Minkowski space description: nucleon and pion

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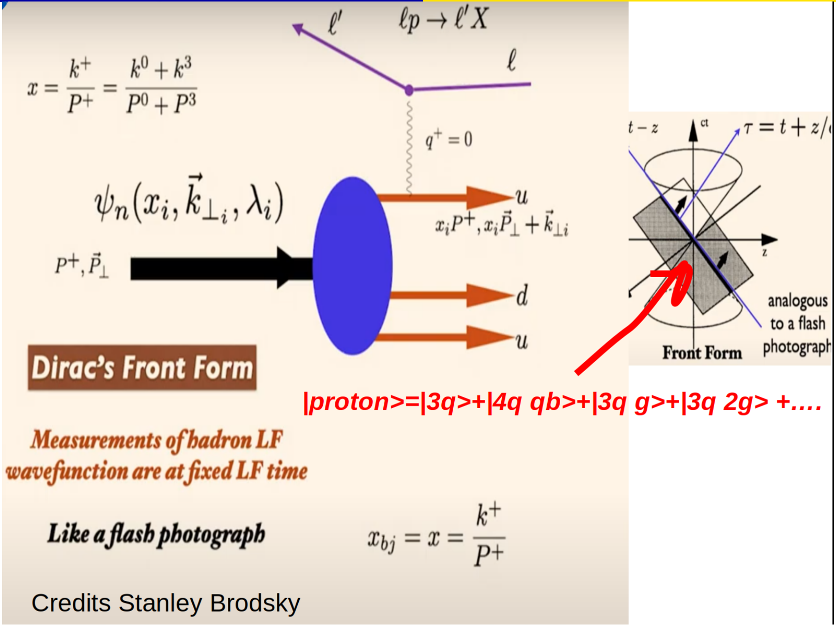
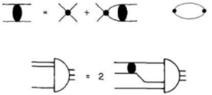
Hadrons 2025: XVI International Workshop on Hadron Physics

UFRGS, Porto Alegre, March 10 to 14, 2025


$\ell p \to \ell' X$
ℓ'
ℓ
$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$
$q^+ = 0$
$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$
$P^+, \vec{P}_\perp$
u
$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$
d
u
Dirac's Front Form
|proton>=|3q>+|4q qb>+|3q g>+|3q 2g> +....
$t - z$
ct
z
analogous
to a flash
photograph
Front Form
Measurements of hadron LF
wavefunction are at fixed LF time
Like a flash photograph
$x_{bj} = x = \frac{k^+}{P^+}$
Credits Stanley Brodsky

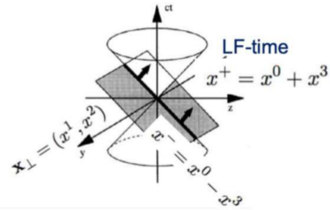
Nucleon Structure in Minkowski space

& LF Fock space decomposition of the nucleon state



$$v(q,p) = 2iF(M_{12}) \int \frac{d^4k}{(2\pi)^4} \frac{i}{[k^2 - m^2 + i\epsilon]} \times \frac{i}{[(p-q-k)^2 - m^2 + i\epsilon]} v(k,p).$$

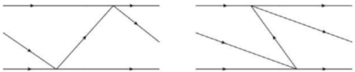
T. Frederico, Phys. Lett. B 282 (1992) 409.



|proton>=|3q>+|4q qb>+|5q 2qb > +....

E. Ydrefors et al. / Physics Letters B 770 (2017) 131–137

|3q>



|4q qb>

Fig. 2. The three-body LF graphs obtained by time-ordering of the Feynman graph shown in right panel of Fig. 1.

|4q qb>

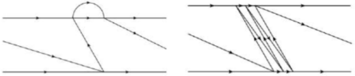


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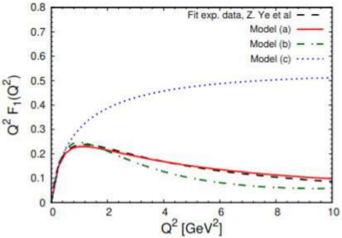
Fig. 3. Examples of many-body intermediate state contributions to the LF three-body forces.

Ydrefors and TF, PLB **838**, 137732 (2023)

Electromagnetic form factor



Model	m [MeV]	a $[m^{-1}]$	μ/m	M_{dq} [MeV]
(a)	366	2.70	1	644
(b)	362	3.60	∞	682
(c)	317	-1.84	∞	-

Valence Proton PDF @ initial and experimental scale

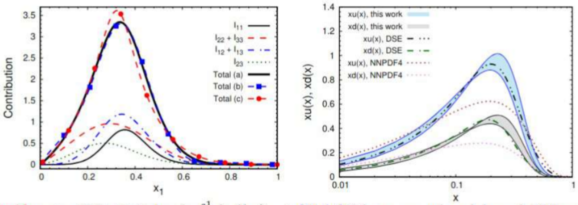
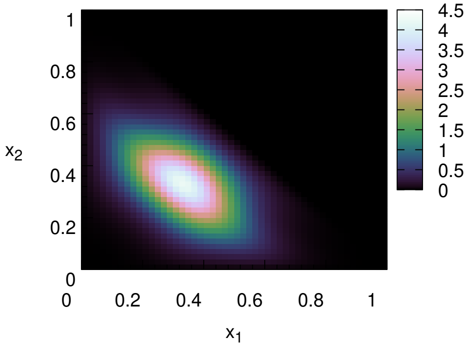


Figure: [Left] proton PDF at initial scale $\int_0^1 dx_1 f(x_1) = 1$. [Right] Valence u-quark and d-quark PDFs evolved to $Q = 3.097\,\text{GeV}$, compared with the DSE results of Lu et al (2203.00753 [hep-ph]) and the results of the NNPDF4 global fit. The shaded areas indicate the uncertainty with respect to the initial scale $Q_0 = 0.33 \pm 0.03\,\text{GeV}$.

Valence Proton Double PDF

$$D_3(x_1, x_2; \vec{\eta}_\perp) = \frac{1}{(2\pi)^6} \int d^2k_{1\perp} d^2k_{2\perp} \Psi_3^\dagger(x_1, \vec{k}_{1\perp} + \vec{\eta}_\perp; x_2, \vec{k}_{2\perp} - \vec{\eta}_\perp; x_3, \vec{k}_{3\perp}) \Psi_3(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}; x_3, \vec{k}_{3\perp}).$$

- Fourier transform in $\vec{\eta}_\perp$: probability of quarks 1 and 2 for x_1 and x_2 with a separation in the transverse direction $\vec{y}_\perp$.
- $D_3 = 0$ for $x_1 + x_2 > 1$ - momentum conservation. (Below $\vec{y}_\perp = \vec{0}_\perp$)



Ioffe-time image - valence state

NS FROM … PHYS. REV. D **104**, 114012 (2021)

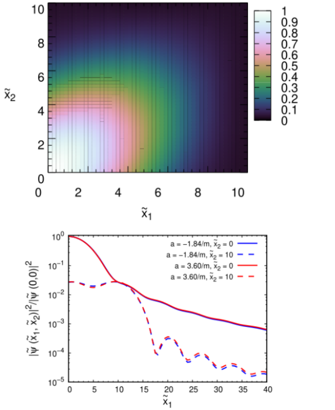
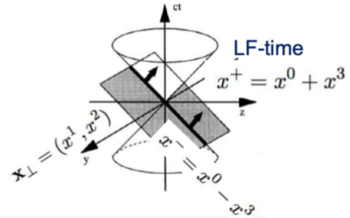


FIG. 3. Upper panel: squared modulus of the Ioffe-time distribution as a function of $\tilde{x}_1$ and $\tilde{x}_2$, for the model I. Lower panel: squared modulus of the Ioffe-time distribution as a function of $\tilde{x}_1$ for two fixed values of $\tilde{x}_2$, namely $\tilde{x}_2 = 0$ (solid line) and $\tilde{x}_2 = 10$ (dashed line). Results shown for the model I (blue line) and model II (red line).

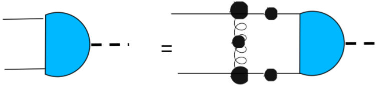


$$\Phi(\tilde{x}_1, \tilde{x}_2) \equiv \tilde{\Psi}_3(\tilde{x}_1, \vec{0}_\perp, \tilde{x}_2, \vec{0}_\perp)$$
$$= \int_0^1 dx_1 e^{i\tilde{x}_1 x_1} \int_0^{1-x_1} dx_2 \int_0^1 dx_3$$
$$\times \delta(1 - x_1 - x_2 - x_3) e^{i\tilde{x}_2 x_2} \phi(x_1, x_2, x_3).$$

W. de Paula, Ydrefors, Alvarenga Nogueira, TF Salmè, PRD 103, 014002 (2021)

How we model:

BSE quark-antiquark & pion model



Ladder approximation (L): suppression of XL for Nc=3 in a bosonic system
[A. Nogueira, CR Ji, Ydrefors, TF, PLB777(2017) 207]

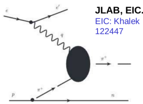
- dressed quark propagator (mass =255MeV) $S(P) = \frac{i}{\not{P} - m + i\epsilon}$
- dressed gluon propagator (mass =637MeV) $i\mathcal{K}_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2 - \mu^2 + i\epsilon}$
- dressed quark-gluon vertex (306 MeV) $\lambda_1 \gamma_\mu \quad F(q) = \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon}$

- Model parameters: quark and gluon masses & quark-gluon vertex

SOLUTION IN MINKOWSKI SPACE [pion mass→ g]

off-shell π:
Leão, de Melo, TF, Choi, Ji, PRD 110, 074035 (2024)

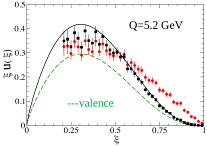
How to look?



JLAB, EIC...
EIC: Khalek et al. NPA 1026 (2022) 122447

FIG. 1. Sullivan process: $ep \to e'\pi^+n$ scattering. The black blob represents the half-on-mass shell photo absorption amplitude. Diagrammatic representation of the pion pole amplitude for $p(e,e')\pi^+n$ process.

Comparison with experimental data



[32] J. Conway *et al.*, Experimental Study of Muon Pairs Produced by 252-GeV Pions on Tungsten, Phys. Rev. D **39**, 92 (1989).

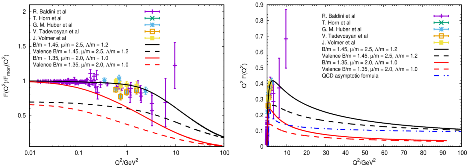
[33] M. Aicher, A. Schäfer, and W. Vogelsang, Soft-gluon resummation and the valence parton distribution function of the pion, Phys. Rev. Lett. **105**, 252003 (2010), arXiv:1009.2481 [hep-ph].

FIG. 2. (Color online). The distribution function $\xi\ u(\xi)$ in a pion. Solid line: full calculation (see Eqs. (7) and (8)), obtained from the BS amplitude solution of the BSE with $m = 255$ MeV, $\mu = 637.5$ MeV and $\Lambda = 306$ MeV, and evolved from the initial scale $Q_0 = 0.360$ GeV to $Q = 5.2$ GeV (see text). Dashed line: the evolved LF valence component, Eq. (9). Full dots: experimental data from Ref. [32]. Full squares: reanalyzed data by using the ratio between the fit 3 of Ref. [33], evolved to 5.2 GeV, and the experimental data [32], at each data point, so that the resummation effects (see text) are accounted for.

π-PDF: de Paula, et al PRD 105, L071505 (2022)

Pion EM Form Factor

Alvarenga Nogueira, de Paula, TF, Ydrefors, Salmè, PLB *820, 136494 (2021)*



$Q^2 F_{\text{asymp}}(Q^2) = 8\pi\alpha_s(Q^2) f_\pi^2$ G. Lepage, S. J. Brodsky Phys. Lett. B 87 (1979) 359

Decomposition of the pion EM form factor

$$F_\pi(Q^2) = \sum_n F_n(Q^2) = F_{val}(Q^2) + F_{nval}(Q^2)$$

qq+gluons

$$r_\pi^2 = P_{val}\, r_{val}^2 + (1 - P_{val})\, r_{nval}^2$$

r_π (fm)	r_{val} (fm)	r_{nval} (fm)
0.663	0.710	0.538

0.657 ± 0.003 fm B. Ananthanarayan, I. Caprini, D. Das, Phys. Rev. Lett. 119 (2017) 132002

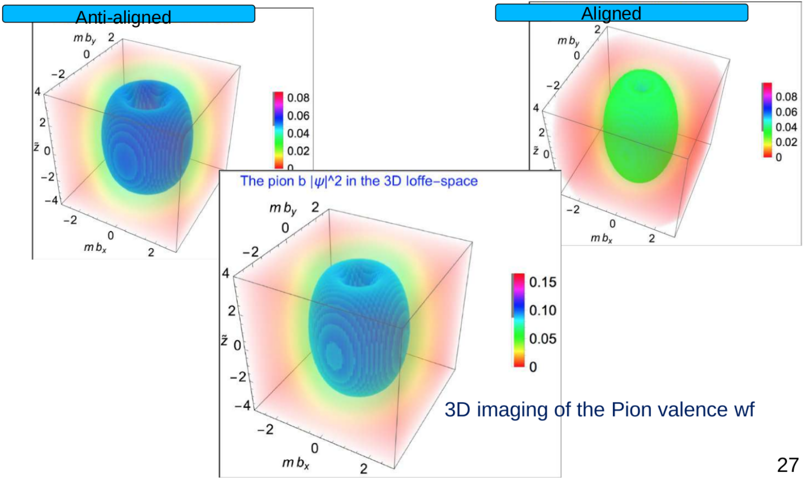
higher Fock-components → large virtuality → more compact

Kharzeev, "Mass radius of the proton" PRD104, 054015 (2021)

$R_m = 0.55 \pm 0.03$ fm $\quad R_C = 0.8409 \pm 0.0004$ fm

3D Pion image on the null-plane: Spin configurations

Space-time structure of the pion in terms o $\tilde{z} = x^{-} p^{+}/2$ and transverse coord. $\{b_x, b_y\}$

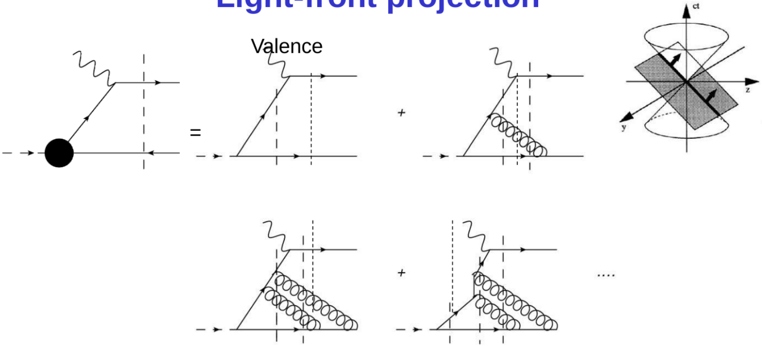


3D imaging of the Pion valence wf

27

Bethe-Salpeter amplitude: beyond the valence states

Light-front projection



- **higher Fock-components** $|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}\,2g\rangle + \cdots$
- **gluon radiation from initial state interaction (ISI)**
-

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001);
Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010(2008)

Pion mass distribution

de Paula, TF, Salmè, EPJC 83, 985 (2023)

Gluon momentum in the pion

$$|\pi\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}\,2g\rangle + \cdots$$

quark momentum distribution

$$u^q(\xi) = \sum_{n=2}^{\infty} \left\{ \prod_i^n \int \frac{d^2k_{i\perp}}{(2\pi)^2} \int_0^1 d\xi_i \right\} \times \delta\left(\xi - \xi_1\right) \delta\left(1 - \sum_{i=1}^n \xi_i\right) \delta\left(\sum_{i=1}^n \mathbf{k}_{i\perp}\right) \times \left|\Psi_n(\xi_1, \mathbf{k}_{1\perp}, \xi_2, \mathbf{k}_{2\perp}, ...)\right|^2,$$

first-moment

$$\langle \xi_q \rangle = P_{val}\ \langle \xi_q \rangle_{val} + \sum_{n>2} P_n\ \langle \xi_q \rangle_n$$

0.471 0.5

$$= P_{val}\ \langle \xi_q \rangle_{val} + (1 - P_{val})\ \langle \xi_q \rangle_{HFS}$$

P_{val}=0.7 0.4

momentum sum-rule in the HFS

$$\langle \xi_q \rangle_{HFS} = 1 - \langle \xi_{\bar{q}} \rangle_{HFS} - \langle \xi_g \rangle$$

0.2

Gluons carry 6% of the longitudinal momentum of the pion!

@ the pion scale

Summary and Prospects

- BSE in Minkowski space: proton and pion
    - PDFs, EM FF, TMDs, Ioffe-time Image

Prospects:

- K, D, B, ρ, Nucleon (spin)...
- T-odd TMDS, GTMDs (SL & TL), GFF...
- dressed constituents in BS equation - **Castro et al PLB845 (2023) 138159**
- Gluon exchange & dressed vertices
- Integral representation to solve the FBS equation
- Expand the applicability of Quantum algorithm for solving the pion BS & FBS equations **Fornetti, et al, PRD 110 (2024) 056012**
- Confinement?

THANK YOU!