Alessandro Vicini

CERN and University of Milano, INFN Milano

CERN, QCD seminar, February 12th 2024

S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.10782

Alessandro Vicini - CERN and University of Milano

Determination of $\sin^2 \hat{\theta}(\mu_R^2)$ at HL-LHC



Outline

Motivations	perform a test of the Standa
Methodologies	assuming the Standard Mode
	theory requirements for a sig
Status	sensitivity study with CMS co

Outlook

- ard Model at very different energy scales
- el validity, how can we fit a Lagrangian input parameter
- gnificant extraction
- olleagues







The weak mixing angle

In the construction of the SM,

identification of the electromagnetic current and electric charge $e = g \sin \theta_W$

 \rightarrow prediction of the second neutral current, coupling the Z boson to fermions

$$Zf\bar{f} \propto i \frac{g}{\cos\theta_W} \gamma^\mu \left(T_3 \frac{1-\gamma_5}{2} - \sin^2\theta_W\right)$$

It is interesting to test both: the strength of the neutral current interaction the mixing of the $SU(2)_L$ and $U(1)_Y$ gauge groups

 $\left(\frac{\partial^2 \theta_W Q_f}{\partial Q_f}\right)$

$$\sin^2 \theta_W = \frac{(g')^2}{g^2 + (g')^2}$$



The weak mixing angle(s)

• the effective leptonic weak mixing angle enters in the definition of the effective $Zf\bar{f}$ vertex at the Z resonance ($q^2 = m_Z^2$), when f is a lepton

$$\mathscr{M}_{Zf\bar{f}}^{eff} = \bar{u}_l \gamma_\alpha \left[\mathscr{G}_v^f(m_Z^2) - \mathscr{G}_a^f(m_Z^2) \gamma_5 \right] v_l \varepsilon_Z^\alpha$$

$$4 |Q_f| \sin^2 \theta_{eff}^f = 1 - \frac{\mathscr{G}_v^f}{\mathscr{G}_a^f}$$



The weak mixing angle(s)

• the effective leptonic weak mixing angle enters in the definition of the effective $Zf\bar{f}$ vertex at the Z resonance ($q^2 = m_Z^2$), when f is a lepton

$$\mathscr{M}_{Zf\bar{f}}^{eff} = \bar{u}_l \gamma_\alpha \left[\mathscr{G}_v^f(m_Z^2) - \mathscr{G}_a^f(m_Z^2) \gamma_5 \right] v_l \varepsilon_Z^\alpha$$

the effective weak mixing angle receives quantum corrections (SM, BSM,...) through

the self-energy corrections



the flavour-dependent vertex corrections

from all those diagram which yield a different corrections to left- and right-handed currents

$$4|Q_f|\sin^2\theta_{eff}^f = 1 - \frac{\mathscr{G}_v^f}{\mathscr{G}_a^f}$$





The weak mixing angle(s)

• the MSbar weak mixing angle stems from the renormalisation of the weak coupling in the MSbar renormalisation scheme

$$\frac{G_{\mu}}{\sqrt{2}} = \frac{g_0^2}{8m_{W,0}^2} \longrightarrow \hat{s}^2 \hat{c}^2 = \frac{\pi\alpha}{\sqrt{2}G_{\mu}m_Z^2(1-\Delta\hat{r})} \qquad \hat{s}^2 \equiv \sin^2\hat{\theta}(\mu_R = m_Z)$$

it is flavour independent

it has a weak dependence on the top-quark corrections \rightarrow precise theoretical prediction



The running of the MSbar weak mixing angle

The electric charge $\hat{\alpha}$ and the vector coupling \hat{v}_f of a Z boson to a fermion f satisfy the RGEs:

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum_i K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$
$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha} Q_f}{24\pi} \left[\sum_i K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q Q_q \right) \left(\sum_q \hat{v}_q \right) \right]$$

with Q_i electric charges, γ_i field type constants, K_i the β -function coefficients, σ 4- and 5-loop corrections

J.Erler, M. Ramsey Musolf, hep-ph/0409169, J.Erler, R.Ferro-Hernandez, arXiv:1712.09146,



The running of the MSbar weak mixing angle

The electric charge $\hat{\alpha}$ and the vector coupling \hat{v}_f of a Z boson to a fermion f satisfy the RGEs:

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum_i K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$
$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha} Q_f}{24\pi} \left[\sum_i K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q Q_q \right) \left(\sum_q Q_q \right) \left(\sum_q Q_q \right) \right]$$

with Q_i electric charges, γ_i field type constants, K_i the β -function coefficients, σ 4- and 5-loop corrections

The solution

$$\hat{s}^{2}(\mu) = \hat{s}^{2}(\mu_{0})\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_{0})} + \lambda_{1}\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_{0})}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_{2}}{3}\ln\frac{\mu^{2}}{\mu_{0}^{2}} + \frac{3\lambda_{3}}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_{0})} + \tilde{\sigma}(\mu_{0}) - \tilde{\sigma}(\mu)\right]$$

expresses the dependence of $\hat{s}^2(\mu) \equiv \sin^2 \hat{\theta}_W(\mu)$ on the renormalisation scale μ and on the coupled running of $\hat{\alpha}(\mu)$

J.Erler, M. Ramsey Musolf, hep-ph/0409169, J.Erler, R.Ferro-Hernandez, arXiv:1712.09146,





The running of the MSbar weak mixing angle and sensitivity to New Physics

Additional (BSM) virtual contributions modify the β -function changing the slope of the running (or even the sign)

At low energies,

there is sensitivity to the effects due to light new particles otherwise swamped at the Z resonance or at higher scales

At high energies, we might hope to have indirect hints of new heavy particles

The reference experimental precision is still set by the LEP value $\sin^2 \hat{\theta}_W(m_Z^2) = 0.23121(4)$ (PDG) or $\Delta \sin^2 \theta_{eff}^{\ell} = 16 \cdot 10^{-5}$

Given the size of the running effects, a SM test achievable with O(1%) determinations

Given the rich literature on the possible studies at low-energy facilities it is natural to investigate the possibility of a determination in the TeV region exploiting the sub-percent precision expected at the end of HL-LHC









Observables sensitive to the weak mixing angle (1)

Neutral Current Drell-Yan is the obvious process to investigate the couplings of the Z boson

The lepton-pair invariant mass distribution offers the possibility of testing the couplings at different energy scales



 $\sin^2 \theta_W$ is related to the ratio of vector and axial-vector couplings \rightarrow link to observables sensitive to parity violation The cross section is mediated by photon and Z exchanges \rightarrow the actual rate depends on $\sin^2 \theta_W$ value



Observables sensitive to the weak mixing angle (2)

Decomposing the invariant mass distribution into a forward (F) and a backward (B) components, $F(M_{\ell\ell}) \equiv \int_{0}^{1} d\cos\theta_{CS} \frac{d\sigma}{d\cos\theta_{CS}} (M_{\ell\ell}) \qquad B(M_{\ell\ell}) \equiv \int_{-1}^{0} d\cos\theta_{CS} \frac{d\sigma}{d\cos\theta_{CS}} (M_{\ell\ell})$

we have two combinations F + B and F - B, with complementary information

$$\rightarrow \text{ we consider } \frac{d\sigma}{dM_{\ell\ell}} \quad \text{and} \quad A_{FB}(M_{\ell\ell}) \equiv \frac{F(M_{\ell\ell}) - F(M_{\ell\ell})}{F(M_{\ell\ell}) + F(M_{\ell\ell})}$$



 $m_{\mu\mu}$ (GeV)

Sensitivity of the observables to the weak mixing angle (1)

The Forward-Backward asymmetry

The $A_{FB}(M_{\ell\ell})$ asymmetry expresses the amount of parity violation due to the presence of an axial-vector coupling but $A_{FB}(M_{\ell\ell}) \neq 0$ does not imply sensitivity to $\sin^2 \theta_W$

At the Z resonance, the Z vector coupling v_f plays a role \rightarrow we have sensitivity to $\sin^2 \theta_W$

Outside the resonance, the Z exchange falls to zero more rapidly than the $\gamma - Z$ amplitudes interference

→ large asymmetry due to the axial-vector Z coupling but very low sensitivity to $\sin^2 \theta_W$

The theoretical sensitivity $\frac{\delta A_{FB}}{\delta \sin^2 \theta_W}$ can be evaluated at LO and must be supplemented by all relevant uncertainty sources



12

M_{II} (GeV)





Sensitivity of the observables to the weak mixing angle (2) The triple-differential cross section at LO

$$\frac{\mathrm{d}^{3}\sigma}{\mathrm{dm}_{\ell\ell}\mathrm{d}y_{\ell\ell}\mathrm{d}\cos\theta_{CS}} = \frac{\pi\alpha^{2}}{\mathrm{3m}_{\ell\ell}s} \left((1 + \cos^{2}\theta_{CS})\sum_{q} S_{q}[f_{q}(x_{1}, Q^{2})f_{\overline{q}}(x_{2}, Q^{2}) + f_{q}(x_{2}, Q^{2})f_{\overline{q}}(x_{1}, Q^{2})] + \cos\theta_{CS}\sum_{q} A_{q}\mathrm{sign}(y_{\ell\ell}) \cdot [f_{q}(x_{1}, Q^{2})f_{\overline{q}}(x_{2}, Q^{2}) - f_{q}(x_{2}, Q^{2})f_{\overline{q}}(x_{1}, Q^{2})] \right)$$

$$S_{q} = e_{\ell}^{2} e_{q}^{2} + P_{\gamma Z} \cdot e_{\ell} v_{\ell} e_{q} v_{q} + P_{ZZ} \cdot (v_{\ell}^{2} + a_{\ell}^{2}) (v_{q}^{2} + a_{q}^{2}) \qquad P_{\gamma Z}(\mathbf{m}_{\ell \ell}) = \frac{2\mathbf{m}_{\ell \ell}^{2}(\mathbf{m}_{\ell \ell}^{2} - \mathbf{m}_{Z}^{2})}{\sin^{2} \theta_{W} \cos^{2} \theta_{W}[(\mathbf{m}_{\ell \ell}^{2} - \mathbf{m}_{Z}^{2})^{2} + \Gamma_{Z}^{2} \mathbf{m}_{Z}^{2}]} A_{q} = P_{\gamma Z} \cdot 2e_{\ell} a_{\ell} e_{q} a_{q} + P_{ZZ} \cdot 8v_{\ell} a_{\ell} v_{q} a_{q}, \qquad P_{ZZ}(\mathbf{m}_{\ell \ell}) = \frac{\mathbf{m}_{\ell \ell}^{4}}{\sin^{4} \theta_{W} \cos^{4} \theta_{W}[(\mathbf{m}_{\ell \ell}^{2} - \mathbf{m}_{Z}^{2})^{2} + \Gamma_{Z}^{2} \mathbf{m}_{Z}^{2}]}$$

The 3D differential xsec exhibits a dependence on the specific $\sin^2 \theta_W$ value, modulated by the different combinations of γ and Z propagators.

At the Z resonance, specific sensitivity to $\sin^2 \theta_W$, via the ratio of vector/axial-vector couplings, assessed from the study of A_{FB} and A_{LR} asymmetries

Also at large invariant masses the cross section features a sensitivity to $\sin^2 \theta_W$, stemming from both normalisation and angular-dependent factors, in non trivial combination!

 \rightarrow at NLO-EW we can study $\sin^2 \hat{\theta}(\mu_R)$, the MSbar renormalised mixing angle, and exploit the large mass range to test the running of this quantity

```
13
```



Determination of the Lagrangian parameters

The input parameters choice We trade (g, g', v, λ) for 4 experimental inputs, e.g. $(\hat{\alpha}(\mu$

The only parameters that we can fit from the kinematical distributions are these 4 inputs

A good description of the data must accompany the identification of the best-fit value of $\sin^2 \hat{\theta}_W(\mu)$

$$(\mu), \sin^2 \hat{\theta}_W(\mu), m_Z, m_H \Big)$$

Any observable that we compute is a function of only these four quantities $\mathcal{O} = \mathcal{O}\left(\hat{\alpha}(\mu), \sin^2\hat{\theta}_W(\mu), m_Z, m_H\right)$

For its determination, $\sin^2 \hat{\theta}_W(\mu)$ must be one of the inputs and, at (N)NLO-EW, must be renormalized



Determination of the Lagrangian parameters

The input parameters choice We trade (g, g', v, λ) for 4 experimental inputs, e.g. $(\hat{\alpha}(\mu$

The only parameters that we can fit from the kinematical distributions are these 4 inputs For its determination, $\sin^2 \hat{\theta}_W(\mu)$ must be one of the inputs and, at (N)NLO-EW, must be renormalized

A good description of the data must be accompany the identification of the best-fit value of $\sin^2 \hat{\theta}_W(\mu)$

Why $\hat{\alpha}(\mu)$ instead of G_{μ} ? It is not mandatory, but one remarks that $g = e/\sin\theta_W$. A simultaneous MSbar renormalisation of e and $\sin \theta_W$ naturally corresponds to the MSbar definition of g, which leads to a more systematic inclusion of the logarithms to be resummed via RGE

Using two observables such as the invariant mass distribution and the A_{FB} asymmetry allows to decouple the two dependencies, accessing precisely the information about $\sin^2 \hat{\theta}_W(\mu)$

$$(\mu), \sin^2 \hat{\theta}_W(\mu), m_Z, m_H$$

Any observable that we compute is a function of only these four quantities $\mathcal{O} = \mathcal{O}\left(\hat{\alpha}(\mu), \sin^2 \hat{\theta}_W(\mu), m_Z, m_H\right)$



Templates to be fitted to the data (I)



The templates are a set of predictions of our observable, computed with different numerical values of the input parameter.



Templates to be fitted to the data (1)

The templates are a set of predictions of our observable, computed with different numerical values of the input parameter. Example I: fitting $\sin^2 \hat{\theta}_W (\mu_R^2 = m_Z^2)$

we compute the NC DY invariant mass distribution in the range $M_{\ell\ell} \in [m_Z, 1 \text{ TeV}]$ we stick to plain NLO-EW and renormalize at $\mu_R = m_Z$ we assign to $\sin^2 \hat{\theta}_W(\mu_R^2 = m_Z^2)$ several values in a range, e.g. [0.22600, 0.23600] by fitting the data, we identify which value globally best describes the data

since the input is defined at $\mu_R = m_Z$, the good description at high invariant masses depends on the diagrammatic content of the fitting formula the fitted parameter is always defined at $\mu_R = m_Z$, but its value is affected also by high-mass data



with this approach we experimentally determine $\sin^2 \hat{\theta}_W(\mu_R^2 = m_Z^2)$, i.e. only the boundary condition of the RGE



Templates to be fitted to the data (2)

The templates are a set of predictions of our observable, computed with different numerical values of the input parameter. Example 2: fitting $\sin^2 \hat{\theta}_W(\mu_R^2 = M_{\ell\ell}^2)$ in several invariant mass bins we compute the NC DY invariant mass distribution in one single bin of invariant mass $[M_{\ell\ell}, M_{\ell\ell} + \delta]$ we stick to plain NLO-EW and we renormalize $\sin^2 \theta_W$ at $\mu_R = M_{\ell\ell}$ we assign to $\sin^2 \hat{\theta}_W(\mu_R^2 = M_{\ell\ell}^2)$ several values in a range, e.g. [0.22600, 0.25600] by fitting only in that $M_{\ell\ell}$ bin, we identify which value best describes that data point

we repeat the above procedure in different mass bins in each bin we might expect to find a different best fit value

the sequence of best fit values, as a function of $M_{\ell\ell}$, can be compared with the solution of the RGE

18

with this approach, we extract information about: 1) the fact that $\sin^2 \hat{\theta}_W(\mu)$ indeed runs



2) the slope of the running (cfr. with SM β function)









At large invariant masses, NNLO QCD and NLO EW corrections are separately large and with opposite signs acertainities: 7-point scale variation we also observe large NNLO QCD-EW corrections $\hat{QCD+EW+MIX}_{MCM}$ is NNLO OCD+EW+ ised approximation of mixed corrections

Which corrections do not contribute to the redefinition of the running coupling? all the QCD corrections (same contribution to left- and right-handed couplings) more delicate breakdown of the EW contributions

```
20
```



Main subsets of EW corrections in the Drell-Yan process

- QED final state radiation
- universal corrections to the LO couplings
- EW Sudakov logarithms

Which ones do / do not contribute to the redefinition of the weak coupling at quantum level ?



Main subsets of EW corrections in the Drell-Yan process

- QED final state radiation

- universal corrections to the LO couplings
- EW Sudakov logarithms



Do not contribute to the redefinition of the LO couplings (same contribution to left- and right-handed currents) Not negligible kinematical effect moving events from higher to lower invariant mass bins Same mechanism, with large effect, at the Z resonance, of $\mathcal{O}(80\%)$





Main subsets of EW corrections in the Drell-Yan process

20

10

0

-10

-20

-30

0.5

I-loop

- QED final state radiation
- universal corrections to the LO couplings
- EW Sudakov logarithms

Born (%)

correction w.r.t





The EW Sudakov logs stem from vertex and box corrections

Their correction can be cast as

- one overall correction to the cross section
- one factor which distinguishes left- and right-handed currents
 - \rightarrow contributes to the definition of an effective mixing angle

Very large in the high-mass tail of the distribution (also at 2-loop level)

PDF-weighted combination of two alternating signs series of terms





Main subsets of EW corrections in the Drell-Yan process

- QED final state radiation
- universal corrections to the LO couplings
- EW Sudakov logarithms

Relevant in the accurate description of the Z resonance The values of the couplings at $\mu_R = m_Z$ are initial conditions of the running of $\hat{\alpha}(\mu)$ and $\sin^2 \hat{\theta}(\mu) \rightarrow$ relevant for our test EW precision tests at the LHC from the simultaneous comparison of 100 and 1000 GeV regions

The impact of different universal corrections to the LO couplings can be illustrated via an Improved Born Approximation The interplay of photon- and Z-exchange diagrams is modulated by the precise values of their respective couplings

In the following slides, the reference is given by LO results in the $(\alpha(0), m_W, m_Z)$ input scheme

Each input replacement effectively introduces higher-order corrections, which should otherwise be computed in pert. theory





Alessandro Vicini - CERN and University of Milano

Running of α only in the photon diagram enhances the photon exchange contribution which grows with the invariant mass

 $\alpha(0) \to \alpha(M_{\ell\ell}^2)$

2







Alessandro Vicini - CERN and University of Milano

Use of G_{μ} only in the Z diagram enhances the peak of the Z resonance

 $\alpha(0) \to G_{\mu}$







Alessandro Vicini - CERN and University of Milano

Use of G_{μ} and ρ only in the Z diagram enhances the peak of the Z resonance

 $\alpha(0)\to G_\mu\;\rho$

7





- running of α only in the photon diagram

Towards an experimental determination of $\sin^2 \hat{\theta}(\mu_R)$

 $\sin^2 \hat{\theta}(\mu_R)$ determination at hadron colliders at large invariant masses

S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.10782

Study at $\sqrt{S} = 13.6$ TeV, with 300 fb⁻¹ and 3 ab⁻¹ POWHEG_Z-EW_BMNNPV implementing the $(\hat{\alpha}(\mu), \sin^2 \hat{\theta}_W(\mu), m_Z, m_H)$ input scheme templates generated at LO-EW and at NLO-EW, including NLO+PS QCD corrections proton PDF: NPDF31_nnlo_as_0118_hessian The running of $\sin^2 \hat{\theta}(\mu_R)$ and of $\hat{\alpha}(\mu_R)$ computed at 1-loop level

Simulations with 10^9 events per invariant mass bin and a simplified detector simulation

Uncertainties estimated at double differen w.r.t. invariant mass and rapidity

bin boundaries: 0.0, 0.4, 0.8, 1.2, 1.6, 2.0, 2.5

leading lepton: $p_{\perp}^{\ell} > 40 \text{ GeV}$ subleading lepton: $p_{\perp}^{\ell} > 30 \text{ GeV}$, both leptons $|\eta^{\ell}| < 2.5$

- experimental systematics
- missing higher-orders
- PDF uncertainties

ntial	level

$\sin^2 \hat{\theta}(\mu_R)$ determination at hadron colliders at large invariant masses

S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.10782

Study at $\sqrt{S} = 13.6 \text{ TeV}$, with 300 fb⁻¹ and 3 ab⁻¹ POWHEG_Z-EW_BMNNPV implementing the $(\hat{\alpha}(\mu), \sin^2 \hat{\theta}_W(\mu), m_Z, m_H)$ input scheme templates generated at LO-EW and at NLO-EW, including NLO+PS QCD corrections proton PDF: NPDF31_nnlo_as_0118_hessian The running of $\sin^2 \hat{\theta}(\mu_R)$ and of $\hat{\alpha}(\mu_R)$ computed at 1-loop level Simulations with 10^9 events per invariant mass bin and a simplified detector simulation

The sensitivity has been studied by fitting the triple-diff. xsec in the xFitter framework determining $\delta \sin^2 \hat{\theta}(\mu_R)$ induced by the uncertainty sources treated as nuisance parameters σ [%]

available PDF parameterisations lead to different estimates of \check{b} the uncertainty, up to 50%

missing higher orders do not spoil the conclusions about sensitivity

Uncertainties estimated at double differential level w.r.t. invariant mass and rapidity

 $^{\prime} > 40 \text{ GeV}$ subleading lepton: $p_{\perp}^{\ell} > 30 \text{ GeV}$, both leptons $|\eta^{\ell}| < 2.5$

- experimental systematics
- missing higher-orders
- PDF uncertainties

$\sin^2 \hat{\theta}(\mu_R)$ determination at hadron colliders at large invariant masses

S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.10782

The running of the MSbar angle can be established at LHC in Run III and at HL-LHC with percent precision.

For the actual measurement the best theoretical predictions will be needed, to avoid interpretation mismatches: full NNLO (QCD, EW and mixed QCDxEW) and leading higher orders

Conclusions

• The running of the MSbar angle can be established at LHC in Run III and at HL-LHC with percent precision. Precision test of the SM, complementary to those at low-energy facilities.

Sensitivity to different BSM physics hypotheses

• Several quantum corrections, which do not contribute to the redefinition of the renormalised / effective coupling, must be computed explicitly to avoid SM biases which would fake a BSM signal

The renormalisation program at NNLO-EW will be needed

• The determination of $\sin^2 \hat{\theta}(\mu_R)$ and its related difficulties are the prototype of a precision determination of a Wilson coefficient in SMEFT (starting from the NNLO-EW renormalisation)

Theoretical best predictions vs templates

To compute the best prediction of one observable, we use

- the best (most precisely measured) available input parameters
- the best theoretical cross section results

Assuming e.g. a NLO-EW calculation,

- the renormalized parameter is a constant, without running.
- the I-loop corrections relevant for the definition of the Z vector coupling would generate the same effect of the running coupling, approximated at $\mathcal{O}(\alpha)$
- provided we remove the double counting with the diagrammatic contribution

The theoretical best prediction is not meant to fit the input parameters, but it rather tests the quality of data description

in our discussion $\sin^2 \hat{\theta}_W(m_z^2) = 0.23121(4)$ would be the value of the renormalized parameter at the scale $\mu_R = m_Z$.

- it is possible to improve the calculation with higher-order corrections, replacing $\sin^2 \hat{\theta}_W(\mu_R^2 = m_Z^2) \rightarrow \sin^2 \hat{\theta}_W(M_{\ell\ell}^2)$

