The structure of the Proton

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Outline

Lecture I: How do we know the nucleon has a structure

$e^- \mu^-$ elastic scattering
$e^- p^+$ elastic and inelastic scattering
Parton model e Bjorken scaling
The proton structure

Lecture II: Factorization

The operator product expansion
The Collins way
The formal definition of PDFs
One loop-corrections
PDFs from first principles?

Lecture IIII: PDFs and the path towards a 3-D picture

PDFs and quasi-PDFs
GPDs
TMD-PDFs
Recent results

Lattice basic definitions

- Replace Euclidian space-time by 4-dimensional hypercubic lattice:

  quark fields on lattice sites,

  gluon fields on lattice links.

- Lattice as a regulator:

  UV cut-off: inverse of lat. spacing a^{-1},

  IR cut-off: inverse of lat. size L^{-1}.

- Remove the regulator:

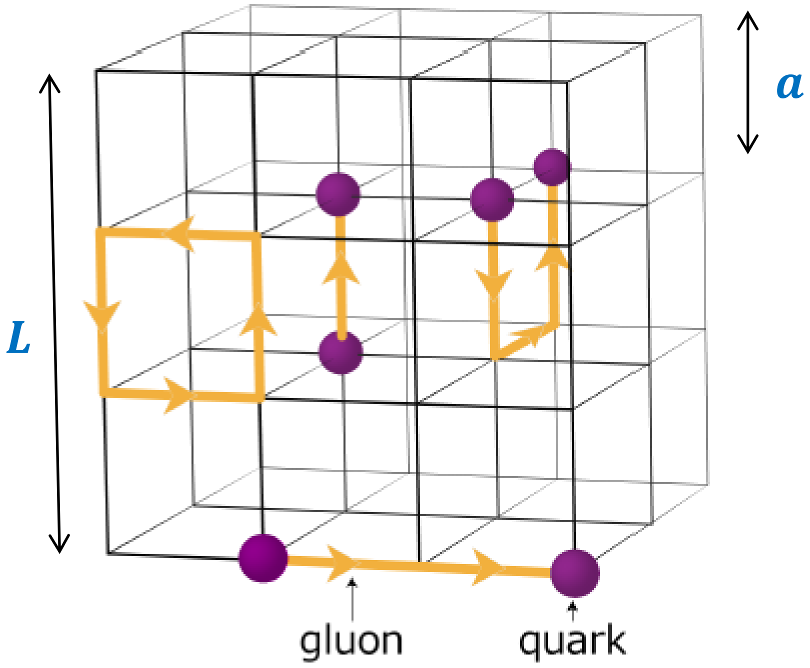
  continuum limit $a \to 0$,

  infinite volume limit $L \to \infty$.

- Gauge invariant objects:

  Wilson line: any path-ordered product of gauge link is gauge covariant,

  Wilson loops: the trace of a closed loop is gauge invariant

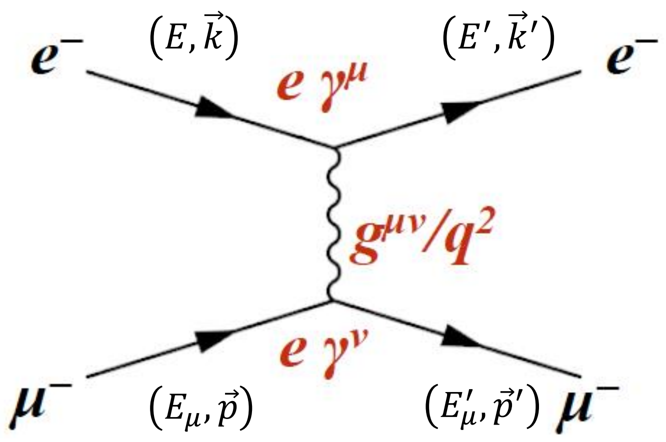


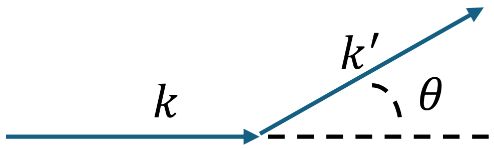
Source: JICFuS, Tsukuba

Elastic electron – muon scatteting

$e^-\mu^-$ is useful for our purpose :

- Both have spin ½
- Both known to be elementary
- One is much heavier than the other





$$\nu = E - E'$$

$$k^2 = k'^2 \cong 0, p^2 = p'^2 = m_\mu^2$$

$$Q^2 = -q^2 = -(k-k')^2 = 4EE'sin^2\theta/2$$

$$\frac{d^2\sigma}{d\Omega dE'} \propto |M|^2 = \frac{1}{2}\frac{\alpha_e^2}{q^4}\frac{E'}{E}L_{\alpha\beta}W^{\alpha\beta}$$

$$L_{\alpha\beta} = \frac{1}{2}Tr[(\gamma_\mu k'^\mu + m_e)\gamma_\alpha(\gamma_\mu k^\mu + m_e)\gamma_\beta]$$

e^- tensor, and a similar one for the μ^-

$$M = \frac{4\pi\alpha_e}{q^2}[\overline{u}(k',s'_e)\gamma_\mu u(k,s_e)][\overline{u}(p',s'_m)\gamma^\mu u(p,s_m)]$$

e^- current $\quad$ μ^- current

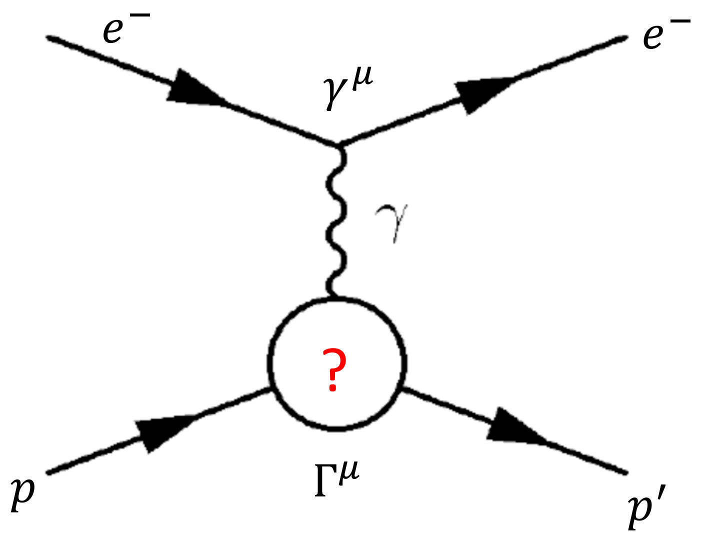
$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4E'\alpha_e^2}{q^4}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_\mu^2}\sin^2\frac{\theta}{2}\right]\delta\left(\nu - \frac{Q^2}{2m_\mu}\right)$$

Elastic scatteting

Elastic e^-p^+ scattering

e^-p^+ as before:

- Both have spin ½
- One is much heavier than the other
- But now only the electron is elementary



Gordon identity:

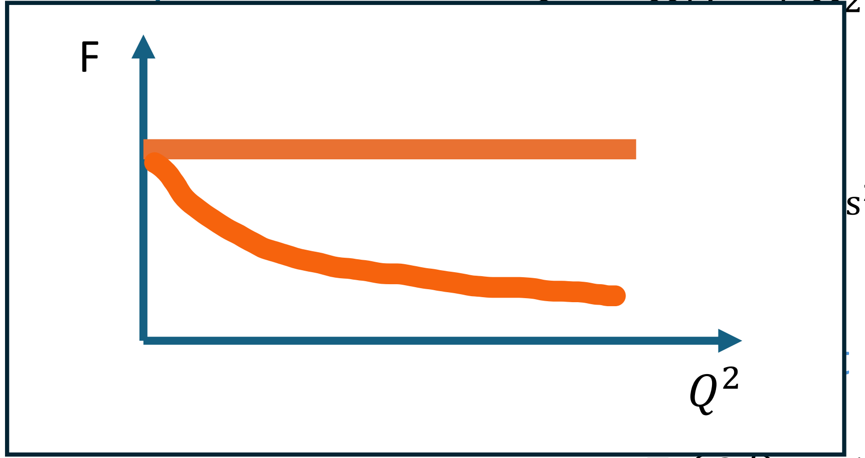
$$\overline{u}(p')\gamma^\mu u(p) = \frac{1}{2m}\overline{u}(p')[(p+p')^\mu + i\sigma^{\mu\nu}q_\nu]u(p)$$

Available 4-vectors: γ^μ, p, p'

$$\Gamma^\mu = A_1\gamma^\mu + A_2(p+p')^\mu + A_3(p'-p)^\mu$$

Current convervation: $q_\mu\Gamma^\mu = 0$

$$\Gamma^\mu = F_1\gamma^\mu + F_2\frac{i\sigma^{\mu\nu}q_\nu}{2m}$$



$$\ldots s^2\frac{\theta}{2} + \frac{Q^2}{2m}(F_1+F_2)^2 \sin^2\frac{\theta}{2}\Big]\,\delta\left(\nu - \frac{Q^2}{2m}\right)$$

... like:

$$F_1(Q^2) = 1$$

Dirac (F_1) and Pauli (F_2) form factors

It is often rewritten as

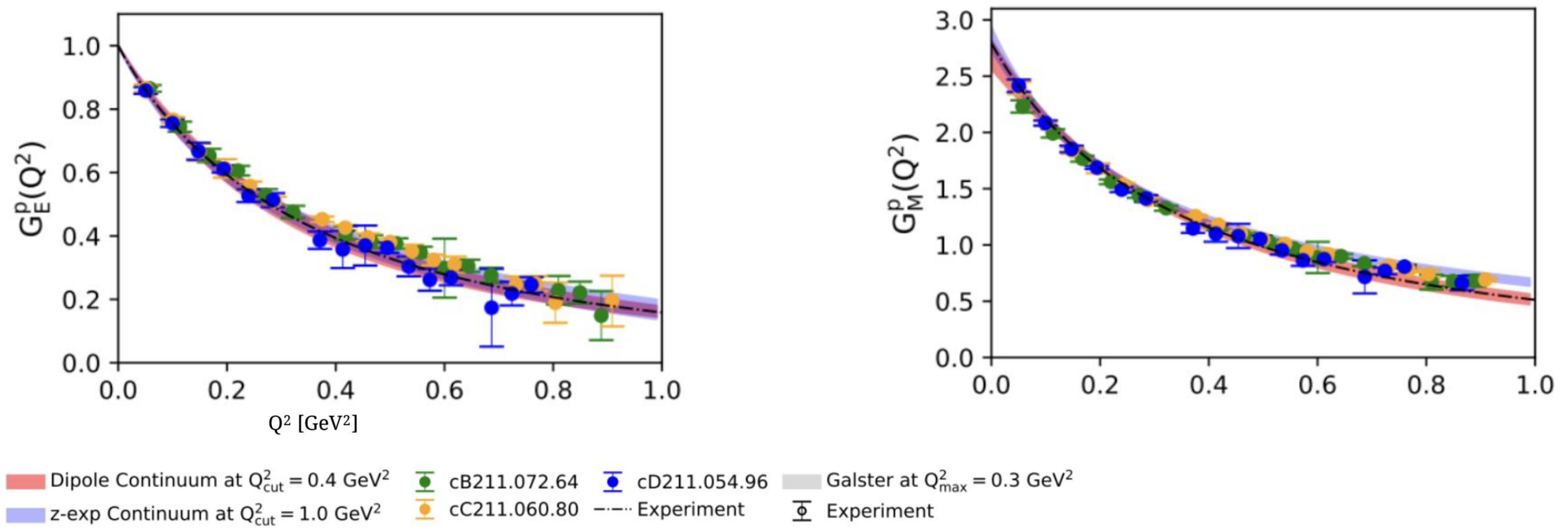
$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha_e^2}{4E^2\sin^4\frac{\theta}{4}}\frac{E'}{E}\left[\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2(Q^2)\,\sin^2\frac{\theta}{2}\right]\delta\left(\nu - \frac{Q^2}{2m_p}\right)$$

$$G_E(Q^2) = F_1^2 - \frac{Q^2}{4m_p^2}F_2^2$$

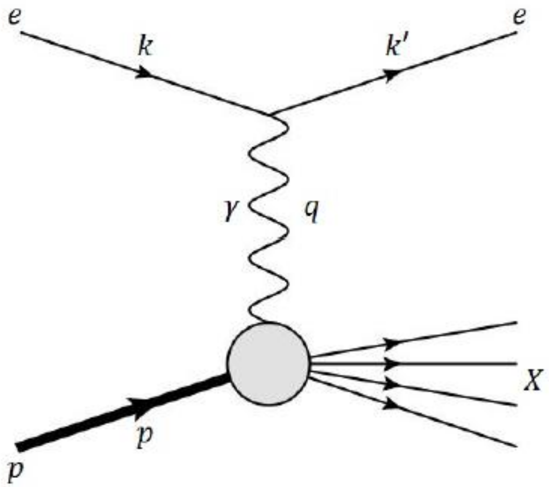
$$G_M(Q^2) = F_1 + F_2$$

$$\tau = \frac{Q^2}{4m_p}$$

arXiv: 2502.11301



Deep inelastic $e^- p^+$ scattering



$e^- p^+$ as before:

- Both have spin ½
- One is much heavier than the other
- Only the electron is elementar
- Inelastic scattering: $\nu \neq Q^2/2M$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4E'\alpha_e^2}{4Q^4}\left[W_2(\nu, Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, Q^2)\sin^2\frac{\theta}{2}\right]$$

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_M W_2(\nu, Q^2)\left[1 + 2\frac{Q^2+\nu^2}{(1+R)Q^2}\tan^2\frac{\theta}{2}\right]$$

$$R = \frac{W_2(\nu, Q^2)}{W_1(\nu, Q^2)}\left(\frac{Q^2+\nu^2}{Q^2}\right) - 1$$

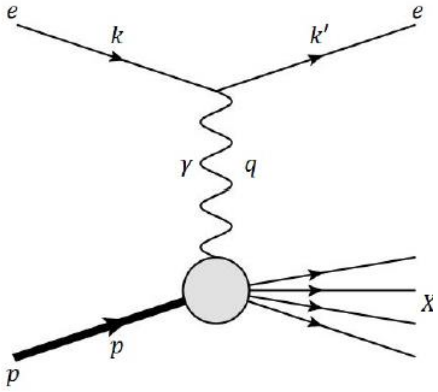
$q^\mu P^\mu + q^\nu P^\mu) + \frac{D}{M^2}P^\mu P^\nu$

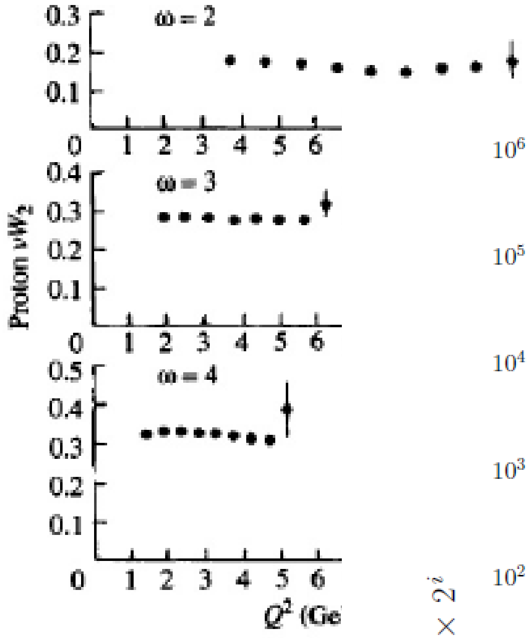
$^\mu - \frac{P.q}{q^2}q^\mu\Big)\Big(P^\mu - \frac{P.q}{q^2}q^\nu\Big)W_2$

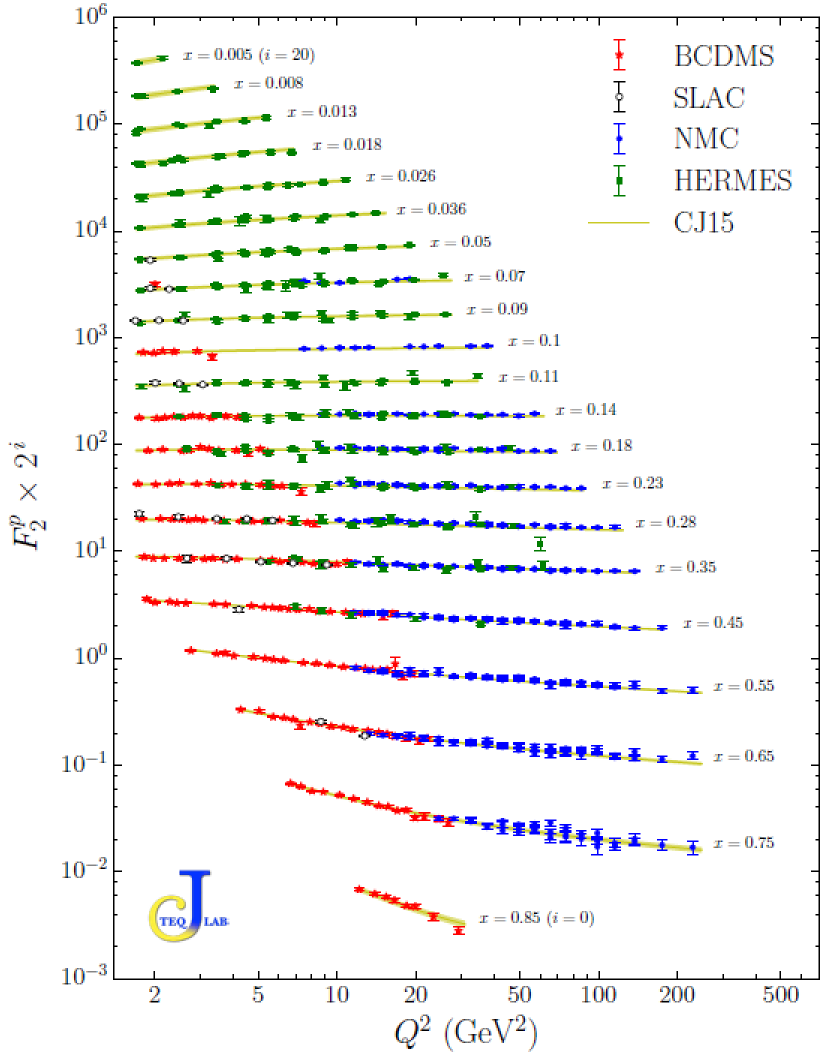
$^2)$ because of $\delta\left(\nu - \frac{Q^2}{2M}\right)$

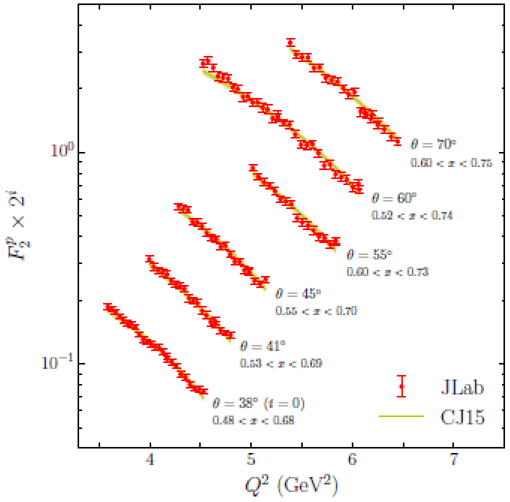
nction, and $W_{1,2} = W_{1,2}(\nu, Q^2)$

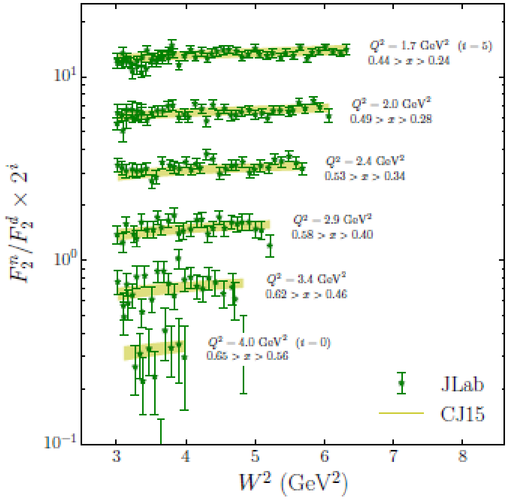
Experimentaly



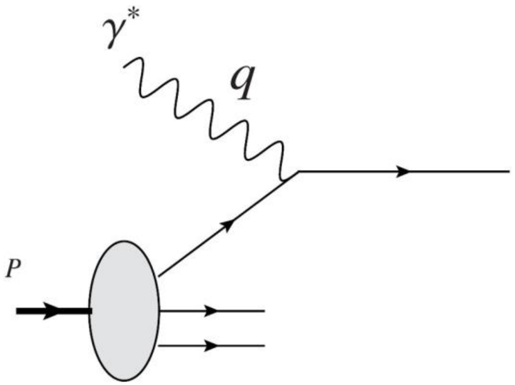








DIS may be seen elastic scattering off spin 1/2 point particles:



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4E'\alpha_e^2}{Q^4}\left[\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right]\delta\left(\nu - \frac{Q^2}{2m}\right)$$

Computed before for $e^-\mu^-$ scattering

$$\left(\frac{d^2\sigma}{\ldots}\right)^{point} \ldots \frac{4E'\alpha^2}{\ldots}\left[\ldots \frac{\theta}{\ldots} \ldots (\nu, Q^2)\sin^2\frac{\theta}{2}\right]$$

This ratio is defined as

$$x_B \equiv \frac{Q^2}{2M\nu}$$ Bjorken x_B

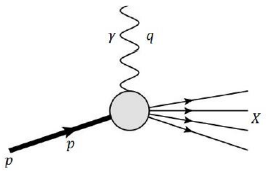
The limit $Q^2, \nu \longrightarrow \infty$ with x_B fixed is known as the Bjorken limit

$$\ldots^2) = \frac{Q^2}{2m^2\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\ldots) = \frac{1}{\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

Depends on one variable only

$$(P+q)^2 \geq M^2 \Longrightarrow \frac{Q^2}{2M\nu} \leq 1$$

$W_{1,2}(\nu, Q^2) = \left|\right.$  $\left.\right|^2$

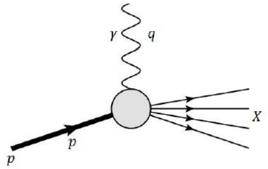
= Incoherent sum of elastic scattering between the photon and one point-like objects: Partons

$$= \sum_j \int dx' e_j^2 f_j(x') W_{1,2}^{point}(\nu, Q^2)$$

Probability to find the parton j with a momentum fraction x' of the parent hadron

Physical interpretation

$$W_{1,2}(\nu, Q^2) = \left| \quad \right|^2$$



$$= \sum_j \int dx' e_j^2 f_j(x') W_{1,2}^{point}(\nu, Q^2)$$

Probability to find the parton j with a momentum fraction x' of the parent hadron

$MW_1 \rightarrow F_1$

$$F_1 = \sum_j \int dx' e_j^2 f_j(x') M \frac{Q^2}{4m^2\nu} \delta\left(1 - \frac{Q^2}{2m\nu}\right) = \sum_j \int dx' e_j^2 f_j(x') \frac{x_B}{2x'} \delta(x_B - x') = \frac{1}{2} \sum_j e_j^2 f_j(x_B)$$

$$m = x'M$$

$\nu W_2 \rightarrow F_2$

$$F_2 = \sum_j \int dx' e_j^2 f_j(x') \delta\left(1 - \frac{Q^2}{2x'M\nu}\right) = \sum_j \int dx' e_j^2 f_j(x') x' \delta(x_B - x') = \sum_j e_j^2 x_B f_j(x_B)$$

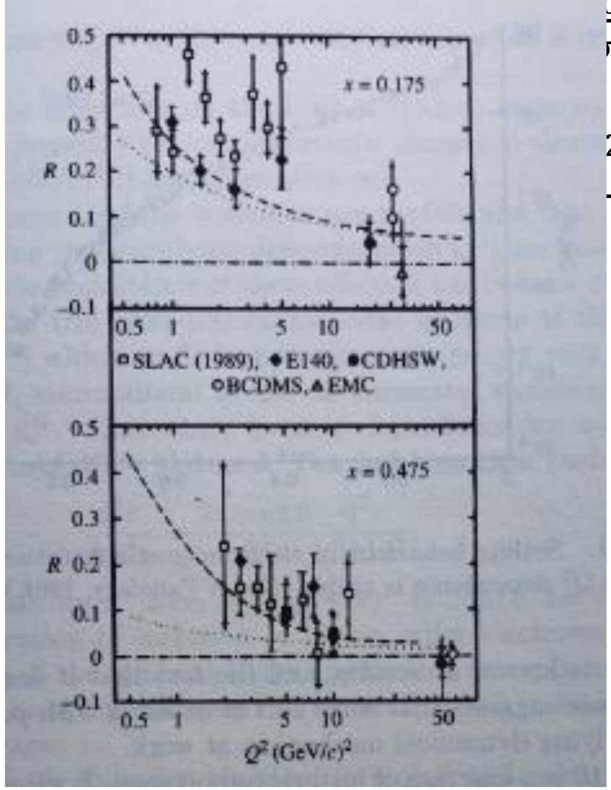
$$F_1(x_B) = \frac{1}{2x_B} F_2(x_B)$$

Callan – Gross relation

Consequence of the Callan-Gross relation

The virtual photon-proton cross section

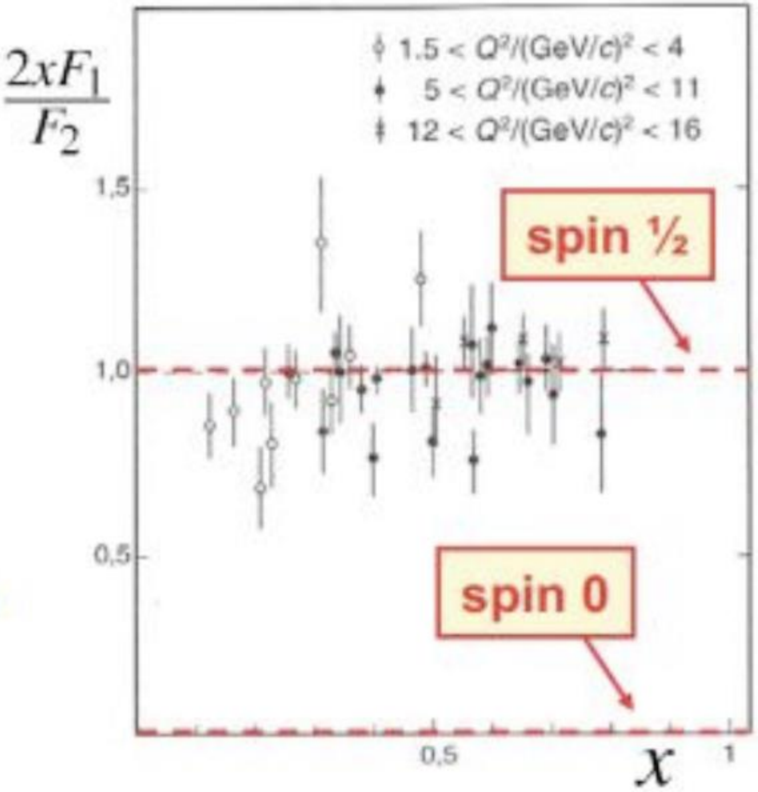
$$\sigma_\lambda(\gamma^* p \longrightarrow X) \propto \epsilon^*_\mu(\lambda) W^{\mu\nu} \epsilon_\nu(\lambda), \qquad \lambda = 0, \pm 1$$



$$\frac{2\sigma_0}{\sigma_{-1}}$$

$$\frac{\ ^2 + \nu^2}{Q^2} - 1 = \frac{M}{\nu}\frac{F_2}{F_1}\frac{Q^2 + \nu^2}{Q^2} -$$

$$R = \frac{F_2 - 2xF_1}{2xF_1}$$



Partons as quarks and gluons

- Partons are identified as quarks
- Quarks have fractional charges

$$e_u = \frac{2}{3}e, e_d = -\frac{1}{3}e, e_s = -\frac{1}{3}e, e_c = \frac{2}{3}e$$

- Proton: $u\,u\,d$
- Neutron: $d\,d\,u$

Crossing symmetry: $\overline{q}(x) = -q(-x)$

$$\int_0^1 dx(q(x) - \overline{q}(x)) = \int_{-1}^{+1} dx q(x)$$

Valence quarks:

$$\int_0^1 dx\,\left(u(x) - \overline{u}(x)\right) = \int_0^1 dx\, u_v(x) = 2$$

$$\int_0^1 dx\,\left(d(x) - \overline{d}(x)\right) = \int_0^1 dx\, d_v(x) = 1$$

$$q(x) = q^{\uparrow}(x) + q^{\downarrow}(x)$$

$$\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$$

The F_2 structure function: $F_2^p(x) = x\left(\frac{4}{9}\left(u(x) + \overline{u}(x)\right) + \frac{1}{9}\left(d(x) + \overline{d}(x)\right) + \frac{1}{9}\left(s(x) + \overline{s}(x)\right) + \cdots\right)$

Sea and valence dominance

The structure function $F_2(x,Q^2) = \sum_q x(q(x)+\bar{q}(x))$ is rewritten as:

$= u_s + \bar{u} + d_s + \bar{d} + s + \bar{s} + \cdots$

$$F_2^p(x) \equiv \frac{x}{9}\left(4u_v(x) + d_v(x) + S(x)\right)$$

Isospin symmetry:

$$u^p(x) = d^n(x) \equiv u(x)$$

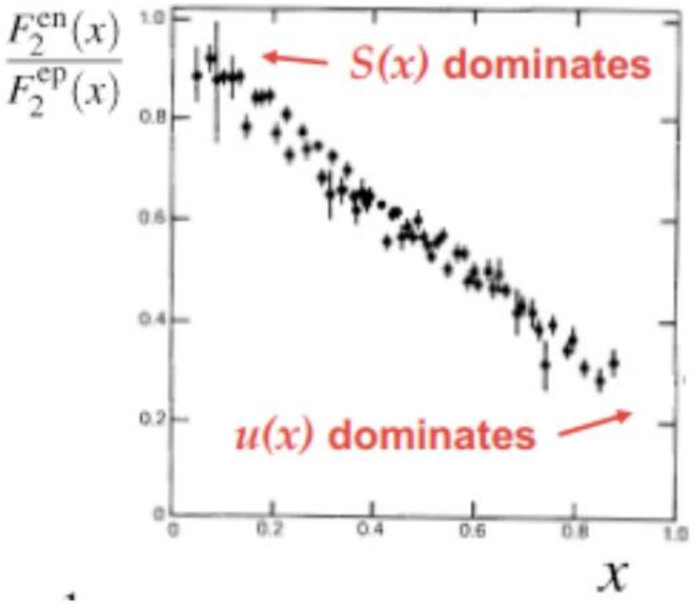
$$d^p(x) = u^n(x) \equiv d(x)$$

$$\Rightarrow \frac{F_2^n(x)}{F_2^p(x)} = \frac{4d_v(x) + u_v(x) + S(x)}{4u_v(x) + d_v(x) + S(x)}$$

$\longrightarrow 1$ if $S(x)$ dominates

$\longrightarrow\ > 1$ if $d_v(x)$ dominates

$\longrightarrow\ \approx 0.25$ if $u_v(x)$ dominates



Momentum sum rule

Do quarks carry all the proton momentum?

$$\int_0^1 dx x \left(u(x) + \overline{u}(x) + d(x) + \overline{d}(x)\right) \cong 1$$

(neglegible contribution from heavier quarks)

Experimental data and lattice data:

$$\int_0^1 dx x \left(u(x) + \overline{u}(x) + d(x) + \overline{d}(x)\right) \sim 1/2$$

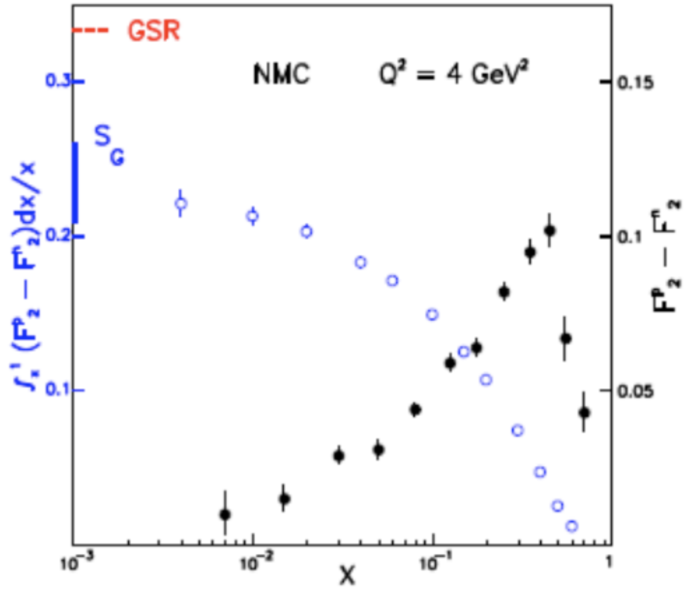
More on these calculations, not only for the proton but also for other hadrons, will be presented in Lecture II

Is the sea symmetric?

$u_s = \overline{u}$, but is $\overline{u} = \overline{d}$?

$$\frac{F_2^p(x) - F_2^n(p)}{x} = \frac{1}{3}\left(u_v(x) - d_v(x)\right) + \frac{2}{3}\left(\overline{u}(x) - \overline{d}(x)\right)$$

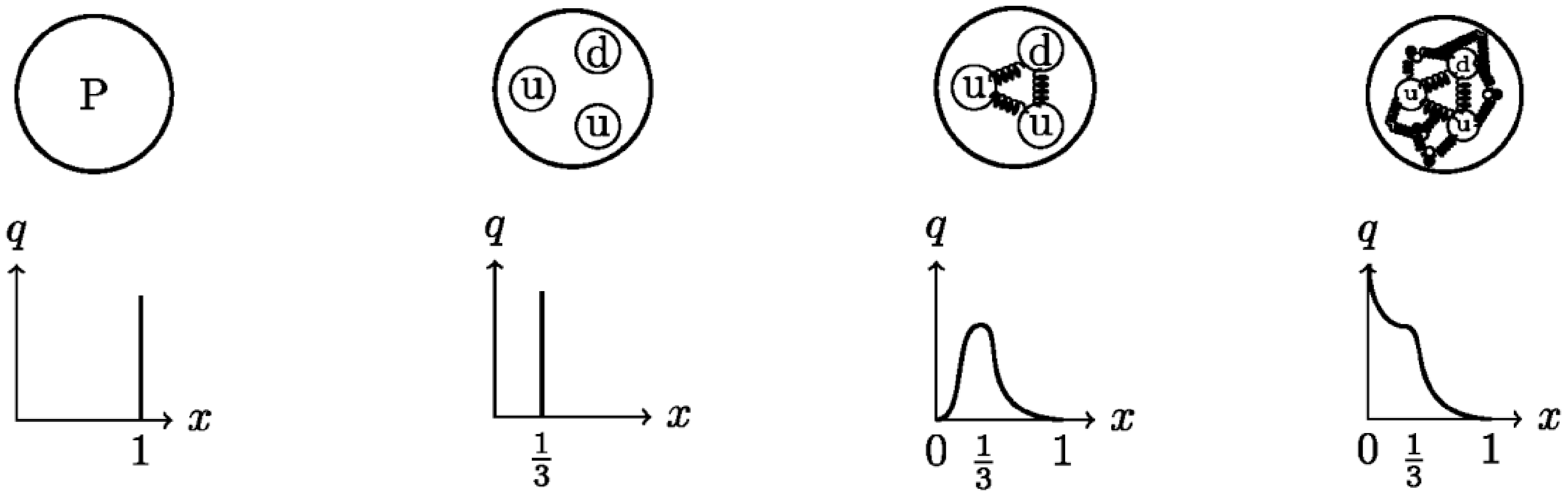
$$\Rightarrow \int_0^1 \frac{dx}{x}\left(F_2^p(x) - F_2^n(x)\right) = \frac{1}{3}$$



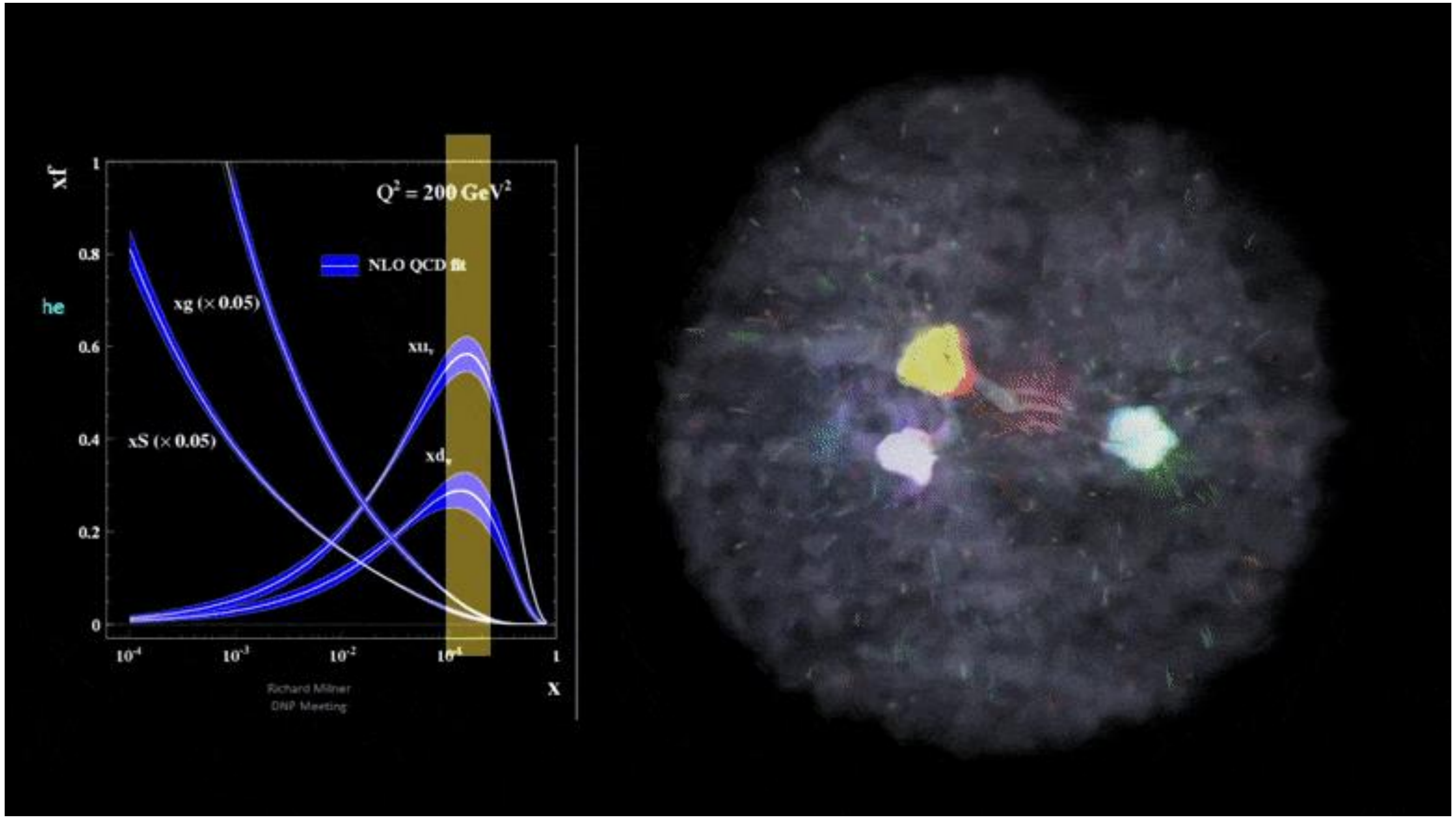
NMC + E866: $\int_0^1 \frac{dx}{x}\left(F_2^p(x) - F_2^n(x)\right) \approx 0.23$

There is an excess of $\overline{d}$ over $\overline{u}$ antiquarks in the proton

These simple tools help us already to see the following proton structure



Elastic | 3 free quarks | 3 bound quarks | Quarks + Gluons



MIT professor of physics Richard Milner, Jefferson Laboratory physicists Rolf Ent and Rik Yoshida, MIT documentary filmmakers Chris Boebel and Joe McMaster, and Sputnik Animation's James LaPlante have teamed up to depict the subatomic world in a new way.

Going beyond the electromanetic current,

$$W^{\mu\nu} = (\dots)W_1 + (\dots)W_2 - \frac{i\epsilon^{\mu\nu\lambda\sigma}p_\lambda q_\sigma}{m^2}W_3$$

Parity violating term

$\Rightarrow$ $$xF_3(x) = xV(x) \pm x(\bar{c}(x) - \bar{s}(x) \pm x\big(c(x) - s(x)\big) \pm \cdots$$

\+ for $\bar{\nu}$
\- for ν

$$\int_0^1 dx x \frac{F_3^{\bar{\nu}}(x) + F_3^{\nu}(x)}{2} = 3$$

Gross-Llewellyn Smithe sum rule

Experimentally, this number is smaller than 3. This is in agreement with the inclusion of QCD corrections

CCFR, J. H. Kim et al., PRL 81 (1998) 3595, hep-exp/9808015