Locating the QCD critical point via contours of constant entropy density

Hitansh Shah, Mauricio Hippert, Jorge Noronha,
Claudia Ratti and Volodymyr Vovchenko

arXiv:2410.16206

12 de Março de 2025

QCD phase transition

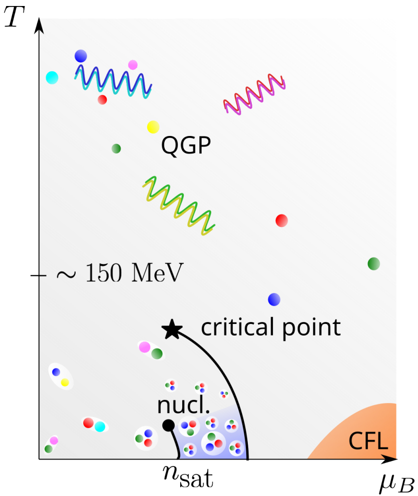
- Extreme temperatures and densities

  $\rightarrow$ color deconfinement

  J. C. Collins and M. J. Perry, PRL **34** (1975)
  N. Cabibbo and G. Parisi, PLB **59** (1975)

- Smooth change for $\mu_B = 0$.

  S. Borsanyi *et al.*, JHEP **09** (2010)
  Aoki, Endrodi, Fodor, Katz, Szabo, Nature **443** (2006)



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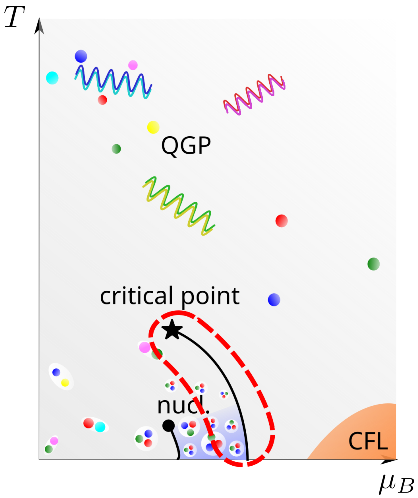

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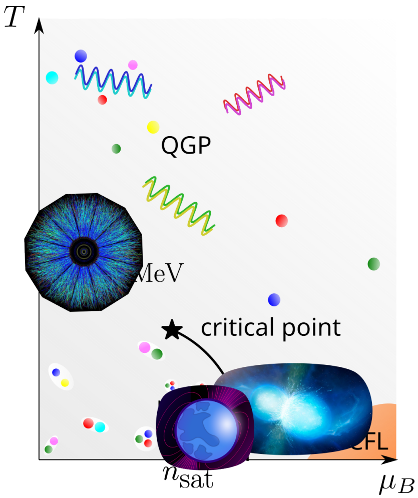
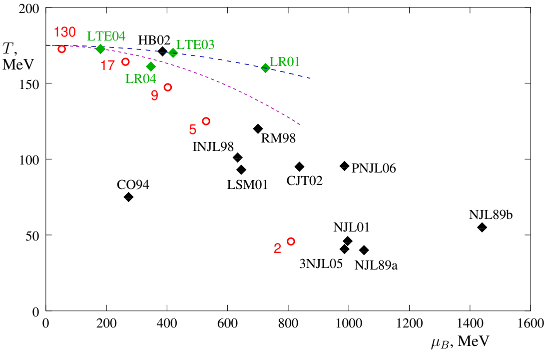
- *Fundamental physics problem!*
  → Empirical consequences?



Predictions for the QCD critical point (CP)

- 2006: Predictions scattered around huge region.



M. A. Stephanov, PoS **LAT2006** (2006) arXiv:hep-lat/0701002.

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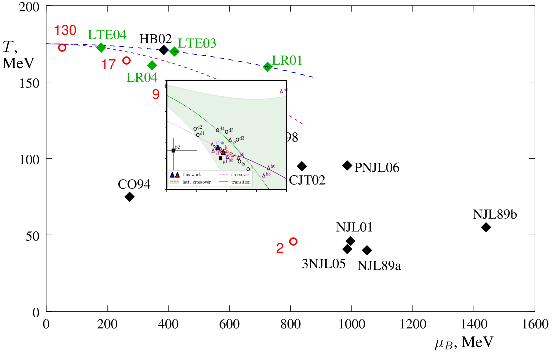

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h3	PRD 96 (2017) 9, 096026
h4	PRD 106 (2022) 12, L121902
h5	Bayesian + lattice constraints!
h6	PRD 109 (2024) 5, L051902
h7	e-Print: 2404.12109
h8	PRD 110 (2024) 12, 126013
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d1	PRD 90 (2014) 3, 034022
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p1	PRC 110 (2024) 1, 015203
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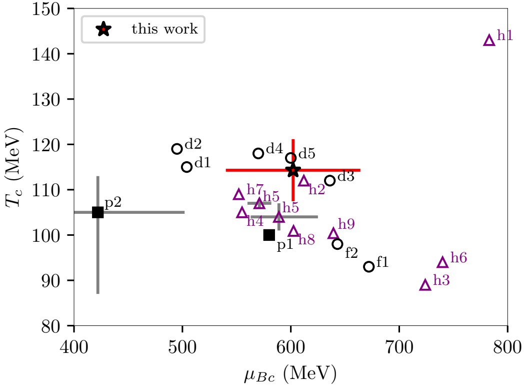

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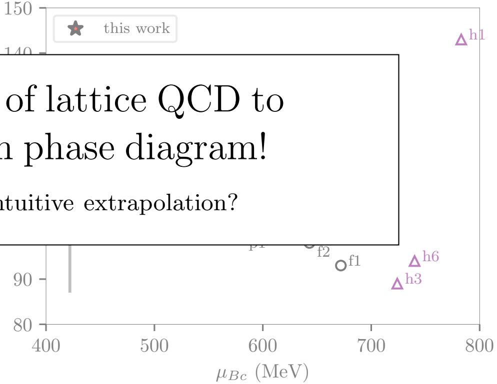


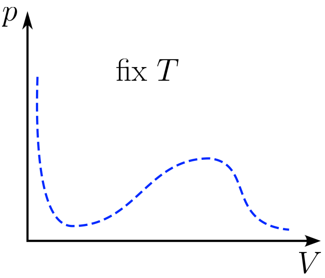
Potential of lattice QCD to constrain phase diagram!

Simpler intuitive extrapolation?

MH, J. Grefa, T. A. Manning, J. Noronha, J. Noronha-Hostler, I. Portillo Vazquez, C. Ratti, R. Rougemont and M. Trujillo, PRD **110** (2024)

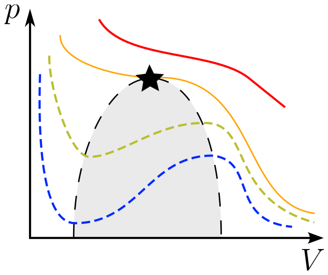
Van der Waals equation of state

- Metastable phases and unstable region.



Van der Waals equation of state

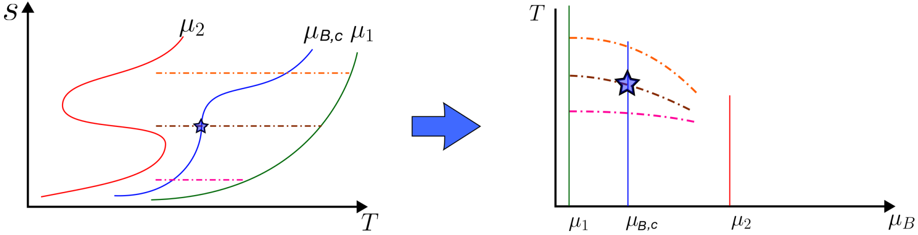
- Metastable phases and unstable region.
- High temperatures ⇒ critical point.
- Spinodal curve: limit of metastability.



Entropy-density contours

Entropy density s *vs.* T:

T *vs.* μ_B phase diagram:



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H. Shah, MH, J. Noronha, C. Ratti, V. Vovchenko arXiv:2410.16206.

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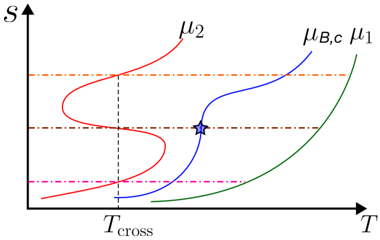

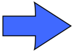

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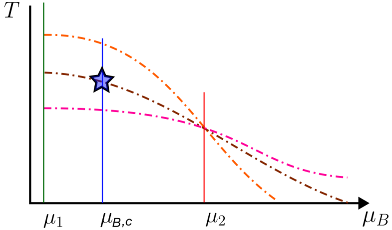

- Horizontal lines $\Rightarrow$ constant s contours.
- 1st–order region $\Rightarrow (s,T) \nleftrightarrow (\mu_B, T) \Rightarrow$ contours cross!

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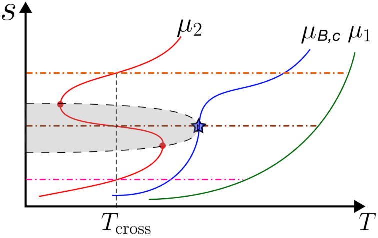

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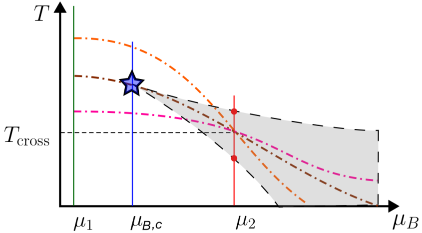

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- Spinodal curve: $(dT/ds)_\mu = 0$. Critical point: also $(d^2T/ds^2)_\mu = 0$.

(crossing of infinitesimally close contours)

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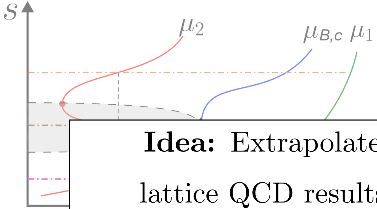

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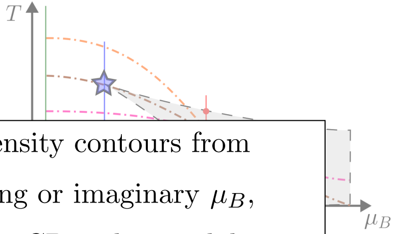

Idea: Extrapolate entropy density contours from lattice QCD results at vanishing or imaginary μ_B, then use crossing conditions for CP and spinodals.

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New expansion

- Phase transition possible for analytical $T(\mu_B, s) = T_s(\mu_B, T_0)$!
- Polynomial extrapolation of contour $T = T_s(\mu_B; T_0)$ with $s = s_0$.
- From the Taylor expansion of $T(\mu_B, s)$,

$$T_s(\mu_B; T_0) \approx T_0 + \sum_{n=1}^{N} \alpha_{2n}(T_0) \frac{\mu_B^{2n}}{(2\,n)!} + \mathcal{O}\left(\mu_B^{2(N+1)}\right), \qquad (1)$$

H. Shah, MH, J. Noronha, C. Ratti, V. Vovchenko arXiv:2410.16206.

Resummation of usual Taylor series with $s = dp/dT$

$$p(\mu_B, T) \approx \sum_{n=1}^{N} \chi_{2n}(T) \frac{\mu_B^{2n}}{(2\,n)!} + \mathcal{O}\left(\mu_B^{2(N+1)}\right)$$

See also Borsányi, Fodor, Guenther et al., PRL **126** (2021)

Proof of principle: $\mathcal{O}(\mu_B^2)$

- Quadratic coefficient:

$$\alpha_2(T_0) = -\frac{2T_0\chi_2^B(T_0) + T_0^2\chi_2^{B'}(T_0)}{s'(T_0)}$$

- Sample thermodynamics from lattice errorbars.

Borsanyi, Fodor, Hoelbling et al., PRL **730** (2014)
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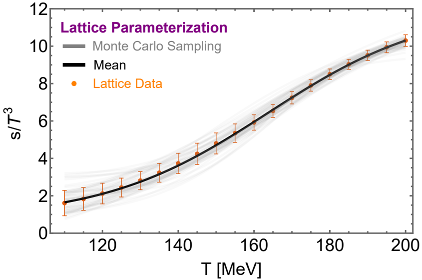

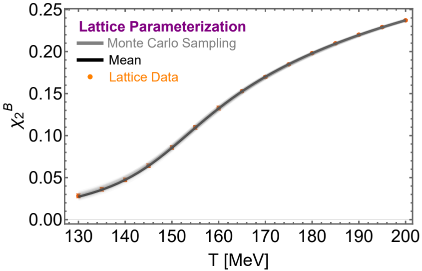

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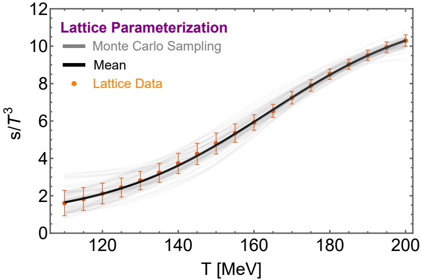

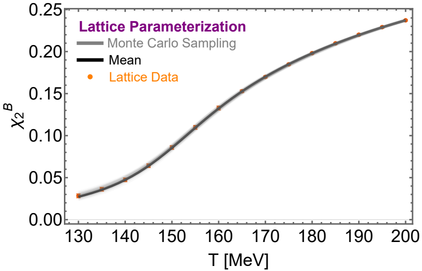

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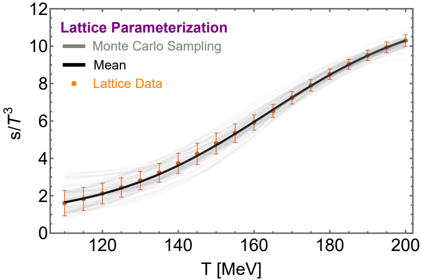
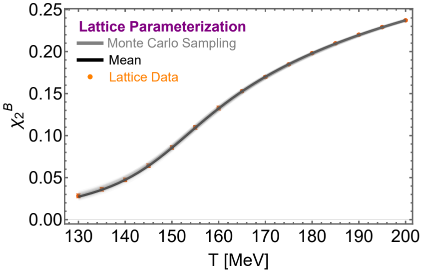




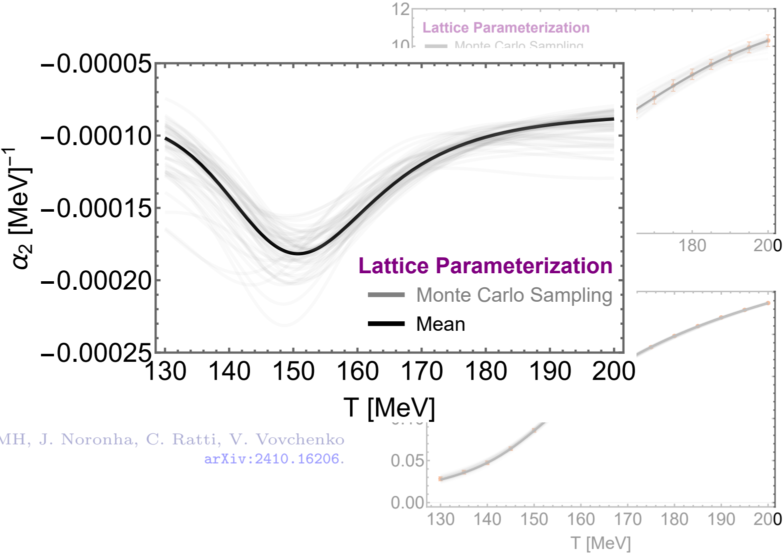
Proof of principle: $\mathcal{O}(\mu_B^2)$

- Quadra

$\alpha_2(T_0)$

- Sample
  lattice

Borsanyi,
Borsányi,



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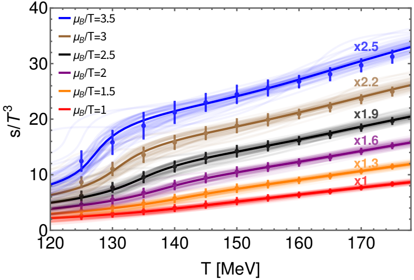
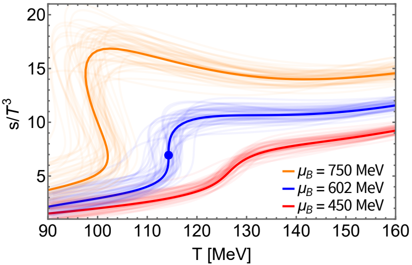
- Validation against lattice QCD T' expansion.

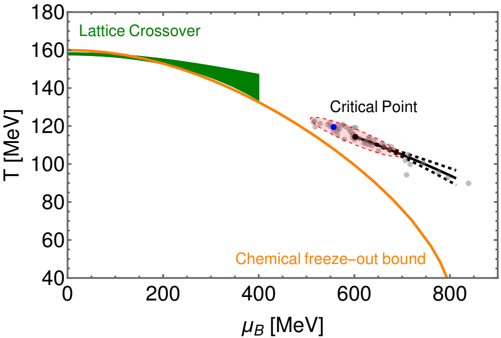
Borsányi, Fodor, Guenther et al., PRL **126** (2021)

- $s(T)$ develops infinite derivative and S-shape.





Results



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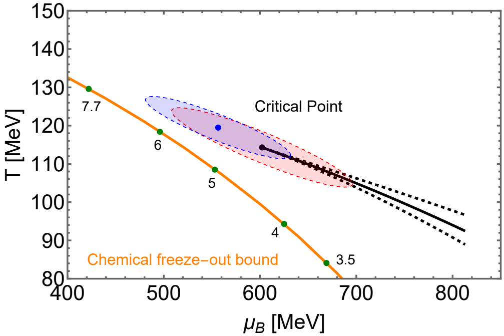

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- Sampling from error bars + smoothing splines:

$$(T_c, \mu_{B,c}) = (119.5 \pm 5.4, 556.5 \pm 49.8) \text{ MeV}.$$

Comments

- Entropy-density contours move towards low temperatures at higher μ_B, reaching the CP at $T_c \sim 120$ MeV.
- Extraction of higher-order coefficients from $\mu_B \approx 0$ Taylor coefficients hindered by high-order derivatives w.r.t. T.

  Alternative: Extrapolate contours from imaginary μ_B.
- Roberge-Weiss critical point found at

$$\tilde{T}_c = 197.1 \pm 7.1 \text{ MeV}, \quad \tilde{\mu}_{B,c}/\tilde{T}_c = i(3.50 \pm 0.30).$$

Ref.: $\tilde{T}_c = 208 \pm 5$ MeV, $\tilde{\mu}_{B,c}/\tilde{T}_c = i\pi$

A. Roberge and N. Weiss, Nucl. Phys. B **275** (1986)

Conclusions

1. New method for predicting CP from lattice QCD results.

2. Simple concept, expansion can capture phase transitions.

3. Proof of principle using both parametrized forms and smoothing splines, and Wuppertal-Budapest results:

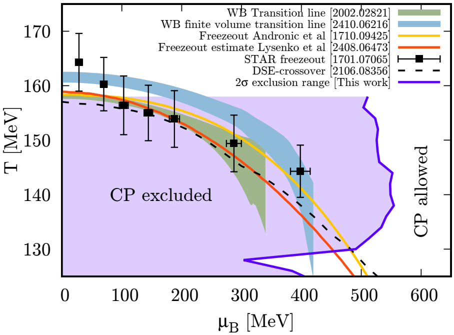
$$\text{CP @} \quad \mu_{B,c} \sim 500 - 700 \text{ MeV}, \quad T_c \sim 100 - 130 \text{ MeV}.$$

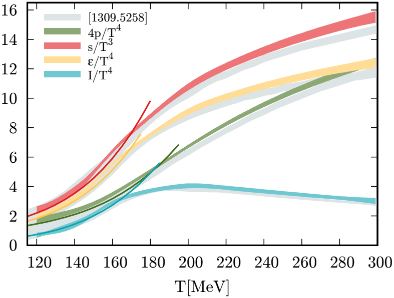
4. Validation from Roberge-Weiss CP and holographic model.

A. Roberge and N. Weiss, Nucl. Phys. B **275** (1986)
R. Rougemont et al., PPNP **135** (2024)

H. Shah, MH, J. Noronha, C. Ratti, V. Vovchenko arXiv:2410.16206.

Application by Wuppertal-Budapest collaboration





- CP exclusion region from entropy density contours, using new lattice QCD results for the strangeness-neutral equation of state.

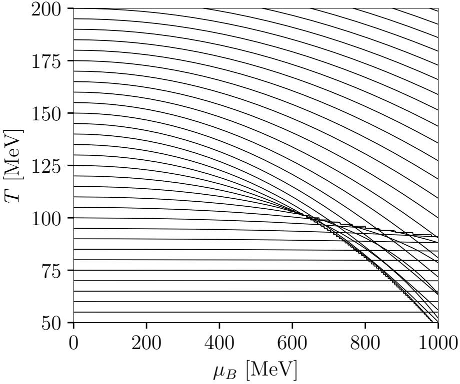
S. Borsanyi, Z. Fodor, J. N. Guenther et al., arXiv:2502.10267.

Backup slides

Controlled test: Holographic model

- Good description of lattice + QGP phenomenology.

R. Rougemont et al., PPNP **135** (2024)
MH, J. Grefa, et al., PRD **110** (2024)



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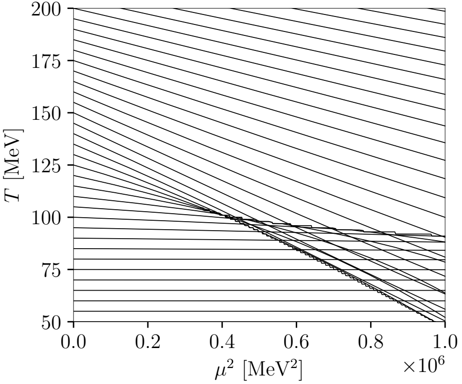

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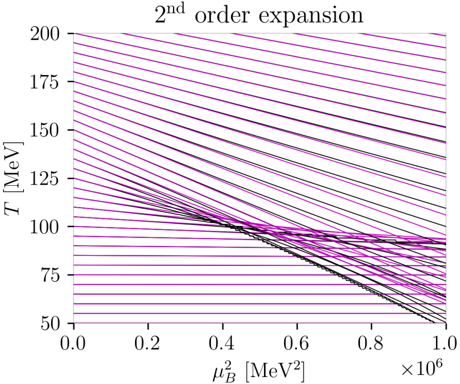

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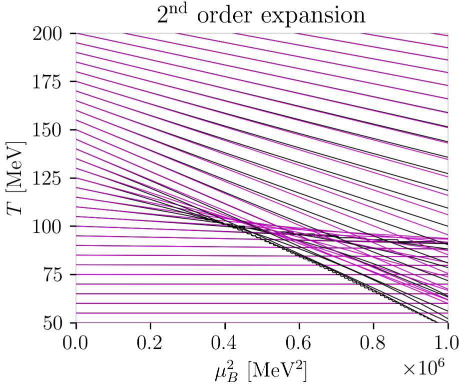

- True CP: $(T_c, \mu_{B,c}) \simeq (103, 599)$ MeV
- Extrapolation: $(T_c, \mu_{B,c}) \simeq (104.5, 637.6)$ MeV