Heavy Flavour Physics

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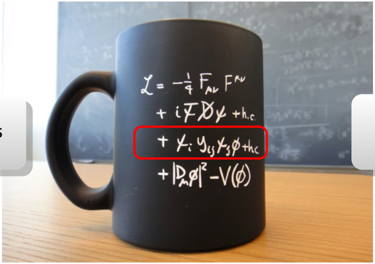
Heavy Flavour Physics

Flavour in the SM

NP searches with flavour



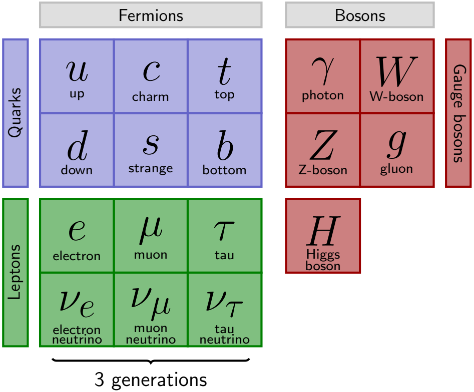
The experiments

CKM metrology and CP violation

Rare decays

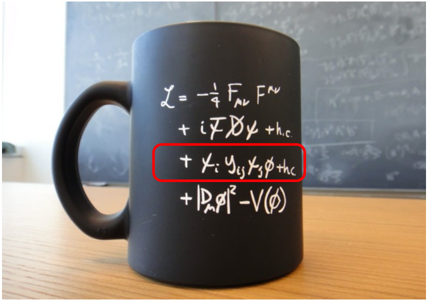
Future prospects

What is flavour?



- Fundamental matter comes in 3 generations in the SM
  (Open question: Why 3?)
- Flavour is the feature that distinguishes the generations
- Flavour physics studies phenomenology of flavour transitions to
  1. Determine SM parameters precisely
  2. Search for physics beyond the SM in precision measurements

The SM Lagrangian: Where is the Flavour?



- Flavour structure of the SM determined by the Yukawa terms: coupling of fermions to Higgs
- After EWSB (put in Higgs expectation ν) for the quark fields:

$$\mathcal{L}^{\text{quarks}}_{\text{Yukawa}} = -\frac{\nu}{\sqrt{2}}\bar{d}_{Li} Y_{d,ij} d_{Rj} - \frac{\nu}{\sqrt{2}}\bar{u}_{Li} Y_{u,ij} u_{Rj} + h.c.$$

- $Y_{d,ij}$, $Y_{u,ij}$ complex 3×3 matrices in generation space, not diagonal!

Mass and weak eigenstates

- Mass eigenstates u^m, d^m obtained by diagonalisation of $Y_{u,d}$ via unitary transformations $V_{(u,d)(L,R)}$ with $VV^\dagger = 1$:

$$d_L = V_{dL} d_L^m \quad d_R = V_{dR} d_R^m \qquad u_L = V_{uL} u_L^m \quad u_R = V_{uR} u_R^m$$

- Yukawa terms in mass basis then diagonal with $M_d = \mathrm{diag}(m_d, m_s, m_b)$ and $M_u = \mathrm{diag}(m_u, m_c, m_t)$

$$\begin{aligned}\mathcal{L}^{\text{quarks}}_{\text{Yukawa}} &= -\frac{\nu}{\sqrt{2}} \bar{d}_{Li} Y_{d,ij} d_{Rj} - \frac{\nu}{\sqrt{2}} \bar{u}_{Li} Y_{u,ij} u_{Rj} + h.c. \\ &= -\bar{d}_L^m \left[\frac{\nu}{\sqrt{2}} V_{dL}^\dagger Y_d V_{dR}\right] d_R^m - \bar{u}_L^m \left[\frac{\nu}{\sqrt{2}} V_{uL}^\dagger Y_u V_{uR}\right] u_R^m + h.c. \\ &= -\bar{d}_{Li}^m M_{d,ii} d_{Ri}^m - \bar{u}_{Li}^m M_{u,ii} u_{Ri}^m + h.c.\end{aligned}$$

- Yukawa terms contain 6 mass parameters from quark sector, 3 (+3 for $m_\nu \neq 0$) from lepton sector
- Spanning several orders of magnitude. Open question: Why?

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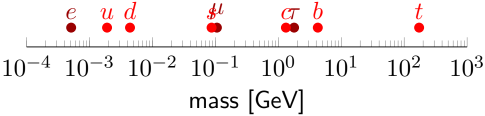

Charged current and CP violation

- Up- and down-type quarks cannot be diagonalised with same matrix ($V_{dA} \neq V_{uA}$) $\rightarrow$ net effect on flavour structure of charged current

$$
\begin{aligned}
\mathcal{L}_{\mathrm{CC}}^{\mathrm{quarks}} &= -\frac{g}{\sqrt{2}}\left(\bar{u}_{Li}\gamma^\mu W_\mu^+ d_{Li} + \bar{d}_{Li}\gamma^\mu W_\mu^- u_{Li}\right) \\
&= -\frac{g}{\sqrt{2}}\left(\bar{u}_{Li}^m V_{uL}^\dagger V_{dL}\gamma^\mu W_\mu^+ d_{Lj}^m + \bar{d}_{Li}^m V_{dL}^\dagger V_{uL}\gamma^\mu W_\mu^- u_{Lj}^m\right) \\
&= -\frac{g}{\sqrt{2}}\left(\bar{u}_{Li}^m V_{\mathrm{CKM},ij}\gamma^\mu W_\mu^+ d_{Lj}^m + \bar{d}_{Lj}^m V_{\mathrm{CKM},ij}^*\gamma^\mu W_\mu^- u_{Li}^m\right)
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with Cabbibo-Kobayashi-Maskawa matrix $V_{\mathrm{CKM}} = V_{uL}^\dagger V_{dL}$

- Applying the CP operation (Charge and parity conjugation) results in

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- Invariant only if $V_{\mathrm{CKM}} = V_{\mathrm{CKM}}^*$, *i.e.* all CKM elements are real
- CP violation possible if CKM elements complex

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The CKM matrix

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad \begin{aligned} V_{ub} &= |V_{ub}|e^{-i\gamma} \\ V_{td} &= |V_{td}|e^{-i\beta} \\ V_{ts} &= |V_{ts}|e^{-i\beta_s} \end{aligned}$$

- $V_{\rm CKM}$ product of unitary matrices $\rightarrow$ unitary itself: $V_{\rm CKM}V_{\rm CKM}^{\dagger} = 1$
- Complex $n \times n$ matrix: n^2 real parameters, n^2 complex phases
- Unitarity cond.: $n(n-1)/2$ real param., $n(n+1)/2$ complex phases
- After removal of 5 unobservable quark phases $\rightarrow$ 4 free parameters: 3 Euler angles $\theta_{12}, \theta_{13}, \theta_{23}$, 1 phase δ

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$

CKM hierarchy and Wolfenstein parameterisation

- Wolfenstein parameterisation uses the parameters λ, A, ρ and η, with η responsible for imaginary entries in $V_{\rm CKM}$

$$s_{12} = \lambda \qquad s_{23} = A\lambda^2 \qquad s_{13}e^{+i\delta} = A\lambda^3\left(\rho + i\eta\right)$$

- parameter $\lambda \approx 0.22$ plays the role of an expansion parameter
- Up to $\mathcal{O}(\lambda^4)$ the CKM matrix in the Wolfenstein param. given by

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3\left(\rho - i\eta\right) \\ -\lambda & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4\left(1+4A^2\right) & A\lambda^2 \\ A\lambda^3\left(1-\rho-i\eta\right) & -A\lambda^2+\frac{1}{2}A\lambda^4\left(1-2\left(\rho+i\eta\right)\right) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

- Diagonal elements close to 1, off-diagonal transitions suppressed $|V_{us}|, |V_{cd}| \sim \lambda$, $|V_{cb}|, |V_{ts}| \sim \lambda^2$ and $|V_{ub}|, |V_{td}| \sim \lambda^3$.
- Imaginary part relative to CKM element largest for V_{ub}

Flavour sector in the SM

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \qquad V_{\mathrm{PMNS}} = \begin{pmatrix} U_{\mathrm{e}1} & U_{\mathrm{e}2} & U_{\mathrm{e}3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- Majority of SM parameters in the flavour sector, in total 13 (20 for $m_\nu \neq 0$) of 19 (26):
  - 6 quark masses
  - 3 quark mixing angles, 1 mixing phase: CKM matrix
  - 3(+3) lepton masses
  - (+3 lepton mixing angles, +1 mixing phase: PMNS matrix[1])
- Many open question:

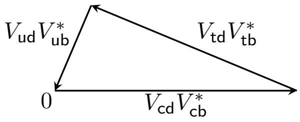
  Why these values? Hierarchical structure? Relations between mixing parameters and masses? Relations between CKM and PMNS matrix?
- If you can answer any of these: 

[1] Pontecorvo, Maki, Nakagawa, Sakata

The unitarity triangle(s)

- Unitarity condition $V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1$ results in 3 equations for the off-diagonal elements[2]:

$$V_{ud}\, V_{ub}^* + V_{cd}\, V_{cb}^* + V_{td}\, V_{tb}^* = 0$$
$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0$$
$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$$

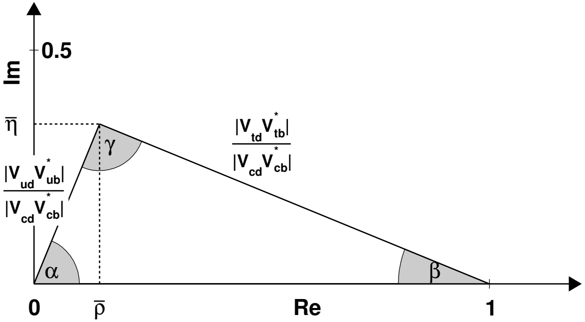


- sum of 3 numbers can be visualized as triangle in the complex plane
- one side of each triangle normalised to coincide with the real axis

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{cd}V_{cb}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0 \leftarrow \text{The } B^0 \text{ unitarity triangle}$$
$$\frac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*} + \frac{V_{cs}V_{cb}^*}{V_{cs}V_{cb}^*} + \frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} = 0$$
$$\frac{V_{ud}V_{us}^*}{V_{cd}V_{cs}^*} + \frac{V_{cd}V_{cs}^*}{V_{cd}V_{cs}^*} + \frac{V_{td}V_{ts}^*}{V_{cd}V_{cs}^*} = 0$$

[2] Other 3 off-diagonal elements result in 3 equations which are complex conjugates.

Overconstrain the B^0 unitarity triangle



- Vertices at $(0,0)$, $(1,0)$, $(\bar{\rho},\bar{\eta})$ with $\bar{\rho}+i\bar{\eta}=-(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$
- Angles: $\alpha=\arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$, $\beta=\arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$, $\gamma=\arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$

- Sides from CP conserving obs.

top left: $B\to\pi\ell^-\bar{\nu}_\ell$
top right: B^0 and B_s^0 mixing
norm: $B\to D\ell^-\bar{\nu}_\ell$

- Angles from CPV observables

α: $B\to\pi\pi,\rho\pi,\rho\rho$
β: $B^0\to J/\psi K_S^0$
γ: $B\to DK$

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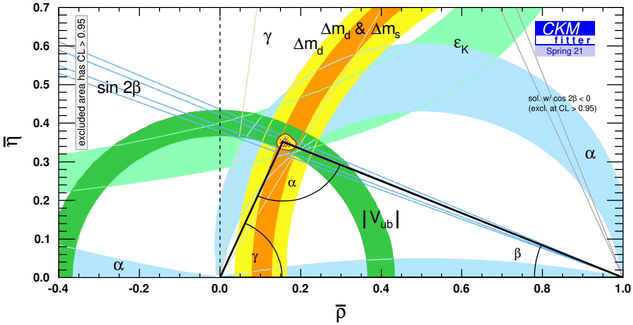

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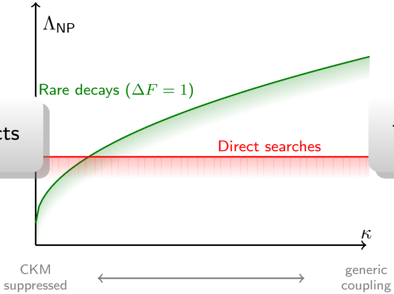
NP searches with flavour

Exclusion limits for NP searches



Future prospects

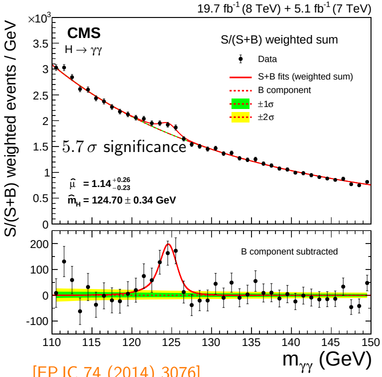
The experiments

Rare decays

CKM metrology and CP violation

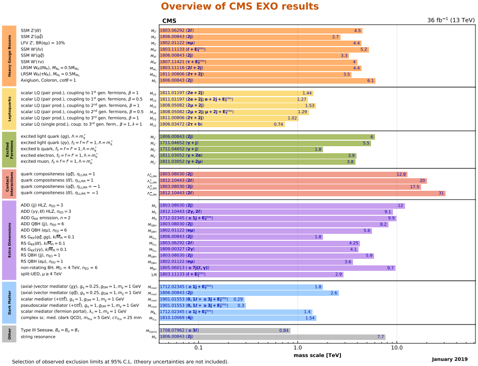
Direct searches for NP

clear signature in direct search for $H \to \gamma\gamma$ (SM)



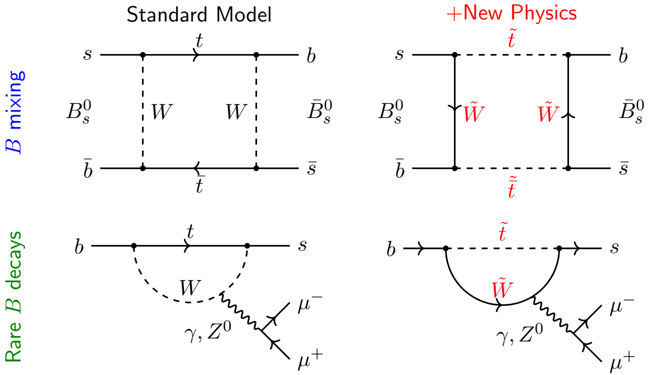
[EPJC 74 (2014) 3076]

Selection of exclusion limits from direct searches for NP



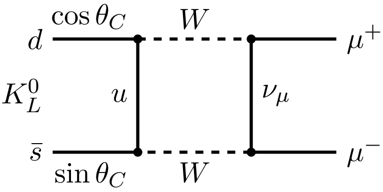
- Direct on-shell production of new heavy particles
- Observation of new particles via their decay products
- Limited by beam energy (LHC Run 2 $\sqrt{s} = 13\,\text{TeV}$)
- Direct searches so far did not result in signs for NP
- Maybe NP is too heavy for current direct searches?

NP searches with precision flavour observables



- Precisely measure processes known in the SM
- Detect virtual contributions of heavy NP particles
- Circumvents possible limitation by beam energy
- Particularly sensitive: Processes that are heavily suppressed in the SM, *e.g.* B mixing and rare decays (*Flavour changing neutral currents*)
- Complementary to direct searches, historically often preceeds direct obs.

Historical example: Prediction of the c quark



$$\mathcal{A} \propto +\cos\theta_C \sin\theta_C \frac{m_u^2}{m_W^2}$$

- Decay $K_\mathrm{L}^0 \to \mu^+\mu^-$ should have significant branching fraction in three quark model (left diagram)
- But experimentally[3]: $\mathcal{B}(K_\mathrm{L}^0 \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$
- Prediction: Existence of c-quark, results in additional diagram (right)!
- Additional diagram leads to partial cancellation through GIM (Glashow, Iliopoulos, Maiani) mechanism [PRD 7 (1970) 2]
- c-quark directly observed (through J/ψ) by Richter and Ting in 1974

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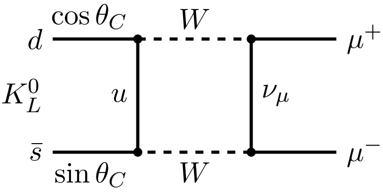

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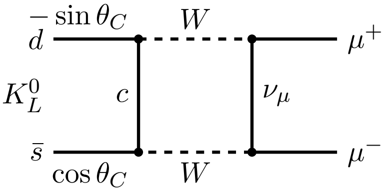

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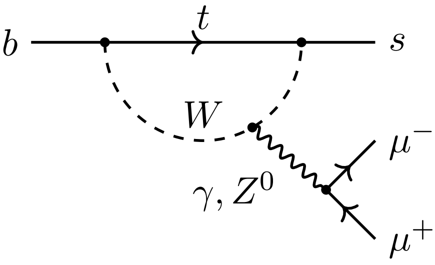
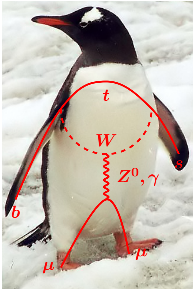
Rare B decays as sensitive probes for New Physics

Rare decays in the SM



Penguin decays



[Ellis et al, Nucl.Phys. B131 (1977) 285]

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- New heavy particles can significantly contribute to rare processes
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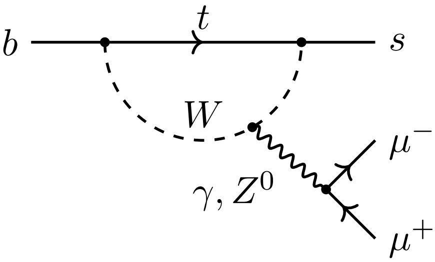
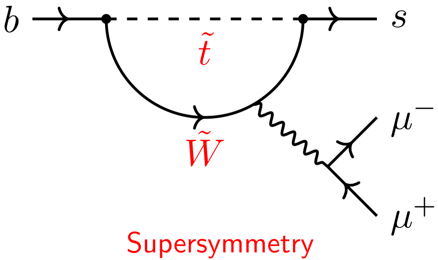


Possible contributions from NP



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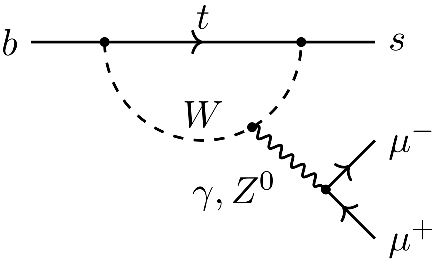

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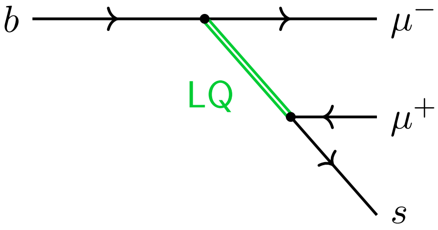
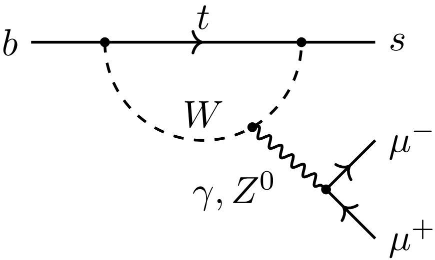


Leptoquarks

- Rare decays are so called Flavour Changing Neutral Currents
- In the SM: Only allowed via quantum fluctuations (loop suppressed)
- New heavy particles can significantly contribute to rare processes
- Search for deviations of observables from their SM predictions

Rare B decays as sensitive probes for New Physics

Rare decays in the SM



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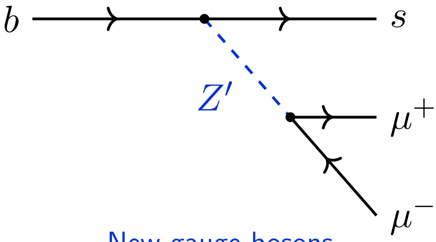

New gauge bosons

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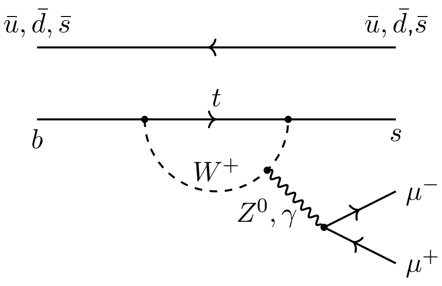
Observables in rare semileptonic B decays



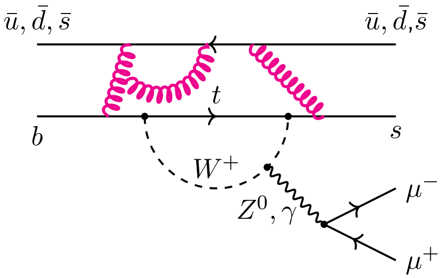
Quarks are bound in hadrons, mesons ($q\bar{q}$) and baryons (qqq)

- Many decay modes allows to check consistency
- Different spin configurations allow complementary probes
- Need to account for QCD, affects observables differently

1. Branching fraction measurements
   Directly affected by hadronic uncertainties
2. Angular distributions, CP asymmetries, Isospin asymmetries
   Observables with reduced hadronic uncertainties (relative measurements)
3. Tests of Lepton Universality
   full cancellation of hadronic uncertainties

Increasing theoretical precision

Observables in rare semileptonic B decays



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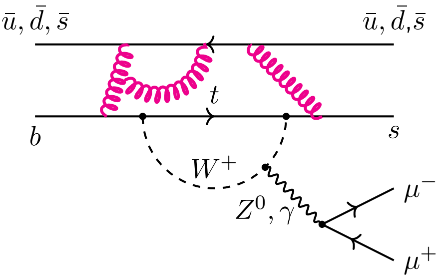

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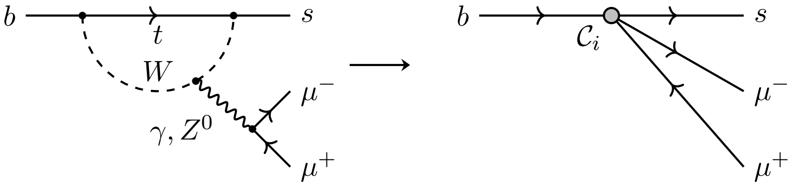
Increasing theoretical precision

Rare B decays in effective field theory

Full theory — Effective description



- Model-independent description in effective field theory

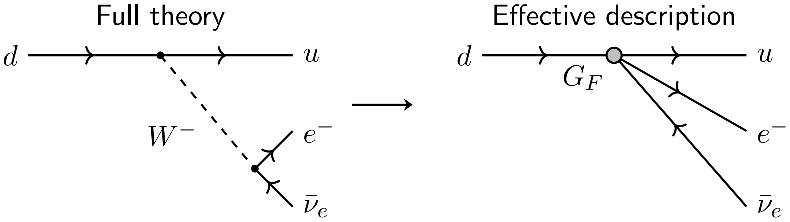
$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{\text{tb}} V_{\text{ts}}^{*} \frac{e^2}{16\pi^2} \sum_i \mathcal{C}_i \mathcal{O}_i$$

$\mathcal{C}_i$: Effective coupling "Wilson coefficient" *short distance physics*

$\mathcal{O}_i$: Local operator with specific Lorentz structure *long distance physics*

- Analogous to β-decay: Integrate out heavy degrees of freedom $\rightarrow$ point-interaction with Fermi-coupling G_F

Rare B decays in effective field theory



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"Wilson coefficient"
short distance physics

Local operator with
specific Lorentz structure
long distance physics

- Analogous to **β-decay**: Integrate out heavy degrees of freedom
$\rightarrow$ point-interaction with Fermi-coupling G_F

NP contributions and reach with indirect searches

- NP can contribute to different operators $\mathcal{O}_i$ depending on its type

$$\mathcal{H}_{\text{eff}} = -\underbrace{\frac{4G_F}{\sqrt{2}} V_{\text{tb}} V_{\text{ts}}^* \frac{e^2}{16\pi^2}}_{\sim 1/(35\,\text{TeV})^2} \sum_i \mathcal{C}_i \mathcal{O}_i \qquad \Delta\mathcal{H}_{\text{NP}} = \frac{\kappa}{\Lambda_{\text{NP}}^2} \mathcal{O}_i$$

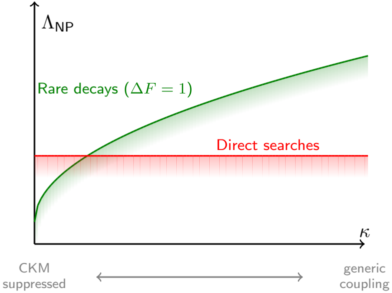
κ: Flavour-viol. coupling

Λ_{NP}^2: NP scale

$$\Rightarrow \boxed{\Lambda_{\text{NP}} \sim 35\,\text{TeV}\sqrt{\kappa/\Delta\mathcal{C}_i}}$$

- NP reach not limited by $\sqrt{s}$, complementarity with direct searches

Exclusion limits for NP searches



NP Scenario	Coupling κ
Tree-level generic	1
Tree-level CKM suppressed	$V_{\text{tb}}V_{\text{ts}}$
Loop-level generic	$\frac{1}{16\pi^2}$
Loop-level CKM suppressed	$\frac{V_{\text{tb}}V_{\text{ts}}}{16\pi^2}$

Heavy Flavour Physics

Flavour in the SM

NP searches with flavour

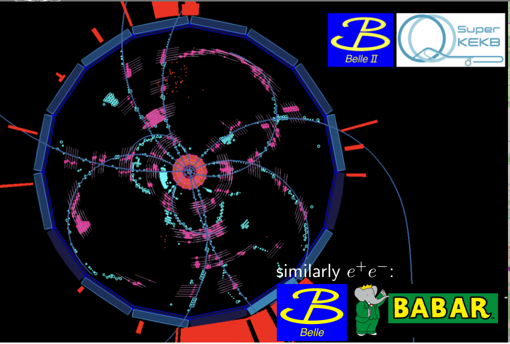
Future prospects

The experiments

Rare decays

CKM metrology and CP violation

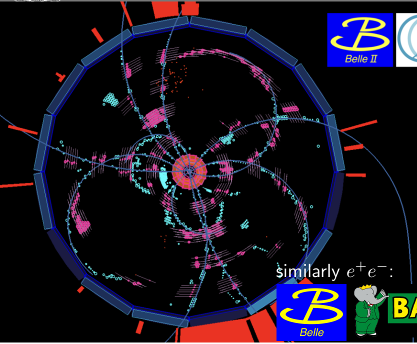
The experiments: e^+e^- machines and hadron colliders



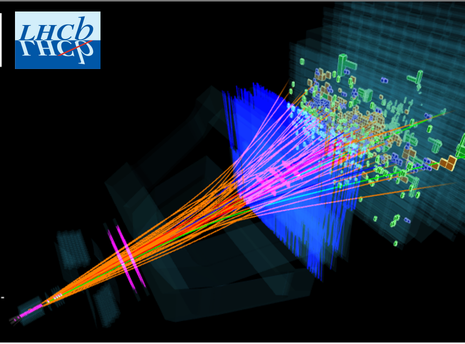
- Low multiplicity e^+e^- environment
  $e^+e^- \to \Upsilon(4S) \to B\bar{B}$
- Well def. initial state with known energy
- Full event reconstruction possible
- Inclusive reconstruction possible
- e/μ experimentally similar
- Leading with challenging signatures involving τ, ν, π^0

- High multiplicity hadronic environment
  $pp \to X + b\bar{b}$
- Large $b\bar{b}$ ($c\bar{c}$) prod. cross-section
- All b-hadrons: B^0, B^+, B_s^0, B_c^+, Λ_b^0, ...
- Initial state kinematics not known
- Trigger and reconstruction challenging (particularly for ATLAS/CMS)
- Leading for charged track final states, in particular involving μ

The experiments: e^+e^- machines and hadron colliders



similarly e^+e^-:



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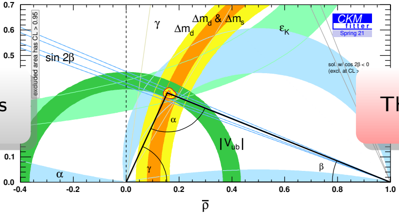
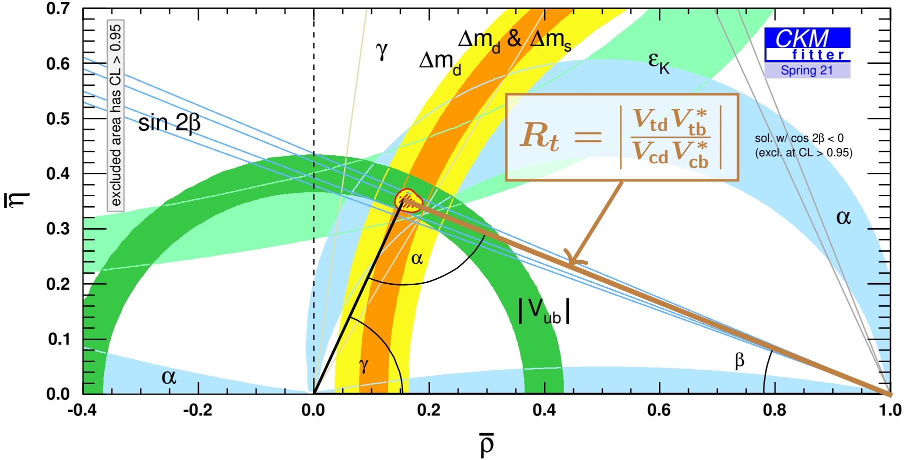
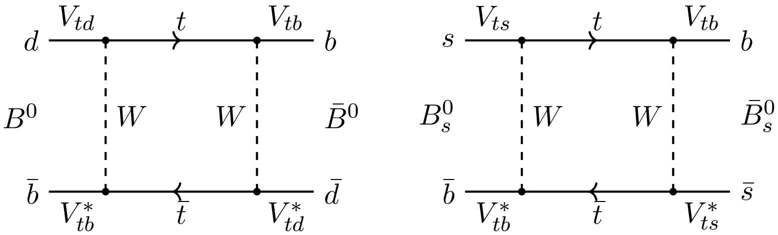


$B^0_{(s)}$ mixing



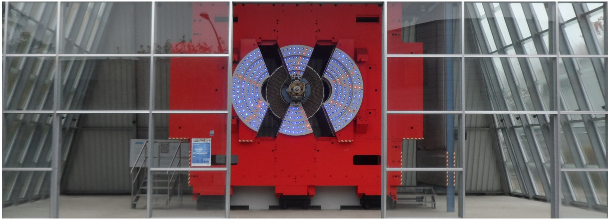
$$R_t = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right|$$

$B^0_{(s)}$ mixing



- Neutral B mesons oscillate $B^0 \leftrightarrow \overline{B}^0$ via loop-level diagrams
- Sensitive to size of CKM matrix elements $|V_{td}|$ and $|V_{ts}|$
  $\rightarrow$ determines top right side of CKM triangle
- SM diagrams dominated by top-quark contributions
  $\rightarrow$ gives information on top mass

History: Discovery of B^0 mixing by ARGUS at DESY

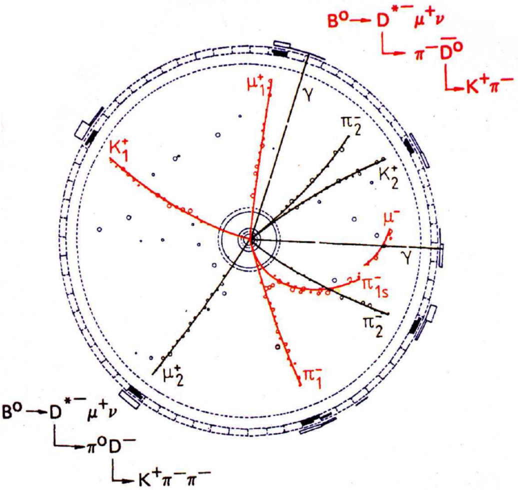


- Production of $B^0\overline{B}^0$ pairs via $e^+e^- \to \Upsilon(4S) \to B^0\overline{B}^0$
- Dominant semileptonic decays $\bar{b} \to \bar{c}\ell^+\nu_\ell$ and $b \to c\ell^-\bar{\nu}_\ell$

unmixed	mixed
$B^0 \to D^{(*)-}\ell^+\nu_\ell$	$B^0 \to \overline{B}^0 \to D^{(*)+}\ell^-\bar{\nu}_\ell$
$\overline{B}^0 \to D^{(*)+}\ell^-\bar{\nu}_\ell$	$\overline{B}^0 \to B^0 \to D^{(*)-}\ell^+\nu_\ell$

- Same-sign high momentum lepton events sign of $B^0 \leftrightarrow \overline{B}^0$ mixing
- ARGUS finds [PLB 192 (1987) 245]
  - 24.8 like-sign lepton pairs $(4\,\sigma)$
  - 4.1 reconstructed B^0 $(\overline{B}^0)$ and additional fast ℓ^+ (ℓ^-) $(3\,\sigma)$
  - One fully reconstructed "golden" event

Golden fully reconstructed event



- Observation of B^0 mixing
- Also set lower limit for top mass $m_t > 50\,\mathrm{GeV}$

Precision measurement of B^0 mixing by LHCb

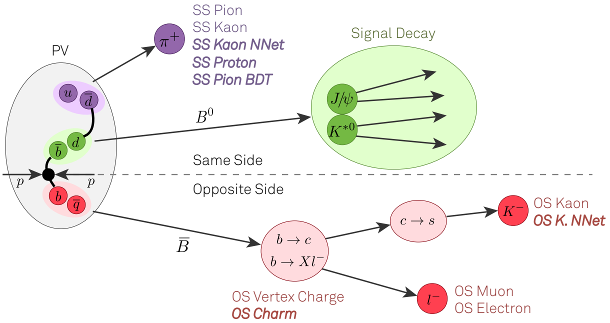
- Precision mixing measurement using $B^0 \to D^{(*)-}\mu^+\nu_\mu$ events [see backup]

$$N^{\text{unmix}} = N(B^0 \to D^{(*)-}\mu^+\nu_\mu X)(t) \quad \propto e^{-\Gamma_d t}[1+\cos(\Delta m_d t)]$$

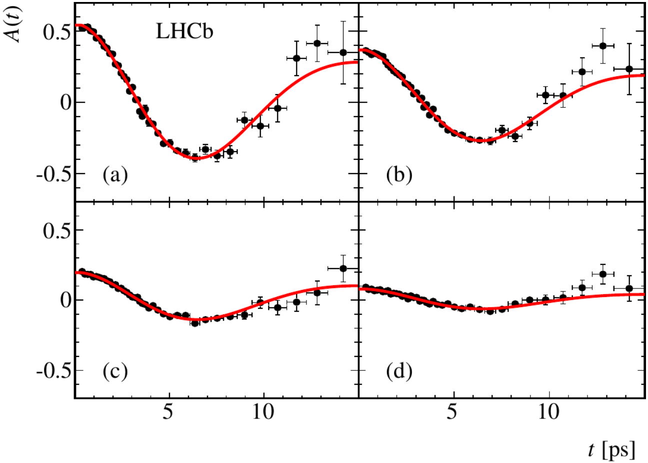
$$N^{\text{mix}} = N(B^0 \to \overline{B}^0 \to D^{(*)+}\mu^-\bar{\nu}_\mu X)(t) \quad \propto e^{-\Gamma_d t}[1-\cos(\Delta m_d t)]$$

$$\mathcal{A}(t) = \frac{N^{\text{unmix}} - N^{\text{mix}}}{N^{\text{unmix}} + N^{\text{mix}}} = \cos(\Delta m_d t)$$

- Production flavour determined using flavour tagging algorithms, exploiting $B^0(\overline{B}^0)$ hadronisation process

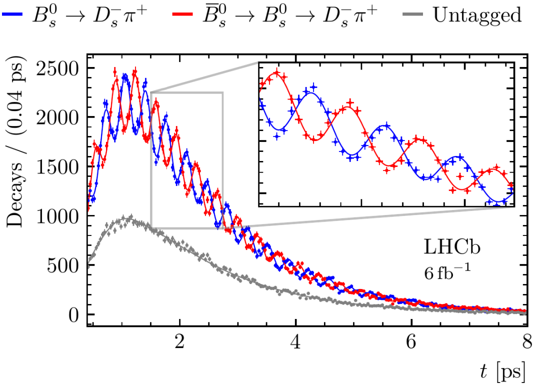


Precision measurement of B^0 mixing by LHCb II



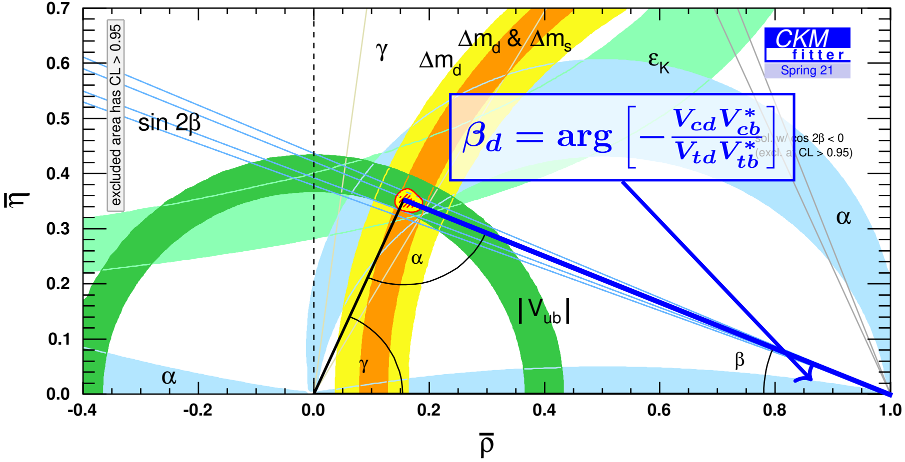
- Time-dep. asymmetry $\mathcal{A}(t)$, in different tagging categories (a)-(e)
- Most precise measurement of B^0 mixing frequency [EPJC 76 (2016) 412]
  $\Delta m_d = (505.0 \pm 2.1_{\text{stat.}} \pm 1.0_{\text{syst.}})\,\text{ns}^{-1}$

Precision measurement of B_s^0 mixing by LHCb



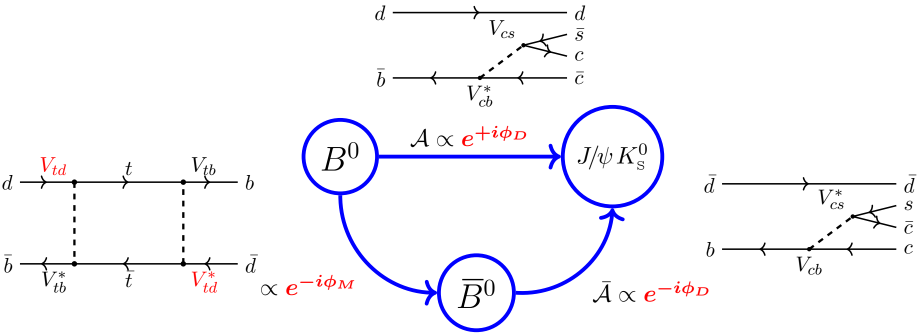
- B_s^0 mixing much faster than B^0 mixing as $|V_{ts}| \sim \lambda^2 \gg |V_{td}| \sim \lambda^3$
- B_s^0 mixing first observed CDF [PRL 97 (2006) 242003]
- Most precise Δm_s mesurement by LHCb [Nature Phys. 18 (2022) 1]
  $\Delta m_s = (17.7683 \pm 0.0051_{\text{stat.}} \pm 0.0032_{\text{syst.}})\,\text{ps}^{-1}$
- SM predictions are much less precise than experiment, theory limited

CP-violating B^0 mixing phase: β_d



$$\beta_d = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

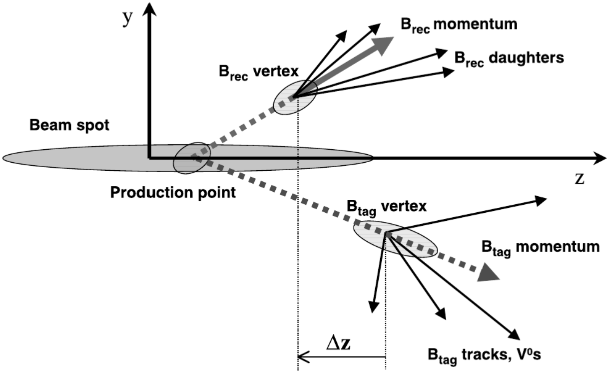
β_d determination with $B^0 \to J/\psi K^0_S$



- Observables are squares of QM amplitudes $\mathcal{A}\bar{\mathcal{A}} = |A|^2 e^{i\phi} e^{-i\phi} = |\mathcal{A}|^2$
  $\to$ Phases can only be measured through interference effects
- Here phase difference $\phi_M - 2\phi_D = 2\beta_d$ with $\beta_d = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$
- Time-dep. CPV in interference between mixing and decay [see backup]

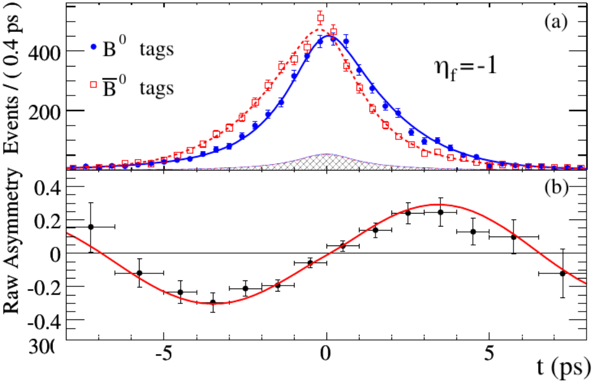
$$\mathcal{A}_{\rm CP}(t) = \frac{\Gamma(\bar{B}^0 \to J/\psi K^0_S, t) - \Gamma(B^0 \to J/\psi K^0_S, t)}{\Gamma(\bar{B}^0 \to J/\psi K^0_S, t) + \Gamma(B^0 \to J/\psi K^0_S, t)} = \sin(\Delta m_d t)\sin(2\beta_d)$$

Measurement of β_d at B factories



- At B factories: $B^0\overline{B}^0$ system correlated until decay
- Perform analysis dependent on $\Delta t = t_{\text{sig}} - t_{\text{tag}}$:
  $$\mathcal{A}_{\text{CP}}(\Delta t) = \frac{\Gamma(\overline{B}^0\to J/\psi\, K^0_{\text{S}},\Delta t)-\Gamma(B^0\to J/\psi\, K^0_{\text{S}},\Delta t)}{\Gamma(\overline{B}^0\to J/\psi\, K^0_{\text{S}},\Delta t)+\Gamma(B^0\to J/\psi\, K^0_{\text{S}},\Delta t)} = \sin(\Delta m_d \Delta t)\sin(2\beta_d)$$
- Observation of CPV in the B system by BaBar [PRL 87 (2001) 091801] and Belle [PRL 87 (2001) 091802]

Precise β_d measurements from the B-factories

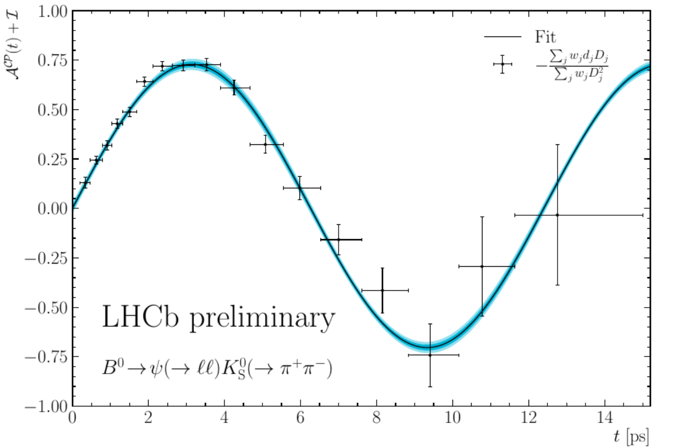


- Top: $\overline{B}^0$ and B^0 tag, Bottom: $\mathcal{A}_{\rm CP}^{\rm raw}(\Delta t)$ [PRD 79 (2009) 072009]
- Precise legacy measurements from the B-factories

$\sin 2\beta_d = 0.687 \pm 0.028_{\rm stat.} \pm 0.012_{\rm syst.}$ BaBar [PRD 79 (2009) 072009]

$\sin 2\beta_d = 0.667 \pm 0.023_{\rm stat.} \pm 0.012_{\rm syst.}$ Belle [PRL 108 (2012) 171802]

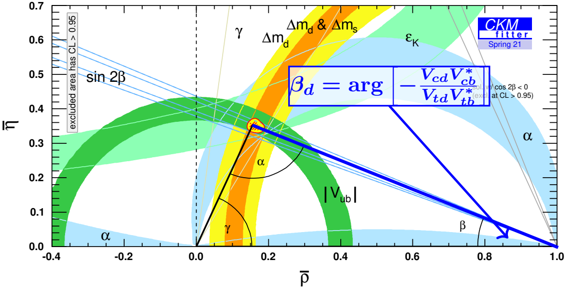
Most precise single β_d measurement by LHCb

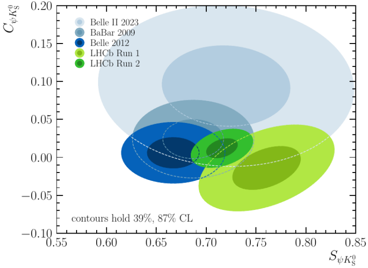


- Most precise single measurement using LHCb Run 2 data

$$\sin 2\beta_d = 0.714 \pm 0.015(\text{stat}) \pm 0.007(\text{syst})$$ (LHCb-PAPER-2023-013, prelim.)

B^0 mixing phase: β_d world average





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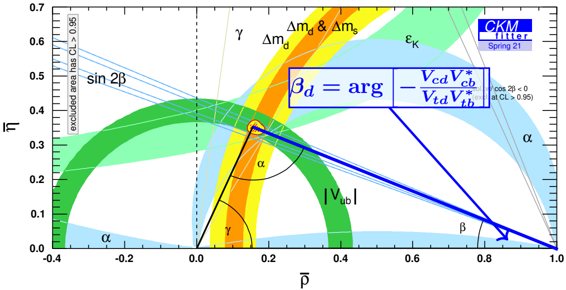
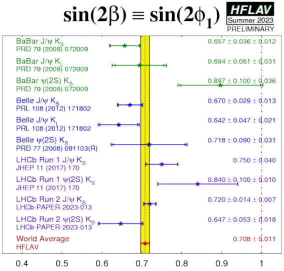
- Run 1+2 combination from LHCb

  $\sin 2\beta_d = 0.723 \pm 0.014(\text{stat+syst})$ (LHCb-PAPER-2023-013, prelim.)

- Prelim. world average $\sin 2\beta_d = 0.708 \pm 0.011$ (HFLAV)

B^0 mixing phase: β_d world average





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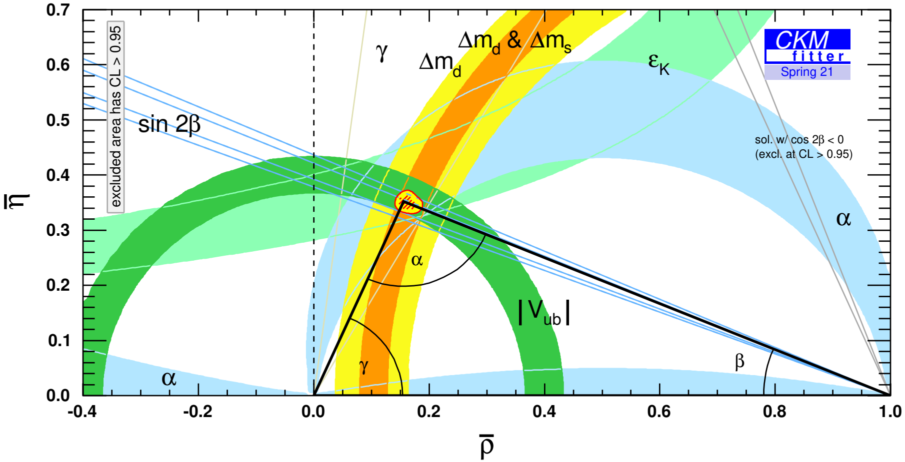
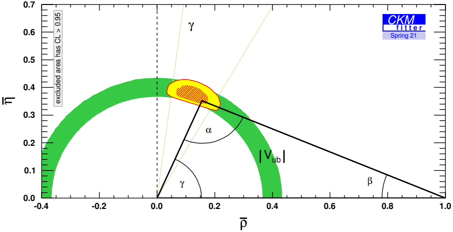
Global combination of CKM measurements



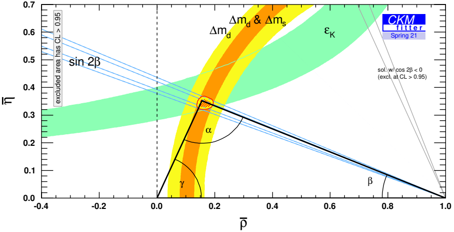
- Overall good compatibility of measurements

NP searches with CKM measurements: Tree *vs.* Loop

Tree-level determinations

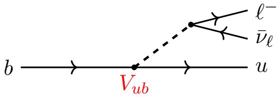


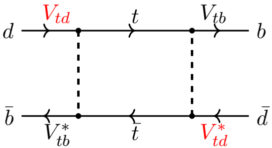
Loop-level determinations



e.g. $|V_{ub}|/|V_{cb}|$, angle γ

e.g. $\Delta m_{(s)}$, $\sin 2\beta_d$





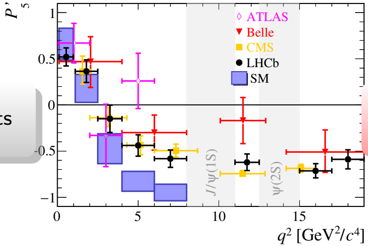
SM contribution dominant

NP contributions could be sign.

- Consistency between tree- and loop-level measurements, but still room for NP
- Need to improve precision, particularly for tree-level determinations
  Aim: $\sigma(\gamma) < 1\%$ in LHCb Upgrade II

Flavour in the SM

NP searches with flavour



The experiments

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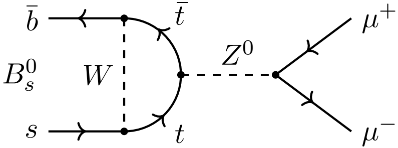
Rare decays

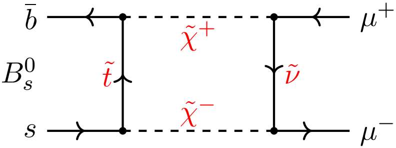
Future prospects

The very rare decay $B_s^0 \to \mu^+\mu^-$

SM diagram



Possible NP contribution



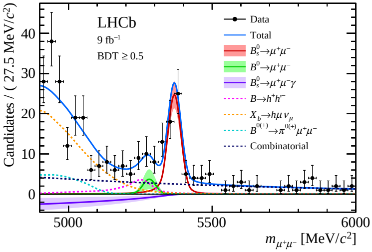
- Loop-, helicity- and CKM suppressed
- Purely leptonic final state, theoretically and experimentally very clean
- Precise SM prediction [PRL 112 (2014) 101801] [JHEP 10 (2019) 232]

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (3.66 \pm 0.14) \times 10^{-9}$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-) = (1.03 \pm 0.05) \times 10^{-10}$$

- Very sensitive to new scalar sector (*e.g.* extended Higgs sector, SUSY)

Measurements of $B^0_{(s)} \to \mu^+\mu^-$

[PRL 128 (2022) 041801] [PRD 105 (2022) 012010]



- Recent LHCb measurement [PRL 128 (2022) 041801] [PRD 105 (2022) 012010]

$$\mathcal{B}(B^0_s \to \mu^+\mu^-) = (3.09^{+0.46+0.15}_{-0.43-0.11}) \times 10^{-9}$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-) = (1.2^{+0.8}_{-0.7} \pm 0.1) \times 10^{-10} \quad (\mathcal{B} < 2.6 \times 10^{-10} \text{ @ } 95\%\,\text{CL})$$

- New precise CMS result (full Run 2) [PLB 842 (2023) 137955]

$$\mathcal{B}(B^0_s \to \mu^+\mu^-) = (3.83^{+0.38}_{-0.36}(\text{stat})^{+0.19}_{-0.16}(\text{syst})^{+0.14}_{-0.13}(f_s/f_u)) \times 10^{-9}$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-) = (0.37^{+0.75+0.08}_{-0.67-0.09}) \times 10^{-10} \quad (\mathcal{B} < 1.9 \times 10^{-10} \text{ @ } 95\%\,\text{CL})$$

- Overall good agreement with SM prediction

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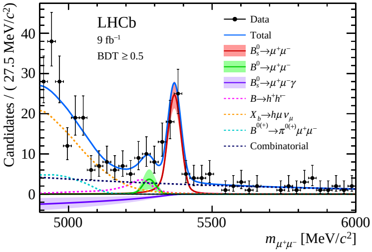
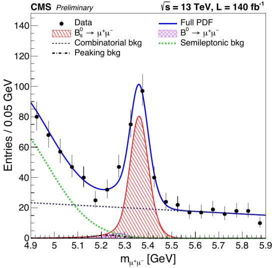


[PLB 842 (2023) 137955]



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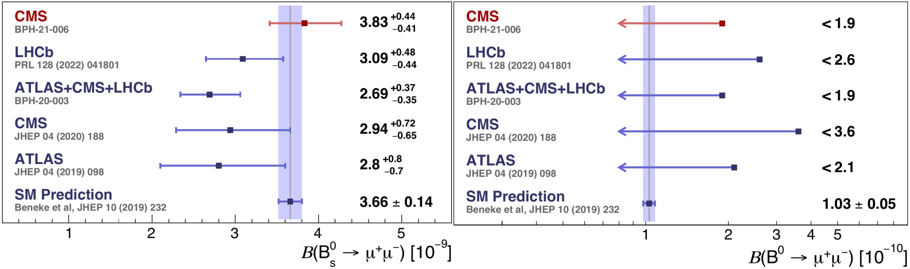

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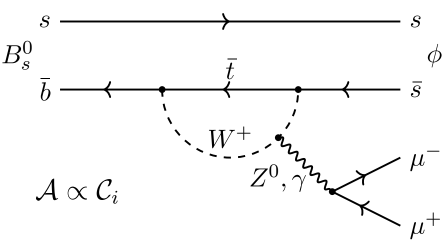
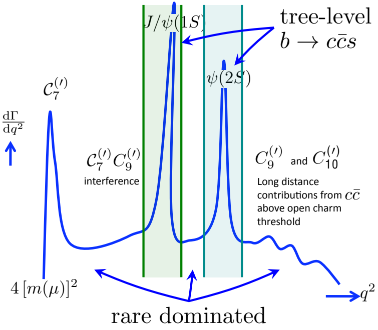
Branching fraction of $B_s^0 \to \phi\mu^+\mu^-$





- $\mathcal{B}$ of semileptonic $b \to s\mu^+\mu^-$ decays can also be affected by NP
- Central: $q^2 = m(\ell^+\ell^-)^2$, different operators contribute depending on q^2
- At $q^2 = m^2_{J/\psi}$ important tree-level $b \to c\bar{c}s$ normalisation mode $B_s^0 \to J/\psi\,\phi$
- SM predictions directly affected by significant form factor uncertainties
  Low q^2: LCSRs [PRD 71 (2005) 014029] [JHEP 08 (2016) 98] [PRD 75 (2007) 054013] [JHEP 09 (2010) 089] High q^2: Lattice [PRD 89 (2014) 094501] [PRD 88 (2013) 054509]

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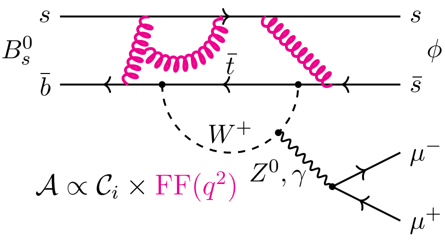

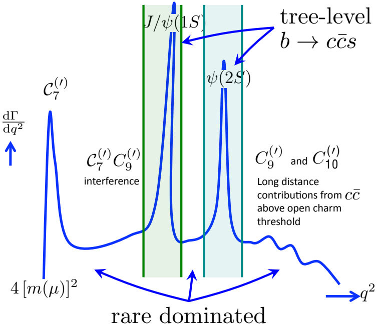

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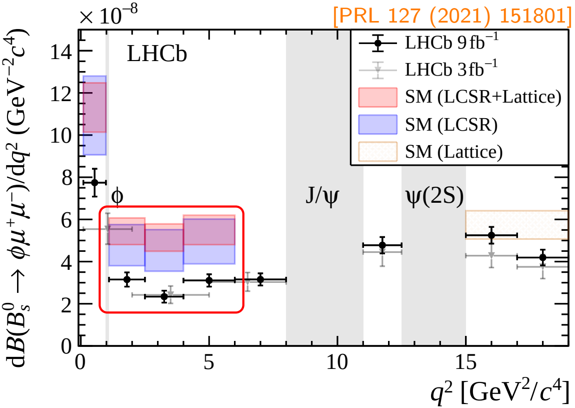
$B_s^0 \to \phi\mu^+\mu^-$ branching fraction

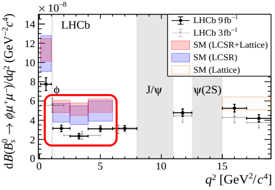


SM LCSR
[JHEP 08 (2016) 098]
[EPJC 75 (2015) 8]
SM LCSR+Lattice
[PRL 112 (2014) 212003]
[PoS Lattice 2014 372]

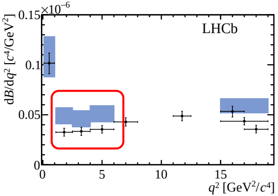
- Recent LHCb measurement using full Run 1+2 sample [PRL 127 (2021) 151801]
- $d\mathcal{B}(B_s^0 \to \phi\mu^+\mu^-, 1.1 < q^2 < 6\,\mathrm{GeV}^2/c^4) = (2.88 \pm 0.21)^{-8}\,\mathrm{GeV}^2/c^4$
- Tension with SM at $3.6\,\sigma$ (LCSR+Lattice) and $1.8\,\sigma$ (LCSR only)

Low $\mathcal{B}$ also found for other $b \to s\mu^+\mu^-$ decays

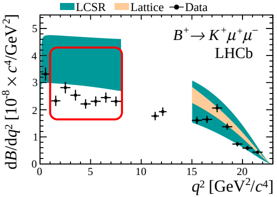
LHCb $B_s^0 \to \phi\mu^+\mu^-$ [PRL 127 (2021) 151801]



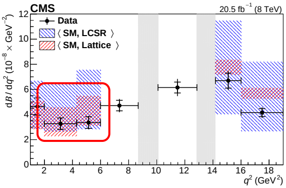
LHCb $B^0 \to K^{*0}\mu^+\mu^-$ [JHEP 11 (2016) 047]



LHCb $B^+ \to K^+\mu^+\mu^-$ [JHEP 06 (2014) 133]



CMS $B^0 \to K^{*0}\mu^+\mu^-$ [PLB 753 (2016) 424]



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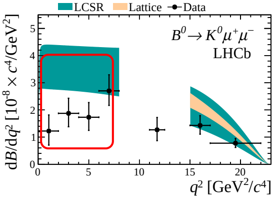
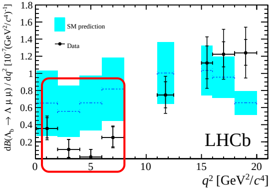


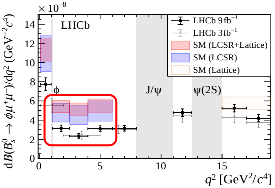
LHCb $\Lambda_b^0 \to \Lambda\mu^+\mu^-$ [JHEP 06 (2015) 115]



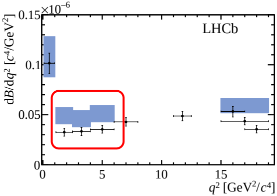
- Data consistently below SM predictions (particularly at low q^2)
- Tensions at $1\text{–}3\sigma$ level, SM predictions exhibit sizeable had. uncertainties
- Exciting recent developements on non-local corrections [JHEP 09 (2022) 133] and new results from Lattice QCD [HPQCD, PRD 107 (2023) 1]

Low $\mathcal{B}$ also found for other $b \to s\mu^+\mu^-$ decays

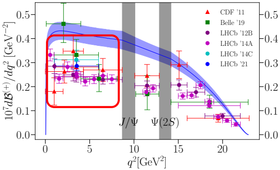
LHCb $B_s^0 \to \phi\mu^+\mu^-$ [PRL 127 (2021) 151801]



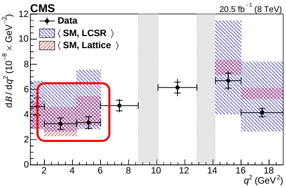
LHCb $B^0 \to K^{*0}\mu^+\mu^-$ [JHEP 11 (2016) 047]



Lattice $B^+ \to K^+\mu^+\mu^-$ [arXiv:2207.13371]



CMS $B^0 \to K^{*0}\mu^+\mu^-$ [PLB 753 (2016) 424]



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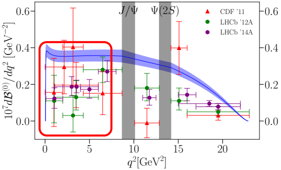

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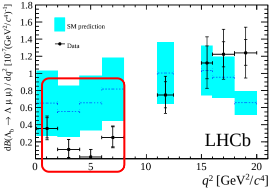

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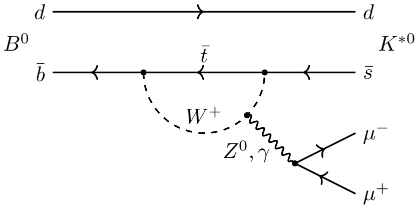
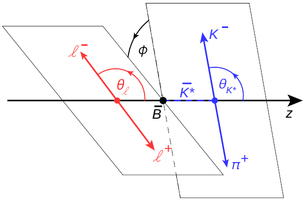
Angular analysis of $B^0 \to K^{*0}[\to K^+\pi^-]\mu^+\mu^-$



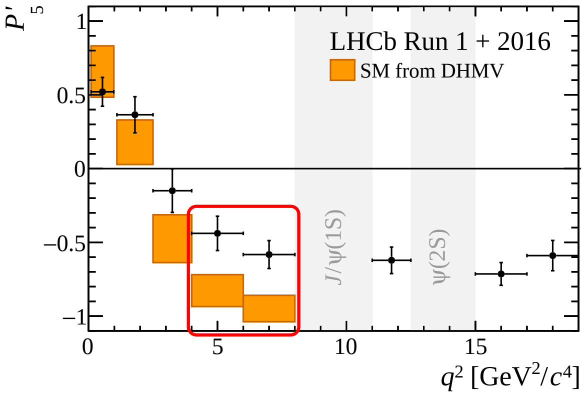


- Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m^2_{\mu\mu}$
- $$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K + \tfrac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell - F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi + \tfrac{4}{3}A_\mathrm{FB}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi + S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$
- Angular observables $F_\mathrm{L}, A_\mathrm{FB}, S_i$ sensitive to NP contributions
- Perform ratios of observables where form factors cancel at leading order
  Example: $P_5' = \frac{S_5}{\sqrt{F_\mathrm{L}(1-F_\mathrm{L})}}$ [S. Descotes-Genon *et al.*, JHEP, 05 (2013) 137]

For completeness

Angular observable P_5' from $B^0 \to K^{*0}\mu^+\mu^-$

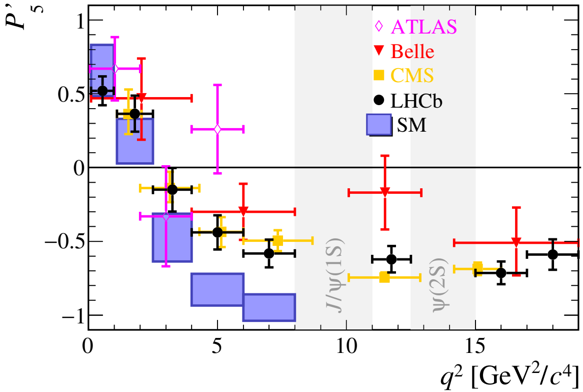
[PRL 125 (2020) 011802]



- In q^2 bins $[4.0, 6.0]$ and $[6.0, 8.0]\,\mathrm{GeV}^2/c^4$ local tensions of $2.5\,\sigma$ and $2.9\,\sigma$
- Global $B^0 \to K^{*0}\mu^+\mu^-$ analysis finds deviation corresponding to $3.3\,\sigma$
- [LHCb, PRL 125 (2020) 011802] consistent with [Belle, PRL 118 (2017) 111801] [CMS-PAS-BPH-21-002] [ATLAS, JHEP 10 (2018) 047]

Angular observable P'_5 from $B^0 \to K^{*0}\mu^+\mu^-$

adapted from [PIPNP 120 (2021) 103885]



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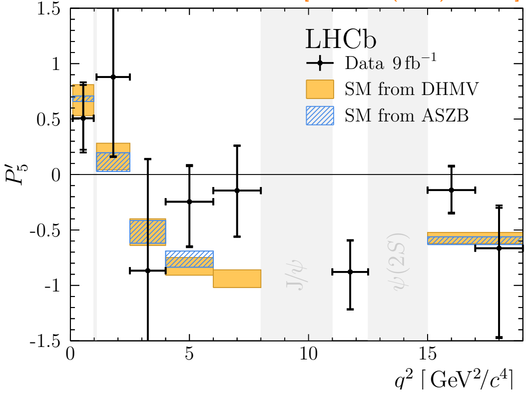
Angular observable P_5' from $B^+ \to K^{*+}(\to K^0_S\pi^+)\mu^+\mu^-$

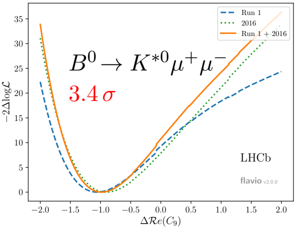
[PRL 126 (2021) 161802]



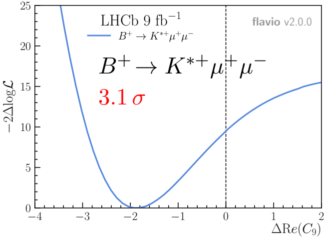
- Recent LHCb measurement using Run 1+2 data [PRL 126 (2021) 161802]
- Global tension corresponding to $3.1\,\sigma$, consistent with $B^0 \to K^{*0}\mu^+\mu^-$
- Angular analysis ($F_{\rm L}$+$A_{\rm FB}$) also by CMS [JHEP 04 (2021) 124]

Consistency of $b \to s\mu^+\mu^-$ angular analyses

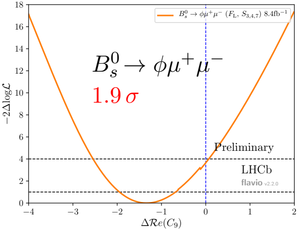
[PRL 125 (2020) 011802]



[PRL 126 (2021) 161802]



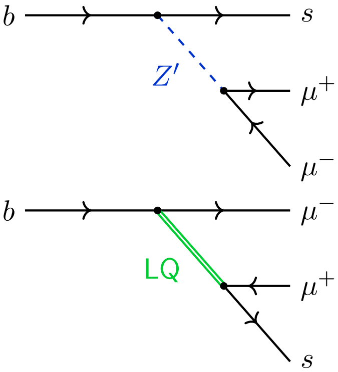
[JHEP 11 (2021) 043]



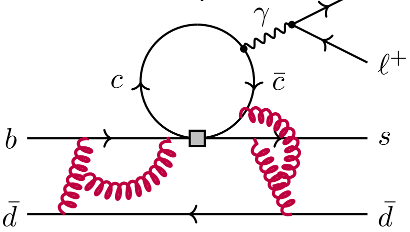
- Use `flavio` [arXiv:1810.08132] to determine tension with SM hypothesis
- Variation of vector coupling $\mathcal{R}e(\mathcal{C}_9)$ results in improved description of data
- Consistent trend for $B^0 \to K^{*0}\mu^+\mu^-$ [PRL 125 (2020) 011802], $B^+ \to K^{*+}\mu^+\mu^-$ [PRL 126 (2021) 161802] and $B_s^0 \to \phi\mu^+\mu^-$ [JHEP 11 (2021) 043] angular observables
- However, interpretation not clear due to significant hadronic uncertainties

New Physics or hadronic effect?

Possible NP



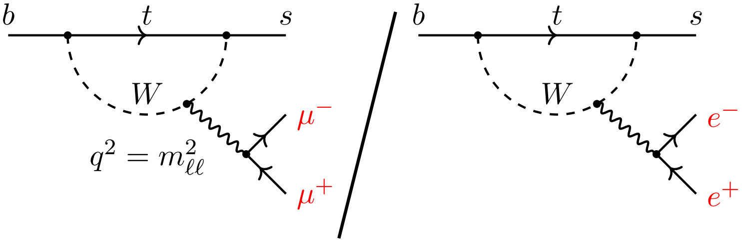
SM $c\bar{c}$ loop



- Tensions received significant attention in theory community
- Possible explanations for shift in $\mathcal{C}_9$
  - NP *e.g.* Z' [Gauld et al.] [Buras et al.] [Altmannshofer et al.] [Crivellin et al.] Leptoquarks [Hiller et al.] [Biswas et al.] [Buras et al.] [Gripaios et al.]
  - Hadronic *charm-loop* contributions
- Look for clean observables without *charm-loop* pollution!

  $\rightarrow$ Lepton Flavour Universality tests

Lepton Flavour Universality tests in $b \to s\ell\ell$ decays



- Lepton flavour universality central property of SM
- Testable using ratios of branching fractions of rare $b \to s\ell^+\ell^-$ decays:

$$R_{K,K^*} = \frac{\mathcal{B}(B^{(+,0)} \to K^{(+,*0)}\mu^+\mu^-)}{\mathcal{B}(B^{(+,0)} \to K^{(+,*0)}e^+e^-)}$$

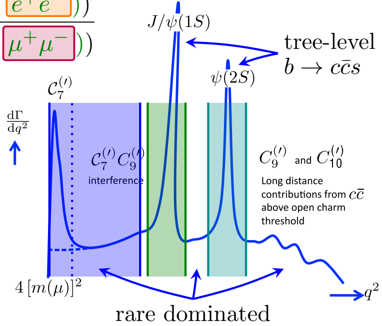
- Exactly unity in SM, differences only through lepton mass effects
- Dominant SM uncertainty: QED corrections $\mathcal{O}(1\,\%)$ [EPJC 76 (2016) 440]
- Hadronic uncertainties (form-factors and $c\bar{c}$-loop) cancel in the ratio

Analysis strategy: Double ratio (Example: R_K)

- Analysis strategy: Double ratio of rare modes $B^+ \to K^+\ell^+\ell^-$ with resonant decays $B^+ \to K^+ J/\psi(\to \ell^+\ell^-)$:

$$R_K = \frac{\mathcal{B}(B^+ \to K^+ \mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+ e^+e^-)} \times \overbrace{\frac{\mathcal{B}(B^+ \to K^+ J/\psi(\to e^+e^-))}{\mathcal{B}(B^+ \to K^+ J/\psi(\to \mu^+\mu^-))}}^{r^{-1}_{J/\psi}=1 \text{ [PRD 88 (2013) 3]}}$$

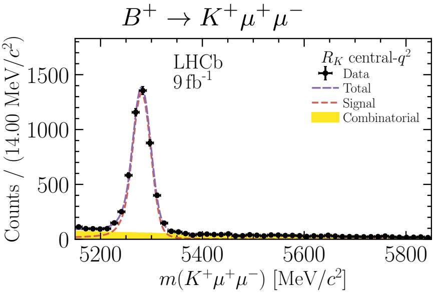
- Electron and Muon reconstruction very different at LHCb
- Efficiencies from corrected simulation
- Double ratio cancels most experimental systematic effects in efficiency ratios

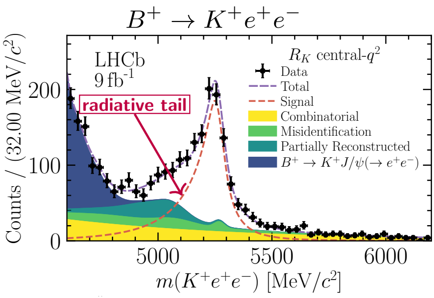


- Important cross-checks: $r_{J/\psi} = \frac{\mathcal{B}(B^+\to K^+ J/\psi(\to\mu^+\mu^-))}{\mathcal{B}(B^+\to K^+ J/\psi(\to e^+e^-))}$ and

$$R_{\psi(2S)} = \frac{\mathcal{B}(B^+\to K^+\psi(2S)(\to\mu^+\mu^-))}{\mathcal{B}(B^+\to K^+\psi(2S)(\to e^+e^-))} \times \frac{\mathcal{B}(B^+\to K^+ J/\psi(\to e^+e^-))}{\mathcal{B}(B^+\to K^+ J/\psi(\to\mu^+\mu^-))}$$

Experimental challenges for electron modes at LHCb



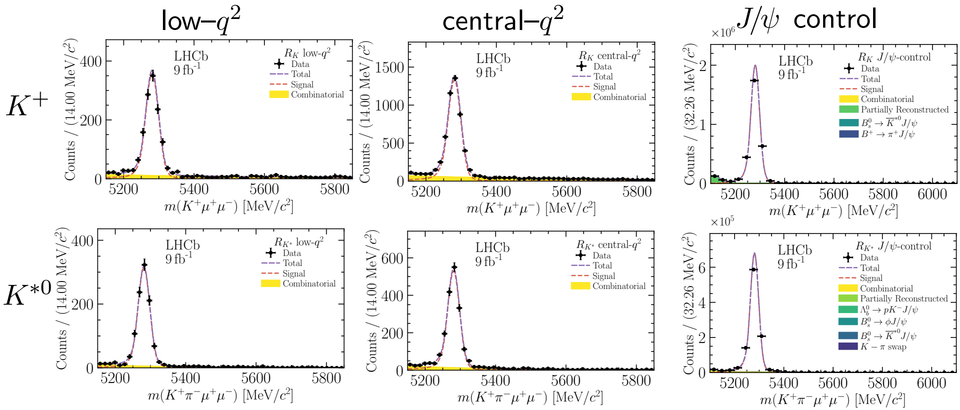


[arXiv:2212.09152] [arXiv:2212.09153]

Experimental Challenges for electron modes:

1. Low e trigger efficiencies due to higher thresholds compared to muons
2. Electrons strongly emit Bremsstrahlung traversing material
   Brem-γ recovery has limited efficiency and degrades mass resolution
3. Contribution from several background sources, bkg. modeling critical

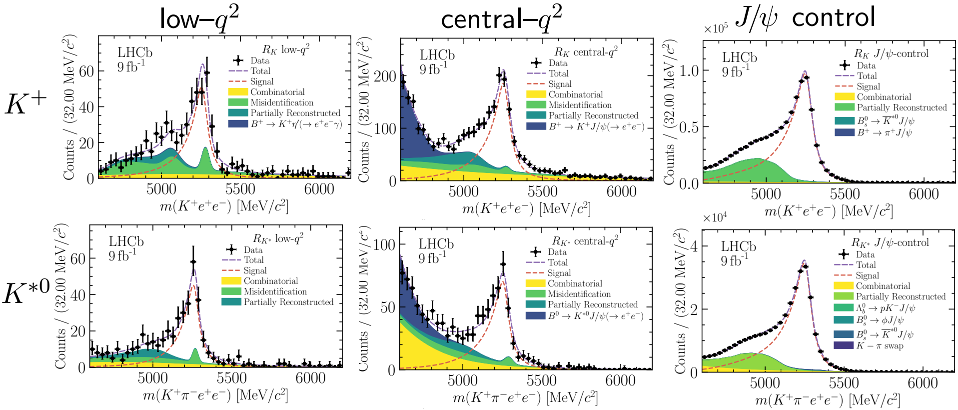
Muon mode fits



- Muon mode is very clean!
- Muon branching fraction compatible with published results

[JHEP 06 (2014) 133] [JHEP 11 (2016) 047]

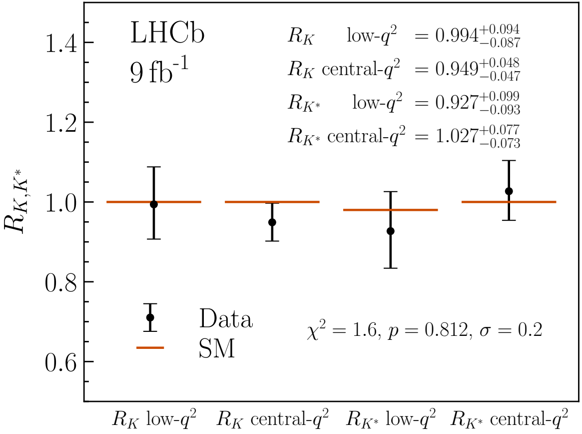
Electron mode fits



- Brems. tails from J/ψ entering rare modes constrained in sim. fit
- Partially reconstructed bgk. from $K^{*0}e^+e^-$ constrained in $K^+e^+e^-$

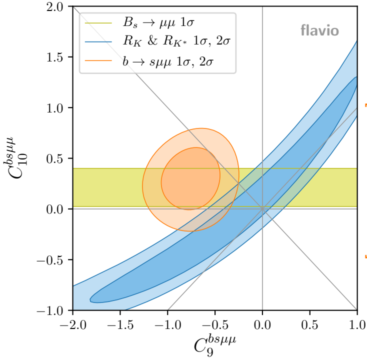
R_K and R_{K^*} results

[arXiv:2212.09152] [arXiv:2212.09153]



- Most precise test of LFU in $b \to s\ell^+\ell^-$ transitions
- Compatible with the SM at $0.2\,\sigma$ using simple χ^2 test

Interpretation in global fits



[arXiv:2212.10497]

- $b \to s\ell^+\ell^-$ data can be interpreted using *global fits* of Wilson coefficients
- Assuming NP only in muon-sector ($\mathcal{R}e(C_9^{bs\mu\mu})$ and $\mathcal{R}e(C_{10}^{bs\mu\mu})$) reveals tension between $b \to s\mu^+\mu^-$ angular and $\mathcal{B}$ measurements and R_{K,K^*}
- Can be resolved in presence of LFU NP which does not affect R_{K,K^*}
- Data prefers negative $C_9^{\text{univ.}}$, tension depends on hadronic uncertainties

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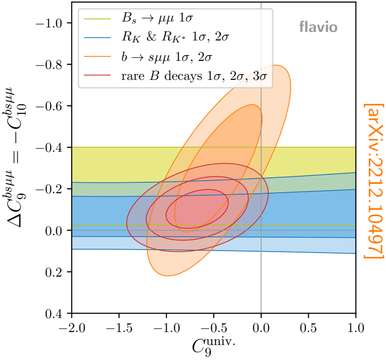

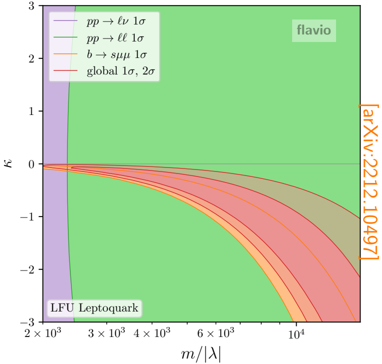

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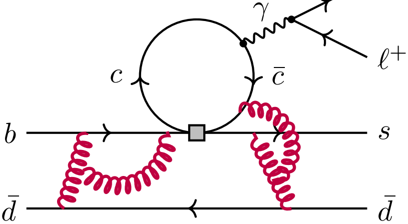
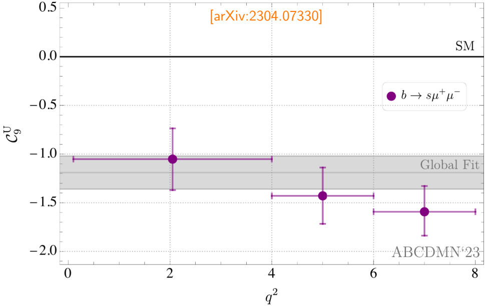
Disentangling hadronic contributions from potential NP

SM *charm-loop*





- Hadronic contributions affect angular observables and branching fractions disentangling them from NP requires work from theory and experiment
- Progress on theory side:
  - Form-factors are systematically improved on the lattice [PRD 107 (2023) 1]
  - Recent more precise estimation of charm-loop effect [JHEP 09 (2022) 133]
  - . . . but still a lot of discussion, see *e.g.* [arXiv:2212.10516]
- Experimental approaches exploiting q^2-dependence underway:
  - charm-loop rises towards $c\bar{c}$-resonances
  - NP q^2-independent

Heavy Flavour Physics

Flavour in the SM

NP searches with flavour

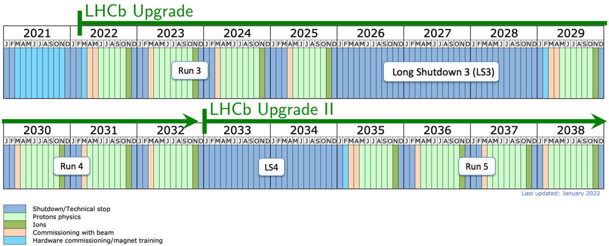
The experiments

CKM metrology and CP violation

Rare decays

Future prospects

Outlook



Integrated luminosity $[\text{fb}^{-1}]$	LHCb	ATLAS/CMS
Run 1	3	25
Run 2	9	+140
Run 3	23	+300
Run 4	50	+300/year
Run 5(+)	300	+300/year

- Measurements largely statistically dominated → requires more data
- Updates with the full Run 1+2 data still ongoing
- Run 3 started in 2022 with upgraded LHCb detector [Upgrade TDR]
- Unprecedented precision in the HL-LHC era following LS3 [Yellow Report 7 (2019) 867]
- LHCb Upgrade II installation during LS4 [arXiv:1808.08865] [arXiv:1808.08865] → $300\,\text{fb}^{-1}$
- Belle II has already taken $> 500\,\text{fb}^{-1}$, aims for $50\,\text{ab}^{-1}$
- Belle II will deliver important complementary results

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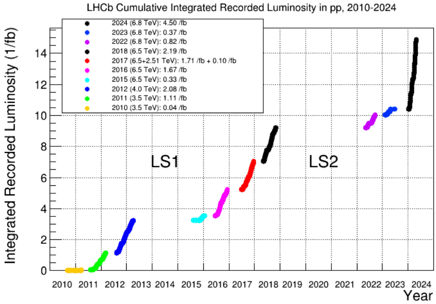

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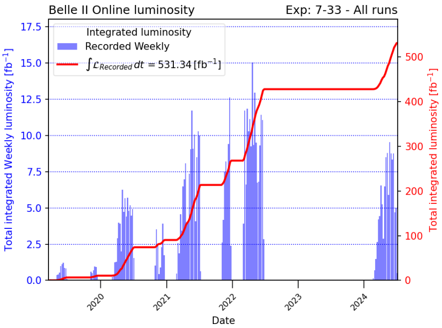

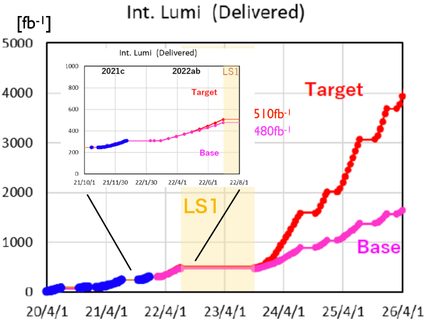

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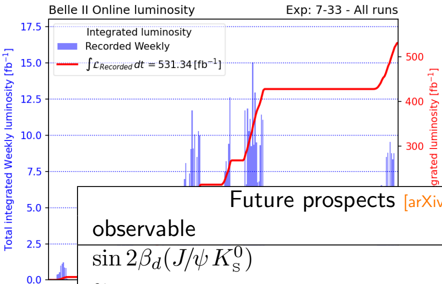

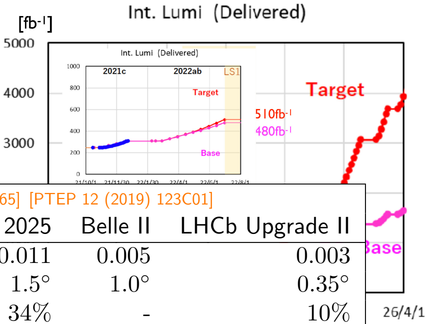

Future prospects [arXiv:1808.08865] [PTEP 12 (2019) 123C01]

observable	LHCb 2025	Belle II	LHCb Upgrade II
$\sin 2\beta_d(J/\psi K^0_S)$	0.011	0.005	0.003
γ	$1.5°$	$1.0°$	$0.35°$
$\mathcal{B}(B^0\to\mu^+\mu^-)/\mathcal{B}(B^0_s\to\mu^+\mu^-)$	34%	-	10%
$R_K(1<q^2<6\,\mathrm{GeV}^2/c^4)$	0.025	0.036	0.007
$R_{K^*}(1<q^2<6\,\mathrm{GeV}^2/c^4)$	0.031	0.032	0.008

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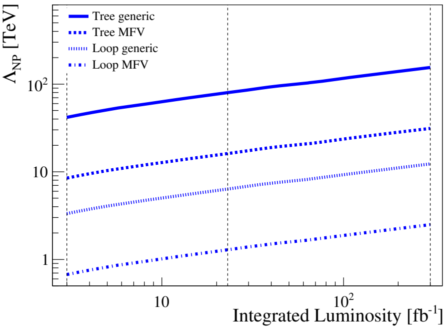
NP reach of rare decays in the LHCb Upgrade(s)

$\Lambda_{\rm NP}$ exclusion limits with $R_{K^{(*)}}$



[Upgrade II Physics case] [Physics of the HL-HLC WG 4]

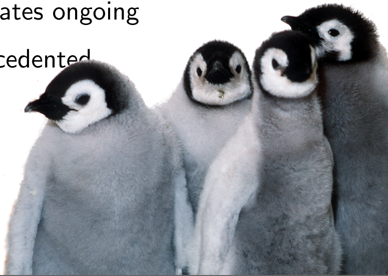
$\int \mathcal{L}dt$	$3\,{\rm fb}^{-1}$	$23\,{\rm fb}^{-1}$	$300\,{\rm fb}^{-1}$
R_K and R_{K^*} measurements			
$\sigma(\mathcal{C}_9)$	0.44	0.12	0.03
$\Lambda_{\rm NP}^{\rm tree\,generic}$ [TeV]	40	80	155
$\Lambda_{\rm NP}^{\rm tree\,MFV}$ [TeV]	8	16	31
$\Lambda_{\rm NP}^{\rm loop\,generic}$ [TeV]	3	6	12
$\Lambda_{\rm NP}^{\rm loop\,MFV}$ [TeV]	0.7	1.3	2.5
$B^0 \to K^{*0}\mu^+\mu^-$ angular analysis			
$\sigma^{\rm stat}(S_i)$	0.034–0.058	0.009–0.016	0.003–0.004
$\sigma(\mathcal{C}'_{10})$	0.31	0.15	0.06
$\Lambda_{\rm NP}^{\rm tree\,generic}$ [TeV]	50	75	115
$\Lambda_{\rm NP}^{\rm tree\,MFV}$ [TeV]	10	15	23
$\Lambda_{\rm NP}^{\rm loop\,generic}$ [TeV]	4	6	9
$\Lambda_{\rm NP}^{\rm loop\,MFV}$ [TeV]	0.8	1.2	1.9

- $\sigma(\mathcal{C}_i)$ from Flavio [arXiv:1810.08132] using extrap. $\sigma_{\rm exp.}$ of current measurements
- Exclusion limits for NP scale[4] $\Lambda_{\rm NP} \propto \sqrt{1/\sigma(\mathcal{C}_{\rm NP})} \propto \sqrt[4]{\int \mathcal{L}dt}$
- Precision flavour observables probe scales far beyond $\sqrt{s} = 14\,{\rm TeV}$

[4] Naive scaling: Assume identical scaling of syst. uncertainties

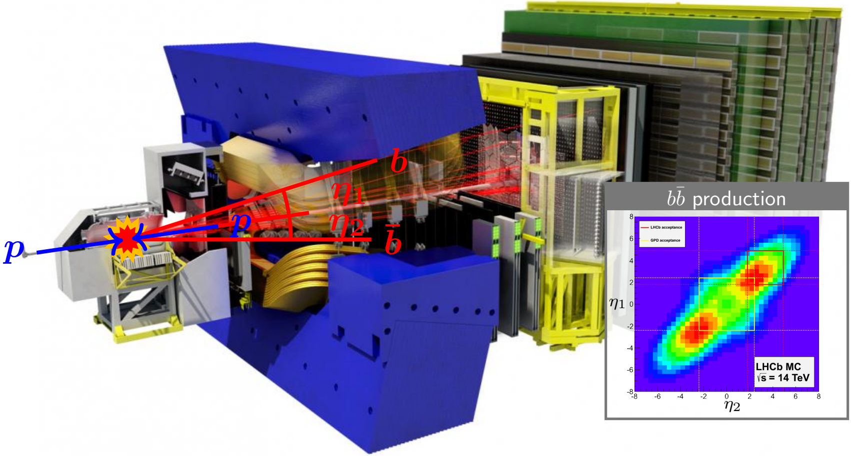
Conclusions

- Heavy flavour physics an extremely interesting field, allows for powerful tests of the SM through precision measurements
- These indirect searches for NP can reach high NP scales (up to $\sim 100\,\mathrm{TeV}$), complementary to direct searches
- Most measurements in excellent agreement with SM, but some tensions exist
- However, tensions in $\mathcal{B}$ and angular observables not theoretically clean, Progress requires synergistic work between experiment and theory!
  - Work ongoing to improve theory predictions
  - Measurements statistically limited, updates ongoing
- LHC Run 3 just started, will allow for unprecedented reach with brand new LHCb detector
- Belle 2 will provide important additional and complementary information




Backup

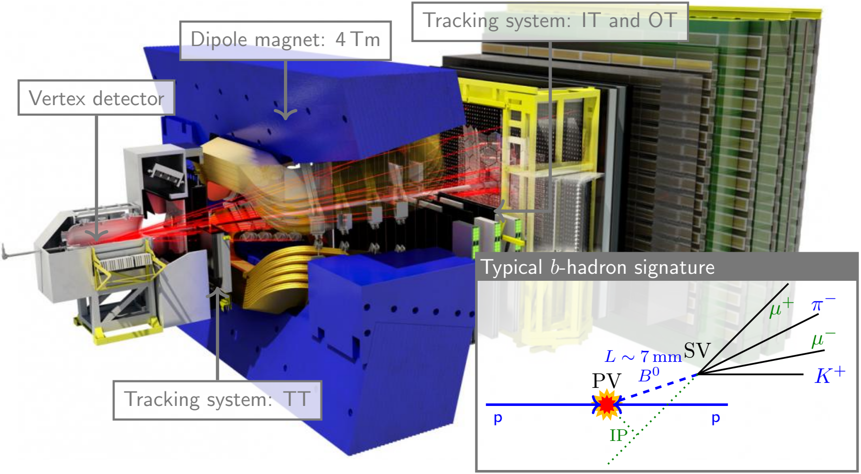
The LHC as heavy flavour factory



- $b\bar{b}$ produced in forward/backward dir. $\rightarrow$ Forward spectrometer $2 < \eta < 5$
- Large $b\bar{b}$ $(c\bar{c})$ production cross-sections allows precision measurements of rare decays
- Run 1: $3\,\mathrm{fb}^{-1}$, Run 2: $6\,\mathrm{fb}^{-1}$

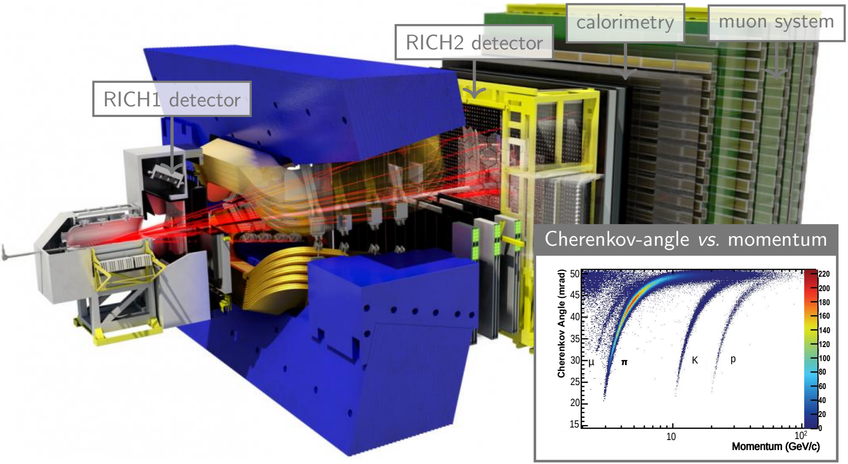
	$\sqrt{s} = 7\,\mathrm{TeV}$	$\sqrt{s} = 13\,\mathrm{TeV}$
$\sigma^{\mathrm{acc.}}_{b\bar{b}}\,[\mu\mathrm{b}]$	75.3 ± 14.1	135.8 ± 14.1
$\sigma^{\mathrm{acc.}}_{c\bar{c}}\,[\mu\mathrm{b}]$	1419 ± 134	2940 ± 241
Refs.	[PLB 694:209 (2010)] [NPB 871 (2013) 1-20]	[JHEP 10 (2015) 172] [arXiv:1510.01707]

The LHCb detector: Tracking



- Excellent *impact parameter* resolution $\sim 20\,\mu\mathrm{m}$
  $\rightarrow$ Identify secondary vertices from heavy flavour decays
- Momentum resolution $\frac{\Delta p}{p} \sim 0.5 - 1\%$ $\rightarrow$ Low combinatorial background

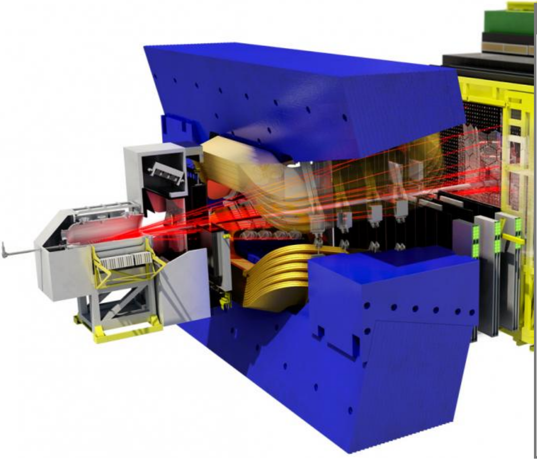
The LHCb detector: Particle identification

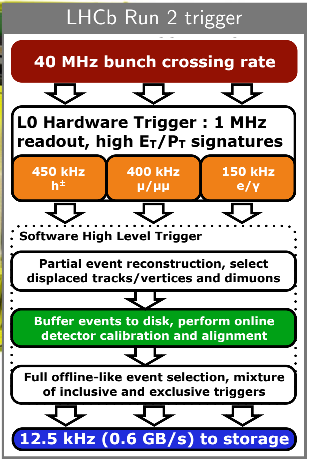


- Good $K\pi$ separation through RICH detectors: $\epsilon_{K\to K}\sim 95\%$, $\epsilon_{\pi\to K}\sim 5\%$
- Excellent muon identification: $\epsilon_{\mu\to\mu}\sim 97\%$, $\epsilon_{\pi\to\mu}\sim 1-3\%$

→ Reject backgrounds from misidentified B decays (peaking backgrounds)

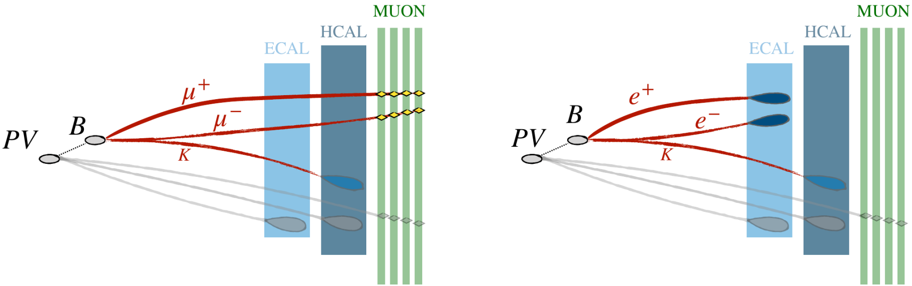
The LHCb detector: Flexible trigger system





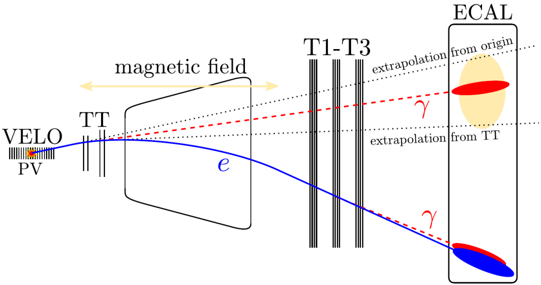
- Low trigger thresholds ($p_{\mathrm{T}}(\mu) > 1.76\,\mathrm{GeV}/c$ in 2012) and high efficiencies: $\epsilon^{\text{trig}}_{B\to\mu\mu X} \sim 90\%$, $\epsilon^{\text{trig}}_{\text{had.}} \sim 30\%$
- Run 2: Full online detector calibration and alignment
- LHCb Upgrade: L0 (hardware) replaced, full software trigger

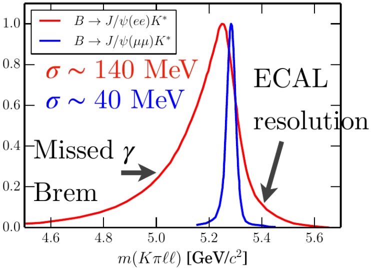
Experimental challenge: 1. Electron trigger



- Trigger signatures for muon and electron modes very different
- Lower L0 p_{T} thresholds for muons $(1.5\text{–}1.8\,\mathrm{GeV}/c)$ compared to electrons $(2.5\text{–}3.0\,\mathrm{GeV})$ $\rightarrow$ challenging for e^+e^- modes
- Combine exclusive trigger categories to improve ϵ for electron modes:
  1. Trigger on rest of event (independent of signal)
  2. Trigger on e/μ from signal

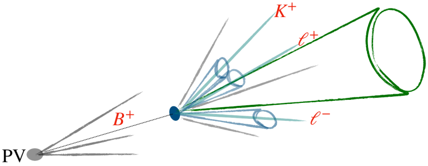
Experimental challenge: 2. Bremsstrahlung

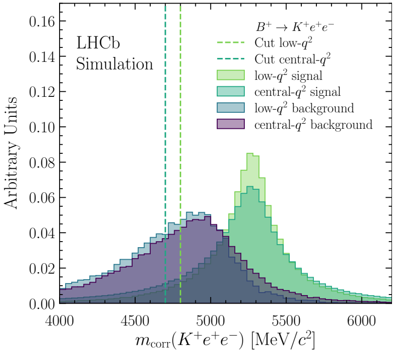




- Correct electron momentum by adding matching photons ($E_\mathrm{T} > 75\,\mathrm{MeV}/c^2$) reconstructed in the ECAL
- Bremsstrahlung recovery $\sim 50\%$ efficient, well simulated
- Bremsstrahlung reconstruction impacts momentum resolution
  $\rightarrow$ higher background pollution and more sensitive to bkg. modeling

Experimental challenge: 3. Background suppression





- Combinatorial backgrounds: suppressed using multivariate classifier using kinematic and vertex quality information
- Partially reconstructed:
  1. MVA using track/vertex isolation
  2. Corrected mass exploiting PV/SV
- Specific backgrounds: vetos combining PID and kinematic criteria

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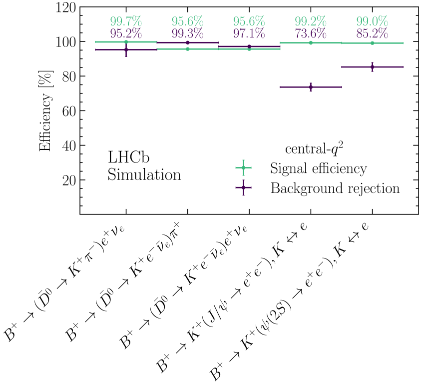

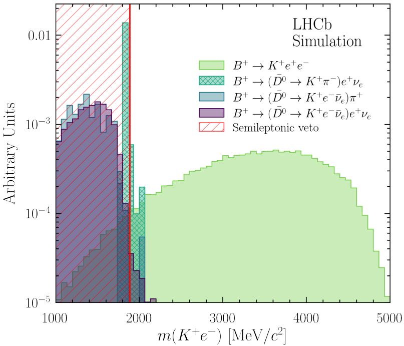

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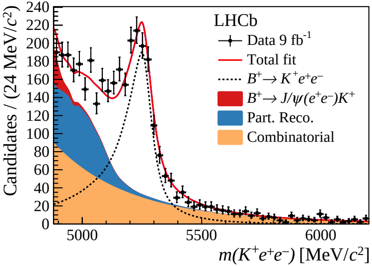
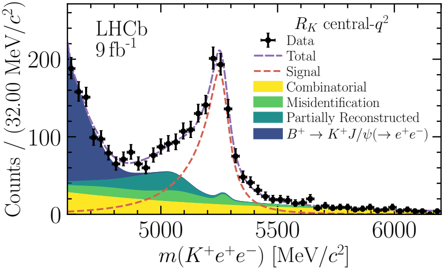
Difference to previous R_K analysis

[Nature Phys. 18 (2022) 277]



[arXiv:2212.09152] [arXiv:2212.09153]



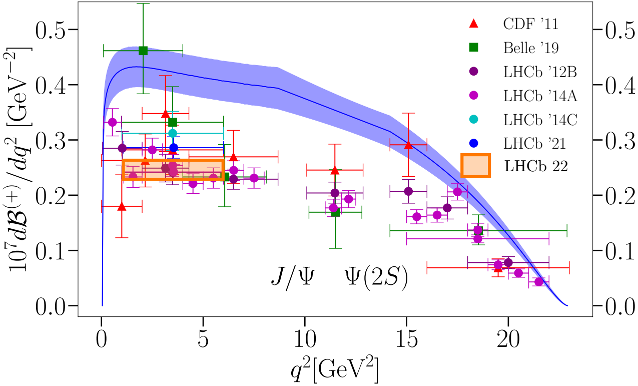
$$R_K = 0.846^{+0.042}_{-0.039}{}^{+0.013}_{-0.012} \quad \rightarrow \quad R_K = 0.949^{+0.042}_{-0.041}{}^{+0.022}_{-0.022}$$

[Nature Phys. 18 (2022) 277] [arXiv:2212.09152] [arXiv:2212.09153]

- Different selection allows for statistical scatter of ± 0.033
- Shift of ~ 0.1 due to pollution by residual misidentified backgrounds present and not accounted for in [Nature Phys. 18 (2022) 277]
  - Tighter particle identification cuts: Shift of $+0.064$
  - Explicit inclusion of residual misid. backgrounds: Shift of $+0.038$

One remark on $B^+ \to K^+ e^+ e^-$ branching fraction

[PRD 107 (2023) 1] with [arXiv:2212.09152]

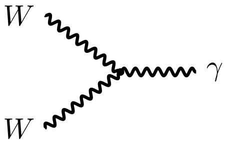


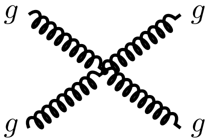
- Scaling R_{K,K^*} with measured muon $\mathcal{B}$ yields [JHEP 06 (2014) 133]

$$\mathrm{d}\mathcal{B}(B^+ \to K^+ e^+ e^-)/\mathrm{d}q^2 = (25.5^{+1.3}_{-1.2} \pm 1.1) \times 10^{-9}\ \mathrm{GeV}^{-2}$$

- Electron $\mathcal{B}$ consistent with muons, also below SM prediction

Terms in the SM Lagrangian





The SM Lagrangian:

$$\mathcal{L} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{self-int.}} + \underbrace{i\bar{\psi}\not{D}\psi}_{\text{kinetic}} + \underbrace{\psi_i Y_{ij}\psi_j\phi}_{\text{Yukawa}} + h.c. + \underbrace{|D_\mu\phi|^2 - V(\phi)}_{\text{Higgs}}$$

- Self-interaction/kinetic term for the gauge bosons (electroweak gauge bosons, gluons)
- Kinetic term for the fermions, interactions with the gauge bosons
- Yukawa-term: Couplings of the fermions to the Higgs
- Higgs potential and self coupling

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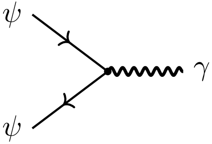

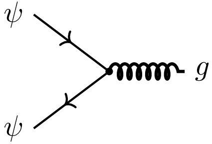

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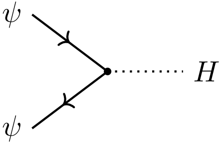

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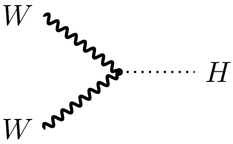

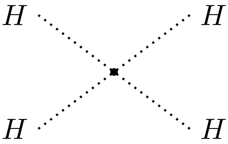

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The particle content

Fermions

Quarks				q	T	T_3	Y
q_{Li}	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$+\frac{2}{3}$ $-\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{1}{3}$ $+\frac{1}{3}$
u_{Ri}	u_R	c_R	t_R	$+\frac{2}{3}$	0	0	$+\frac{4}{3}$
d_{Ri}	d_R	s_R	b_R	$-\frac{1}{3}$	0	0	$-\frac{2}{3}$
Leptons				q	T	T_3	Y
ℓ_{Li}	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	0 -1	$\frac{1}{2}$	$+\frac{1}{2}$ $-\frac{1}{2}$	-1 -1
e_{Ri}	e_R	μ_R	τ_R	-1	0	0	-2

- q el. charge
- T weak isospin, T_3 third component
- Y weak hypercharge, $Y = 2(q - T_3)$

In detail: the kinetic term

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{kinetic}}^{\text{quarks}} = \sum_{i=1}^{3} i\bar{q}_{Li}\not{D}_q q_{Li} + i\bar{u}_{Ri}\not{D}_u u_{Ri} + i\bar{d}_{Ri}\not{D}_d d_{Ri} \quad \text{with} \tag{1}$$

$$\begin{aligned} D_{q\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig\tfrac{1}{2}\tau^a W^a_\mu + ig'\tfrac{1}{2}YB_\mu \\ D_{u\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig'\tfrac{1}{2}YB_\mu \\ D_{d\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig'\tfrac{1}{2}YB_\mu, \end{aligned} \tag{2}$$

- $D_{q\mu}$ cov. derivatives arising from invariance under local gauge trans.[5]
- T^a generators of $SU(3_\text{C})$, 3×3 Gell-Mann matrices
- τ^a generators of $SU(2)_\text{L}$, Pauli matrices

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3}$$

- couplings to gauge bosons *flavour-universal* and *flavour-diagonal*

[5] Reminder: $\not{D}_q = \gamma^\mu D_{q\mu}$

For completeness: the kinetic term for leptons

$$\mathcal{L}_{\text{kinetic}}^{\text{leptons}} = \sum_{i=1}^{3} i\bar{\ell}_{Li} \not{D}_{\ell} \ell_{Li} + i\bar{e}_{Ri} \not{D}_{e} e_{Ri} \text{ with} \quad (4)$$

$$D_{\ell\mu} = \partial_\mu + ig\tfrac{1}{2}\tau^a W_\mu^a + ig'\tfrac{1}{2} Y B_\mu$$

$$D_{e\mu} = \partial_\mu + ig'\tfrac{1}{2} Y B_\mu. \quad (5)$$

- Analogous to the kinetic term for quarks
- No coupling to gluons

For completeness: Higgs term

- Higgs potential

$$\mathcal{L}_{\mathrm{Higgs}} = \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \quad (6)$$

with $\lambda > 0$ for vacuum stability and $\mu^2 > 0$ to achieve non-zero vacuum expectation value $\langle \phi \rangle = (0, v/\sqrt{2})$

- Gauge bosons acquire mass through the cov. derivatives

$$\mathcal{L}^{\mathrm{Higgs}}_{\mathrm{kinetic}} = (D^\mu \phi)^\dagger (D_\mu \phi) \,. \quad (7)$$

Central for Flavour Physics: Yukawa couplings

- Couplings of quarks to Higgs central to flavour physics

$$\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} = \sum_{i,j=1}^{3} -\bar{q}_{Li} Y_{u,ij} \tilde{\phi} u_{Rj} - \bar{q}_{Li} Y_{d,ij} \phi d_{Rj} + h.c., \tag{8}$$

- $Y_{u,ij}$ and $Y_{d,ij}$ a priori arbitrary complex 3×3 Yukawa matrices
- Yukawa interactions can be expressed in different bases:
  mass basis, where the Yukawa interactions are diagonal
  interaction basis (as in Eq. 33), where the W interactions are diagonal
- Rotation between these two bases $\rightarrow$ CKM matrix

Yukawa interactions in the mass basis

- Replace Higgs field with exp. value (note $\tilde{\phi} = i\tau^2\phi^\dagger$, $\langle\tilde{\phi}\rangle = (v/\sqrt{2}, 0)$)

$$\mathcal{L}^{\text{quarks}}_{\text{Yukawa}} = \sum_{i,j=1}^{3} -\bar{q}_{Li} Y_{u,ij} \tilde{\phi} u_{Rj} - \bar{q}_{Li} Y_{d,ij} \phi d_{Rj} + h.c.,$$

$$= \sum_{i,j=1}^{3} -\bar{d}_{Li} M_{d,ij} d_{Rj} - \bar{u}_{Li} M_{u,ij} u_{Rj} + h.c. \qquad (9)$$

$$\text{with the mass matrices } M_{d,ij} = \frac{v}{\sqrt{2}} Y_{d,ij} \text{ and } M_{u,ij} = \frac{v}{\sqrt{2}} Y_{u,ij}$$

- Diagonalise the mass matrices using unitary matrices V_{dL} and V_{dR} (V_{uL} and V_{uR})

$$M_d^{\text{diag}} = V_{dL}^\dagger M_d V_{dR} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}, \quad M_u^{\text{diag}} = V_{uL}^\dagger M_u V_{uR} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}. \qquad (10)$$

Yukawa interactions in the mass basis II

- Diagonalisation results in mass eigenstates (superscript m)
- Mass eigenstates are connected to the interaction eigenstates via

$$\begin{aligned} d_{Li} &= V_{dL,ij} d^m_{Lj} & d_{Ri} &= V_{dR,ij} d^m_{Rj} \\ u_{Li} &= V_{uL,ij} u^m_{Lj} & u_{Ri} &= V_{uR,ij} u^m_{Rj}, \end{aligned} \tag{11}$$

- In the mass basis the Yukawa interactions become diagonal

$$\mathcal{L}^{\text{quarks}}_{\text{M}} = \sum_{i=1}^{3} -\bar{d}^m_{Li} M^{\text{diag}}_{d,ii} d^m_{Ri} - \bar{u}^m_{Li} M^{\text{diag}}_{u,ii} u^m_{Ri} + h.c. \tag{12}$$

$W^\pm$ interactions in the mass basis

- $W^\pm$ interactions in Eq. 33 in the interaction basis
- Writing the term instead in the mass basis results in

$$\begin{aligned}
\mathcal{L}_{W^\pm}^{\text{quarks}} &= \sum_{i=1}^{3} i\bar{q}_{Li}\gamma^\mu ig\tfrac{1}{2}\left[\tau^1 W_\mu^1 + \tau^2 W_\mu^2\right] q_{Li} \\
&= \sum_{i=1}^{3} i\bar{q}_{Li}\gamma^\mu ig\tfrac{1}{2}\left[W_\mu^1 \begin{pmatrix} d_{Li} \\ u_{Li} \end{pmatrix} + W_\mu^2 \begin{pmatrix} -id_{Li} \\ iu_{Li} \end{pmatrix}\right] \\
&= \frac{g}{\sqrt{2}} \sum_{i=1}^{3} -\bar{u}_{Li}\gamma^\mu W_\mu^+ d_{Li} - \bar{d}_{Li}\gamma^\mu W_\mu^- u_{Li} \\
&= \frac{g}{\sqrt{2}} \sum_{i,j,k=1}^{3} -\bar{u}_{Li}^m \underbrace{V_{uL,ij}^\dagger V_{dL,jk}}_{V_{\text{CKM},ik}} \gamma^\mu W_\mu^+ d_{Lk}^m - \bar{d}_{Li}^m V_{dL,ij}^\dagger V_{uL,jk}\gamma^\mu W_\mu^- u_{Lk}^m
\end{aligned} \tag{13}$$

where $W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$ was used

The CKM matrix

- CKM (Cabibbo Kobayashi Maskawa) matrix is defined as

$$V_{\mathrm{CKM,ik}} = V^{\dagger}_{uL,ij} V_{dL,jk} \tag{14}$$

- The CKM matrix describes the misalignment of the left-handed up- and down-type mass eigenstates
- The off-diagonal elements result in flavour violating transitions between the different generation in the charged weak interaction
- The CKM matrix elements are denoted as

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{15}$$

- CKM element V_{ub} gives coupling strength of the b to the u-quark

The Unitarity of the CKM matrix

- CKM matrix is a product of unitary matrices, therefore unitary itself
- Unitarity condition

$$V_{\mathrm{CKM}} V_{\mathrm{CKM}}^{\dagger} = 1 \tag{16}$$

- General complex $n \times n$ matrices can be described by n^2 real parameters and n^2 complex phases
- Unitarity condition reduces the number of free parameters to $n(n-1)/2$ real parameters, $n(n+1)/2$ phases
- $n = 3$: 3 real parameters, 6 phases
- For the CKM matrix, 5 of the phases can be absorbed as unphysical (unobservable) quark phases
- In total, the CKM matrix is therefore given by 3 real parameters and 1 complex phase

The CKM matrix: PDG parameterisation

- Standard PDG parameterisation with three (real) Euler angles θ_{12}, θ_{23}, θ_{13} and one complex phase δ

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (17)$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$

- Experimentally, it is known that the CKM matrix is hierarchichal with

$$s_{12} \ll s_{23} \ll s_{13} \ll 1 \quad (18)$$

- PDG parameterisation is exact, but hierarchical nature more clear in Wolfenstein parameterisation

The CKM matrix: Wolfenstein parameterisation

- Wolfenstein parameterisation uses the parameters λ, A, ρ and η, with η responsible for imaginary entries in V_{CKM}

$$s_{12} = \lambda \tag{19}$$
$$s_{23} = A\lambda^2 \tag{20}$$
$$s_{13}e^{+i\delta} = A\lambda^3\left(\rho + i\eta\right) \tag{21}$$

- parameter $\lambda \approx 0.22$ plays the role of an expansion parameter
- Up to $\mathcal{O}(\lambda^4)$ the CKM matrix in the Wolfenstein param. given by

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3\left(\rho - i\eta\right) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4\left(1 + 4A^2\right) & A\lambda^2 \\ A\lambda^3\left(1 - \rho - i\eta\right) & -A\lambda^2 + \frac{1}{2}A\lambda^4\left(1 - 2\left(\rho + i\eta\right)\right) & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} \tag{22}$$

- Diagonal elements close to 1, off-diagonal transitions suppressed $|V_{us}|, |V_{cd}| \sim \lambda$, $|V_{cb}|, |V_{ts}| \sim \lambda^2$ and $|V_{ub}|, |V_{td}| \sim \lambda^3$.
- Imaginary part relative to CKM element largest for V_{ub}

Symmetries

- Symmetries play a central role in physics, instructive to study SM Lagrangian under the discrete transformations C, P and T.

$$\text{Parity (space inversal)}: \quad \text{P}: \ \psi(r,t) \to \gamma^0\psi(-r,t) \tag{23}$$

$$\text{Charge conjugation}: \quad \text{C}: \ \psi \to i(\bar{\psi}\gamma^0\gamma^2)^T \tag{24}$$

$$\text{Time reversal}: \quad \text{T}: \ \psi(r,t) \to \gamma^1\gamma^3\psi(r,-t) \tag{25}$$

- P transforms left-handed fermions to right-handed fermions
- C transforms left-handed quarks to right-handed antiquarks
- Lagrangian not invariant under C and P (left-handed and right-handed fields with different representations in the SM)
- combined CP operation central for Flavour Physics

CP operation on Lagrangian

- Apply CP operation on Eq. 13

$$
\begin{aligned}
\mathcal{L}_{W^\pm}^{\text{quarks}} &= \frac{g}{\sqrt{2}} \sum_{i,j=1}^{3} -\bar{u}_{Li}^{m} V_{\mathrm{CKM},ij} \gamma^\mu W_\mu^+ d_{Lj}^m - \bar{d}_{Li}^m V_{\mathrm{CKM},ij}^\dagger \gamma^\mu W_\mu^- u_{Lj}^m \\
&= \frac{g}{\sqrt{2}} \sum_{i,j=1}^{3} -\bar{u}_{Li}^{m} V_{\mathrm{CKM},ij} \gamma^\mu W_\mu^+ d_{Lj}^m - \bar{d}_{Lj}^m V_{\mathrm{CKM},ij}^* \gamma^\mu W_\mu^- u_{Li}^m \\
&\overset{\mathrm{CP}}{\rightarrow} \frac{g}{\sqrt{2}} \sum_{i,j=1}^{3} -\bar{d}_{Lj}^{m} V_{\mathrm{CKM},ij} \gamma^\mu W_\mu^- u_{Li}^m - \bar{u}_{Li}^m V_{\mathrm{CKM},ij}^* \gamma^\mu W_\mu^+ d_{Lj}^m \\
&= \frac{g}{\sqrt{2}} \sum_{i,j=1}^{3} -\bar{u}_{Li}^{m} V_{\mathrm{CKM},ij}^* \gamma^\mu W_\mu^+ d_{Lj}^m - \bar{d}_{Lj}^m V_{\mathrm{CKM},ij} \gamma^\mu W_\mu^- u_{Li}^m. \qquad (26)
\end{aligned}
$$

- Invariant under the CP transformation only if $V_{\mathrm{CKM}}^* = V_{\mathrm{CKM}}$, *i.e.* all CKM matrix elements are real.
- CP-violation has been experimentally established in the K, D, B systems
- Any local lorentz-invariant QFT conserves CPT

A measure for CP violation: The Jarlskog invariant

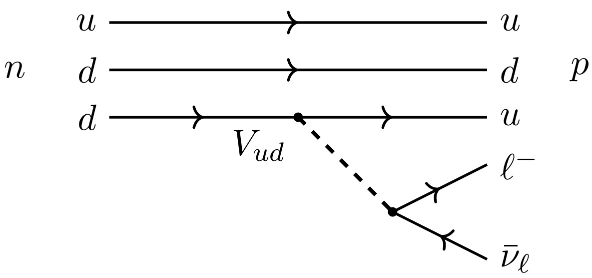
- Any non-trivial phase δ leads to CP violation
- Phase δ is not convention independent as quarks can be rephased
- A convention-independent measure of CP-violation is given by the Jarlskog-invariant, defined by

$$\mathrm{Im}\left(V_{ij}V_{kl}V_{il}^*V_{kj}^*\right) = J\sum_{m,n}\epsilon_{ikm}\epsilon_{jln} \tag{27}$$

$$\overset{e.g.}{\rightarrow} J = \mathrm{Im}\left(V_{us}V_{cb}V_{ub}^*V_{cs}^*\right)$$
$$= s_{12}c_{13}s_{23}c_{13}s_{13}\sin\delta c_{12}c_{23} \approx A^2\lambda^6\eta \tag{28}$$

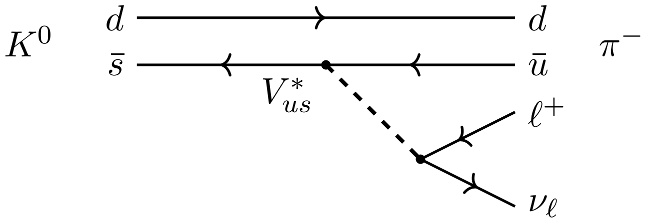
- Experimentally $J = (3.18 \pm 0.15) \times 10^{-5}$

CKM element magnitudes: $|V_{ud}|$



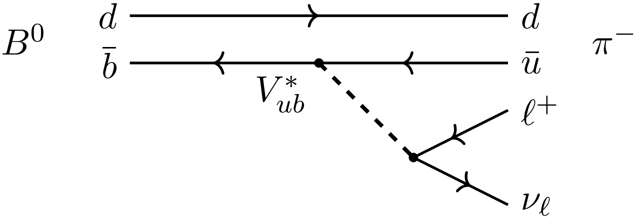
- $|V_{ud}|$ is determined most precisely in nuclear β decays
- The current world average is $|V_{ud}| = 0.97420 \pm 0.00021$

CKM element magnitudes: $|V_{us}|$



- $|V_{us}|$ is determined in (semi)leptonic kaon decays
- A combined average yields $|V_{us}| = 0.2243 \pm 0.0005$

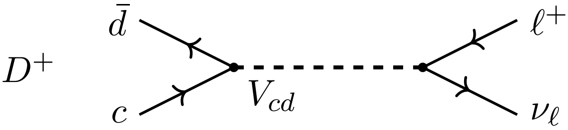
CKM element magnitudes: $|V_{ub}|$



- $|V_{ub}|$ can be determined in exclusive or inclusive decays of B mesons to light mesons[6]
- Some tension between inclusive/exclusive determination ($\approx 3\,\sigma$)
- Average yields $|V_{ub}| = (3.94 \pm 0.36) \times 10^{-3}$

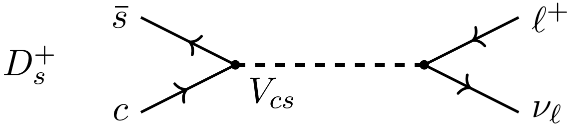
[6] Inclusive here means to include all $b \to u\ell\nu$ decays, exclusive refers to the analysis of specific decay modes like $B^0 \to \pi^- \ell^+ \nu_\ell$

CKM element magnitudes: $|V_{cd}|$



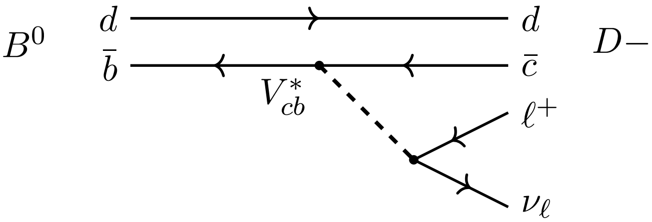
- $|V_{cd}|$ is determined in (semi)leptonic decays of charmed D mesons
- Current world average from direct measurements is $|V_{cd}| = 0.218 \pm 0.004$.

CKM element magnitudes: $|V_{cs}|$



- $|V_{cs}|$ is extracted from semileptonic decays of D mesons to strange mesons and leptonic decays of D_s^+ mesons
- Average yields $|V_{cs}| = 0.997 \pm 0.017$.

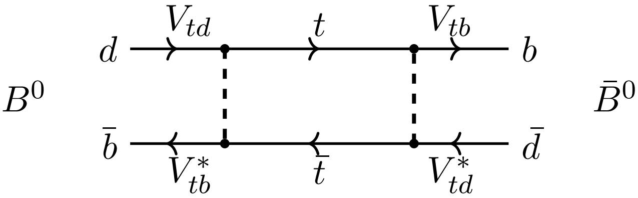
CKM element magnitudes: $|V_{cb}|$



- $|V_{cb}|$ can be determined using exclusive or inclusive decays of B mesons to charm mesons[7]
- Combination yields $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$

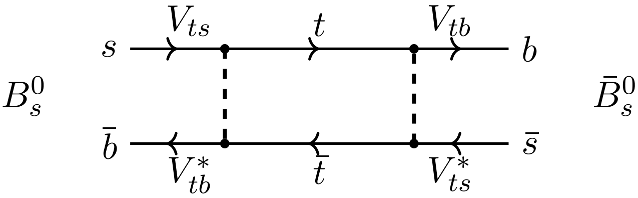
[7] Inclusive here means to include all $b \to c\ell\nu$ decays, whereas exclusive refers to the analysis of specific decay modes like $B^0 \to D^+\ell^-\nu_\ell$

CKM element magnitudes: $|V_{td}|$



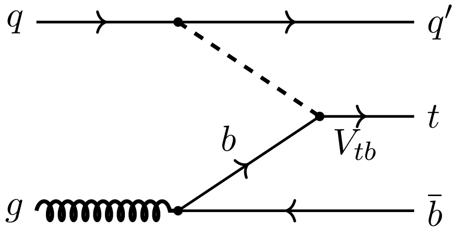
- $|V_{td}|$ is determined in B^0 mixing
- World average $|V_{td}| = (8.1 \pm 0.5) \times 10^{-3}$.

CKM element $|V_{ts}|$



- $|V_{ts}|$ is determined in B_s^0 mixing
- World average $|V_{ts}| = (39.4 \pm 2.3) \times 10^{-3}$

CKM element $|V_{tb}|$



- $|V_{tb}|$ can be determined from the single-top production cross-section
- World average $V_{tb} = 1.019 \pm 0.025$

Resulting CKM matrix from direct measurements

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97420 \pm 0.00021 & 0.2243 \pm 0.0005 & 0.00394 \pm 0.00036 \\ 0.218 \pm 0.004 & 0.997 \pm 0.017 & 0.0422 \pm 0.0008 \\ 0.0081 \pm 0.0005 & 0.0394 \pm 0.0023 & 1.019 \pm 0.025 \end{pmatrix} \tag{29}$$

- Shows hierarchical nature of CKM elements
- Note that the CKM matrix in SM determined by only 4 parameters
  - PDG convention: 3 Euler angles, 1 complex phase
  - Wolfenstein param.: λ, A, ρ, η
- Assuming unitarity, CKM elements can be determined more precisely in a global fit (later)

Phases of CKM matrix elements I

- Information on phases of CKM elements is obtained from measuring CP violating quantitites
- Will discuss these measurements in detail later,
  here mostly for completeness

ϵ quantifies CP violation in $K^0 \leftrightarrow \bar{K}^0$-mixing,
i.e. $\mathcal{P}(K^0 \to \bar{K}^0) \neq \mathcal{P}(\bar{K}^0 \to K^0)$
$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$

ϵ' describes CP violation in decay,
i.e. $\mathcal{P}(K^0 \to f) \neq \mathcal{P}(\bar{K}^0 \to \bar{f}))$
$\mathrm{Re}(\epsilon'/\epsilon) = (1.67 \pm 0.23) \times 10^{-3}$

angle β appears in $B^0 \leftrightarrow \overline{B}^0$ mixing
flagship measurement of B-factories (using $B^0 \to J/\psi\, K^0_\mathrm{S}$),
discovery mode for CPV in the B sector
world average $\sin 2\beta = 0.691 \pm 0.017$

Phases of CKM matrix elements II

angle γ	can be determined in tree-level $B^{\pm}\to DK^{\pm}$ decays world average $\gamma = \left(73.5^{+4.2}_{-5.1}\right)^{\circ}$
angle β_s	appears in $B_s^0 \leftrightarrow \bar{B}_s^0$ mixing time-dependent angular analysis of $B_s^0\to J/\psi\,\phi$ decays world average $-2\beta_s = (-0.021 \pm 0.031)\,\mathrm{rad}$
angle α	appears in $B^0 \leftrightarrow \overline{B}^0$ mixing from time-dependent $b\to u\bar{u}d$ decays ($B\to\pi\pi$) world average $\alpha = \left(84.5^{+5.9}_{-5.2}\right)^{\circ}$

Global fits of CKM parameters

- Global fits [CKMfitter] [UTfit] of all experimental inputs allow to determine the four CKM parameters assuming CKM unitarity
- For Wolfenstein parameterisation

$$\lambda = 0.22453 \pm 0.00044 \qquad A = 0.836 \pm 0.015$$
$$\bar{\rho} = 0.122^{+0.018}_{-0.017} \qquad \bar{\eta} = 0.355^{+0.012}_{-0.011} \tag{30}$$

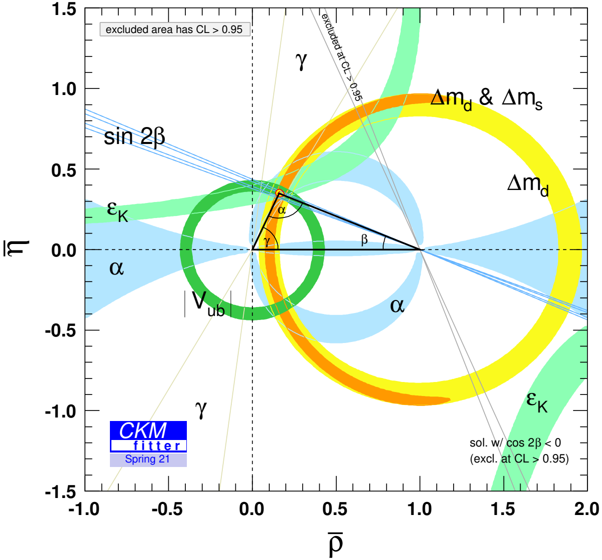
- Resulting magnitudes of the CKM matrix elements determined to be

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix} \tag{31}$$

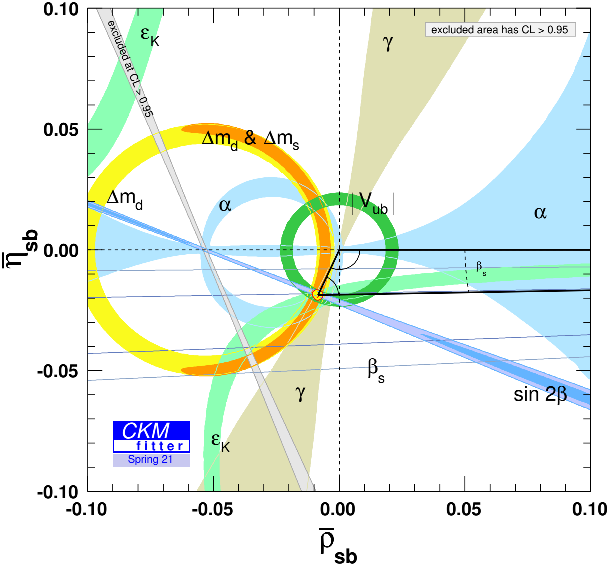
- Compare direct measurements

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97420 \pm 0.00021 & 0.2243 \pm 0.0005 & 0.00394 \pm 0.00036 \\ 0.218 \pm 0.004 & 0.997 \pm 0.017 & 0.0422 \pm 0.0008 \\ 0.0081 \pm 0.0005 & 0.0394 \pm 0.0023 & 1.019 \pm 0.025 \end{pmatrix} \tag{32}$$

Resulting CKM triangles: B^0 triangle



Resulting CKM triangles: B_s^0 triangle



Flavour Changing Neutral Currents

- Flavour Changing Neutral currents (quark which undergoes transition to a different quark of same charge) central for Flavour Physics
- Return to the kinetic term for quarks in the SM

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} &= \mathcal{L}_{\mathrm{kinetic}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Yukawa}} \\ \mathcal{L}_{\mathrm{kinetic}}^{\mathrm{quarks}} &= \sum_{i=1}^{3} i\bar{q}_{Li}\not{D}_q q_{Li} + i\bar{u}_{Ri}\not{D}_u u_{Ri} + i\bar{d}_{Ri}\not{D}_d d_{Ri} \quad \text{with} \end{aligned} \tag{33}$$

$$\begin{aligned} D_{q\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig\tfrac{1}{2}\tau^a W^a_\mu + ig'\tfrac{1}{2}YB_\mu \\ D_{u\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig'\tfrac{1}{2}YB_\mu \\ D_{d\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig'\tfrac{1}{2}YB_\mu, \end{aligned} \tag{34}$$

- Neutral current interactions

$$\mathcal{L}_{Z,\gamma} = \sum_i - \bar{q}_{Li}\gamma^\mu \left(g\tfrac{1}{2}\tau^3 W^3_\mu + g'\tfrac{1}{2}YB_\mu\right) q_{Li} - \bar{u}_{Ri}\gamma^\mu g'\tfrac{1}{2}YB_\mu u_{Ri} - \bar{d}_{Ri}\gamma^\mu g'\tfrac{1}{2}YB_\mu d_{Ri} \tag{35}$$

Flavour Changing Neutral Currents II

$$\begin{aligned}\mathcal{L}_{Z,\gamma} &= \sum_i -\bar{q}_{Li}\gamma^\mu\left(g\tfrac{1}{2}\tau^3 W^3_\mu + g'\tfrac{1}{2}YB_\mu\right)q_{Li} - \bar{u}_{Ri}\gamma^\mu g'\tfrac{1}{2}YB_\mu u_{Ri} - \bar{d}_{Ri}\gamma^\mu g'\tfrac{1}{2}YB_\mu d_{Ri}\\ &= \sum_i -\bar{u}^m_{Li}V^\dagger_{uL}\gamma^\mu\left(g\tfrac{1}{2}W^3_\mu + g'\tfrac{1}{2}YB_\mu\right)V_{uL}u^m_{Li} - \bar{d}^m_{Li}V^\dagger_{dL}\gamma^\mu\left(-g\tfrac{1}{2}W^3_\mu + g'\tfrac{1}{2}YB_\mu\right)V_{dL}d^m_{Li}\\ &\quad -\bar{u}^m_{Ri}V^\dagger_{uR}\gamma^\mu g'\tfrac{1}{2}YB_\mu V_{uR}u^m_{Ri} - \bar{d}^m_{Ri}V^\dagger_{dR}\gamma^\mu g'\tfrac{1}{2}YB_\mu V_{dR}d^m_{Ri}\end{aligned}$$

- where we put in the mass eigenstates and

$$\begin{aligned}\mathcal{L}_{Z,\gamma} = \sum_i &-\bar{u}^m_{Li}\gamma^\mu\left(g\tfrac{1}{2}\sin\theta_{\mathrm{W}} + g'\tfrac{1}{2}Y\cos\theta_{\mathrm{W}}\right)A_\mu u^m_{Li} - \bar{u}^m_{Li}\gamma^\mu\left(g\tfrac{1}{2}\cos\theta_{\mathrm{W}} - g'\tfrac{1}{2}Y\sin\theta_{\mathrm{W}}\right)Z^0_\mu u^m_{Li}\\ &-\bar{d}^m_{Li}\gamma^\mu\left(-g\tfrac{1}{2}\sin\theta_{\mathrm{W}} + g'\tfrac{1}{2}Y\cos\theta_{\mathrm{W}}\right)A_\mu d^m_{Li} - \bar{d}^m_{Li}\gamma^\mu\left(-g\tfrac{1}{2}\cos\theta_{\mathrm{W}} - g'\tfrac{1}{2}Y\sin\theta_{\mathrm{W}}\right)Z^0_\mu d^m_{Li}\\ &-\bar{u}^m_{Ri}\gamma^\mu g'\tfrac{1}{2}Y\cos\theta_{\mathrm{W}}A_\mu u^m_{Ri} + \bar{u}^m_{Ri}\gamma^\mu g'\tfrac{1}{2}Y\sin\theta_{\mathrm{W}}Z^0_\mu u^m_{Ri}\\ &-\bar{d}^m_{Ri}\gamma^\mu g'\tfrac{1}{2}Y\cos\theta_{\mathrm{W}}A_\mu d^m_{Ri} + \bar{d}^m_{Ri}\gamma^\mu g'\tfrac{1}{2}Y\sin\theta_{\mathrm{W}}Z^0_\mu d^m_{Ri}\end{aligned}$$

- where we replaced B_μ, W^3_μ by A_μ, Z_μ

$$\begin{pmatrix}A_\mu\\ Z^0_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_{\mathrm{W}} & \sin\theta_{\mathrm{W}}\\ -\sin\theta_{\mathrm{W}} & \cos\theta_{\mathrm{W}}\end{pmatrix}\begin{pmatrix}B_\mu\\ W^3_\mu\end{pmatrix} \rightarrow \begin{pmatrix}B_\mu\\ W^3_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_{\mathrm{W}} & -\sin\theta_{\mathrm{W}}\\ \sin\theta_{\mathrm{W}} & \cos\theta_{\mathrm{W}}\end{pmatrix}\begin{pmatrix}A_\mu\\ Z_\mu\end{pmatrix}$$

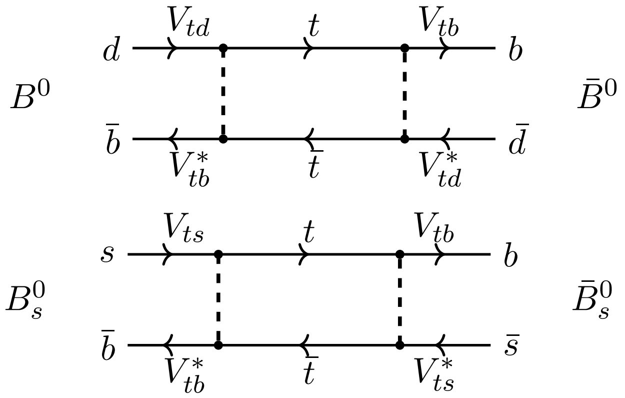
Flavour Changing Neutral Currents III

- Finally remember for Weinberg angle θ_W: $g\sin\theta_W = g'\cos\theta_W = e$

$$
\begin{aligned}
\mathcal{L}_{Z,\gamma} = \sum_i & - \left(+\tfrac{2}{3}e\right)\bar{u}^m_{Li}\gamma^\mu A_\mu u^m_{Li} - \tfrac{g}{\cos\theta_\mathrm{W}}\left(+\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_\mathrm{W}\right)\bar{u}^m_{Li}\gamma^\mu Z^0_\mu u^m_{Li} \\
& - \left(-\tfrac{1}{3}e\right)\bar{d}^m_{Li}\gamma^\mu A_\mu d^m_{Li} - \tfrac{g}{\cos\theta_\mathrm{W}}\left(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_\mathrm{W}\right)\bar{d}^m_{Li}\gamma^\mu Z^0_\mu d^m_{Li} \\
& - \left(+\tfrac{2}{3}e\right)\bar{u}^m_{Ri}\gamma^\mu A_\mu u^m_{Ri} - \tfrac{g}{\cos\theta_W}\left(-\tfrac{2}{3}\sin^2\theta_\mathrm{W}\right)\bar{u}^m_{Ri}\gamma^\mu Z^0_\mu u^m_{Ri} \\
& - \left(-\tfrac{1}{3}e\right)\bar{d}^m_{Ri}\gamma^\mu A_\mu d^m_{Ri} - \tfrac{g}{\cos\theta_W}\left(+\tfrac{1}{3}\sin^2\theta_\mathrm{W}\right)\bar{d}^m_{Ri}\gamma^\mu Z^0_\mu d^m_{Ri}
\end{aligned}
$$

- Interaction moderated by A_μ/Z_μ flavour diagonal
- Photon A_μ couples to el. charge
- Coupling strength of Z_μ proportional to $T_3 - q\sin^2\theta_W$
- No Flavour Changing Neutral Currents at tree-level in the SM
  $\rightarrow$ Only allowed at loop-level
- Important example: Neutral meson mixing

Neutral Meson Mixing



- Only allowed at loop-level as FCNCs are forbidden at tree-level
- Above examples are for $B^0 \leftrightarrow \overline{B}^0$ and $B_s^0 \leftrightarrow \overline{B}_s^0$ mixing
  Principle the same for $K^0 \leftrightarrow \bar{K}^0$ and $D^0 \leftrightarrow \bar{D}^0$ (details differ)
- We will derive the general case ($M^0 \leftrightarrow \overline{M}^0$ mixing)

Decay Amplitudes and Definitions

- General decay amplitudes for a meson[8] M and CP-conjugate $\bar{M}$ to the final state f and CP-conjugate $\bar{f}$

$$\begin{aligned} A_f &= \langle f|\mathcal{H}|M\rangle \\ \bar{A}_f &= \langle f|\mathcal{H}|\bar{M}\rangle \\ A_{\bar{f}} &= \langle \bar{f}|\mathcal{H}|M\rangle \\ \bar{A}_{\bar{f}} &= \langle \bar{f}|\mathcal{H}|\bar{M}\rangle \end{aligned} \tag{36}$$

  with interaction Hamiltonian $\mathcal{H}$

- For neutral mesons we define

$$\begin{aligned} \mathrm{CP}|M^0\rangle &= -|\overline{M}^0\rangle \\ \mathrm{CP}|\overline{M}^0\rangle &= -|M^0\rangle \end{aligned} \tag{37}$$

  where an arbitrary non-physical phase factor has been omitted.

- If the final state is a CP-eigenstate we have (η_f CP-eigenvalue)

$$\begin{aligned} \mathrm{CP}|f\rangle &= \eta_f|\bar{f}\rangle \\ \mathrm{CP}|\bar{f}\rangle &= \eta_f|f\rangle, \end{aligned} \tag{38}$$

[8] charged or neutral

Phenomenological Schrödinger equation

- Time development for flavour eigenstates M^0 and $\overline{M}^0$ given by phenomenological Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} = \begin{pmatrix} \mathrm{M} - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix}$$
$$= \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}\begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} \quad (39)$$

- with hermitean matrices M and Γ ($M_{21} = M_{12}^*$, $\Gamma_{21} = \Gamma_{12}^*$), off-diagonal elements responsible for mixing
- From CPT invariance we have $\Gamma_{11} = \Gamma_{22} = \Gamma$ and $M_{11} = M_{22} = M$

$$i\frac{\partial}{\partial t}\begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} \quad (40)$$

Diagonalisation I

- Diagonalisation of the Hamiltonian results in mass eigenstates

$$
\begin{aligned}
|M_L\rangle &= p|M^0\rangle + q|\overline{M}^0\rangle \\
|M_H\rangle &= p|M^0\rangle - q|\overline{M}^0\rangle
\end{aligned}
\tag{41}
$$

with $|p|^2 + |q|^2 = 1$, M_L (M_H) light (heavy) mass eigenstate

- Mass eigenstates develop in time according to

$$
\begin{aligned}
|M_L(t)\rangle &= e^{-im_L t} e^{-\frac{\Gamma_L}{2} t} |M_L\rangle \\
|M_H(t)\rangle &= e^{-im_H t} e^{-\frac{\Gamma_H}{2} t} |M_H\rangle
\end{aligned}
\tag{42}
$$

- With Eq. 41 eigenvectors of Hamiltonian are $(p, q)^T$ and $(p, -q)^T$

Diagonalisation II

- Hamiltonian can be diagonalised by matrix V with eigenvectors as columns, *i.e.*

$$\begin{pmatrix} m_L - \frac{i}{2}\Gamma_L & 0 \\ 0 & m_H - \frac{i}{2}\Gamma_H \end{pmatrix} = V^{-1} \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} V \tag{43}$$

$$V = \begin{pmatrix} p & p \\ q & -q \end{pmatrix} \quad \text{and} \quad V^{-1} = -\frac{1}{2pq} \begin{pmatrix} -q & -p \\ -q & p \end{pmatrix} = \frac{1}{2pq} \begin{pmatrix} q & p \\ q & -p \end{pmatrix}$$

- The time development for M^0 and $\overline{M}^0$ is given by[9]

$$\begin{aligned} |M^0(t)\rangle &= \frac{1}{2p}\left[|M_L(t)\rangle + |M_H(t)\rangle\right] \\ |\overline{M}^0(t)\rangle &= \frac{1}{2q}\left[|M_L(t)\rangle - |M_H(t)\rangle\right]. \end{aligned} \tag{44}$$

[9] From Ansatz Eq. 41

Time development for M^0 and $\overline{M}^0$

- Inserting the time-development of $|M_L\rangle$ and $|M_H\rangle$ we find

$$\begin{aligned}
|M^0(t)\rangle &= \frac{1}{2p}\left[|M_L(t)\rangle + |M_H(t)\rangle\right] \\
&= \frac{1}{2}\left(e^{-im_L t}e^{-\frac{\Gamma_L}{2}t} + e^{-im_H t}e^{-\frac{\Gamma_H}{2}t}\right)|M^0\rangle + \frac{q}{2p}\left(e^{-im_L t}e^{-\frac{\Gamma_L}{2}t} - e^{-im_H t}e^{-\frac{\Gamma_H}{2}t}\right)|\overline{M}^0\rangle \\
&= g_+(t)|M^0\rangle + \frac{q}{p}g_-(t)|\overline{M}^0\rangle \\
|\overline{M}^0(t)\rangle &= \frac{1}{2q}\left[|M_L(t)\rangle - |M_H(t)\rangle\right] \\
&= \frac{p}{2q}\left(e^{-im_L t}e^{-\frac{\Gamma_L}{2}t} - e^{-im_H t}e^{-\frac{\Gamma_H}{2}t}\right)|M^0\rangle + \frac{1}{2}\left(e^{-im_L t}e^{-\frac{\Gamma_L}{2}t} + e^{-im_H t}e^{-\frac{\Gamma_H}{2}t}\right)|\overline{M}^0\rangle \\
&= \frac{p}{q}g_-(t)|M^0\rangle + g_+(t)|\overline{M}^0\rangle \qquad (45)
\end{aligned}$$

with

$$g_\pm(t) = \frac{1}{2}\left(e^{-im_L t}e^{-\frac{\Gamma_L}{2}t} \pm e^{-im_H t}e^{-\frac{\Gamma_H}{2}t}\right) \qquad (46)$$

Time development for M^0 and $\overline{M}^0$ II

- The following expressions are useful for transition probabilities:

$$g_\pm(t) = \frac{1}{2}\left(e^{-im_L t}e^{-\frac{\Gamma_L}{2}t} \pm e^{-im_H t}e^{-\frac{\Gamma_H}{2}t}\right)$$
$$|g_\pm(t)|^2 = \frac{1}{2}e^{-\Gamma t}\left(+\cosh\frac{\Delta\Gamma}{2}t \pm \cos\Delta m t\right)$$
$$g_+(t)g_-^*(t) = \frac{1}{2}e^{-\Gamma t}\left(-\sinh\frac{\Delta\Gamma}{2}t - i\sin\Delta m t\right), \qquad (47)$$

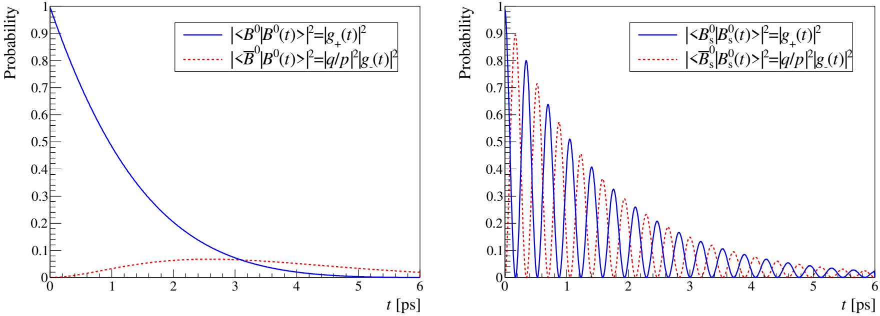
where we used the definitions

$$\Gamma = \frac{\Gamma_L + \Gamma_H}{2} \qquad \Delta\Gamma = \Gamma_L - \Gamma_H$$
$$M = \frac{m_L + m_H}{2} \qquad \Delta m = m_H - m_L. \qquad (48)$$

- We then have for the transition probabilities

$$|\langle M^0|M^0(t)\rangle|^2 = |g_+(t)\langle M^0|M^0\rangle + \tfrac{q}{p}g_-(t)\langle M^0|\overline{M}^0\rangle|^2 = |g_+(t)|^2$$
$$|\langle \overline{M}^0|M^0(t)\rangle|^2 = |g_+(t)\langle \overline{M}^0|M^0\rangle + \tfrac{q}{p}g_-(t)\langle \overline{M}^0|\overline{M}^0\rangle|^2 = |\tfrac{q}{p}|^2|g_-(t)|^2$$

Resulting transition probabilities in the B system



- Using the experimental world averages $\tau(B^0) = 1.52\,\text{ps}$, $\Delta\Gamma_d = 0$, $\Delta m_d = 0.5064\,\text{ps}^{-1}$ and $\tau(B_s^0) = 1.527\,\text{ps}$, $\Delta\Gamma_s/\Gamma_s = 0.132$, $\Delta m_s = 17.757\,\text{ps}^{-1}$

Solving Diagonalisation problem

- Diagonalisation problem (Eq. 43)

$$\begin{pmatrix} m_L - \frac{i}{2}\Gamma_L & 0 \\ 0 & m_H - \frac{i}{2}\Gamma_H \end{pmatrix} = V^{-1} \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} V$$

- Solving explicitly yields

$$\frac{q}{p} = -\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \tag{49}$$

$$\begin{aligned} m_{L(H)} - \frac{i}{2}\Gamma_{L(H)} &= M - \frac{i}{2}\Gamma \mp \sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \\ &= M - \frac{i}{2}\Gamma \mp \sqrt{|M_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2 - i|M_{12}||\Gamma_{12}|\cos\left(\phi_\Gamma - \phi_M\right)} \end{aligned} \tag{50}$$

$$\text{where} \quad \phi_\Gamma = \arg(\Gamma_{12}) \quad \text{and} \quad \phi_M = \arg(M_{12})$$

- Rewriting Eq. 50 in terms of Δm and $\Delta\Gamma$, squaring and taking Re/Im:

$$\Delta m^2 - \frac{1}{4}\Delta\Gamma^2 = +4|M_{12}|^2 - |\Gamma_{12}|^2 \tag{51}$$

$$\Delta m \Delta\Gamma = -4|M_{12}||\Gamma_{12}|\cos(\phi_\Gamma - \phi_M). \tag{52}$$

Approximations in the B system

- In the B system we have experimentally $\Gamma_{12} \ll M_{12}$ and we can expand in $|\Gamma_{12}|/|M_{12}|$

$$\Delta m \approx 2|M_{12}| \quad \text{and} \quad \Delta\Gamma \approx -2|\Gamma_{12}|\cos(\phi_\Gamma - \phi_M) \tag{53}$$

- Also q/p (Eq. 49) can be expanded in $|\Gamma_{12}|/|M_{12}|$

$$\begin{aligned} \frac{q}{p} &= -\sqrt{\frac{M_{12}^*}{M_{12}} \frac{1 - \frac{i}{2}\frac{|\Gamma_{12}|}{|M_{12}|} e^{-i(\phi_\Gamma - \Phi_M)}}{1 - \frac{i}{2}\frac{|\Gamma_{12}|}{|M_{12}|} e^{+i(\phi_\Gamma - \Phi_M)}}} \\ &= -e^{-i\phi_M}\left[1 - \tfrac{1}{2}\sin(\phi_\Gamma - \phi_M)\frac{|\Gamma_{12}|}{|M_{12}|} + \mathcal{O}\left(\frac{|\Gamma_{12}|^2}{|M_{12}|^2}\right)\right] \\ &\approx -e^{-i\phi_M}. \end{aligned} \tag{54}$$

i.e. $|q/p| = 1$ and q/p only determined by mixing phase ϕ_M

Time dependent decay rates I

- We can now write down time-dependent decay rates of (produced) M^0 and $\overline{M}^0$ to final states f and $\bar{f}$, accounting for $M^0 \leftrightarrow \overline{M}^0$ mixing
- Before we do we define a central quantity for CP-violation

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}. \tag{55}$$

- We then have for the decay rate of the process $M^0 \to f$

$$\begin{aligned}
\frac{\mathrm{d}\Gamma(M^0 \to f)}{\mathrm{d}t\mathcal{N}_f} &= \left|\langle f|M^0(t)\rangle\right|^2 = \left|g_+(t)\langle f|M^0\rangle + \frac{q}{p}g_-(t)\langle f|\overline{M}^0\rangle\right|^2 \\
&= \left(g_+(t)A_f + \frac{q}{p}g_-(t)\bar{A}_f\right)\left(g_+(t)A_f + \frac{q}{p}g_-(t)\bar{A}_f\right)^* \\
&= |A_f|^2\left[|g_+(t)|^2 + |\lambda_f|^2|g_-(t)|^2 + \lambda_f^* g_+(t)g_-^*(t) + \lambda_f g_+^*(t)g_-(t)\right] \\
&= \tfrac{1}{2}|A_f|^2 e^{-\Gamma t}\left[(1+|\lambda_f|^2)\cosh\left(\frac{\Delta\Gamma}{2}t\right) + (1-|\lambda_f|^2)\cos(\Delta m t)\right. \\
&\left. - 2\sinh\left(\frac{\Delta\Gamma}{2}t\right)\mathrm{Re}\lambda_f - 2\sin(\Delta m t)\mathrm{Im}\lambda_f\right]
\end{aligned} \tag{56}$$

Time dependent decay rates II

- And for the decay of a produced $\overline{M}^0 \to f$

$$\frac{\mathrm{d}\Gamma(\overline{M}^0 \to f)}{\mathrm{d}t\mathcal{N}_f} = \left|\langle f|\overline{M}^0(t)\rangle\right|^2 = \left|\frac{p}{q}g_-(t)\langle f|M^0\rangle + g_+(t)\langle f|\overline{M}^0\rangle\right|^2$$
$$= |A_f|^2\left|\frac{p}{q}\right|^2\left[|g_-(t)|^2 + |\lambda_f|^2|g_+(t)|^2 + \lambda_f^* g_+^*(t)g_-(t) + \lambda_f g_+(t)g_-^*(t)\right]$$
$$= \tfrac{1}{2}|A_f|^2\left|\frac{p}{q}\right|e^{-\Gamma t}\Bigg[(1+|\lambda_f|^2)\cosh\left(\frac{\Delta\Gamma}{2}t\right) - (1-|\lambda_f|^2)\cos(\Delta m t)$$
$$-2\sinh\left(\frac{\Delta\Gamma}{2}t\right)\mathrm{Re}\lambda_f + 2\sin(\Delta m t)\mathrm{Im}\lambda_f\Bigg]. \quad (57)$$

- For the decays to the CP-conjugated final state $\bar{f}$ replace $A_f \to A_{\bar{f}}$, $\bar{A}_f \to \bar{A}_{\bar{f}}$ and $\lambda_f \to \lambda_{\bar{f}} = \frac{q}{p}\bar{A}_{\bar{f}}/A_{\bar{f}}$

Prerequisites for CP violation I

- CP violation can only be observed if there are two amplitudes interfering with different *strong* and *weak* phases
- *weak phases* are phases caused by complex CKM matrix elements, which are complex-conjugated under CP
- *strong phases* are phases that do not change sign under CP (QCD or simple time evolution)
- For a process with two contributing amplitudes a_1 and a_2

$$\begin{aligned} A_f &= |a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)} \\ \bar{A}_{\bar{f}} &= |a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{58}$$

- Physically observable: squares of amplitudes

Prerequisites for CP violation II

- Squaring amplitudes results in

$$|A_f|^2 = \left(|a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)}\right)\left(|a_1|e^{-i(\delta_1+\phi_1)} + |a_2|e^{-i(\delta_2+\phi_2)}\right)$$
$$= |a_1|^2 + |a_2|^2 + |a_1||a_2|e^{+i(+\delta_1-\delta_2+\phi_1-\phi_2)} + |a_1||a_2|e^{-i(+\delta_1-\delta_2+\phi_1-\phi_2)}$$
$$= |a_1|^2 + |a_2|^2 + 2|a_1||a_2|\cos\left(\Delta\delta + \Delta\phi\right) \quad (59)$$
$$|\bar{A}_{\bar{f}}|^2 = \left(|a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)}\right)\left(|a_1|e^{-i(\delta_1-\phi_1)} + |a_2|e^{-i(\delta_2-\phi_2)}\right)$$
$$= |a_1|^2 + |a_2|^2 + |a_1||a_2|e^{+i(+\delta_1-\delta_2-\phi_1+\phi_2)} + |a_1||a_2|e^{-i(+\delta_1-\delta_2-\phi_1+\phi_2)}$$
$$= |a_1|^2 + |a_2|^2 + 2|a_1||a_2|\cos\left(\Delta\delta - \Delta\phi\right), \quad (60)$$

- with the phase differences $\Delta\delta = \delta_1 - \delta_2$ and $\Delta\phi = \phi_1 - \phi_2$
- $|A_f|^2 \neq |\bar{A}_{\bar{f}}|^2$ if $\Delta\delta \neq 0$ and $\Delta\phi \neq 0$

Types of CP violation

- When studying decays of neutral mesons, mixing amplitudes and decay amplitudes can give rise to CP-violating effects
- This gives rise to three types of CP violation:
  1. CP violation in decay
  2. CP violation in mixing
  3. CP violation in interference between mixing and decay

1. CP violation in decay

- CPV in decay occurs when $\left|\bar{A}_{\bar{f}}/A_f\right| \neq 1$,
  i.e. the amplitudes for the process $M \to f$
  and its CP conjugate $\bar{M} \to \bar{f}$ differ
- CP violation then manifests itself as asymmetry

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}} = \frac{\Gamma\left(M^- \to f^-\right) - \Gamma\left(M^+ \to f^+\right)}{\Gamma\left(M^- \to f^-\right) + \Gamma\left(M^+ \to f^+\right)} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|\bar{A}_{\bar{f}}|^2 + |A_f|^2} = \frac{|\bar{A}_{\bar{f}}/A_f|^2 - 1}{|\bar{A}_{\bar{f}}/A_f|^2 + 1}. \quad (61)$$

- This type of CP violation is also called *direct* CP violation.
- The strong phase contributing is due to rescattering
- Only type of CP violation possible for charged meson decays

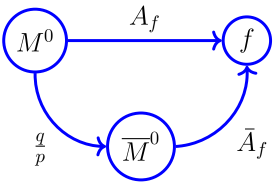
2. CP violation in mixing

- CP violation in mixing occurs when $|q/p| \neq 1$
- In this case $\mathcal{P}(M^0 \to \overline{M}^0) \neq \mathcal{P}(\overline{M}^0 \to M^0)$.
- The resulting asymmetry assuming no direct CP violation, *i.e.* $A_f = \bar{A}_{\bar{f}}$ and $A_{\bar{f}} = \bar{A}_f = 0$, is given by

$$\begin{aligned} \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}} &= \frac{\Gamma(\overline{M}^0 \to f) - \Gamma(M^0 \to \bar{f})}{\Gamma(\overline{M}^0 \to f) + \Gamma(M^0 \to \bar{f})} \\ &= \frac{\left|\frac{p}{q} g_-(t) A_f\right|^2 - \left|\frac{q}{p} g_-(t) \bar{A}_{\bar{f}}\right|^2}{\left|\frac{p}{q} g_-(t) A_f\right|^2 + \left|\frac{q}{p} g_-(t) \bar{A}_{\bar{f}}\right|^2} = \frac{1 - \left|\frac{q}{p}\right|^4}{1 + \left|\frac{q}{p}\right|^4}. \end{aligned} \quad (62)$$

- Here, the strong phase is due to the time evolution of the oscillation $(\exp(iEt))$.

3. CPV in interference between mixing and decay



- Can occur when the direct decay $M^0 \to f$ interferes with mixing from M^0 to $\overline{M}^0$ followed by the decay $\overline{M}^0 \to f$
- If λ_f (Eq. 56 and 57) has a non-trivial phase, *i.e.* $\mathrm{Im}\,(\lambda_f) = \mathrm{Im}\left(q/p\,\bar{A}_f/A_f\right) \neq 0$, this gives rise to this type of CPV

$$\mathcal{A}_{\mathrm{CP}}(t) = \frac{\Gamma(\overline{M}^0 \to f)(t) - \Gamma(M^0 \to f)(t)}{\Gamma(\overline{M}^0 \to f)(t) + \Gamma(M^0 \to f)(t)} = \frac{-\left(1-|\lambda_f|^2\right)\cos\left(\Delta m t\right) + 2\sin\left(\Delta m t\right)\mathrm{Im}\lambda_f}{\left(1+|\lambda_f|^2\right)\cosh\left(\frac{\Delta\Gamma}{2}t\right) - 2\sinh\left(\frac{\Delta\Gamma}{2}t\right)\mathrm{Re}\lambda_f}. \quad (63)$$

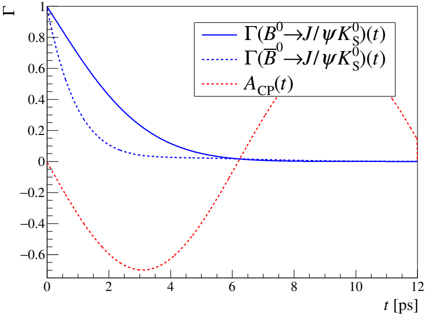
- Strong phase is due to the time evolution of the oscillation.

CPV in interference between mixing and decay (example)

- For $\Delta\Gamma = 0$ and $|\lambda_f| = 1$ (B^0 system) the asymmetry simplifies to

$$\mathcal{A}_{\rm CP}(t) = \sin\left(\Delta m t\right) {\rm Im}\lambda_f. \tag{64}$$

- Example: time-dependent CP-asymmetry in the decay $B^0 \to J/\psi K^0_{\rm S}$, where $\Delta\Gamma_d = 0$ and ${\rm Im}(\lambda_f) = -\sin(2\beta_d)$.



B-mixing and the CKM triangle I

- Δm_d and Δm_s constrain unitarity triangle (UT)

$$\Delta m_d = \frac{G_{\rm F}^2}{6\pi^2} M_W^2 \eta_B M_{B_d} f_{B_d}^2 \hat{B}_{B_d} (V_{tb} V_{td}^*)^2 S(x_t) \tag{65}$$

$$\Delta m_s = \frac{G_{\rm F}^2}{6\pi^2} M_W^2 \eta_B M_{B_s} f_{B_s}^2 \hat{B}_{B_s} (V_{tb} V_{ts}^*)^2 S(x_t), \tag{66}$$

- Δm_d and Δm_s theory dominated, dominant uncertainty from decay constant and bag factor:

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = (225 \pm 9)\,\text{MeV} \quad f_{B_s}\sqrt{\hat{B}_{B_s}} \quad = (274 \pm 8)\,\text{MeV} \tag{67}$$

$$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.206 \pm 0.017 \tag{68}$$

- $B^0_{(s)}$ mixing measurements used to determine length of right leg of UT

$$R_t = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| = \frac{1}{\lambda}\frac{|V_{td}|}{|V_{cb}|} \tag{69}$$

B-mixing and the CKM triangle II

- From Δm_d can determine $|V_{td}|$ and thus R_t, remaining dependency on $|V_{cb}|$ and uncertainty from $f_{B_d}\sqrt{B}_{B_d}$ is significant.
- Instead, we can use both Δm_d and Δm_s according to

$$
\begin{aligned}
R_t = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| = \left|\frac{V_{td}}{V_{ts}}\right|\left|\frac{V_{ts}V_{tb}^*}{V_{cd}V_{cb}^*}\right| &= \left|\frac{V_{td}}{V_{ts}}\right|\frac{1}{\lambda}\frac{|V_{ts}|}{|V_{cb}|} \\
&\approx \frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right| \quad \text{using} \qquad (70) \\
\left|\frac{V_{ts}}{V_{cb}}\right| &= \frac{|-A\lambda^2 + \frac{1}{2}A\lambda^4(1-2(\rho+i\eta))|}{|A\lambda^2|} \\
&= |-1+\tfrac{1}{2}\lambda^2(1-2(\rho+i\eta))| \approx 1. \qquad (71)
\end{aligned}
$$

- Due to the reduced theory uncertainty on $|V_{td}/V_{ts}|$ and no dependency on $|V_{cb}|$, this results in a more precise determination of R_t