$U_A(1)$ symmetry breaking quark-antiquark interactions mediated by gluons

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March 9, 2025

Hadron Physics
March 2025
Porto Alegre, RS

Talk based on:

* FLB, $U_A(1)$ symmetry breaking quark interactions from vacuum polarization, EPJA 60 (2024)
* FLB, Charged pions asymmetry from the decay of light neutral axial mesons due to interference, JPG. 52 (2024)
* A.P.Jr.+F.L.B., NJL Vacuum polarization corrections to low energy quark effective couplings PRD90 (2014)
* FLB, Constituent quark axial current couplings to light vector mesons..., PRD105 (2022)
* FLB, SU(2) Higher-order effective quark interactions from polarization, PLB761 (2016).

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CNPq

The number of works in the field is very large
Relevant works may not be quoted

Presentation Overview

1. Motivations/context
2. Quark interactions
   - Quark-antiquark interaction- dynamical calculation
3. Expansion of the quark determinant
4. 6^{th}-order quark interactions
5. Summary

Motivations/context

$U_A(1)$ symmetry is exact at the tree level (QED and QCD)

Axial anomaly (ABJ): (l.h.) - (r.h.) fermion currents not conserved
topological charge - 't Hooft

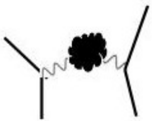
Axial anomaly $\rightarrow \eta - \eta'$ puzzle solved - Witten
$\rightarrow$ 't Hooft interaction for NJL-type models

Recently: the behavior of this symmetry at high T $\rightarrow$ instantons
* is it restored with Chiral Symmetry? Flavor? Are they related?...

Several recent works

Here: gluon-exchange interactions break $U_A(1)$

Quark-antiquark interaction $\to$ dynamics



quark-gluon interaction

Leading term for quark - QCD effective action

$$Z[\eta,\bar{\eta}] = N \int \mathcal{D}[\bar{\psi},\psi]$$

$$\exp\ i \int d^4x \left[\bar{\psi}\left(i\not{D} - m\right)\psi - \frac{g^2}{2}\int_y j^\beta_\mu(x)\tilde{R}^{\mu\nu}_{\beta\alpha}(x-y) j^\alpha_\nu(y) + \bar{\psi}\eta + \bar{\eta}\psi \right]$$

color quark current $j^\mu_\alpha = \bar{\psi}\lambda_\alpha\gamma^\mu\psi$,

Fierz transformation $\to$ all flavor-Dirac channels (mesons)

By introducing quark background currents J_ϕ

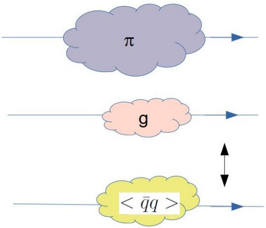
By integrating out quark field $\to$ Quark determinant

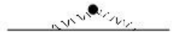
Constituent quarks:
quark-antiquark and meson-quark interactions

Quark determinant $\to$ constituent quark' Gluon cloud

* Gluon cloud:
It dresses background quark currents $\to$ constituent quarks

* By including meson states $\to$ Pion cloud from the *Goldstone boson* couplings (FLB, EPJA-2016, PRD-2018, PRD-2019)





Expansion of the quark determinant: 4^{th}-order

LARGE quark and gluon effective masses $\to$ local limit

Second order (quark currents):

$$\mathcal{L}_{eNJL} \;=\; \bar\psi S_0^{-1}\psi + (G_{ij} + \Delta G_{s,ij})(\bar\psi\lambda_i\psi)(\bar\psi\lambda_j\psi) + G_{ij}(\bar\psi i\gamma_5\lambda_i\psi)(\bar\psi i\gamma_5\lambda_j\psi)$$

Pseudoscalar and scalar (and vector, axial etc) channels with different interactions

* There are (FSB) mixing interactions (adj. rep) $G_{i\neq j}$ (i,j=0,3,8)

$$G_{i\neq j} \propto (M_{f_1} - M_{f_2})^{n=1,2}$$

* Mechanism FSB lead to mixing is know for long time.

All the resulting G_{ij} are 1-loop integrals of quark (and gluon) effective propagators

Quark sector-polarization 4-point GF

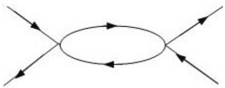


Figure: Polarization in the NJL model, solid lines are quarks, $P = 0$

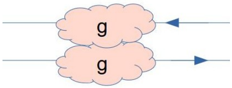


Figure: One loop Two gluon exchange - Fierz transformation in quark det

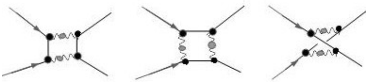


Figure: CQM and QCD-Two gluon exchange

6^{th} order quark interactions $\to$ $U_A(1)$ breaking

Third order expansion in the quark currents:
$U_A(1)$-breaking interactions

$$\mathcal{L}_1 = G_{sb,ps} T^{ijk} (J_i^S J_j^{PS} J_k^{PS} + J_i^{PS} J_j^{PS} S_k + J_i^{PS} J_j^S J_k^{PS}) + G_{sb,s} T^{ijk} J_i^S J_j^S J_k^S ,$$

$$T_{ijk} \equiv d_{ijk} + i f_{ijk}$$

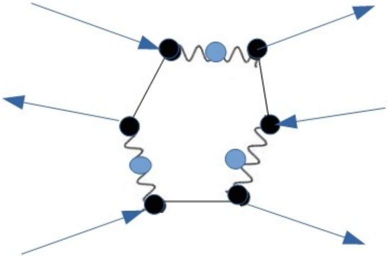
All the resulting G_{ij} are 1-loop integrals of quark (and gluon) effective propagators

Comparable to 't Hooft (determinant) interaction: SU(3) = 6^{th} order

$$\mathcal{L}_{'tHooft} = K \, d_{ijk} \left[(J_i^S J_j^{PS} J_k^{PS} + J_i^{PS} J_j^{PS} S_k + J_i^{PS} J_j^S J_k^{PS}) - J_i^S J_j^S J_k^S \right]$$

6^{th} order vacuum polarization constituent quarks or gluon exchange

* Diagrammatic interpretation



+ other channels

The calculated loop so far: in terms of constituent quark vertices

Mixed interactions
vector and axial currents (V-A), with a scalar or pseudoscalar (S-PS)

$$\begin{aligned}\mathcal{L}_{6,sb,SVV} &= T^{ijk}\Big[3\ G_{sb1} J_i^{V,\mu} J^j_{V,\mu} J^S_k + 3\ G_{sb2} J_i^{A,\mu} J^j_{A,\mu} J^S_k \\ &+ 3i\ G_{sb1}\left(J_i^{V,\mu} J^j_{A,\mu} J^{PS}_k - J_i^{A,\mu} J^j_{V,\mu} J^{PS}_k\right)\Big], \qquad (1)\end{aligned}$$

Last term antisymmetric structure

* And many other momentum dependent interactions

For example:

$$
\begin{aligned}
\mathcal{L}_{6d,SSV} &= 3\ T^{ijk} \left\{ \left(\partial_\mu J_i^{PS}\right) \left[-G_{d1} J_j^S J_k^{A,\mu} + G_{d2} J_j^{A,\mu} J_k^S \right] \right. \\
&- G_{d2} (\partial_\mu J_i^S) \left[J_j^{A,\mu} J_k^{PS} + J_j^{PS} J_k^{A,\mu} \right] \\
&+ (\partial_\mu A_i^\mu) \left[G_{d1} J_j^S J_k^{PS} + G_{d2} J_j^{PS} J_k^S \right] \\
&+ i \left[(G_{d1} - G_{d2}) J_i^{V,\mu} J_j^P \left(\partial_\mu J_{PS}^k\right) \right. \\
&- \left. \left. 2 G_{d2} J_i^{V,\mu} \left(\partial_\mu J_{PS}^j\right) J_k^P , \right] \right\} \qquad (2)
\end{aligned}
$$

1) And terms with unusual parity structure

$$\mathcal{L}_{6,SVA,sb} = -3\, G_{sva,sb} T^{ijk} \epsilon_{\mu\nu\rho\sigma} \left\{ 2(\partial^\mu J_i^{V,\nu})(\partial^\rho J_j^{A,\sigma}) J_S^k + (\partial^\mu J_i^{V,\nu}) J_S^j (\partial^\rho J_k^{A,\sigma}) \right\} + \mathcal{O}(\partial\partial), \quad (3)$$

(topological) properties akin to Wess-Zumino-Witten interactions PRD105 (2022)

2) Flavor-dependencies $G^{ijk}_{sp,ps}$,
* Mixing interactions also appear $G^{mix}_{ijk} \propto (M_{f_1} - M_{f_2})^n$

Few phenomenological consequences

Up and down quark dynamics T_{118} FLB, EPJA (2024)

Three-meson interactions from 6-quark interactions

Longwavelength limit

$M_u = M_d = 0.391$GeV, $M_s = 0.600$ GeV

$G_{sb,s}/G_{sb,ps}$	G_{sb1}/G_{sb2}	G_{d1}/G_{d2}	G_{v1}/G_{v2}	$G_{sva,sb}$
GeV^{-5}	GeV^{-5}	GeV^{-6}	GeV^{-6}	GeV^{-7}
2.9/86.7	2.9/-86.7	59.3/221.7	118.7/443.5	-459.7

M.f.	G^{ps}_{03}/G^{s}_{03}	G^{ps}_{08}/G^{s}_{08}	$G^{ps}_{38}/\ G^{s}_{38}$	G^{v}_{03}/G^{a}_{03}
	GeV^{-2}	GeV^{-2}	GeV^{-2}	GeV^{-2}
	-0.08/-0.40	-0.79/-3.85	-0.05/-0.23	-0.06/0.28

Small values and GeV^{-n}

However at mean field $\rightarrow$ sizeable 4th order NJL

Mean field for

$$J^k_s \sim < \psi \lambda^k \psi > .$$

Example $\rho - \pi - \pi$ vertex
With $T^{ijk} = d^{ijk} + if^{ijk}$

$$\begin{aligned}\mathcal{L}^{mes}_{VPP} &= i\,3T^{ijk}G_{d2}\left(\left\{(\partial^\mu V^i_\mu)P^*_j P_k\right\} - \left\{V^i_\mu(\partial^\mu P^*_j)P_k\right\}\right)\\ &+ i\,3T^{ijk}G_{d1}\left\{V^i_\mu P^*_j(\partial^\mu P_k)\right\}, \end{aligned} \tag{4}$$

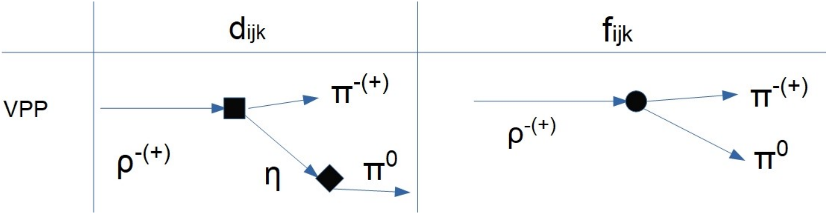


Figure: Three leg meson decays of rho into two pions $\rho^\pm \to \pi^\pm \pi^0$.

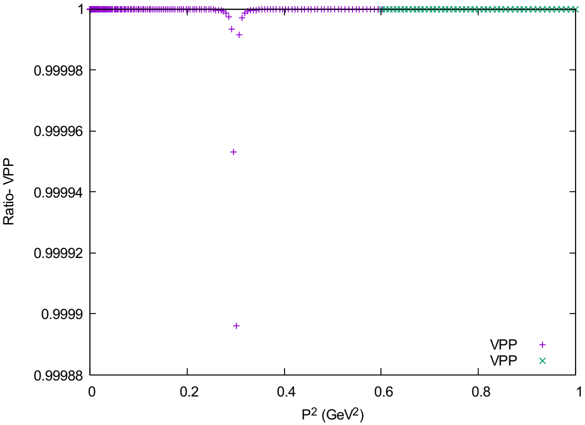
Extremely Simplified prescription for rho-polarization
Momentum independent limit of interactions/form factors

Effective gluon propagator from SDE (C.Roberts, et al)

$$R_T(k) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2} + \frac{8\pi^2 \gamma_m E(k^2)}{\ln\left[\tau + (1 + k^2/\Lambda_{QCD}^2)^2\right]}, \tag{5}$$

The set of parameters $X - 20$ - $D_{I,2}$ of FLB, PRD 2021,

$$M_u = 0.392\ GeV, \quad M_d = 0.396\ GeV, \quad M_s = 0.600\ GeV, \tag{6}$$



$$A^{\rho} \;=\; \frac{\Gamma^{V}_{\pi^+} - \Gamma^{V}_{\pi^-}}{\Gamma^{V}_{\pi^+} + \Gamma^{V}_{\pi^-}} \simeq 2.5 \times 10^{-4}, \tag{7}$$

Summary

- Vacuum polarization leads to $U_A(1)$ breaking dynamics
- In S-PS channel: 't Hooft-like interactions ($G_s \neq G_{ps}$)
- Part of their effects: mean field level in NJL
- Momentum dependent corrections
  $\rightarrow$ interference in meson decays ?

Looking forward:

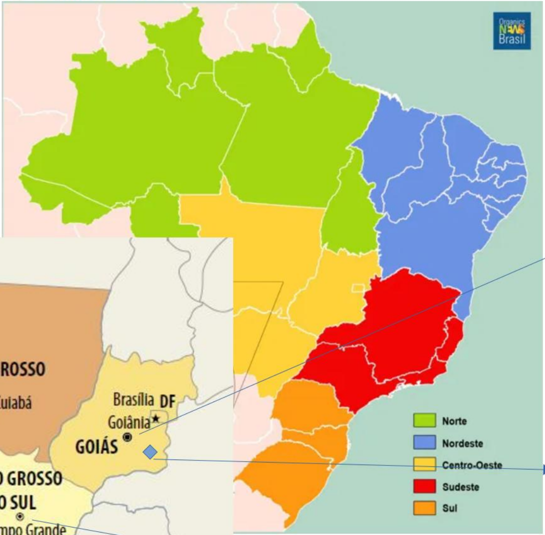
- Behavior at high temperatures and densities?
- To compare with strict 2 gluon exchange
- Effects in topological charges?
- Looking for further consequences/tests, etc

Thank you for your attention!

Network of researchers Strong Interactions:
Hadrons, QCD, Fís. Nuclear

(..)=not confirmed

Others?



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