Light pseudo-scalar meson mass: effect of temperature and magnetic field

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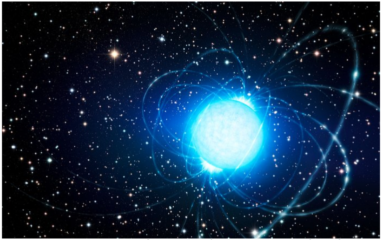
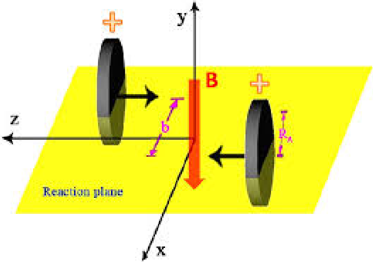
Presentation Outline

Motivation for the study of strong magnetic fields



(a) Magnetar in the star cluster Westerlund 1



(b) collision Au+Au, b=10fm, COM-Energy = $\sqrt{s}$= 200 GeV

- Magnetars: special class of neutron stars(surface field B $\sim 10^{15}$ Gauss)
- Non-central heavy-ion collisions (B $\sim 10^{20}$ Gauss)
  But probably the duration of the field is short ($\sim$ 1fm/c)
- To contribute to a better understanding of the Meson structure under strong magnetic fields
- Validate effective models used in the non-perturbative domain of QCD $\rightarrow$ exploring recent lattice QCD calculations of Meson Masses

credits: ESO, heavy-ion collision figure from: *"Electromagnetic fields and anomalous transports in heavy-ion collisions - A pedagogical review"* , Xu-Guang Huang - arxiv: 1509.04073

Effective Nambu-Jona-Lasinio (NJL) QCD Model

NJL Lagrangian as a phenomenological effective model for the QCD:
$\rightarrow$ it has to reflect the symmetries of the strong interaction!

Positive points:

- Invariant under global phase transformation $\rightarrow$ baryon number conservation
- chiral symmetric Lagrangian(in the limit of zero current quark masses)
- spontaneous chiral symmetry breaking mechanism (dynamical mass generation)
- The whole QCD phase diagram can be described using just one effective model
- The NJL model is reasonably friendly for theoretical and numerical calculations

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Negative points:

- Model is non-renormalizable (needs regularization, i. e., Λ-cutoff, part of the model)
- Interaction is not confining (no gluons or color charge)
  Meson mass is unstable against decay into a pair $q\bar{q}$ above a certain threshold.

Lagrangian Density of the Two-flavor NJL model:

$$\mathcal{L} = \overline{\psi}\left(i\not{D} - \hat{m}\right)\psi + \mathcal{L}_{int,SU(2)} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\ ,\ F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}\ ,\ D^{\mu} = (i\partial^{\mu} - QA^{\mu})\ .$$

$4 -$ point interaction : scalar $-$ isoscalar $+$ pseudoscalar $-$ isovector ,

$$\mathcal{L}_{int,SU(2)} = G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2\right]\ ,\ \psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}\ ,\ \hat{m} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}\ ,$$

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Lagrangian Density of the Three-flavor NJL model:

$$\mathcal{L} = \overline{\psi}\left(i\not{D} - \hat{m}\right)\psi + \mathcal{L}_{sym} + \mathcal{L}_{t'Hooft} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

interaction terms : $\mathcal{L}_{sym} + \mathcal{L}_{t'Hooft}$, (4 $-$ point interaction $+$ 6 $-$ point interaction)

$$\mathcal{L}_{sym} = G\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_a\psi\right)^2\right] \quad (U(N_f)_L \times U(N_f)_R \text{ symmetric})$$

$$\mathcal{L}_{t'Hooft} = -K\left[\det_f\left(\bar{\psi}\left(1+\gamma_5\right)\psi\right) + \det_f\left(\bar{\psi}\left(1-\gamma_5\right)\psi\right)\right] \quad (\text{breaks } U_A(1) \text{ symmetry})\ ,$$

$$\psi = \begin{pmatrix}\psi_u \\ \psi_d \\ \psi_s\end{pmatrix}\ ,\ \hat{m} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s\end{pmatrix}\ ,\ Q = \begin{pmatrix} q_u = 2e/3 & 0 & 0 \\ 0 & q_d = -e/3 & 0 \\ 0 & 0 & q_s = -e/3\end{pmatrix}\ ,$$

$A^{\mu} = (0, 0, Bx, 0) \rightarrow \vec{B} = B\hat{z}$(Landau gauge) and we take$m = m_u = m_d \neq m_s$.

NJL in the Mean Field Approximation

MFA $\rightarrow$ linearization of the $\mathcal{L}_{NJL}$ interaction terms disregarding quadratic fluctuations:

$$\hat{O} \equiv \langle \hat{O} \rangle + \underbrace{(\hat{O} - \langle \hat{O} \rangle)}_{\delta \hat{O}} \rightarrow \hat{O}_1 \hat{O}_2 \equiv (\langle \hat{O}_1 \rangle + \delta \hat{O}_1)(\langle \hat{O}_2 \rangle + \delta \hat{O}_2) \approx \hat{O}_1 \langle \hat{O}_2 \rangle \; + \; \langle \hat{O}_1 \rangle \hat{O}_2 \; - \; \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle \,,$$

$$\hat{O}_1 \hat{O}_2 \hat{O}_3 \approx \hat{O}_1 \langle \hat{O}_2 \rangle \langle \hat{O}_3 \rangle \; + \; \langle \hat{O}_1 \rangle \hat{O}_2 \langle \hat{O}_3 \rangle \; + \; \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle \hat{O}_3 - 2 \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle \langle \hat{O}_3 \rangle \,.$$

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Two-Flavor NJL Lagrangian Density in the MFA

$$(\overline{\psi}\psi)^2 \cong 2\left\langle\overline{\psi}\psi\right\rangle\overline{\psi}\psi - \left\langle\overline{\psi}\psi\right\rangle^2 \quad , \quad (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2 \cong 0 \quad , \quad \left(\langle\overline{\psi}i\gamma_5\vec{\tau}\psi\rangle = 0 \text{ (symmetry)}\right) \, ,$$

$$\boxed{\mathcal{L}_{NJL}^{MFA} = \overline{\psi}\left(i\not{D} - M\right)\psi - \frac{(M-m)^2}{4G} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}} \, , \; M \equiv m - 2G\left\langle\overline{\psi}\psi\right\rangle \, , \text{ CONSTITUENT QUARK MASS}$$

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$$\hat{O}_1 \hat{O}_2 \hat{O}_3 \approx \hat{O}_1 \langle \hat{O}_2 \rangle \langle \hat{O}_3 \rangle \; + \; \langle \hat{O}_1 \rangle \hat{O}_2 \langle \hat{O}_3 \rangle \; + \; \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle \hat{O}_3 - 2 \langle \hat{O}_1 \rangle \langle \hat{O}_2 \rangle \langle \hat{O}_3 \rangle \, .$$

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$$\boxed{\mathcal{L}_{NJL}^{MFA} = \overline{\psi} \left(i \not{D} - M \right) \psi - \frac{(M-m)^2}{4G} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}} \; , \; M \equiv m - 2G \left\langle \overline{\psi}\psi \right\rangle \; , \text{ CONSTITUENT QUARK MASS}$$

Three-Flavor NJL Lagrangian Density in the MFA

$$\boxed{\mathcal{L}^{MFA} = \bar{\psi} \left(\gamma_\mu \left(i \partial^\mu - Q A^\mu \right) - \hat{M} \right) \psi - 2G(\phi_u^2 + \phi_d^2 + \phi_s^2) + 4K \phi_u \phi_d \phi_s} \; .$$

$$\hat{M} = diag(M_u, M_d, M_s) \quad , \quad \phi_f \equiv \left\langle \bar{\psi}_f \psi_f \right\rangle \quad , \quad \mathrm{f} \in (\mathrm{u}, \mathrm{d}, \mathrm{s}) \, , \text{ quark condensate}$$

$$M_i = m_i - 4G\phi_i + 2K\phi_j\phi_k \quad , \quad (i, j, k) \text{ any permutation of } (\mathrm{i}, \mathrm{j}, \mathrm{k})$$

Expanding the bosonized action around the MF values:

$$S_{bos} = -\ln\det\tilde{D} + \frac{1}{4G}\int d^4x\,(\sigma(x)\sigma(x) + \vec{\pi}(x)\cdot\vec{\pi}(x)) \equiv S_{bos}^{MFA} + S_{bos}^{quad}\,,$$

$$S_{bos}^{MFA} = -\frac{N_c}{V^{(4)}}\sum_{f=u,d}\int d^4x\, d^4x'\; tr_D \ln(\mathcal{S}_f)^{-1}(x,x') + \frac{\bar{\sigma}^2}{4G}\,,$$

$$S_{bos}^{quad} = \frac{1}{2}\sum_{M=\sigma,\pi^0,\pi^\pm}\int d^4x\int d^4x'\,\delta M(x)^*\left[\frac{1}{2G}\delta^{(4)}(x-x') - J_M(x,x')\right]\delta M(x')\,,$$

$$S_{\pi^0}^{quad} = \frac{1}{2}\int d^4x\int d^4x'\;\delta\pi_0(x)^*\left[\frac{1}{2G}\delta^{(4)}(x-x') - J_{\pi^0}(x,x')\right]\delta\pi_0(x')$$

$$= \frac{1}{2}\int d^4q\;\delta\pi_0(-q)\underbrace{\left[\frac{1}{2G} - J_{\pi^0}(q_\perp^2, q_\parallel^2)\right]}_{inverse\ propagator}\delta\pi_0(q)$$

Polarization Function

$$J_{\pi^0}(q_\perp^2, q_\parallel^2) = \sum_{f=u,d} N_c\int\frac{d^4p}{(2\pi)^4}\, tr_D\left[\tilde{S}_f(p+\frac{q}{2})\,\gamma_5\tilde{S}_f(p-\frac{q}{2})\gamma_5\right] = \sum_{f=u,d} c_{ff}(q)$$

(ref. M. Coppola, D. G. Dumm and N. Scoccola, PLB 782 (2018), 155.)

SU(3)-NJL Model: Meson Pole Mass Calculation

$$S_E = \int d^4x \left[\bar{\psi}\left(-i\not{D} + \hat{m}\right)\psi - G\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_a\psi\right)^2\right] + K\left(d_+ + d_-\right)\right] ,$$

$$\psi = \left(\psi_u, \psi_d, \psi_s\right)^T \; , \; d_\pm = \det\left[\bar{\psi}\left(1 \pm \gamma_5\right)\psi\right] \text{ and } \hat{m} = \mathrm{diag}\left(m_u, m_d, m_s\right)$$

Expanding the bosonized action around the MF values:

$$S_{bos} = S_{bos}^{MFA} + S_{bos}^{quad}$$

$$S_{bos}^{MFA} = -\frac{N_c}{V^{(4)}}\sum_{f=u,d,s}\int d^4x\, d^4x'\; tr_D \ln(\mathcal{S}_f)^{-1}(x,x') + 2G\sum_{f=u,d,s}\phi_f^2 - 4K\,\phi_u\,\phi_d\,\phi_s ,$$

$$S_{bos}^{quad} = \frac{1}{2}\int d^4x'\, d^4x \sum_{P,P'} \delta P^*(x)\; \underbrace{\mathcal{G}_{P,P'}(x,x')}_{Inverse\;\; Propagator}\; \delta P'(x') , \quad P, P' = \pi_3, \pi^\pm, K^0, \bar{K}^0, K^\pm, \eta_0, \eta_8 .$$

For details see: Sidney S. Avancini, Máximo Coppola, Norberto N. Scoccola, Joana C. Sodré, Phys. Rev. D 104, 094040 (2021)

Polarization Function - Neutral Mesons

$$c_{ff'}(q) \equiv c^B_{ff'}(q_\perp^2, q_\parallel^2) = N_c \int \frac{d^4p}{(2\pi)^4} \, tr_D \left[\tilde{S}_f(p + \frac{q}{2}) \, \gamma_5 \, \tilde{S}_{f'}(p - \frac{q}{2})\gamma_5 \right] =$$

$$= 2N_c \int_0^\infty d\tau \int_0^\infty d\tau' \int \frac{d^4p}{(2\pi)^4}$$

$$\times \exp\left[-\tau\left(M^2 + p^2_{\parallel +} + p^2_{\perp +} \frac{\tanh(B_f\tau)}{\tau B_f}\right)\right] \times \exp\left[-\tau'\left(M^2 + p^2_{\parallel -} + p^2_{\perp -} \frac{\tanh(B_{f'}\tau')}{\tau' B_{f'}}\right)\right]$$

$$\times tr_D\left(\left[(M - p_{\parallel +} \cdot \gamma_\parallel) \, \Pi^f_+(\tau) - \frac{p_{\perp +} \cdot \gamma_\perp}{\cosh^2(B_f\tau)} \right] \gamma_5 \right.$$

$$\left. \times \left[(M - p_{\parallel -} \cdot \gamma_\parallel) \, \Pi^{f'}_+(\tau') - \frac{p_{\perp -} \cdot \gamma_\perp}{\cosh^2(B_{f'}\tau')} \right] \gamma_5 \right) \quad , \quad p_\parallel \equiv (p_3, p_4) \quad , \quad p_\perp \equiv (p_1, p_2) \, .$$

$$c_{ff'}(q) = \frac{N_c}{(2\pi)^2} \int_0^\infty dz \int_{-1}^1 dx \; e^{-\frac{z}{B_f}\left(M^2 + \frac{(1-x^2)}{4} \, q^2_\parallel\right)}$$

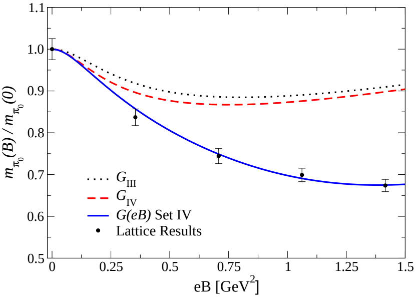
$$\times \left\{ \left(M^2 - (1 - x^2)\frac{q^2_\parallel}{4} + \frac{B_f}{z} \right) \times \coth z + \frac{1}{\sinh^2 z}\left[B_f - \frac{1}{2}\left(\coth z - \frac{\cosh zx}{\sinh z} \right) q^2_\perp \right] \right\}$$

$$c^B_{ff'}(q^2_\perp, q^2_\parallel) = \left(c^B_{ff'}(q^2_\perp, q^2_\parallel) - \lim_{B\to 0} c^B_{ff'}(q^2_\perp, q^2_\parallel) \right) + \lim_{B\to 0} c^B_{ff'}(q^2_\perp, q^2_\parallel)$$

$$= c^{mag}_{ff'}(q^2_\perp, q^2_\parallel) + \lim_{B\to 0} c^B_{ff'}(q^2_\perp, q^2_\parallel) \quad \text{(MFIR -- Magnetic Field Independent Regularization).}$$

The π^0 pole mass:

$$S^{quad}_{\pi^0} = \frac{1}{2}\int d^4q\, \delta\pi_0(-q)\left[\frac{1}{2G} - J_{\pi^0}(q_\perp^2, q_\parallel^2)\right]\delta\pi_0(q)\ ,$$
$$q_\perp^2 = 0\ ,\ q_3^2 = 0\ ,\ q_4^2 = -m^2_{\pi^0}\ , 1 - 2G\, J_{\pi^0}(0, -m^2_{\pi^0}) = 0$$



Sidney S. Avancini, Ricardo L.S. Farias, Marcus Benghi Pinto, William R. Tavares, Varese S. Timóteo, PLB 767, 247 (2017)

π^0 pole mass calculation

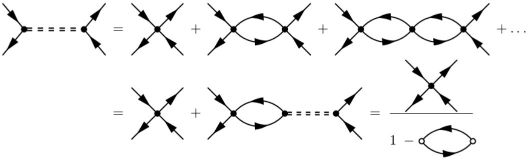


Figure: Diagrammatic representation of the RPA approximation.

$$1 - 2G\, J_{\pi^0}(q^2)|_{q^2=m^2_{\pi^0}} = 0\,.$$

$$J_{\pi^0}(q^2)|_{q_0^2=m^2_{\pi^0}} = J_1(q_0^2) + q_0^2\, J_2(q_0^2) = 2N_cN_f \int \frac{d^3p}{(2\pi)^3}\left(\frac{1}{E} + \frac{q_0^2}{E(4E^2 - q_0^2)}\right)(1 - 2\, n(E))$$

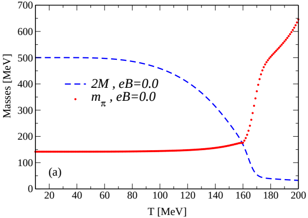
When $B > 0$

$$J_{\pi^0}(q^2, B)|_{q_0^2=m^2_{\pi^0}} = J_1(q_0^2, B) + q_0^2\, J_2(q_0^2, B)$$

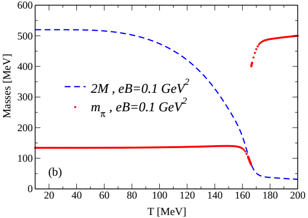
$$= N_c \sum_{f=u,d} \beta_f \sum_{n=0}^{\infty} g_n \int_{-\infty}^{\infty} \frac{dp_3}{(2\pi)^2}\left(\frac{1}{E_n} + \frac{q_0^2}{4E_n\,(4E_n^2 - q_0^2)}\right)(1 - 2\, n(E_n))\,,$$

$$E_n = \sqrt{p_3^2 + M^2 + 2n|eB|}\quad,\quad E = \sqrt{\vec{p}^2 + M^2}\quad,\quad n(E) = \frac{1}{1 + \exp(E/T)}\quad,\quad g_n = (2 - \delta_{n0})$$

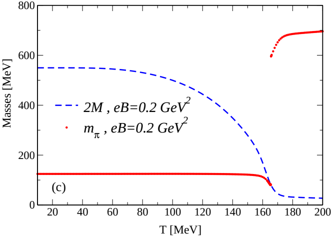
π^0 Meson Pole Mass Calculation - SU(2)-NJL Finite T Results



(a) π_0 pole mass, G(eB,T), eB=0



(b) π_0 pole mass, G(eB,T),eB=0.1GeV^2



(c) π_0 pole mass, G(eB,T), eB=0

Sidney S. Avancini, Ricardo L.S. Farias, William R. Tavares, PRD 99, 056009 (2019) neutral meson contribution to the quadratic action:

$$S^{\rm quad}_{neut.mes} = \frac{1}{2}\int_q \sum_{A=K^0,\bar{K}^0} \Pi_A^*(-q)\,\mathcal{G}_A(q_\perp^2, q_\parallel^2)\,\Pi_A(q) + \frac{1}{2}\int_q \sum_{A,A'=\pi_3,\eta_0,\eta_8} \Pi_A^*(-q)\,\mathcal{G}_{A,A'}(q_\perp^2, q_\parallel^2)\,\Pi_{A'}(q)\,. \tag{1}$$

Inverse neutral kaon propagator is given by

$$\mathcal{G}_{K^0}(q_\perp^2, q_\parallel^2) = \mathcal{G}_{\bar{K}^0}(q_\perp^2, q_\parallel^2) = [2G - K\phi_u]^{-1} + c_{ds}(q_\perp^2, q_\parallel^2) \to \mathcal{G}_{K^0}(q_\perp^2 = 0, q_\parallel^2 = -m_{K^0}^2) = 0\,.$$

$$\mathcal{M} = \begin{pmatrix} \mathcal{G}_{\pi_3\pi_3} & \mathcal{G}_{\pi_3\eta_0} & \mathcal{G}_{\pi_3\eta_8} \\ \mathcal{G}_{\eta_0\pi_3} & \mathcal{G}_{\eta_0\eta_0} & \mathcal{G}_{\eta_0\eta_8} \\ \mathcal{G}_{\eta_8\pi_3} & \mathcal{G}_{\eta_0\eta_8} & \mathcal{G}_{\eta_8\eta_8} \end{pmatrix}\,,$$

The physical meson π^0, η, η' pole-masses and widths $\to$ roots of

$$\det[\mathcal{M}(m_A, \Gamma_A)] = 0 \tag{2}$$

Sidney S. Avancini, Máximo Coppola, Norberto N. Scoccola, Joana C. Sodré, Phys. Rev. D 104, 094040 (2021)

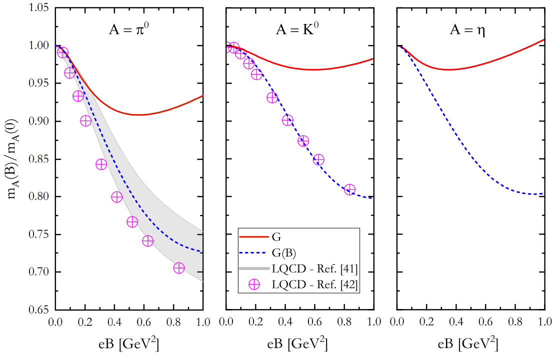
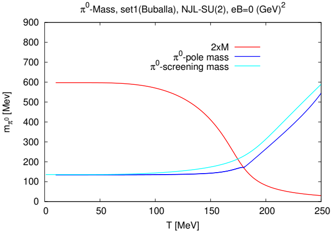


Figure: Normalized neutral meson masses as functions of *eB* for constant (red solid lines) and *B*-dependent (blue dashed lines) coupling *G*. LQCD results from Ref.[1] (grey band) and Ref.[2] (magenta circles) are added for comparison.

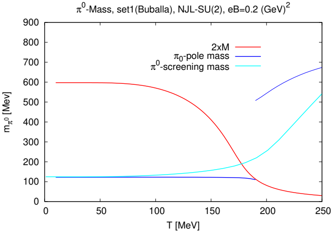
[1] G. Bali, B. Brandt, G. Endrődi, and B. Gläßle, Phys. Rev. D 97, 034505 (2018)
[2] H.-T. Ding, S.-T. Li, A. Tomiya, X.-D. Wang, and Y. Zhang, Phys. Rev. D 104, 014505 (2021).

The π^0 screening mass:

$$\frac{1}{2G} - J_{\pi^0}(q_\perp^2, q_\parallel^2) = 0 \ , \ \begin{cases} m_{sc,\parallel} \rightarrow q_3^2 = -m_{sc,\parallel}^2 \ , \ q_4^2 = 0 \ , \ q_\perp^2 = 0, \\ m_{sc,\perp} \rightarrow q_\perp^2 = -m_{sc,\perp}^2 \ , \ q_4^2 = 0 \ , \ q_3^2 = 0 \end{cases} .$$



(a) π_0 pole and parallel screening masses, eB=0



(b) π_0 pole and parallel screening masses, eB=0.2GeV^2

W. Florkowski and B. L. Friman, Acta Physica Polonica B, vol. 25(1994), p.49.

Masahiro Ishii et al, PRD 89 (2014)), 071901R (the formalism is exposed in a clear way).

Maximo Coppola, William Tavares, Sidney S. Avancini, Norberto Scoccola, Joana C. Sodré, PRD 110 (2024), 114036.

see Joana′s poster for details!

Conclusions

- The light pseudo-scalar meson masses in a strong magnetic field were obtained from the expansion of the bosonized Euclidean action (RPA calculation).
- For calculating the polarization functions, we use (for most of the calculations) the Schwinger proper-time formalism (performs the sum over the Landau levels analytically).
- We always use the MFIR regularization procedure, thus avoiding spurious solutions which are often found in the literature.
- Our calculation, when it is possible to compare, shows the same trend as the lattice.

Collaborators (in Magnetized Matter):

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