Ensemble of topological defects and the confining flux tube

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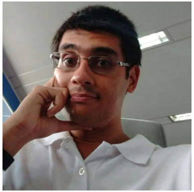

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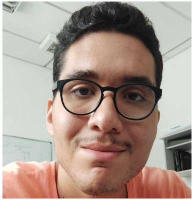


Rafael Carlos da Silva Tonhon

Main Refs.

- A. L. L. de Lemos, L. E. Oxman, and B. F. I. Teixeira, Physical Review **D85** (2012) 125014.
- L. E. Oxman, Journal of High Energy Physics, **3** (2013) 1.
- L. E. Oxman, G. C. Santos-Rosa, and B. F. I. Teixeira, Journal of Physics A: Mathematical and Theoretical **47** (2014) 305401.
- L. E. Oxman, and G. C. Santos-Rosa, Physical Review **D92** (2015) 125025.
- L. E. Oxman, D. Vercauteren, Physiscal Review **D95** (2017) 025001.
- L. E. Oxman, and H. Reinhardt, The European Physical Journal **C78** (2018) 1.
- L. E. Oxman, Physical Review **D98** (2018) 036018.
- L. E. Oxman, and G. M. Simões Physical Review **D99** (2019) 016011.
- D. R. Junior, L. E. Oxman, and G. M. Simões, Journal of High Energy Physics **01** (2020) 1.
- D. Fiorentini, D. R. Junior, L. E. Oxman, and R. F. Sobreiro, Phys. Rev. **D101** (2020) 085007.
- D. R. Junior, L. E. Oxman, and G. M. Simões, Phys. Rev. **D102** (2020) 074005.
- D. Fiorentini, D. R. Junior, L. E. Oxman, G. M. Simões, and R. F. Sobreiro, Phys. Rev. **D103** (2021) 114010.
- D. R. Junior, L. E. Oxman, and G. M. Simões Universe **7** (2021) 253.
- D. Fiorentini, D. R. Junior, L. E. Oxman, and R. F. Sobreiro, Physical Review **D105**, (2022) 125015.
- D. R. Junior, L. E. Oxman, and H. Reinhardt, Physical Review **D106**, (2022) 114021.

General Strategy

Yang Mills theory $Z_{YM} = \int [DA]\, e^{-S_{YM}}$	$\rightarrow$	Sectorization $Z_{YM} = \sum_{S_0} Z_{YM}^{(S_0)}$	$\rightarrow$	???
				$\downarrow$
Observables $\sigma_k,\ T_{\mu\nu}$	$\leftarrow$	Effective Model $S_{YMH}(\Lambda_\mu, \psi_A)$	$\leftarrow$	Ensemble $\langle W(\mathcal{C}) \rangle = \sum_\omega e^{-S(\omega)} z_N^L(\omega, \mathcal{C})$

Wilson Loop

- $\mathcal{W}_{\mathrm{D}}(\mathcal{C}_{\mathrm{e}}) = \frac{1}{\mathscr{D}} \operatorname{tr} \mathrm{D} \left(P \left\{ e^{i \int_{\mathcal{C}_{\mathrm{e}}} dx_\mu \, A_\mu(x)} \right\} \right)$
- In the heavy-quark limit, $\langle \mathcal{W}_{\mathrm{D}}(\mathcal{C}_{\mathrm{e}}) \rangle \sim e^{-T \, V_{\mathrm{D}}(R)}$
- Lattice: $V_D(R) = \sigma_D R + \frac{\gamma}{R} + O(1/R^2)$
- In general, confinement leads to an Area Law
- Formation of a flux tube

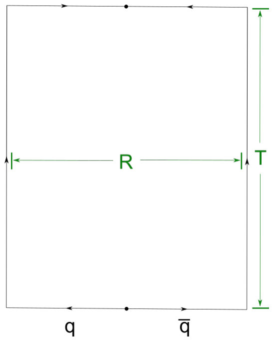


Figure: Rectangular Wilson Loop

The role of center vortices

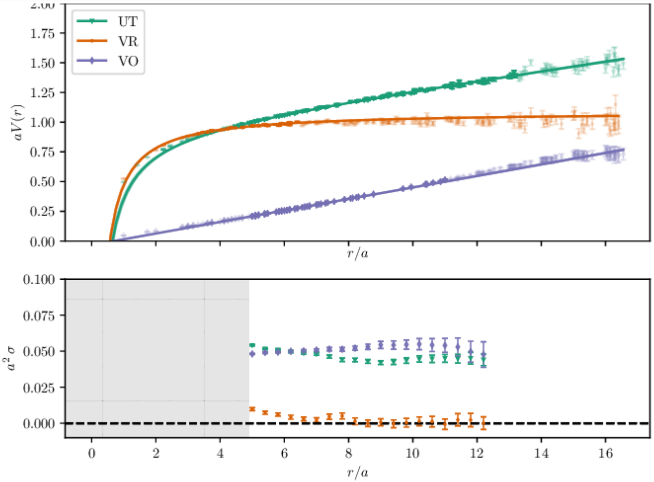
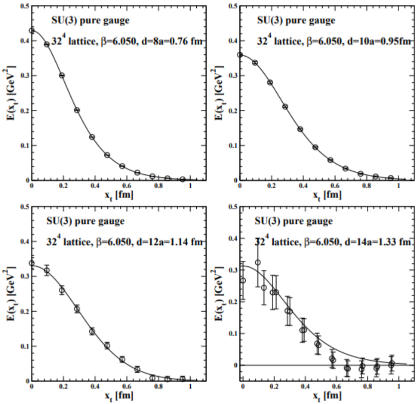
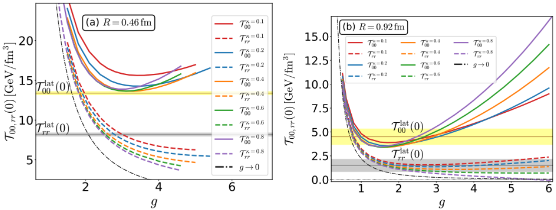


Figure: Static quark-antiquark potential for the untouched, vortex-removed and vortex-only lattice simulations. From Biddle, Kamleh, and Leinweber (2022).



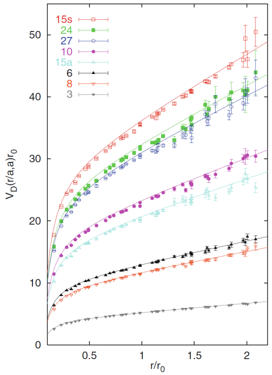
Transverse profiles of the confining string. From Cosmai et al. (2020)



Energy-momentum tensor of the confining string. From M. Kitazawa and R. Yanagihara (2019)

Scaling Laws

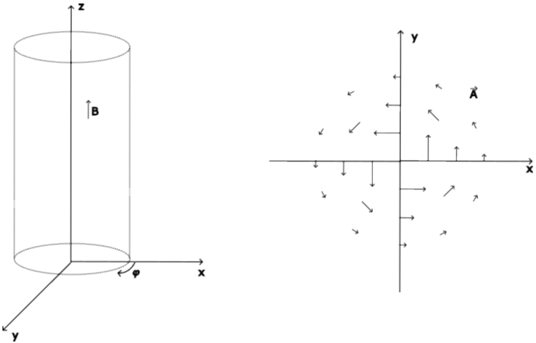
- Fundamental quarks: $\psi \to S\psi$
- Higher irrep. quarks: $\psi \to D(S)\psi$
- Intermidiate distances: $\frac{\sigma_{\mathrm{I}}(\mathrm{D})}{\sigma_{\mathrm{I}}(\mathrm{F})} = \frac{C_2(\mathrm{D})}{C_2(\mathrm{F})}$
- Gluonic screening for large r
- N-ality: $\mathrm{D}\left(e^{i\frac{2\pi}{N}} I\right) = \left(e^{i\frac{2\pi}{N}}\right)^k \mathbb{I}_{\mathscr{D}}$
- k-Antisymmetric irrep has the smallest quadratic Casimir.
- Large r in 3D: $\sigma_k^{(3)} = \frac{k(N-k)}{N-1} = \frac{C_2(k\text{-A})}{C_2(\mathrm{F})}$
- Large r in 4D: $\sigma_k^{(4)} = \frac{k(N-k)}{N-1}$ or $\sigma_k^{(4)} = \frac{\sin k\pi/N}{\sin \pi/N}$



Casimir scaling for $SU(3)$, from Bali (2000). The length $r_0 \approx 0.5 fm$

Simplest Vortex

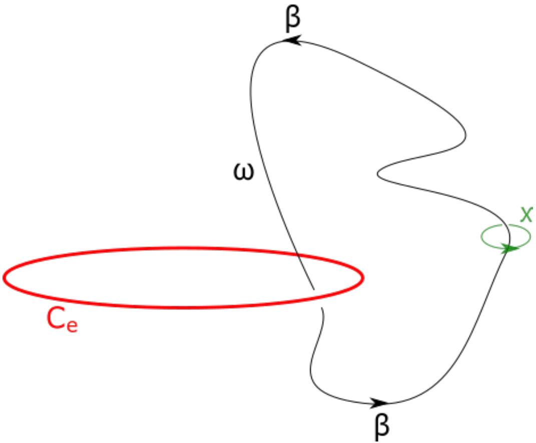
- $A_\mu = \frac{1}{g}\partial_\mu \varphi \beta \cdot T$
- $\vec{B} = \frac{1}{g}\nabla \times (\nabla\varphi)\beta \cdot T = \frac{\hat{z}}{g}\delta^{(2)}(x,y)\beta \cdot T$
- $\partial_\mu \varphi$ gives the vorticity while $\beta \cdot T$ gives the center...icity (???)
- Can be created with a singular-valued gauge transformation: $A_\mu = \frac{i}{g} S \partial_\mu S^{-1}$, $S = e^{i\varphi\beta\cdot T}$
- $\varphi \rightarrow \chi$ changes the vortex core location
- $\frac{1}{g} \rightarrow \frac{a(\rho)}{g}$ gives thickness to the vortex
- Non-Abelian d.o.f.



Straight vortex around the z axis.

Center vortex and Wilson loop

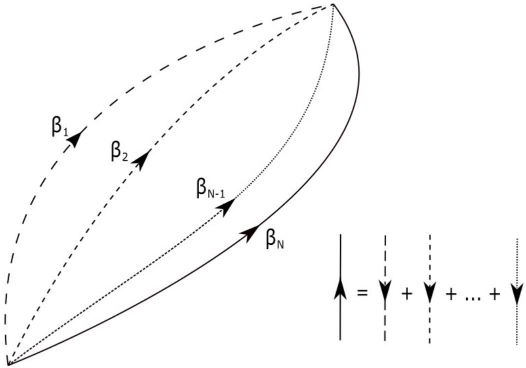
- $\mathcal{W}_{\mathcal{C}_e}[A_\mu] = \frac{1}{N}\,\mathrm{tr}\left(P\left\{e^{i\int_{\mathcal{C}_e} dx_\mu\, A_\mu(x)}\right\}\right)$
- $\langle \mathcal{W}_{\mathcal{C}_e} \rangle = \int [DA]\, \mathcal{W}_{\mathcal{C}_e}[A_\mu]\, e^{-S_{YM}}$
- Center vortices are worldsheets/worldlines in 4D/3D
- $\mathcal{W}_{\mathcal{C}_e} = \left(e^{i\frac{2\pi}{N}}\right)^{\mathrm{L}(\omega, \mathcal{C}_e)}$
- $Z(N) = \{e^{i\frac{2k\pi}{N}} | k = 0, 1, ..., N-1\}$



Center vortex line linking a Wilson loop.

N-lines correlations

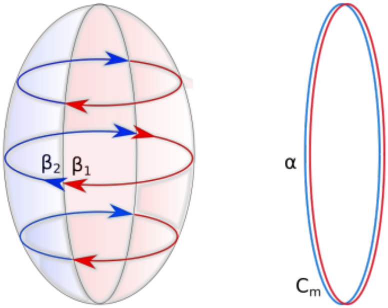
- Can be regarded as $(N-1)$ loops, although with different probability.
- $A_\mu = \sum_{i=1}^{N-1} \beta_i \cdot T\, \partial_\mu \chi_i$
- $W_{\mathcal{C}}[A_\mu] = \left(e^{i\,2\pi\, \beta_1 \cdot w_e}\right)^{L_1} \dots \left(e^{i\,2\pi\, \beta_{N-1} \cdot w_e}\right)^{L_N}$
- Only works because $\beta_1 + \beta_2 + ... + \beta_N = 0$



N-vortex matching configuration. From Júnior, Oxman, and Simões (2019)

Monopoles

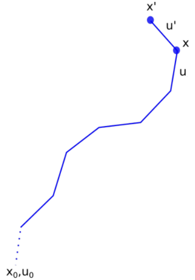
- Monopoles are worldlines/instantons in 4D/3D
- Cannot exist isolated. It must be either attached to a Dirac string or something else.
- In $SU(N)$, something else could be center vortices, carrying weights β_1, β_2.
- In this case, the monopole carries a root $\alpha = \beta_1 - \beta_2$ as its charge
- Can also be created with a singular-valued gauge transformation:
  $A_\mu = \frac{i}{g} S \partial_\mu S^{-1}$, $S = e^{i\varphi\beta_1 \cdot T} e^{i\sqrt{N}\theta T_\alpha}$
- Notice that $S(\varphi, \theta = 0) = e^{i\varphi\beta_1 \cdot T}$ and $S(\varphi, \theta = \pi) = e^{i\varphi\beta_2 \cdot T}$



Non-oriented vortex with a monopole in the middle. From Oxman (2018)

Worldline ensemble

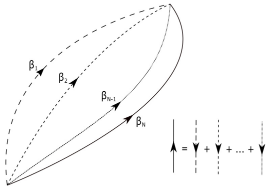
- Polymer Techniques
- $Z_{\text{loops}}[b_\mu] =$
  $\sum_{n=0}^{\infty} \frac{1}{n!} \prod_{k=1}^{n} \int_0^\infty \frac{dL_k}{L_k} \int dv_k \int [dx^{(k)}]_{v_k,v_k}^{L_k}$
  $e^{-\int_0^{L_k} ds_k \left[\frac{1}{2\kappa} \dot{u}_\mu^{(k)} \dot{u}_\mu^{(k)} + \mu\right]} W_{l_k}[b_\mu]$
- μ is the string mass density and κ is the stiffness. The percolating regime is realized when $\mu < 0$ and $\kappa > 0$.
- It can be shown that $Z_{\text{loops}}[b_\mu] \approx (\det\ O)^{-1} = \int [d\phi]\, e^{-\int d^3x\, \phi^\dagger O \phi}$, where $O = -\frac{1}{3\kappa}(\partial_\mu - ib_\mu)^2 + \mu I_N$
- Extended models can include more exotic interactions



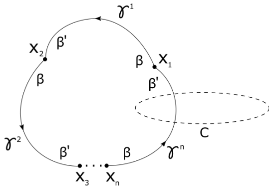
Polymer Growth. From Oxman (2017)

Ensemble in (2+1)-D

- $W_{\mathcal{C}}[A_\mu] =$
  $\left(e^{i\,2\pi\ \beta_1\cdot w_e}\right)^{L(S(\mathcal{C}),l_1)}\ldots\left(e^{i\,2\pi\ \beta_{N-1}\cdot w_e}\right)^{L(S(\mathcal{C}),l_{(N-1)})}$
- N charges $\to N\times N$ matrix field Φ
- $S_{\rm eff}(\Phi,b_\mu) =$
  $\int d^3x\ \left(\mathrm{Tr}\,(D_\mu\Phi)^\dagger D^\mu\Phi + V(\Phi,\Phi^\dagger)\right)$
- $V(\Phi,\Phi^\dagger) = \frac{\lambda}{2}\mathrm{Tr}\,(\Phi^\dagger\Phi - a^2 I_N)^2 - \xi\big(\det\Phi + \det\Phi^\dagger\big) - \vartheta\mathrm{Tr}\,(\Phi^\dagger T_A\Phi T_A) + c$
- Vacuum in the center: $\Phi = vZ$ with $z\in\mathcal{Z}_N$



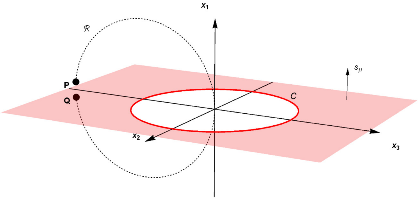
An N center-vortex creation-annihilation process.



A chain configuration, with n correlated instantons, linking a Wilson Loop $\mathcal{C}$

Domain Walls

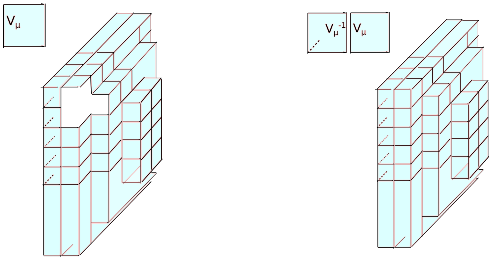
- Φ must be in the vacuum at **P** and **Q**
- $\lim_{x_1 \to -\infty} \Phi(x_1, x_2, x_3) = v\, I_N$
- $\lim_{x_1 \to +\infty} \Phi(x_1, x_2, x_3) = v\, e^{i2\pi \beta_e \cdot T}$
- Area Law: $S_{\text{eff}} \approx \varepsilon A$
- $\varepsilon = \int dx \left(\text{Tr} \left(\partial_x \Phi \right)^\dagger \partial_x \Phi + V(\Phi, \Phi^\dagger) \right)$
- Different irreps.: $\varepsilon_k = \frac{k(N-k)}{N-1} \varepsilon$



A ring $\mathcal{R}$ that goes through the center of the Wilson loop. The red surface is the one where s_μ is concentrated.

Ensemble in 4D

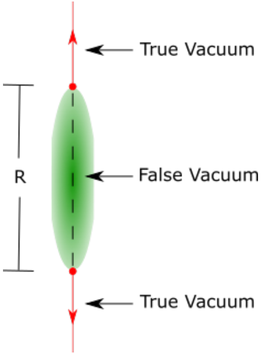
- Ensemble of worldsurfaces is still lacking.
- The resulting order parameter would be a string field, which, in certain conditions, can give rise to a dual gauge field Λ_μ
- In analogy with the 3D case, the resulting action is $S = \int d^4x \langle F_{\mu\nu}(\Lambda) - s_{\mu\nu} \rangle^2$
- Monopoles are represented by adjoint ψ_I



An ensemble of vortices is realized by a sum over closed worldsurfaces.

Effective model in 4D

- $S = \int d^4x \left(\frac{1}{4} \langle F_{\mu\nu} - s_{\mu\nu} \rangle^2 + \frac{1}{2} \langle D_\mu \psi_I, D_\mu \psi_I \rangle + V_{\mathrm{H}}(\psi) \right)$
- $V_{\mathrm{H}}(\psi) = c + \frac{\mu^2}{2} \langle \psi_A, \psi_A \rangle + \frac{\kappa}{3} f_{ABC} \langle \psi_A \wedge \psi_B, \psi_C \rangle + \frac{\lambda}{4} \langle \psi_A \wedge \psi_B, \psi_A \wedge \psi_B \rangle$
- Vacua: $\psi_A = vST_AS^{-1}$, i.e. a $SU(N) \to Z_N$ spontaneous symmetry breaking.
- Area Law, Casimir law, and Nielsen-Olesen tranverse profiles.



Flux tube between static charges (Wilson loop)

Remember?

Yang Mills theory $Z_{YM} = \int [DA]\, e^{-S_{YM}}$	$\rightarrow$	Sectorization $Z_{YM} = \sum_{S_0} Z_{YM}^{(S_0)}$	$\rightarrow$	???
				$\downarrow$
Observables $\sigma_k,\ T_{\mu\nu}$	$\leftarrow$	Effective Model $S_{YMH}(\Lambda_\mu, \psi_A)$	$\leftarrow$	Ensemble $\langle W(\mathcal{C})\rangle = \sum_{\omega} e^{-S(\omega)} z_N^L(\omega, \mathcal{C})$

Local gauge fixing

- Singer theorem doesn't hold if the fixing is made by sectors and if those sectors are disjointed $\vartheta_\alpha \subset \{A_\mu\}$:

$$\{A_\mu\} = \cup_\alpha \vartheta_\alpha \ , \ \vartheta_\alpha \cap \vartheta_\beta = \emptyset \ \text{ se } \ \alpha \neq \beta$$

The procedure

$$S_{\rm aux}[A, \psi]$$

$$\frac{\delta S_{\rm aux}}{\delta \psi_I} = 0 \text{ e } D_\mu \psi \to 0 \ , \ |x| \to \infty \Rightarrow \psi_I[A]$$

Polar decomposition: $\psi_I = S q_I S^{-1}$

Gauge transformation: $A_\mu \to A_\mu^U \ , \ q[A^U] = q[A] \ , \ S[A^U] = US[A]$

Fixing $S = S_0$ also fixes the gauge

Important requirements

- Uniqueness of the solution ψ_A
- Injectivity: $\psi(A^U) = \psi(A) \Rightarrow U \in Z(N)$
- Unique polar decomposition

Thanks for your attention!