The infrared regime of the quark-gluon vertex in general kinematics

Arlene Cristina Aguilar
University of Campinas, São Paulo - Brazil

Based on:
ACA., M. N. Ferreira, B. M. Oliveira, J. Papavassiliou and G. T. Linhares, Eur. Phys. J. C 84, no.11, 1231 (2024)

IFGW Raios Cósmicos e Cronologia

Supported by
CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

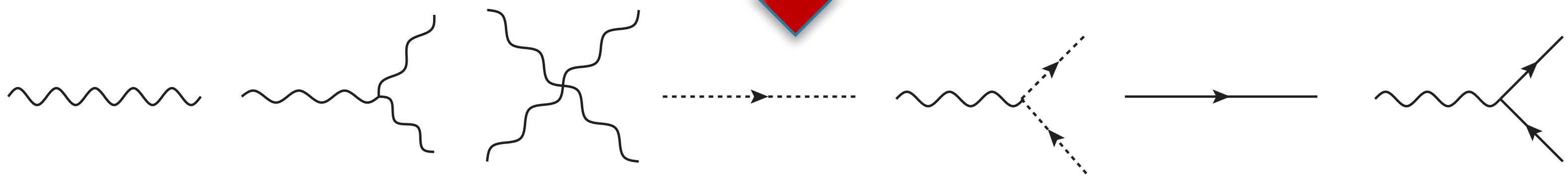
XVI International Workshop on Hadron Physics (Hadrons 2025)
March, 10-14, 2025 – Porto Alegre – Brazil

QCD Lagrangian

From the QCD Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 + \bar{c}^a(-\partial^\mu D^{ac}_\mu)c^c + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

The elementary interactions are encoded in the Lagrangian
QFT → Feynman Diagrams → Perturbative Expansions (in α_s)

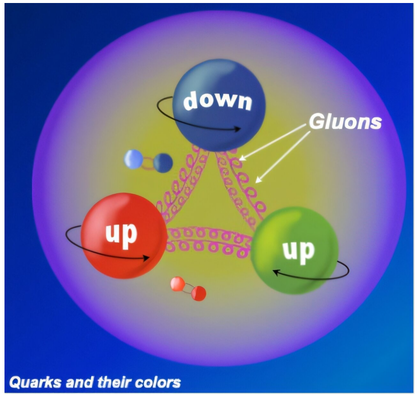


However

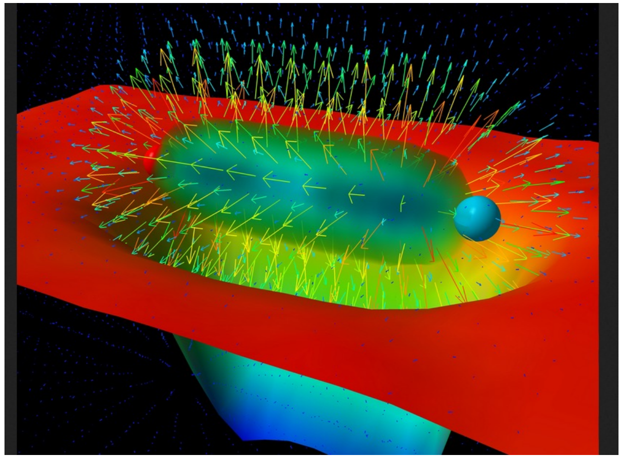
Emergent phenomena in QCD

Cannot be "guessed'' directly from Lagrangian

Emergent phenomena in QCD



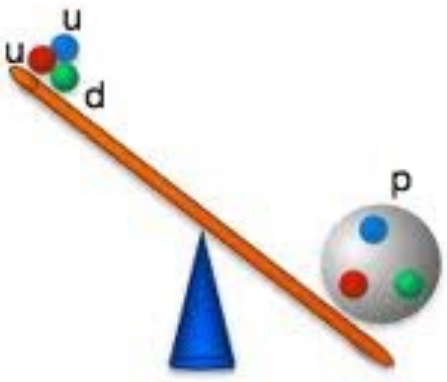
Bound states (Hadrons ⟷ Spectroscopy)



Confinement

Dynamical mass generation

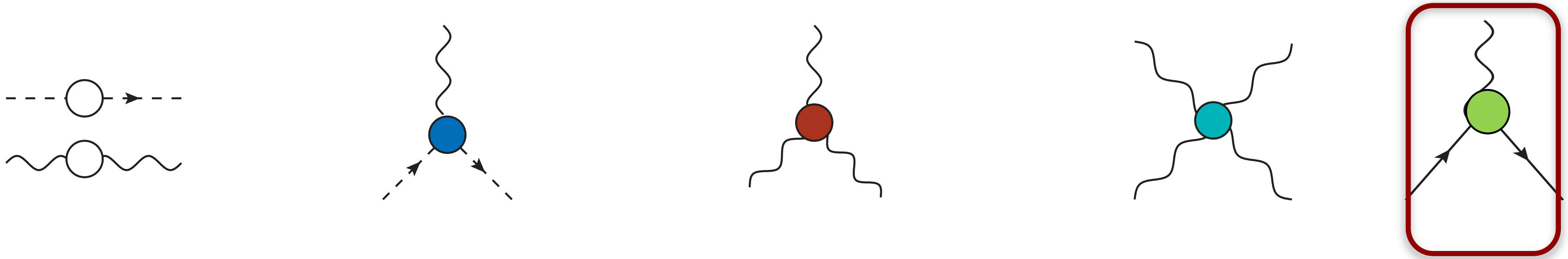
(quark & gluon mass scales)



How to get information about the emergent QCD phenomena?

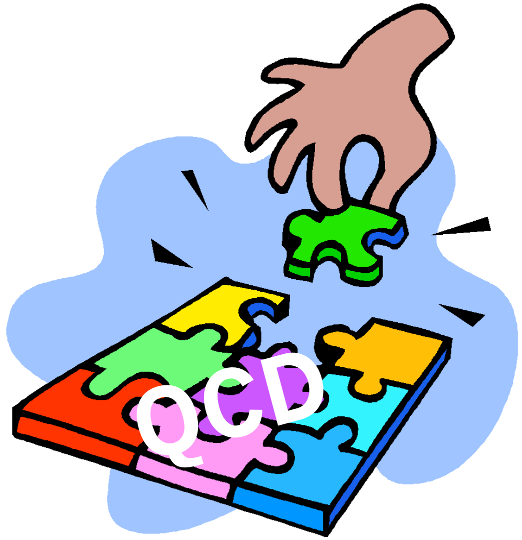
$$\mathcal{L}_{QCD} = -\frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a} + \frac{1}{2\xi}(\partial^{\mu}A^{a}_{\mu})^{2} + \bar{c}^{a}(-\partial^{\mu}D^{ac}_{\mu})c^{c} + \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi$$

They are **encoded** in the **full Green's functions** → determined using **nonperturbative methods**



Nonperturbative QCD Green's functions:
Basic building blocks

Off-shell QCD Green's functions



Green's functions:

Propagators and vertices

Although they are:

- Gauge-dependent
- Renormalization point (μ) and scheme-dependent

However

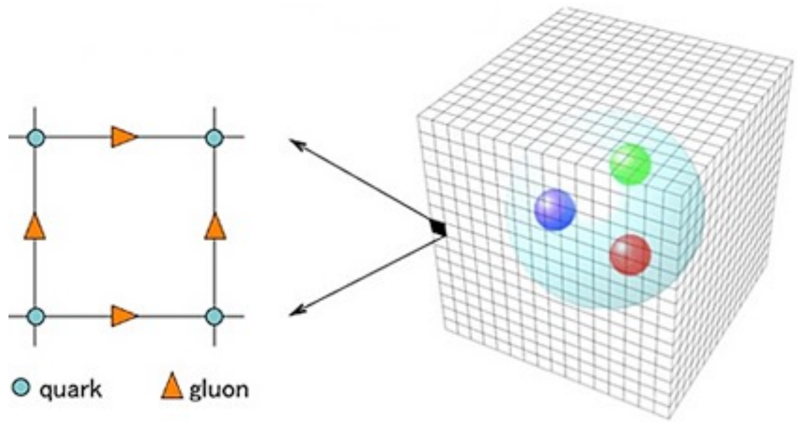
- They capture characteristic features of the underlying dynamics, both perturbative and non-perturbative.
- When appropriately combined they give rise to physical observables.

Crucial pieces for completing the QCD puzzle

Non-pertubative tools

- Non-perturbative physics requires special tools.
- For QCD we have (first principles: employing only fundamental degrees of freedom, i.e., quark, gluon and ghost fields of the QCD lagrangian)

Lattice simulations

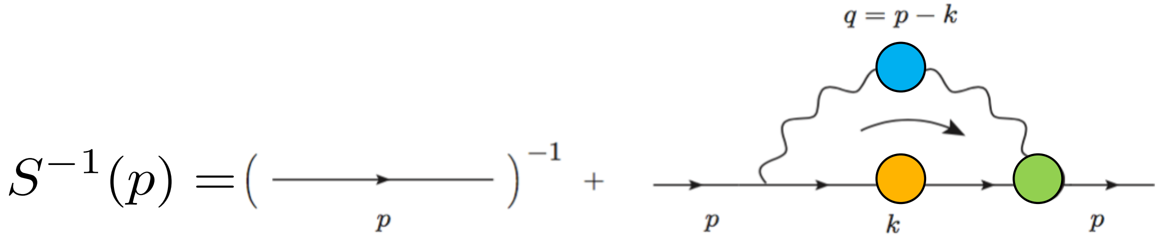


- Space-time is discretized;
- The precision depends on the lattice spacing parameter and volume. (See: Fernanda's lectures & Tereza's talk on friday)

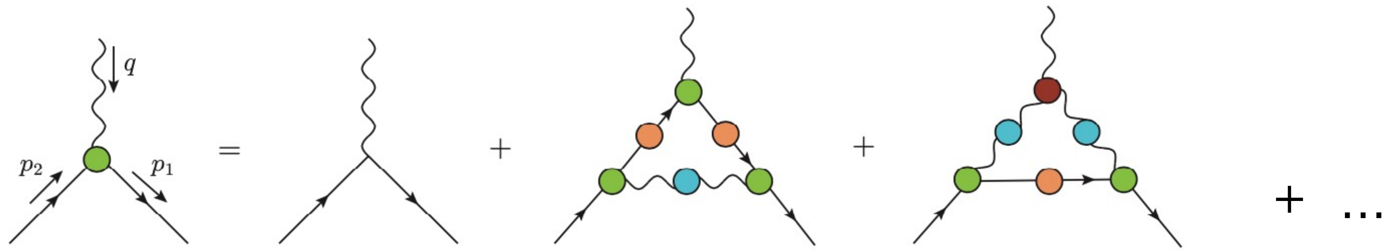
Non-pertubative tools

Schwinger-Dyson equations

- Equations of motion for the Green's functions (propagators/vertices) → Euler-Lagrange equation for a field theory
- Derived formally from the generating functional.

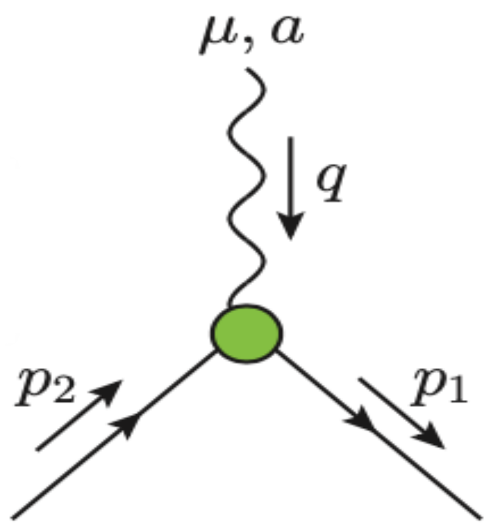


- Infinite system of coupled non-linear integral equations

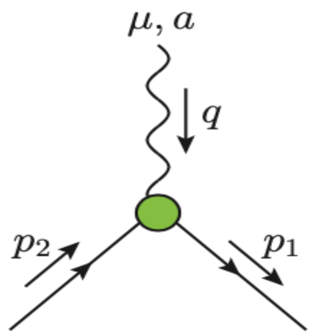


- Inherently non-perturbative, but at the same time captures the perturbative behavior → It accommodates the *full range of physical momenta.*

The full quark-gluon vertex



The quark-gluon vertex



- The full vertex has ***twelve tensorial structures***. It can be decomposed in a ***"longitudinal"*** and a ***transverse*** parts

$$\Gamma_\mu(q,p_2,-p_1) = \Gamma_\mu^{\rm STI}(q,p_2,-p_1) + \Gamma_\mu^{\rm T}(q,p_2,-p_1)\,,$$

- The ***transverse*** part has ***eight tensorial*** structures and it satisfies

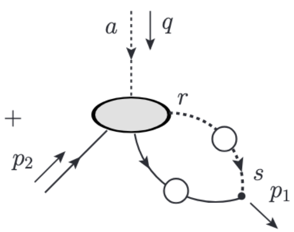
$$q^\mu \Gamma_\mu^{\rm T}(q,p_2,-p_1) = 0\,,$$

- The other ***four saturates the Slavnov-Taylor identity (STI)***

$$q^\mu \Gamma_\mu^{\rm STI}(q,p_2,-p_1) = F(q)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)S^{-1}(p_2)]\,.$$

where S(p) → quark propagator $S^{-1}(p) = A(p^2)\not{p} - B(p^2)\,,$

H(q,p_2,-p_1) → quark-ghost scattering kernel $H^a(q,p_2,-p_1) = -gt^a +$



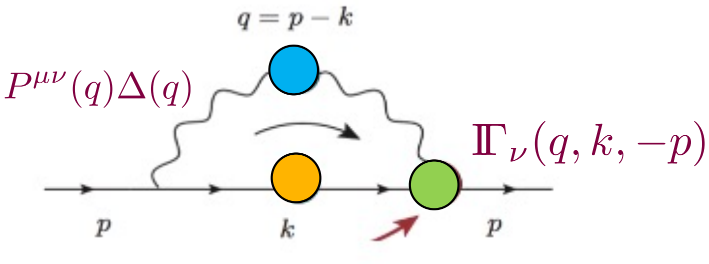
Transversely-projected vertex

- In Landau gauge, the full ***gluon propagator is transverse.***

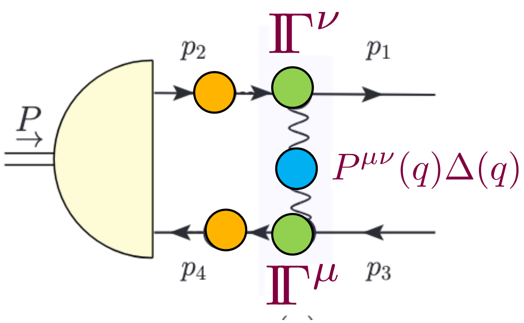
$$\Delta_{\mu\nu}(q) = \Delta(q^2) P_{\mu\nu}(q)\,,$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu / q^2\,.$$

- For ***most phenomenological applications***, the quark-gluon vertex appears ***contract with the transverse projector*** of the gluon propagator.



Gap equation



Bethe-Salpeter

$$\overline{\mathrm{I\Gamma}}_\mu(q,p_2,-p_1) := P_{\mu\nu}(q)\mathrm{I\Gamma}^\nu(q,p_2,-p_1)\,,$$

Transversely projected quark-gluon vertex

The tranverse projectly quark-gluon basis

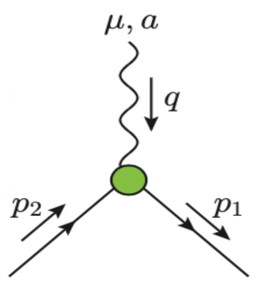
- Decomposing the vertex in a basis in terms of the ***eight form factors:***

$$\overline{\mathrm{I\Gamma}}_\mu(q,p_2,-p_1) = \sum_{i=1}^{8} \lambda_i(q,p_2,-p_1) P_{\mu\nu}(q)\tau_i^\nu(p_2,-p_1)\,,$$

Determine these form factors

- where the tensorial elements of the basis given by

$$\tau_1^\nu = \gamma^\nu \qquad \tau_2^\nu = (p_1+p_2)^\nu$$
$$\tau_3^\nu = (\not{p}_1+\not{p}_2)\gamma^\nu \qquad \tau_4^\nu = (\not{p}_1-\not{p}_2)\gamma^\nu$$
$$\tau_5^\nu = (\not{p}_1-\not{p}_2)(p_1+p_2)^\nu \qquad \tau_6^\nu = (\not{p}_1+\not{p}_2)(p_1+p_2)^\nu$$
$$\tau_7^\nu = -\frac{1}{2}[\not{p}_1,\not{p}_2]\gamma^\nu \qquad \tau_8^\nu = -\frac{1}{2}[\not{p}_1,\not{p}_2](p_1+p_2)^\nu$$



- Separate in two subsets: ***chirally symmetric (cs)*** and ***chirally symmetric breaking (csb)*** – tensors with na odd (even) number of γ matrices

$$\tau_{cs} = \{\tau_1^\nu, \tau_5^\nu, \tau_6^\nu, \tau_7^\nu\} \qquad \tau_{csb} = \{\tau_2^\nu, \tau_3^\nu, \tau_4^\nu, \tau_8^\nu\}$$

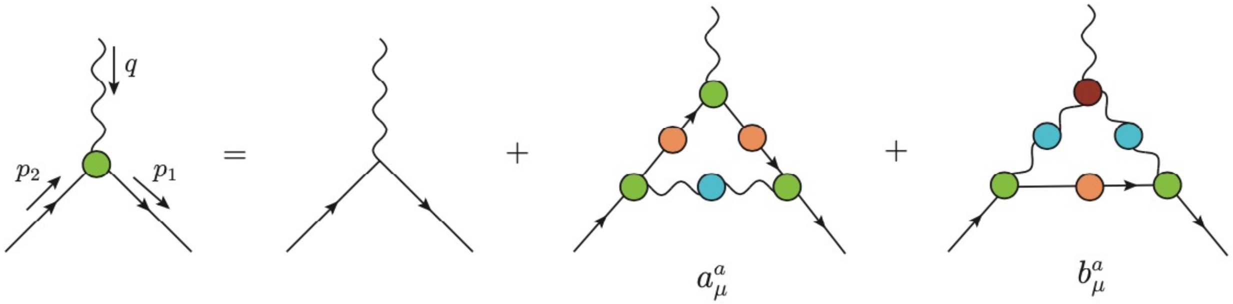
M. Mitter, J. M. Pawlowski, and N. Strodthoff, Phys. Rev. D91, 054035 (2015)
F. Gao, J. Papavassiliou, and J. M. Pawlowski, Phys. Rev. D 103, 094013 (2021)
A. K. Cyrol, M. Mitter, J. M. Pawlowski, and N. Strodthoff, Phys. Rev. D97, 054006 (2018)

The SDE for the quark-gluon vertex

- We are going to determine the form factors of the transversely projected quark-gluon vertex in general kinematics, ***with two degenerate light dynamical quarks*** ($N_f = 2$):

$$\overline{\Gamma}_\mu(q, p_2, -p_1) = \sum_{i=1}^{8} \lambda_i(q, p_2, -p_1) P_{\mu\nu}(q) \tau_i^\nu(p_2, -p_1) \, ,$$

- Use the SDE - 3PI effective action, at the three-loop level



J. Berges, Phys. Rev. D 70, 105010 (2004).
M. E. Carrington and Y. Guo, Phys. Rev. D 83, 016006 (2011).
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D93, 034026 (2016)

- Charge conjugation symmetry preserved

$$C\overline{\Gamma}_\mu(q, p_2, -p_1)C^{-1} = -\overline{\Gamma}_\mu^T(q, -p_1, p_2)$$

Charge conjugation constraints

$$C\overline{\Gamma}_\mu(q,p_2,-p_1)C^{-1} = -\overline{\Gamma}^T_\mu(q,-p_1,p_2)$$

The charge conjugation imposes the following constrains on the form factors:

- ✓ *Symmetric* under the exchange of $p_1 \to -p_2$

$$\lambda_i(q,p_2,-p_1) = \lambda_i(q,-p_1,p_2)\,, \quad i = 1,4,6,7,8\,,$$

- ✓ *Antisymmetric* under the exchange of $p_1 \to -p_2$

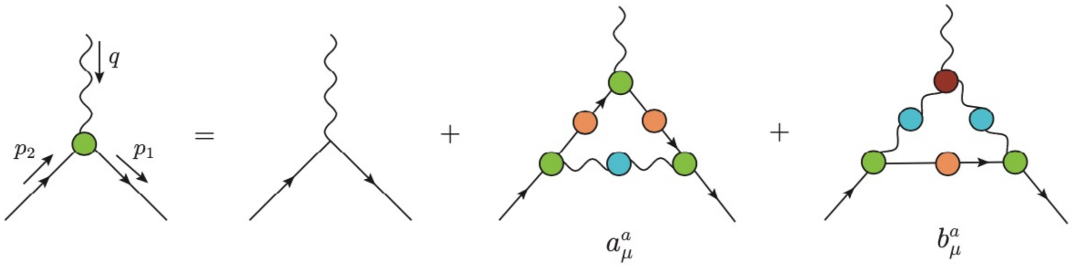
$$\lambda_3(q,p_2,-p_1) = -\lambda_3(q,-p_1,p_2)$$

- ✓ *Mixture* under the exchange of $p_1 \to -p_2$

$$\lambda_2(q,p_2,-p_1) + 2\lambda_3(q,p_2,-p_1) = \lambda_2(q,-p_1,p_2)\,,$$
$$\lambda_5(q,p_2,-p_1) - \lambda_7(q,p_2,-p_1) = -\lambda_5(q,-p_1,p_2)\,.$$

Approximations for SDE

- We solve the SDE under the following simplifications:



- Quark-gluon vertex

$$\overline{\Gamma}_\mu(q, p_2, -p_1) \to \lambda_1(q, p_2, -p_1) P_{\mu\nu}(q) \gamma^\nu \,.$$

- 8 equations decouple – one integral equation for λ_1, and conventional integral depending on λ_1 for the others seven form factors λ_i

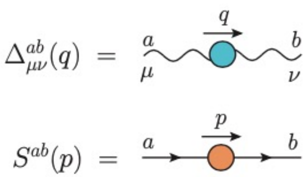
- Solve in Euclidean space

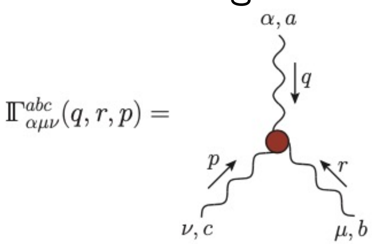
$$\lambda_i(q, p_2, -p_1) \to \lambda_i(p_1^2, p_2^2, \theta)$$

$$\lambda_i(p_1^2, p_2^2, \theta) = \delta_{i1} + \underbrace{\int_E \mathcal{K}_{i\mathbb{A}} \lambda_1^3}_{\text{Abelian Diagram}} + \underbrace{\int_E \mathcal{K}_{i\mathbb{B}} \lambda_1^2}_{\text{Non-Abelian Diagram}}$$

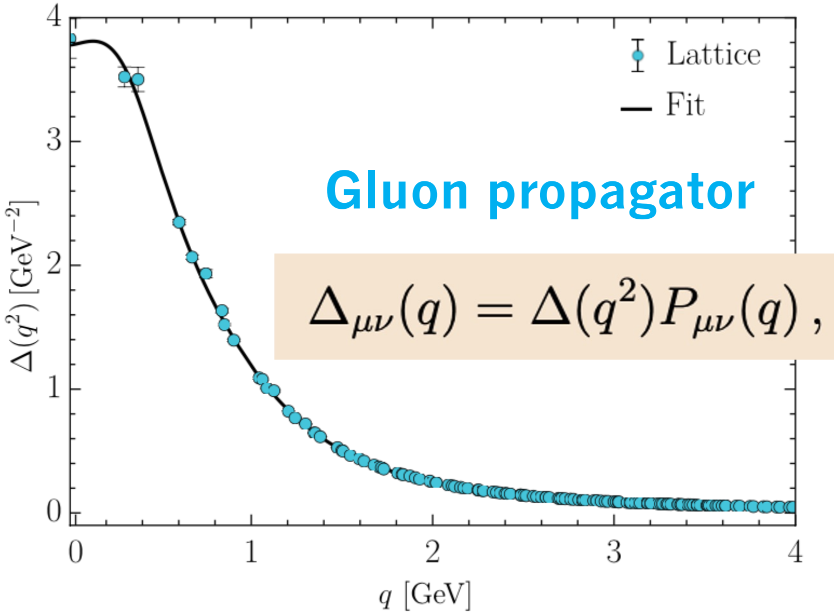
- External inputs:

From unquenched lattice

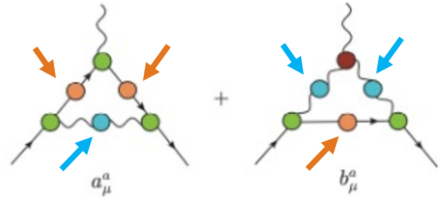


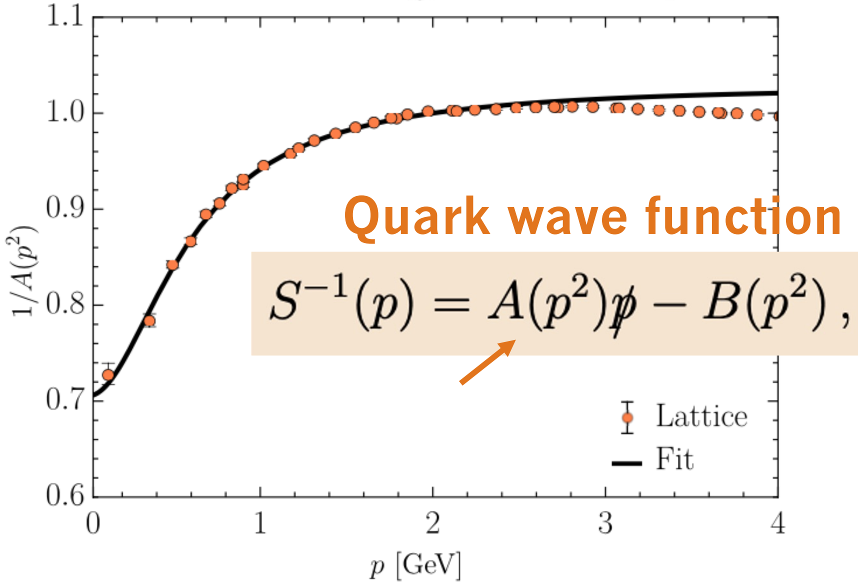


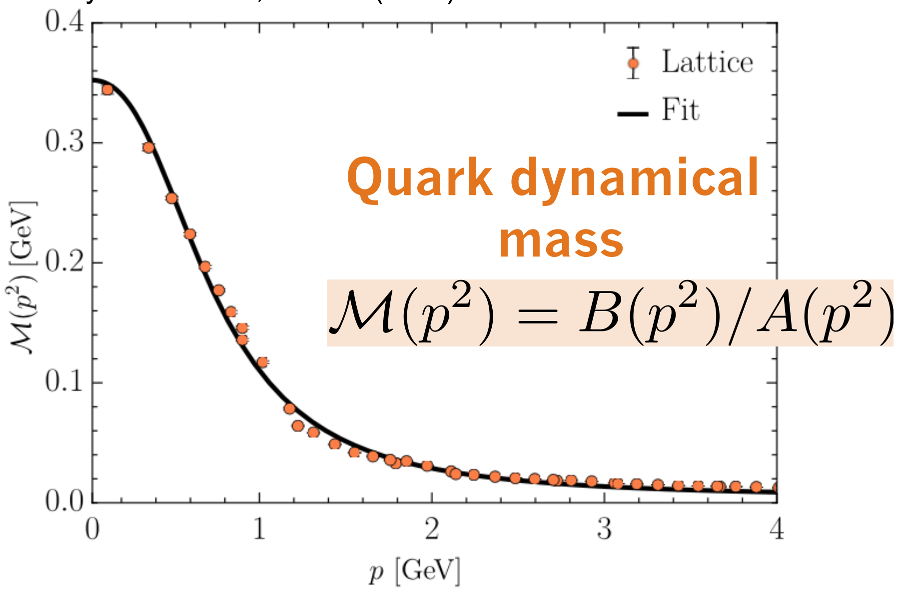
External inputs



Gluon→ Lattice data from:
A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti, and J. Rodriguez-Quintero, Phys. Rev. D86, 074512 (2012)
D. Binosi, C. D. Roberts, and J. Rodriguez-Quintero, Phys. Rev. D 95, 114009 (2017)



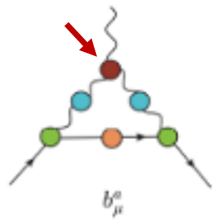




Quark → Lattice data from:
A. Kızılersü, O. Oliveira, P. J. Silva, J. I. Skullerud, and A. Sternbeck, Phys. Rev. D 103, 114515 (2021)
O. Oliveira, P. J. Silva, J.-I. Skullerud, and A. Sternbeck, Phys. Rev. D 99, 094506 (2019).

Setup "L08"
m_q= 6.2 MeV, and m_π = 280 MeV

External inputs



- The three-gluon vertex appears transversely-projected

$$\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) = P_{\alpha}^{\alpha'}(q)P_{\mu}^{\mu'}(r)P_{\nu}^{\nu'}(p)\Gamma_{\alpha'\mu'\nu'}(q,r,p)\,.$$

- Planar degeneracy approximation:

$$\overline{\Gamma}^{\mu\alpha\beta}(q,r,p) = L_{sg}(s^2)\overline{\Gamma}_0^{\mu\alpha\beta}(q,r,p)$$

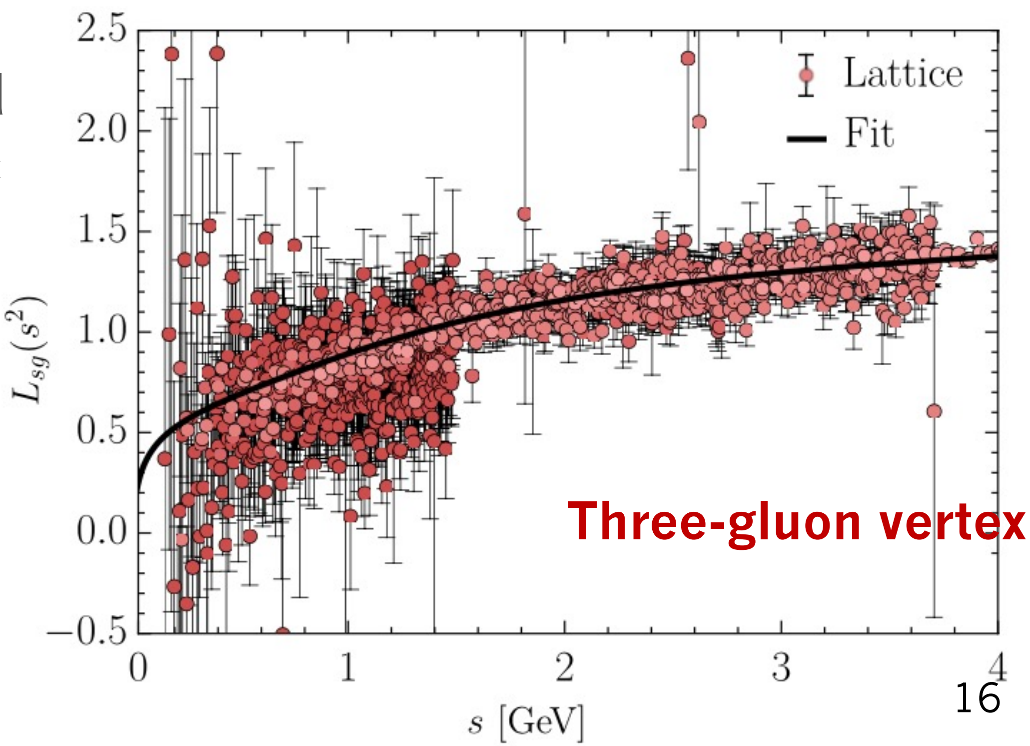
G. Eichmann, R. Williams, R. Alkofer, and M. Vujinovic, Phys. Rev. D89, 105014 (2014);
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D93, 034026 (2016);
F. Pinto-Gómez, F. De Soto, M. N. Ferreira, J. Papavassiliou, and J. Rodríguez-Quintero, Phys. Lett. B 838, 137737 (2023);
ACA, M.N.Ferreira, J.Papavassiliou and L.R.Santos, Eur. Phys. J. C 83, no.6, 549 (2023).

- The three-level tensor is dominant and can be expressed in terms of one Bose symmetric variable:

$$s^2 = \frac{1}{2}(q^2 + r^2 + p^2)$$

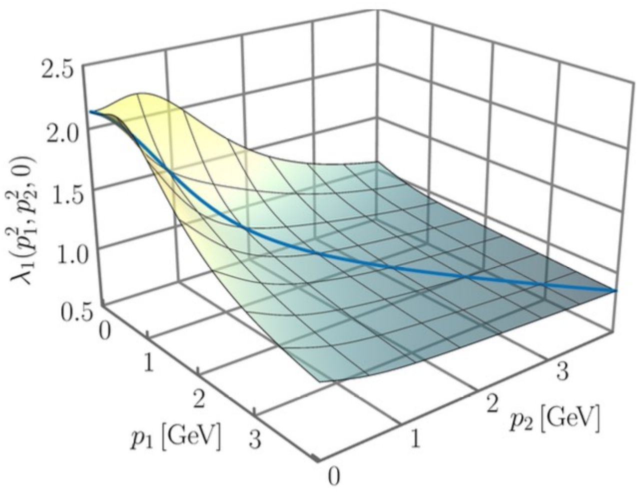
- Lattice data for N_f=2+1

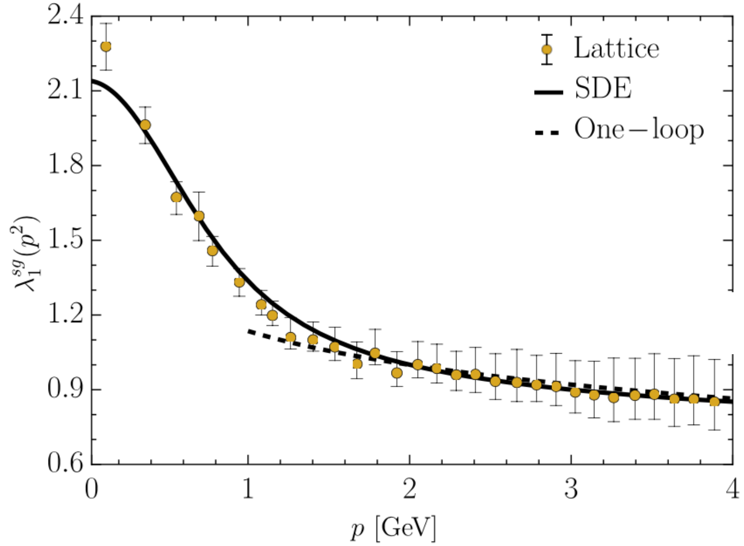
ACA, F. De Soto, M. N. Ferreira, J. Papavassiliou, J. Rodrıíguez-Quintero, and S. Zafeiropoulos, Eur. Phys. J. C80, 154 (2020)



Numerical Results for $\lambda_1(p_1^2, p_2^2, \theta)$

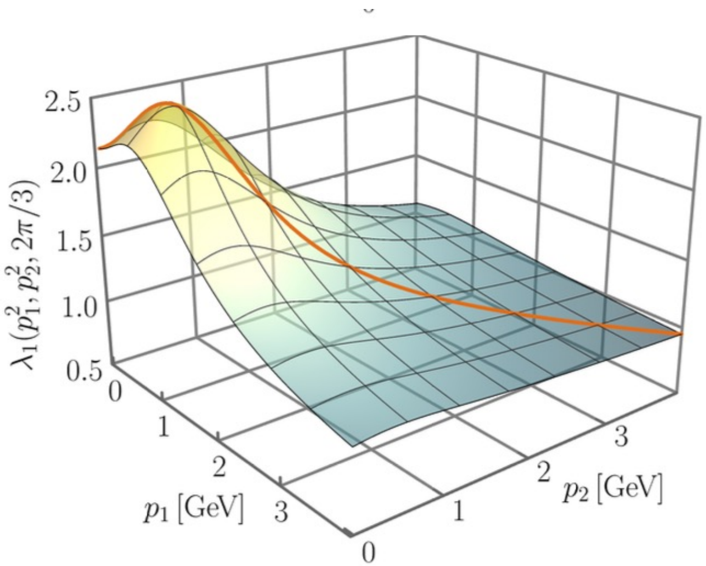
- 3D for θ = 0 and soft-gluon config.

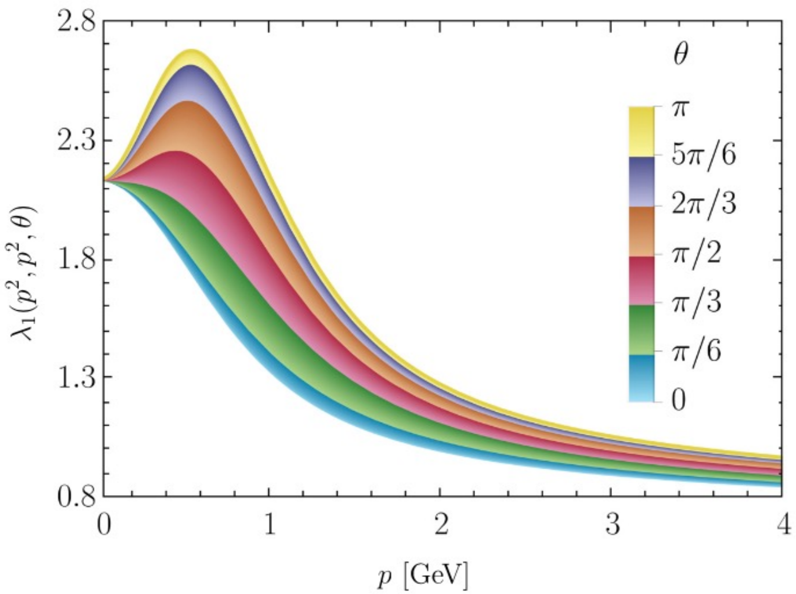




- Strong angular dependence
- 27% difference between the peaks
- Precludes planar degeneracy
- C symmetric

- 3D for θ =π and asymmetric config

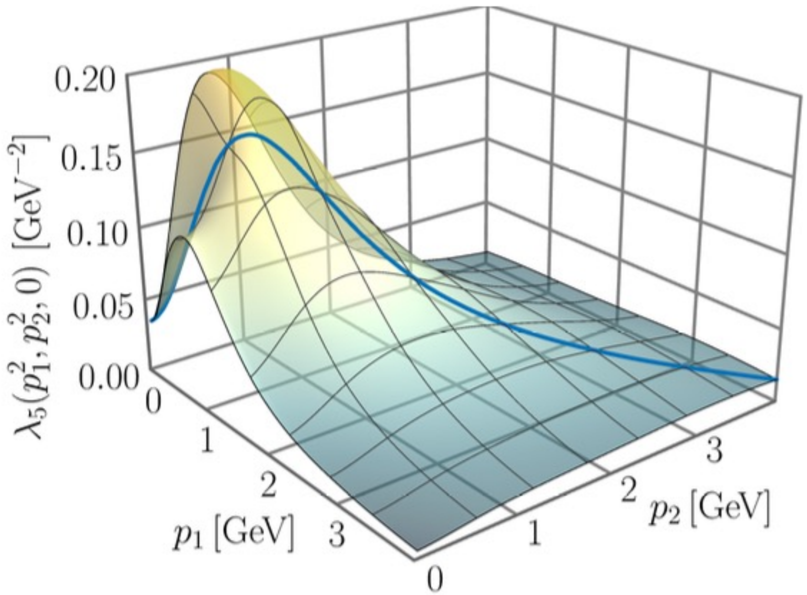


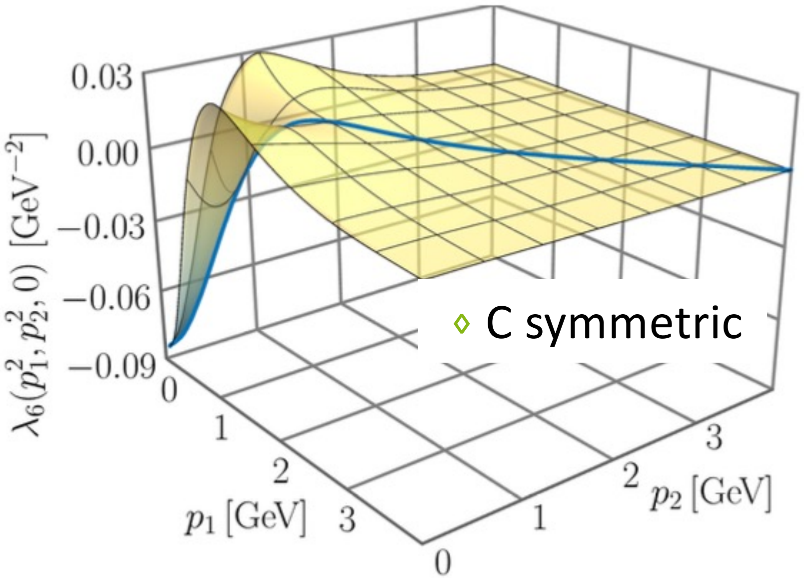


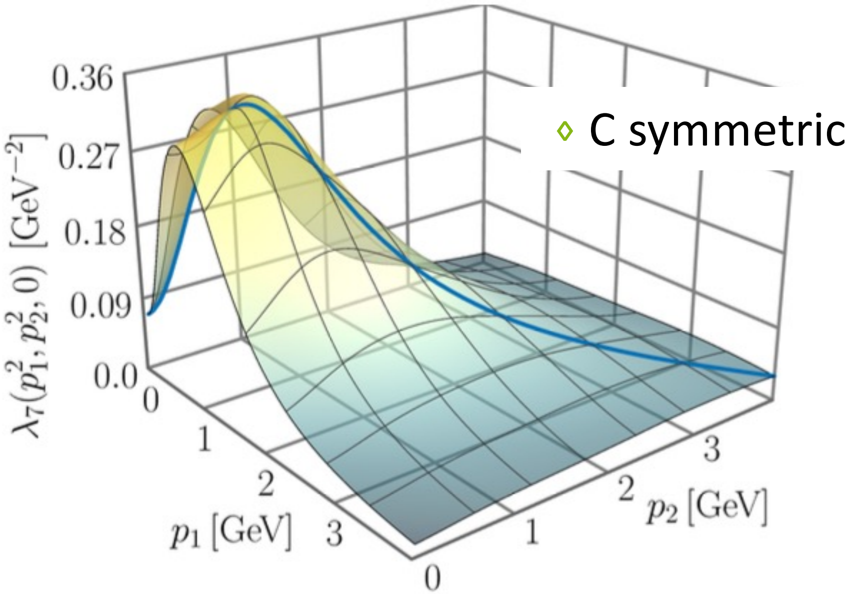
Results for quantum form factors

Chirally symmetric form factors

- With the results for λ_1 , we compute the other form-factors as one conventional integral
- $\theta = 0$ → Soft-gluon limit are highlighted in the diagonal
- The charge conjugation relations are all numerically satisfied
- IR finite and perturbative behavior recovered in the UV

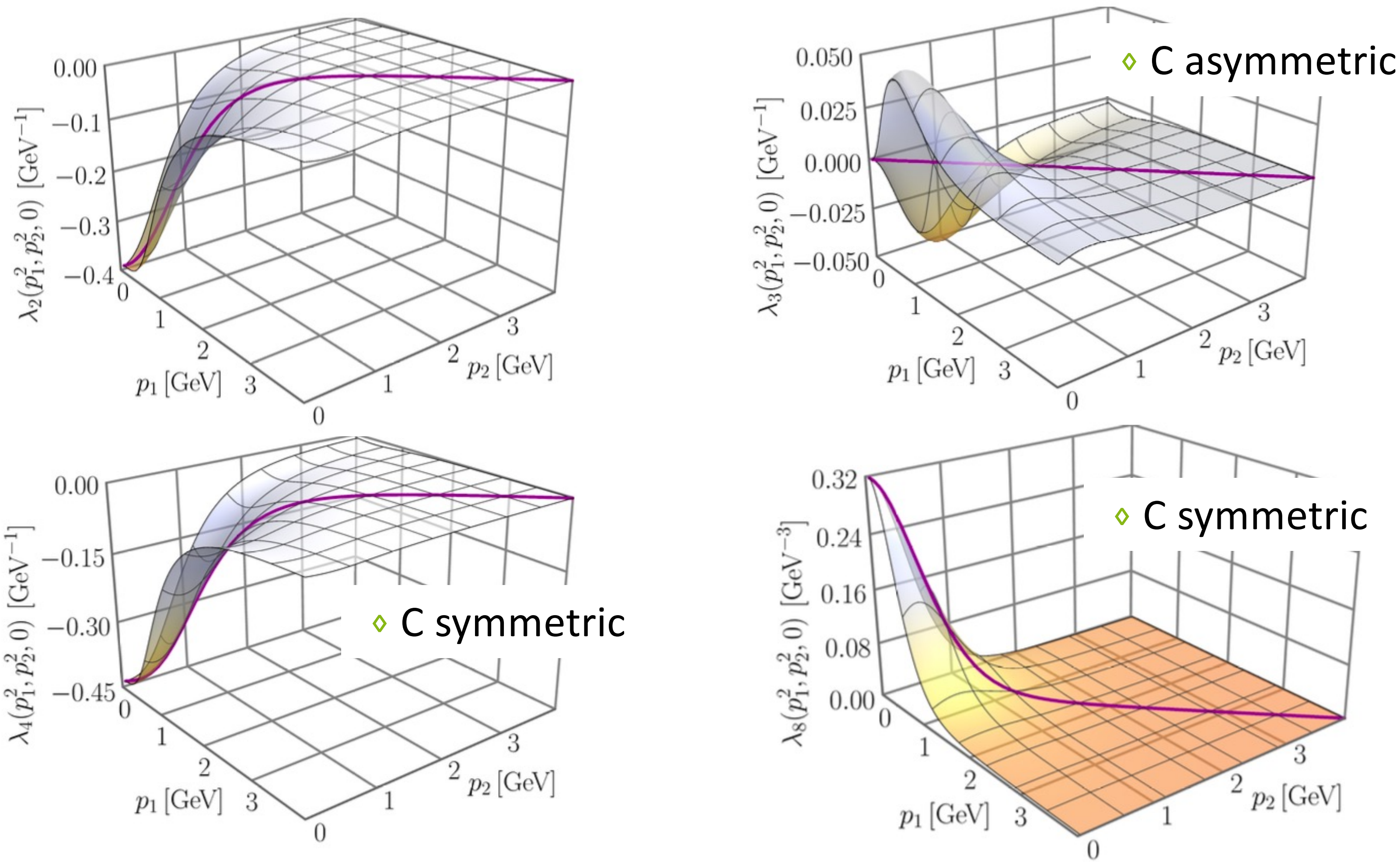






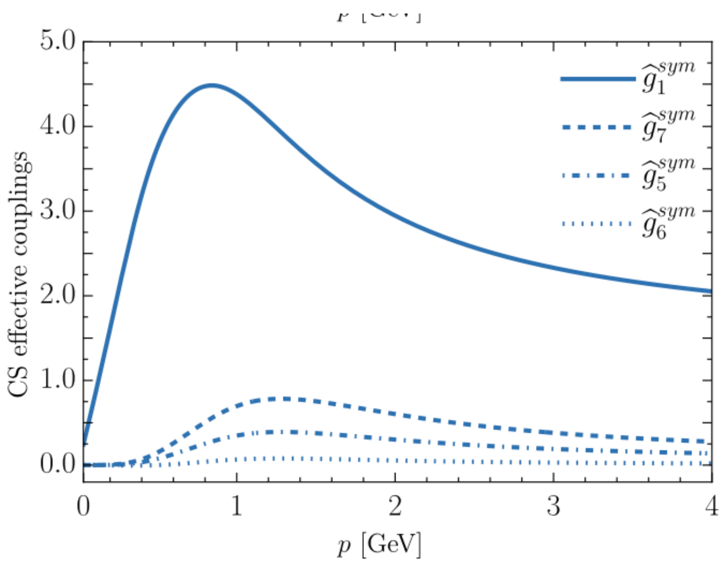
Results for quantum form factors

Chiral symmetry breaking form factors



- λ_2 and λ_4 have the Abelian contribution very suppressed.
- For the others, for p ≤ 1 GeV, the Abelian contribution is significant, but still smaller in magnitude then the non-Abelian.

Effective coupling



- Comparison through dimensionless RGI combinations:

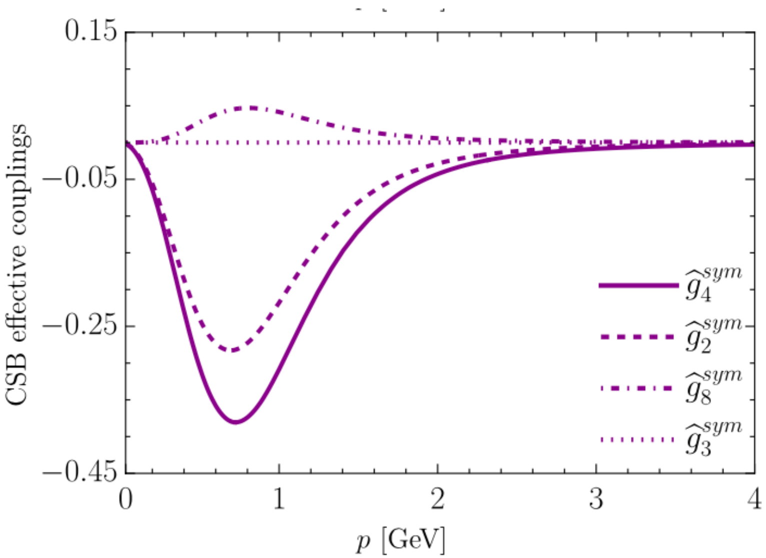
$$\widehat{g}_i^{sym}(p^2) = g(\mu^2)\,[p^{n_i}\lambda_i^{sym}(p^2)]\,A^{-1}(p^2)\mathcal{Z}^{1/2}(p^2)$$

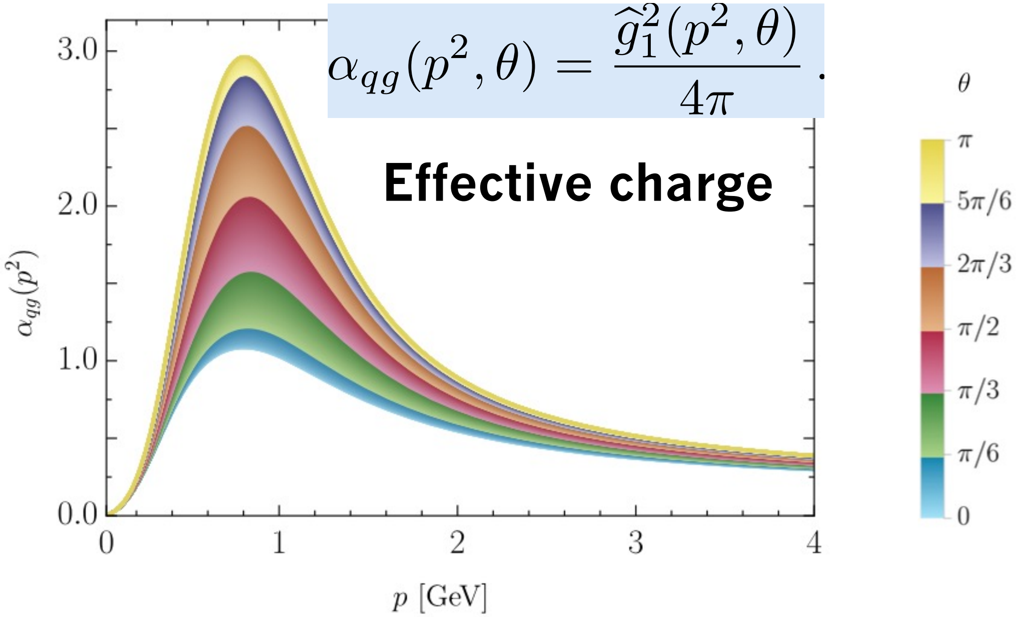
$$\mathcal{Z}(q^2) = q^2\Delta(q^2)$$

- Same hierarchy found in literature

$$\widehat{g}_1^{sym}(p^2), \lambda_1^{sym}(p^2) \to \widehat{g}_1(p^2,\theta), \lambda_1(p^2,p^2,\theta)$$

M. Mitter, J. M. Pawlowski, and N. Strodthoff, Phys. Rev. D91, 054035 (2015)
F. Gao, J. Papavassiliou, and J. M. Pawlowski, Phys. Rev. D 103, 094013 (2021)





$$\alpha_{qg}(p^2,\theta) = \frac{\widehat{g}_1^2(p^2,\theta)}{4\pi}\,.$$

Conclusions

- In this work, we have ***determined the eight form factors of the transversely projected quark-gluon vertex*** two light degenerate quarks.
- Our analysis is ***based on the SDE*** derived within the 3PI formalism at the three-loop level, ***where lattice results have been employed*** for all SDE components,
- We obtain ***all eight form factors, λ_i,*** of this vertex ***for arbitrary momenta.***
- ***λ_1 exhibits a*** considerable ***angular dependence***, displaying a large peak at the "asymmetric limit", which is absent in the "soft-gluon" configuration.
- Our results ***confirm the hierarchy of the RGI effective couplings*** $\widehat{g}_1^{sym}(p^2)$ ***is clearly dominant***.
- The ***strong angular dependence of λ_1 precludes the possibility*** of accurately describing λ_1 in terms of a ***single variable, s^2***, as we approximate the three-gluon vertex → "planar degeneracy".