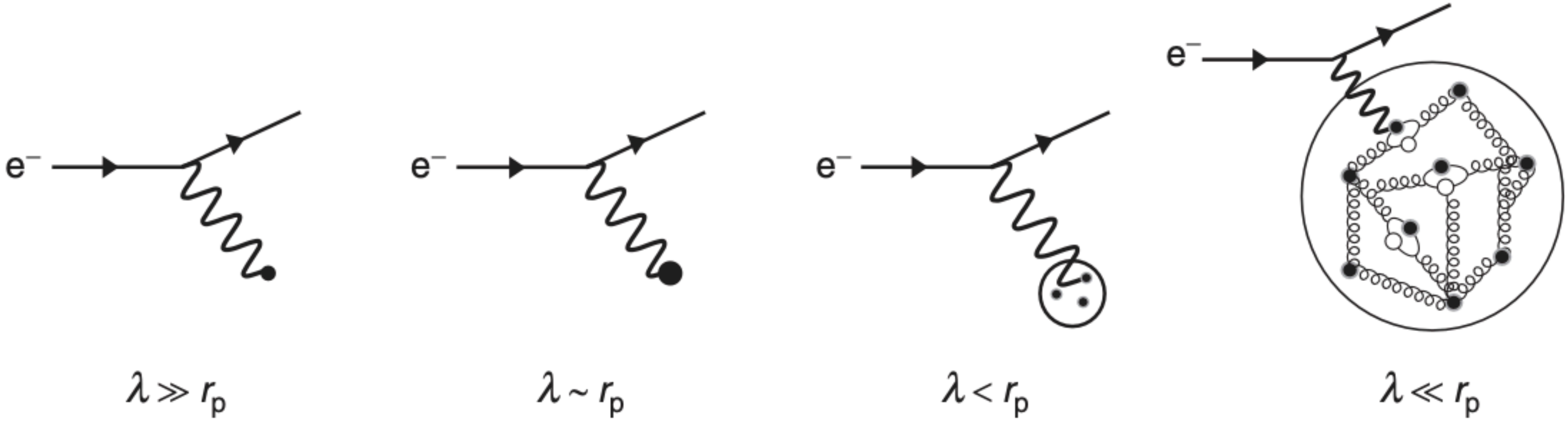
Proton structure

In hadron colliders we collide protons with (anti-)protons.

Protons (and hadrons in general) are not elementary particles, but composite.
→ need to understand the proton structure first!

We need a clean “projectile” to break up messy proton → **electron beams**



elastic

inelastic

deep inelastic

To resolve proton substructure we need electron wave lengths $\lambda \ll r_\mathrm{p}$
→ **deep inelastic scattering**

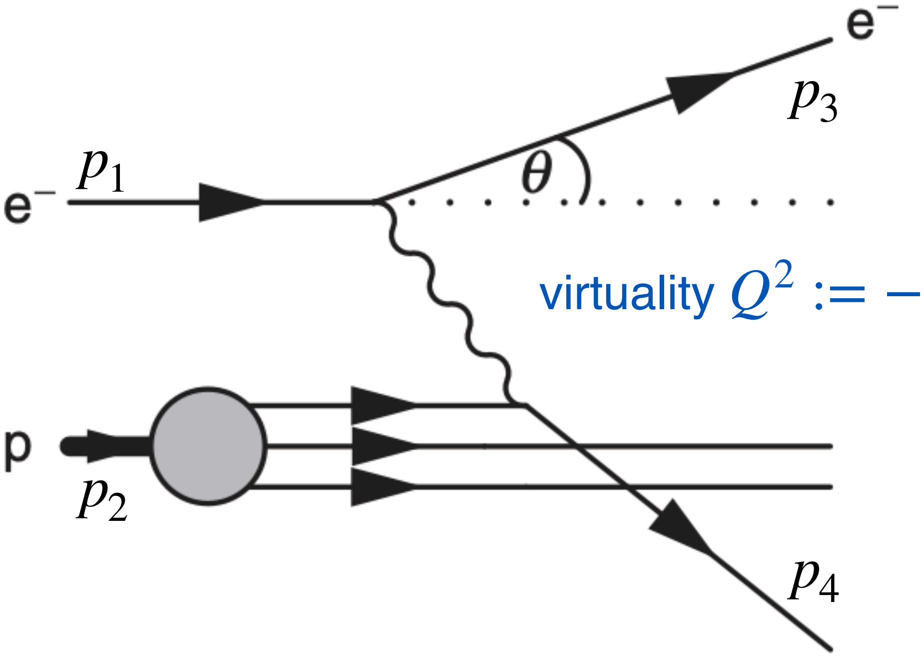
Deep inelastic scattering (DIS) I

Assumption: point-like electrons scatter from point-like quarks inside the proton

Kinematics parametrised as:

Bjørken-x: $x = \dfrac{Q^2}{2p_2q}$

inelasticity: $y = \dfrac{p_2q}{p_1p_2}$



virtuality $Q^2 := -q^2 > 0$

Ansatz* for double-differential cross section in terms of **structure functions**:

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left((1-y)\frac{F_2(x,Q^2)}{x} + \frac{y}{2}2F_1(x,Q^2)\right)$$

electric + magnetic

purely magnetic

*its derivation is beyond the scope of these lectures

Deep inelastic scattering (DIS) II

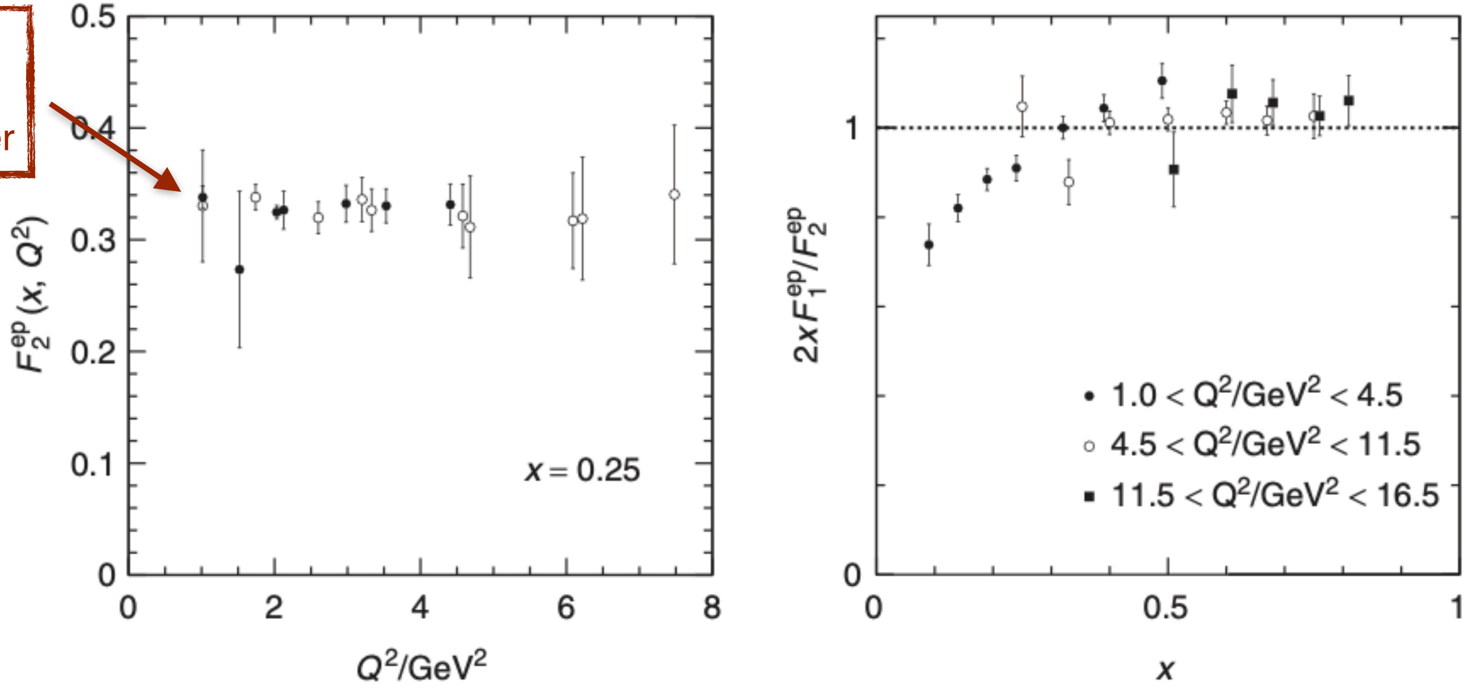
Two important observations:

Bjørken scaling: to first approximation, structure functions are independent of Q^2

$$F_1(x, Q^2) \to F_1(x) \text{ and } F_2(x, Q^2) \to F_2(x)$$

Callan-Gross relation: electrons scatter from point-like spin-$\frac{1}{2}$ constituents (quarks)

$$F_2(x) = 2xF_1(x)$$



will see how true this is later

Adapted from Thomson, data from [Friedman, Kendall ARNPS 22 (1972) 203] and [Bodek et al. PRD 20 (1979) 1471]

Flavour symmetry I

The strong interaction treats all quarks equally
→ assume **approximate** flavour symmetry between light quarks u, d, and s

Not exact, because $m_u \neq m_d \neq m_s$, but differences at most $\sim 100\ \mathrm{MeV}$, which is a lot smaller than typical hadron binding energies $\sim 1\ \mathrm{GeV}$.

Express symmetry by unitary rotation in flavour space

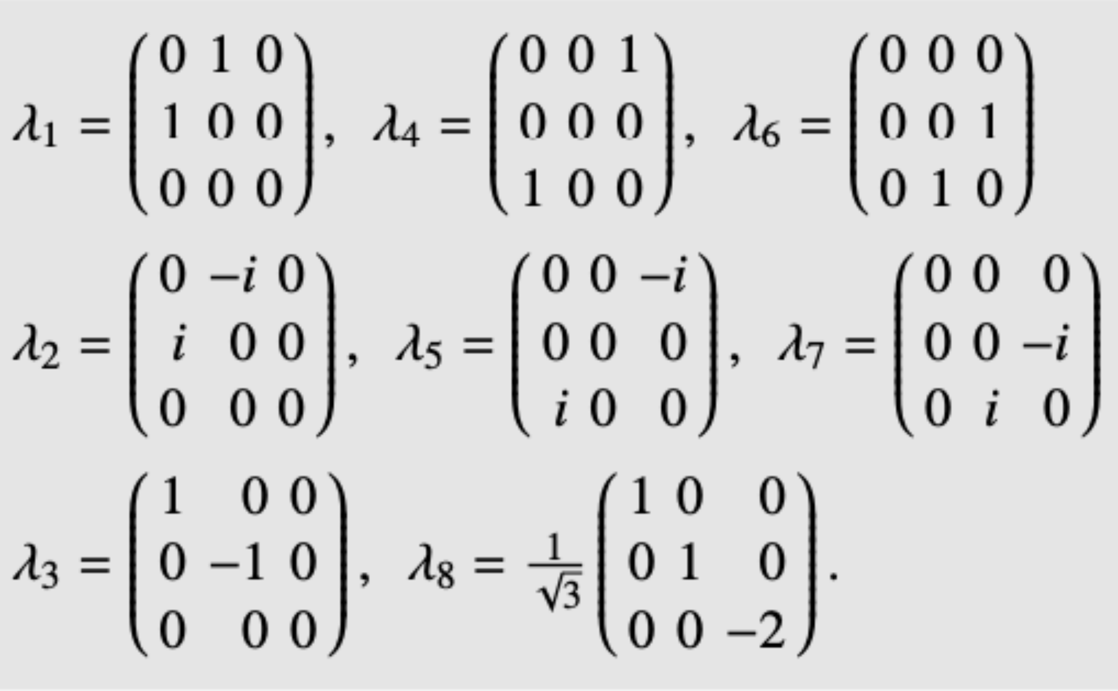
$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

sum convention!

- ignoring a trivial “rotation” $U = \mathbf{1}\mathrm{e}^{\mathrm{i}\phi}$, the matrices U form the group $\mathrm{SU}(3)$
- each matrix can be written in terms of the eight **generators** T^a as $U = \mathrm{e}^{\mathrm{i}\lambda^a T^a}$
- the generators T^a form a Lie algebra with commutator $[T^a, T^b] = \mathrm{i}f^{abc}T^c$

Flavour symmetry II

The $\mathrm{SU}(3)$ generators can be written in terms of Gell-Mann matrices as $T^a = \dfrac{1}{2}\lambda^a$



$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Only two of these commute ⇒ two observable quantities ("quantum numbers")

→ third component of isospin: eigenstates of $T_3 = \dfrac{1}{2}\lambda_3$

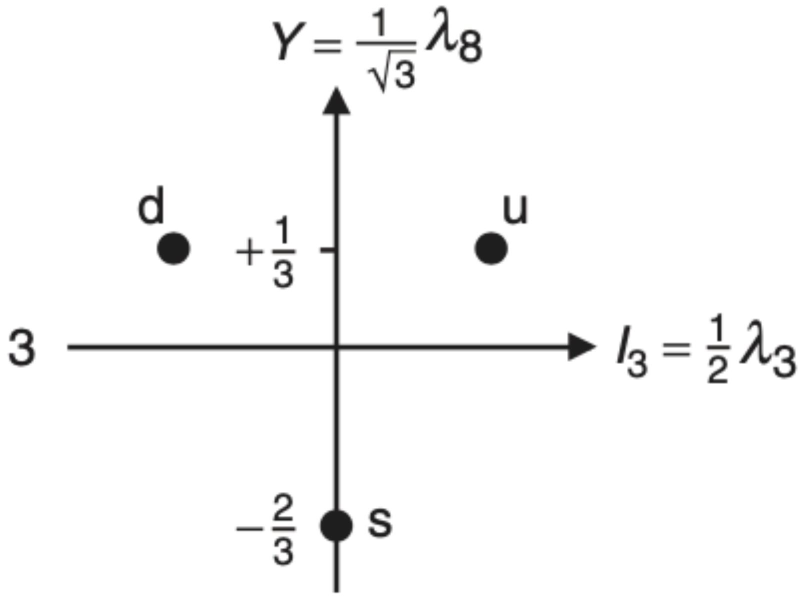
→ flavour hypercharge: eigenstates of $Y = \dfrac{1}{\sqrt{3}}\lambda_8$

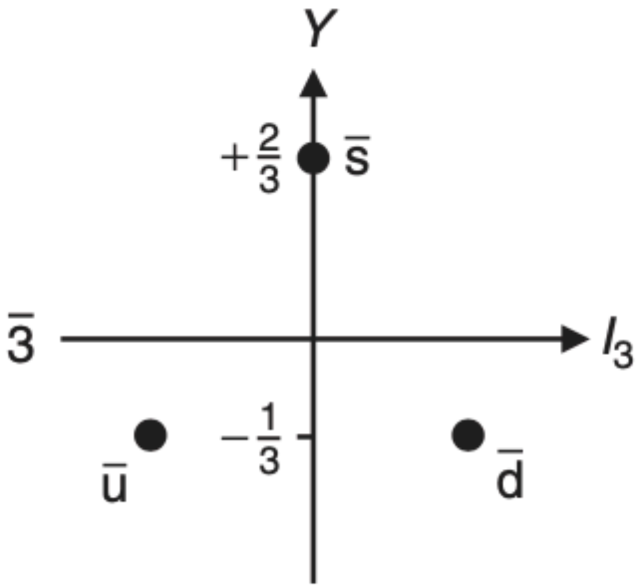
Flavour symmetry III

The light quarks are identified with the eigenstates of T_3 and Y

$$u \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, d \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, s \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

→ light quarks form a flavour triplet, antiquarks a flavour anti-triplet:

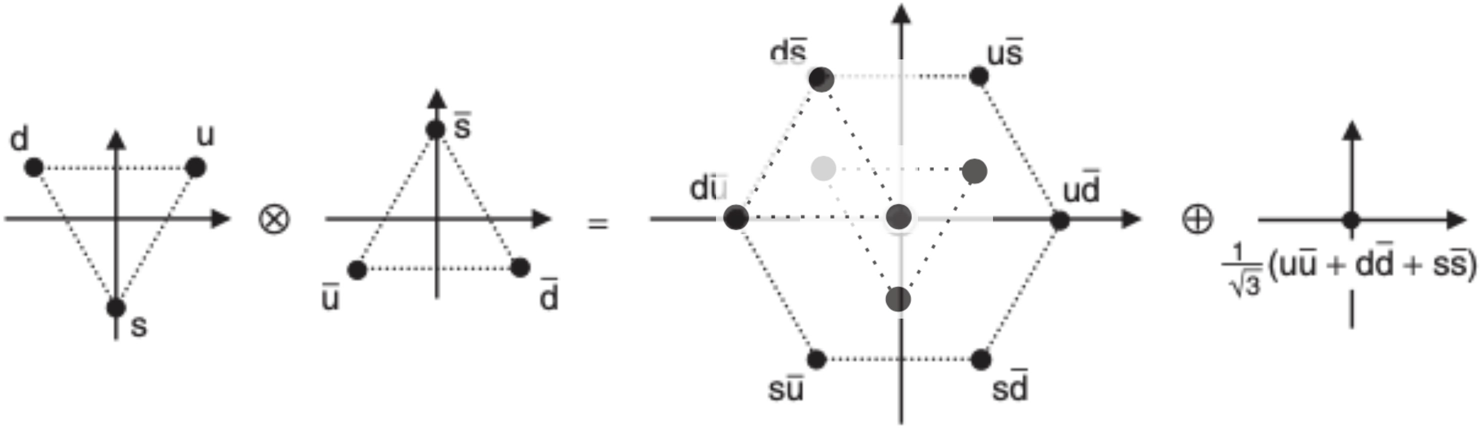


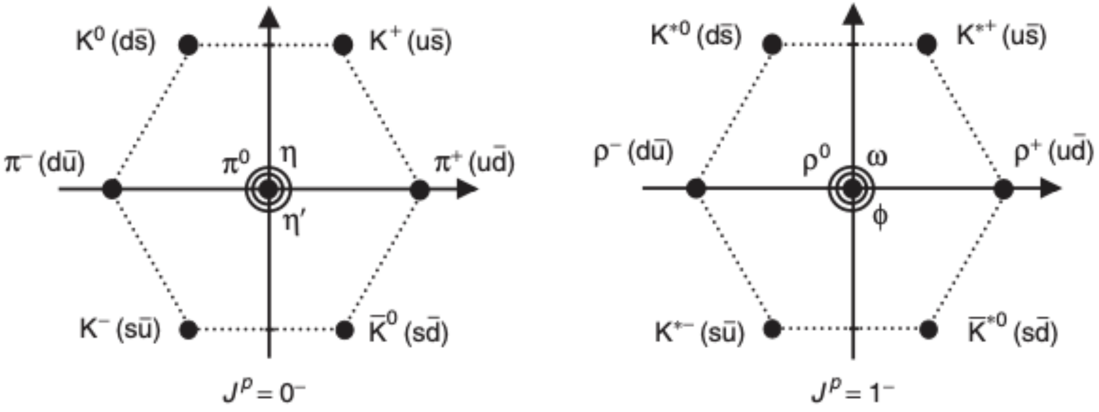


Light mesons

Mesons: bound $q\bar{q}$ states

Structure of light mesons described by Clebsch-Gordan decomposition of direct product of a flavour triplet and a flavour anti-triplet:





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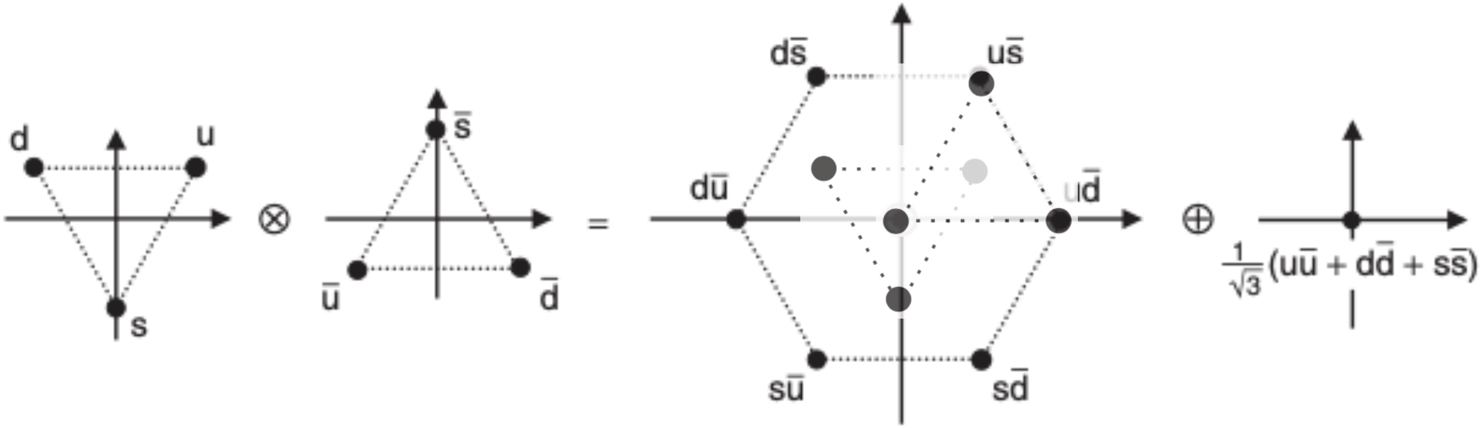

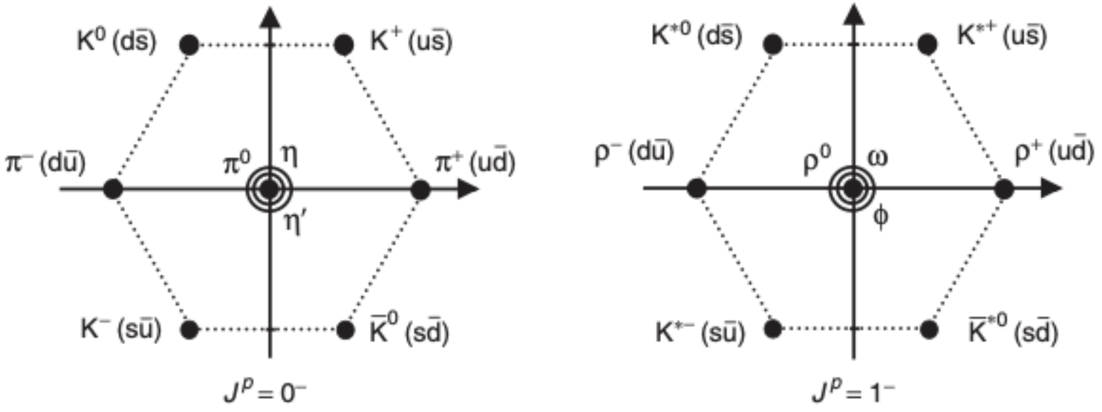

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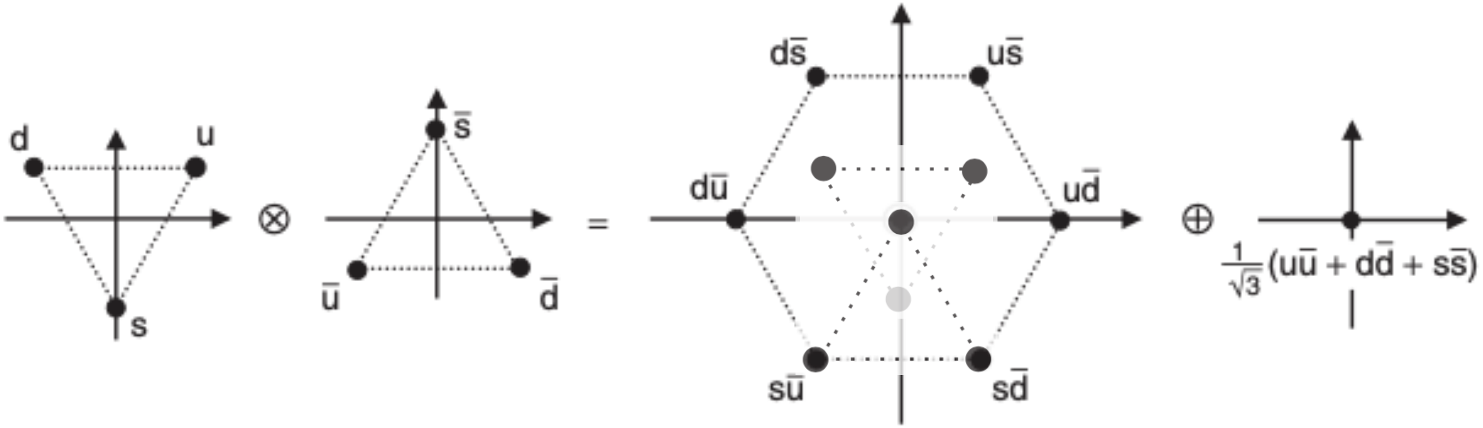

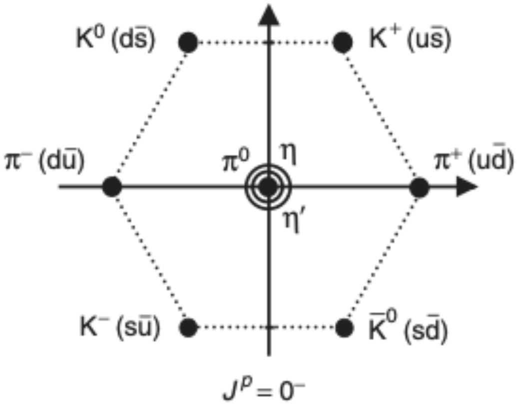

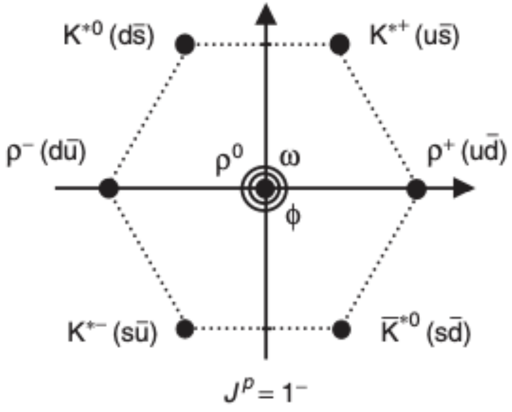

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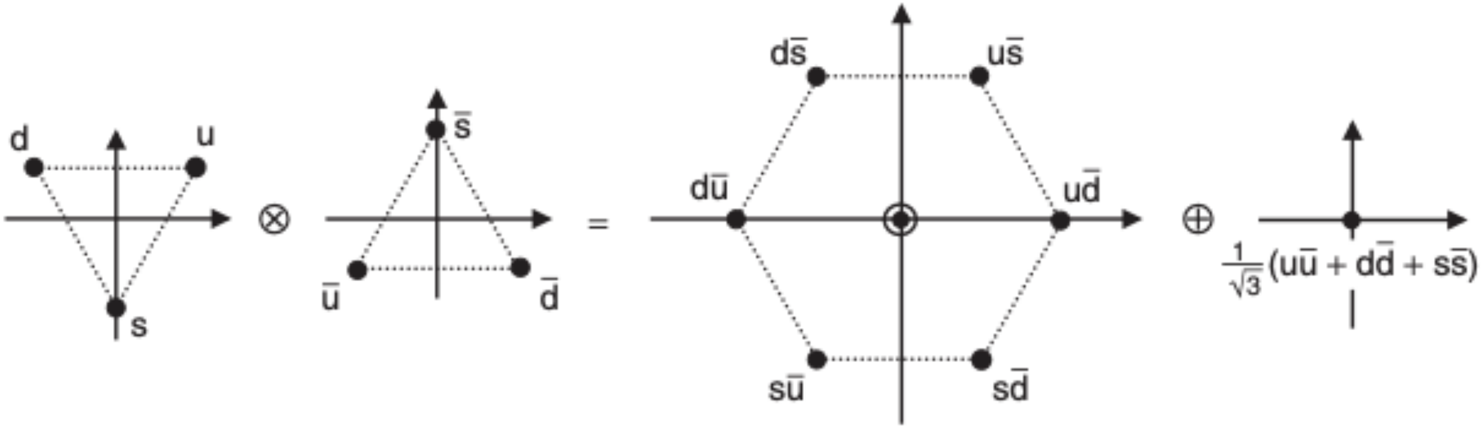
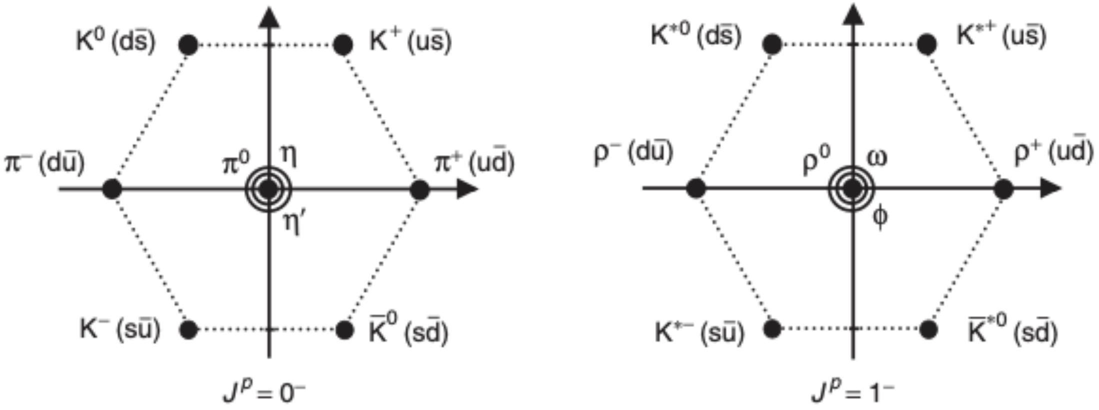
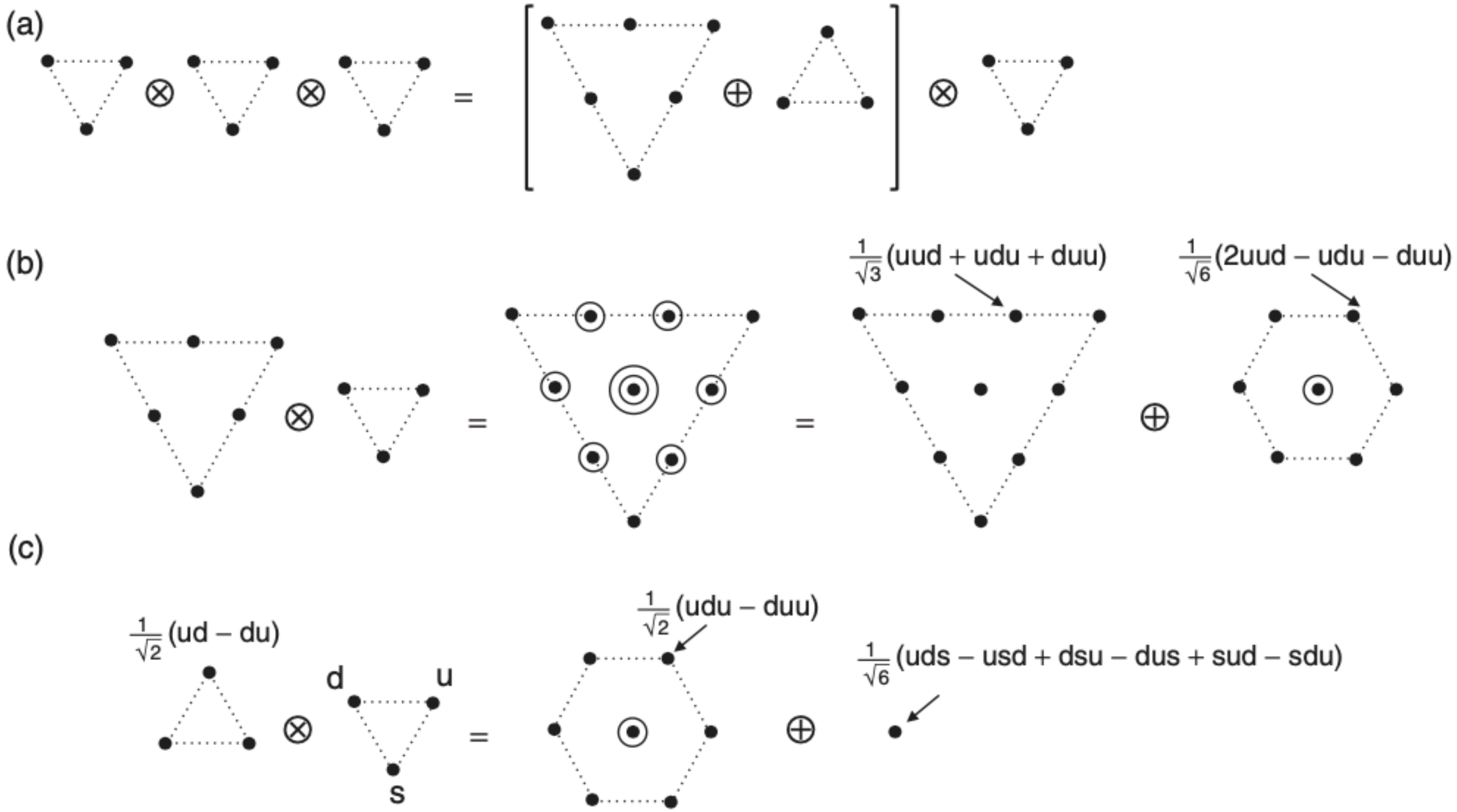




Light baryons

Baryons: bound qqq (or $\bar{q}\bar{q}\bar{q}$) states

Light baryon structure arises from Clebsch-Gordan decomposition of direct product of three flavour triplets:



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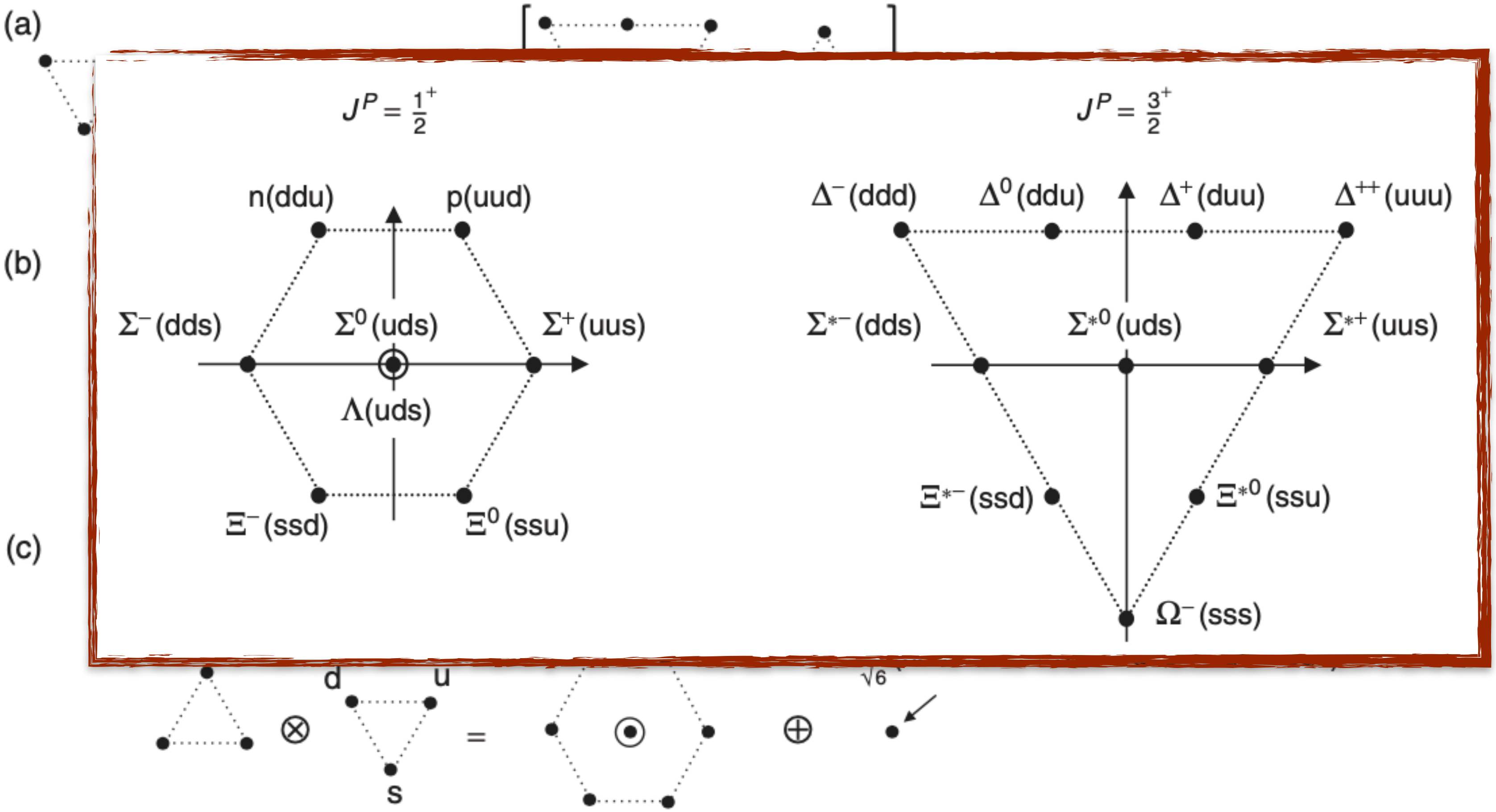

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Colour

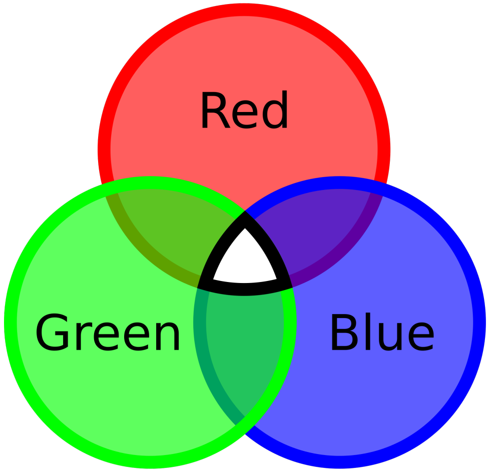
Problem: we observe hadrons in states such as $\left|\Delta^{++}\right\rangle \equiv \left|u_\uparrow u_\uparrow u_\uparrow\right\rangle$

- this is a completely symmetric state
- does this violate Fermi-Dirac statistics?

Solution: with an additional quantum number ("**colour**") with three states, this can be anti-symmetrised as

$$\left|\Delta^{++}\right\rangle = \varepsilon_{ijk}\left|u^i_\uparrow u^j_\uparrow u^k_\uparrow\right\rangle$$

All observed particles are **colour-neutral**, i.e., this quantum number is not observed directly!
→ confinement



Colour gauge group I

The three colour charges of quarks can be represented as

$$r \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, g \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, b \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

colour

and each quark wave function carries an additional colour index ψ_q^i

flavour

Just as in QED, we impose invariance under a **local** gauge transformation

$$\psi_q^i(x) \rightarrow U_j^i \psi_q^j(x) = \left(\mathrm{e}^{\mathrm{i}\lambda^a(x)T^a}\right)_j^i \psi_q^j$$

- ignoring a trivial “rotation” $U = \mathbf{1}\mathrm{e}^{\mathrm{i}\phi}$, the matrices U form the group $\mathrm{SU}(3)$
- each matrix can be written in terms of the eight **generators** T^a as $U = \mathrm{e}^{\mathrm{i}\lambda^a(x)T^a}$
- the generators T^a form a Lie algebra with commutator $[T^a, T^b] = \mathrm{i}f^{abc}T^c$

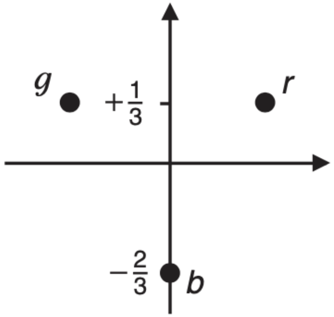
We have seen this before!

But: despite the same structure, flavour $\mathrm{SU}(3)$ and colour $\mathrm{SU}(3)$ describe very different concepts!

Colour gauge group II

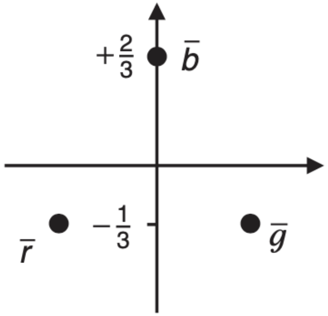
Quarks transform according to the fundamental representation of $\mathrm{SU}(3)$

$$(T^a)^i_{\ j} = \frac{1}{2}(\lambda^a)^i_{\ j} \quad \text{with indices } i, j \in \{1{,}2{,}3\}$$



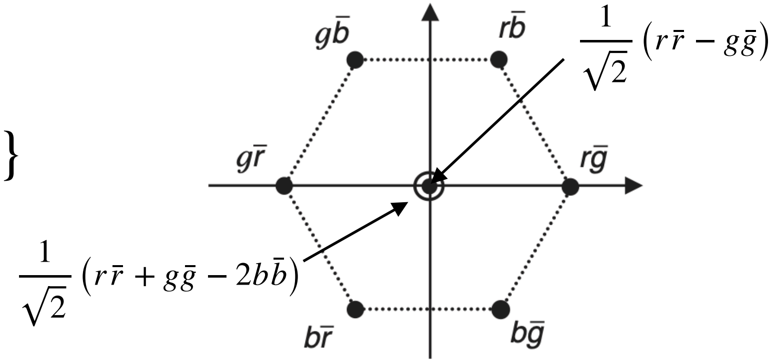
Antiquarks transform according to the anti-fundamental representation of $\mathrm{SU}(3)$

$$(T^a)^{\ j}_{i} = -\frac{1}{2}(\lambda^{a\,*})^{\ j}_{i} \quad \text{with indices } i, j \in \{1{,}2{,}3\}$$



Gluons transform according to the adjoint representation of $\mathrm{SU}(3)$

$$(T^a_{\text{adj}})^{bc} = -\,\mathrm{i}\,f^{abc} \quad \text{with indices } b, c \in \{1,\ldots,8\}$$



Here, λ^a are the 3×3 Gell-Mann matrices

Colour algebra I

Choose normalisation of generators as $\mathrm{Tr}\left(T^aT^b\right) = T_\mathrm{R}\delta^{ab}$ with $T_\mathrm{R} = \dfrac{1}{2}$

- **Casimir invariants**

Fundamental Casimir: $(T^a)^i_j(T^a)^j_k = C_\mathrm{F}\delta^i_k$ with $C_\mathrm{F} = T_\mathrm{R}\dfrac{N_\mathrm{C}^2 - 1}{N_\mathrm{C}}$

Adjoint Casimir: $f^{acd}f^{bcd} = C_\mathrm{A}\delta^{ab}$ with $C_\mathrm{A} = N_\mathrm{C}$

- **Fierz identity**:

$$(T^a)^i_j(T^a)^k_l = T_\mathrm{R}\left(\delta^i_l\delta^k_j - \frac{1}{N_\mathrm{C}}\delta^i_j\delta^k_l\right)$$

Example: calculate fundamental Casimir using Fierz identity

$$(T^a)^i_j(T^a)^j_k = T_\mathrm{R}\left(\delta^i_k\delta^j_j - \frac{1}{N_\mathrm{C}}\delta^i_j\delta^j_k\right) = T_\mathrm{R}\left(N_\mathrm{C} - \frac{1}{N_\mathrm{C}}\right)\delta^i_k$$

$$= T_\mathrm{R}\frac{N_\mathrm{C}^2 - 1}{N_\mathrm{C}}\delta^i_k = C_\mathrm{F}\delta^i_k$$

Colour algebra II

Task: express structure constant in terms of fundamental generators

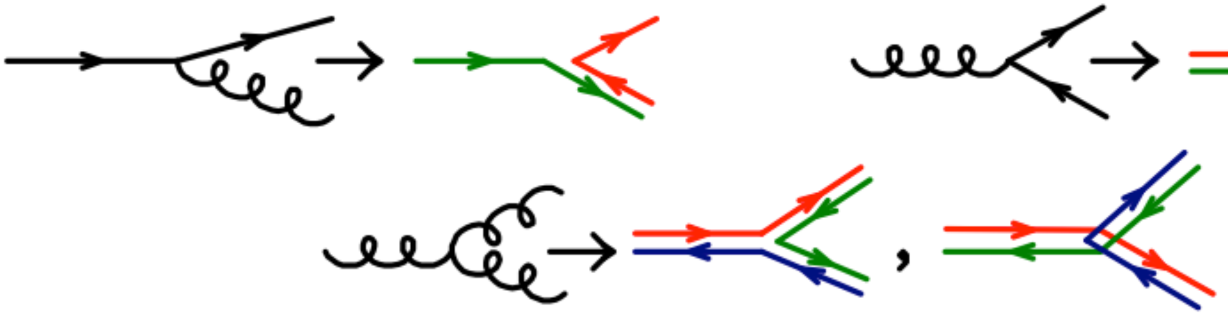
Colour algebra II

Task: express structure constant in terms of fundamental generators

Solution:

$$\begin{aligned}
 & [T^a, T^b] = \mathrm{i} f^{abc} T^c \\
\Rightarrow \quad & [T^a, T^b] T^d = \mathrm{i} f^{abc} T^c T^d \\
\Rightarrow \quad & \mathrm{Tr}\left([T^a, T^b] T^d\right) = \mathrm{i} f^{abc} \mathrm{Tr}\left(T^c T^d\right) = \mathrm{i} T_\mathrm{R} f^{abc} \delta^{cd} = \mathrm{i} T_\mathrm{R} f^{abd} \\
\Leftrightarrow \quad & f^{abd} = -\frac{\mathrm{i}}{T_\mathrm{R}} \mathrm{Tr}\left([T^a, T^b] T^d\right)
\end{aligned}$$

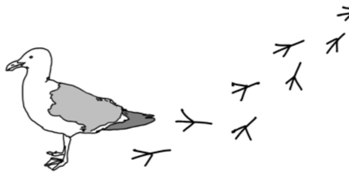
+ Fierz identity: all colour factors can be expressed in terms of (anti-)colour lines δ^i_j



[PDG]

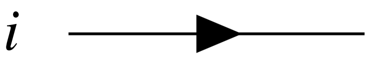
Birdtracks I

Colour factors can most easily be calculated using birdtrack diagrams

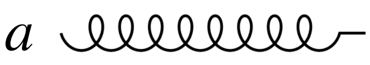


[Keppeler 1707.07280]

- Kronecker deltas are represented by colour lines
  (+ implicit summation over colour indices)

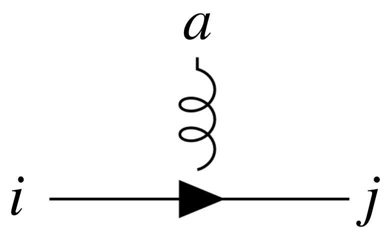


$= \delta^i_j$

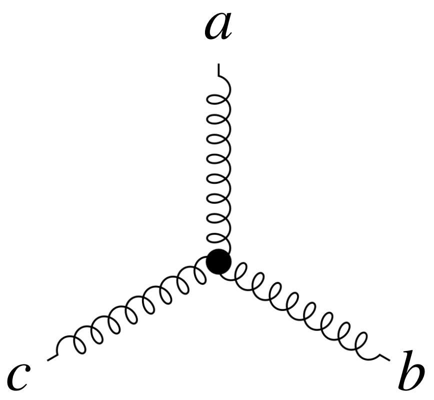


$= \delta^{ab}$

- generators and structure constants are represented by vertices

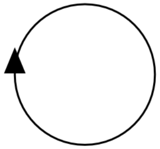


$= (T^a)^i_j$

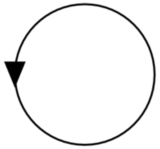


$= \mathrm{i} f^{abc}$

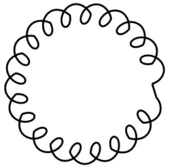
- there are N_C quark colours and $N_\mathrm{C}^2 - 1$ gluon colours



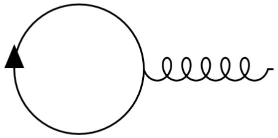
$=$



$= N_\mathrm{C}$



$= N_\mathrm{C}^2 - 1$

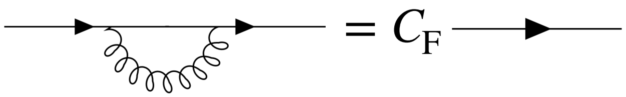
- generators are traceless  $= 0$

Birdtracks II

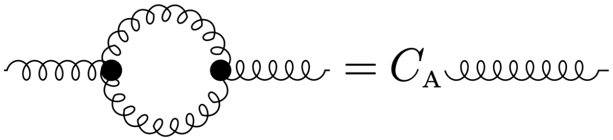
Rewrite $\mathrm{SU}(N_\mathrm{C})$ identities as birdtracks

- **Casimir invariants**

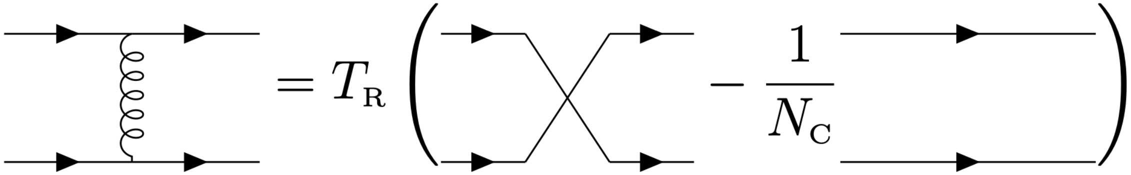
Fundamental Casimir:



Adjoint Casimir:

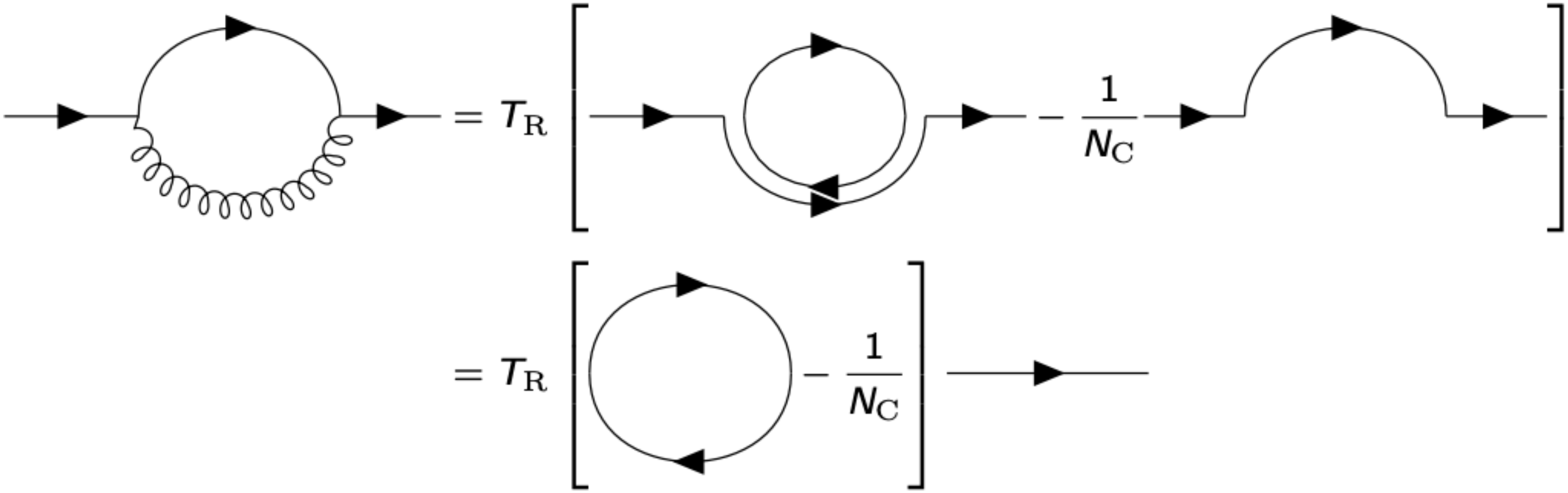


- **Fierz identity**:



Birdtracks III

Example: calculate fundamental Casimir using Fierz identity with birdtracks



$$= T_R \left[N_C - \frac{1}{N_C} \right] \longrightarrow$$

$$= T_R \frac{N_C^2 - 1}{N_C} \longrightarrow$$

$$= C_F \longrightarrow$$

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The quark Lagrangian

Free quark Lagrangian:

$$\mathscr{L}_{\text{quark}} = \overline{\psi}_{q\,i} \mathrm{i}\gamma^\mu \partial_\mu \psi_q^i - m_q \overline{\psi}_{q\,i} \psi_q^i$$

- yields the free Dirac equation $(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi_q^i = 0$
- not invariant under $\mathrm{SU}(3)$ transformations $\psi_q \rightarrow \mathrm{e}^{\mathrm{i}g_\mathrm{s}\lambda^a(x)T^a}\psi_q$

Introduce covariant derivative to restore local gauge invariance:

$$(D_\mu)^i_j := \delta^i_j \partial_\mu - \mathrm{i}g_\mathrm{s}(T^a)^i_j A^a_\mu$$

This introduces **interactions** between the quark spinors ψ_q^i and the gluon field A^a_μ:

$$(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi_q^i = -\, g_\mathrm{s}\gamma^\mu T^a A^a_\mu \psi_q^i$$

The gluon Lagrangian

The field-strength tensor is defined as the commutator of the covariant derivative:

$$F^c_{\mu\nu} = \frac{1}{\mathrm{i}g_\mathrm{s}}\left[D_\mu, D_\nu\right]$$

$$= \partial_\mu A^c_\nu - \partial_\nu A^c_\mu - g_\mathrm{s} f^{abc} A^a_\mu A^b_\nu$$

It obeys the Yang-Mills equation of motion

$$\partial_\mu F^{a\,\mu\nu} + g_\mathrm{s} f^{abc} A^b_\mu F^{c\,\mu\nu} = g_\mathrm{s} \overline{\Psi}_q \gamma^\nu T^a \psi_q$$

compare to Maxwell eqs. in Lorenz gauge: $\partial_\mu \partial^\mu A^\nu = j^\nu$

The corresponding Lagrangian is given by

$$\mathscr{L}_\mathrm{gluon} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

which is invariant under $\mathrm{SU}(3)$ transformations due to gauge freedom of A^a_μ

QCD as a non-abelian gauge theory

Quark Lagrangian:

$$\mathscr{L}_{\text{quark}} = \overline{\psi}_{q\,i} \mathrm{i}\gamma^{\mu}(D_{\mu})^{i}_{j}\psi^{i}_{q} - m_{q}\overline{\psi}_{q\,i}\psi^{i}_{q}$$

Gluon Lagrangian:

$$\mathscr{L}_{\text{gluon}} = -\frac{1}{4}F^{a}_{\mu\nu}F^{a\,\mu\nu} \quad \text{with} \quad F^{c}_{\mu\nu} = \partial_{\mu}A^{c}_{\nu} - \partial^{\nu}A^{c}_{\mu} - g_{\mathrm{s}}f^{abc}A^{a}_{\mu}A^{b}_{\nu}$$

Final **QCD Lagrangian**:

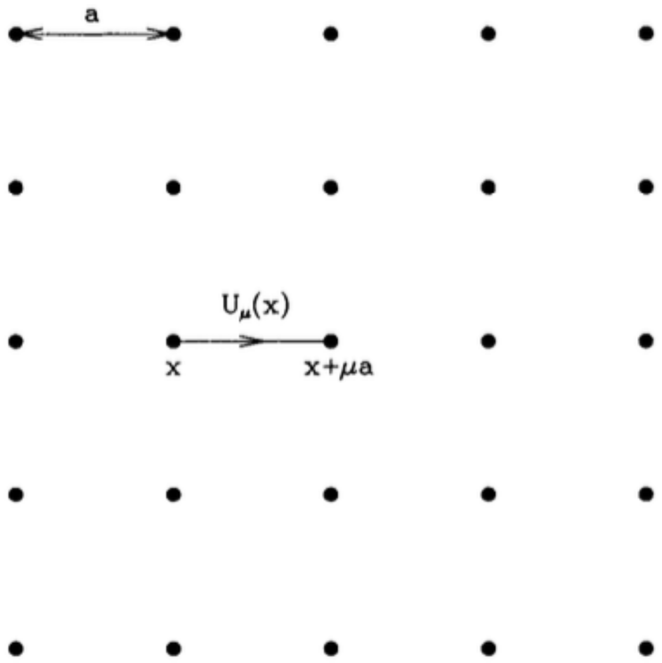
$$\boxed{\mathscr{L}_{\text{QCD}} = -\frac{1}{4}F^{a}_{\mu\nu}F^{a\,\mu\nu} + \overline{\psi}_{q\,i}\left(\mathrm{i}\gamma^{\mu}(D_{\mu})^{i}_{j} - \delta^{i}_{j}m_{q}\right)\psi^{j}_{q}}$$

Note: quantisation requires gauge-fixing terms and so-called ghost fields (ignored for now).

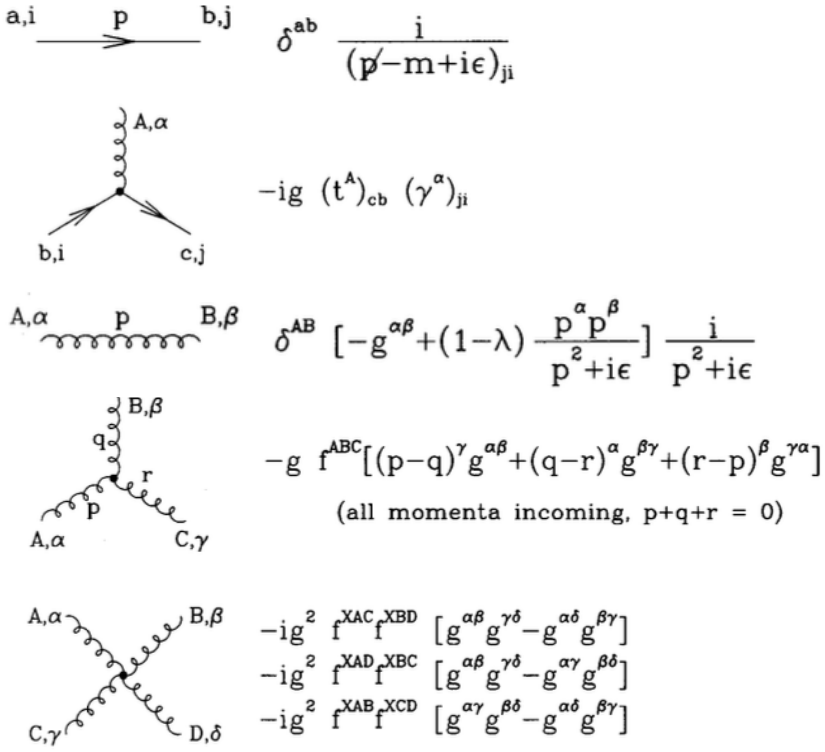
More than one way to skin a cat…

Two approaches to solve equations of motions as governed by QCD Lagrangian

Lattice QCD



Perturbative QCD



$$\delta^{ab}\,\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$$

$$-ig\ (t^A)_{cb}\ (\gamma^\alpha)_{ji}$$

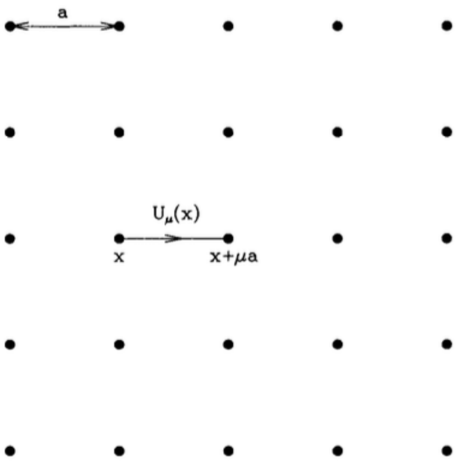
$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^\alpha p^\beta}{p^2+i\epsilon}\right]\frac{i}{p^2+i\epsilon}$$

$$-g\ f^{ABC}[(p-q)^\gamma g^{\alpha\beta}+(q-r)^\alpha g^{\beta\gamma}+(r-p)^\beta g^{\gamma\alpha}]$$

(all momenta incoming, $p+q+r=0$)

$$-ig^2\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^2\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^2\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$

Lattice QCD

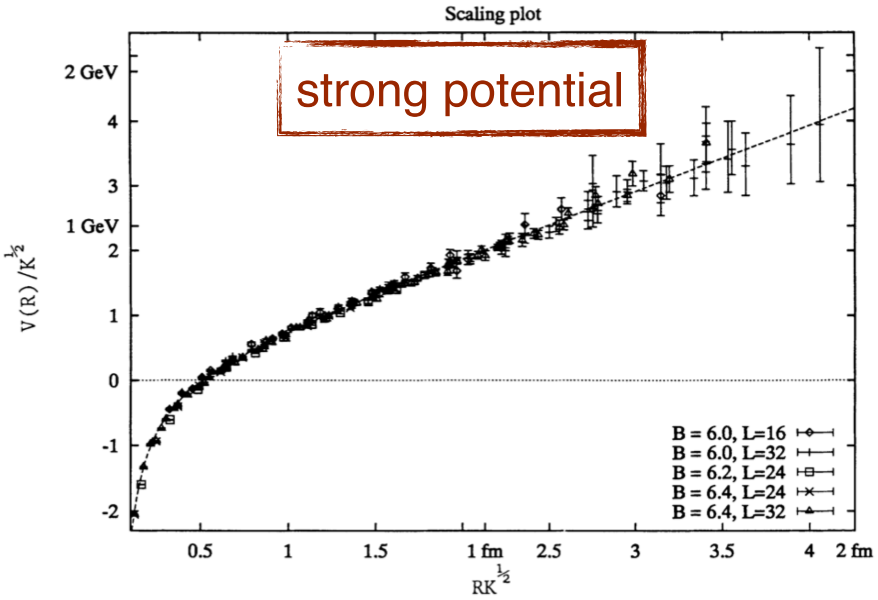


Idea: [Wilson PRD 10 (1974) 2445]

- quantise QCD on a discrete lattice in euclidean space time
- finite lattice spacing a acts as infrared regulator
- solve path integrals numerically

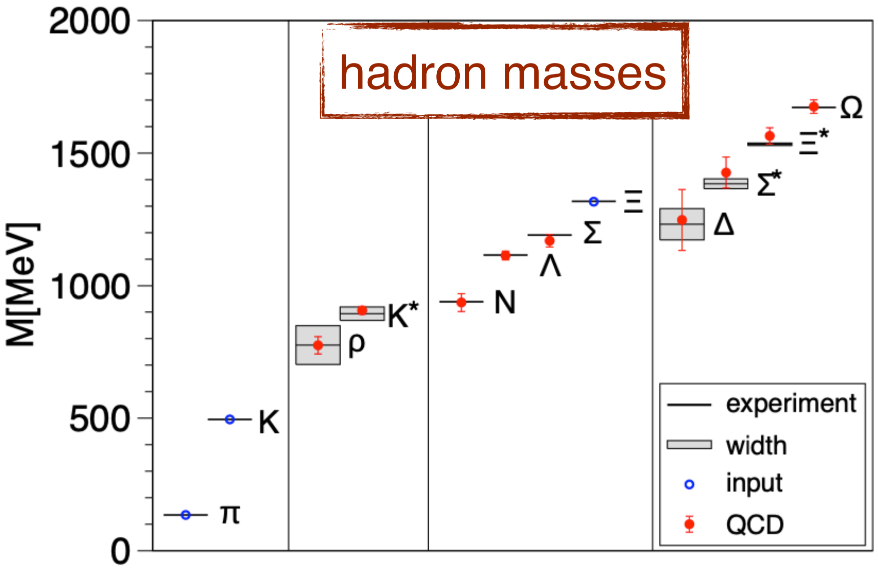
→ suitable to calculate non-perturbative hadron properties

→ not suitable for large-scale collider processes



strong potential

[Bali, Schilling PRD 46 (1992) 2636-2646]



hadron masses

[BMW collaboration 0906.3599]

Perturbative QCD

Idea:

at high scales $\alpha_s \approx 0.1 \Rightarrow$ series expansion in powers of the strong coupling α_s

$$d\sigma \sim C_0 + \alpha_s C_1 + \underbrace{\alpha_s^2 C_2}_{\text{small}} + \underbrace{\alpha_s^3 C_3}_{\text{smaller}} + \underbrace{\quad \ldots \quad}_{\text{negligible?}}$$

→ improve prediction by successively correcting leading-order approximation

(leading order, next-to-leading order, next-to-next-to-leading order, …)

Example:

$$\boxed{\frac{1}{7} = \frac{1}{10}\left(1 - \frac{3}{10}\right)^{-1} \approx \frac{1}{10}(\overset{\text{LO}}{1} + \overset{\text{NLO}}{0.3} + \overset{\text{NNLO}}{0.09} + \overset{\text{N3LO}}{0.027} + \ldots)}$$

Exact: 0.142857143

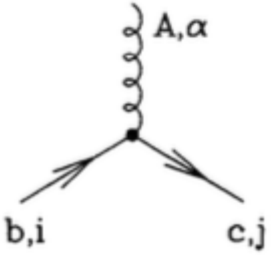
Sum:	0.1	0.13	0.139	0.1417
Error:	30%	9%	3%	1%

Need: set of universal rules to calculate cross sections order by order

Feynman rules of QCD — vertices

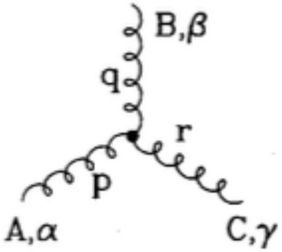
Three types of vertices in QCD

- quark-gluon vertex (∼ fermion-photon vertex in QED)



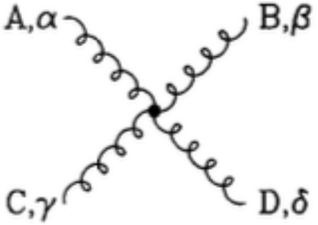
$$-ig\ (t^A)_{cb}\ (\gamma^\alpha)_{ji}$$

- pure gluon vertices (result of non-abelian structure of $\mathrm{SU}(3)$)



$$-g\ f^{ABC}[(p-q)^\gamma g^{\alpha\beta}+(q-r)^\alpha g^{\beta\gamma}+(r-p)^\beta g^{\gamma\alpha}]$$

(all momenta incoming, p+q+r = 0)

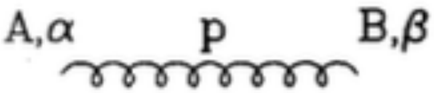


$$-ig^2\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^2\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^2\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$

Feynman rules of QCD — propagators

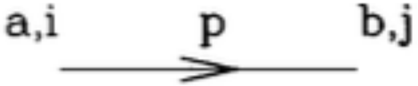
Propagators $\hat{=}$ Green's functions of inhomogeneous equations of motion

- gluon propagator (vector propagator)



$$= \delta^{AB} \frac{-g^{\alpha\beta} + (1-\lambda)\frac{p^\alpha p^\beta}{p^2 + \mathrm{i}\varepsilon}}{p^2 + \mathrm{i}\varepsilon} \overset{\lambda=1}{=} \delta^{AB} \frac{-g^{\alpha\beta}}{p^2 + \mathrm{i}\varepsilon}$$

- quark propagator (spinor propagator)



$$= \delta^a_b \frac{\mathrm{i}\not{p} + m}{p^2 - m^2 + \mathrm{i}\varepsilon}$$

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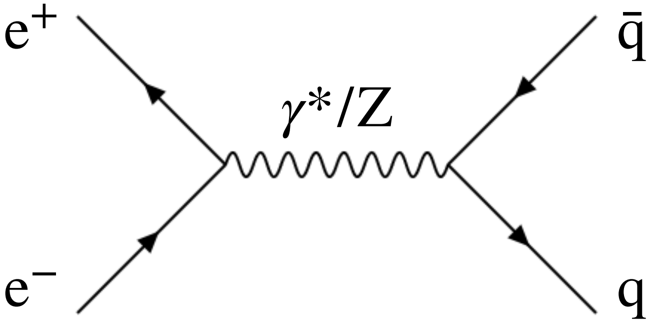
Quark production at lepton colliders

Simplest process involving quarks: $\mathrm{e}^+\mathrm{e}^- \to \gamma^*/Z \to \mathrm{q}\bar{\mathrm{q}}$

(does not actually involve any QCD!)

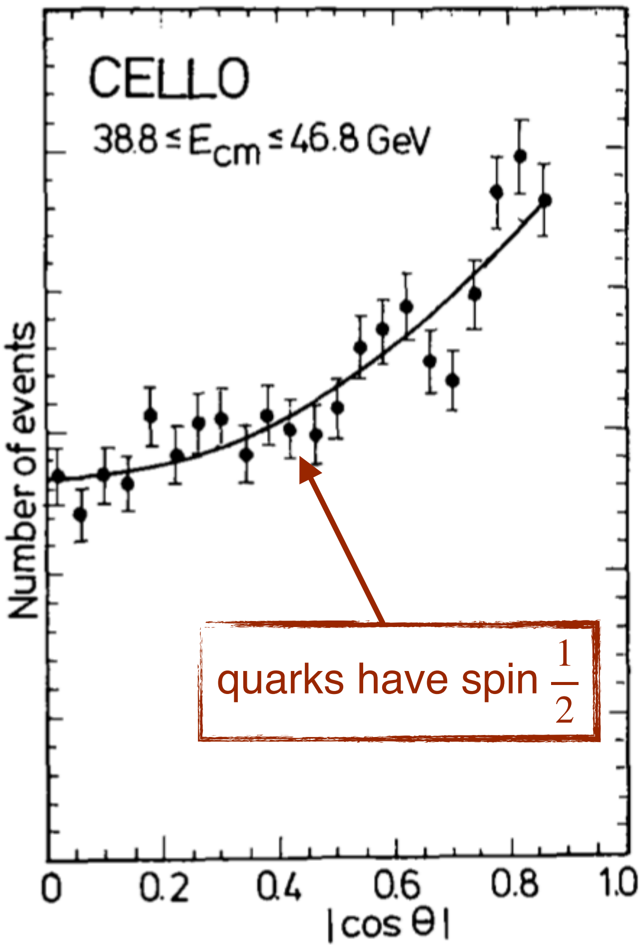


Differential cross section:

$$\frac{d\sigma_{\mathrm{e}^+\mathrm{e}^-\to \mathrm{q}\bar{\mathrm{q}}}}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)\, N_\mathrm{C} \sum_q Q_q^2$$

Inclusive cross section via integration over solid angle $\int d\Omega = \int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi$:

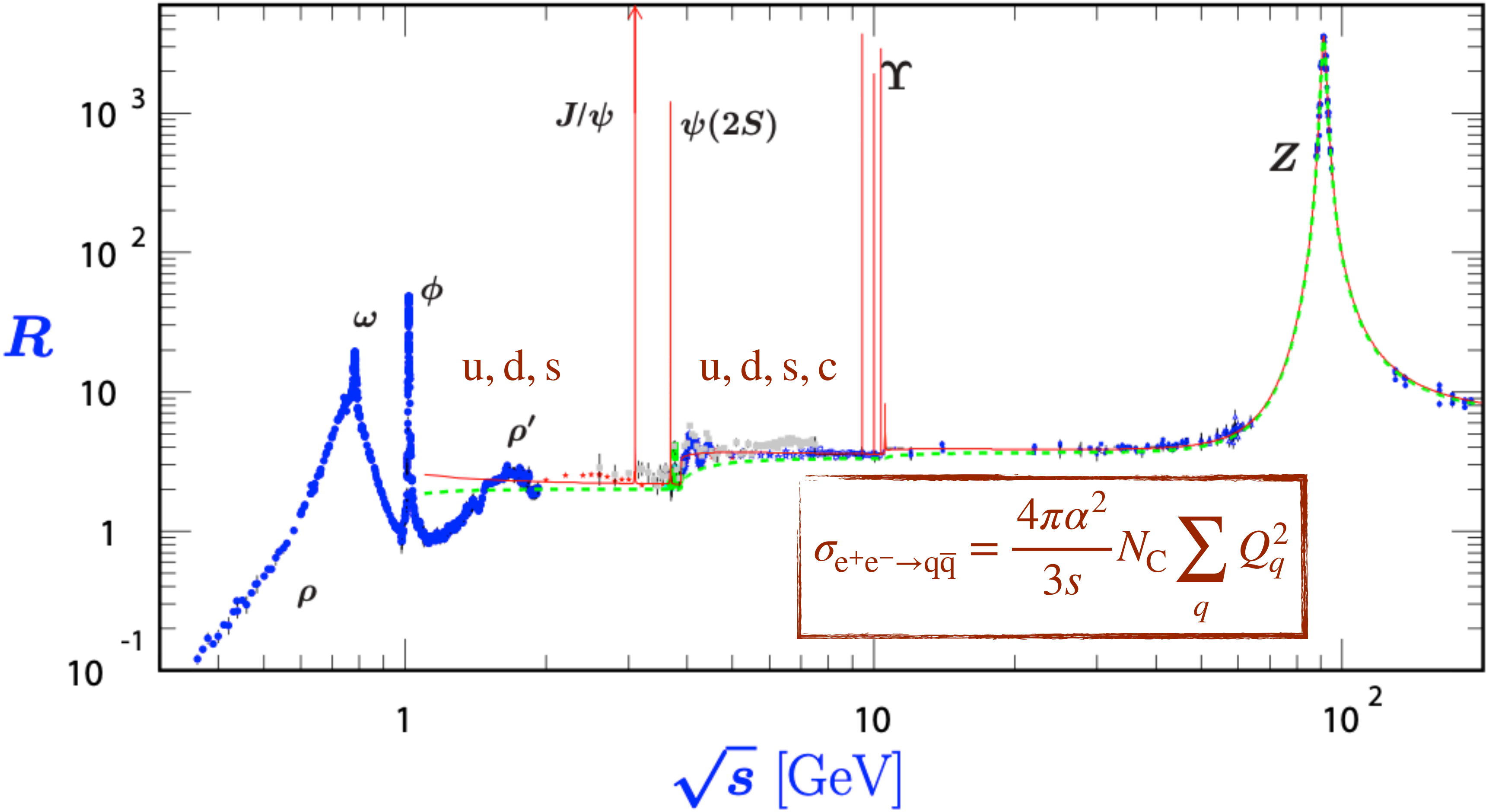
$$\sigma_{\mathrm{e}^+\mathrm{e}^-\to \mathrm{q}\bar{\mathrm{q}}} = \frac{4\pi\alpha^2}{3s} N_\mathrm{C} \sum_q Q_q^2$$



quarks have spin $\frac{1}{2}$

[CELLO collaboration PLB183 (1987) 400]

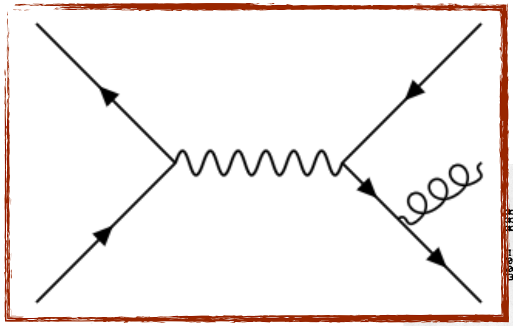
The R-ratio

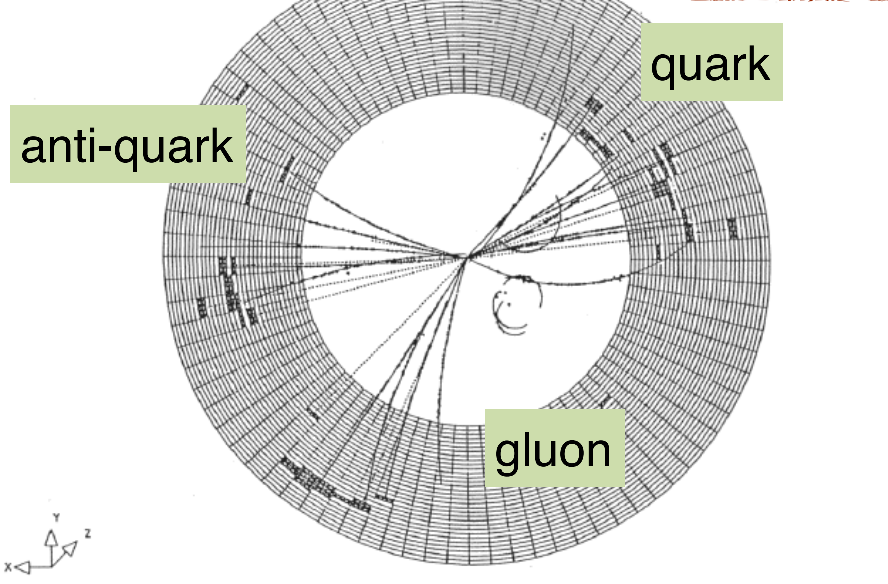


$$\sigma_{\mathrm{e^+e^-\to q\bar{q}}} = \frac{4\pi\alpha^2}{3s} N_{\mathrm{C}} \sum_q Q_q^2$$

[Particle Data Group]

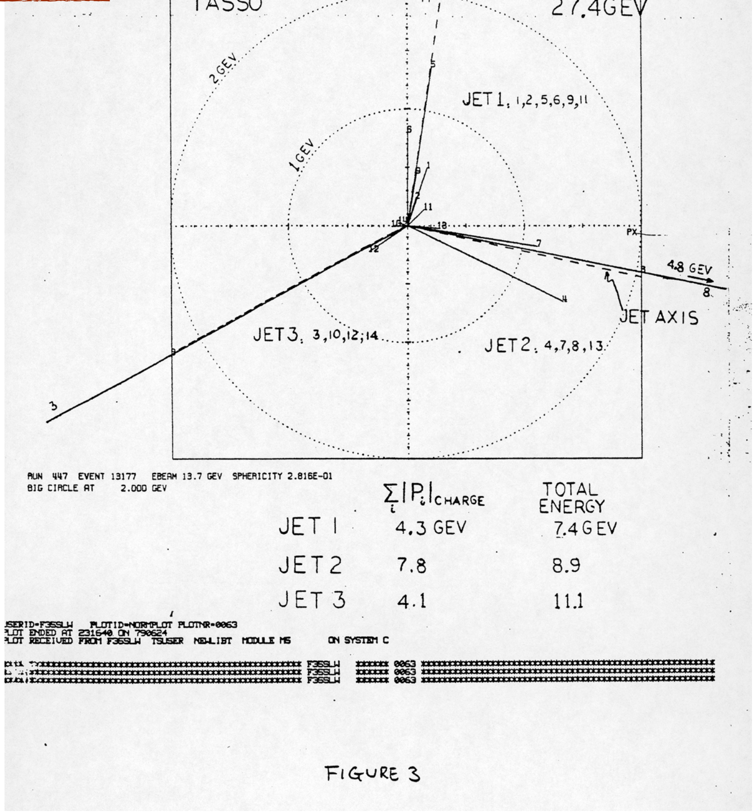
Existence of the gluon





```
*** SUMS (GEV) ***     PTOT  35.768   PTRANS  29.964 PLONG  15.700 CHARGE  -2
TOTAL CLUSTER ENERGY  15.169  PHOTON ENERGY   4.893 NR OF PHOTONS  11
```

```
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx F35SLW   xxxxxx 0063 xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx F35SLW   xxxxxx 0063 xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx F35SLW   xxxxxx 0063 xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

ID=NORMPLOT PLOTNR=0063
01 ON 79175
08 ON 790624
35SLW  TSUSER  NEWLIBT  MODULE MS       ON SYSTEM C
```



```
RUN  447  EVENT 13177   EBEAM 13.7 GEV  SPHERICITY 2.816E-01
BIG CIRCLE AT      2.000 GEV
```

	$\sum_i \lvert P_i \rvert_{\text{CHARGE}}$	TOTAL ENERGY
JET 1	4.3 GEV	7.4 GEV
JET 2	7.8	8.9
JET 3	4.1	11.1

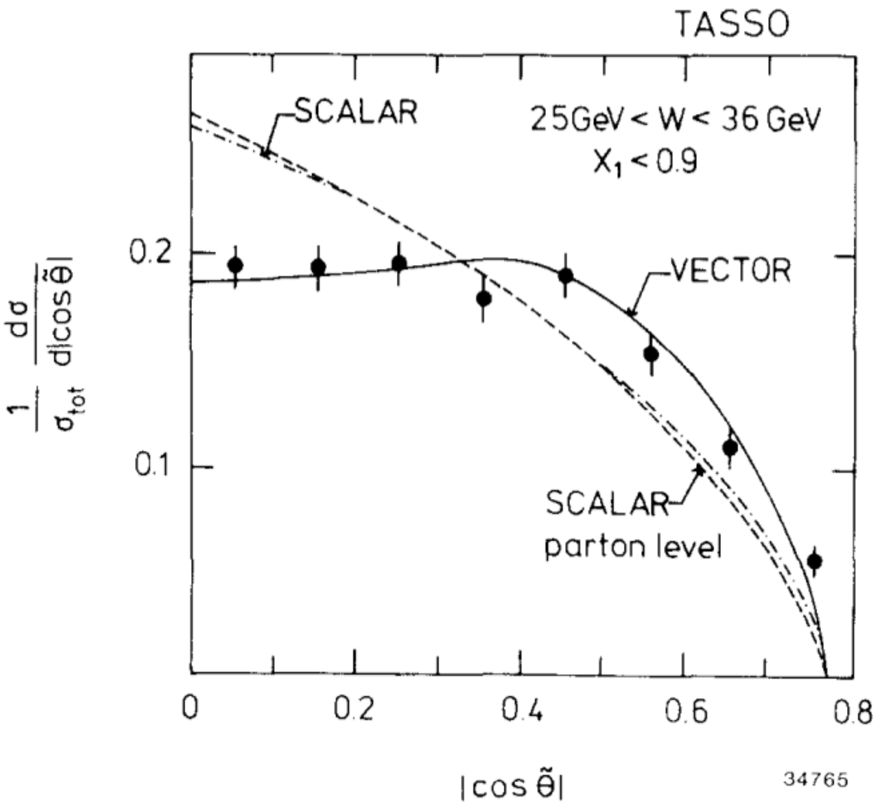
```
SERID=F35SLW   PLOTID=NORMPLOT PLOTNR=0063
LOT ENDED AT 231640 ON 790624
LOT RECEIVED FROM F35SLW  TSUSER  NEWLIBT  MODULE MS       ON SYSTEM C

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx F35SLW   xxxxxx 0063 xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx F35SLW   xxxxxx 0063 xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx F35SLW   xxxxxx 0063 xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

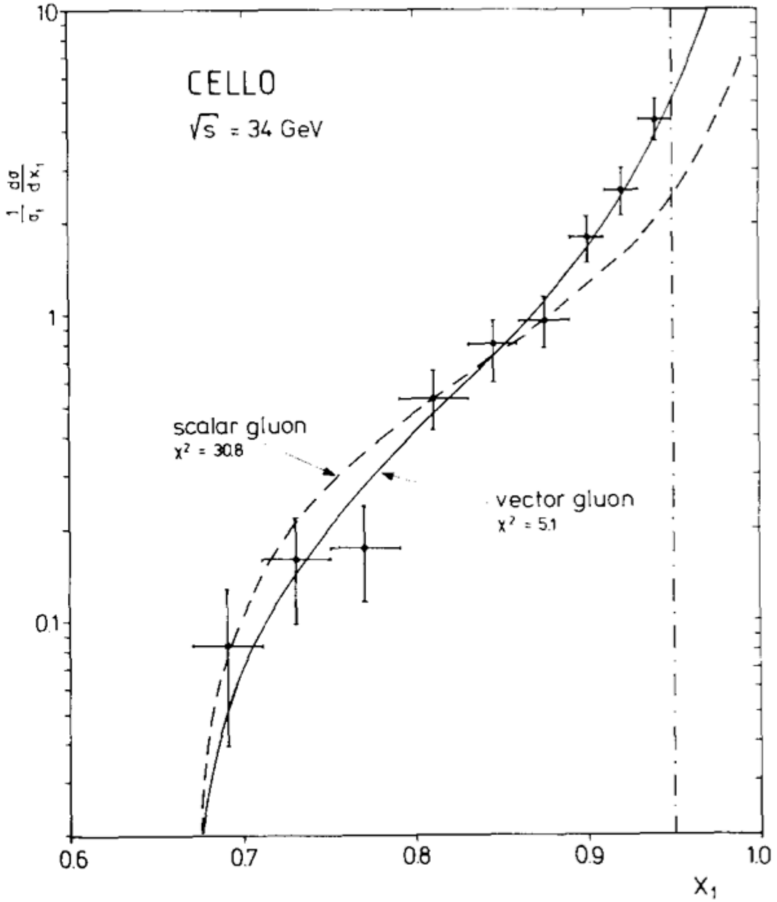
FIGURE 3

[JADE]

[Wu, Zobernig TASSO Note No. 84]

Spin of the gluon

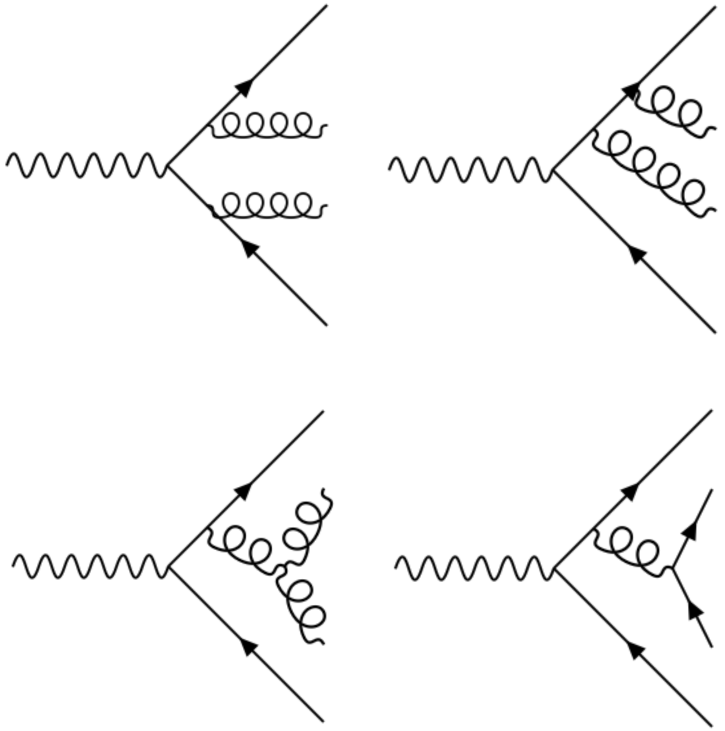


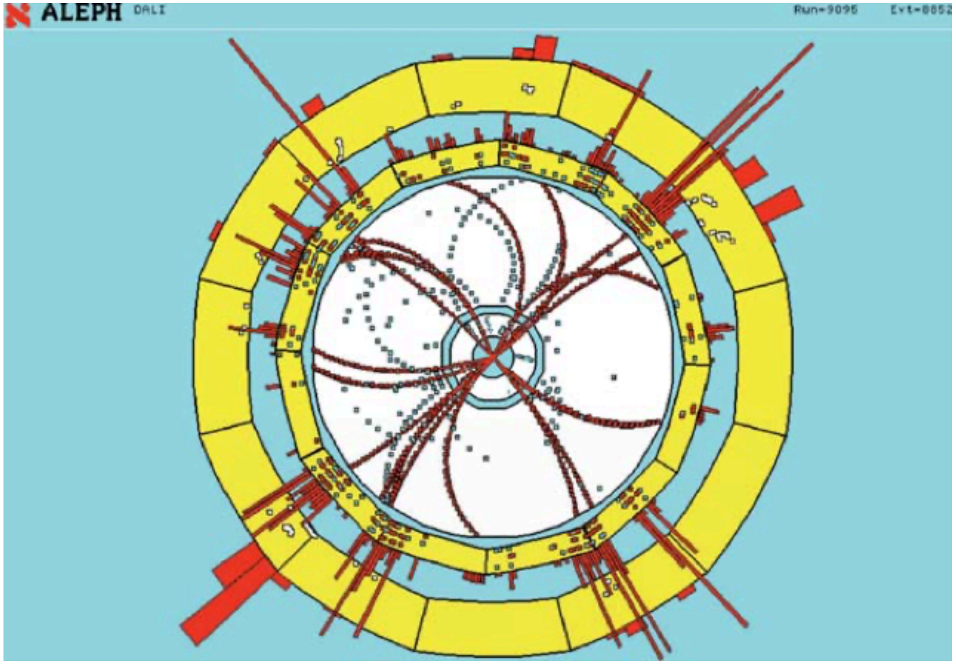


[Wu PR 107 (1984) 59]

[CELLO Collaboration PLB 110 (1982) 329]

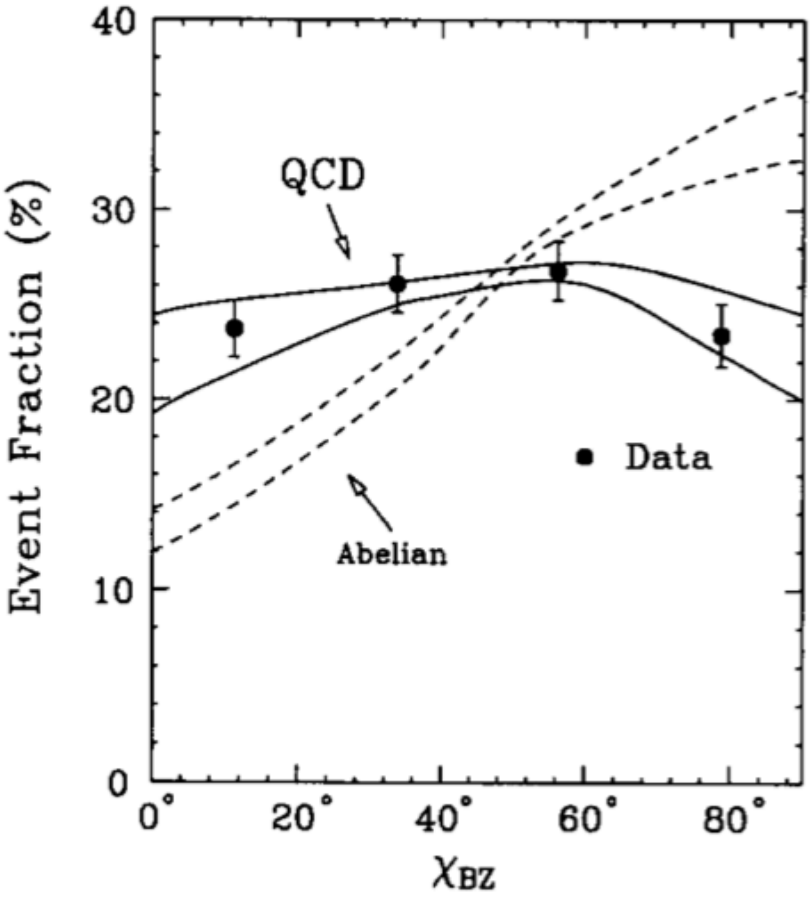
Gluon self-coupling



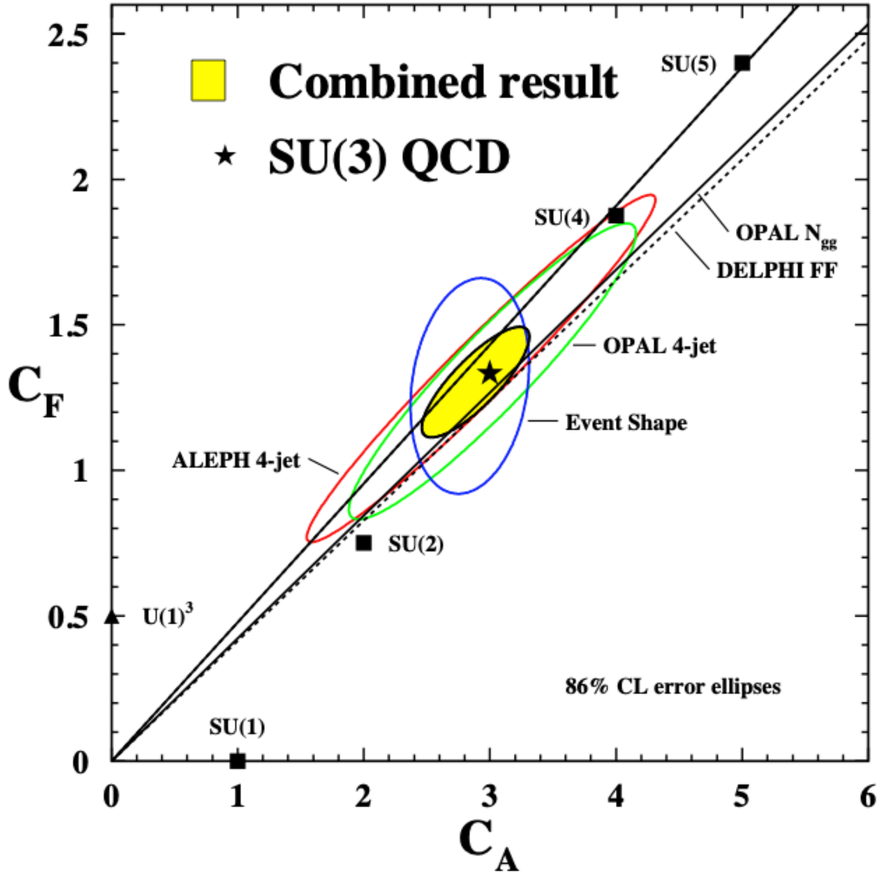


[Bethke PR 403-404 (2004) 203]

Non-abelian structure and Casimirs



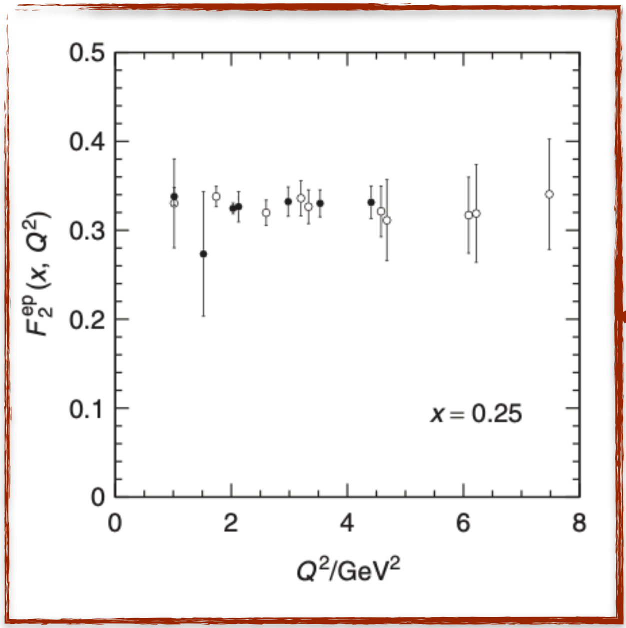
[L3 Collaboration PLB 248 (1990) 227]

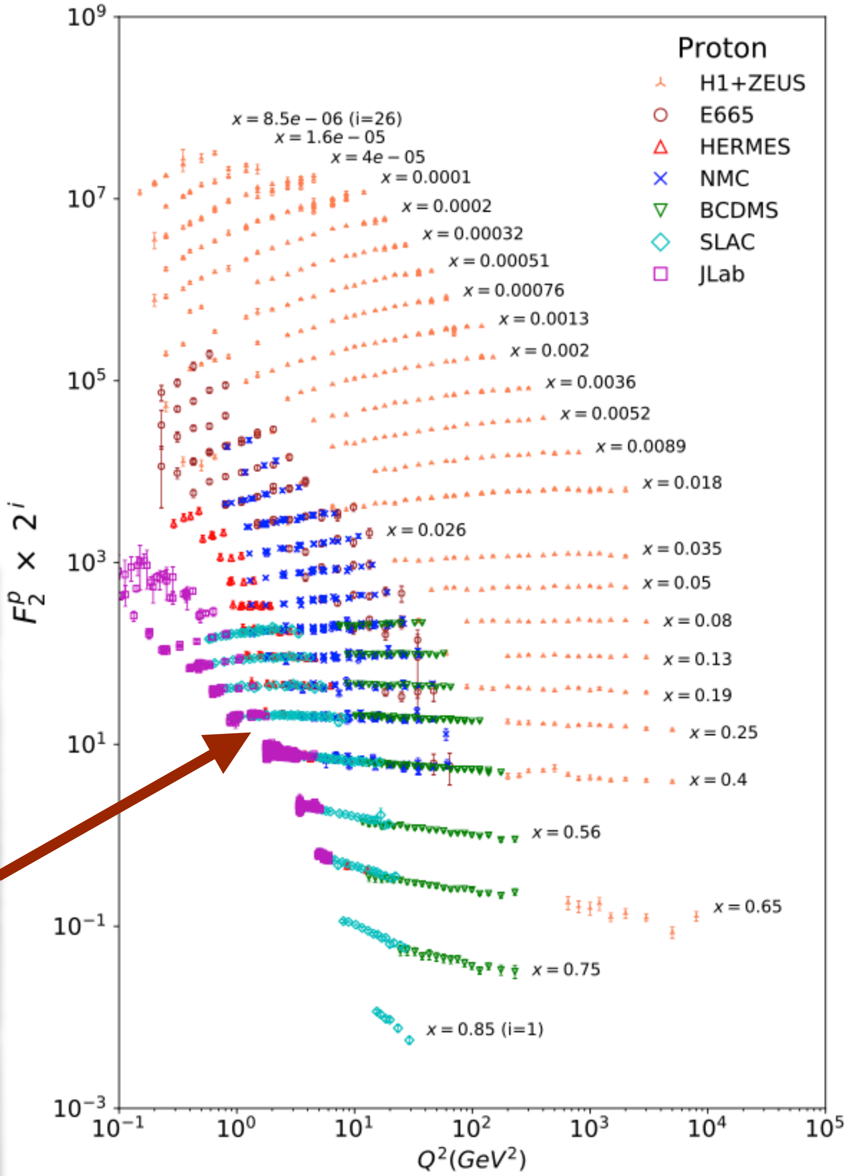


[Kluth NPB Suppl. 133 (2004) 36]

Proton structure revisited

Independence of structure functions $F_1(Q^2, x)$ and $F_2(Q^2, x)$ on Q^2 only approximately true at medium x and Q^2.





[PDG]

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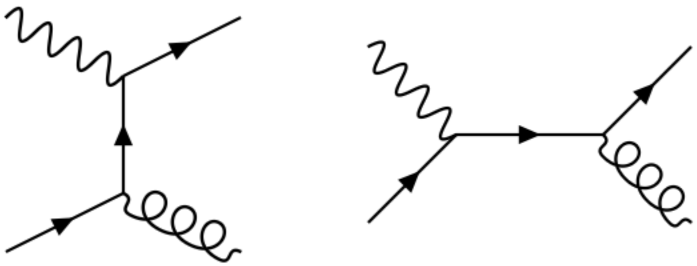
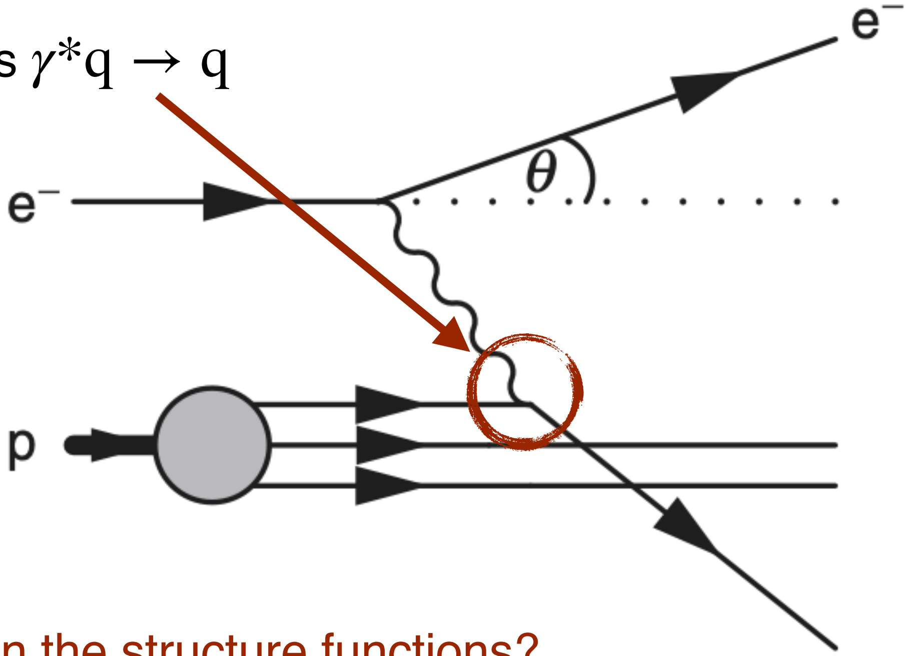
QCD dynamics in the proton

In our simple DIS model we neglected QCD dynamics in the proton!

At first order in QCD, the DIS sub-process $\gamma^{*}\mathrm{q} \to \mathrm{q}$ receives a correction from $\gamma^{*}\mathrm{q} \to \mathrm{qg}$

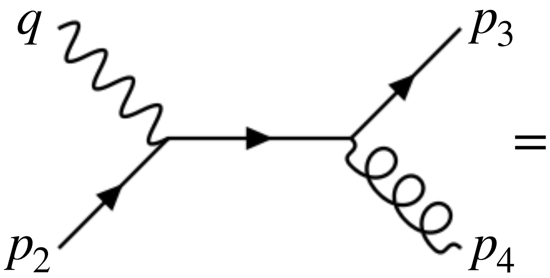




What is the effect of the gluon emission on the structure functions?

First-order corrections in DIS

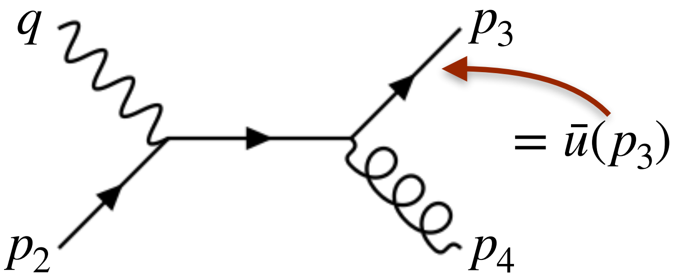
Consider the s-channel contribution to $\gamma^*\mathrm{q} \rightarrow \mathrm{qg}$



$=$

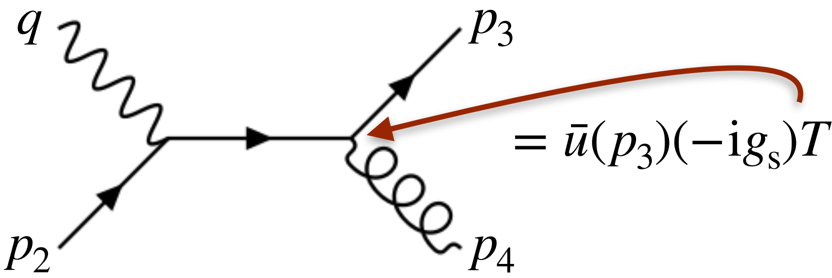
First-order corrections in DIS

Consider the s-channel contribution to $\gamma^* \mathrm{q} \rightarrow \mathrm{qg}$



First-order corrections in DIS

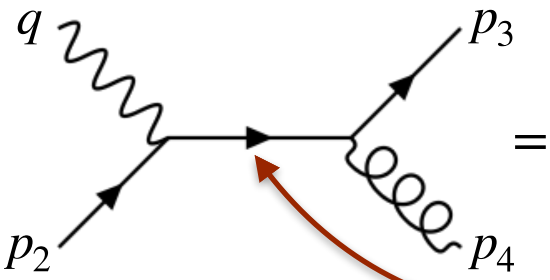
Consider the s-channel contribution to $\gamma^{*}\mathrm{q} \to \mathrm{qg}$



$= \bar{u}(p_3)(-\mathrm{i}g_\mathrm{s})T^a\gamma^\mu$

First-order corrections in DIS

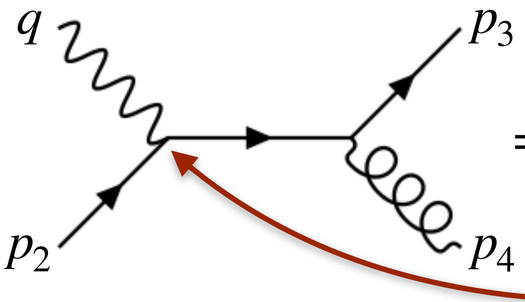
Consider the s-channel contribution to $\gamma^* \mathrm{q} \rightarrow \mathrm{qg}$



$$= \bar{u}(p_3)(-\mathrm{i}g_\mathrm{s})T^a\gamma^\mu \frac{\mathrm{i}(\not{p}_3 + \not{p}_4)}{(p_3 + p_4)^2}$$

First-order corrections in DIS

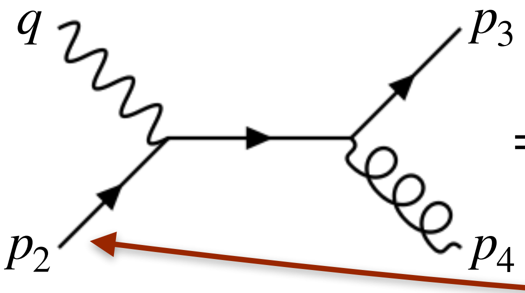
Consider the s-channel contribution to $\gamma^* \mathrm{q} \rightarrow \mathrm{qg}$



$$= \bar{u}(p_3)(-\mathrm{i}g_{\mathrm{s}})T^a\gamma^\mu \frac{\mathrm{i}(\not{p}_3 + \not{p}_4)}{(p_3+p_4)^2}(-\mathrm{i}e)\gamma^\nu$$

First-order corrections in DIS

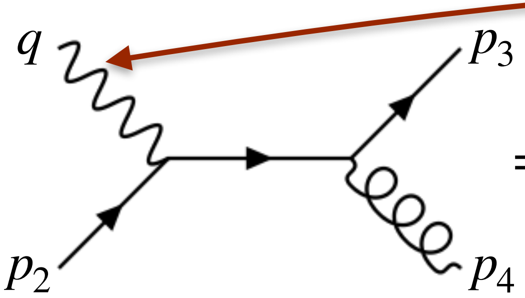
Consider the s-channel contribution to $\gamma^*\mathrm{q} \to \mathrm{qg}$



$$= \bar{u}(p_3)(-\mathrm{i}g_\mathrm{s})T^a\gamma^\mu \frac{\mathrm{i}(\not{p}_3 + \not{p}_4)}{(p_3+p_4)^2}(-\mathrm{i}e)\gamma^\nu u(p_2)$$

First-order corrections in DIS

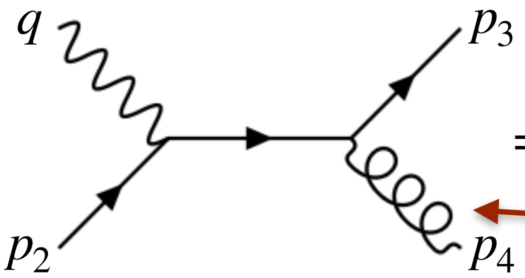
Consider the s-channel contribution to $\gamma^* \mathrm{q} \rightarrow \mathrm{qg}$



$$= \bar{u}(p_3)(-\mathrm{i}g_\mathrm{s})T^a\gamma^\mu \frac{\mathrm{i}(\not{p}_3 + \not{p}_4)}{(p_3+p_4)^2}(-\mathrm{i}e)\gamma^\nu u(p_2)\varepsilon_\nu(q)$$

First-order corrections in DIS

Consider the s-channel contribution to $\gamma^*\mathrm{q} \to \mathrm{qg}$



$$= \bar{u}(p_3)(-\mathrm{i}g_s)T^a\gamma^\mu \frac{\mathrm{i}(\not{p}_3 + \not{p}_4)}{(p_3+p_4)^2}(-\mathrm{i}e)\gamma^\nu u(p_2)\varepsilon_\nu(q)\varepsilon^*_\mu(p_4)$$

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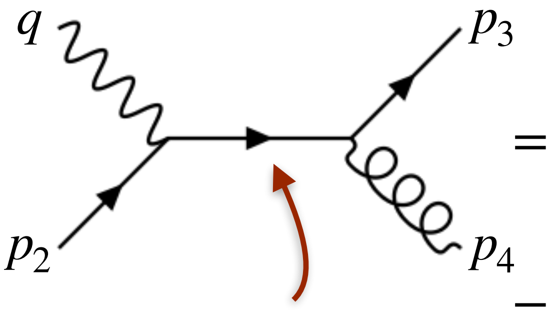

$$= \bar{u}(p_3)(-\mathrm{i}g_\mathrm{s})T^a\gamma^\mu \frac{\mathrm{i}(\not{p}_3 + \not{p}_4)}{(p_3+p_4)^2}(-\mathrm{i}e)\gamma^\nu u(p_2)\varepsilon_\nu(q)\varepsilon^*_\mu(p_4)$$

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In the collinear limit $3 \parallel 4$, the intermediate quark goes on-shell $p_{34}^2 \to 0$

First-order corrections in DIS

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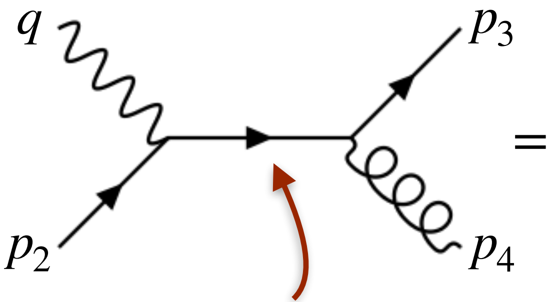

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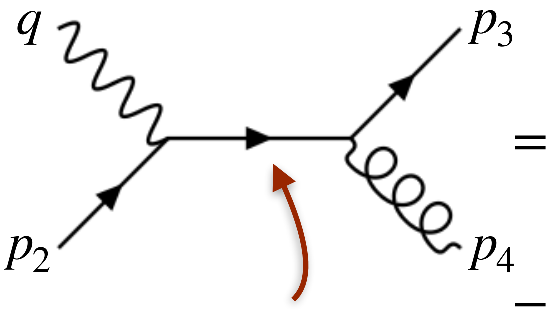
$$\to \bar{u}(p_3)(-\mathrm{i}g_s)T^a\gamma^\mu \frac{\mathrm{i}\sum_\lambda u_\lambda(p_{34})\bar{u}_\lambda(p_{34})}{p_{34}^2}(-\mathrm{i}e)\gamma^\nu u(p_2)\varepsilon_\nu(q)\varepsilon^*_\mu(p_4)$$

for on-shell p

$$\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) = \not{p}$$

First-order corrections in DIS

Consider the s-channel contribution to $\gamma^*\mathrm{q} \to \mathrm{qg}$



In the collinear limit $3 \parallel 4$, the intermediate quark goes on-shell $p_{34}^2 \to 0$

$$= \bar{u}(p_3)(-\mathrm{i}g_\mathrm{s})T^a\gamma^\mu \frac{\mathrm{i}(\not{p}_3 + \not{p}_4)}{(p_3+p_4)^2}(-\mathrm{i}e)\gamma^\nu u(p_2)\varepsilon_\nu(q)\varepsilon^*_\mu(p_4)$$

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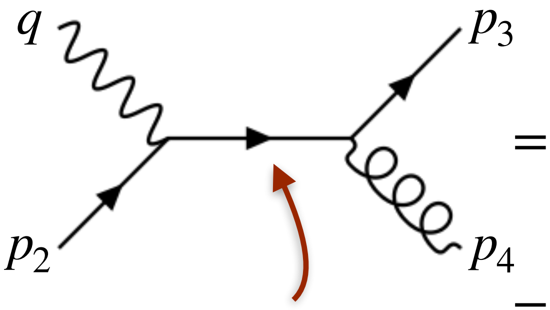
$$= g_\mathrm{s}T^a\frac{1}{p_{34}^2}\sum_\lambda \left[\bar{u}(p_3)\not{\varepsilon}^*(p_4)u_\lambda(p_{34})\right]\left[\bar{u}_\lambda(p_{34})(-\mathrm{i}e)\not{\varepsilon}(q)u(p_2)\right]$$

First-order corrections in DIS

Consider the s-channel contribution to $\gamma^* \mathrm{q} \to \mathrm{qg}$



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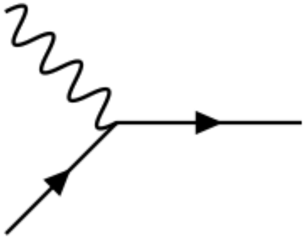
collinear emissions factorise!

$$= g_\mathrm{s}T^a\frac{1}{p_{34}^2}\sum_\lambda \left[\bar{u}(p_3)\not{\varepsilon}^*(p_4)u_\lambda(p_{34})\right] \times$$



Scaling violations

In the collinear limit, the squared amplitude becomes

$$|\mathscr{M}_{\gamma^* \mathrm{q} \to \mathrm{qg}}|^2 \sim g_\mathrm{s}^2 \frac{1}{p_{34}^2} P_{qg}(z) |\mathscr{M}_{\gamma^* \mathrm{q} \to \mathrm{q}}|^2, \;\; P_{qg}(z) = C_\mathrm{F} \frac{1+z^2}{1-z}$$

The effect of the gluon emission on the cross section is given by

$$\sigma_{\gamma^* \mathrm{q} \to \mathrm{qg}} \sim \sigma_{\gamma^* \mathrm{q} \to \mathrm{q}} g_\mathrm{s}^2 \frac{1}{8\pi^2} \int_{\mu^2}^{Q^2} \frac{\mathrm{d}s_{34}}{s_{34}} P_{qg}(z) = \sigma_{\gamma^* \mathrm{q} \to \mathrm{q}} \frac{\alpha_\mathrm{s}}{2\pi} P_{qg}(z) \log \frac{Q^2}{\mu^2}$$

reference scale

We can now write the structure functions as

$$2F_1(x, Q^2) = \sum_i Q_i^2 \int_x^1 \frac{\mathrm{d}y}{y} f_i(y) \left(\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_\mathrm{s}}{2\pi} P_{qg}\left(\frac{x}{y}\right) \log \frac{Q^2}{\mu^2} \right)$$

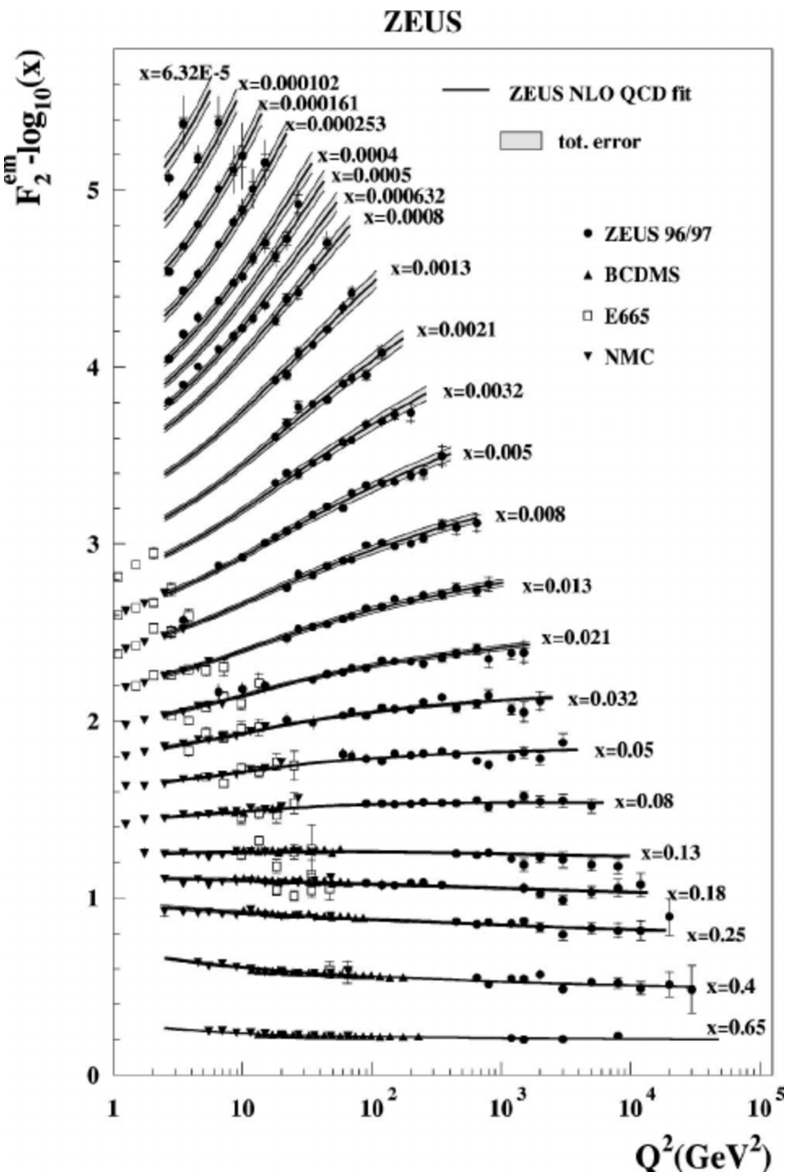
parton distribution function

no scaling

logarithmic scaling

Scaling violations

logarithmic scaling ✓



[ZEUS Collaboration PRD 67 (2003) 012007]

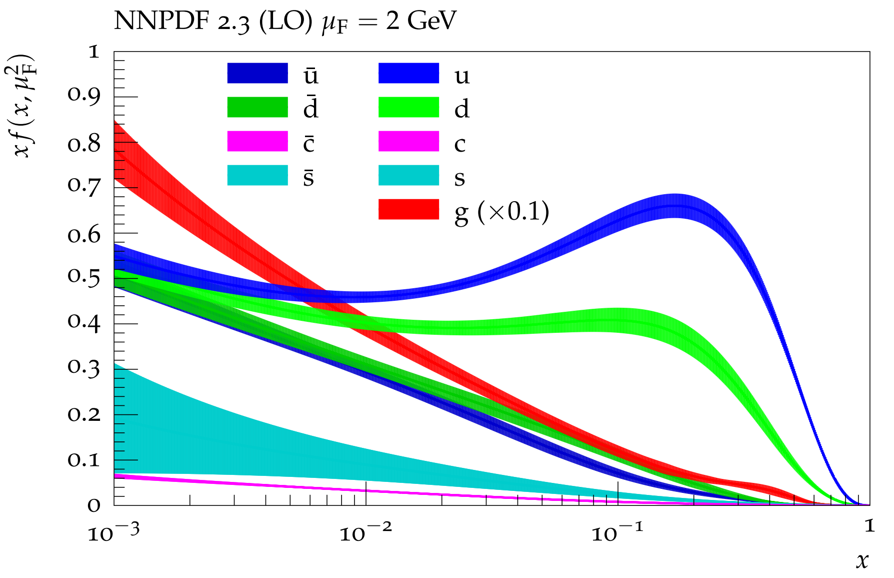
Parton distribution functions

Parton distribution functions (PDFs) $f_a(x_a, Q^2)$ parametrise the probability to find a parton a with energy fraction x_a at scale Q^2 in the proton.

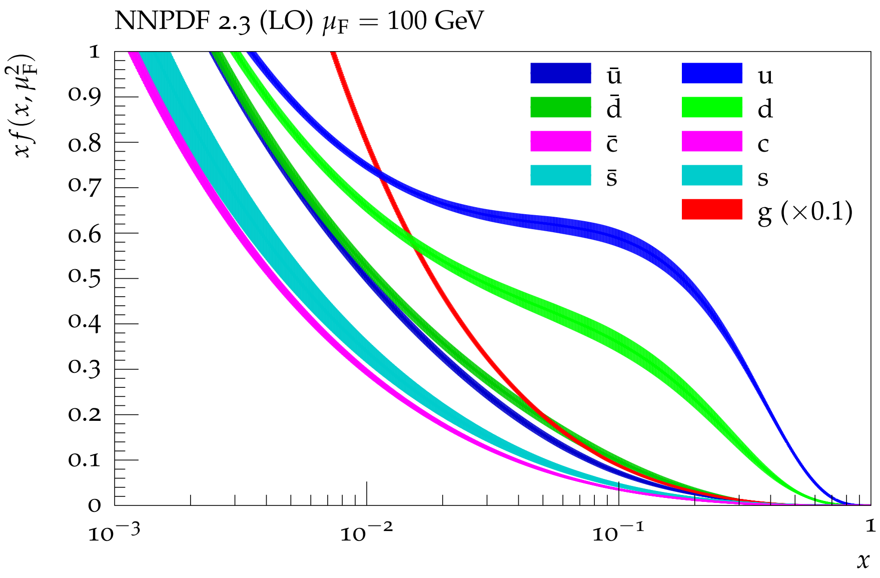
PDFs are intrinsically non-perturbative and have to be measured at some scale Q^2.

“Running” of PDFs with scale Q^2 described by collinear evolution.

see previous slides



NNPDF 2.3 (LO) $\mu_F = 2$ GeV



NNPDF 2.3 (LO) $\mu_F = 100$ GeV

Factorisation

Assume that at scale μ_F the hadronic cross section **factorises** into long-distance (hadronic) and short-distance (partonic) parts:

$$\mathrm{d}\sigma_{\mathrm{pp}\to X} = \sum_{a,b} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; f_a(x_a, \mu_\mathrm{F}) f_b(x_b, \mu_\mathrm{F}) \, \mathrm{d}\hat{\sigma}(x_a, x_b, \mu_\mathrm{F})$$

Factorisation in terms of transverse momentum p_T of partons

- emissions with $p_\mathrm{T} \lesssim \mu_\mathrm{F}$ implicitly included in PDFs
- emissions with $p_\mathrm{T} \gtrsim \mu_\mathrm{F}$ explicitly described by hard process

Factorisation

Assume that at scale μ_{F} the hadronic cross section **factorises** into long-distance (hadronic) and short-distance (partonic) parts:

$$\mathrm{d}\sigma_{\mathrm{pp}\to X} = \sum_{a,b} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b\, f_a(x_a, \mu_{\mathrm{F}}) f_b(x_b, \mu_{\mathrm{F}})\, \mathrm{d}\hat{\sigma}(x_a, x_b, \mu_{\mathrm{F}})$$

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- emissions with $p_{\mathrm{T}} \gtrsim \mu_{\mathrm{F}}$ explicitly described by hard process

Tomorrow we'll see how we can utilise this to simulate collider events…