PROTON-DEUTERON INTERACTION AND FEMTOSCOPY UNDER THE TWO-BODY PICTURE

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Porto Alegre, March 13, 2025,

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Collaborators:

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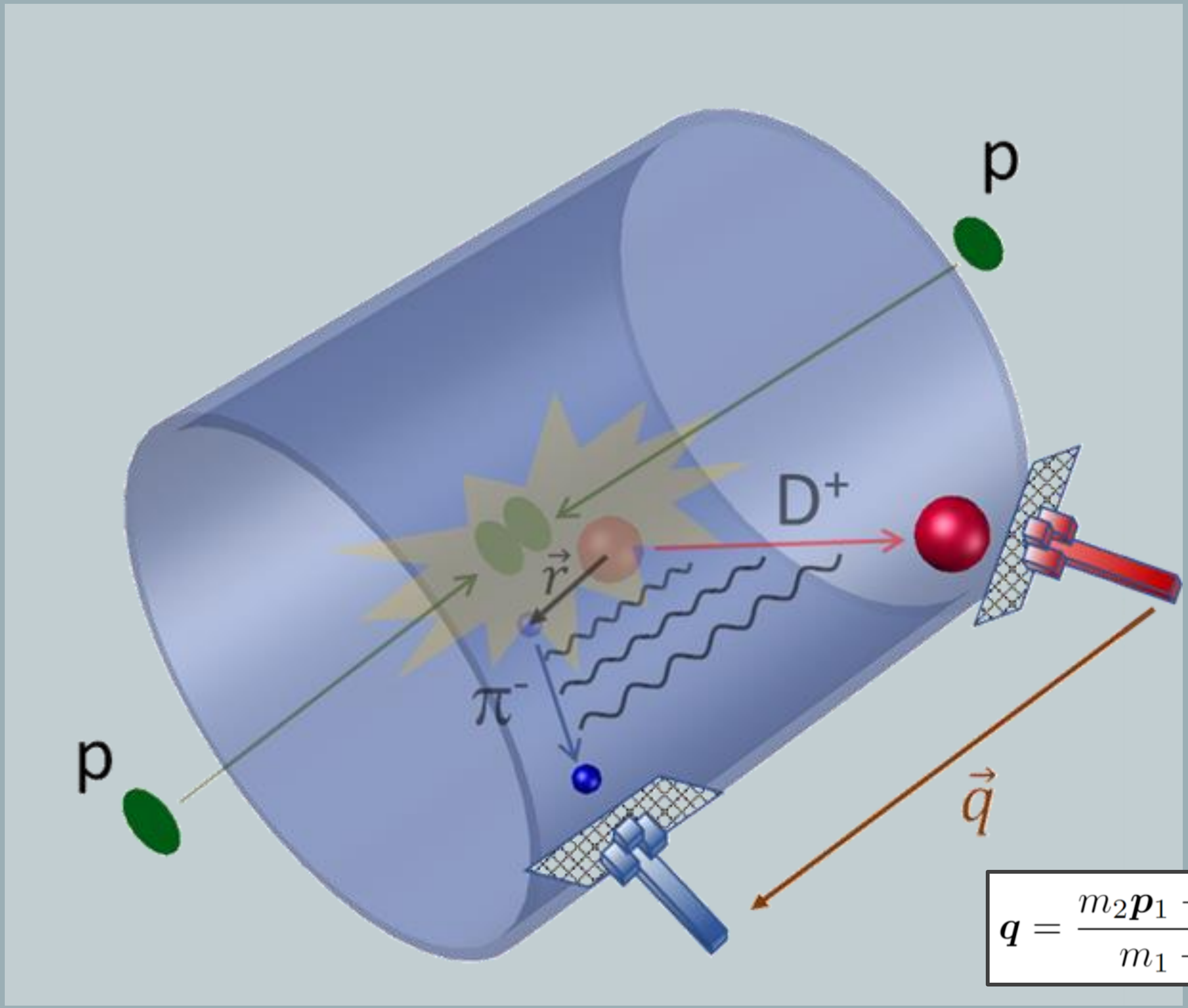
Femtoscopy in RHICs



$$\boldsymbol{q} = \frac{m_2 \boldsymbol{p}_1 - m_1 \boldsymbol{p}_2}{m_1 + m_2}$$

Heinz, Jacak, *Ann.Rev.Nucl.Part.Sci.* 49 (1999) 529-579
Lisa, Pratt, Wiedemann,
Ann.Rev.Nucl.Part.Sci. 55 (2005) 357

Pair Correlation Function

$$C(\boldsymbol{q}) = \mathcal{N} \frac{N_{\text{same}}(\boldsymbol{q})}{N_{\text{mixed}}(\boldsymbol{q})}$$

$$C(\boldsymbol{q}) > 1 : \text{correlation}$$
$$C(\boldsymbol{q}) < 1 : \text{anticorrelation}$$

Femtoscopy in RHICs

Pair Correlation Function

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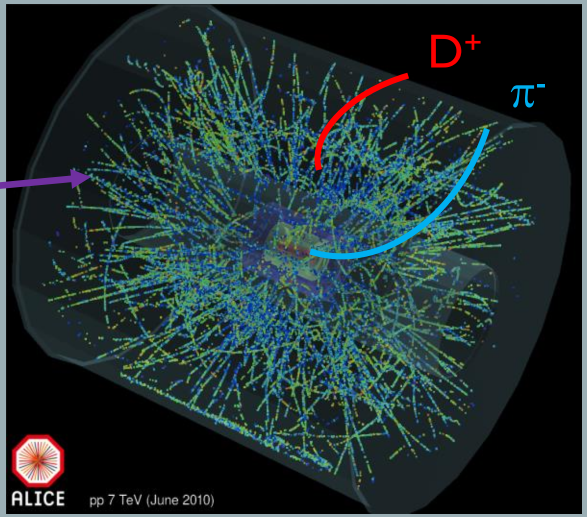
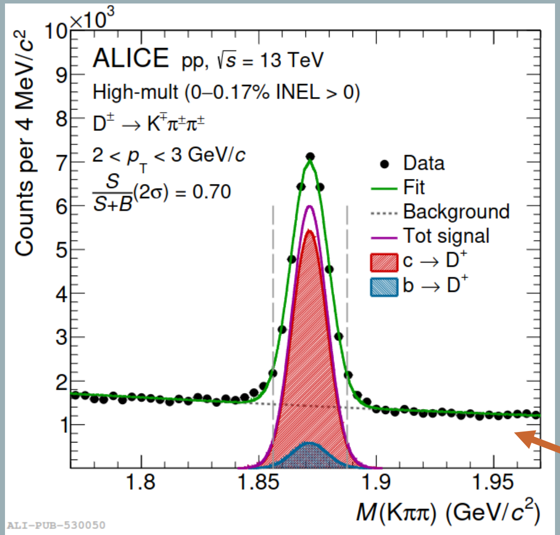
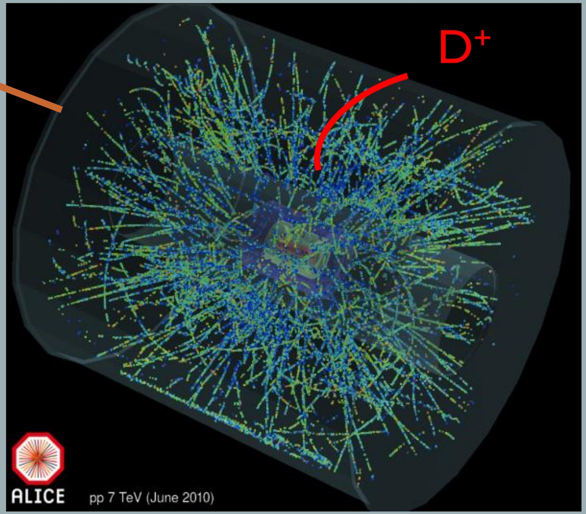
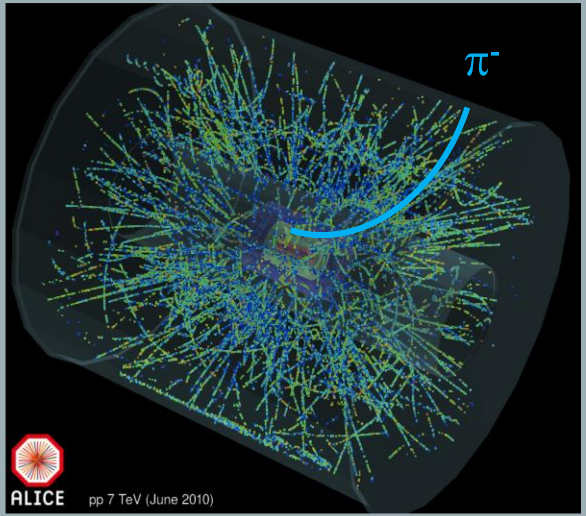




D± reconstruction

'Event mixing' technique





10^6-10^7 events

Koonin-Pratt formula

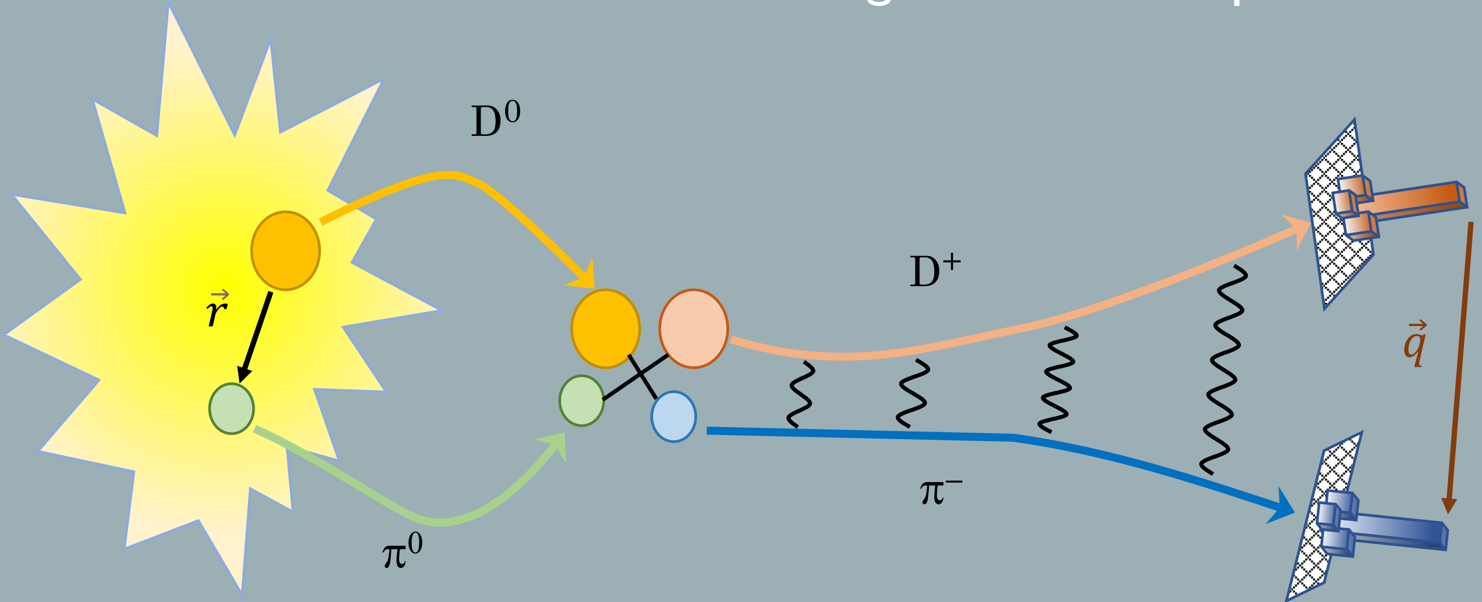
Koonin, *Phys.Lett.B*, 70, 43 (1977)
Pratt, Csorgo, Zimanyi, *Phys.Rev.C*, 42, 2646(1990)

Koonin-Pratt formula

$$C(\boldsymbol{q}) = \int d^3r \sum_i w_i \; S_i(\boldsymbol{r}) \; |\Psi_i(\boldsymbol{q};\boldsymbol{r})|^2$$

Wave function connecting initial channel with observed one

weights related to production mechanism of channels



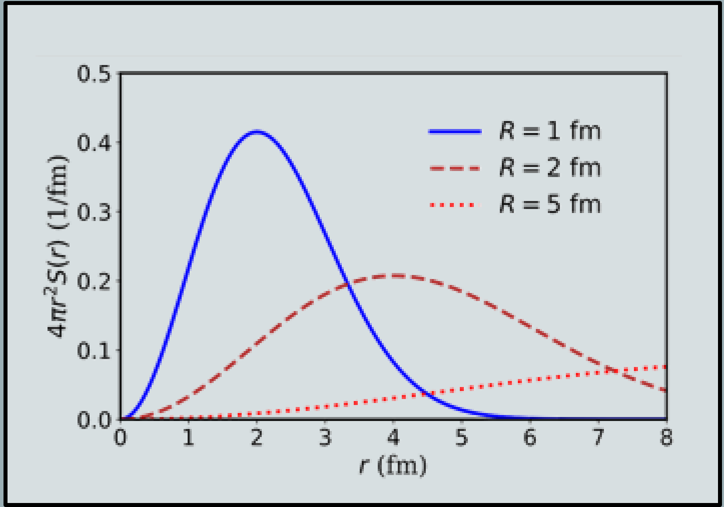
$C(\boldsymbol{q}) > 1$: attraction
$C(\boldsymbol{q}) < 1$: repulsion

Fabbietti, Mantovani Sarti, Vazquez Doce, *Ann. Rev. Nucl. Part. Sci*, 71, 377 (2021)

Correlation function

Gaussian source function

$$S(r) = \frac{1}{(2\sqrt{\pi}R)^3} \exp\left(-\frac{r^2}{4R^2}\right)$$



Lednický-Lyuboshitz approximation

$$\psi(\boldsymbol{r};\boldsymbol{k}) = e^{i\boldsymbol{k}\cdot\boldsymbol{z}} + f(k)\frac{e^{ikr}}{r}$$

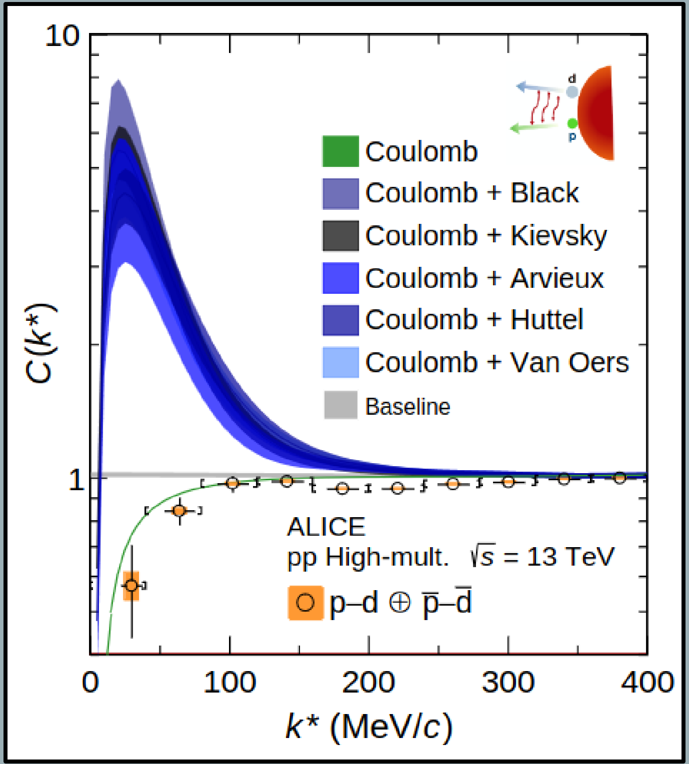
$$f^{-1}(k) = -\frac{1}{a_0} + \frac{1}{2}d_0k^2 - ik + \mathcal{O}(k^4)$$

$$C_{\rm LL}(k) = 1 + \frac{|f(k)|^2}{2R^2} + 2{\rm Re}\, f(k)\frac{F_1(2kR)}{\sqrt{\pi}R} - {\rm Im}\, f(k)\frac{F_2(2kR)}{R}$$

$F_1(x) = x^{-1}\exp(-x^2)\int_0^x \exp(y^2)dy$ and $F_2(x) = (1-\exp(-x^2))/x$

a_0, d_0: s-wave scattering length and effective range

ALICE pd correlation function



ALICE Coll.
Phys. Rev X14, 031051 (2024)

The LL formalism, when applied to pd case, completely fails in the description of the correlation function

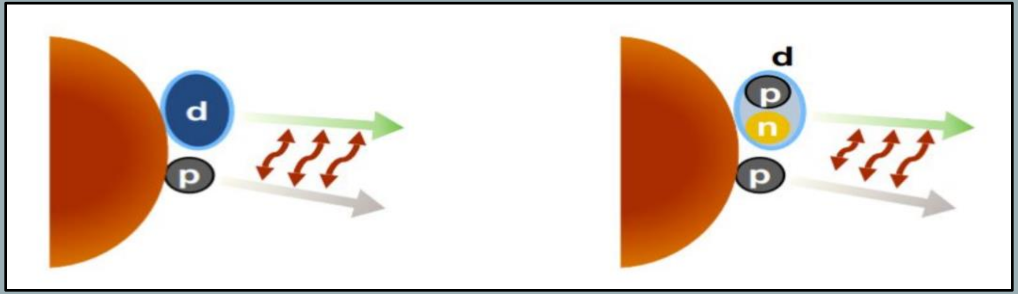
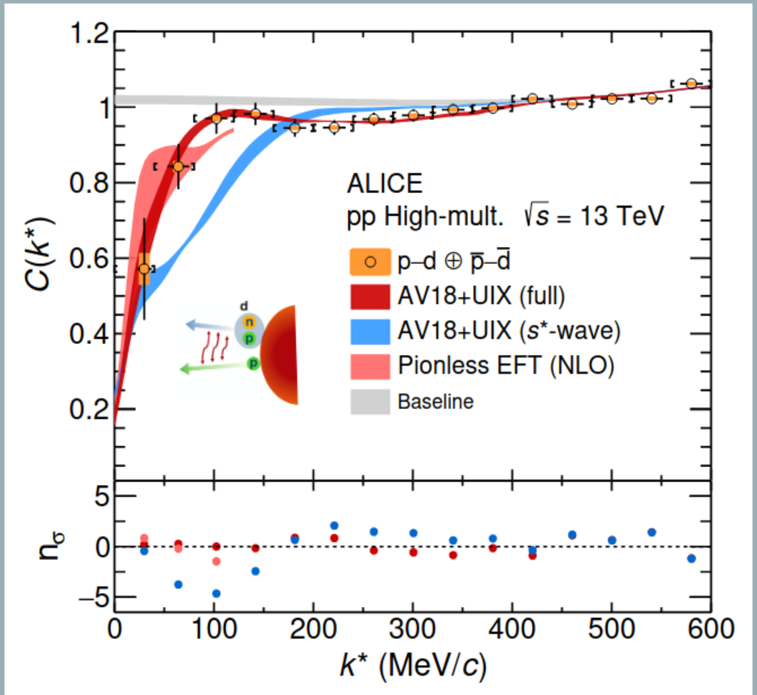


Image Credit:
ALICE coll.

ALICE pd correlation function



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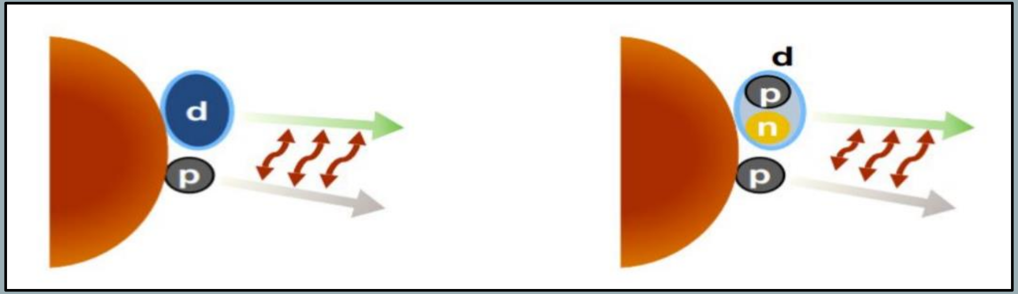
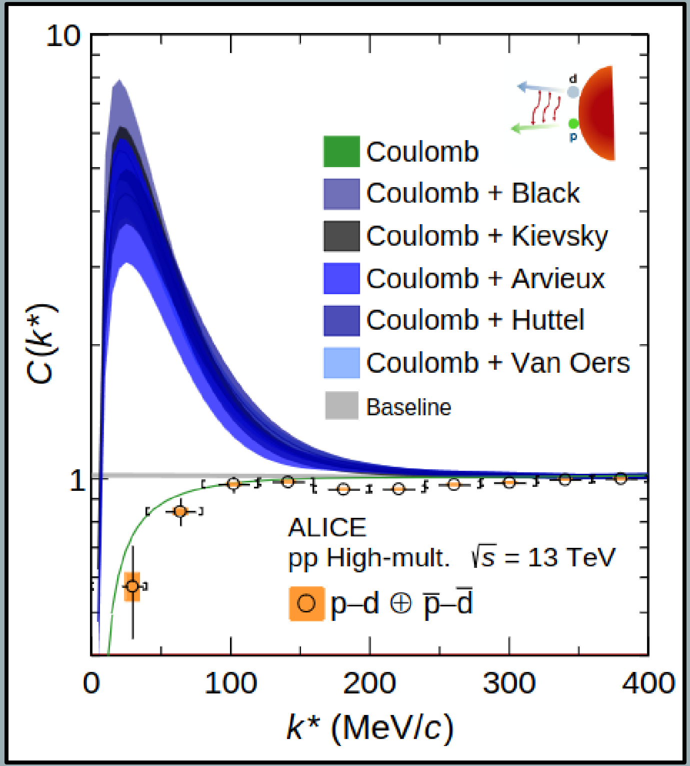


Image Credit:
ALICE coll.

The 3-body approach is the most rigorous description and it works very well to describe the data.
M.Viviani *et al. Phys. Rev. C 108*, 064002 (2023)

ALICE pd correlation function



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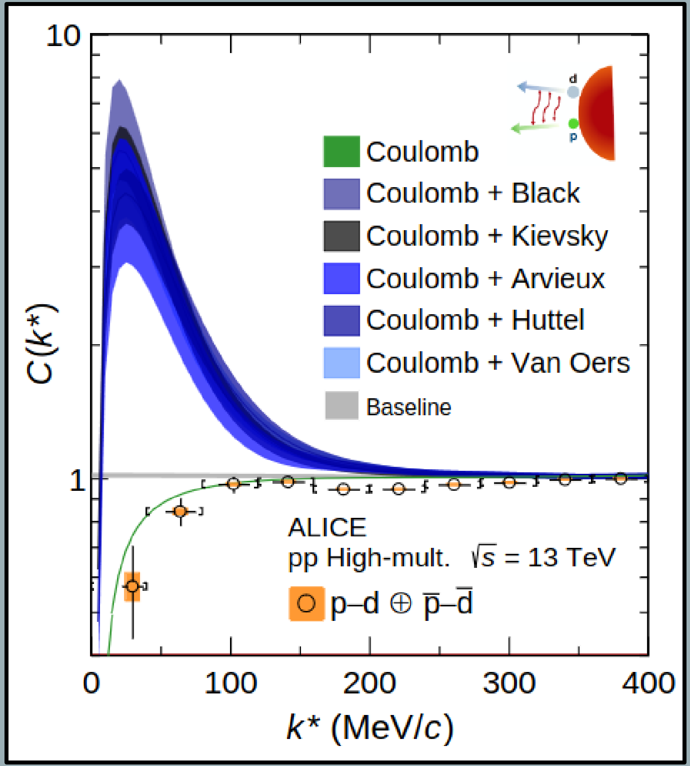
Is there a **shortcoming of the two-body dynamics**, or just a **failure of the LL approximation**?

ALICE pd correlation function



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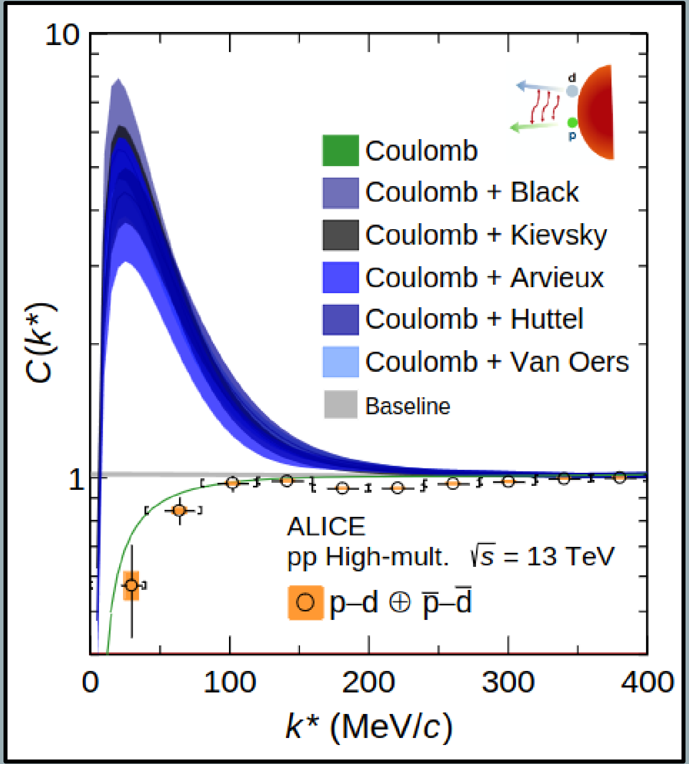

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(2-body) (LL)

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p-d 2-body potentials

Schrödinger equation

$$\left(\frac{d^2}{dr^2} + k^2 - U(r) - \frac{l(l+1)}{r^2}\right) u_l(r,k) = 0$$

Potential: strong plus Coulomb

$$U(r) = 2\mu\left[V_{WS}(r) + \frac{\alpha}{r}\right]$$

$$V_{WS}(r) = -\frac{V_0}{1 + \mathrm{e}^{(r-R)/a}}$$

Woods-Saxon potentials for different S, L channels from B.K. Jennings *et al.* *Phys.Rev.*C33, 1303 (1986)

Asymptotic form:

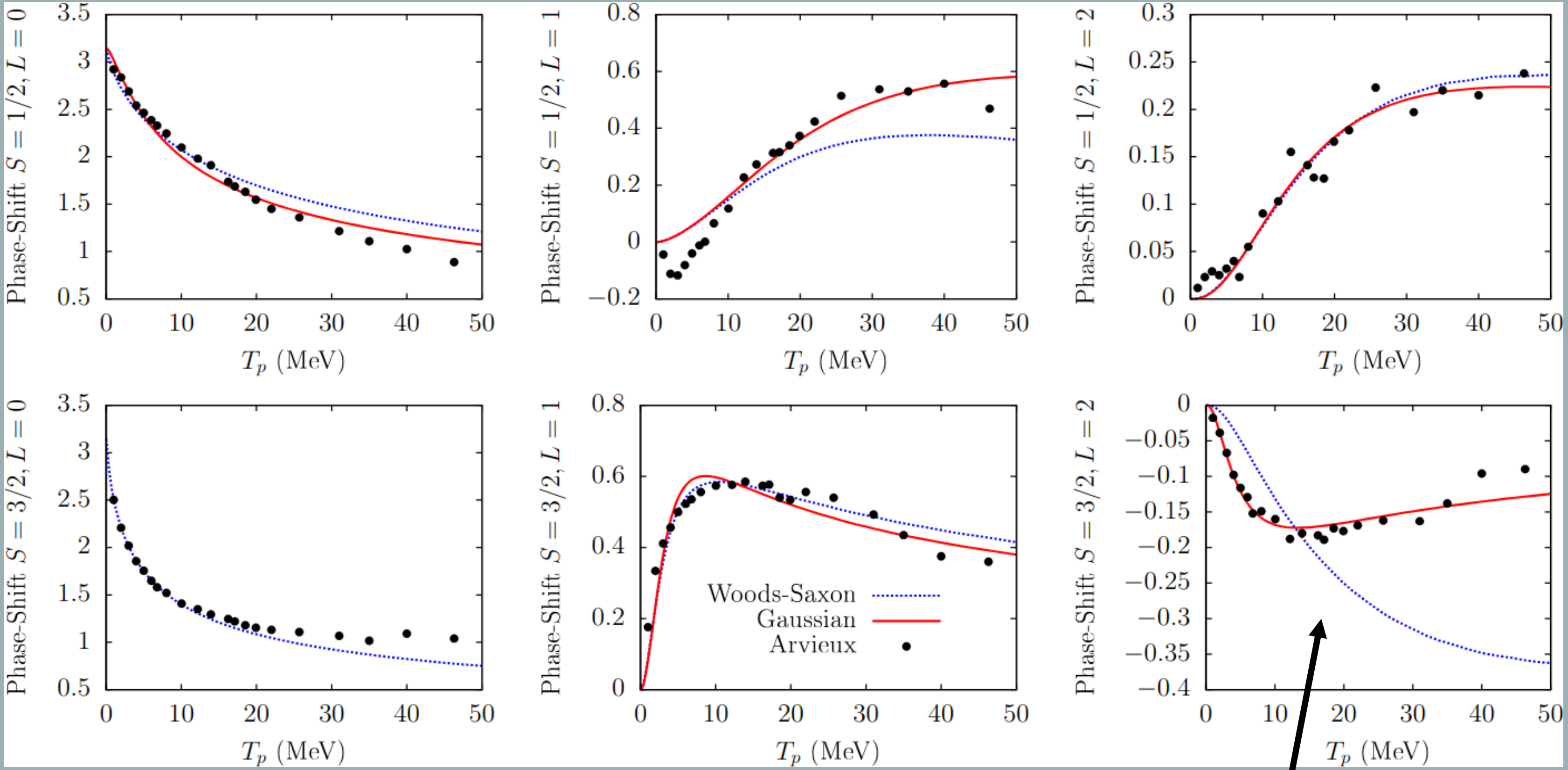
$$u_l^{\mathrm{C+S}}(r,k) = k^{-1} e^{i\Delta_l}\left[\cos\hat{\delta}_l F_l(\gamma;kr) - \sin\hat{\delta}_l G_l(\gamma;kr)\right]$$

$$\gamma = \mu\alpha/k$$

$$\Delta_l = \sigma_l + \hat{\delta}_l$$

$$\sigma_l = \arg\Gamma(1+l+i\gamma)$$

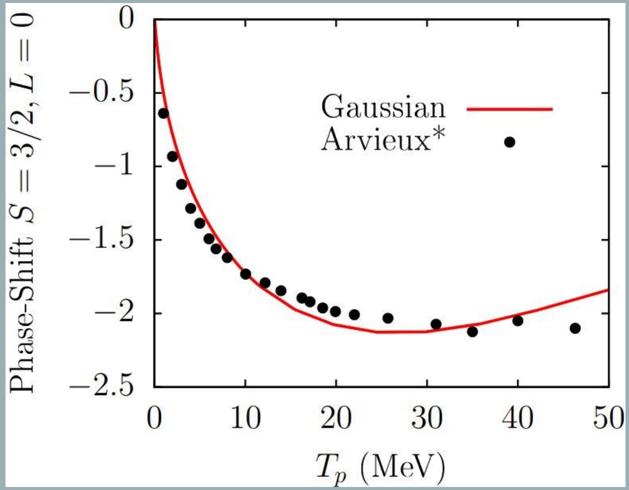
p-d 2-body potentials



Experimental Data from
J.Arvieux, *Nucl.Phys.A221* (1974) 253 -268

B.K. Jennings et al. potential does not describe correctly the S=3/2 L=2 phase shifts

p-d 2-body potentials



We also test Gaussian potential:

1. Study systematics
2. Improve S=3/2, L=2 channel
3. Interpret S=3/2, L=0 as repulsive

$$V_{\text{Gauss}}(r) = \bar{V}_0 \ \exp\left(-\frac{r^2}{r_G^2}\right)$$

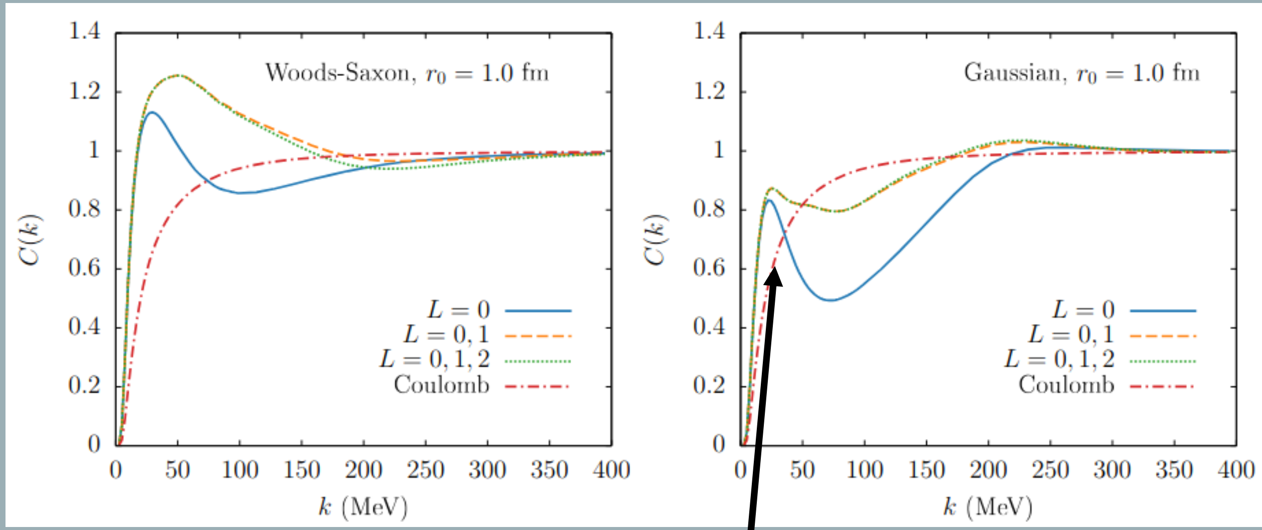
After solving Schrödinger equation for $u_L(r,k)$ we apply the Koonin-Pratt formula:

$$C(k) = \int S(r)\left|\Phi^{\mathrm{C}}(r,z;k)\right|^2 d^3r + \int 4\pi r^2 S(r) \sum_{L=0}^{L_{\max}} (2L+1)\left[\left|\frac{u_L(r,k)}{r}\right|^2 - \left|\Phi_L^{\mathrm{C}}(kr)\right|^2\right] dr$$

$$\Phi_f^C(k;r,z) = e^{-\pi\gamma/2}\Gamma(1+i\gamma)e^{ikz}\,{}_1F_1(-i\gamma;1;ik(r-z))$$

C.J. Joachain, *Quantum Collision Theory* (1975)

p-d 2-body correlation function



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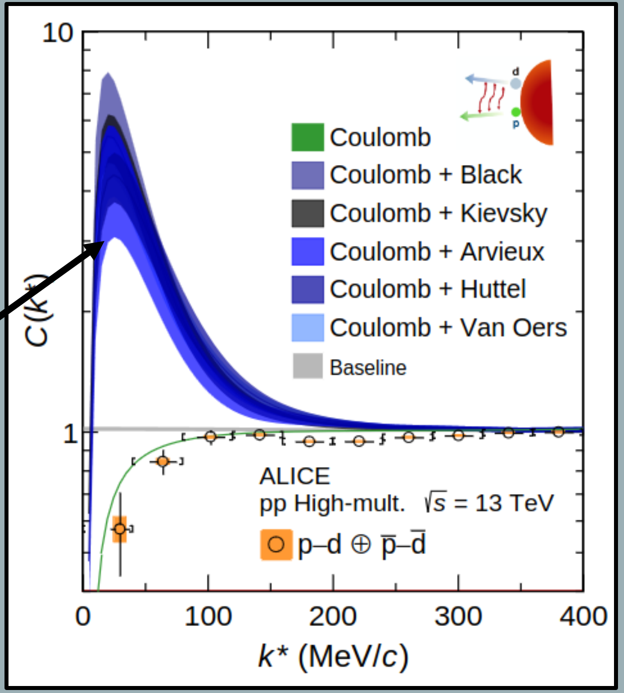
Partial waves component *C(k)*

We see convergence at L=2



We see the peak at low momentum, but no as huge as in LL

p-d 2-body correlation function

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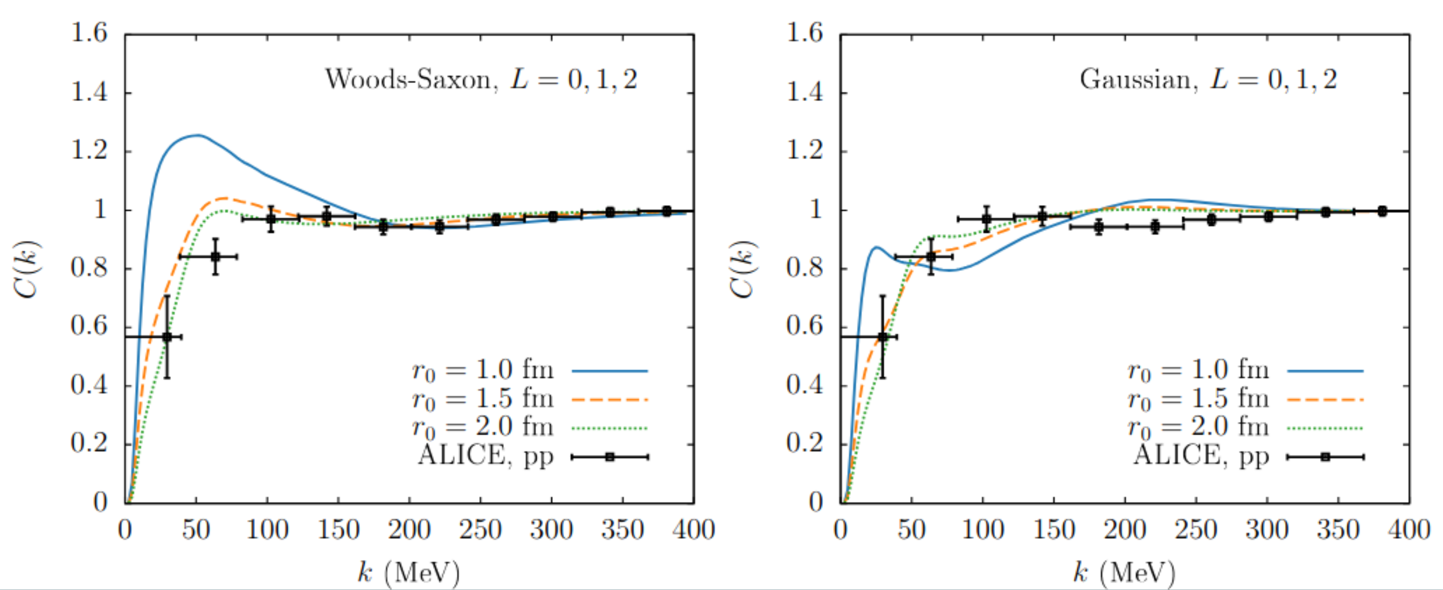


Reasonable agreement with data for R=1.5 fm

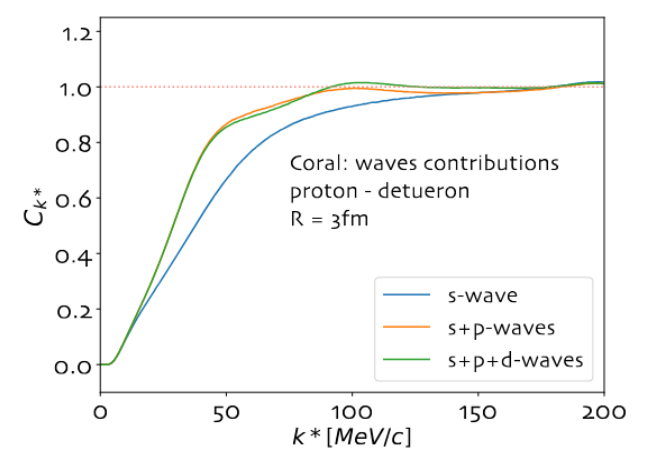
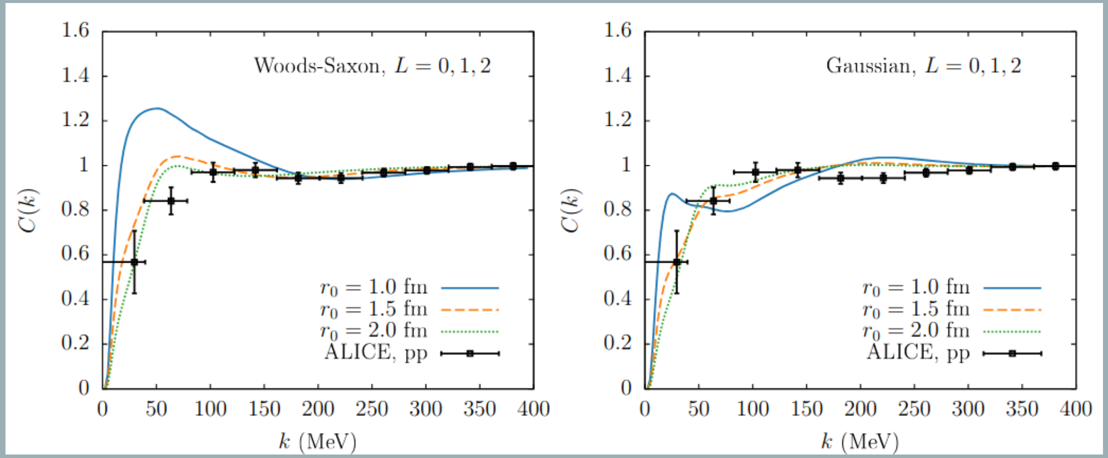


FIG. 3. Correlation functions of proton–deuteron pairs calculated with the CorAL package. The functions take into account Coulomb and strong interactions as well as partial wave contributions: s waves only (blue), s+p waves (orange) and s+p+d waves (green).

In full accordance with
W. Rzesa, M. Stefaniak, S. Pratt
2410.13983 [nucl-th]

p-d 2-body Lednicky formalism

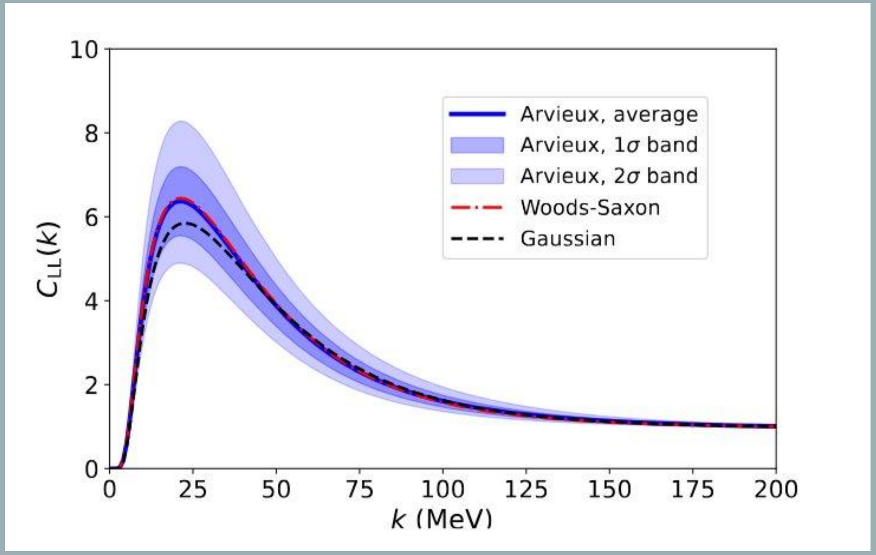


J.Torres-Rincon, A. Ramos, J. Rufi,
2410.23853 [nucl-th]

Extract low-energy parameters, a_0 & d_0

$$f^{-1}(k) = -\frac{1}{a_0} + \frac{1}{2} d_0 k^2 - ik + \mathcal{O}(k^4)$$

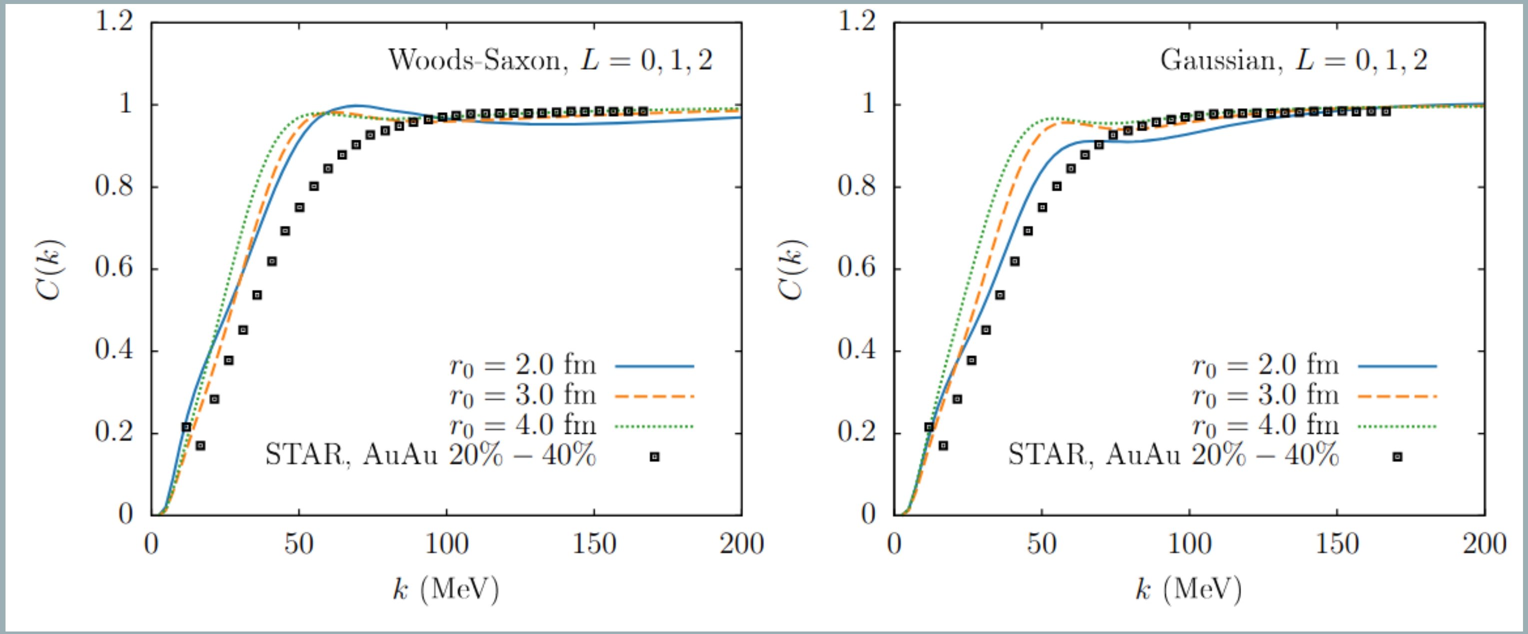
Apply Lednický-Lyuboshitz



We obtain consistent results
with ALICE's LL calculation

p-d 2-body correlation in Au+Au collisions

STAR Coll. 2410.03436 [nucl-ex]



In addition, our results are also compatible with bigger systems measured by STAR collaboration!

Summary

- While 3-body calculation is the most appropriate scheme to deal with the proton-deuteron femtoscopy...
- ...a two-body approach is still a reasonable description and provides satisfactory results when compared to experimental data...
- ...as long as Lednický-Lyuboshitz formalism is not used for this particular system

"*Make the model as simple as possible, but not simpler*"

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LL approximation with Coulomb

$$C_{\mathrm{LL}}(k) = G(\gamma)\left\{1 + \frac{2\mathrm{Re}f_{\mathrm{C}}}{\sqrt{\pi}r_0}F_1(2qr_0) - \mathrm{Im}f_{\mathrm{C}}\frac{G(\gamma)}{r_0}F_2(2qr_0) + \frac{|f_C|^2}{2r_0^2}\left[1 + \frac{1}{2}[G(\gamma)^2 - 1](1 - e^{-4q^2r_0^2})\right]\right\}$$

Lednický-Lyuboshitz approximation

$$f_{\mathrm{C}}^{-1}(k) = -\frac{1}{a_0} + \frac{1}{2}d_0k^2 - 2k\gamma h(\gamma^{-1}) - ikG(\gamma)$$

$$G(\gamma) = \frac{2\pi\gamma}{e^{2\pi\gamma} - 1}$$

$$\gamma = Z_1Z_2\mu\alpha/k$$

$$h(x^{-1}) = -\log(|x|) + \frac{1}{2}\psi(1 - ix) + \frac{1}{2}\psi(1 + ix)$$