Impact of rotational effects on the QCD phase diagram: chiral vortical catalysis constrained by Lattice QCD

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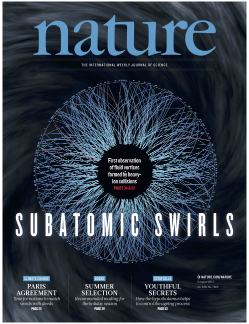
The most vortical system so far observed!



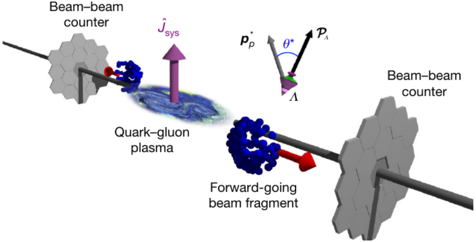


Figure: Ref: The STAR collaboration. Nature 548 (2017) 62-65.

- Alignment between global angular momentum and the spin of emitted particles (Λ and $\bar{\Lambda}$ hyperons).

The most vortical system so far observed!

By using the hydrodynamical relation[1], we obtain the estimated angular velocity for QGP

$$\omega \approx \frac{k_B T(\mathcal{P}_{\Lambda'} + \mathcal{P}_{\Lambda})}{\hbar} \sim (9 \pm 1) \times 10^{21} s^{-1}, \quad (1)$$

This value far surpasses the vorticity of other fluids[2]

- solar subsurface flow $\rightarrow \omega \sim 10^{-7} s^{-1}$;
- super-cell tornado cores $\rightarrow \omega \sim 10^{-1} s^{-1}$;
- the great red spot of Jupiter $\rightarrow \omega \sim 10^{-4} s^{-1}$;
- turbulent flows in bulk superfluid He II, up to $\omega \sim 150 s^{-1}$;
- super fluids nanodroplets $\omega \sim 10^{7} s^{-1}$;

[1] Becattini, F., Karpenko, I., Lisa, M., Upsal, I. and Voloshin, S. Global hyperon polarization at local thermodynamic equilibrium with vorticity, magnetic field, and feed-down. Phys. Rev. C 95, 054902 (2017).

[2] The STAR collaboration. Nature 548 (2017) 62-65.

How the QCD phase diagram changes with rotation?

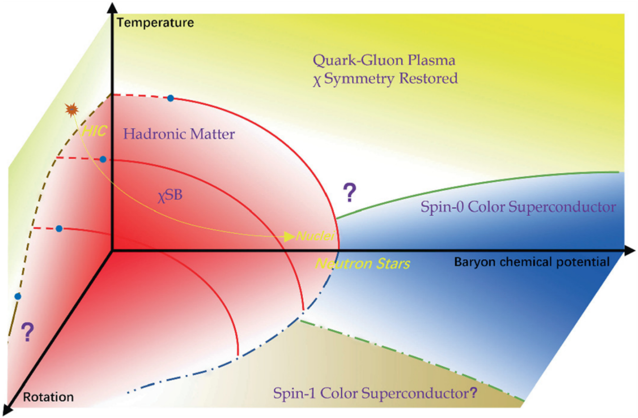


Figure: Ref:Lect.Notes Phys. 987 (2021) 349-379.

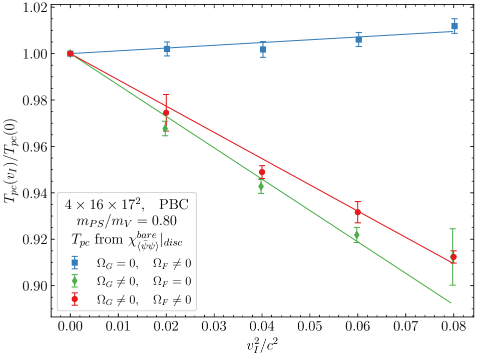
What does LQCD tell us about vortical effects?

In lattice QCD ($N_f = 2$, clover-improved Wilson fermions)[3] the full action with imaginary angular velocities **(sign problem)** is given by

$$S = S_G[\Omega_G] + S_f[\Omega_f]. \qquad (2)$$

Different regimes of rotation are studied:

- Only quarks ($\Omega_G = 0$ and $\Omega_f \neq 0$);
- Only gluons ($\Omega_G \neq 0$ and $\Omega_f = 0$);
- All contributions ($\Omega_G \neq 0$ and $\Omega_f \neq 0$);



Chiral pseudocritical temperature as a function of the imaginary angular velocity.

[3] V. V. Braguta, A. Yu. Kotov, A. A. Roenko, and D. A. Sychev, PoS LATTICE2022 (2023) 190

What does LQCD tell us about vortical effects?

The LQCD data[4] for the chiral pseudocritical temperature transition are well described by

$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 - B_2 v_I^2, \tag{3}$$

where the coefficient B_2 depends on the ratio $M_{PS}/M_V = 0.65, 0.7, 0.75, 0.8$. Upon analytic continuation to real angular velocity, one can obtain

$$\frac{T_{pc}(v)}{T_{pc}(0)} = 1 + B_2 v^2, \tag{4}$$

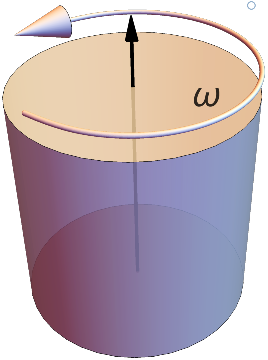
then, the pseudocritical temperature of the chiral phase transition **increases as a function of the angular velocity**.

[4] V. V. Braguta, A. Yu. Kotov, A. A. Roenko, and D. A. Sychev, PoS LATTICE2022 (2023) 190

Basic Theoretical aspects

1. Boundary conditions[5,6,7,8]: **infinite size approximation**[8].
2. $\omega R \leqslant 1$ to guarantee **causality**[5,6,7,8].
3. The chiral condensate $\langle\bar{\psi}\psi\rangle$ is **homogeneous**[9].
4. The metric tensor of the system is given by[9]

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\omega^2 & y\omega & -x\omega & 0 \\ y\omega & -1 & 0 & 0 \\ -x\omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$



Fermionic homogeneous medium, inside a cylinder with radius R and constant angular velocity ω.

[5] S. Ebihara, K. Fukushima, K. Mameda. Phys.Lett.B 764 (2017) 94-99
[6] M.N. Chernodub, Shinya Gongyo. Phys.Rev.D 95 (2017) 9, 096006
[7] Minghua Wei, Mei Huang. Chin.Phys.C 47 (2023) 10, 104105
[8] Yin Jiang, and Jinfeng Liao.Phys.Rev.Lett. 117 (2016) 19, 192302
[9] M.N. Chernodub and Shinya Gongyo. JHEP 01 (2017) 136

SU(2) Nambu–Jona-Lasinio model

The lagrangian of the two-flavor NJL model within a rigid cylinder with constant angular velocity is given by

$$\mathcal{L} = \overline{\psi}\left[i\bar{\gamma}^{\mu}(\partial_{\mu} + \Gamma_{\mu}) - \tilde{m} + \gamma^{0}\mu\right]\psi + G\left[(\overline{\psi}\psi)^{2} + (\overline{\psi}i\gamma_{5}\vec{\tau}\psi)^{2}\right], \tag{5}$$

- the current quark masses matrix $\tilde{m}$ =diag(m_u, m_d) in the isospin symmetry approximation, $m_u = m_d = m$; τ are the Pauli matrices; ψ is the spinor representing the quark fields $\psi = (\psi_u \quad \psi_d)^T$; G is the coupling constant and μ is the quark chemical potential.
- The affine connection is Γ_{μ} and $\bar{\gamma}^{\mu} = e_i^{\mu}\gamma^i$, where γ^i is the usual gamma matrices and e_i^{μ} the *vierbein*.
  Recommended References: M.N. Chernodub and Shinya Gongyo. JHEP 01 (2017) 136

  Yin Jiang, and Jinfeng Liao.Phys.Rev.Lett. 117 (2016) 19, 192302

Thermodynamical potential

The thermodynamical potential in the mean field approximation is given by

$$\begin{aligned}\Omega_{NJL} &= \frac{(M-m)^2}{4G} - \frac{3}{2\pi^2}\sum_{n=-\infty}^{\infty}\int_0^{\Lambda} p_t dp_t \int_{-\sqrt{\Lambda^2-p_t^2}}^{\sqrt{\Lambda^2-p_t^2}} dp_z \mathcal{J}_n(p_t,r)\epsilon_n \\ &- \frac{3}{2\pi^2}\sum_{n=-\infty}^{\infty}\int_0^{\infty} p_t dp_t \int_{-\infty}^{\infty} dp_z \mathcal{J}_n(p_t,r) T\left[\ln\left(e^{-\frac{\epsilon_n+\mu}{T}}+1\right)+\ln\left(e^{-\frac{\epsilon_n-\mu}{T}}+1\right)\right],\end{aligned}$$

where we define $\mathcal{J}_n(p_t,r) = \left(J_{n+1}(p_t r)^2 + J_n(p_t r)^2\right)$ with $J_n(x)$ being the Bessel function of the first kind and $\epsilon_n = \sqrt{M^2+p_z^2+p_t^2} - (\frac{1}{2}+n)\omega$ is the quark energy dispersion. The gap equation is given by $\partial\Omega/\partial M = 0$.

Mismatch between LQCD and Effective Models

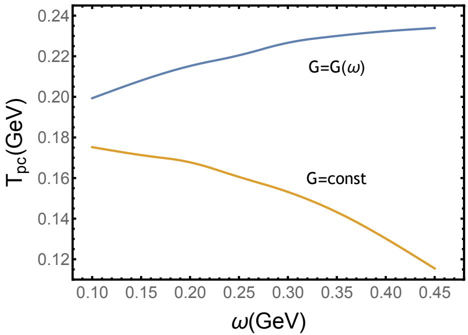
Effective models that lack gluonic degrees of freedom, (e.g., LSMq and NJL) can mismatch the $T_{pc} \times \omega$ relation with LQCD. In order to improve the models, it has been suggested:

- Corrections on the coupling constant (NJL), i.e., G=const$\longrightarrow G = G(\omega)$, as[10]

$$G(\omega) = G_0 \left(1 + 0.32 \frac{\omega}{\Lambda_{NJL}}\right),$$

which induces the recently dubbed chiral vortical catalysis[10] effect.

chiral vortical catalysis (blue line)

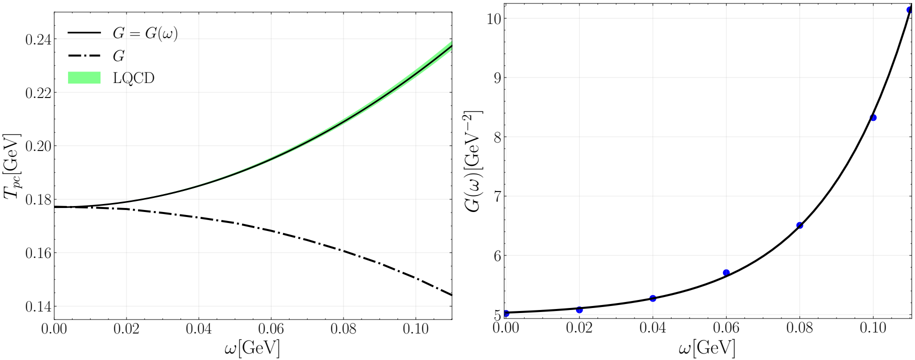


Pseudocritical temperature of chiral phase transition as a function of the angular velocity.

[10] Yin Jiang. Eur.Phys.J.C 82 (2022) 10, 949

Effective coupling and Parametrizations

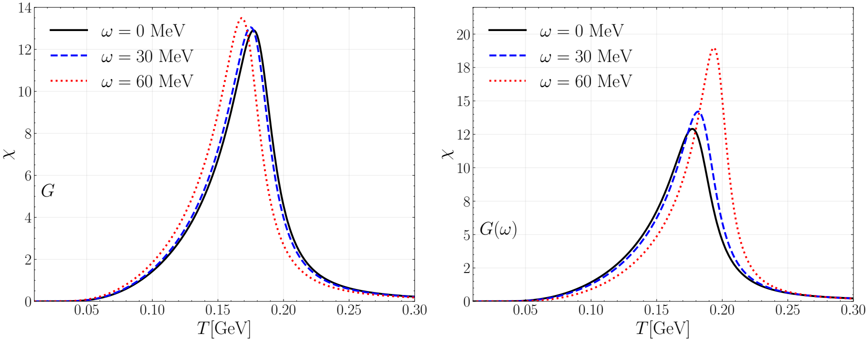
$$G \to G(\omega) = G_\alpha + G_\beta \exp\left(\frac{\omega}{\Omega}\right) \ , \quad (6)$$



R.M. Nunes, R.L.S. Farias, **W.R. Tavares**, V.S. Timóteo. ArXiv: 2412.14541 [hep-ph]. **To appear in PRD.**

Effective coupling and Parametrizations

$$\chi = \frac{1}{m_\pi}\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial T} \tag{7}$$



R.M. Nunes, R.L.S. Farias, **W.R. Tavares**, V.S. Timóteo. ArXiv: 2412.14541 [hep-ph]. **To appear in PRD.**

Effective coupling and Parametrizations

We propose a coupling that depends on the angular velocity as

$$G \to G(\omega) = G_\alpha + G_\beta \, \exp\left(\frac{\omega}{\Omega}\right) . \tag{8}$$

G_α [GeV^{-2}]	G_β [GeV^{-2}]	Ω [GeV]	R [GeV^{-1}]	ω_{max} [GeV]
4.97667	0.05840	0.02457	5.00	0.2

The set of parameters with 3D sharp cutoff are given by:

Λ [MeV]	$G(0)$ [GeV^{-2}]	m [MeV]	$\langle \bar{u}u \rangle^{1/3}$ [MeV]	f_π [MeV]	m_π [MeV]
651	5.04	5.5	−251.322	94.03	141.715

R.M. Nunes, R.L.S. Farias, **W.R. Tavares**, V.S. Timóteo. ArXiv: 2412.14541 [hep-ph]. **To appear in PRD.**

Numerical Results

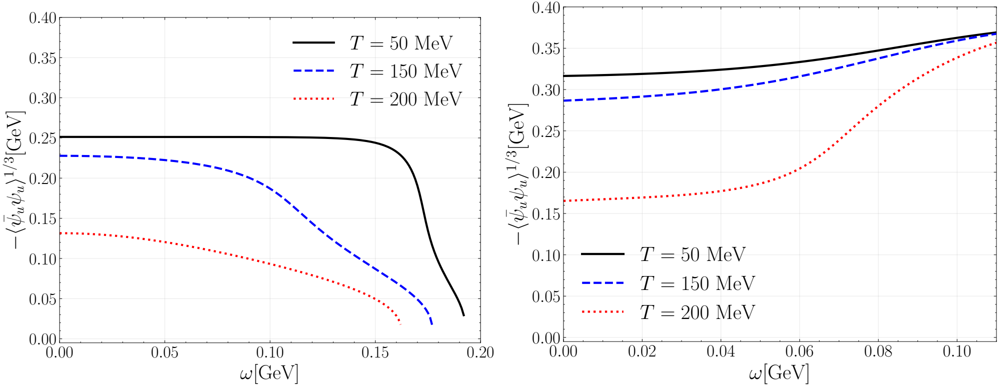


Figure: Chiral condensate as a function of the angular velocity with constant G (left) and running $G(\omega)$ (right).

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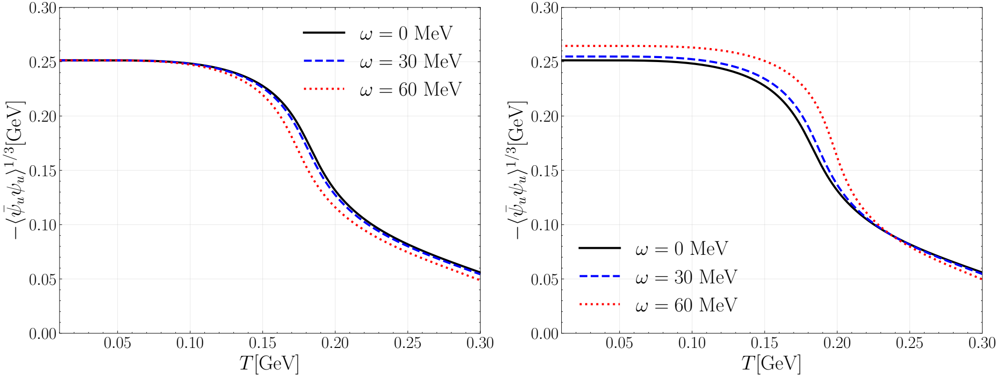

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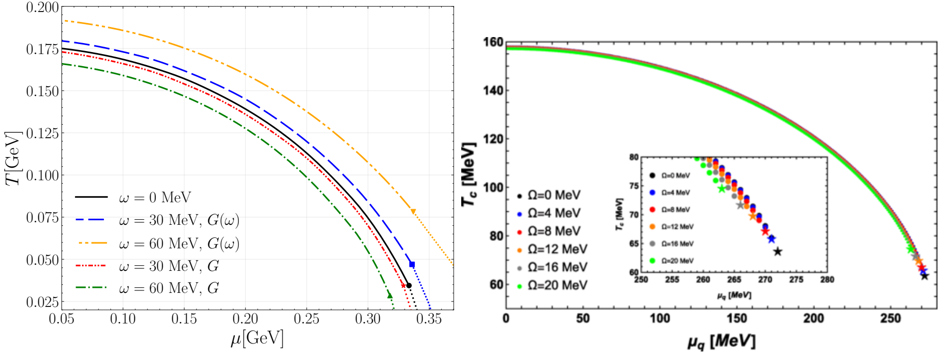

Figure: Phase diagram of NJL model for different angular velocities with constant coupling G and $G(\omega)$[11] (left). Phase diagram of LSMq model for different angular velocities[12] (right).

[11] R.M. Nunes, R.L.S. Farias, **W.R. Tavares**, V.S. Timóteo. ArXiv: 2412.14541 [hep-ph]. **To appear in PRD.**

[12] L. A. Hernández and R. Zamora. Phys.Rev.D 111 (2025) 3, 036003

Thanks for your attention!

Collaborators:

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- Rodrigo M. Nunes (Ph.D student at UFSM)

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Main Reference:

- R.M. Nunes, R.L.S. Farias, **W.R. Tavares**, V.S. Timóteo. ArXiv: 2412.14541 [hep-ph]. **To appear in PRD.**