FE Reduced Order Models for Superconducting Wires and Cables

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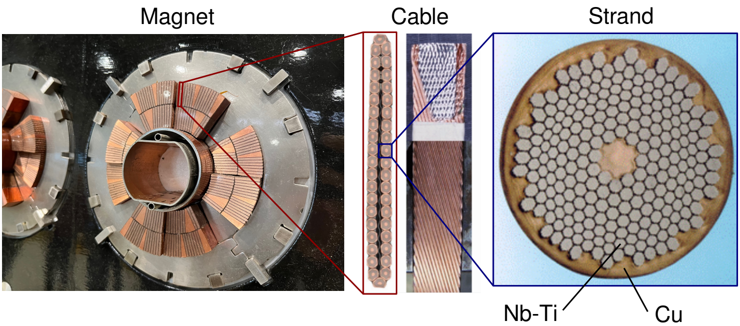
Outline

Superconducting magnet modelling

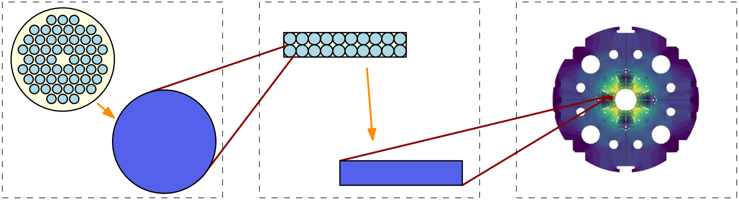


- Quench protection design requires good AC loss models.
- Example: CLIQ (coupling-loss induced quench) devices.
- Magnet geometry is multi-scale and small-scale effects contribute significantly to AC loss.
  $\Rightarrow$ Need for accurate strand and cable models.

From strand to magnet

Fully discretized magnet models are too heavy to solve.
$\Rightarrow$ Intermediate models are necessary.

Homogenization of small-scale properties in two steps:



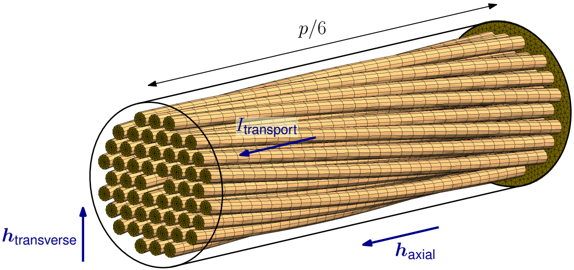
Homogenized parameters: magnetization and lumped R and L.

Back to the small-scales, today's focus is AC losses in strands.

$\Rightarrow$ Linked-flux method applied on LTS strands.

Problem statement

Multifilamentary strand subject to transport current and magn. field.



Coupling currents [Morgan, 1970]

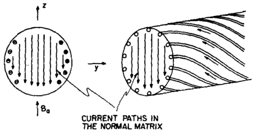


FIG. 2. Current paths in some of the superconducting filaments at the surface and the normal metal matrix of a twisted, multifilament wire which is exposed to a uniform changing field. The interior filaments are not shown since they carry no current.

Loss contributions:

- Coupling current losses,
- Eddy current in the matrix,
- Losses in SC filaments.

Magnetization (hysteresis).

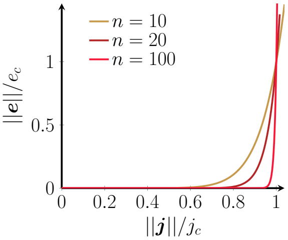
Equations and FE formulation

Magneto-quasistatic equations and constitutive laws:

$$\begin{cases} \operatorname{div} \boldsymbol{b} = 0, & \text{(Gauss)} \\ \textbf{curl}\, \boldsymbol{h} = \boldsymbol{j}, & \text{(Ampère)} \\ \textbf{curl}\, \boldsymbol{e} = -\partial_t \boldsymbol{b}, & \text{(Faraday)} \end{cases} \quad \text{with} \quad \begin{cases} \boldsymbol{b} = \mu_0 \boldsymbol{h}, \\ \boldsymbol{e} = \rho(\boldsymbol{j}, \boldsymbol{b})\, \boldsymbol{j}, \end{cases}$$

with the (nonlinear) power law for the resistivity in SC filaments:

$$\rho(\boldsymbol{j}, \boldsymbol{b}) = \frac{e_c}{j_c(\boldsymbol{b})} \left(\frac{\|\boldsymbol{j}\|}{j_c(\boldsymbol{b})} \right)^{n-1}.$$



Efficient choice for SC: h-ϕ-formulation

- Weak form of Faraday's law,
- Find $\boldsymbol{h} \in \mathcal{H}(\Omega)$ such that, $\forall \boldsymbol{h}' \in \mathcal{H}_0(\Omega)$:

$$\left(\partial_t(\mu_0 \boldsymbol{h})\, , \boldsymbol{h}'\right)_\Omega + \left(\rho\, \textbf{curl}\, \boldsymbol{h}\, , \textbf{curl}\, \boldsymbol{h}'\right)_{\Omega_c} = 0,$$

- It ensures $\textbf{curl}\, \boldsymbol{h} = \boldsymbol{0}$ in Ω_c^C ("cuts").

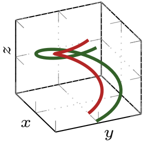
Modelling approaches

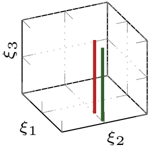
- **3D model**
  $\Rightarrow$ Most general but also most expensive (CPU).

- **Helicoidal transformation** with a change of variables

$$\begin{cases} \xi_1 = x\cos(\alpha z) + y\sin(\alpha z), \\ \xi_2 = -x\sin(\alpha z) + y\cos(\alpha z), \quad \alpha = 2\pi/p. \\ \xi_3 = z, \end{cases}$$

[Nicolet et al., 2004.] [Dular et al., 2023.]





  $\Rightarrow$ Efficient and exact with linear materials,
  $\Rightarrow$ Too complex with transv. field + nonlinear materials.

- **Linked-flux method**, two coupled 2D models
  $\Rightarrow$ Approached but very fast with good accuracy.

Outline

Introduction

Linked-Flux

Verification

Applications

Conclusions

Additional slides

Linked-flux method - Genesis

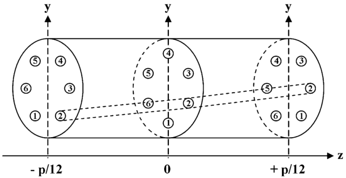


Fig. 2. Position of six filaments in the planes $z = 0$ and $z = \pm p/12$.

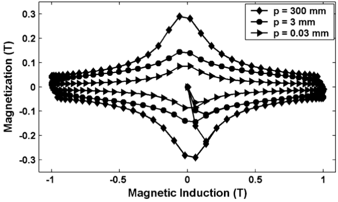
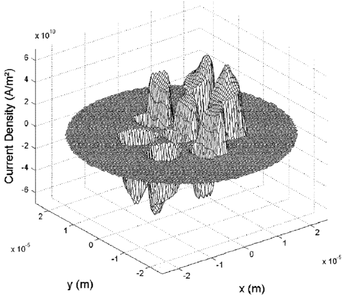


Fig. 9. Magnetization cycles obtained from Kim's model.



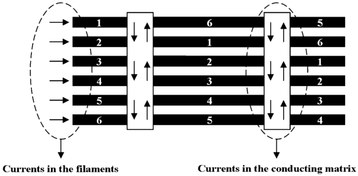
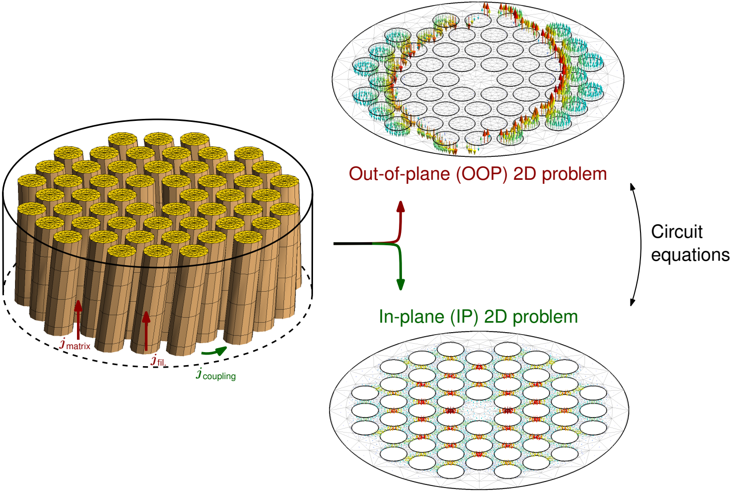


Fig. 5. 2D Drawing of currents in wire with twisted filaments.

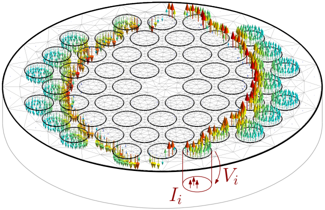
[Satiramatekul, Bouillault, 2005]
[Satiramatekul, Bouillault, 2007]

Linked-flux method - Two 2D problems



Linked-flux method - Two 2D problems (Cont'd)

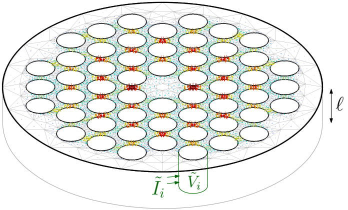
OOP



- Magnetodynamics
- Tilt of filaments neglected
- Currents: I_i (and I_t)
- Voltages: V_i (and V_t)

$$\left(\ell\,\partial_t(\mu\,\boldsymbol{h})\,,\boldsymbol{h}'\right)_\Omega + \left(\ell\,\rho\,\mathbf{curl}\,\boldsymbol{h}\,,\mathbf{curl}\,\boldsymbol{h}'\right)_{\Omega_\mathrm{c}} = V_\mathrm{t}\mathcal{I}_\mathrm{t}(\boldsymbol{h}') + \sum_{i\in F} V_i\mathcal{I}_i(\boldsymbol{h}').$$

IP



- Electrokinetics (static!)
- In matrix Ω_m only
- Currents: $\tilde{I}_i$
- Voltages: $\tilde{V}_i$

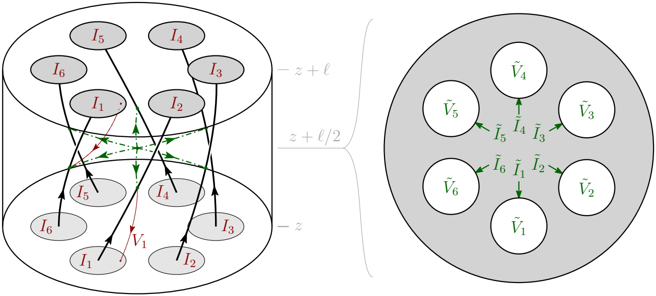
$$\left(\ell\,\sigma\,\mathbf{grad}\,v\,,\mathbf{grad}\,v'\right)_{\Omega_\mathrm{m}} + \sum_{i\in F}\tilde{I}_i\tilde{\mathcal{V}}_i(v') = 0.$$

with Ω_c cond. domain, $F=\{1,\dots,N_\mathrm{f}\}$, N_f nb fil., and $\ell = p/6$ here.

Linked-flux method - Coupling equations

Number of global variables: $4N_\mathsf{f}+2$.

- OOP and IP problems: $2N_\mathsf{f}+1$ equations for them,
- Transport current (or voltage) imposed: 1 equation,
- Remaining $2N_\mathsf{f}$ equations:



After ℓ along z, filament i becomes filament $j=\mathcal{S}(i)$:

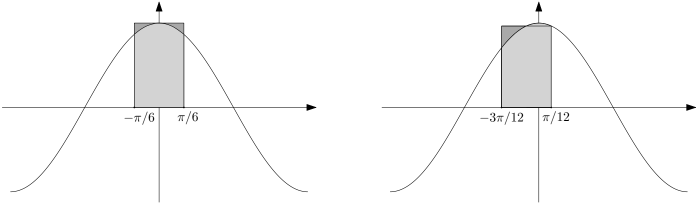
$$I_j = I_i + \tilde{I}_i, \qquad V_j = \tilde{V}_j - \tilde{V}_i.$$

(can be written as an electric circuit, see appendix.)

Linked-flux method - Equivalent length

With no correction: overestimation of the flux and current.

- Quantities vary sinusoidally:



- We are assuming constant values over ℓ.
- Let us reduce ℓ to ℓ^* to acccount for it:

$$\ell^* = \frac{\sin(\pi\ell/p)}{\pi\ell/p}\,\ell.$$

- For $\ell = p/6$ (common case), $\ell^*/\ell = 0.9549$.

Linked-flux method - Dynamic coupling currents

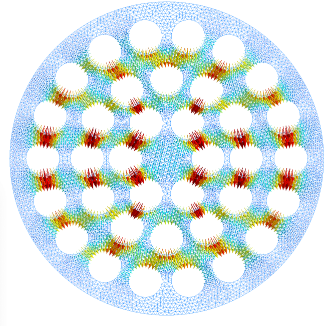
The IP problem solves a static current flow:

$$\left(\ell\,\sigma\,\mathbf{grad}\ v\ ,\mathbf{grad}\ v'\right)_{\Omega_\mathrm{m}} + \sum_{i\in F} \tilde{I}_i \tilde{\mathcal{V}}_i(v') = 0.$$

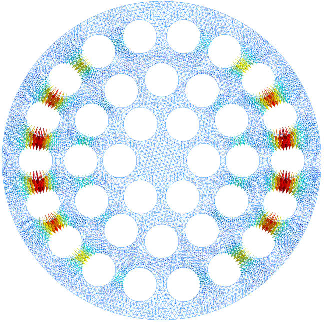
Dynamic coupling current flows are not reproduced

(unless we correct for them, see appendix.)

Static current flow:



Dynamic current flow:



Outline

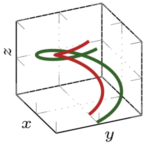
Verification

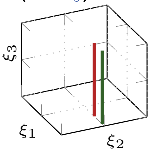
The approach seems reasonable, but does it actually work?

- Ideally, validation against experimental measurements.
- Here, verification with reference models:
  - Linear case: helicoidal transformation method (2D-ξ)

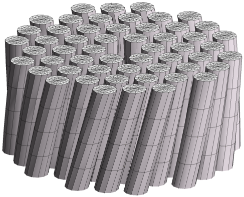
$$\begin{cases} \xi_1 = x\cos(\alpha z) + y\sin(\alpha z), \\ \xi_2 = -x\sin(\alpha z) + y\cos(\alpha z), \qquad \alpha = 2\pi/p. \\ \xi_3 = z, \end{cases}$$

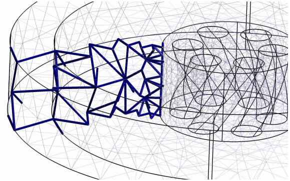
[Nicolet et al., 2004.] [Dular et al., 2023.]





  - Nonlinear case: 3D model in GetDP





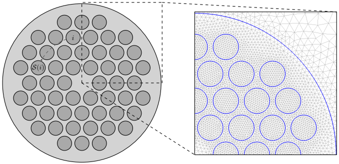
[Dular, 2023.]

NB: tilted filaments in helicoidal and 3D models make the reference problems slightly different by construction.

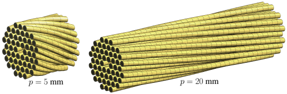
Verification - Linear

Linear 54-filament problem

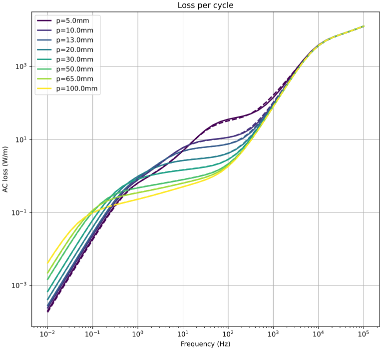
- $d_{\text{wire}} = 1$ mm
- $p \in [5, 100]$ mm
- $\sigma_{\text{fil}} = 10^5\ \sigma_{\text{matrix}}$



- DOFs linked-flux: 62k
- DOFs helicoidal: 110k

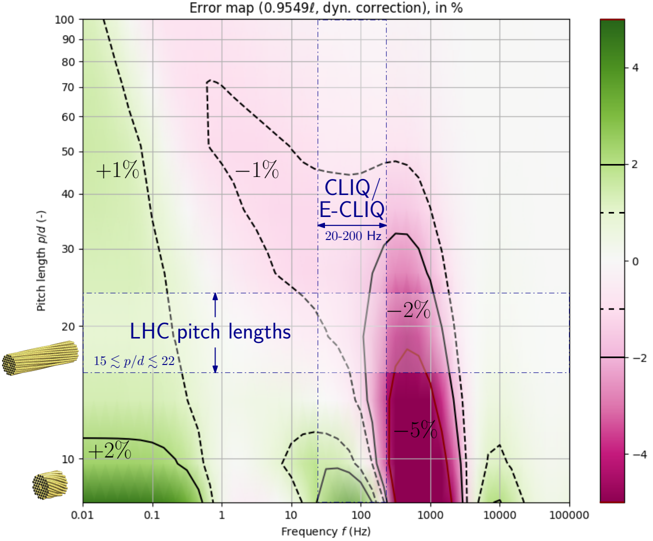


Total AC loss:



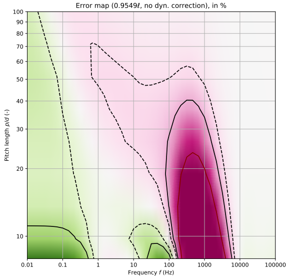
- Solid lines: linked-flux
- Dashed lines: helicoidal

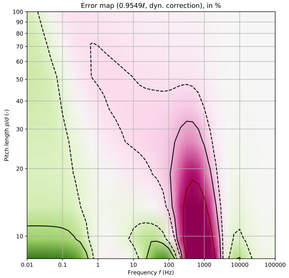
Verification - Linear (ref. helicoidal)



Verification - Dynamic correction

Effect of the dynamic correction on coupling currents:

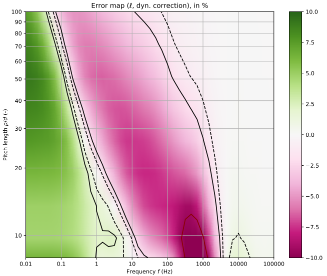


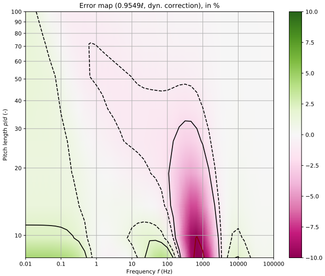


- Biggest effect around $f \approx 1$ kHz,
  - Below 100 Hz, mostly static coupling currents,
  - Above 10 kHz, mostly eddy current losses.

Verification - Length correction

Effect of the length correction:

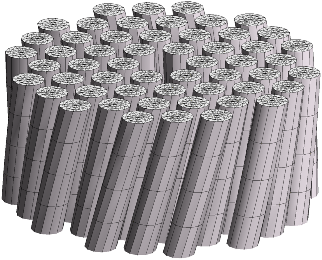


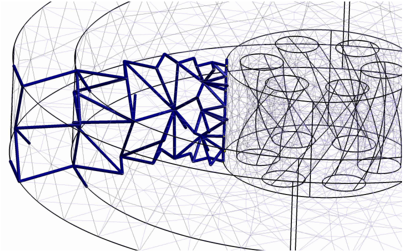


- Without the correction, poor estimate of the interfilament time constant $\Rightarrow$ wrong coupling losses,
- Corrected length $\ell^* = 0.9549\ell$ shifts this time constant.

3D model for nonlinear materials

Twisted filaments over length $p/6$. Hybrid 3D mesh.





- Strong periodic boundary conditions $\Rightarrow$ periodic mesh.
- Periodic support for the cut shape function (for ϕ).
- Structured (and periodic) mesh in the filaments.
- Standard h-ϕ-formulation.

Verification - Nonlinear (first result)

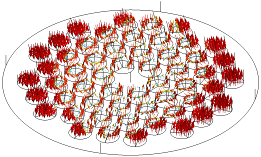
- GetDP 3D model not optimized
  $\Rightarrow$ slow ($\approx$ 30 h per simu).
- Running on HPC cluster.
- Further verifications (magnetiz., field maps) are coming.

First test (3D: 666k DOFs)

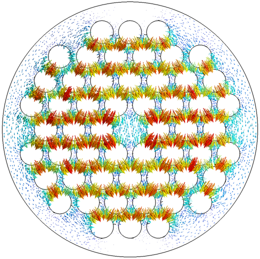
- 54 Nb-Ti filaments,
- $j_c(\boldsymbol{b})$, $\sigma_{\text{Cu}}(\boldsymbol{b})$, $T = 1.9$ K,
- $p = 19$ mm, $I_{\text{t}} = 0$ A,
- $f = 10$ Hz, $\|\boldsymbol{b}\| = 0.2$ T,

  $\Rightarrow$ 6.3% error on AC loss.

Filament current density



Matrix current density



Verification - Analytical models?

Back to linear case, what do analytical models predict?

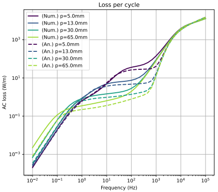
- Coupling currents (loss per cycle): [Campbell, 1982]

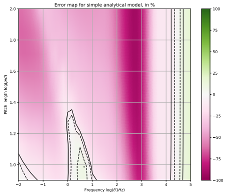
$$\tau_{\mathrm{c}} = \frac{\mu_0}{2\rho_{\mathrm{eff}}}\left(\frac{p}{2\pi}\right)^2, \quad q_{\mathrm{coupling}} = \pi R_{\mathrm{w}}^2 \frac{b_{\mathrm{max}}^2}{2\mu_0}\,\frac{\pi\omega\tau_{\mathrm{c}}}{(\omega^2\tau_{\mathrm{c}}^2+1)} \quad \text{(J/m)}$$

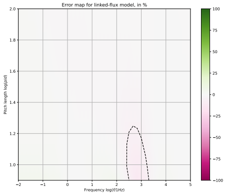
- Filament and matrix loss:

  $\Rightarrow$ simple analytical solutions (low and high f limits).

(First attempt of a model: could be improved!)







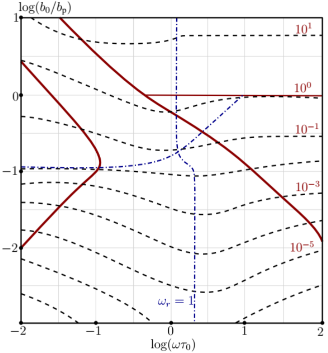
Dashed contours: $\pm$ 5% Solid contours: $\pm$ 10%

Outline

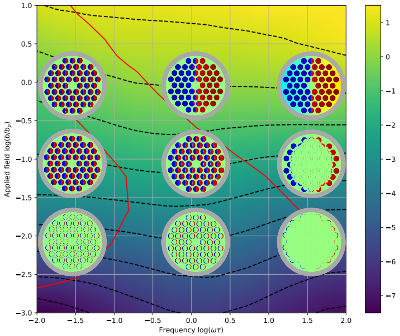
Application - Loss map

The linked-flux method is 2D ⇒ much faster than 3D.
⇒ Allows for efficient parameter sweep studies.

Loss per cycle w.r.t. transverse field f and amplitude:



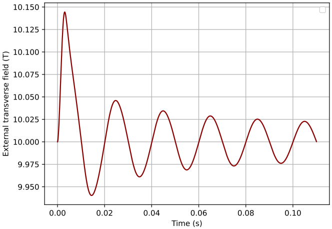
[Campbell, 1982]

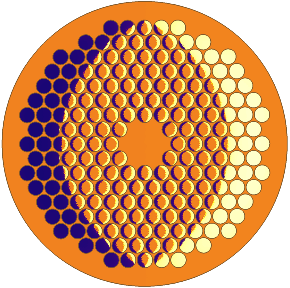


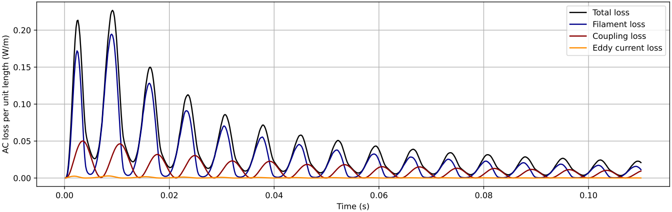
Linked-flux 2D method

Application - E-CLIQ study

192-filament strand subject to transverse field:

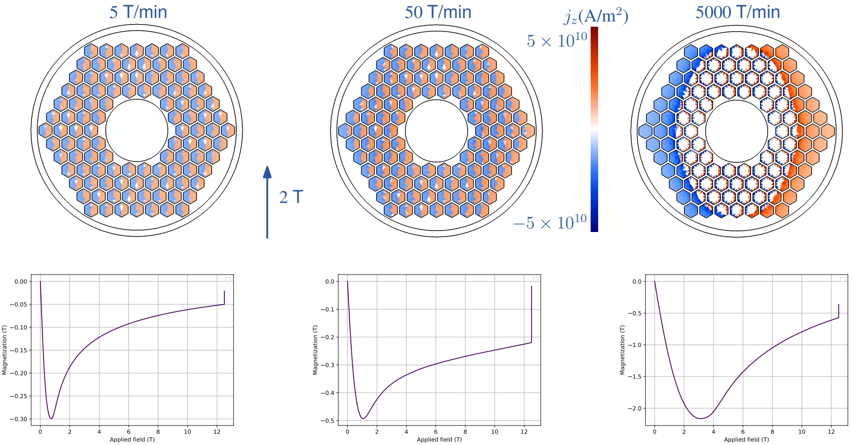






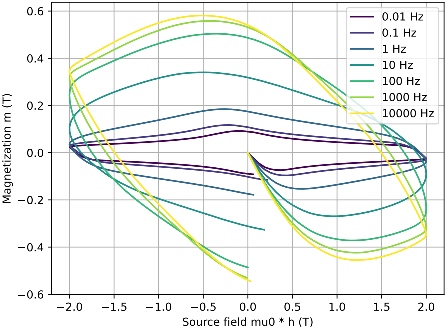
Application - Magnetization curves

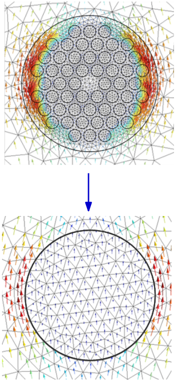
Nb$_3$Sn 108/127 geometry, ramp-up field with different rates:



Application - Magnetization curves (Cont'd)

Strand homogenization:





⇒ Dynamic vector hysteresis model (energy-based).

[Jacques, 2018]

Outline

Conclusions and perspectives

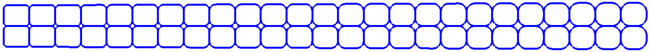
Three models for AC losses in strands (h-ϕ-formulation)

- 3D model: good reference,
- Helicoidal: 2D model, fast and exact in some cases,
- Linked-flux: 2D model, lightest and fairly accurate.

Implemented in GetDP/Gmsh or FiQuS: open-source.

Outlooks

1. Extend the linked-flux method to cable level,



2. Use linked-flux solutions to feed homogenized models,
3. Extend to HTS conductor geometries.

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Many thanks to Fredrik Magnus, Mariusz Wozniak, and STEAM team!

References

Analytical AC loss

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- Campbell, A. M. (1982). *A general treatment of losses in multifilamentary superconductors*. Cryogenics, 22(1), 3-16.

Helicoidal transformation and linked-flux first papers

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- Satiramatekul, T., and Bouillault, F. (2005). *Magnetization of coupled and noncoupled superconducting filaments with dependence of current density on applied field*. IEEE transactions on magnetics, 41(10), 3751-3753.
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Recent contributions

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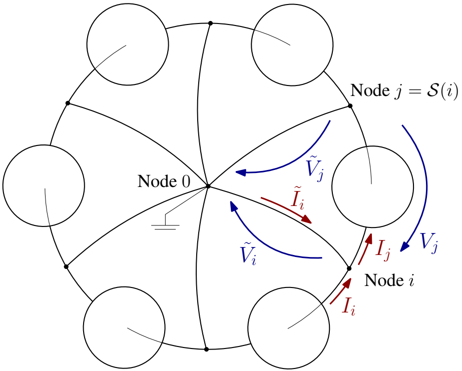
- Dular, J. (2023). *Standard and mixed finite element formulations for systems with type-II superconductors*. PhD dissertation, University of Liège, Belgium.
- Dular, J., Henrotte, F., Nicolet, A., Wozniak, M., Vanderheyden, B., and Geuzaine, C. (2023). *Helicoidal Transformation Method for Finite Element Models of Twisted Superconductors*. In press.

Thank you!

Outline

Linked-flux method - Equivalent circuit

The coupling equations $I_j = I_i + \tilde{I}_i$, $V_j = \tilde{V}_j - \tilde{V}_i$ are that of the following equivalent circuit (which is implemented in GetDP):



Linked-flux method - Dynamic correction

IP problem is static, we can add a dynamic correction.

- Let $\boldsymbol{h}_{\mathrm{s}}$ be one static axial field related to the coupling currents.

  (Attention, these currents are not associated with an axial field only, but also with some variation along z, this is neglected here, but it it not negligible. The proposed method is only approximate.)

- We extract this field from the solution of the IP problem:

$$(\sigma\,\mathbf{grad}\,v\,,\mathbf{curl}\,\boldsymbol{h}_{\mathrm{s}}')_{\Omega_{\mathrm{m}}}+(\mathbf{curl}\,\boldsymbol{h}_{\mathrm{s}}\,,\mathbf{curl}\,\boldsymbol{h}_{\mathrm{s}}')_{\Omega_{\mathrm{m}}}=0.$$

- We introduce a dynamic axial field component $\boldsymbol{h}_{\mathrm{d}}$, obtained from:

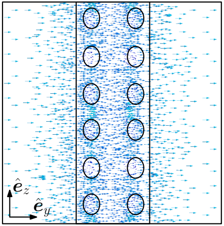
$$(\mu_0\partial_t(\boldsymbol{h}_{\mathrm{s}}+\boldsymbol{h}_{\mathrm{d}})\,,\mathbf{curl}\,\boldsymbol{h}_{\mathrm{d}}')_{\Omega_{\mathrm{m}}}+(\rho\,\mathbf{curl}\,\boldsymbol{h}_{\mathrm{d}}\,,\mathbf{curl}\,\boldsymbol{h}_{\mathrm{d}}')_{\Omega_{\mathrm{m}}}=0.$$

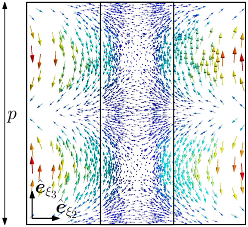
  We used $\mathbf{curl}\,(\rho\,\mathbf{curl}\,\boldsymbol{h}_{\mathrm{s}})=\mathbf{0}$ (as $\mathbf{curl}\,\boldsymbol{h}_{\mathrm{s}}$ is not an eddy current).

- $\mathbf{curl}\,\boldsymbol{h}_{\mathrm{d}}$ can then be used to correct the IP current.

Helicoidal transformation method

Transverse field does not lead to helicoidally symmetric BC.





But, there is a periodicity along ξ_3:

$$\boldsymbol{h}(\xi_1, \xi_2, \xi_3) = \sum_{k=-\infty}^{\infty} \boldsymbol{h}_k(\xi_1, \xi_2)\; f_k(\xi_3),$$

$$\text{with } \begin{cases} f_k(\xi_3) = \sqrt{2}\cos(\alpha k \xi_3), & k < 0, \\ f_0(\xi_3) = 1, & \\ f_k(\xi_3) = \sqrt{2}\sin(\alpha k \xi_3), & k > 0. \end{cases}$$

Special treatment in $\Omega_\mathrm{c}^\mathrm{C}$ to satisfy strongly $\mathbf{curl}\, \boldsymbol{h} = \mathbf{0}$.

Transverse field (linear material) - Cont'd

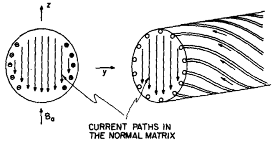
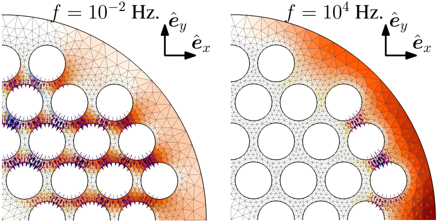
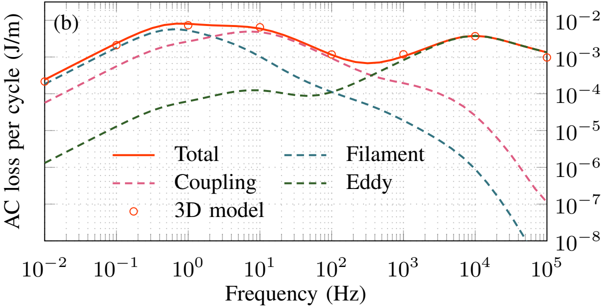


FIG. 2. Current paths in some of the superconducting filaments at the surface and the normal metal matrix of a twisted, multifilament wire which is exposed to a uniform changing field. The interior filaments are not shown since they carry no current.

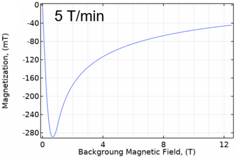
[Morgan, 1970]

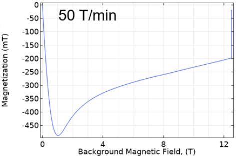


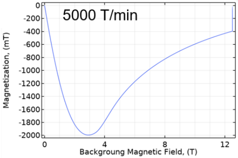


Comsol comparison - RRP 108/127 geometry

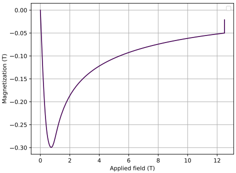
3D results from Comsol (courtesy of Bernardo Bordini):

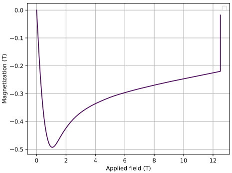


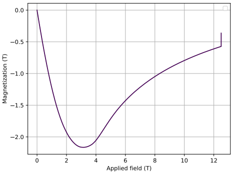




2D results from linked-flux method:

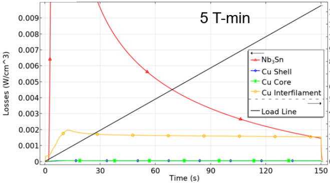


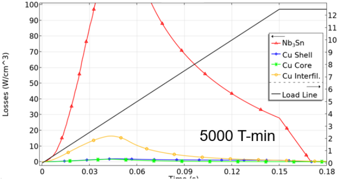




Comsol comparison - RRP 108/127 geometry (Cont'd)

3D results from Comsol (courtesy of Bernardo Bordini):





2D results from linked-flux method: