Dark matter effects in hadronic and strange stars

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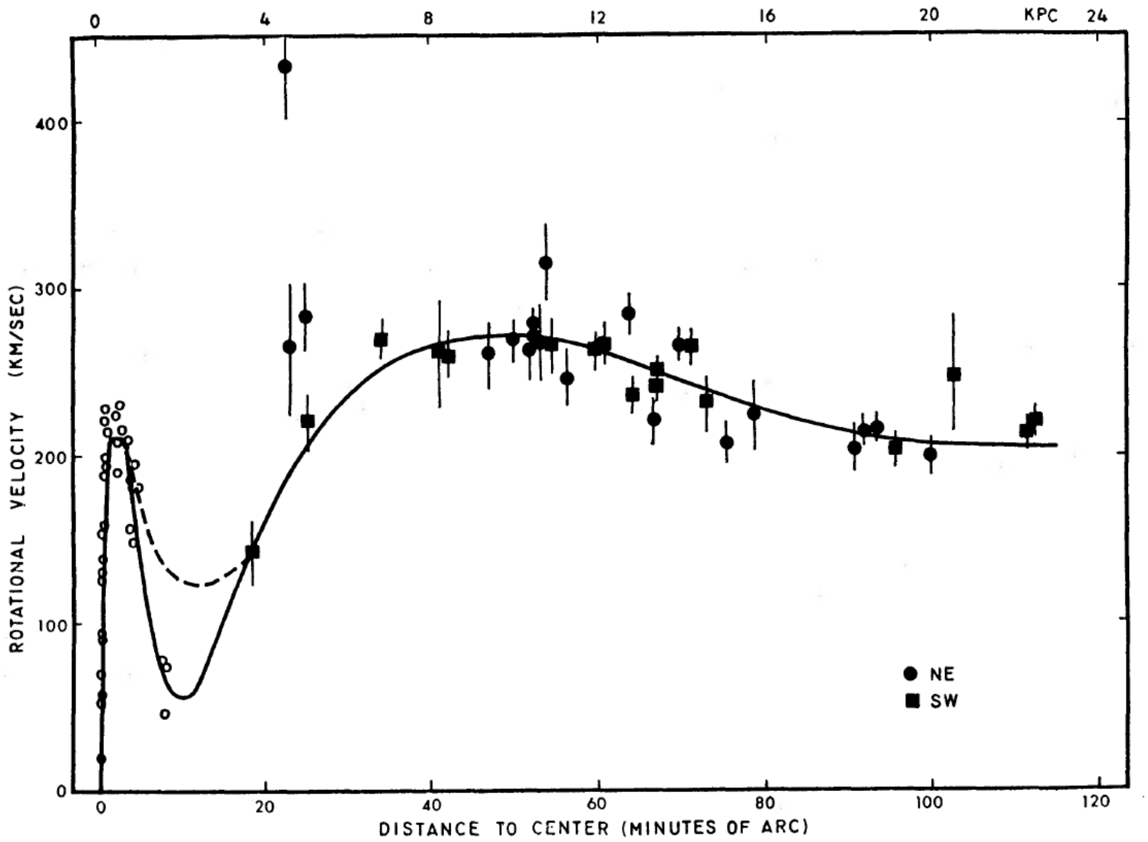
OUTLINE

- Introduction
- Dark matter coupled to hadronic models
- DM coupled to ordinary matter: 2-fluid formalism
  - "Visible" matter models
  - Dark matter models
- Strange stars with dark matter content
- Main conclusions

- Observational evidences of non luminous matter.

Rotation curves of spiral galaxies, for instance:

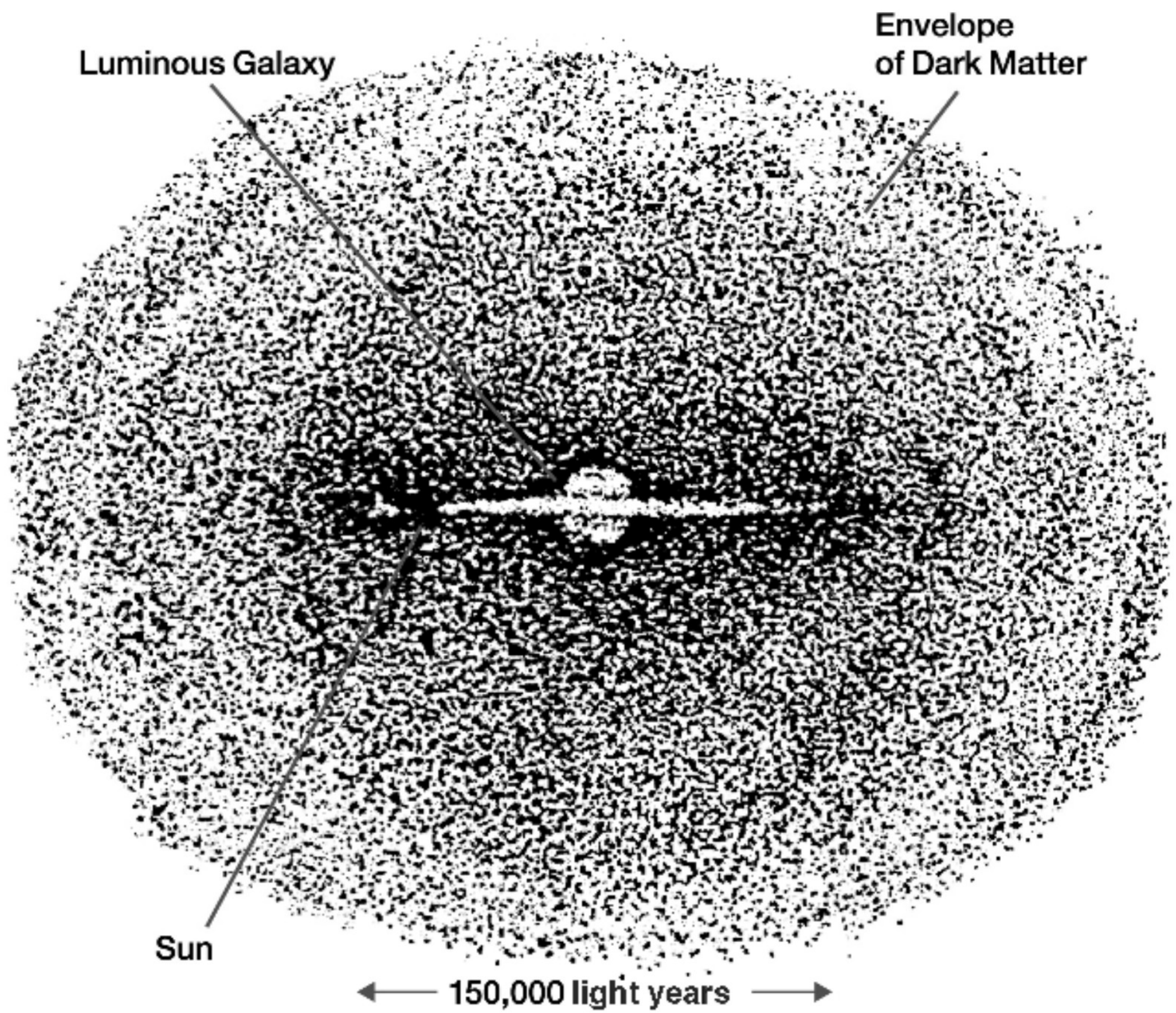


Kepler's law

$$v = \frac{2\pi a}{T} = \sqrt{\frac{\gamma M}{a}} \sim \frac{1}{\sqrt{a}}$$

Vera Cooper Rubin, Astrophys. J. 159, (1970)

- Observational evidences of non luminous matter.



- The “missing mass” (dark matter) was suggested by F. Zwicky in 1937.

- The dark fermion is represented by the Dirac field χ, that interacts with nucleons through the exchange of the Higgs boson (field h, mass m_h).

$$\mathcal{L} = \bar{\chi}(i\gamma^{\mu}\partial_{\mu} - M_{\chi})\chi + yh\bar{\chi}\chi + \frac{1}{2}(\partial^{\mu}h\partial_{\mu}h - m_h^2 h^2) + f\frac{M_{\text{nuc}}}{v}h\bar{\psi}\psi + \mathcal{L}_{\text{HAD}},$$

fM_{nuc}/v: Higgs-nucleon coupling,

$M_\chi = 200$ GeV: neutralino mass (DM particle),

$v = 246$ GeV: Higgs vacuum expectation value,

y: Higgs-dark particle coupling strength;

M_{nuc}: nucleon mass, Ψ: nucleon Dirac field.

- This formalism has been used recently. See, for instance:

  - *A. Das, T. Malik, and A. C. Nayak, PRD105, 123034 (2022)*
  - *G. Panotopoulos and I. Lopes, PRD96, 083004 (2017)*
  - *A. Das, T. Malik, and A. C. Nayak, PRD99, 043016 (2019)*
  - *S. A. Bhat and A. Paul, EPJC80, 544 (2020)*
  - *A. Quddus, G. Panotopoulos, B. Kumar, S. Ahmad, and S. K. Patra, JPG47, 095202 (2020)*
  - *H. C. Das, A. Kumar, and S. K. Patra, MNRAS507, 4053 (2021)*

- Hadronic sector. Typical Lagrangian density:

$$\mathcal{L}_{\mathrm{HAD}} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - M_{\mathrm{nuc}})\psi + g_{\sigma}\sigma\bar{\psi}\psi - g_{\omega}\bar{\psi}\gamma^{\mu}\omega_{\mu}\psi - \frac{g_{\rho}}{2}\bar{\psi}\gamma^{\mu}\vec{\rho}_{\mu}\vec{\tau}\psi + \frac{1}{2}(\partial^{\mu}\sigma\partial_{\mu}\sigma - m_{\sigma}^{2}\sigma^{2}) - \frac{A}{3}\sigma^{3} - \frac{B}{4}\sigma^{4} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu} + \frac{C}{4}(g_{\omega}^{2}\omega_{\mu}\omega^{\mu})^{2} - \frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}\alpha_{3}' g_{\omega}^{2}g_{\rho}^{2}\omega_{\mu}\omega^{\mu}\vec{\rho}_{\mu}\vec{\rho}^{\mu} + \frac{1}{2}m_{\rho}^{2}\vec{\rho}_{\mu}\vec{\rho}^{\mu},$$

$$F_{\mu\nu} = \partial_{\nu}\omega_{\mu} - \partial_{\mu}\omega_{\nu} \quad \vec{B}_{\mu\nu} = \partial_{\nu}\vec{\rho}_{\mu} - \partial_{\mu}\vec{\rho}_{\nu} - g_{\rho}(\vec{\rho}_{\mu} \times \vec{\rho}_{\nu}).$$

- red quantities: coupling constants (free parameters). Found through bulk parameters of infinite nuclear matter, for instance.
- mean-field approximation:

$$\sigma \to \langle\sigma\rangle \equiv \sigma, \qquad \omega_{\mu} \to \langle\omega_{\mu}\rangle \equiv \omega_{0},$$

$$\vec{\rho}_{\mu} \to \langle\vec{\rho}_{\mu}\rangle \equiv \bar{\rho}_{0(3)}, \qquad h \to \langle h\rangle \equiv h,$$

- Field equations:

$$m_\sigma^2\sigma = g_\sigma\rho_s - A\sigma^2 - B\sigma^3,$$

$$m_\omega^2\omega_0 = g_\omega\rho - Cg_\omega(g_\omega\omega_0)^3 - \alpha_3' g_\omega^2 g_\rho^2 \bar{\rho}_{0(3)}^2\omega_0,$$

$$m_\rho^2\bar{\rho}_{0(3)} = \frac{g_\rho}{2}\rho_3 - \alpha_3' g_\omega^2 g_\rho^2 \bar{\rho}_{0(3)}\omega_0^2,$$

$$[\gamma^\mu(i\partial_\mu - V) - M^*]\psi = 0,$$

$$m_h^2 h = y\rho_s^{\rm DM} + f\frac{M_{\rm nuc}}{v}\rho_s$$

$$(\gamma^\mu i\partial_\mu - M_\chi^*)\chi = 0,$$

$$V = g_\omega\omega_0 + \tfrac{g_\rho}{2}\bar{\rho}_{0(3)}\tau_3$$

$$\rho_s = \langle\bar{\psi}\psi\rangle = \rho_{sp} + \rho_{sn},$$

$$\rho = \langle\bar{\psi}\gamma^0\psi\rangle = \rho_p + \rho_n,$$

$$\rho_3 = \langle\bar{\psi}\gamma^0\tau_3\psi\rangle = \rho_p - \rho_n$$

$$\rho_s^{\rm DM} = \langle\bar{\chi}\chi\rangle,$$

$$M^* = M_{\rm nuc} - g_\sigma\sigma - f\frac{M_{\rm nuc}}{v}h,$$

$$M_\chi^* = M_\chi - yh.$$

- From the energy-momentum tensor we have

$$\mathcal{E} = \langle T_{00}\rangle \text{ and } P = \langle T_{ii}\rangle/3,$$

- Energy density:

$$\mathcal{E} = \frac{m_\sigma^2\sigma^2}{2} + \frac{A\sigma^3}{3} + \frac{B\sigma^4}{4} - \frac{m_\omega^2\omega_0^2}{2} - \frac{Cg_\omega^4\omega_0^4}{4} - \frac{m_\rho^2\bar{\rho}_{0(3)}^2}{2}$$
$$+ g_\omega\omega_0\rho + \frac{g_\rho}{2}\bar{\rho}_{0(3)}\rho_3 - \frac{1}{2}\alpha_3' g_\omega^2 g_\rho^2 \omega_0^2 \bar{\rho}_{0(3)}^2 + \mathcal{E}_{\rm kin}^p + \mathcal{E}_{\rm kin}^n$$
$$+ \frac{m_h^2 h^2}{2} + \mathcal{E}_{\rm kin}^{\rm DM},$$

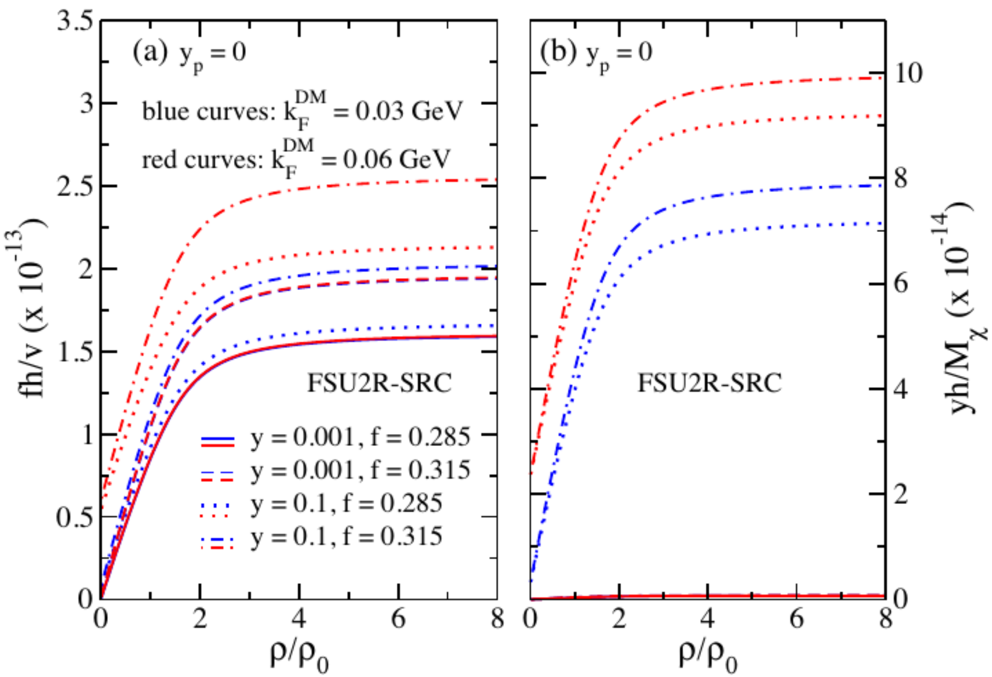
$$\mathcal{E}_{\rm kin}^{p,n} = \frac{\gamma}{2\pi^2}\int_0^{k_{F\,p,n}} k^2\left(k^2 + M_{p,n}^{*2}\right)^{1/2} dk \qquad \mathcal{E}_{\rm kin}^{\rm DM} = \frac{\gamma}{2\pi^2}\int_0^{k_F^{\rm DM}} k^2(k^2 + M_\chi^{*2})^{1/2} dk,$$

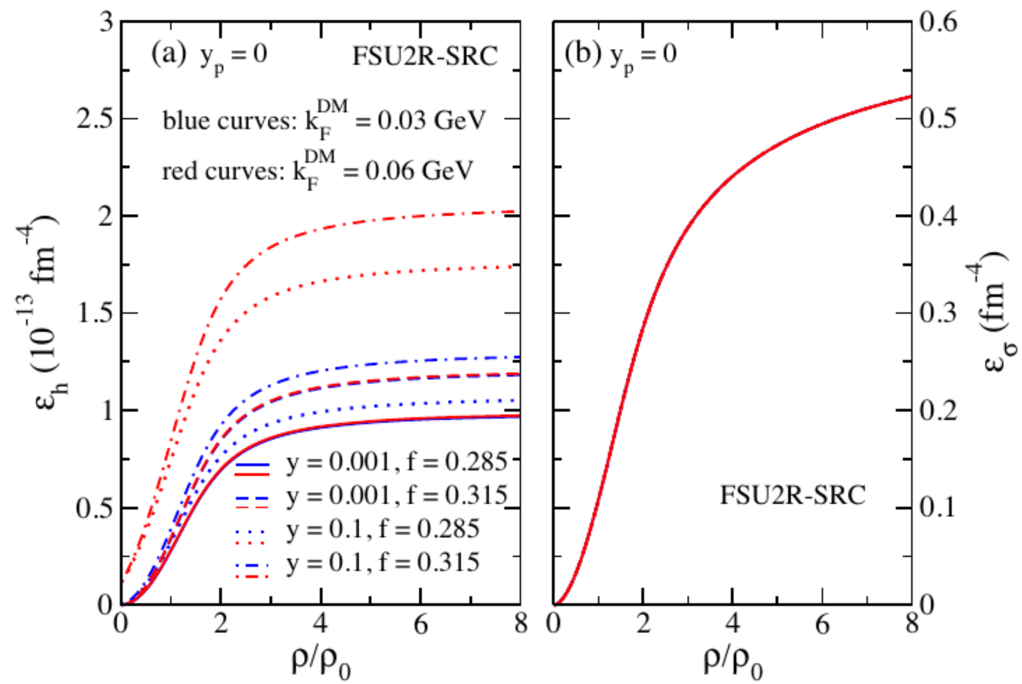
- Pressure:

$$P = -\frac{m_\sigma^2\sigma^2}{2} - \frac{A\sigma^3}{3} - \frac{B\sigma^4}{4} + \frac{m_\omega^2\omega_0^2}{2} + \frac{Cg_\omega^4\omega_0^4}{4}$$
$$+ \frac{m_\rho^2\bar{\rho}_{0(3)}^2}{2} + \frac{1}{2}\alpha_3' g_\omega^2 g_\rho^2 \omega_0^2 \bar{\rho}_{0(3)}^2 + P_{\rm kin}^p + P_{\rm kin}^n - \frac{m_h^2 h^2}{2}$$
$$+ P_{\rm kin}^{\rm DM},$$

$$P_{\rm kin}^{p,n} = \frac{\gamma}{6\pi^2}\int_0^{k_{F\,p,n}} \frac{k^4 dk}{\left(k^2 + M_{p,n}^{*2}\right)^{1/2}} \qquad P_{\rm kin}^{\rm DM} = \frac{\gamma}{6\pi^2}\int_0^{k_F^{\rm DM}} \frac{k^4 dk}{(k^2 + M_\chi^{*2})^{1/2}}.$$

- Major effects from kinetic terms





$$M^* = M_{\text{nuc}} - g_\sigma \sigma - f\frac{M_{\text{nuc}}}{v}h,$$

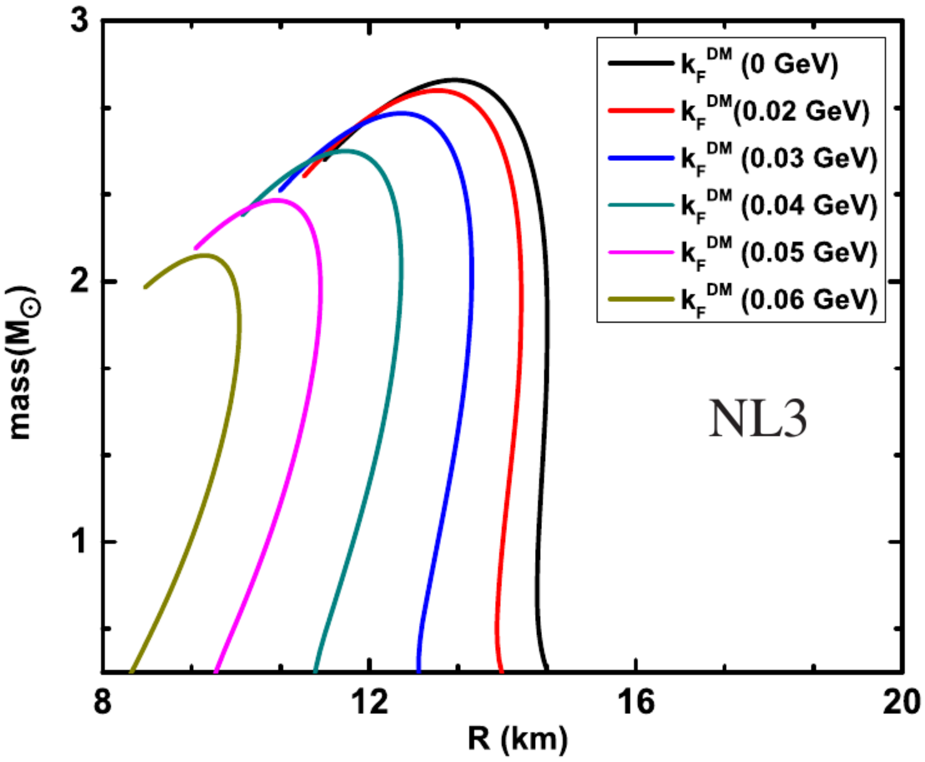
$$M^*_\chi = M_\chi - yh.$$

$$\mathcal{E} = \frac{m_\sigma^2\sigma^2}{2} + \frac{A\sigma^3}{3} + \frac{B\sigma^4}{4} - \frac{m_\omega^2\omega_0^2}{2} - \frac{Cg_\omega^4\omega_0^4}{4} - \frac{m_\rho^2\bar{\rho}_{0(3)}^2}{2} + g_\omega\omega_0\rho + \frac{g_\rho}{2}\bar{\rho}_{0(3)}\rho_3 - \frac{1}{2}\alpha_3' g_\omega^2 g_\rho^2 \omega_0^2 \bar{\rho}_{0(3)}^2 + \mathcal{E}_{\text{kin}}^p + \mathcal{E}_{\text{kin}}^n + \frac{m_h^2 h^2}{2} + \mathcal{E}_{\text{kin}}^{\text{DM}}$$

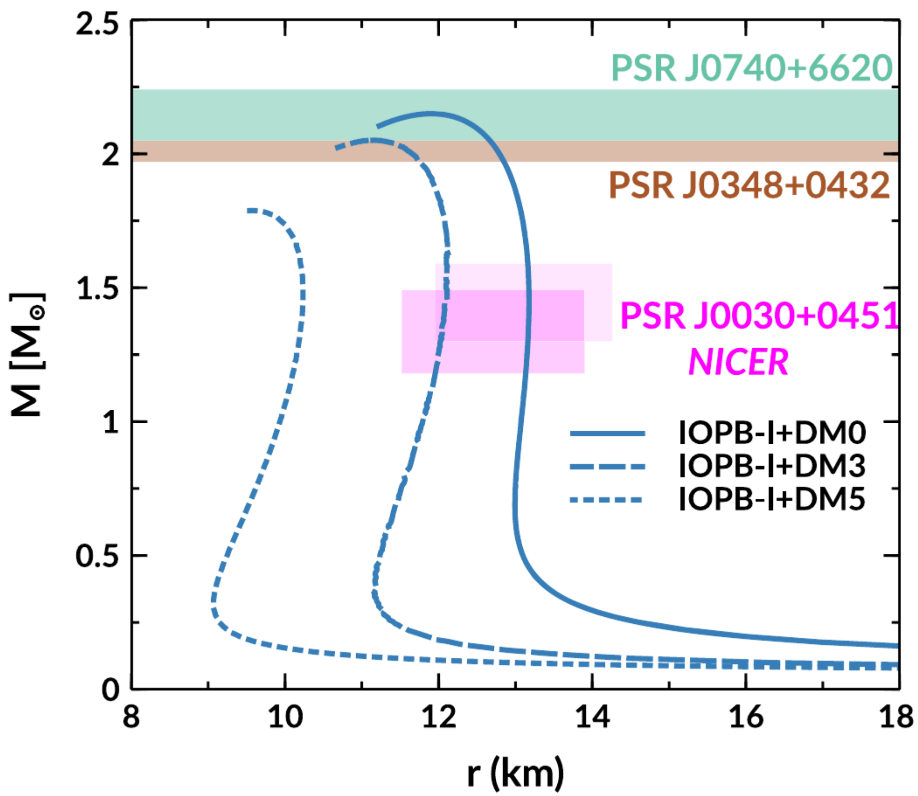
O. Lourenço , T. Frederico, M. Dutra
PRD105, 023008 (2022)

- mass-radius diagrams:

$$\frac{dp(r)}{dr} = -\frac{[\epsilon(r) + p(r)][m(r) + 4\pi r^3 p(r)]}{r^2[1 - 2m(r)/r]}, \qquad \frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r)$$



A. Das, T. Malik, and A. C. Nayak
PRD99, 043016 (2019)



H. C. Das, A. Kumar, and S. K. Patra
MNRAS507, 4053 (2021)

- reduction in the NS mass due to the increase of DM content.

- It considers the inter-fluid interaction coming uniquely from gravitational effects

- In this approach, each fluid satisfies the conservation of energy-momentum separately, which is equivalent to having

$$\mathcal{E}(r) = \mathcal{E}_{\rm vis}(r) + \mathcal{E}_{\rm DM}(r) \qquad P(r) \; = \; P_{\rm vis}(r) + P_{\rm DM}(r)$$

- Generalized TOV equations:

$$\frac{dP_{\rm vis}(r)}{dr} = -\frac{\left[\mathcal{E}_{\rm vis}(r) + P_{\rm vis}(r)\right]\left[m(r) + 4\pi r^3 P(r)\right]}{r\left[r - 2m(r)\right]}, \qquad m(r) \; = \; m_{\rm vis}(r) + m_{\rm DM}(r)$$

$$\frac{dP_{\rm DM}(r)}{dr} = -\frac{\left[\mathcal{E}_{\rm DM}(r) + P_{\rm DM}(r)\right]\left[m(r) + 4\pi r^3 P(r)\right]}{r\left[r - 2m(r)\right]},$$

$$\frac{dm_{\rm vis}(r)}{dr} = 4\pi r^2 \mathcal{E}_{\rm vis}(r),$$

$$\frac{dm_{\rm DM}(r)}{dr} = 4\pi r^2 \mathcal{E}_{\rm DM}(r),$$

- initial conditions:

$$m_{\rm vis}(0) \; = \; m_{\rm DM}(0) \; = \; 0, \;\; P_{\rm vis}(0) \; = \; P^c_{\rm vis}, \;\; P_{\rm DM}(0) = P^c_{\rm DM}$$

Qian-Fei Xiang, Wei-Zhou Jiang, Dong-Rui Zhang, Rong-YaoYang, PRC 89, 025803 (2014)

- We use a model in which the constituent quark masses are density dependent:

$$H_{\mathrm{QCD}} = H_k + \sum_{i=u,\,d,\,s} m_{i0}\bar{q}q + H_I, \qquad H_{\mathrm{eqv}} = H_k + \sum_{i=u,\,d,\,s} m_i\bar{q}q.$$

$$\langle\Psi|H_{\mathrm{eqv}}|\Psi\rangle=\langle\Psi|H_{\mathrm{QCD}}|\Psi\rangle$$

the same eigenenergy for any eigenstate $|\Psi\rangle$

$$\langle\rho_b|H_{\mathrm{eqv}}|\rho_b\rangle-\langle 0|H_{\mathrm{eqv}}|0\rangle=\langle\rho_b|H_{\mathrm{QCD}}|\rho_b\rangle-\langle 0|H_{\mathrm{QCD}}|0\rangle.$$

↓

$$m_i = m_{i0}+\frac{\langle H_{\mathrm{I}}\rangle_{\rho_b}-\langle H_{\mathrm{I}}\rangle_0}{\sum_q[\langle\bar{q}q\rangle_{\rho_b}-\langle\bar{q}q\rangle_0]} \equiv m_{i0}+m_I,$$

$$\langle H_{\mathrm{I}}\rangle_{\rho_b}\equiv\langle\rho_b|H_{\mathrm{I}}|\rho_b\rangle,\ \langle H_{\mathrm{I}}\rangle_0\equiv\langle 0|H_{\mathrm{I}}|0\rangle,\ \text{and}\ \langle\bar{q}q\rangle_{\rho_b}\equiv\langle\rho_b|\bar{q}q|\rho_b\rangle,\ \langle\bar{q}q\rangle_0\equiv\langle 0|\bar{q}q|0\rangle.$$

G. X. Peng, H. C. Chiang, J. J. Yang, L. Li, B. Liu
PRC61, 015201 (1999)

- Interaction energy:

$$\langle H_I \rangle_{\rho_b} - \langle H_I \rangle_0 = \frac{1}{2V} \int \int_V v(r)(3n_b d\vec{r}_1)(3n_b d\vec{r}_2) = 18\pi \rho_b^2 \int_0^R v(r) r^2 \, dr,$$

$r = |\vec{r}_1 - \vec{r}_2|$, $v(r)$ is the quark-quark interaction $\qquad V = 4/3\pi R^3$

- Quark condensate:

$$\frac{\langle \bar{q}q \rangle_{\rho_b}}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho_b}{\rho'_q} \longrightarrow \sum_q [\langle \bar{q}q \rangle_{\rho_b} - \langle \bar{q}q \rangle_0] = \sum_q [-\langle \bar{q}q \rangle_0 / \rho'_q] \rho_b \equiv A \rho_b .$$

- By taking a potential that incorporates confinement and asymptotic freedom:

$$V(r) = -\frac{\beta}{r} + \sigma r$$

$$\downarrow$$

$$m_i = m_{i0} + \frac{\langle H_I \rangle_{\rho_b} - \langle H_I \rangle_0}{\sum_q [\langle \bar{q}q \rangle_{n_b} - \langle \bar{q}q \rangle_0]} = m_{i0} + m_I = m_{i0} + \frac{D}{\rho_b^{1/3}} + C \rho_b^{1/3}$$

- Equations of state for the model:

$$P = -\Omega_0 + \rho_b \frac{\partial m_I}{\partial \rho_b} \frac{\partial \Omega_0}{\partial m_I}, \qquad \mathcal{E} = \Omega_0 - \sum_i \mu_i^* \frac{\partial \Omega_0}{\partial \mu_i^*}, \qquad \mu_i = \mu_i^* + \frac{1}{3} \frac{\partial m_I}{\partial \rho_b} \frac{\partial \Omega_0}{\partial m_I}.$$

$$\Omega_0 = -\sum_i \frac{\gamma}{24\pi^2} \left[\mu_i^* k_{Fi} \left(k_{Fi}^2 - \frac{3}{2} m_i^2 \right) + \frac{3}{2} m_i^4 \ln \frac{\mu_i^* + k_{Fi}}{m_i} \right],$$

$$k_{Fi} = \sqrt{\mu_i^{*2} - m_i^2}, \qquad \rho_i = \frac{\gamma k_{Fi}^3}{6\pi^2}, \qquad \rho_b = \frac{1}{3} \sum_i \rho_i.$$

- Stellar matter conditions:

$$\mu_u + \mu_e = \mu_d = \mu_s \qquad \tfrac{2}{3}\rho_u - \tfrac{1}{3}\rho_d - \tfrac{1}{3}\rho_s - \rho_e = 0.$$

Electrons contribution:

$$P_{\rm vis} = P + \frac{\mu_e^4}{12\pi^2}, \qquad \mathcal{E}_{\rm vis} = \mathcal{E} + \frac{\mu_e^4}{4\pi^2} \qquad \mu_e = (3\pi^2 \rho_e)^{1/3}.$$

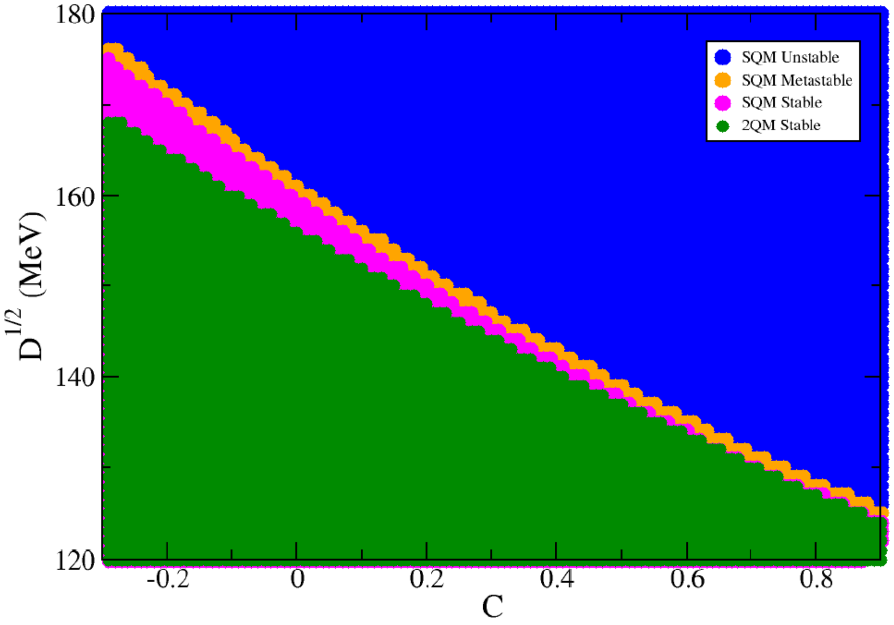
- Stability window.

- Bodmer-Witten hypothesis:

SQM stable: $(E/A)_{\text{min}} \lesssim 930$ MeV. The minimum energy per baryon is lower than the binding energy of ^{56}Fe.

$$m_i = m_{i0} + m_I = m_{i0} + \frac{D}{\rho_b^{1/3}} + C\rho_b^{1/3}$$

$m_{u0} = 2.16$ MeV , $m_{d0} = 5.15$ MeV, and $m_{s0} = 90$ MeV



For the pair (C, D) we take $C = 0.81$ and $D^{1/2} = 127$ MeV

Isabella Marzola , Sérgio B. Duarte, César H. Lenzi, Odilon Lourenço
PRD 108, 083006 (2023)

- Fermionic dark matter model.

- Dirac Lagrangian density of a single fermionic component, along with a vector meson coupled to the Dirac spinor:

$$\mathcal{L}_{\rm FDM} = \bar{\chi}\left[\gamma_\mu(i\partial^\mu - g_V V^\mu) - m_\chi\right]\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu$$

- Mean-field approach leads to

$$\mathcal{E}_{\rm FDM} = \frac{1}{\pi^2}\int_0^{k_{F\chi}} dk\, k^2(k^2+m_\chi^2)^{1/2} + \frac{1}{2}C_V^2\rho_\chi^2,$$

$$P_{\rm FDM} = \frac{1}{3\pi^2}\int_0^{k_{F\chi}} dk\,\frac{k^4}{(k^2+m_\chi^2)^{1/2}} + \frac{1}{2}C_V^2\rho_\chi^2,$$

with $C_V = g_V/m_V$ and $\rho_\chi = k_{F\chi}^3/(3\pi^2)$.

$m_\chi = 1.9$ GeV, and $C_V = 3.26$ fm.

- Bosonic dark matter model.

- Lagrangian density:

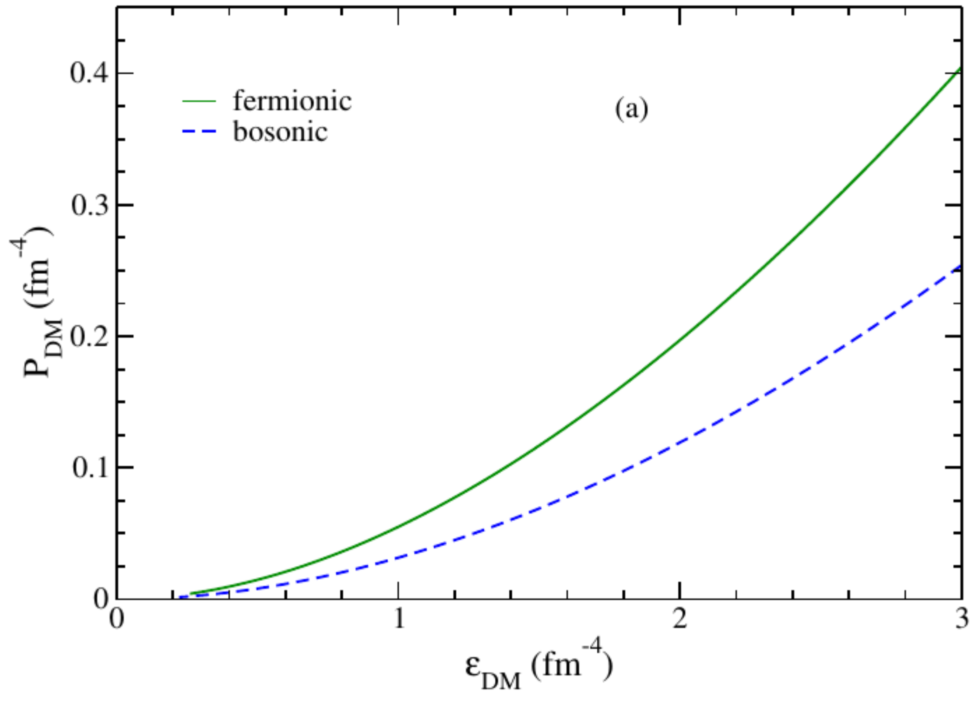
$$\mathcal{L}_{\rm BDM} = -\sqrt{-g}\left(D^*_\mu \sigma^* D^\mu \sigma - m_\sigma^2 \sigma^* \sigma - \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu}\right)$$

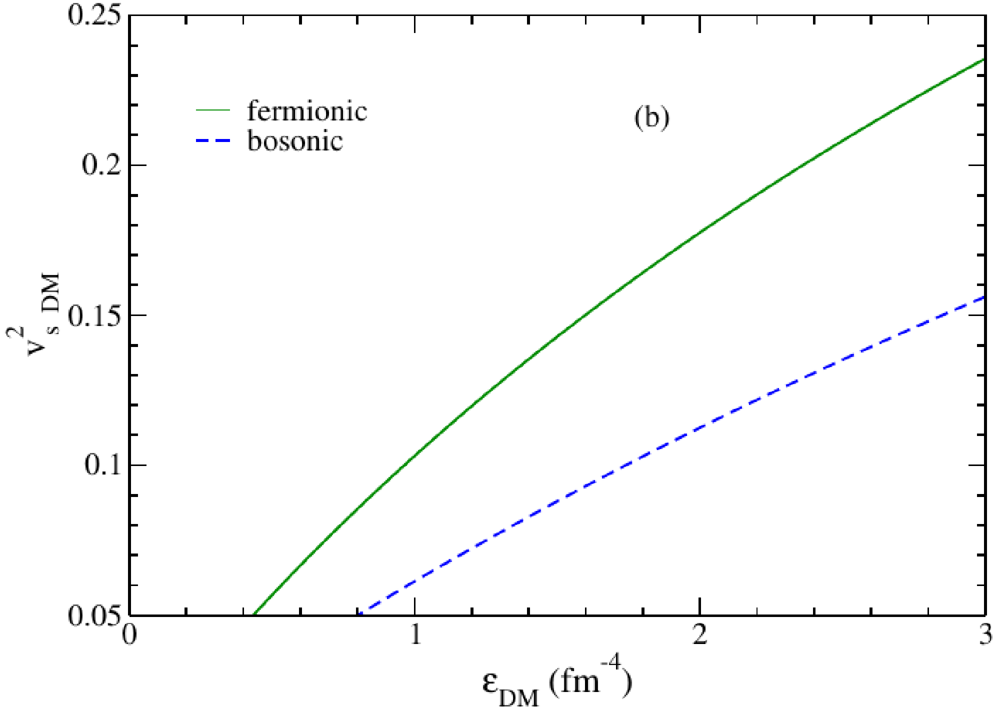
- Mean-field approach:

$$\mathcal{E}_{\rm BDM} = m_\sigma \rho_\sigma + \frac{1}{2} C_{\sigma\phi}^2 \rho_\sigma^2,$$
$$P_{\rm BDM} = \frac{1}{2} C_{\sigma\phi}^2 \rho_\sigma^2,$$

$$C_{\sigma\phi} = g_\sigma / m_\phi \qquad \phi_0 = (g_\sigma / m_\phi^2)\rho_\sigma, \qquad \rho_\sigma = 2 m_\sigma \sigma^* \sigma$$

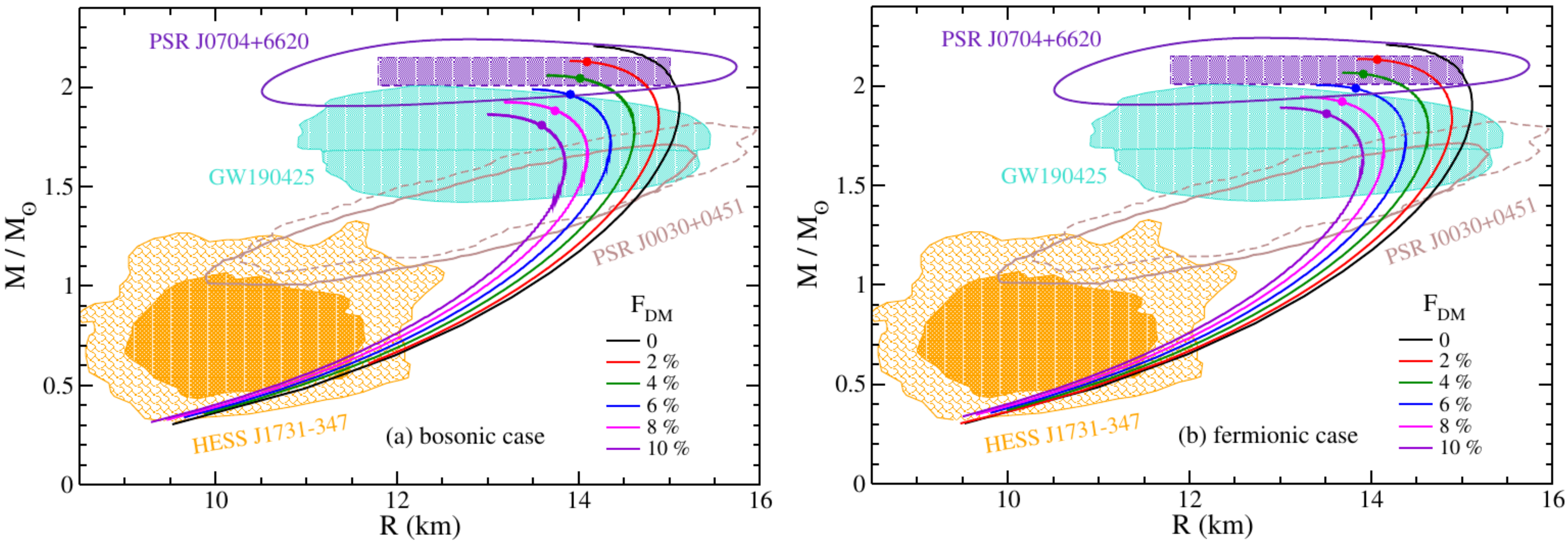
$$m_\sigma = 15 \text{ GeV} \qquad C_{\sigma\phi} = 0.1 \text{ MeV}^{-1}$$





Isabella Marzola, Everson H. Rodrigues, Anderson F. Coelho, Odilon Lourenço
arXiv:2408.16583, accepted for publication in PRD

- Solving the two-fluid TOV equations:



$$F_{\rm DM} = M_{\rm DM}/M$$

$$M_{\rm vis} \equiv m_{\rm vis}(R_{\rm vis}), \text{ and } M_{\rm DM} \equiv m_{\rm DM}(R_{\rm DM}) \qquad M = M_{\rm vis} + M_{\rm DM},$$

$$R = R_{\rm vis} \text{ if } R_{\rm vis} > R_{\rm DM}, \text{ or } R = R_{\rm DM} \text{ if } R_{\rm DM} > R_{\rm vis}$$

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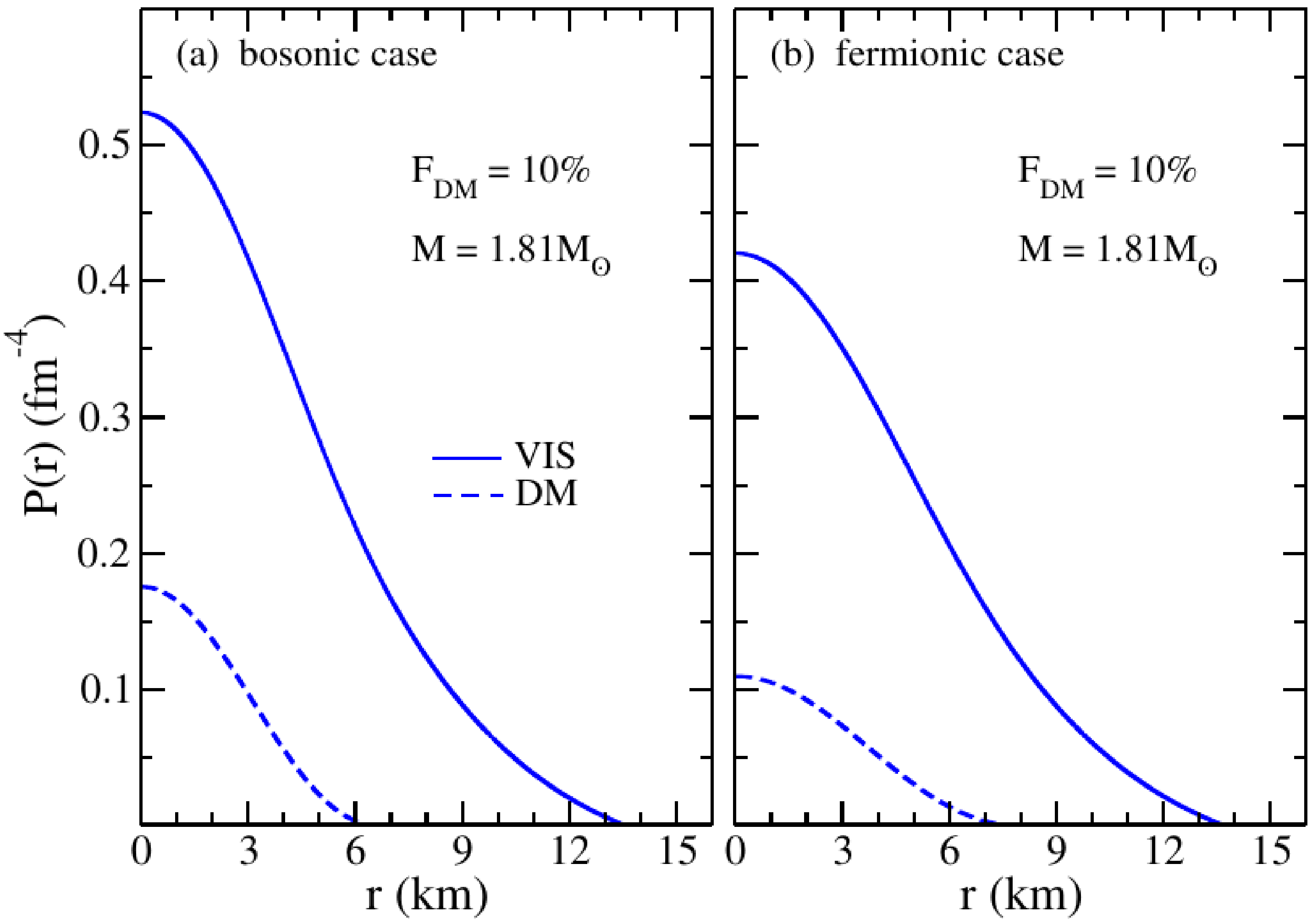
arXiv:2408.16583, accepted for publication in PRD



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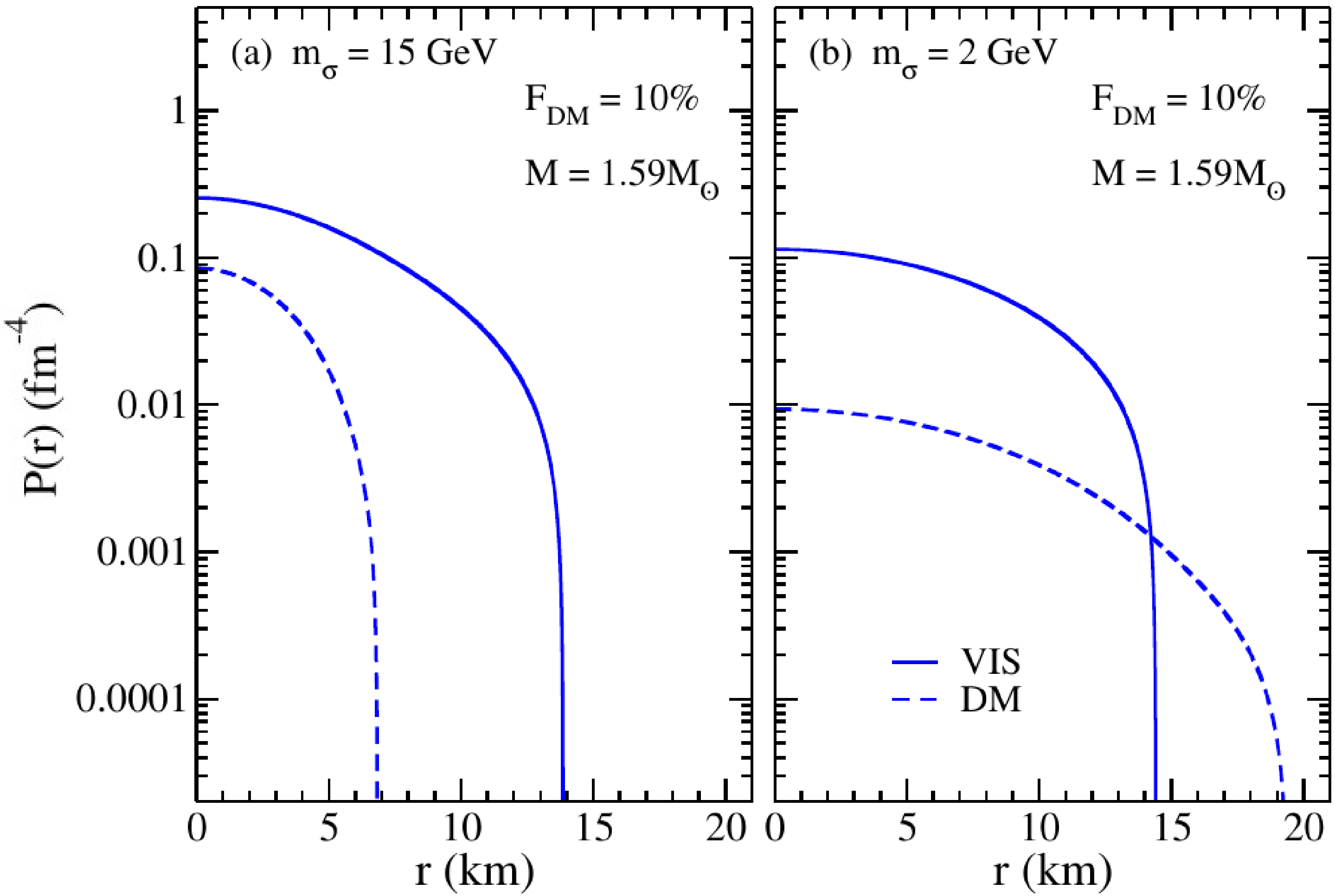
arXiv:2408.16583, accepted for publication in PRD



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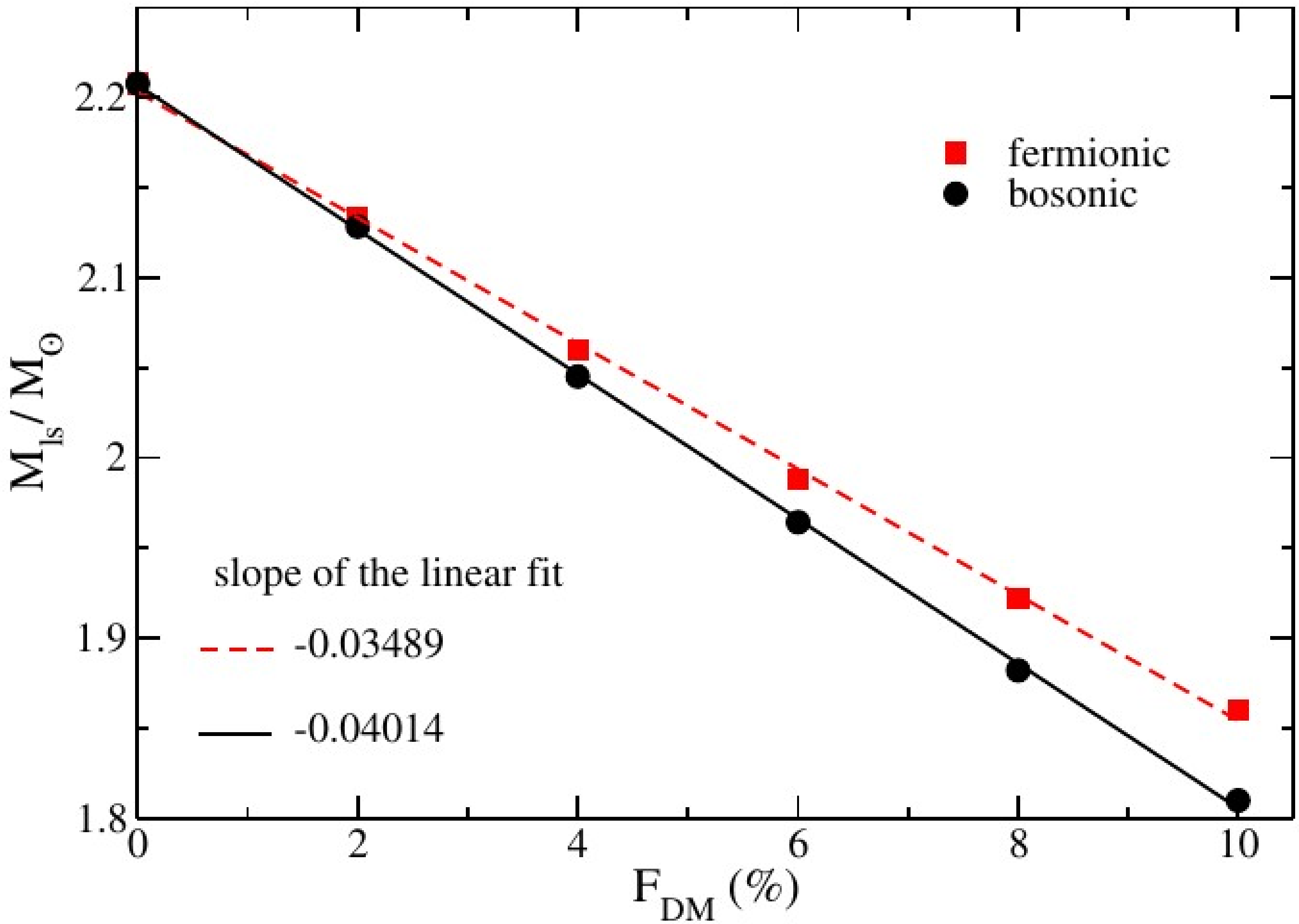
arXiv:2408.16583, accepted for publication in PRD

- Last stable star:



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arXiv:2408.16583, accepted for publication in PRD

- Stability analysis:

$$\frac{dN_{\rm vis}}{dr} = \frac{4\pi r^2 \rho_{\rm vis}}{[1-2m(r)/r]^{1/2}}, \quad \frac{dN_{\rm DM}}{dr} = \frac{4\pi r^2 \rho_{\rm DM}}{[1-2m(r)/r]^{1/2}},$$

$$\begin{pmatrix} \delta N_{\rm vis} \\ \delta N_{\rm DM} \end{pmatrix} = \begin{pmatrix} \partial N_{\rm vis}/\partial \mathcal{E}^c_{\rm vis} & \partial N_{\rm vis}/\partial \mathcal{E}^c_{\rm DM} \\ \partial N_{\rm DM}/\partial \mathcal{E}^c_{\rm vis} & \partial N_{\rm DM}/\partial \mathcal{E}^c_{\rm DM} \end{pmatrix} \begin{pmatrix} \delta \mathcal{E}^c_{\rm vis} \\ \delta \mathcal{E}^c_{\rm DM} \end{pmatrix} = 0$$

The associate eigenvalues, named here as κ_1 and κ_2, are claimed to satisfy the condition

$$\kappa_1 > 0, \qquad \kappa_2 > 0$$

M. Hippert, E. Dillingham, H. Tan, D. Curtin, J. Noronha-Hostler, and N. Yunes, Phys. Rev. D **107**, 115028 (2023).
S. L. Pitz and J. Schaffner-Bielich, Generating ultra-compact neutron stars with bosonic dark matter (2024), arXiv:2408.13157 [astro-ph.HE].
C. Biesdorf, J. Schaffner-Bielich, and L. Tolos, Masquerading hybrid stars with dark matter (2024), arXiv:2412.05207 [hep-ph].

- Strange stars admixed with dark matter constitute a possible scenario consistent with astrophysical data from:

  - Massive millisecond pulsars, such as PSR J0030+0451and PSR J0740+6620,

  - Observations from NASA's Neutron Star Interior Composition Explorer (NICER) x-ray telescope,

  - Compact object HESS J1731-347.

- More data coming from GW events (LIGO-Virgo-Kagra), and programs such as ATHENA, STROBE-X, and the Enhanced X-ray Timing and Polarimetry Mission (eXTP) will play an important role in advancing our understanding of CS and their potential DM content.

Dedicated to Kau Dalfovo Marquez

Thank you very much!