Searching for femtoscopic signatures of the $D\bar{D}(I=0)$ $[X(3700)]$ bound state

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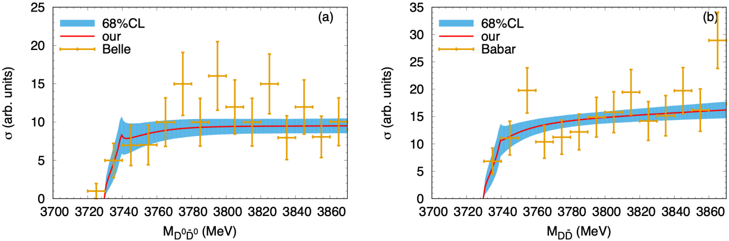
Motivation

- Exotic hadron states: one of the most relevant topics in the contemporary hadron physics
- Most famous example: $X(3872)$ observed in 2003 by Belle, $I^G(J^{PC}) = 0^+(0^{++})$, its nature remains a matter of debate $((D\bar{D}^* + c.c.), c\bar{c}q\bar{q}, ...?)$
- Speculation: existence of its spin partner $D\bar{D}$ state, $X(3700)$
- Gamermann, Oset (2007): Prediction of $X(3700)$ using a coupled channel unitary approach
- Theoretical investigations in different contexts:
    - Gamermann et al. (2008), Wang et al.(2019): $e^+e^- \to J/\psi D\bar{D}$
    - Xiao et al. (2012): $e^+e^- \to J/\psi\eta\eta'$
    - Dai et al. (2020): $\psi(3770) \to \gamma D^0\bar{D}^0$
    - Wang et al.(2020), Deineka at al. (2021): $\gamma\gamma \to D\bar{D}$ reaction
    - Brandao at al. (2023:)$B^+ \to K^+\eta\eta$
    - Sobrinho et al. (2024): $\gamma\gamma \to D^+D^-$ in UPCs

Motivation

- Experimental searches of $X(3700)$ reported in the literature are scarce:
  - Belle (2007,2017): $e^+e^- \to J/\psi D\bar{D}$
  - Babar (2010): $e^+e^- \to D\bar{D}$
- Gamermann et al. (2007), Xiao et al. (2012), Wang et al. (2020), Deineka et al. (2021): claim reproduction of the data by means of a $D\bar{D}$ bound state



Wang et al. (2020): structure located at 3720 MeV

- Controversy: $X(3700)$ not yet listed in the RPP

Our work

- Proposal of an alternative way for searching the $X(3700)$ state
- Investigation of femtoscopic $D\bar{D}$ correlations to predict the signature of the $X(3700)$
- $X(3700)$: interpreted as a bound state dynamically generated by solving the coupled-channel Bethe-Salpeter equations
- Analysis of its properties (pole position, scattering lengths and compositeness)
- Estimation of the correlation functions of the $D^0\bar{D}^0$ and D^+D^- pairs
- Discussion of how their behavior encode the features of the $X(3700)$

Scattering amplitudes

The model

Meson-meson interactions: coupled-channel Bethe-Salpeter equations with the local hidden-gauge formalism

- Interactions between vector and pseudoscalar mesons based on SU(4) symmetry (Bayar (2022), Abreu (2023)): simplified version

$$V_{ij} = -C_{ij}\frac{g^2}{2}\left[3s - \left(m_1^2 + m_2^2 + m_3^2 + m_4^2\right) - \frac{1}{s}(m_1^2 - m_2^2)(m_3^2 - m_4^2)\right]$$

$$C = \begin{pmatrix} \frac{1}{2}\left(\frac{3}{m_\rho^2} + \frac{1}{m_\omega^2} + \frac{2}{m_{J\psi}^2}\right) & \frac{\sqrt{2}}{m_{K^*}^2} & v \\ \frac{\sqrt{2}}{m_{K^*}^2} & \frac{1}{m_\phi^2} + \frac{1}{m_{J\psi}^2} & 0 \\ v & 0 & 0 \end{pmatrix}$$

Channels: $\{|D\bar{D}; I = 0\rangle, |D_s^+ D_s^-\rangle, |\eta\eta\rangle\}$

v: free parameter (yields Γ)

- Search for solutions of the Bethe-Salpeter equations:

$$T = V + VGT,$$

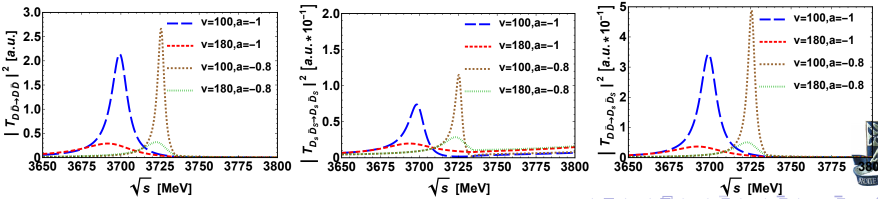
G: loop function (dimensional regularization: $a(\mu)$; $\mu = 1500$ MeV)

Mass and width of the lower energy pole dynamically generated, and the couplings g_i:

$a_{D\bar{D}}(\mu)$	v	(m,Γ) [MeV]	$g_{D\bar{D}}$ [MeV]	$g_{D_s\bar{D}_s}$ [MeV]	$g_{\eta\eta}$ [MeV]
-1	0	$(3699.1, -)$	$-14508.2 - i0.3$	$-5697.1 - i0.5$	-
-1	100	$(3699.5, 12.6)$	$-14503.7 + i894.1$	$-5766.3 - i896.2$	$1540.1 + i198.5$
-1	180	$(3694.2, 39)$	$-15577.4 + i2349.5$	$-4923.8 - i2963.3$	$2528.6 + i1259.4$
-0.8	0	$(3725.4, -)$	$-10065.1 - i1.3$	$-4172.2 - i0.4$	-
-0.8	100	$(3725.9, 5.2)$	$-10011.2 + i852.1$	$-4284.2 - i490.4$	$1010.2 + i102.2$
-0.8	180	$(3724.5, 17.9)$	$-10990.8 - i2442.4$	$-4146.5 - i19140.8$	$1806.7 + i737.9$

Scattering lengths $\tilde{a}_i$ of the lower energy pole dynamically generated and the probabilities P_is:

$(a_{D\bar{D}}(\mu), v)$	(m,Γ) [MeV]	$\tilde{a}_{D\bar{D}}$ [fm]	$\tilde{a}_{D_s\bar{D}_s}$ [fm]	$\tilde{a}_{\eta\eta}$ [fm]	P_1	P_2	P_3
$(-1, 0)$	$(3699.1, -)$	0.81	$1.42 + i0.62$	$-$	0.93	0.04	$-$
$(-1, 100)$	$(3699.5, 12.6)$	$0.80 + i0.06$	$0.86 + i0.61$	-0.25	0.93	0.04	0.001
$(-1, 180)$	$(3694.2, 39)$	$0.73 + i0.13$	$0.62 + i0.31$	1.66	0.97	0.02	0.002
$(-0.8, 0)$	$(3725.4, -)$	1.57	$1.34 + i0.54$	$-$	0.96	0.02	$-$
$(-0.8, 100)$	$(3725.9, 5.2)$	$1.56 + i0.22$	$0.86 + i0.58$	-0.23	0.96	0.03	0.001
$(-0.8, 180)$	$(3724.5, 17.9)$	$1.23 + i0.43$	$0.61 + i0.31$	2.76	0.97	0.02	0.001



- Low-energy pole couples more strongly to the $D\bar{D}$ channel
- Increase of $a_{D\bar{D}}(\mu) \rightarrow$ peak shifted to the right and more prominent
- Increase of $v \rightarrow$ spread of the peak
- $v \neq 0 \rightarrow \tilde{a}_{D\bar{D}}$ acquires a complex contribution because the channel $D\bar{D} \rightarrow \eta\eta$ is open at the $D\bar{D}$ threshold
- Augmentation of v and the decrease of $a_{D\bar{D}} \rightarrow$ increase of $\tilde{a}_{D\bar{D}}$ and $\tilde{a}_{D_s\bar{D}_s}$
- Pole closer to the $D\bar{D}$ threshold $\rightarrow$ growth of $\tilde{a}_{D\bar{D}}$ and $\tilde{a}_{D_s\bar{D}_s}$
- $\tilde{a}_{D\bar{D}}$ of a shallow $D\bar{D}(I=0)$ bound state is larger than the typical length scale of 1 fm for the strong interactions
- P_1, P_2 and $P_3 \rightarrow$ molecular state dominantly made of the $D\bar{D}(I=0)$ component

Correlation Functions

Generalized coupled-channel CF for a specific channel i

$$
\begin{aligned}
C_i(k) &\simeq \int d^3\vec{r}\, S_{12}(\vec{r})\, |\Psi(\vec{r},\vec{k})|^2 \\
&= 1 + 4\pi\theta(\Lambda - k)\int_0^\infty dr r^2 S_{12}(\vec{r}) \left(\sum_j |j_0(kr)\delta_{ji} + T_{ji}(\sqrt{s})\widetilde{G}_j(r;s)|^2 - j_0^2(kr) \right),
\end{aligned}
$$

$\vec{k}$: relative momentum;
$E = \sqrt{s}$: the CM energy;
T_{ji}: elements of the scattering matrix encoding the meson–meson interactions;

$$
\widetilde{G}_j(r;s) = \int_{|\vec{q}|<\Lambda} \frac{d^3q}{(2\pi)^3} \frac{\omega_1^{(j)} + \omega_2^{(j)}}{2\omega_1^{(j)}\omega_2^{(j)}} \frac{j_0(qr)}{s - \left(\omega_1^{(j)} + \omega_2^{(j)}\right)^2 + i\varepsilon},
$$

($\omega_a^{(j)} \equiv \omega_a^{(j)}(k) = \sqrt{k^2 + m_a^2}$; $\Lambda = 700$ MeV);
$S_{12}(\vec{r})$: source function,

$$
S_{12}(\vec{r}) = \frac{1}{(4\pi)^{\frac{3}{2}} R^3} \exp\left(-\frac{r^2}{4R^2}\right),
$$

(R: source size parameter)

$D\bar{D}(I=0)$ system in terms of of the fixed charge states:

$$|D\bar{D}, I=0\rangle \;\; = \;\; \sqrt{\frac{1}{2}}\left[\left|D^0\bar{D}^0\right\rangle + \left|D^+D^-\right\rangle\right]$$

Experimentally accessible CFs:

$$C_{D^0\bar{D}^0}(k) \;\; = \;\; C^{(S)}_{D^0\bar{D}^0}(k),$$
$$C_{D^+D^-}(k) \;\; = \;\; C^{(S)}_{D^+D^-}(k) + C^{(C)}_{D^+D^-}(k),$$

$C^{(S)}_i(k)$: the pure strong contribution calculated with the projections of the amplitudes into the fixed charge basis;

$C^{(C)}_{D^+D^-}(k)$: calculated using the complete Coulomb wave function,

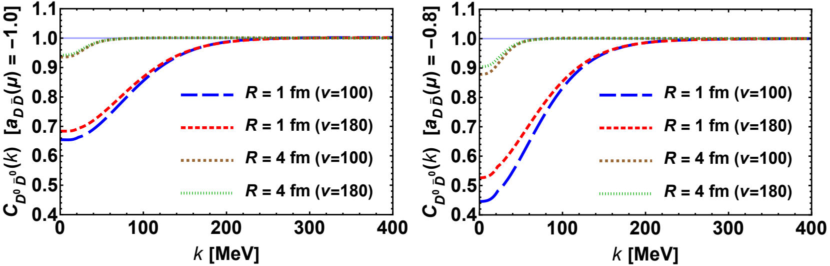
$$\Phi^{\mathrm{C}}(r,z;k) = e^{-\pi\gamma/2}\Gamma(1+i\gamma)e^{ikz}\; {}_1F_1(-i\gamma;1;ik(r-z)) \;,$$

$\Gamma(z)$: Euler gamma function

${}_1F_1(x,y;z)$ confluent hypergeometrical function

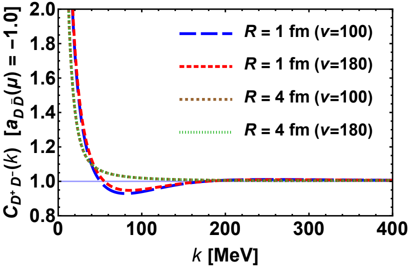
γ: Sommerfeld factor, $\gamma = Z_1 Z_2 \frac{\mu\alpha}{k}$

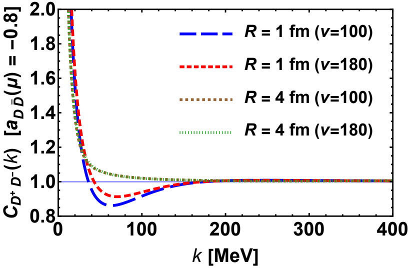
CF of the $D^0\bar{D}^0$ pair



- Low k: dip and increase with the augmentation of R → Signature of a bound state
  (Bound state: $\tilde{a}_{D\bar{D}}/R \sim 1$: CF with a stronger dip at $k \sim 0$;
  Larger R: $\tilde{a}_{D\bar{D}} < R$, and dip weakened
  Weak bound state: enhancement in the CF, similarly to the $X(3872)$ and $T_{cc}^{+}(3875)$)
- Measurements in systems with different sizes (pp, pA, and AA): help us to elucidate if this interpretation is compatible with the data
- Larger width (larger v) → weaker dip
- Pole nearer the $D\bar{D}$ threshold (higher $a_{D\bar{D}}(\mu)$) → more prominent dips → more intense correlations

CF of the D^+D^- pair





- Low k $\rightarrow$ the attractive Coulomb interaction yields a sizable enhancement and is the dominant contribution
- Strong contribution $\rightarrow$ appears only at moderate values of k by means of a dip
- Strong contribution $\rightarrow$ suppressed for large sources
- Dip $\rightarrow$ more pronounced for shallow bound states (higher $a_{D\bar{D}}(\mu)$) with smaller widths (smaller v)
- D^+D^- CF $\rightarrow$ more sensitive to the signature of the $X(3700)$ if it is a narrow and weakly bound structure, produced in a smaller source environment

Summary

- $D\bar{D}$ correlations have been studied to predict the signature of the $D\bar{D}$ [$X(3700)$] bound state in the isoscalar channel
- $X(3700)$: dynamically generated by solving the coupled-channel Bethe-Salpeter equations with the local hidden-gauge formalism
- Pole close to the $D\bar{D}$ threshold: $\tilde{a}_{D\bar{D}} < R$ grows and becomes larger than the typical length scale of 1 fm for the strong interactions
- $C_{D^0\bar{D}^0}(k)$ at low k: dip weakened with the increase of $R \rightarrow$ signature of a bound state
- $C_{D^+D^-}(k)$ at low k: Coulomb interaction $\rightarrow$ dominant contribution Strong interaction $\rightarrow$ dip at moderate values of k
- D^+D^- CF is more sensitive to the sign of the $X(3700)$ if it is a narrow and weakly bound structure, yielded in a smaller source environment
- Future measurements of the $D\bar{D}$ correlations will shed some light on the existence of the $X(3700)$ and its nature

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