Infrared study of the transversely projected quark-gluon vertex

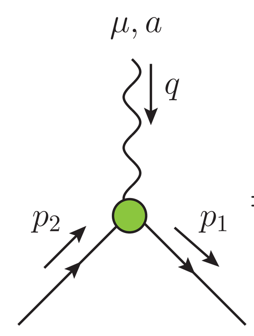
G. L. Teixeira [1] A. C. Aguilar [1] M. N. Ferreira [2] B. M. Oliveira [1] J. Papavassiliou [3]

[1]University of Campinas - UNICAMP, Institute of Physics Gleb Wataghin,
[2]School of Physics, Nanjing University
[3]Department of Theoretical Physics and IFIC, University of Valencia and CSIC

1. Introduction

The quark-gluon vertex is a fundamental component of QCD, playing a central role in various nonperturbative phenomena, including chiral symmetry breaking, dynamical quark mass generation, and the formation of bound states that constitute the QCD spectrum [1]. Due to its complexity, studying this vertex in general kinematics remains a significant challenge.

Diagrammatically, it is represented as



$$= igt^a\Gamma^\mu(q,p_2,-p_1)$$

In this work, we focus on the transversely projected vertex, $\overline{\Gamma}^\mu(q,p_2,-p_1)$, defined as

$$\overline{\Gamma}^\mu(q,p_2,-p_1) := P^\mu_\nu(q)\Gamma^\nu(q,p_2,-p_1)\,,$$
$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\,.$$

and its tree-level reduces to

$$\Gamma^\mu_0(q,p_2,-p_1) = \gamma^\mu\,.$$

Our goal is to investigate the infrared properties of the $\overline{\Gamma}^\mu(q,p_2,-p_1)$ in general kinematics using its Schwinger-Dyson equation (SDE). Consequently, we determine its form factors in this regime.

2. General structure

The vertex, $\overline{\Gamma}_\mu(q,p_2,-p_1)$, is decomposed into eight independent tensors, each associated with a **form factor**,

$$\overline{\Gamma}_\mu(q,p_2,-p_1) = \sum_{i=1}^{8}\lambda_i(q,p_2,-p_1)\,P_{\mu\nu}(q)\,\tau^\nu_i(p_2,-p_1)\,, \qquad (1)$$

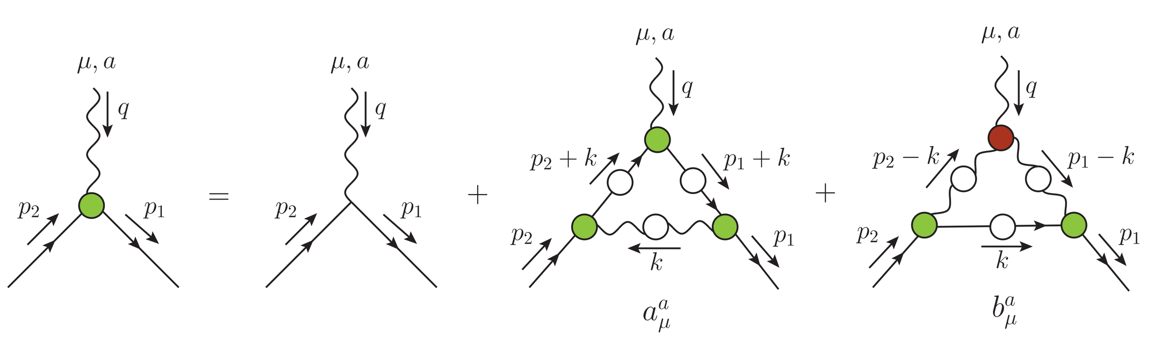
where the basis elements are given by [2]

$$\tau^\nu_1 = \gamma^\nu\,, \qquad \tau^\nu_2 = (p_1+p_2)^\nu\,,$$
$$\tau^\nu_3 = (\not{p}_1+\not{p}_2)\gamma^\nu\,, \qquad \tau^\nu_4 = (\not{p}_1-\not{p}_2)\gamma^\nu\,,$$
$$\tau^\nu_5 = (\not{p}_1-\not{p}_2)(p_1+p_2)^\nu\,, \qquad \tau^\nu_6 = (\not{p}_1+\not{p}_2)(p_1+p_2)^\nu\,,$$
$$\tau^\nu_7 = -\frac{1}{2}[\not{p}_1,\not{p}_2]\gamma^\nu\,, \qquad \tau^\nu_8 = -\frac{1}{2}[\not{p}_1,\not{p}_2](p_1+p_2)^\nu\,.$$

- Chirally symmetric (CS) and chiral symmetry breaking (CSB) subsets
$$\tau_{cs} = \{\tau^\nu_1,\tau^\nu_5,\tau^\nu_6,\tau^\nu_7\}\,, \qquad \tau_{csb} = \{\tau^\nu_2,\tau^\nu_3,\tau^\nu_4,\tau^\nu_8\}\,.$$
- Charge conjugation
$$C\overline{\Gamma}_\mu(q,p_2,-p_1)C^{-1} = -\overline{\Gamma}^T_\mu(q,-p_1,p_2)\,.$$
- Form factors under charge conjugation symmetry,
$$\lambda_i(q,p_2,-p_1) = \lambda_i(q,-p_1,p_2)\,, \quad i=1,4,6,7,8\,,$$
$$\lambda_3(q,p_2,-p_1) = -\lambda_3(q,-p_1,p_2)\,,$$
$$\lambda_2(q,p_2,-p_1) = \lambda_2(q,-p_1,p_2) + 2\lambda_3(q,-p_1,p_2)\,,$$
$$\lambda_5(q,p_2,-p_1) = -\lambda_5(q,-p_1,p_2) + \lambda_7(q,-p_1,p_2)\,. \qquad (2)$$

3. The Schwinger-Dyson Equation

In our study, we employ the SDE for $\overline{\Gamma}_\mu(q,p_2,-p_1)$ within the 3PI effective action formalism [3,4], which is represented diagrammatically by



and expressed as

$$\overline{\Gamma}_\mu(q,p_2,-p_1) = Z_1 P_{\mu\nu}(q)\gamma^\nu + \overline{a}_\mu(q,p_2,-p_1) + \overline{b}_\mu(q,p_2,-p_1) \qquad (3)$$

- $\overline{a}_\mu$ and $\overline{b}_\mu$ are the Abelian and non-Abelian diagrams;
- All components are fully dressed;
- Renormalization procedure simplified;
- Nonperturbative structure of the vertex ⇒ Determine the form factors.

4. Equations for form factors

To access the form factors, we construct basis projectors $\mathcal{P}^\mu_i$ using Eq. (1)

$$\lambda_i(q,p_2,-p_1) = \mathrm{Tr}\left[\mathcal{P}^\mu_i(q,p_2,-p_1)\overline{\Gamma}_\mu(q,p_2,-p_1)\right]\,,$$

Using the SDE from Eq. (3), we express the dynamical equations for $\lambda_i(q,p_2,-p_1)$ in Minkowski space

$$\lambda_i(q,p_2,-p_1) = Z_1\delta_{i1} + \mathbb{A}_i(q,p_2,-p_1) + \mathbb{B}_i(q,p_2,-p_1)\,.$$

To solve this equation, two major simplifications are applied,

- In the rhs of the SDE, we assume that,
$$\overline{\Gamma}_\mu(q,p_2,-p_1) \to \lambda_1(q,p_2,-p_1)P_{\mu\nu}(q)\gamma^\nu\,.$$
- We adopt planar degeneracy for the three-gluon vertex, which is approximated by the following compact form [5,6]
$$\overline{\Gamma}^{\mu\alpha\beta}(q,r,p) = L_{sg}(s^2)\overline{\Gamma}^{\mu\alpha\beta}_0(q,r,p)\,, \qquad s^2 = \frac{1}{2}(q^2+r^2+p^2)\,.$$

Then, λ_i is expressed in Euclidean space,

$$\lambda_i(q,p_2,-p_1) \to \lambda_i(p_1^2,p_2^2,\theta)\,.$$

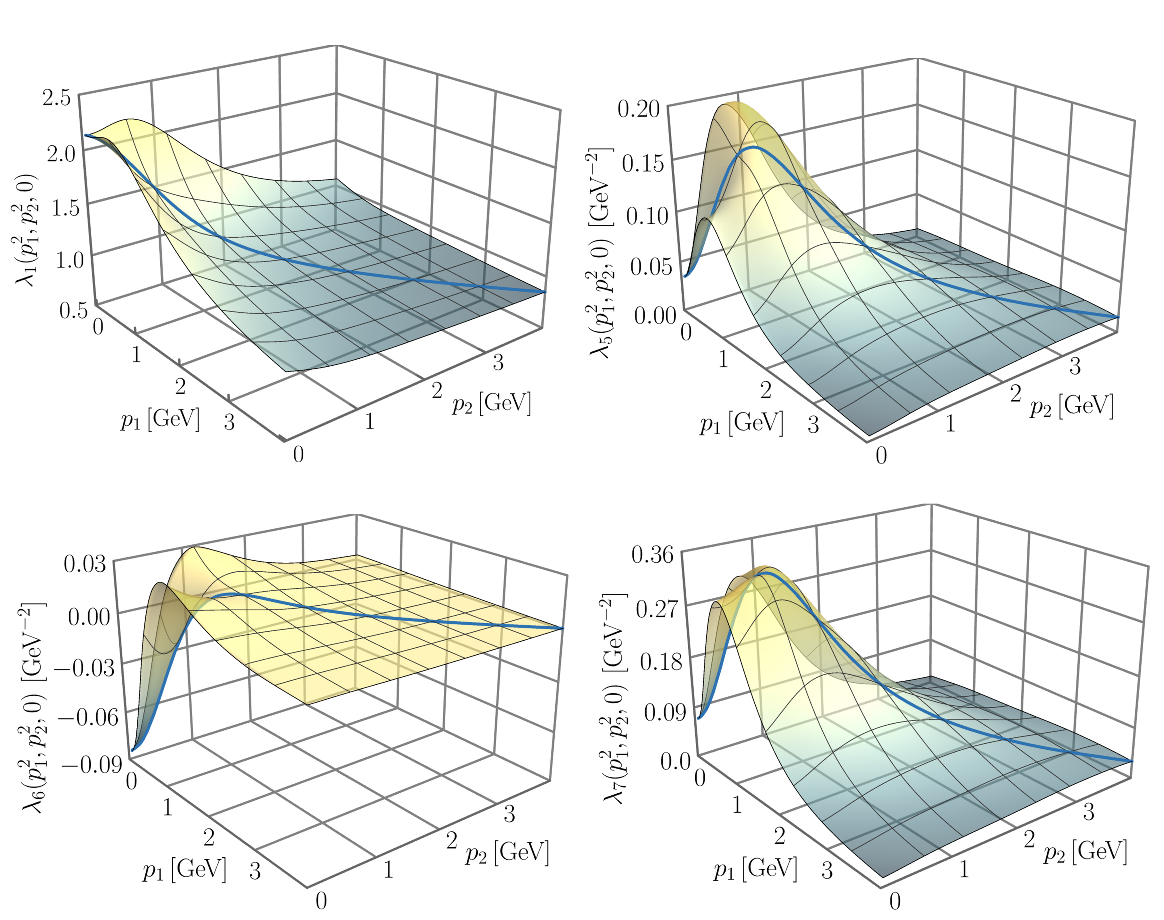
With the simplifications, the λ_i assumes the schematic form,

$$\lambda_i(p_1^2,p_2^2,\theta) = Z_1\delta_{i1} + \int_E \mathcal{K}_{i\mathbb{A}}\lambda_1^3 + \int_E \mathcal{K}_{i\mathbb{B}}L_{sg}\lambda_1^2\,,$$

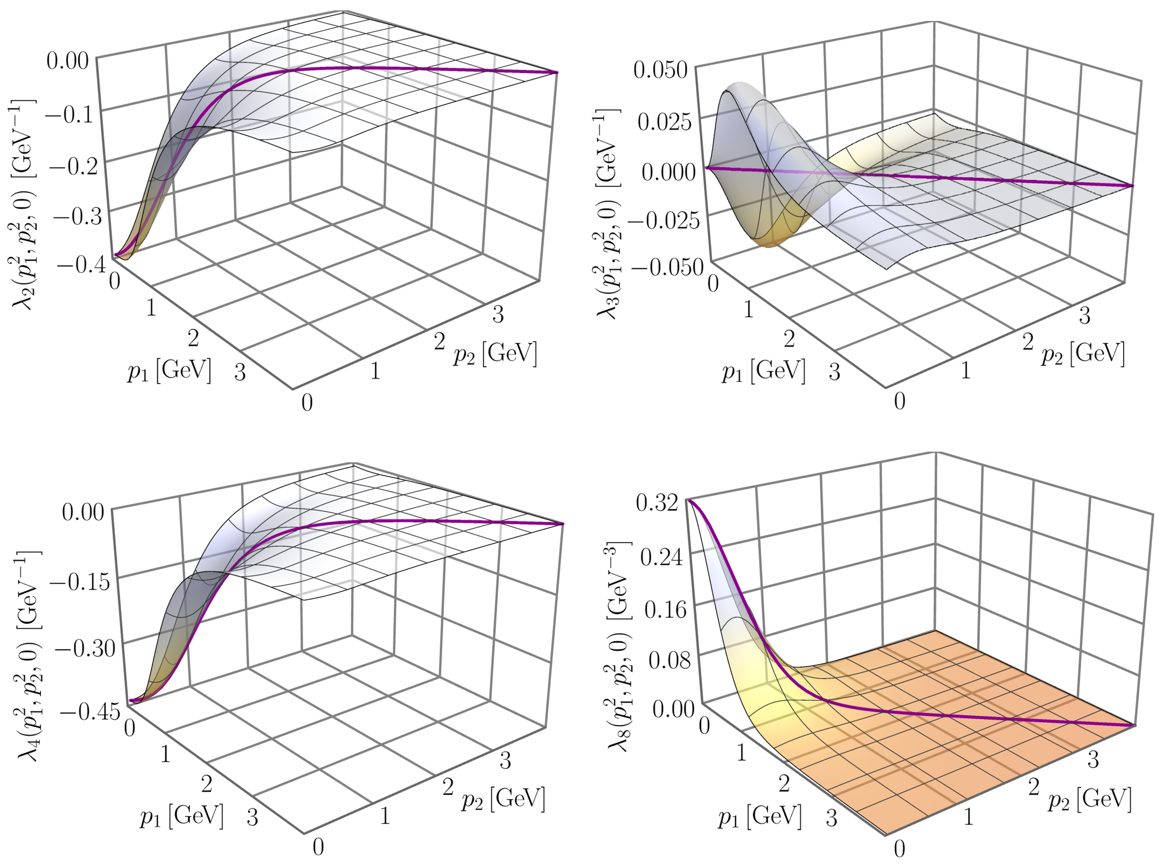
- This equation is numerically solved for the eight form factors.

5. General kinematics results

Chiral symmetric form factors



Chiral symmetry breaking form factors

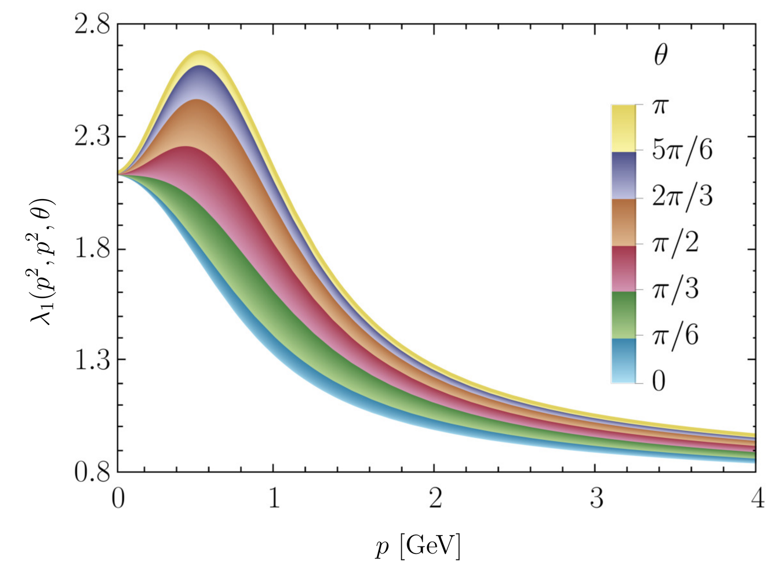


All form factors

- Exhibit the charge conjugation relations as given in Eq. (2);
- Are finite and deviate significantly from their tree-level values in the IR regime;
- Recover the perturbative behavior in the UV regime,
$$\lambda_1 \to 1\,, \qquad \lambda_i \to 0\,;$$
- The blue and purple curves correspond to the soft gluon limit ($i.e.$, $q\to 0$) of each form factor.

6. Angular dependence

The λ_1 displays a considerable dependence on the angle θ



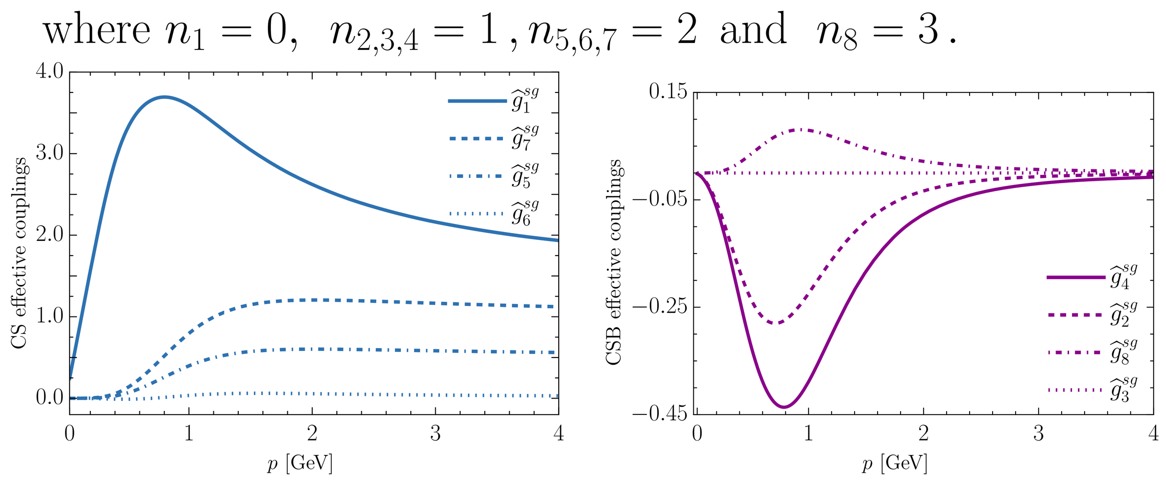
- The peak becomes increasingly pronounced as the angle increases.
- 27% difference in the maximum peak between the angles $\theta = 0$ (soft-gluon) and $\theta = \pi$ (asymmetric configuration).
- The remaining λ_i exhibit a comparably milder dependence [4].

7. Effective couplings

We compare the λ_i constructing RGI dimensionless effective couplings in the soft-gluon limit

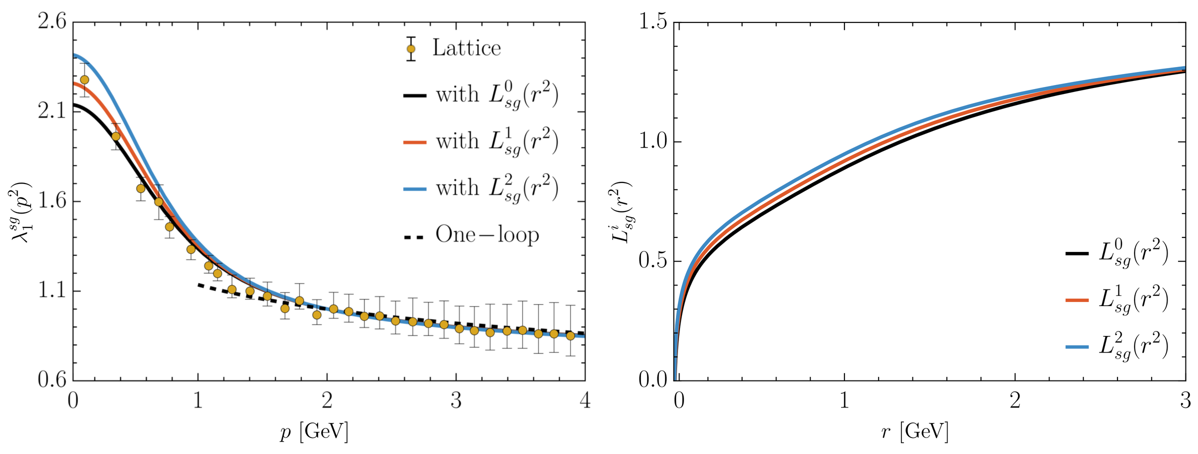
$$\widehat{g}^{sg}_i(p^2) = g(\mu^2)\,[p^{n_i}\lambda^{sg}_i(p^2)]A^{-1}(p^2)\mathcal{Z}^{1/2}(p^2)\,,$$

where $n_1 = 0$, $n_{2,3,4} = 1\,, n_{5,6,7} = 2$ and $n_8 = 3\,.$



- The $\widehat{g}^{sg}_i$ for each form factor reveals a clear hierarchy amongst them,
$$\widehat{g}^{sg}_1 > \widehat{g}^{sg}_7 > \widehat{g}^{sg}_5 > |\widehat{g}^{sg}_6|\,, \qquad |\widehat{g}^{sg}_4| > |\widehat{g}^{sg}_2| > \widehat{g}^{sg}_8\,.$$
- λ_1, λ_7 and λ_4 are the dominant form factors.

8. Comparison with the lattice



- Comparison with lattice data [7] shows excellent agreement for λ^{sg}_1, with a 7% deviation in the deep IR region;
- λ^{sg}_2 and λ^{sg}_6 display significant discrepancies with the lattice data [4].

Conclusions

Using the SDE within the 3PI formalism, we determined the eight form factors of the transversely projected quark-gluon vertex in general kinematics. Our results reveal a clear hierarchy among the form factors and show excellent agreement with lattice data for λ_1 in the soft-gluon limit. We intend to apply these findings in future phenomenological studies, particularly in the context of chiral symmetry breaking and the generation of a dynamical quark mass.

References

[1] F. Gao, J. Papavassiliou, J. M. Pawlowski, Phys. Rev. D 103, 094013 (2021).
[2] M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D91, 054035 (2015).
[3] R. Williams, C. S. Fischer, W. Heupel, Phys. Rev. D 93, 034026 (2016).
[4] A. C. Aguilar, M. N. Ferreira, B. M. Oliveira, J. Papavassiliou, G. L. Teixeira, Eur. Phys. J. C 84, 1231 (2024).
[5] G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89, 105014 (2014).
[6] A. C. Aguilar, M. N. Ferreira, J. Papavassiliou, and L. R. Santos, Eur. Phys. J. C 83, 549 (2023).
[7] A. Kızılersü, O. Oliveira, P. J. Silva, J.-I. Skullerud, A. Sternbeck, Phys. Rev. D 103, 114515 (2021).

Acknowledgments

CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico
CAPES
UNICAMP

guslit2x@ifi.unicamp.br