Critical QFTs with spontaneous breaking of scale symmetry

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3d critical theories

many applications in condensed matter physics
Dirac materials
Wilson-Fisher fixed points
mass generation and symmetry breaking

AdS/CFT conjecture

3d critical bosons and critical fermions
relate to higher-spin gauge theories on AdS4

Klebanov, Polyakov '02
Sezgin, Sundell '03

Giombi, Yin '12

CFT vs higher spin symmetry

Maldacena, Zhiboedov '11, '12

Chern-Simons-matter dualities

Aharony, Giombi, Gur-Ari, Maldacena, Yacoby, '12
Seiberg, Senthil, Wang, Witten, '16

today:

3d QFTs with strongly interacting FPs
and *spontaneous* scale symmetry breaking

scalars	O(*N*)
fermions	Gross-Neveu
Yukawa	Gross-Neveu—Yukawa

based on 2207.10115, 2212.06815, 2311.16246
and ongoing work with **Charlie Cresswell-Hogg**

recap: O(N) symmetric scalars

3d: **super**-renormalisable $(\phi^*\phi)^3_{3\text{d}}$

free UV fixed point
Wilson-Fisher IR fixed point

exactly solvable at infinite N

main tool: functional RG

Wilson '71
Polchinski '84
Wetterich '92

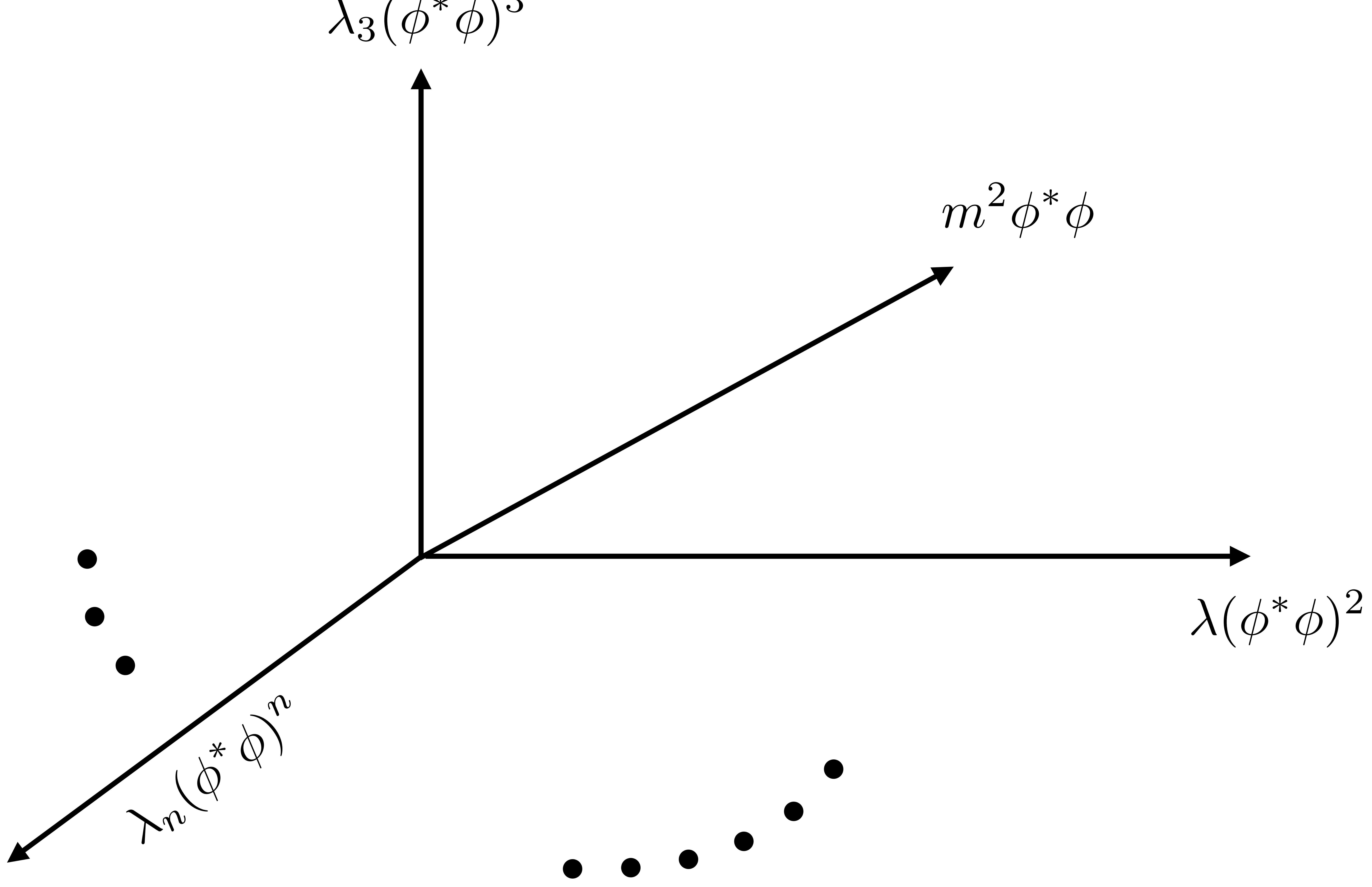
$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{tr}\left\{\left[\Gamma_k^{(2)} + R_k\right]^{-1} \cdot \partial_t R_k\right\}$$

$t = \ln k$

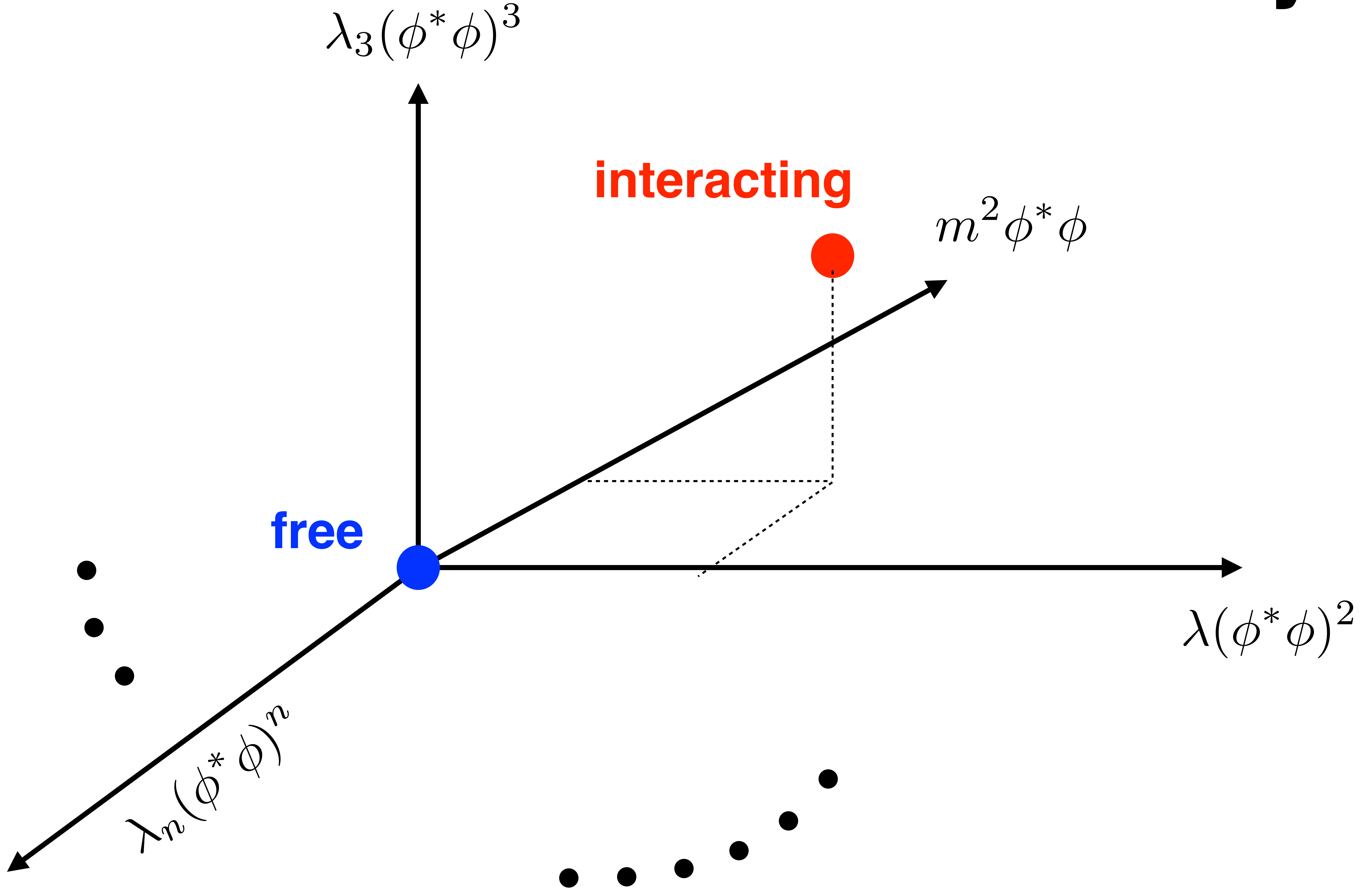
UV
$k = \Lambda$

IR
$k = 0$

“theory space”



“theory space”



“theory space”

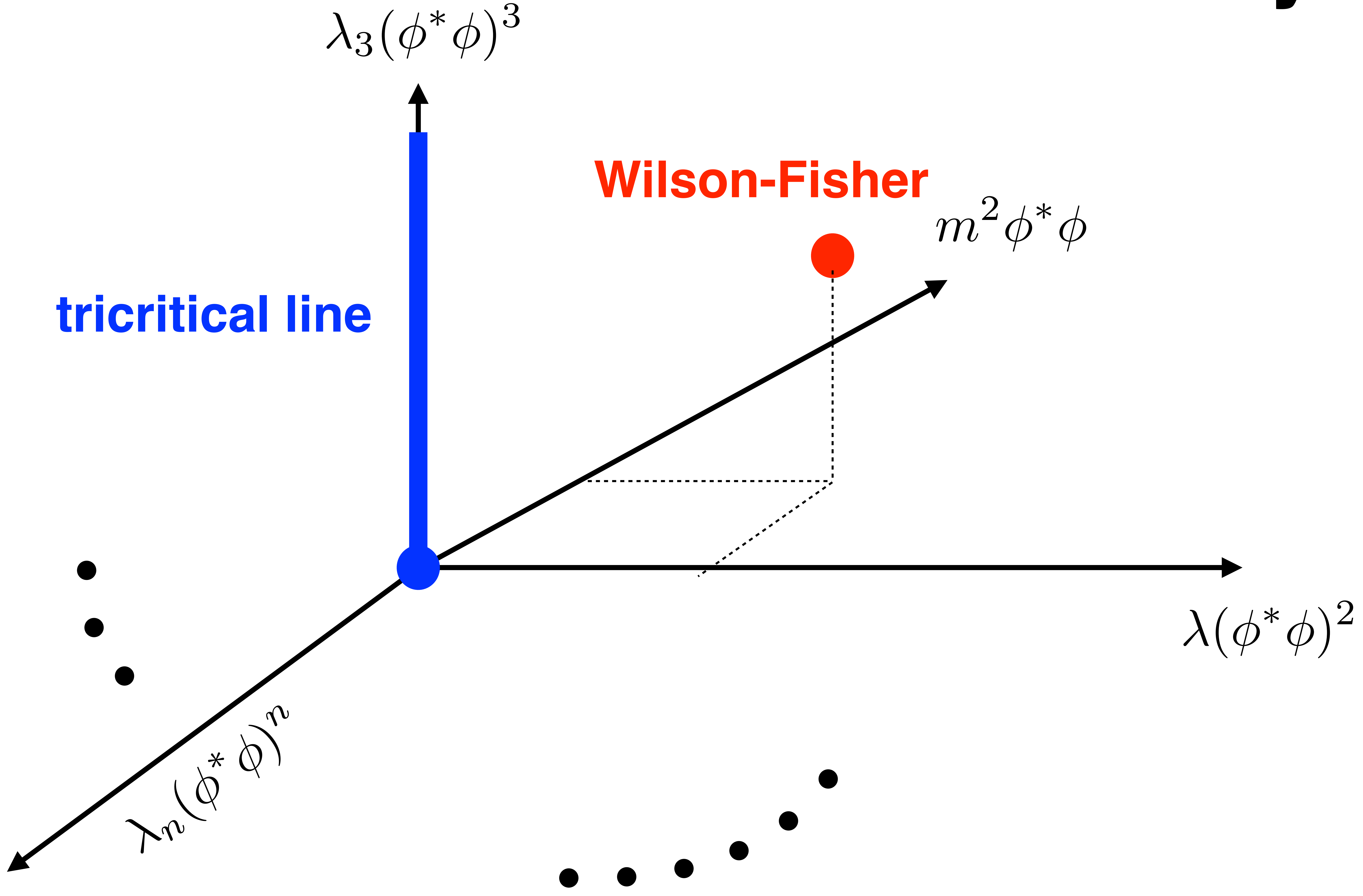
interactions	classical
$m^2\phi^*\phi$	relevant
$\lambda(\phi^*\phi)^2$	relevant
$\lambda_3(\phi^*\phi)^3$	marginal
$\vdots$	
$\lambda_n(\phi^*\phi)^n$	irrelevant
$\vdots$	
	UV

“theory space”

interactions	classical	quantum
$m^2\phi^*\phi$	relevant	relevant
$\lambda(\phi^*\phi)^2$	relevant	
$\lambda_3(\phi^*\phi)^3$	marginal	irrelevant
$\vdots$		
$\lambda_n(\phi^*\phi)^n$	irrelevant	
$\vdots$		

UV ⟷ IR

“theory space”



functional RG study

Polchinski:
UV cutoff

$$P_{\rm UV} = \frac{K(q^2/k^2)}{q^2}$$

Polchinski '84

exact map
R=R_opt

$$K(q^2/k^2) = \frac{R_k(q^2)}{q^2 + R_k(q^2)}$$

$$P_{\rm UV} + P_{\rm IR} = \frac{1}{q^2}$$

Wetterich:
IR cutoff

$$P_{\rm IR} = \frac{1}{q^2 + R_k(q^2)}$$

Wetterich '92

DF Litim, E Marchais, P Mati, 1702.05749/PRD
DF Litim, M Trott, 1810.01678 /PRD

functional RG study

Polchinski:
UV cutoff

exact map
R=R_opt

$$
\begin{cases}
\partial_t u &= -du + (d-2)\rho u' + 2\rho(u')^2 - (N-1)u' - (u' + 2\rho u'') \\
\partial_t w &= -dw + (d-2)zw' + (N-1)\left(\dfrac{1}{1+w'} - 1\right) + \left(\dfrac{1}{1+w'+2zw''} - 1\right)
\end{cases}
$$

Wetterich:
IR cutoff

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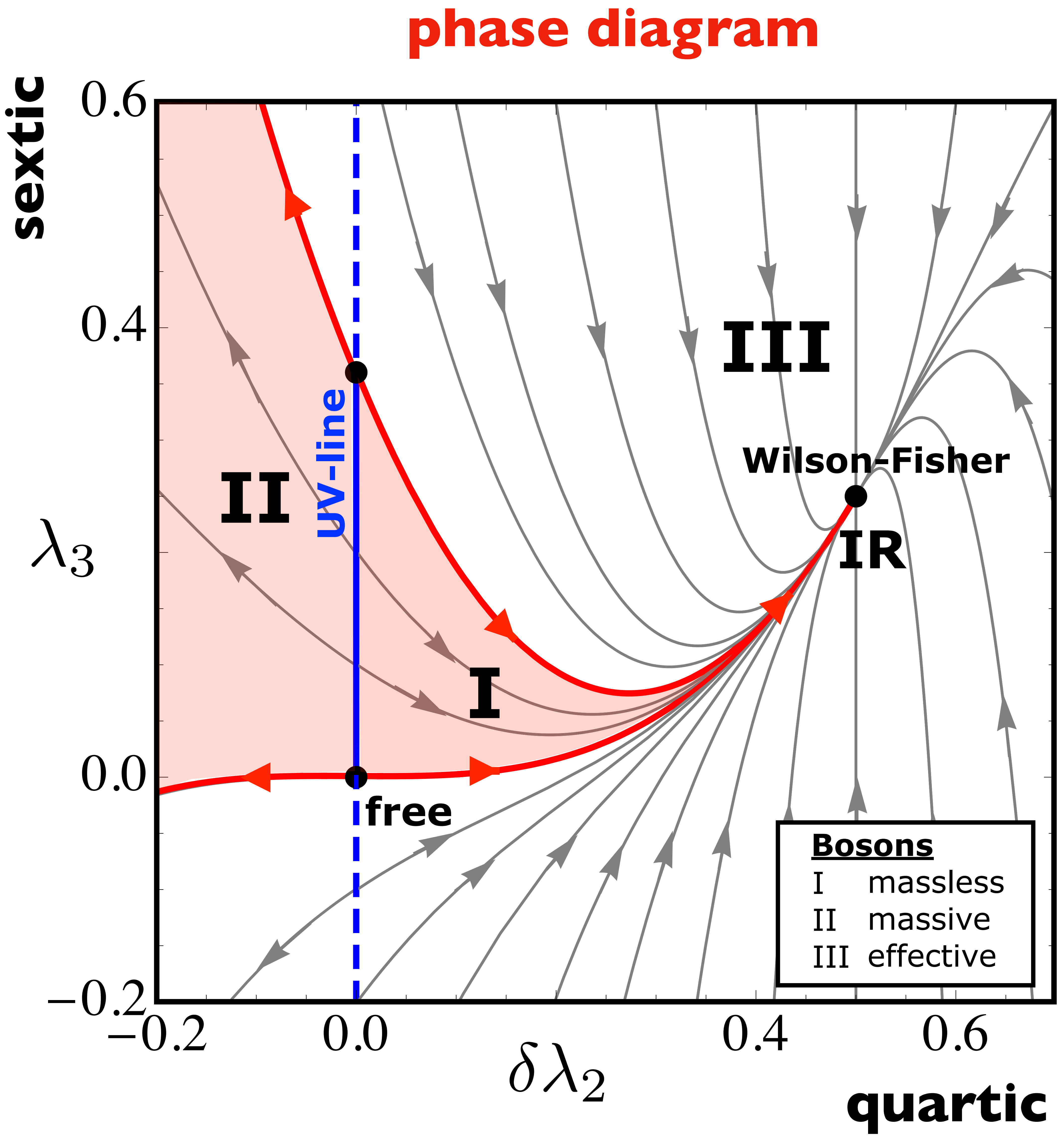
DF Litim, M Trott, 1810.01678 /PRD

functional RG



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functional RG

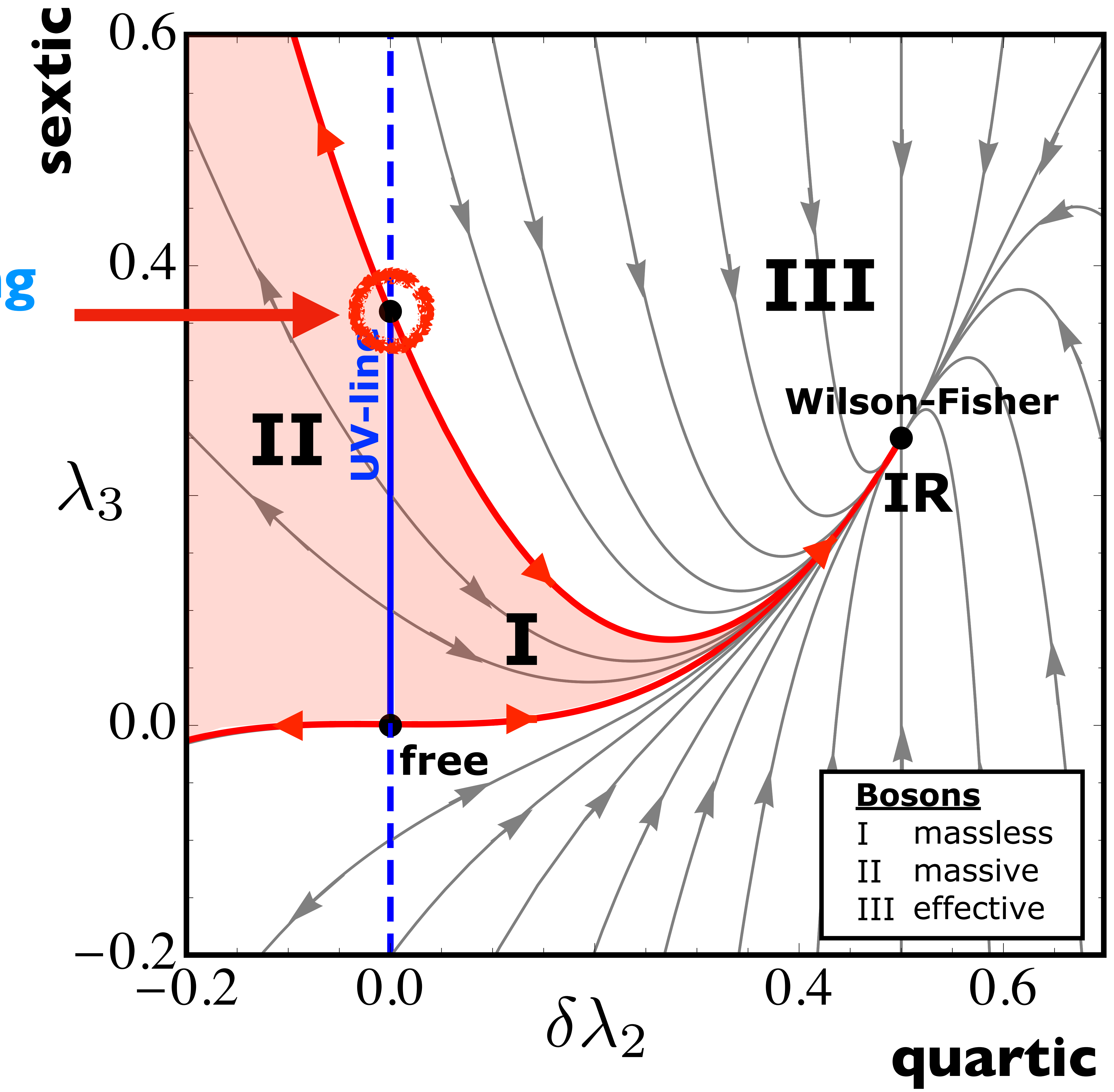
BMB phenomenon Bardeen, Moshe, Bander '84
David, Kessler, Neuberger '84

spontaneous scale symmetry breaking
breaking of hyperscaling

→ **compact conformal manifold**
physical mass = free parameter

→ **non-perturbative**
infinite-order in local couplings

phase diagram



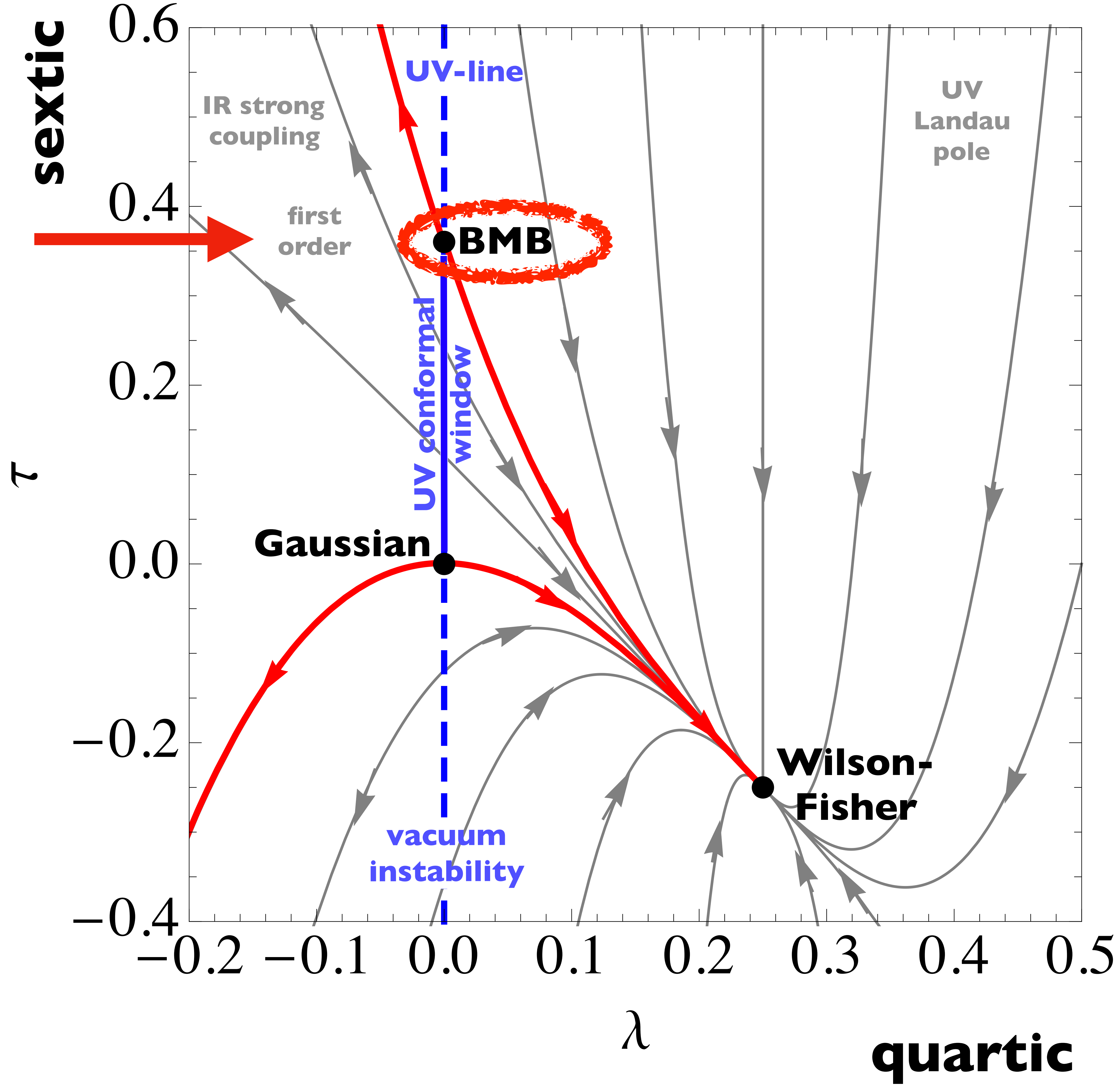
DF Litim, E Marchais, P Mati, 1702.05749/PRD

Polchinski RG

spontaneous scale symmetry breaking

breaking of hyperscaling

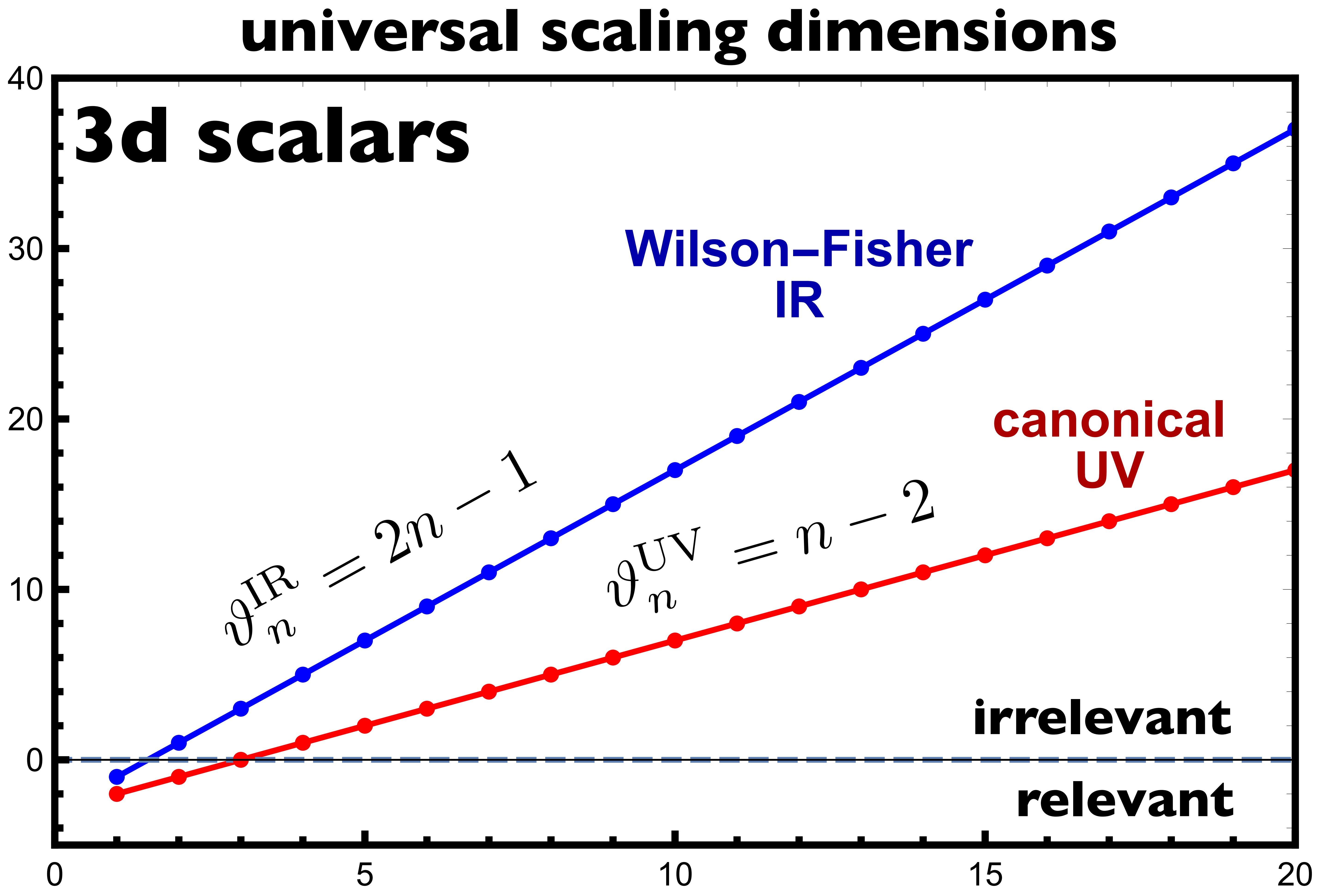
phase diagram



DF Litim, M Trott, 1810.01678 /PRD

large quantum effects

$$\frac{\vartheta_n^{(\mathrm{IR})} - \vartheta_n^{(\mathrm{UV})}}{\vartheta_n^{(\mathrm{IR})}} = \frac{n+1}{2n-1}$$



Gross-Neveu

U(N) symmetric fermions

4-fermion interactions $G(\bar{\psi}\psi)^2$

Gross, Neveu '74

chiral symmetry $\psi \to \gamma^5\psi \quad \bar{\psi} \to -\bar{\psi}\gamma^5$

3d: perturbatively non-renormalisable… $[G] = 2 - d$

…yet non-perturbatively renormalisable
interacting UV fixed point

Gawedzki, Kupiainen '85
Rosenstein, War, Park '89
de Calan, Faria da Veiga, Magnen, de Seneor '91

functional RG

Jakovac, Patkos '13, '14
Cresswell-Hogg, Litim,'22, '23 and in prep '24

Gross-Neveu+

relax chiral symmetry

$$S=\int \mathrm{d}^d x\left\{\bar{\psi}_a \partial\!\!\!/ \psi_a+\frac{1}{2} G(\bar{\psi}_a \psi_a)^2+\frac{1}{3!} H(\bar{\psi}_a \psi_a)^3\right\}$$

mass term permitted $m\bar{\psi}\psi$

6-fermion interactions permitted $[H]=3-2d$

functional RG

exactly solvable at infinite Nf

Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115
Critical fermions with spontaneous broken sale symmetry 2212.06815
Scale-symmetry breaking and generation of mass at quantum critical points 2311.16246

“theory space”

interactions	classical
$\lambda_1 \bar{\psi}\psi$	relevant
$\lambda_2 (\bar{\psi}\psi)^2$	irrelevant
$\lambda_3 (\bar{\psi}\psi)^3$	.
$\lambda_n (\bar{\psi}\psi)^n$	.
.	.
.	
.	IR

“theory space”

interactions

	classical	quantum
$\lambda_1 \bar{\psi}\psi$	relevant	relevant
$\lambda_2(\bar{\psi}\psi)^2$	irrelevant	relevant
$\lambda_3(\bar{\psi}\psi)^3$	.	marginal
$\lambda_n(\bar{\psi}\psi)^n$	.	irrelevant
.	.	.
.		.
.	IR	UV

Gross-Neveu+

functional RG

$$\Gamma_k = \int d^d x \left(\sum_{i=1}^{N_\mathrm{f}} \bar{\Psi}_i \partial\!\!\!/ \Psi_i + V_k(\Psi, \bar{\Psi}) \right)$$

local potential

$$\partial_t v = -dv + (d-1) z v' - (4N_\mathrm{f} + 1)\, \ell_d[(v')^2] + \ell_d[v' \cdot (v' + 2 z v'')],$$

$v(\bar{\psi}\psi)$
fermion potential

classical

$z \propto \bar{\psi}\psi$
field variable

quantum

Gross-Neveu+

$$v(z) = \sum_n \frac{\lambda_n}{n!} z^n$$

large Nf:

mass $$\beta_1 = -\lambda_1 + \frac{2\lambda_1\lambda_2}{(1+\lambda_1^2)^2}$$

4F $$\beta_2 = (d-2)\lambda_2 + \frac{2\lambda_1\lambda_3}{(1+\lambda_1^2)^2} + \frac{(2-6\lambda_1^2)\lambda_2^2}{(1+\lambda_1^2)^3}$$

6F $$\beta_3 = (2d-3)\lambda_3 + \frac{2\lambda_1\lambda_4}{(1+\lambda_1^2)^2} + \frac{6\lambda_2\lambda_3(1-3\lambda_1^2)}{(1+\lambda_1^2)^3} + \frac{24\lambda_1\lambda_2^3(\lambda_1^2-1)}{(1+\lambda_1^2)^4}.$$

mass=0 is an exact RG fixed point

generation of mass

Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115
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Gross-Neveu+

mass=0:

4F fixed point

4F $$\tilde{\beta}_2 = (d - 2 + 2\lambda_2)\lambda_2,$$

6F $$\tilde{\beta}_3 = (2d - 3 + 6\lambda_2)\lambda_3,$$

mass = 0 **renders 4F and 6F betas homogeneous**

Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115

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Gross-Neveu+

mass=0:

4F fixed point

4F $$\tilde{\beta}_2 = (d - 2 + 2\lambda_2)\lambda_2,$$

6F $$\tilde{\beta}_3 = (2d - 3 + 6\lambda_2)\lambda_3,$$

3d: **4F fixed point** **renders** **6F coupling** **exactly marginal**

scheme independent

Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115
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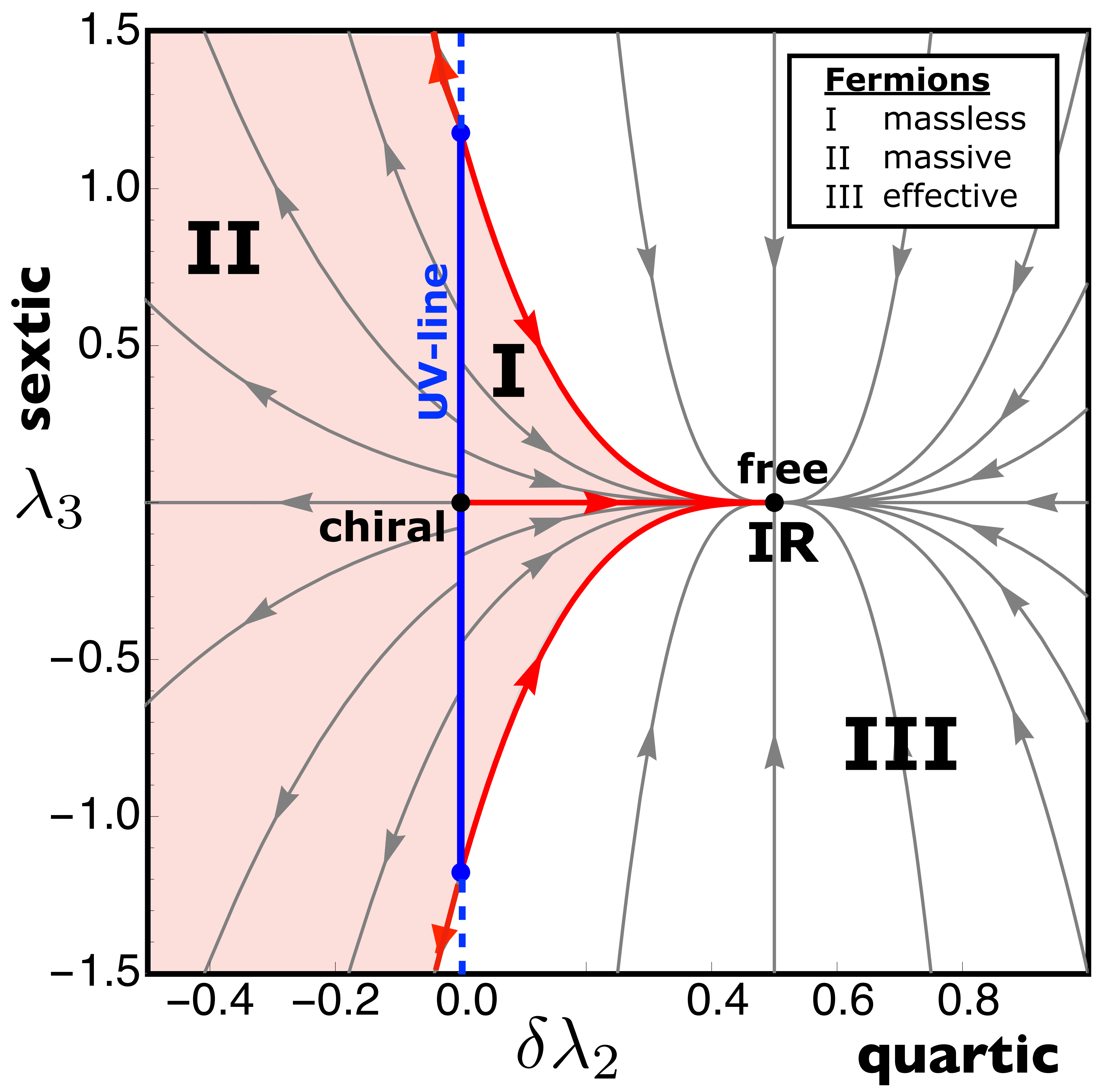
Gross-Neveu+

UV-IR connecting trajectories

exactly marginal sextic coupling



Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115

Gross-Neveu+

spontaneous scale symmetry breaking

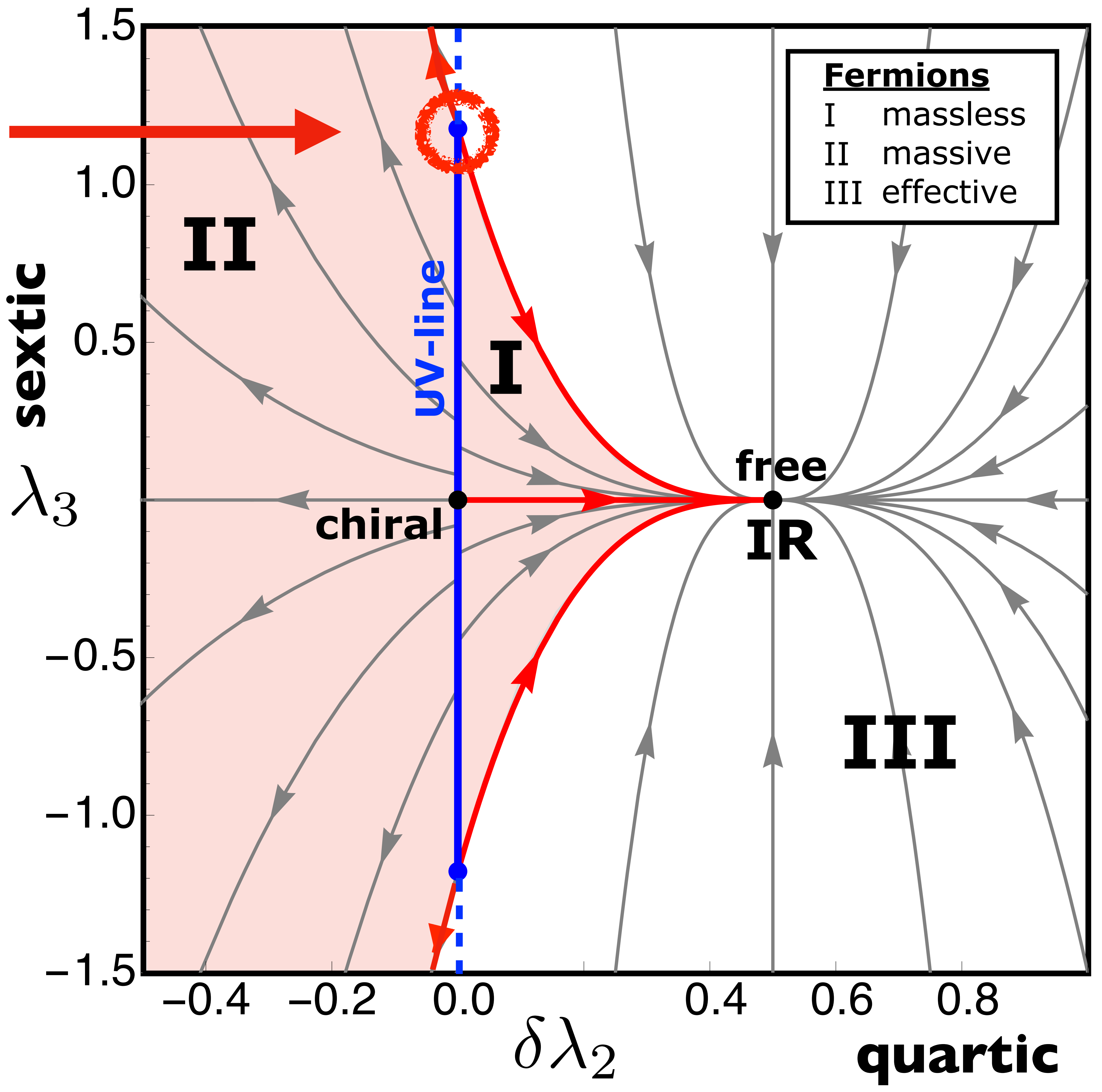
breaking of hyperscaling

compact conformal manifold

physical mass = free parameter

non-perturbative

infinite-order in local couplings



Cresswell-Hogg, Litim, **Line of fixed points in Gross-Neveu theories** 2207.10115

Gross-Neveu+

global fixed points

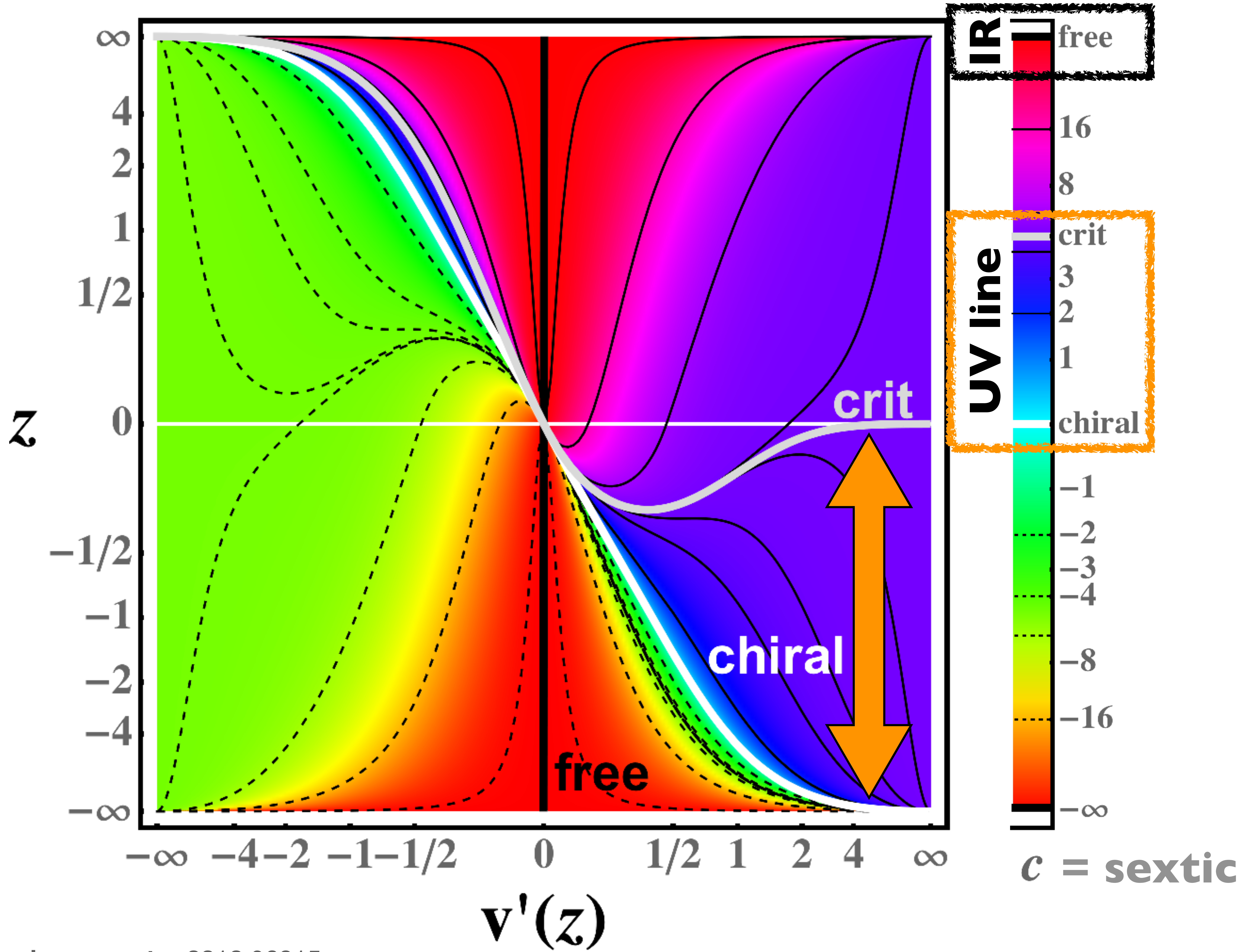
$v' = v'(z)$ for all z

$z=0$: $v' = 0$ **mass=0**

$v'' = -1/2$ **4F**

$v''' = c$ **6F**

finite UV conformal manifold



Cresswell-Hogg, Litim, **Critical fermions with spontaneous broken sale symmetry** 2212.06815

Gross-Neveu+

spontaneous generation of mass

$$M = \lim_{k \to 0} k \cdot v'(0)$$

gap equation

$$\left[c - \frac{3\pi}{2} \,\mathrm{sgn}(M) \right] M^2 = 0$$

two solutions

[...] = 0

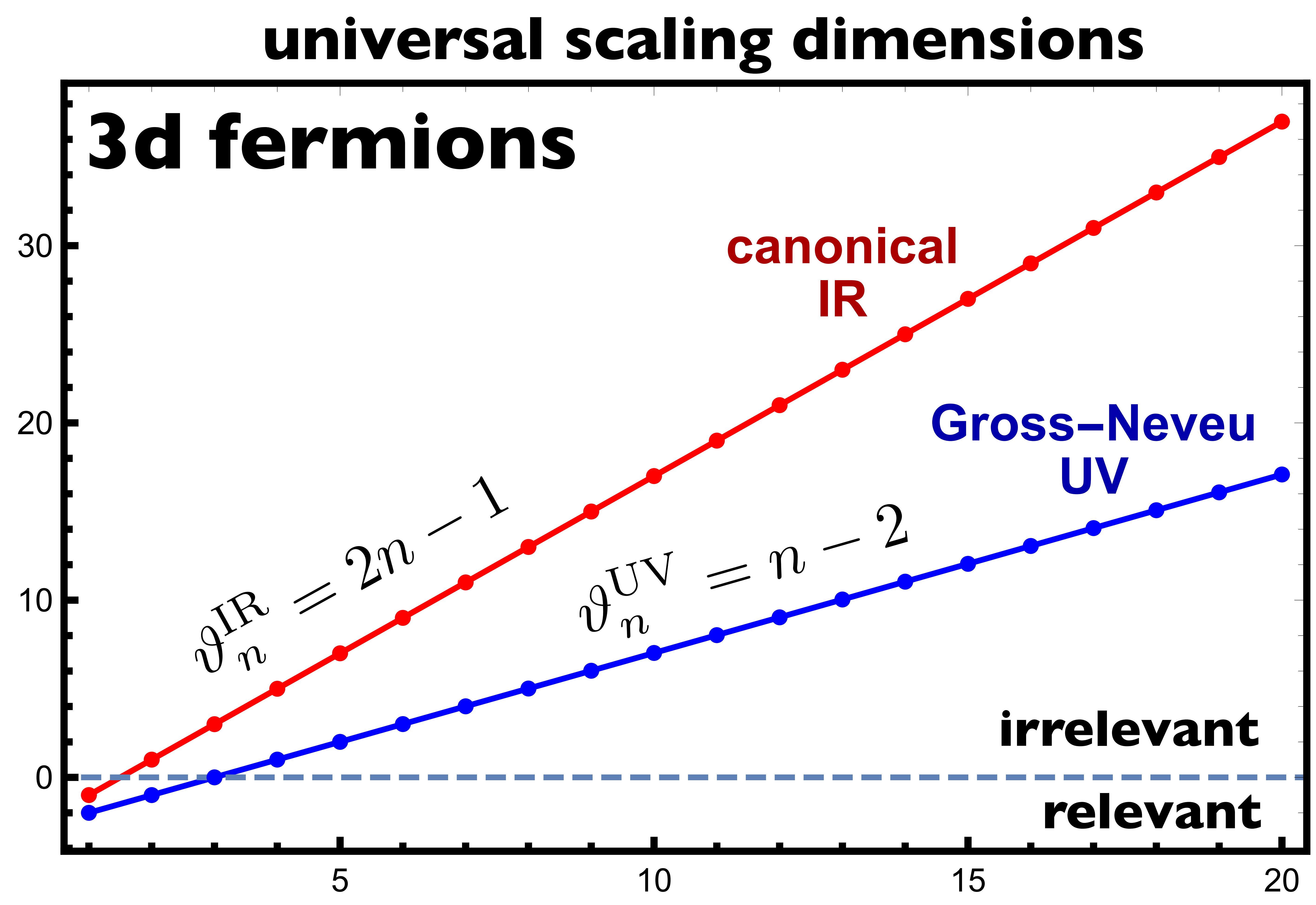
M = free parameter

M = 0

→ scale symmetry broken **spontaneously**

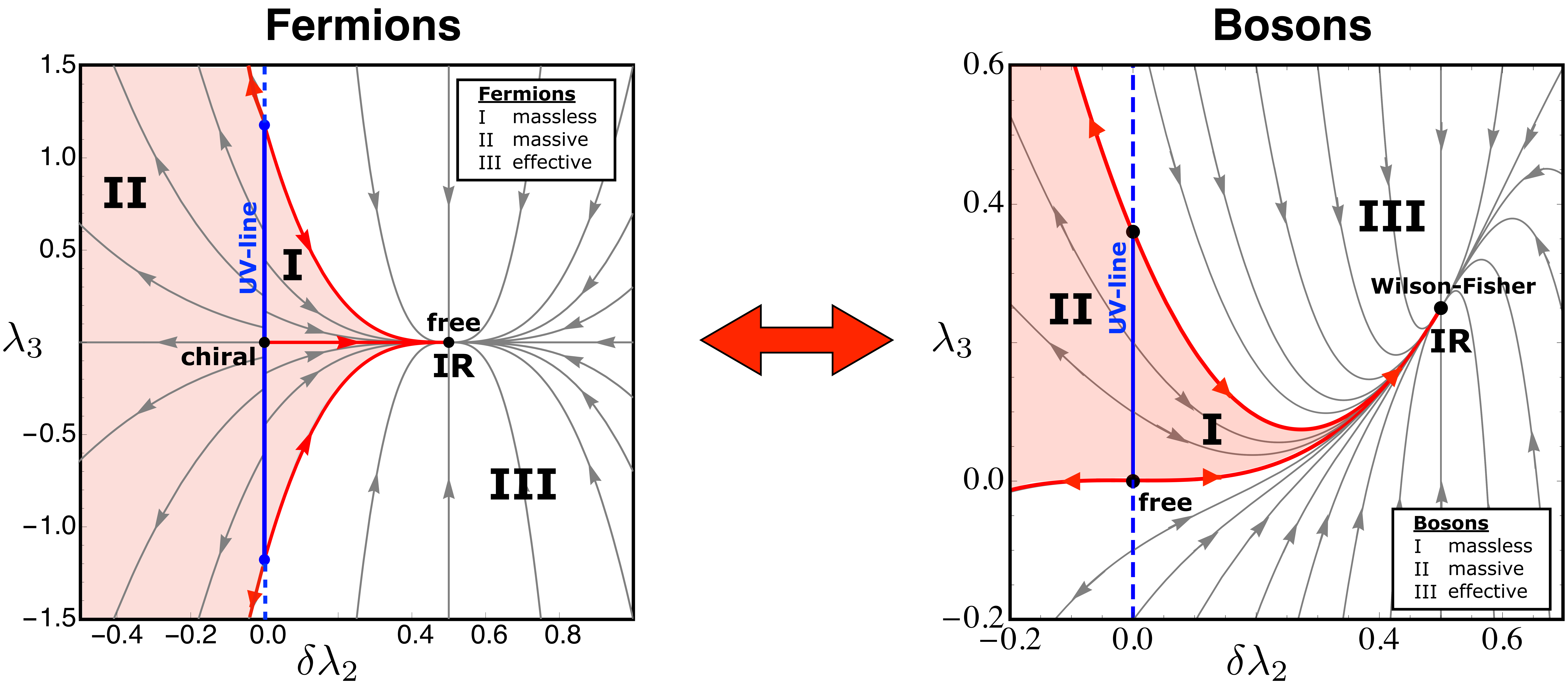
prerequisite: **6F interactions,** hence **no** chiral symmetry breaking

Cresswell-Hogg, Litim, **Critical fermions with spontaneous broken sale symmetry** 2212.06815



universal scaling dimensions
3d fermions
canonical
IR
Gross–Neveu
UV
$\vartheta_n^{\mathrm{IR}} = 2n - 1$
$\vartheta_n^{\mathrm{UV}} = n - 2$
irrelevant
relevant
30
20
10
0
5
10
15
20


3d fermions
canonical
IR
Gross–Neveu
UV
irrelevant
relevant
3d scalars
Wilson–Fisher
IR
canonical
UV
$\vartheta_n^{\rm IR} = 2n - 1$
$\vartheta_n^{\rm UV} = n - 2$
irrelevant
relevant



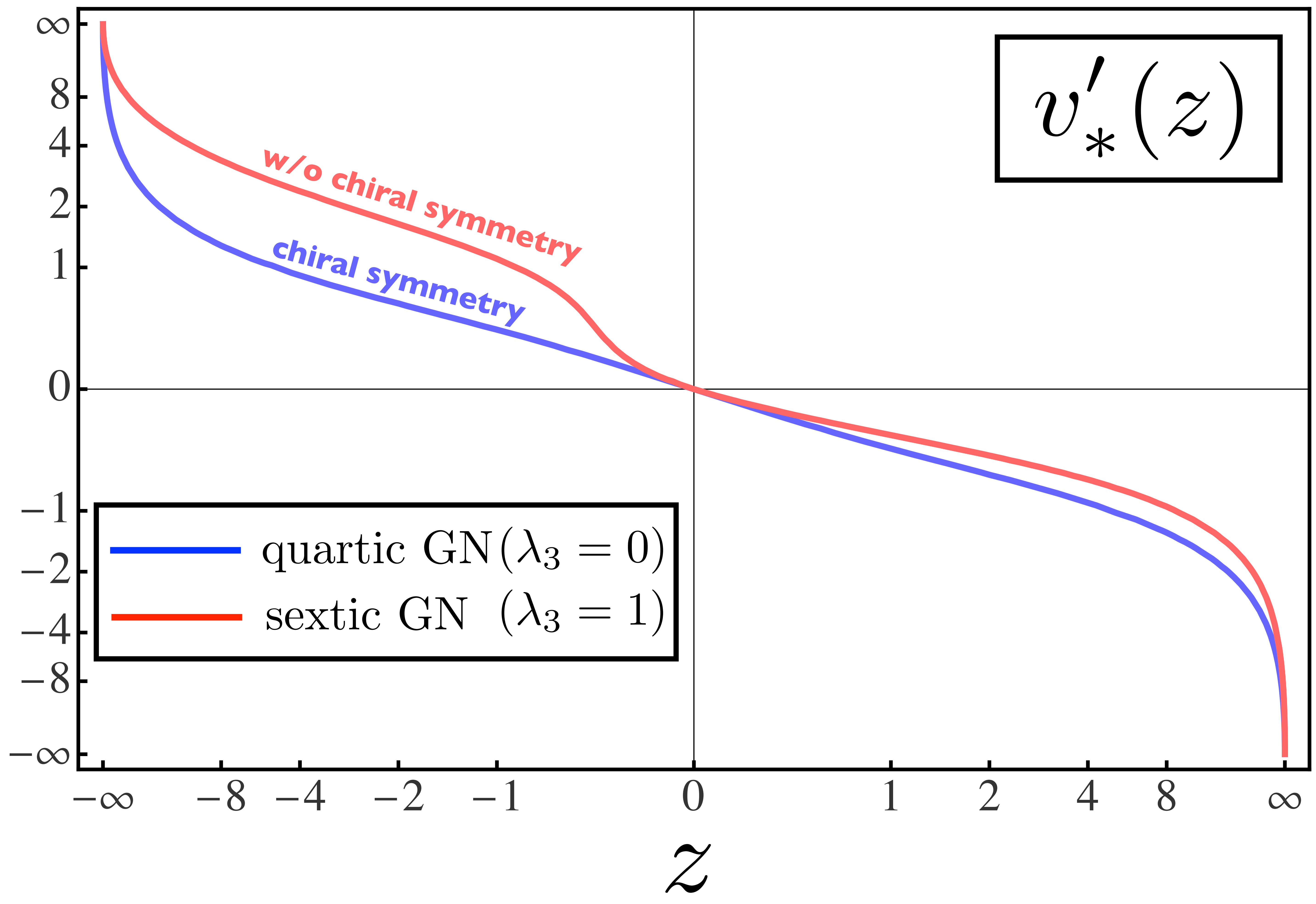
AdS/CFT
higher spin gauge theories

Klebanov, Polyakov, hep-th/0210114, Szegin, Sundell, hep-th/0305040
Maldacena, Zhiboedov, 1112.1016, 1204.3882
Giombi, Zin, 1208.4036

Chern-Simons-Matter

Aharony, Giombi, Gur-Ari, Maldacena, Yacoby, 1211.4843
Seiberg, Senthil, Wang, Witten, 1606.01989

non-chiral CFT duals for higher-spin GTs on AdS4



Gross-Neveu—Yukawa

$$S_{\text{GNY}} = \int_x \left\{ \bar\psi_a \not{\partial} \psi_a + \frac{1}{2}(\partial\phi)^2 + H\phi\bar\psi_a\psi_a + U(\phi) \right\}$$

Yukawa

even / odd

perturbatively renormalisable

"integrate-out the scalar"

PT, large Nf

Gracey '90
Moshe Moshe, Zinn-Justin '03

functional RG

Gies, MM Scherer '10
Braun, Gies, DD Scherer '12

relax chiral symmetry

odd phi powers + fermion mass term permitted

functional RG
exactly solvable at infinite Nf

Cresswell-Hogg, Litim, **Scale-symmetry breaking and generation of mass at quantum critical points** 2311.16246

Gross-Neveu—Yukawa

large Nf

IR fixed point

mass

$$\partial_t m_F = -m_F \left(1 + \frac{2h^2}{(1+m_F^2)^2 \lambda_2}\right)$$

Yukawa

$$\partial_t h = -\tfrac{1}{2}\left(1-\eta_\phi\right) h \qquad \eta_\phi = \frac{5}{2}h^2$$

cubic

$$\partial_t \lambda_3 = -\tfrac{3}{2}(1-\eta_\phi)\lambda_3$$

Cresswell-Hogg, Litim, **Scale-symmetry breaking and generation of mass at quantum critical points** 2311.16246

Gross-Neveu—Yukawa

large Nf

IR fixed point

mass = 0 is an exact fixed point

mass

$$\partial_t m_F = -m_F \left(1 + \frac{2h^2}{(1+m_F^2)^2 \lambda_2}\right)$$

Yukawa

$$\partial_t h = -\frac{1}{2}\left(1 - \eta_\phi\right) h$$

$$\eta_\phi = \frac{5}{2} h^2$$

cubic

$$\partial_t \lambda_3 = -\frac{3}{2}(1 - \eta_\phi)\lambda_3$$

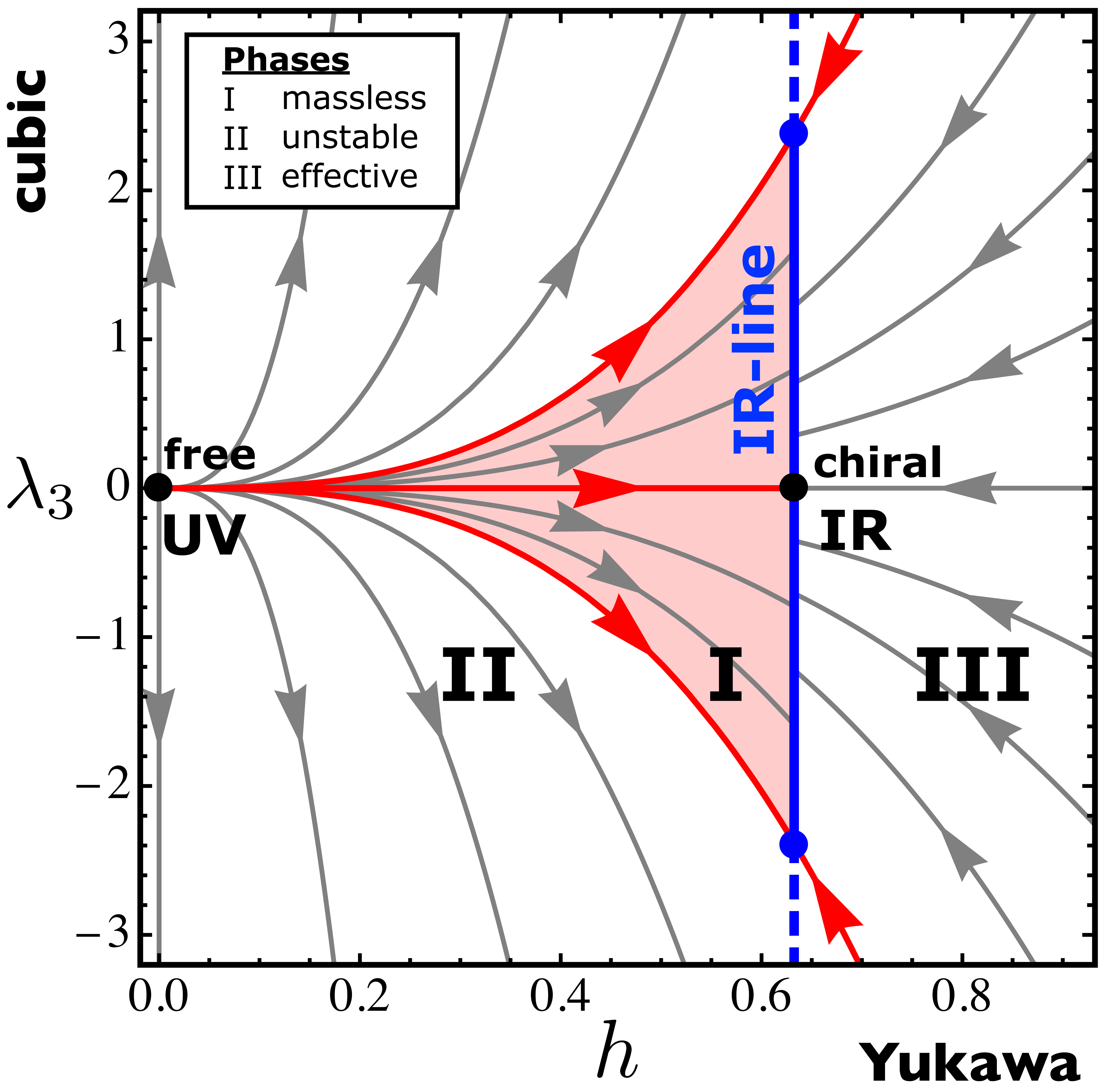
Yukawa fixes **eta = 1**

eta = 1 renders the cubic **exactly marginal**

→ conformal manifold

Cresswell-Hogg, Litim, **Scale-symmetry breaking and generation of mass at quantum critical points** 2311.16246
and in prep. 240x.yyyy

Gross-Neveu—Yukawa



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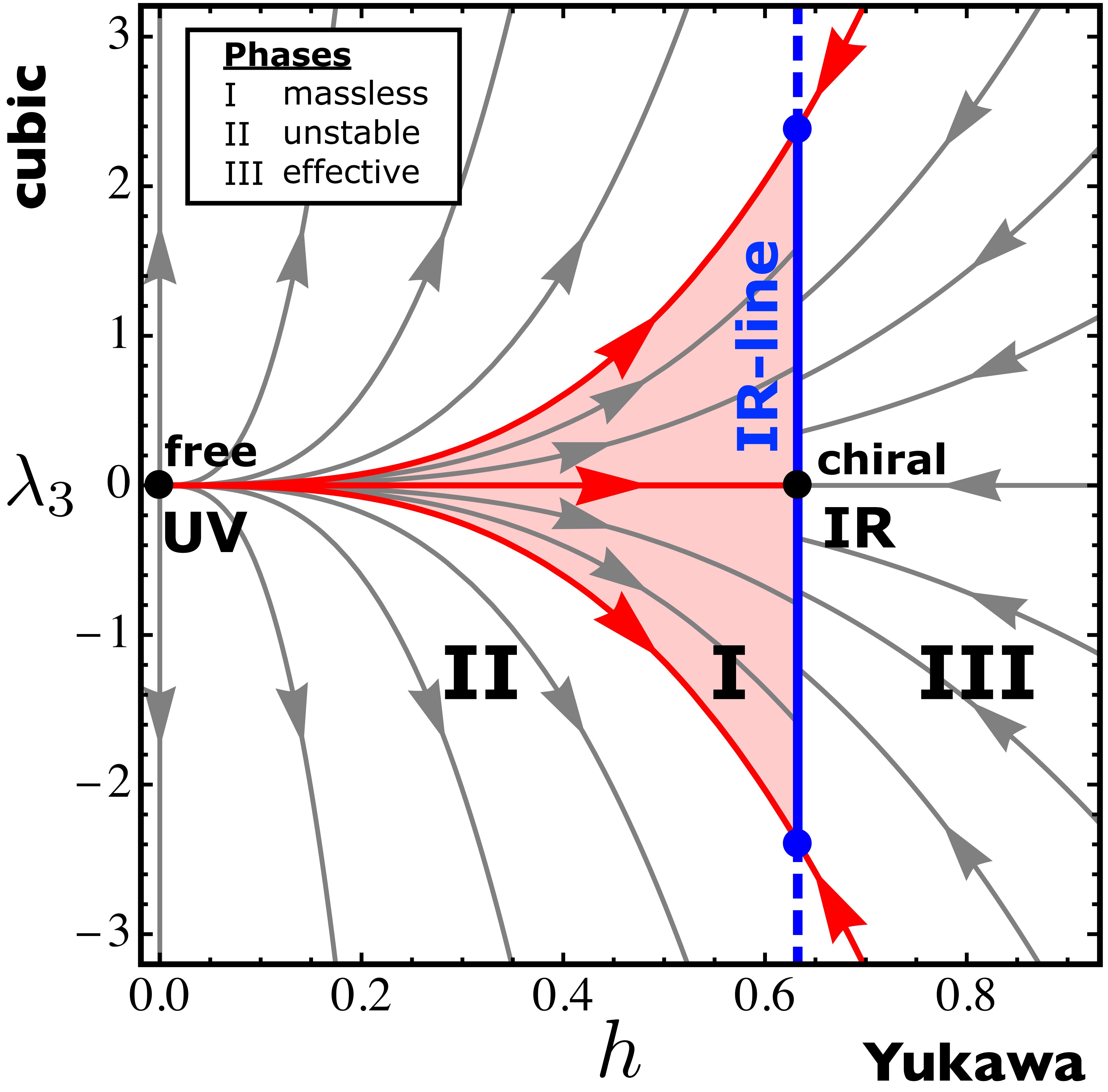
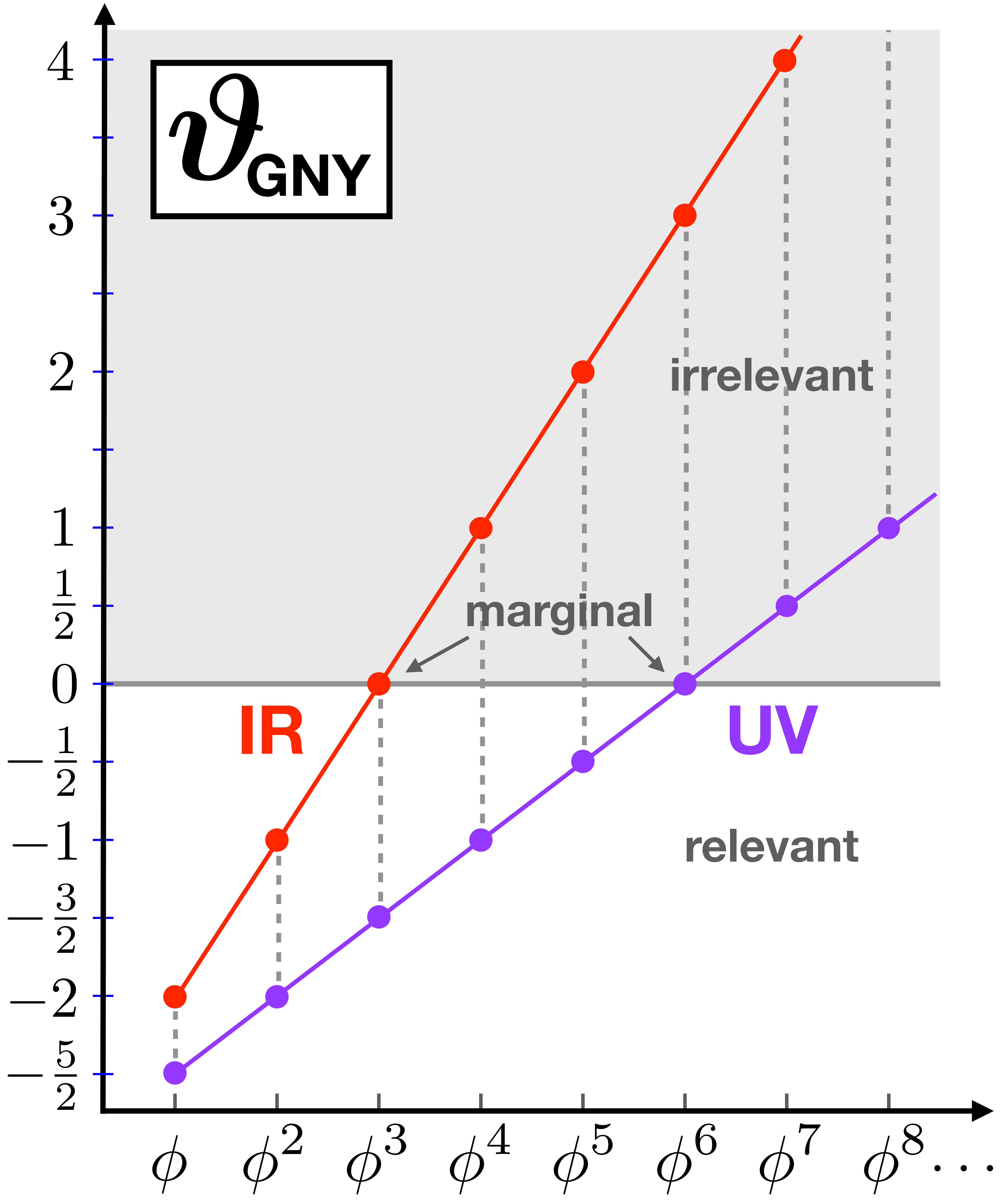
Gross-Neveu—Yukawa





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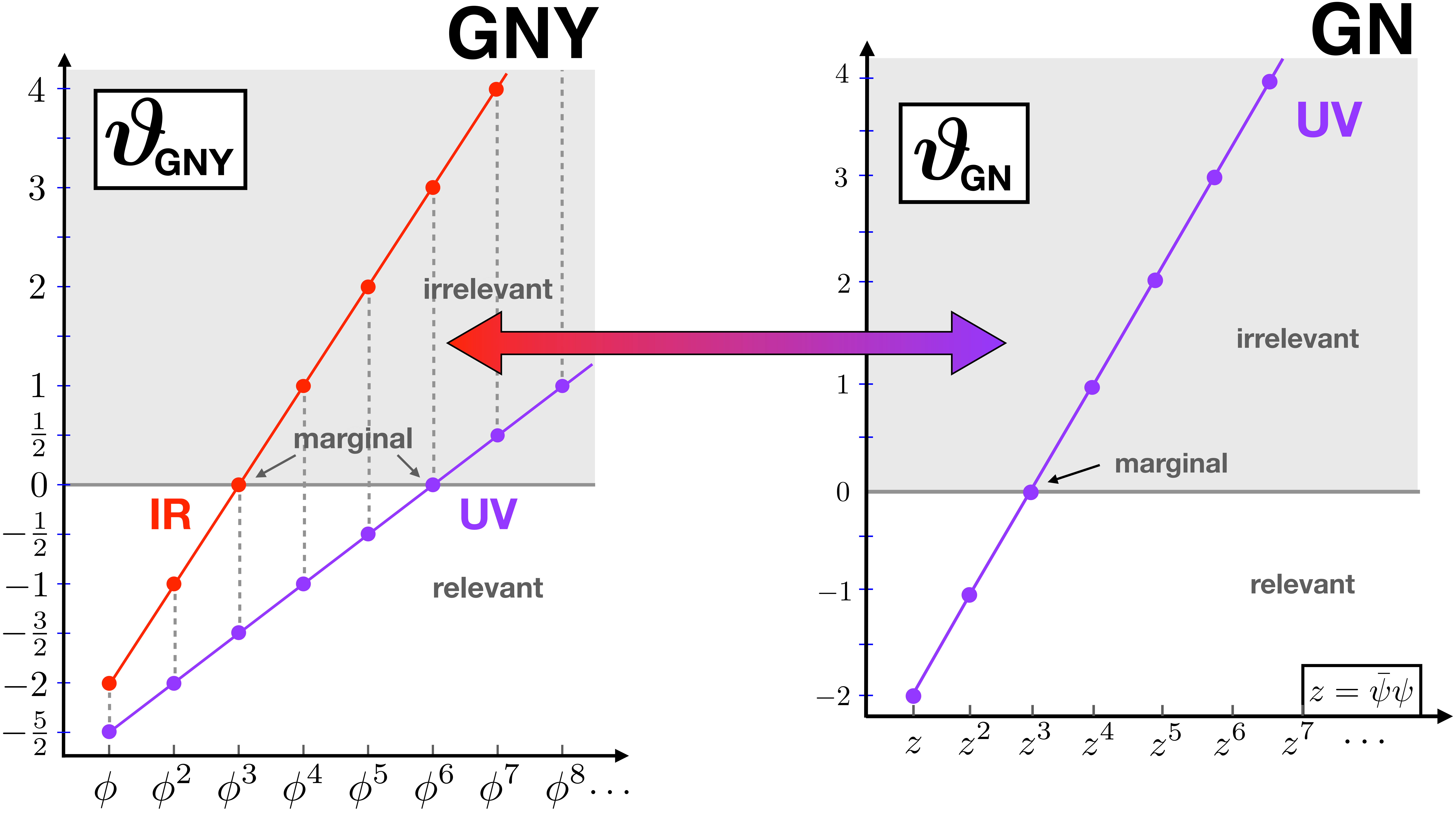
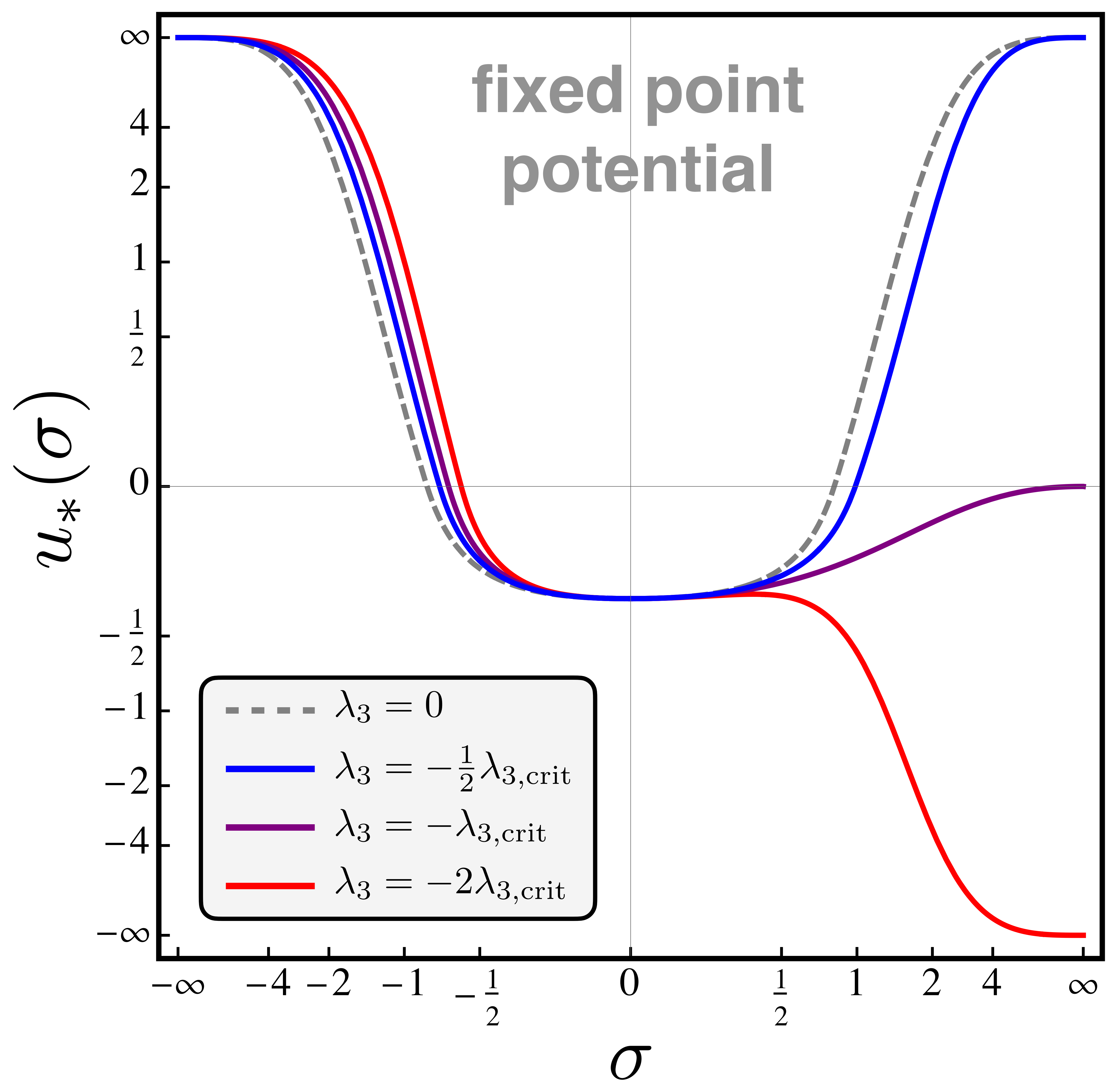


Cresswell-Hogg, Litim, **Scale-symmetry breaking and generation of mass at quantum critical points** 2311.16246

Gross-Neveu—Yukawa



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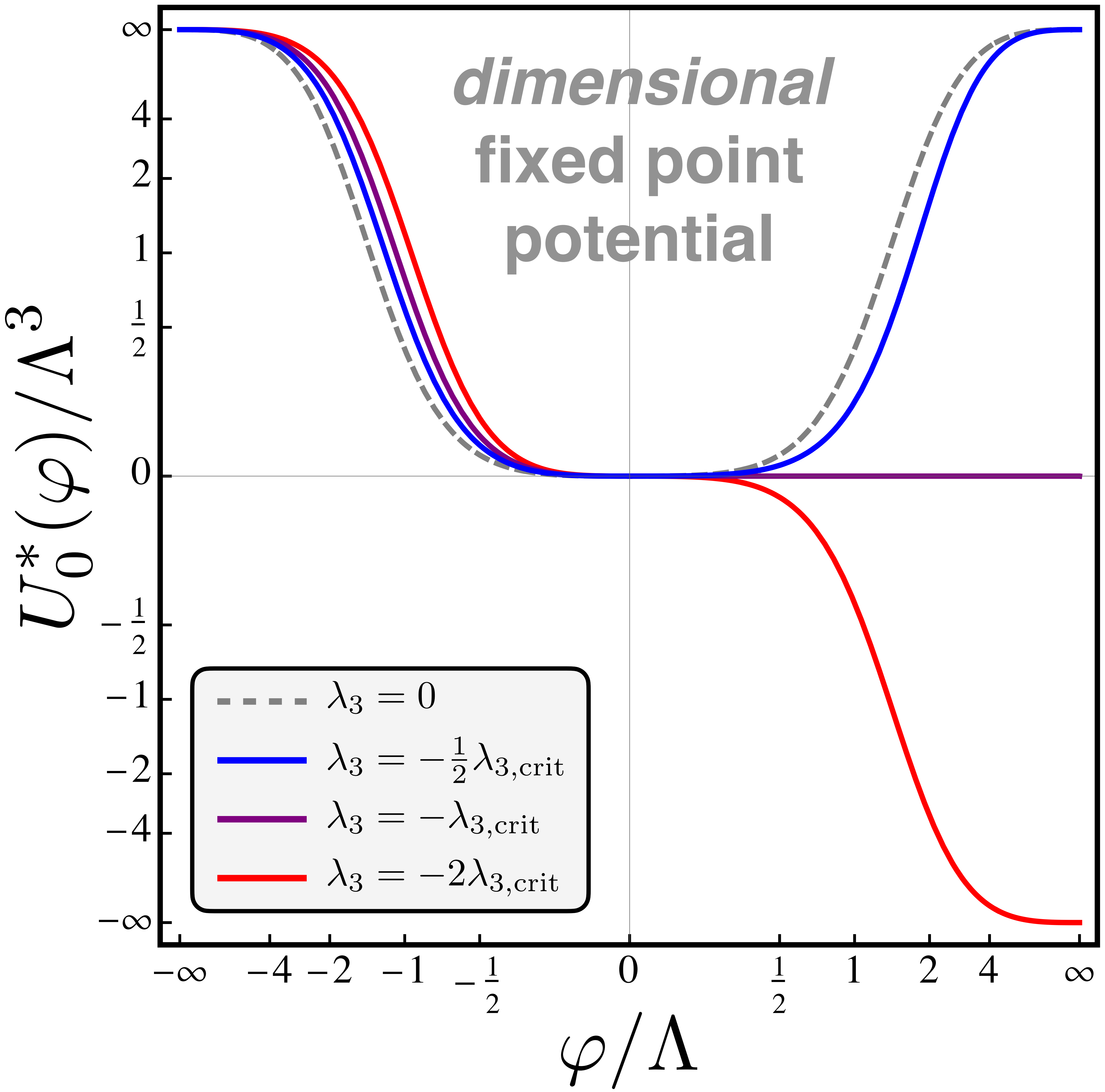
Gross-Neveu—Yukawa



$$\propto [\lambda_3^* + \lambda_3^{\mathrm{crit}}\,\mathrm{sgn}(\phi)]\phi^3$$

conformal window $|\lambda_3^*| \le \lambda_3^{\mathrm{crit}}$

scale symmetry broken spontaneously

$$|\lambda_3^*| = \lambda_3^{\mathrm{crit}}$$

→ $\langle\phi\rangle$ **= free parameter**

→ $M_F = H_* \langle\phi\rangle$ **= free parameter**

$M_s = 0$

Cresswell-Hogg, Litim, **Scale-symmetry breaking and generation of mass at quantum critical points** 2311.16246

bosonisation duality

large N fermionic theory

$$\int_x \bar{\psi}_i \not{\partial} \psi_i + F[\bar{\psi}_i \psi_i]$$

equivalent to

$$\int_x \left\{ \bar{\psi}_i \not{\partial} \psi_i + \sigma \bar{\psi}_i \psi_i \right\} + G[\phi]$$

functional Legendre transform

S Weinberg, **Effective field theories in the large N limit**, hep-th/9706042

bosonisation duality

$$\bar{\psi}_a \partial\!\!\!/ \psi_a + V_k(\bar{\psi}_a \psi_a)$$ **GN**

$$\bar{\psi}_a \partial\!\!\!/ \psi_a + \tfrac{1}{2} Z_\phi (\partial \phi)^2 + H_k \phi \, \bar{\psi}_a \psi_a + U_k(\phi)$$ **GNY**

map

$$V_k(\bar{\psi}_a \psi_a) = H_k \, \phi \, \bar{\psi}_a \psi_a + U_k(\phi)$$

$$U_k'(\phi) = -H_k \, \phi \, \bar{\psi}_a \psi_a$$

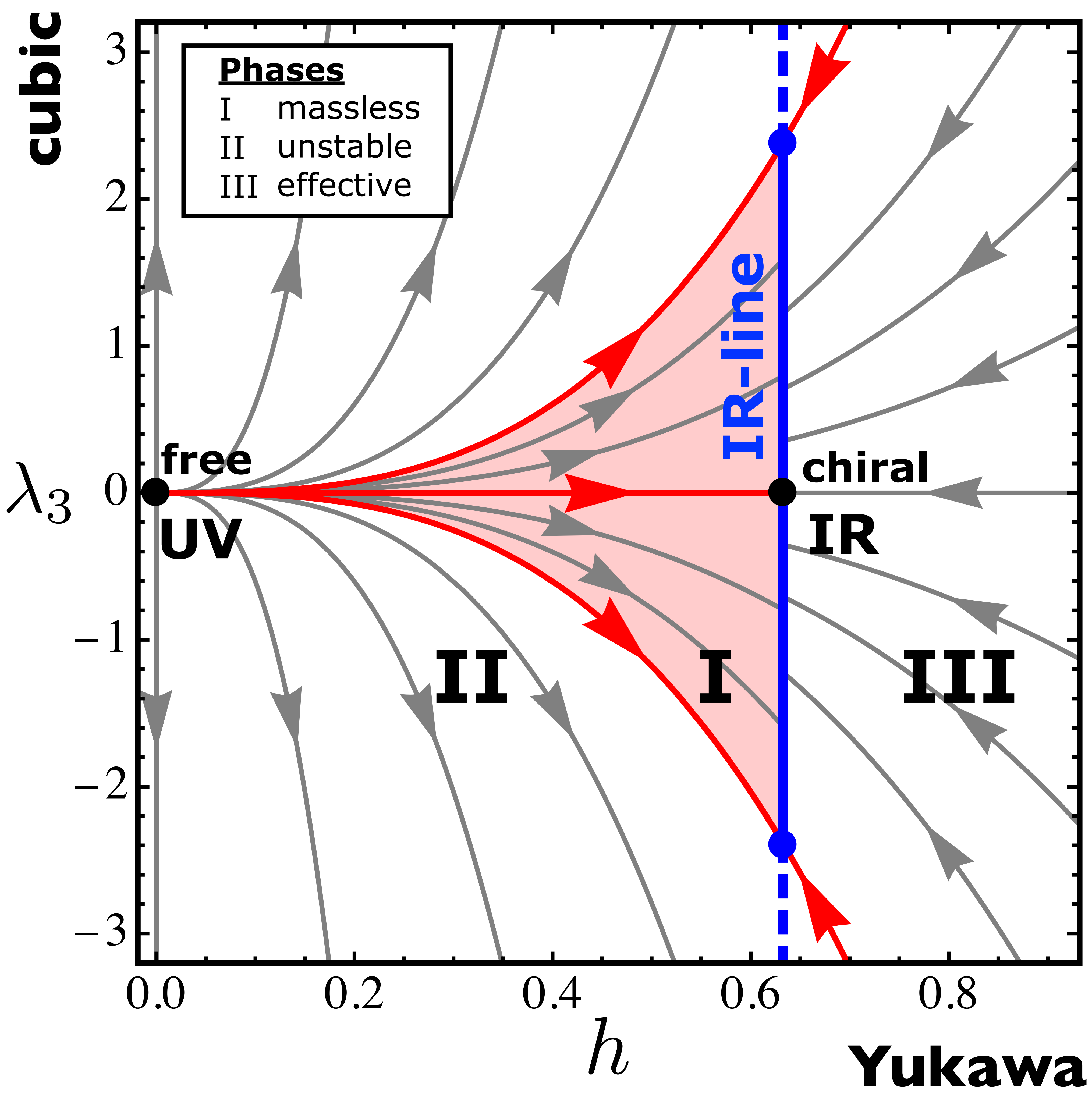
$$\begin{aligned}
\lambda_{2\mathrm{F}} &= \quad h \, \sigma_0 \,, \\
\lambda_{4\mathrm{F}} &= -h^2/\lambda_2 \,, \\
\lambda_{6\mathrm{F}} &= -h^3 \lambda_3/\lambda_2^3 \,, \\
\lambda_{8\mathrm{F}} &= h^4 \left(\lambda_2 \lambda_4 - 3\lambda_3^2\right)/\lambda_2^5 \,, \\
&\vdots
\end{aligned}$$

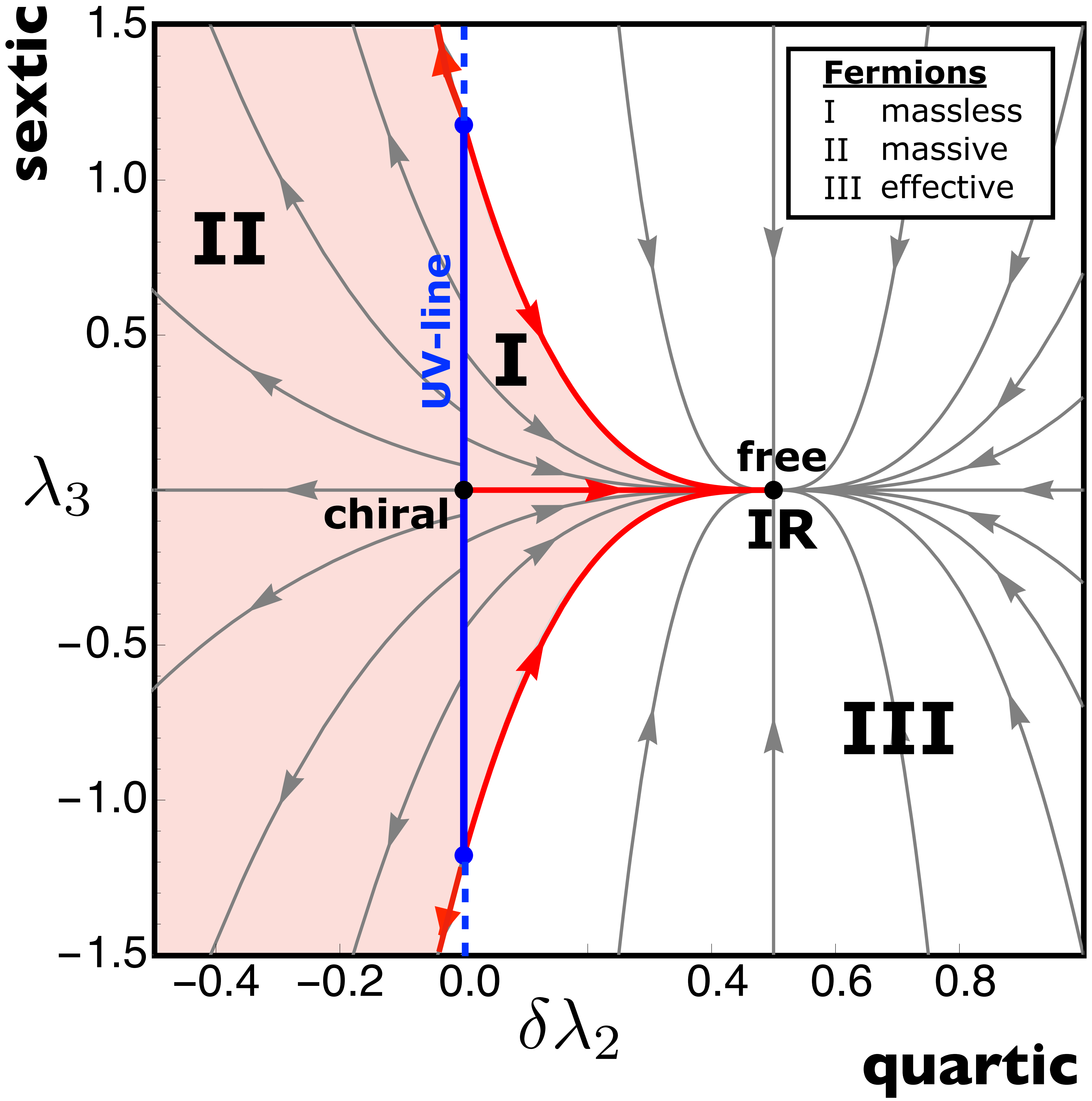
valid along RG trajectories

Cresswell-Hogg, Litim, 2311.16246

GNY

GN





Cresswell-Hogg, Litim, **Scale-symmetry breaking and generation of mass at quantum critical points** 2311.16246

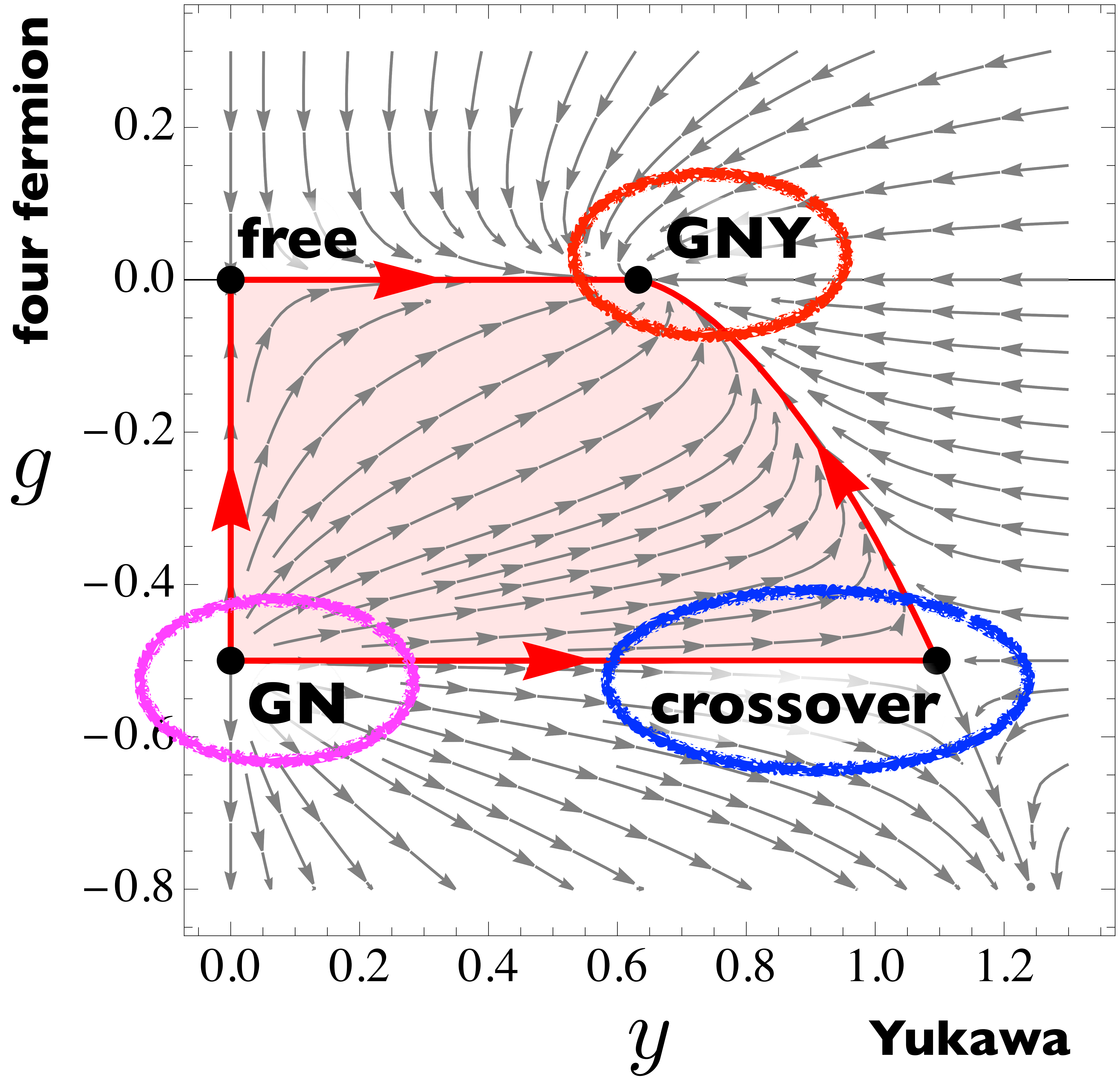
outlook I

action

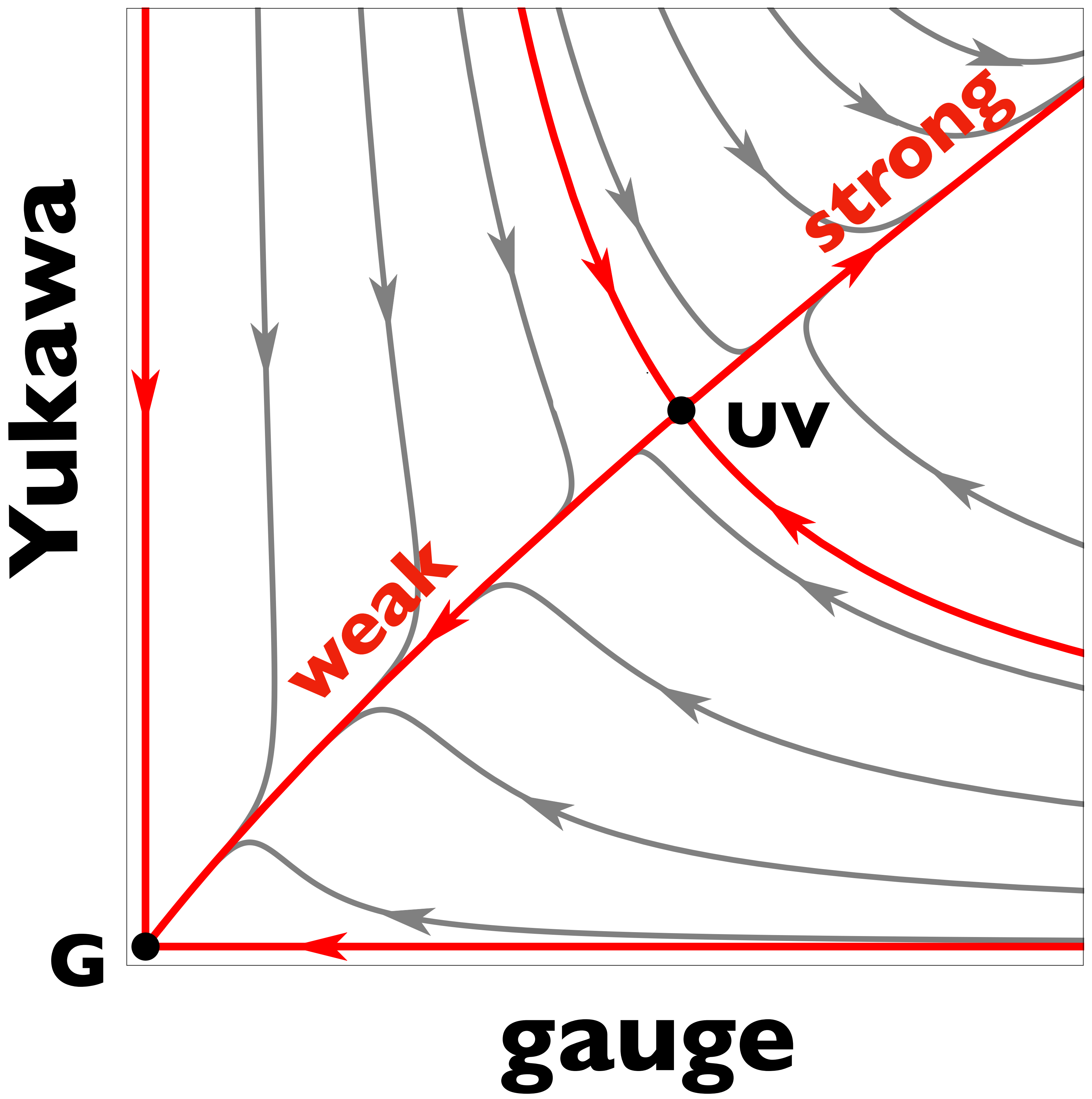
$$\bar{\psi}_i \partial\!\!\!/ \psi_i + \frac{1}{2}(\partial\phi)^2 + W_k(\phi, \bar{\psi}_i\psi_i)$$

e.g.

$$W_k = \frac{1}{2}G(\bar{\psi}_i\psi_i)^2 + Y_k\,\phi\,\bar{\psi}_i\psi_i$$



outlook II



SU(N) + Diracs
+ mesons

SO(N) + Majoranas
+ mesons

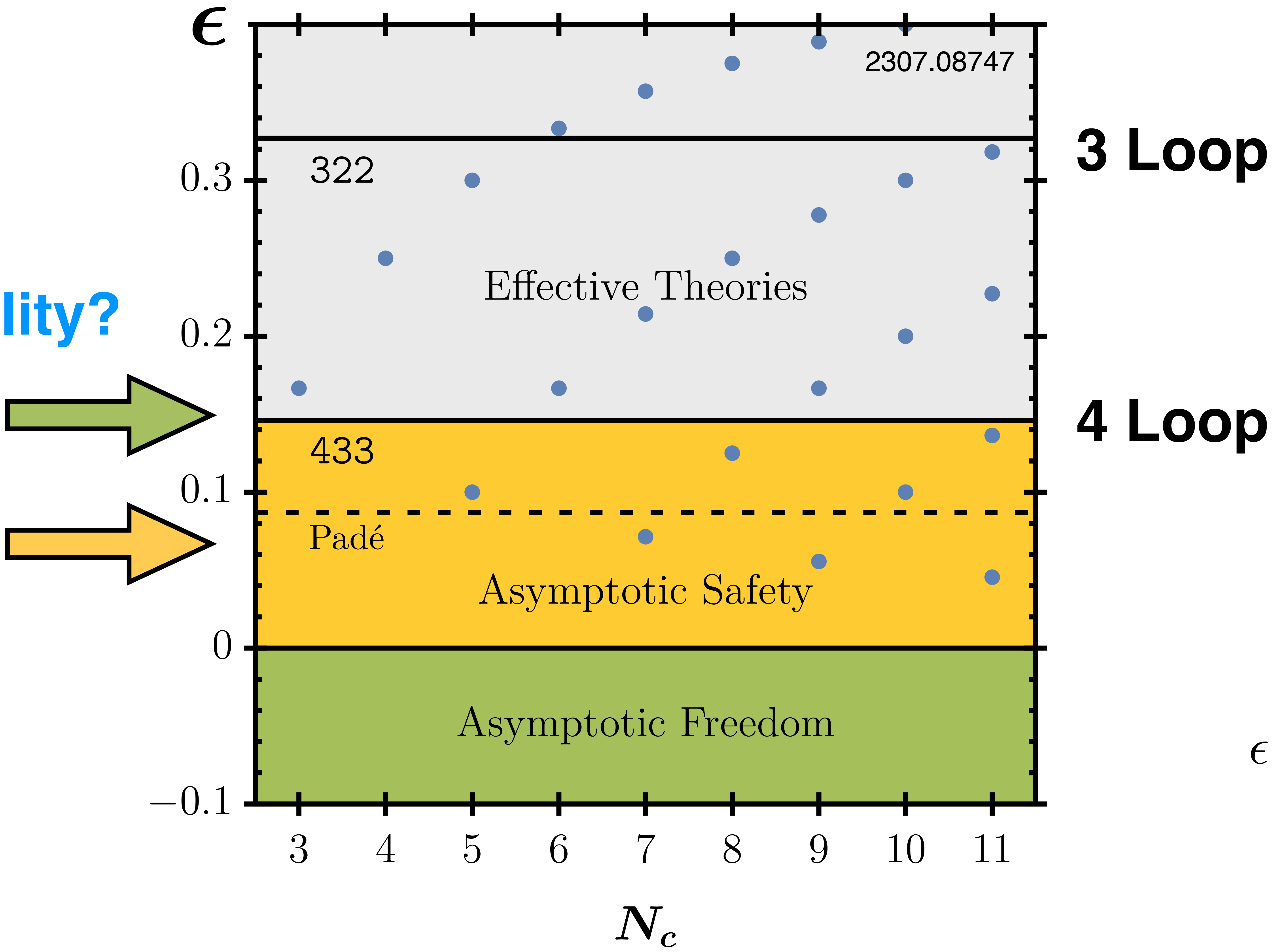
Sp(N) + Majoranas
+ mesons

DF Litim, F Sannino, **Asymptotic safety guaranteed,** 1406.2337
AD Bond, DF Litim, G Medina Vazquez, T Steudtner, **Conformal window for asymptotic safety**, 1710.07615
AD Bond, DF Litim, T Steudtner, **Asymptotic safety with Majorana fermions and new large N equivalences** 1911.11168

4d

loss of vacuum stability?
merger?

conformal
window



$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

AD Bond, DF Litim, G Medina Vazquez, T Steudtner, **Conformal window for asymptotic safety**, 1710.07615 (PRD)
AD Bond, DF Litim, G Medina Vazquez, **Conformal windows beyond asymptotic freedom**, 2107.13020 (PRD)
DF Litim, N Riyaz, E Stamou, T Steudtner, **Asymptotic safety guaranteed at four loop**, 2307.08747 (PRD)

Summary

critical points in 3d QFTs

fermionic theories from first principles, UV / IR fixed points,
mass generation & **spontaneous scale symmetry breaking**
prerequisite: interactions break chiral symmetry
links with CFTs and AdS/CFT

large N dualities

maps between seemingly different 3d / 4d QFTs
new critical points, **fermions vs bosons**, **GN vs GNY**

what's next?

4F interactions **beyond GN**, more dualities,
spontaneous scale symmetry breaking in 4d,
gauge fields, finite *N* …

Thank you!