Medium Separation Scheme and the Proper Description of Extreme Environments

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XVI International Workshop on Hadron Physics

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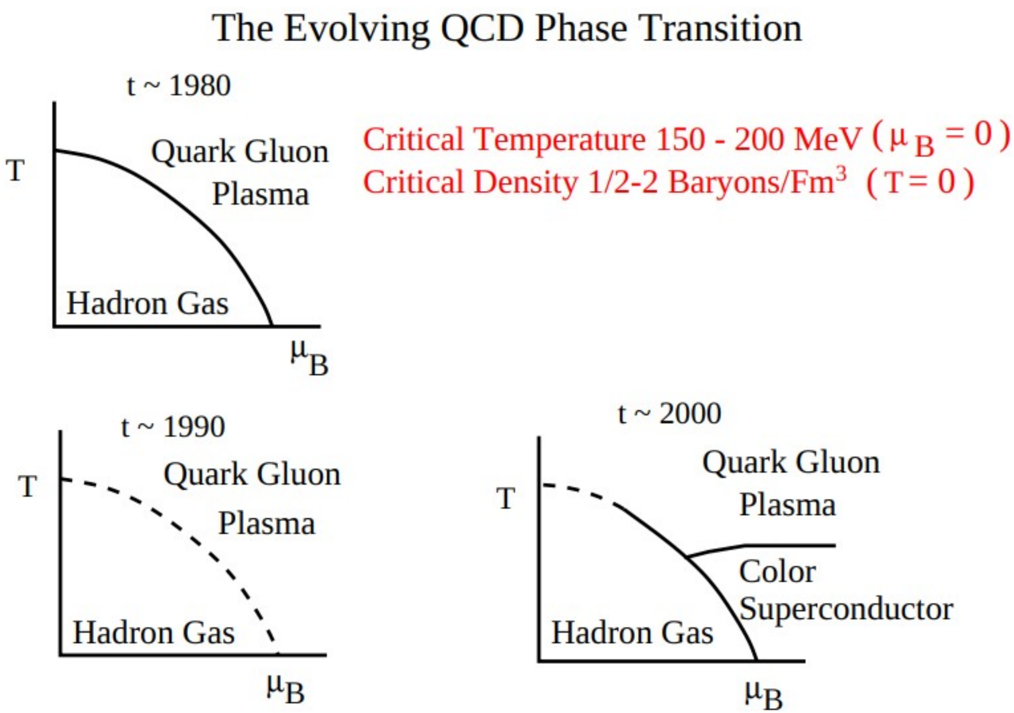
The QCD phase diagram



McLerran (2002).

- Approaches: perturbative, $1/N_c$ Expansion, LQCD, DSE, effective models...

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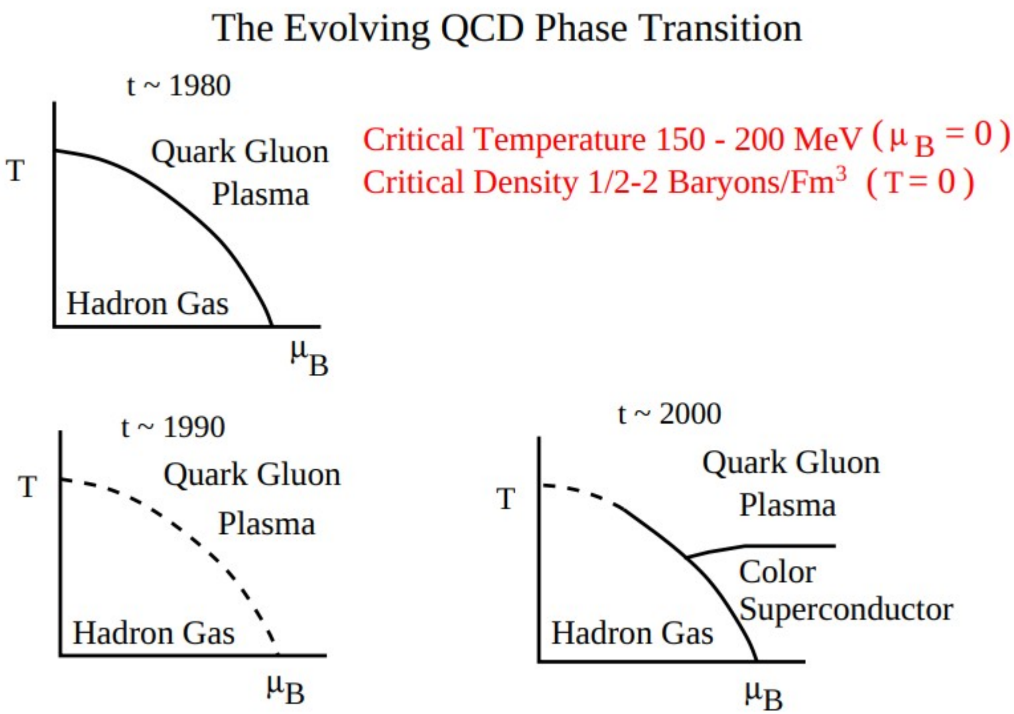

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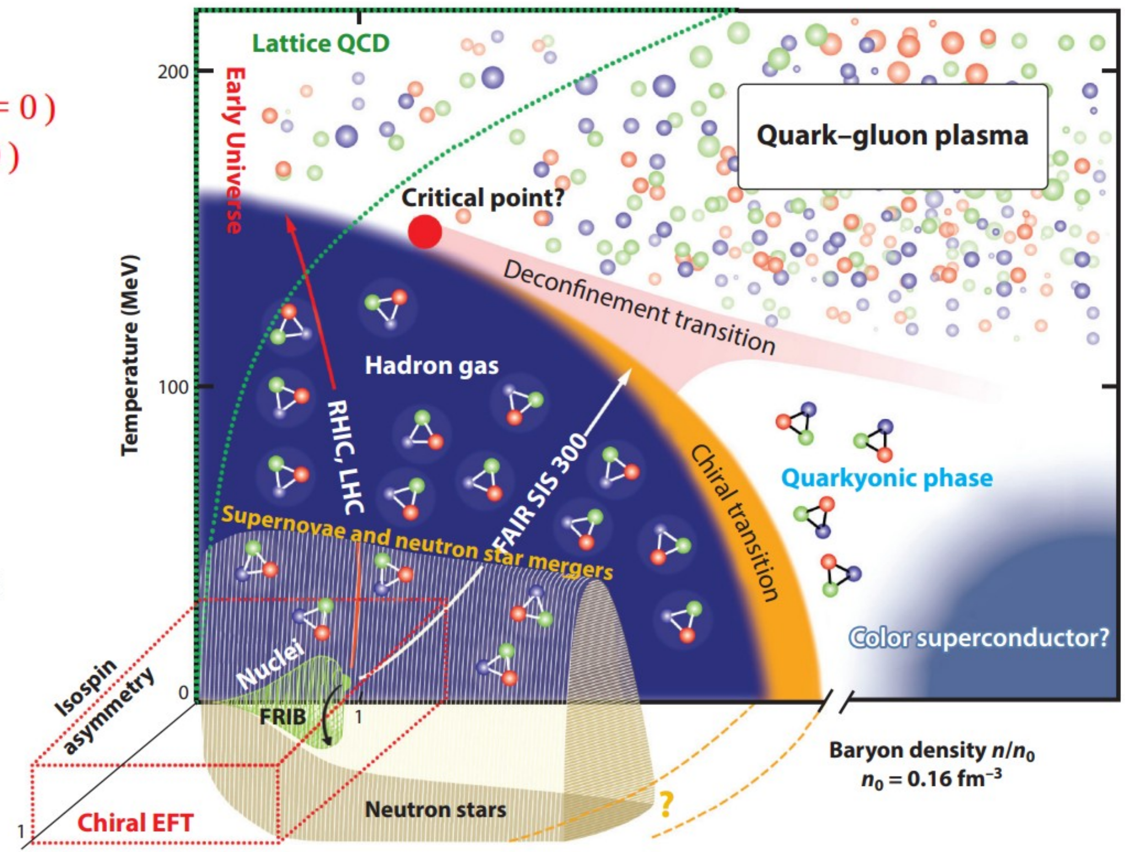
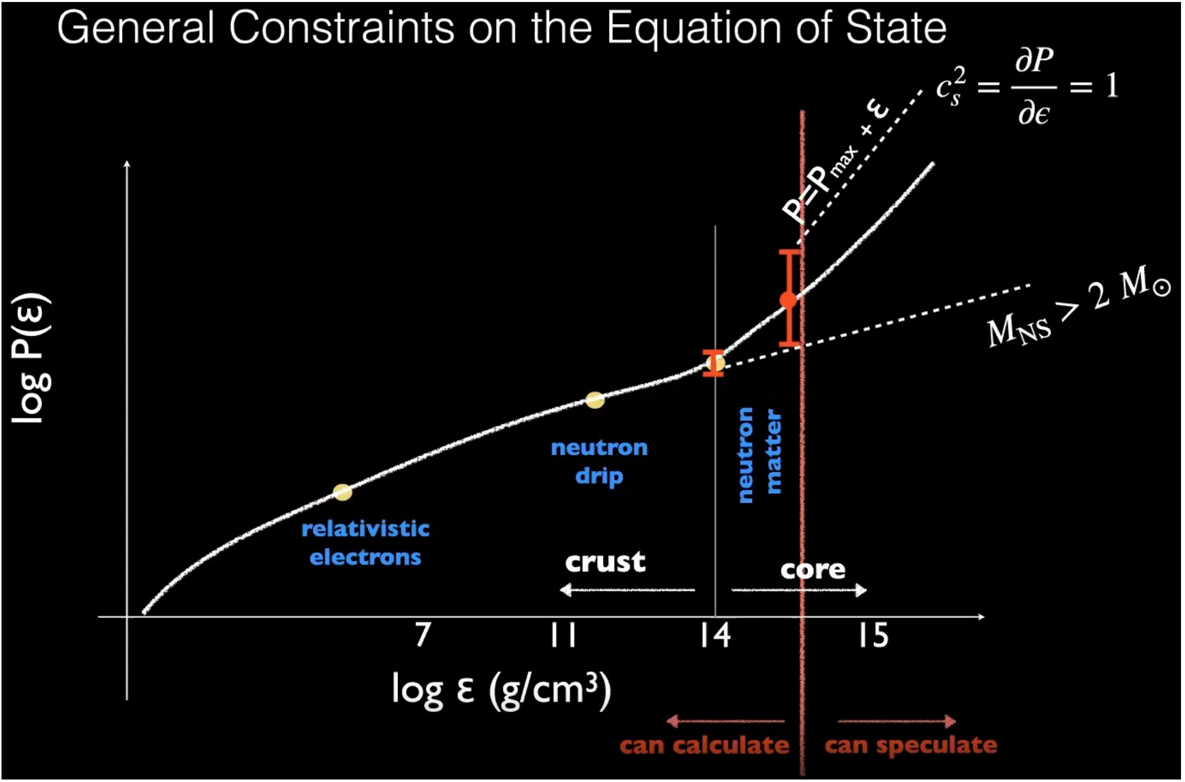


Drischler et al (2021).

- Approaches: perturbative, $1/N_c$ Expansion, LQCD, DSE, effective models...



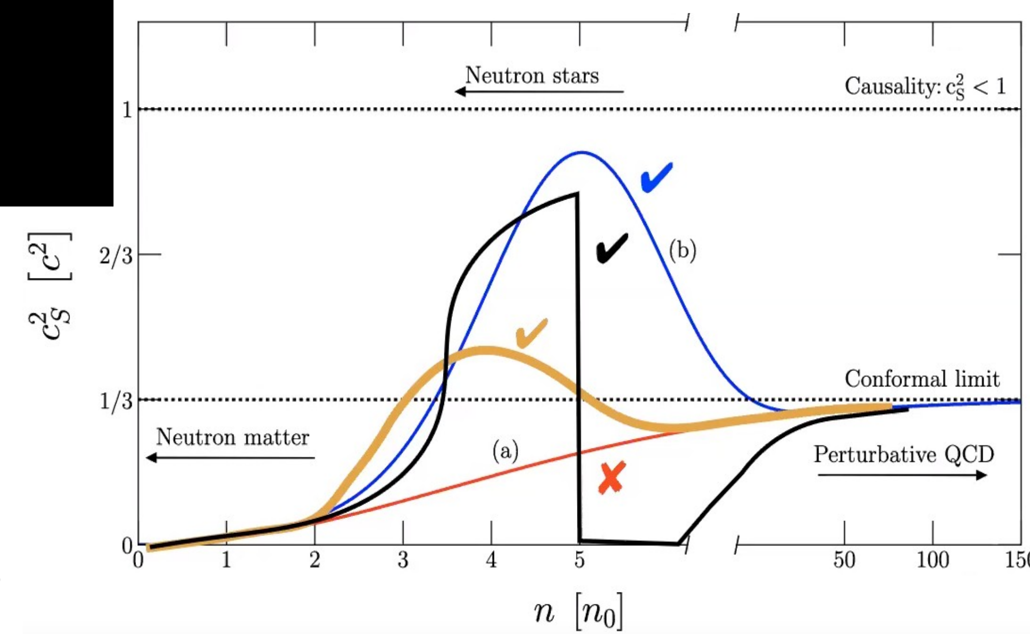
GW provides constraints to the EoS of dense matter

GW170817

- $R_{M=1.4M_\odot} \leq 13.5$ km and $M_{max} > 2M_\odot$

GW190425

- $R_{M=1.4M_\odot} \leq 15$ km and M_{max} $(2-3)M_\odot$



Breakdown of the models!

Figs. from S. Reddy presentation @BNL (2021)

No perfect solution—yet... But is there one?

- This talk focus on a specific approach: the chiral models, sometimes nonrenormalizable.

- In this case, a regularization prescription is required to define the validity scales of the model, as well as the necessary parameters.

- Regularization issues have been widely discussed in the literature:

  ✗ Behavior of order parameters as functions of magnetic field; Menezes et al., PRC 79, 035807 (2009).

  ✗ Nonphysical oscilations at finite chemical potential; Allen et al., PRD 92, 074041(2009).

  ✗ Incorrect prediction of a CEP in the phase diagram; Yu et al., PRD 94, 014026(20016).

  ✗ Magnetic/inverse magnetic catalysis; Avancini et al., PRD 99, 116002 (2019) and PRC103, 056009 (2021).

  ...

The original “Cutoff-independent regularization”

$$1 = \lambda G i \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k_0^2-(k+\mu)^2-\Delta^2} + (\mu \to -\mu)\right], \quad (2)$$

The regularization method we advocate here, proposed in Ref. [11] and used in different contexts [12], avoids the use of an explicit regulator and the calculation of any divergent integrals. Specifically, instead of introducing a cutoff in the integrals of Eq. (2), the integrands are manipulated in such a way that divergences are isolated in well-known, μ-independent one-loop divergent integrals that can be related to divergent integrals of the problem of DχSB in vacuum. Finite integrals are integrated without imposing any restriction on their integrands and the remaining divergent integrals are fitted to physical quantities at the DχSB scale in vacuum.

Farias, Dollabona, Krein, Battistel, PRC73, 018201 (2006).

- Algebraic manipulations leads to

$$1 = 8\lambda G\{2[i I_{\text{quad}}(\Delta^2)] - 4\mu^2[i I_{\log}(\Delta^2)] + I_{\text{fin}}(\Delta^2,\mu) + I_{\text{fin}}(\Delta^2,-\mu)\},$$

where

$$I_{\text{quad}}(\Delta^2) = \int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k_0^2-k^2-\Delta^2},$$

$$I_{\log}(\Delta^2) = \int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{(k_0^2-k^2-\Delta^2)^2},$$

$$I_{\text{fin}}(\Delta^2,\mu) = i\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k_0^2-k^2-\Delta^2)^3} \times\left[2\mu^2(\mu^2-4\Delta^2) + \frac{(\mu^2+2k\mu)^3}{(k_0^2-(k+\mu)^2-\Delta^2)}\right]$$

The original "Implicit Regularization Scheme"

$$1 = \lambda G i \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k_0^2 - (k+\mu)^2 - \Delta^2} + (\mu \to -\mu)\right], \quad (2)$$

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Related to physical quantities in vacuum

$$I_{\text{fin}}(\Delta^2, \mu) = i\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k_0^2 - k^2 - \Delta^2)^3} \times \left[2\mu^2(\mu^2 - 4\Delta^2) + \frac{(\mu^2 + 2k\mu)^3}{(k_0^2 - (k+\mu)^2 - \Delta^2)}\right]$$

Integrated without restrictions

The original "Implicit Regularization Scheme"

- Problem: $\Delta = 0$ in the vacuum $\longrightarrow$ Not a good scale parameter.

$$I_{\text{quad}}(\Delta^2) = I_{\text{quad}}(M^2) + (\Delta^2 - M^2) I_{\log}(M^2) + \frac{i}{(4\pi)^2}\left[\Delta^2 - M^2 - \Delta^2 \ln\left(\frac{\Delta^2}{M^2}\right)\right]$$

$$I_{\log}(\Delta^2) = I_{\log}(M^2) - \frac{i}{(4\pi)^2} \ln\left(\frac{\Delta^2}{M^2}\right).$$

Divergent integrals in terms of the quark mass in the vacuum

Finite terms

- In chiral models I_{quad} and $I_{\log}$ are related to the chiral condensate and pion decay constant as

$$iI_{\text{quad}}(M^2) = -\frac{\langle \bar{q}q \rangle}{12M}$$

$$iI_{\log}(M^2) = -\frac{f_\pi^2}{12M^2}$$

- Back to the gap equation: No divergent integrals!

$$1 = \lambda G \left\{ -\frac{\langle \bar{q}q \rangle}{6M} - (\Delta^2 - M^2)\frac{f_\pi^2}{6M^2} - \frac{1}{8\pi^2}\left[\Delta^2 - M^2 - \Delta^2 \ln\left(\frac{\Delta^2}{M^2}\right)\right] - \mu^2 \left[\frac{f_\pi^2}{3M^2} - \frac{1}{4\pi^2} 2\ln\left(\frac{\Delta^2}{M^2}\right)\right] + I_{\text{fin}}(\Delta^2, \mu) + I_{\text{fin}}(\Delta^2, -\mu)\right.$$

MSS implementation

- Modern implementation: Medium Separation Scheme (MSS).
- Let's suppose an arbitrary energy relation ω including some chemical potential μ and function(s) of a condensate(s) ⟨...⟩:

$$\omega(k) = \sqrt{[f(\vec{k}) \pm \mu]^2 + \langle \cdots \rangle}$$

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Chiral chemical potential:

$$\mu \to \mu_5$$
$$\langle \cdots \rangle \to M_q (\sim \langle \bar{q} q \rangle)$$
$$\omega(k) = \sqrt{(|\vec{k}| \pm \mu_5)^2 + M_q^2}$$

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Quark chemical potential:

$$\mu \to \mu_q$$
$$\langle \cdots \rangle \to \Delta_c(\sim \langle qq \rangle)$$
$$\omega(k) = \sqrt{\left(\sqrt{k^2 + M_q^2} \pm \mu_q\right)^2 + \Delta_c^2}$$

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Isospin chemical potential:

$$\mu \to \mu_I$$
$$\langle \cdots \rangle \to \Delta_\pi(\sim \langle \bar{q} i \gamma_5 \tau_\pm q \rangle)$$
$$\omega(k) = \sqrt{\left(\sqrt{k^2 + M_q^2} \pm \mu_I\right)^2 + \Delta_\pi^2}$$

Quark chemical potential:

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$$\omega(k) = \sqrt{\left(\sqrt{k^2 + M_q^2} \pm \mu_q\right)^2 + \Delta_c^2}$$

MSS implementation: an example

- In general one needs to compute integrals of the form:

$$I = \sum_{s=\pm 1} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{\left(\sqrt{k^2+M^2} \pm \mu\right)^2 + \Delta^2}}$$

- First we rewrite:

$$E_k = \sqrt{k^2+M^2}$$

$$I = \sum_{s=\pm 1} \frac{1}{\pi} \int_{-\infty}^{+\infty} dx \int \frac{d^3k}{(2\pi)^3} \frac{1}{x^2 + \left(E_k \pm \mu\right)^2 + \Delta^2}$$

- Manipulation:

$$A = M_0^2 - \Delta^2 - M^2 - \mu^2$$

$$\frac{1}{x^2 + \left(E_k \pm \mu\right)^2 + \Delta^2} = \frac{1}{x^2 + k^2 + M_0^2} + \frac{A - 2\mu E_k}{(x^2 + k^2 + M_0^2)[x^2 + \left(E_k \pm \mu\right)^2 + \Delta^2]}$$

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Effective quark mass $M \equiv M(T, \mu, eB, \ldots)$

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Vacuum quark mass $M_0 \equiv M(T = \mu = eB, \ldots = 0)$

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MSS implementation: an example

- After some algebra

$$I = 2I_{\text{quad}}(M_0) - (M^2 - M_0^2 + \Delta^2 - 2\mu^2) I_{\log}(M_0) + \left[\frac{3(A^2 + 4\mu^2 M^2)}{4} - 3\mu^2 M_0^2\right] I_1 + 2I_2$$

with the definitions

$$I_{\text{quad}}(M_0) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{\vec{p}^2 + M_0^2}}$$

$$I_{\log}(M_0) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{(\vec{p}^2 + M_0^2)^{\frac{3}{2}}}$$

Related to the quark condensate and f_π^2

$$I_1 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{(\vec{p}^2 + M_0^2)^{\frac{5}{2}}}$$

$$I_2 = \frac{15}{32} \sum_{j=\pm 1} \int \frac{d^3p}{(2\pi)^3} \int_0^1 dt (1-t)^2 \frac{(A - 2j\mu E_p)^3}{[(2j\mu E_p - A)t + \vec{p}^2 + M_0^2]^{\frac{7}{2}}}$$

Finite

1- Nonvanishing Δ at high μ

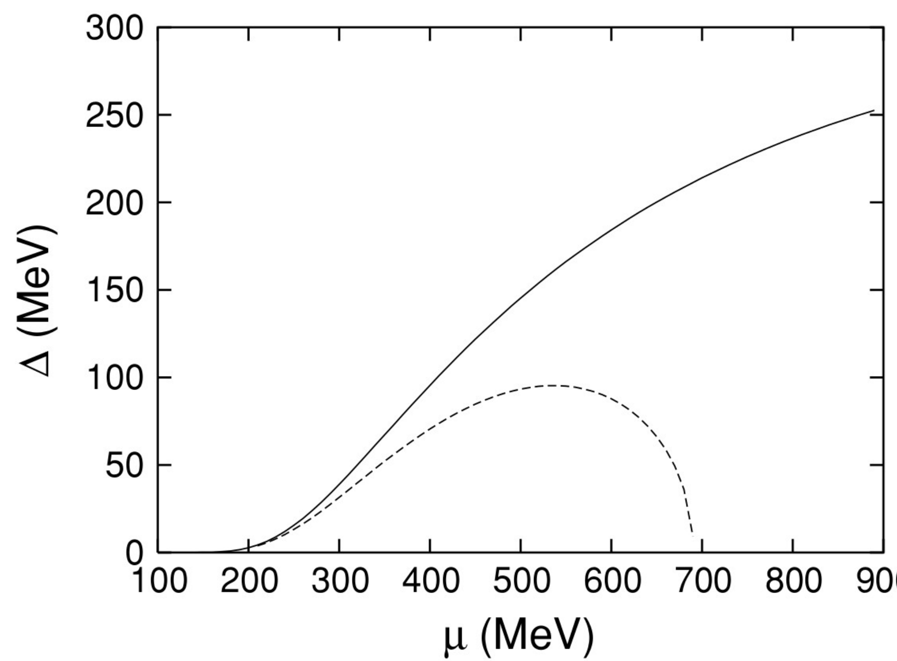
- Two-flavor spin-0 superconducting gap prediction using the weak-coupling renormalization group techniques:

$$\Delta \sim \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

D. T. Son, PRD59, 094019 (1999)

- Δ is an increasing function of μ, and corrections to this formula does not seem to change this behavior.

Hong, Miransky,Shovkovy, Wijewardhana, PRD61 056001 (2000), Hsu, Schwetz, Nucl. Phys. B572, 211(2000).



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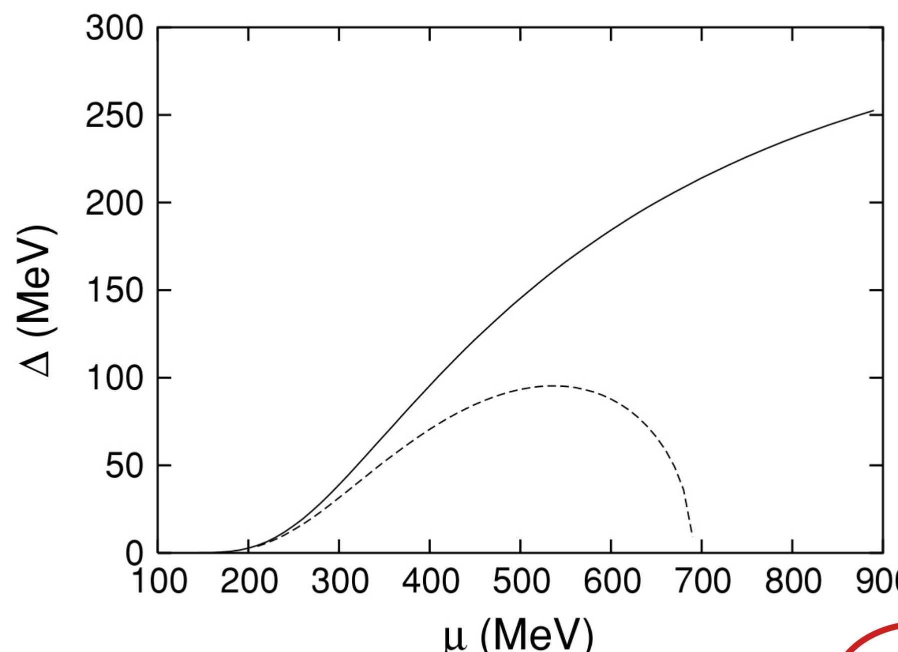

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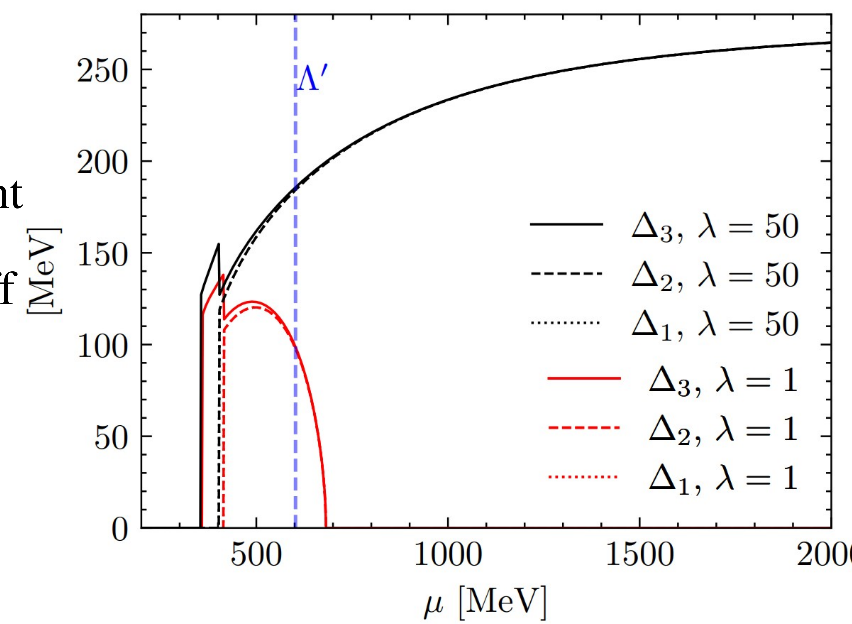
Gholami, Hofmann, Buballa, arXiv:2408.06704

RG -consistent treatment remove cutoff artifacts!

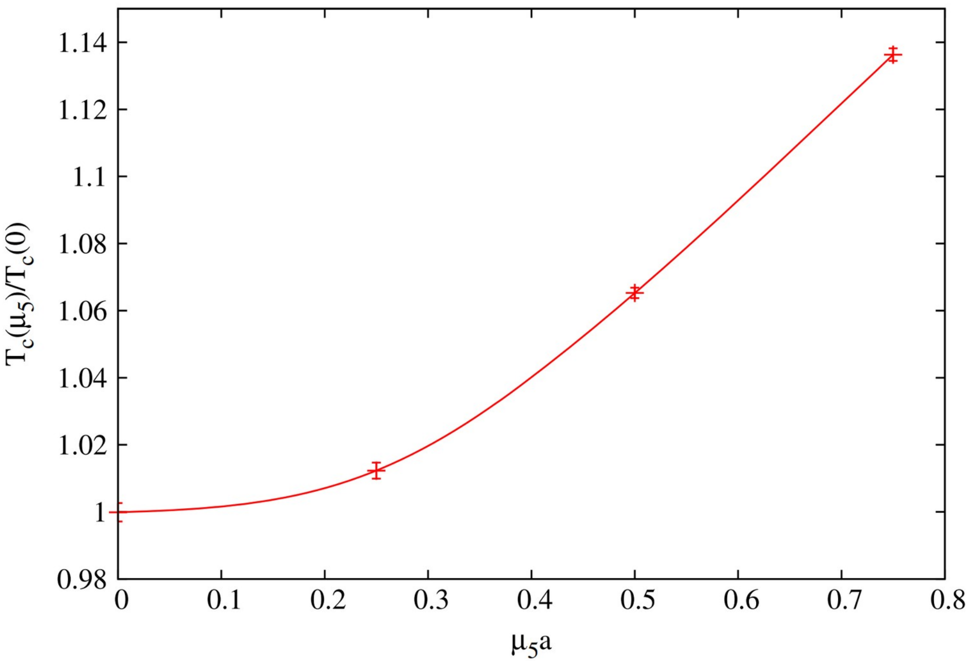


λ = Λ/Λ'

$$\Delta_{ur,dg} = \Delta_{ug,dr} = \Delta_3$$
$$\Delta_{ur,sb} = \Delta_{ub,sr} = \Delta_2$$
$$\Delta_{dg,sb} = \Delta_{db,sg} = \Delta_1$$

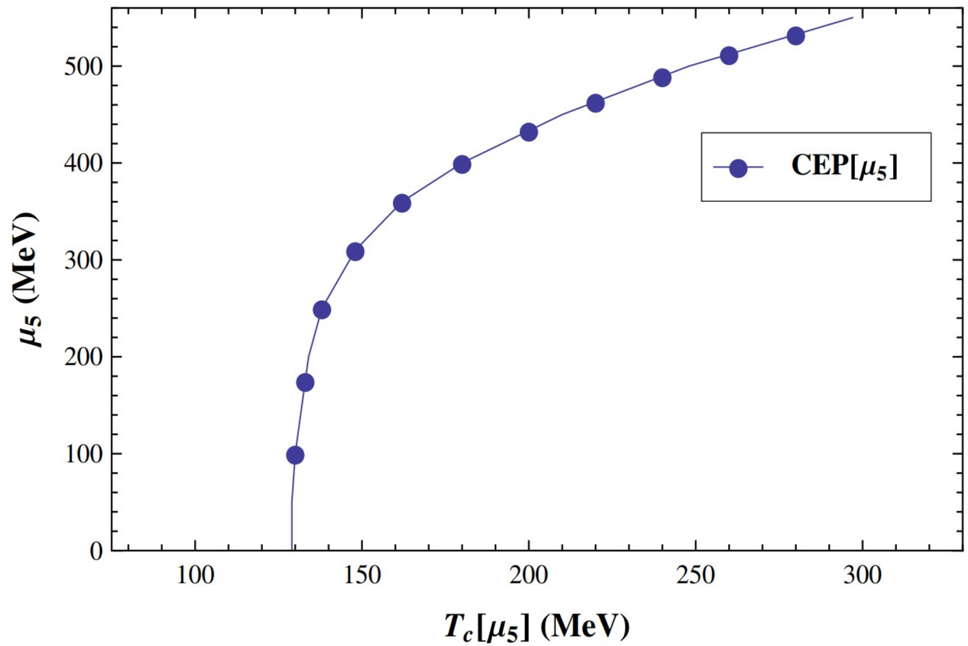
2- Finite chiral chemical potential: agreement with other approaches

- Universality arguments of large N_c, DSE and lattice simulations predicts an increasing T_{pc} with μ_5, with NO CEP...



Braguta et. al., PRD93, 034509 (2016)

(Lattice)

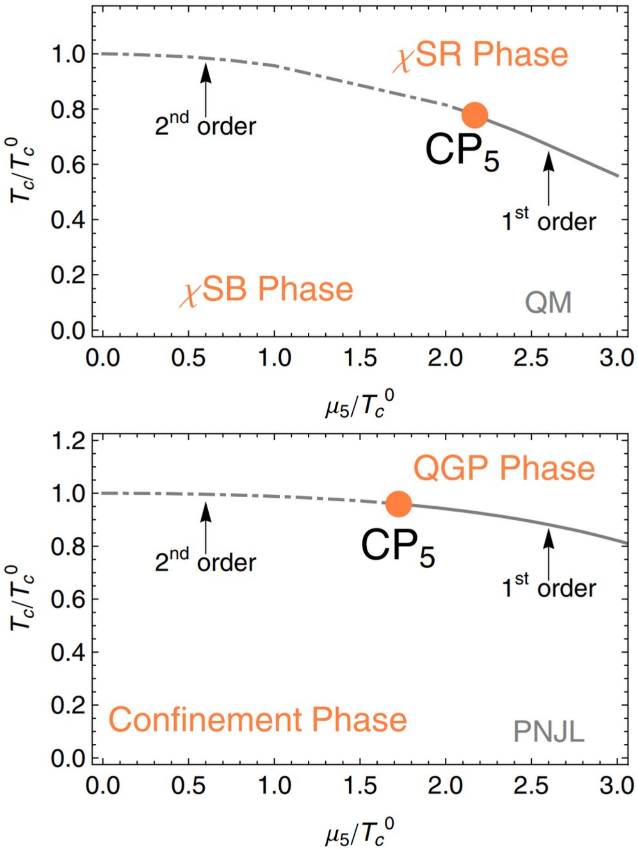


Xu et.al., PRD 91, 056003 (2015)

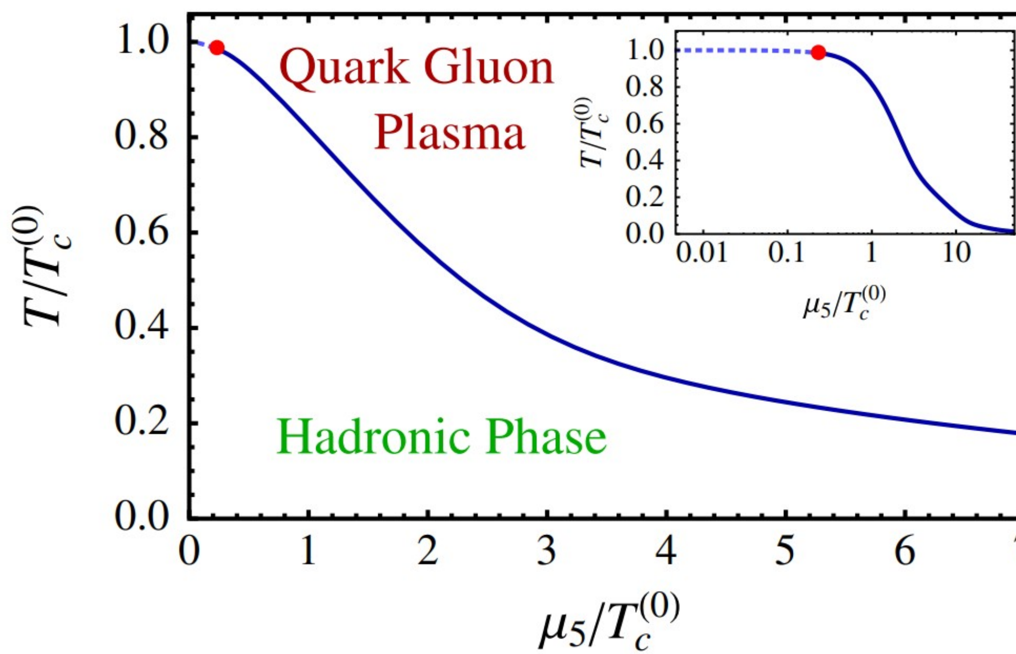
(DSE)

2- Finite chiral chemical potential: agreement with other approaches

... but chiral models (e.g. NJL and LSM) found the opposite.



Ruggieri, PRD 91, 056003 (2015)

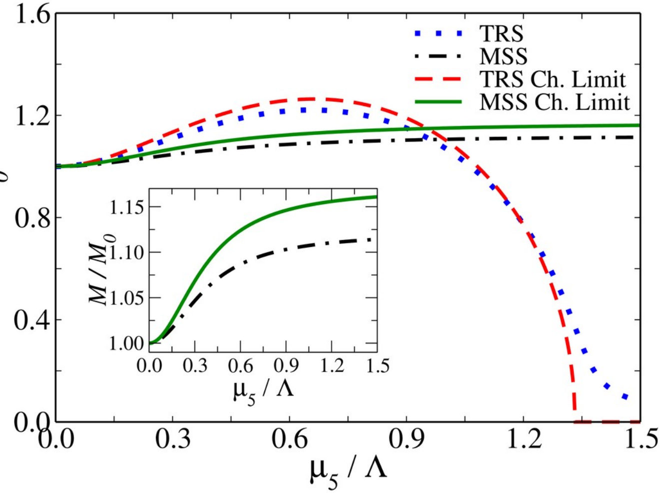


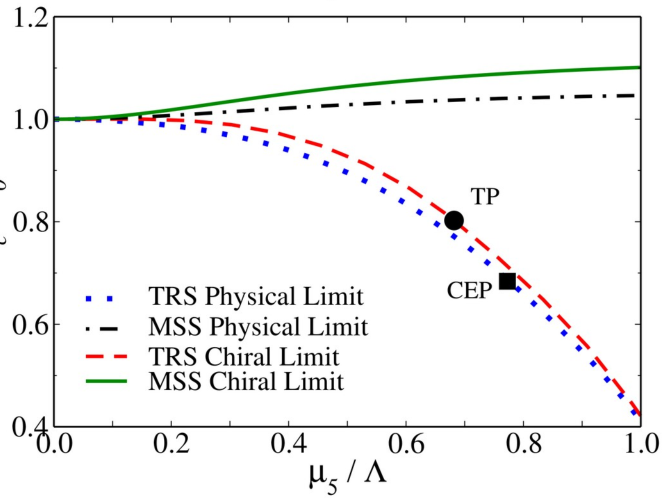
Chernodub, Nedelin., PRD83, 105008 (2011)

(PLSMq)

2- Finite chiral chemical potential: agreement with other approaches

- MSS cure the problem, and more!



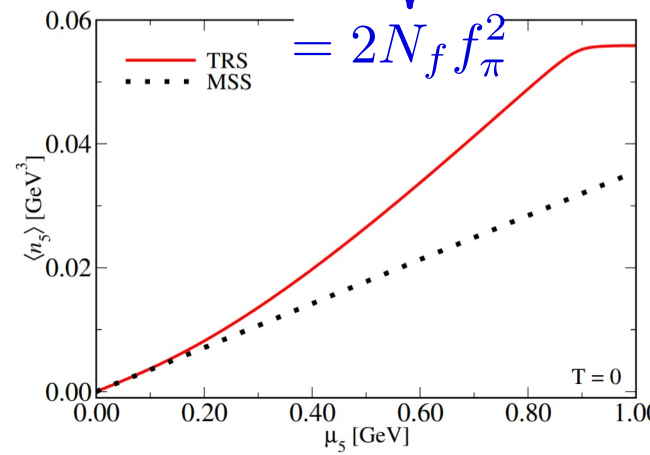


Farias, DD, Krein,Ramos, PRD 91, 056003 (2015)

- ChPT at finite μ_5: $\langle n_5 \rangle = \frac{1}{\beta V} \frac{\partial \log[Z(\mu_5)]}{\partial \mu_5} = 2N_f f_\pi^2 \mu_5$

$$\langle n_5 \rangle^{\rm TRS} = 2N_c \sum_{s=\pm1} \int_0^\Lambda \frac{dp\, p^2}{2\pi^2} \frac{s(|\mathbf{p}| + s\mu_5)}{\sqrt{(|\mathbf{p}| + s\mu_5)^2 + M^2}}$$

$$\langle n_5 \rangle^{\rm MSS} = \underbrace{4N_c \left[M^2 I_{\log}(M_0) - \frac{M^2}{4\pi^2} \ln\left(\frac{M^2}{M_0^2}\right)\right]}_{= 2N_f f_\pi^2} \mu_5$$



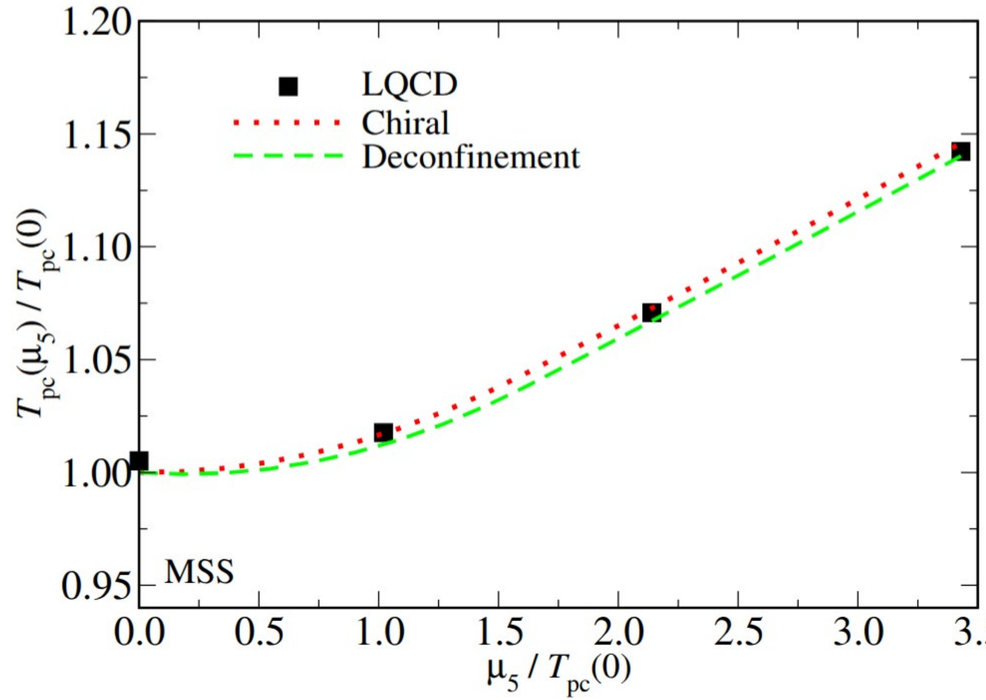
Azeredo, DD, Farias, Krein,Ramos, PRD110 076007 (2024)

2- Finite chiral chemical potential: agreement with other approaches

- A similar result can be obtained in the PNJL model IF we include an explicit dependence of μ_5 on the Polyakov loop potential.

$$\mathcal{U}(\Phi, T)$$

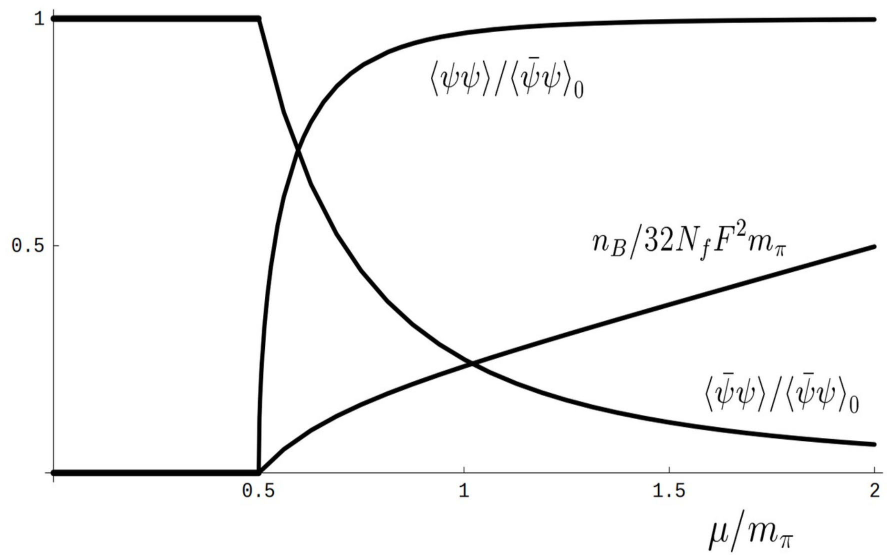
$$\mathcal{U}(\Phi, T, \mu_5) = T^4\left[-\frac{\bar{b}_2(T,\mu_5)}{2}\Phi\Phi^\dagger - \frac{b_3}{6}(\Phi^3 + {\Phi^\dagger}^3) + \frac{b_4}{4}(\Phi\Phi^\dagger)^2 \right]$$



Azeredo, DD, Farias, Krein,Ramos, PRD110 076007 (2024)

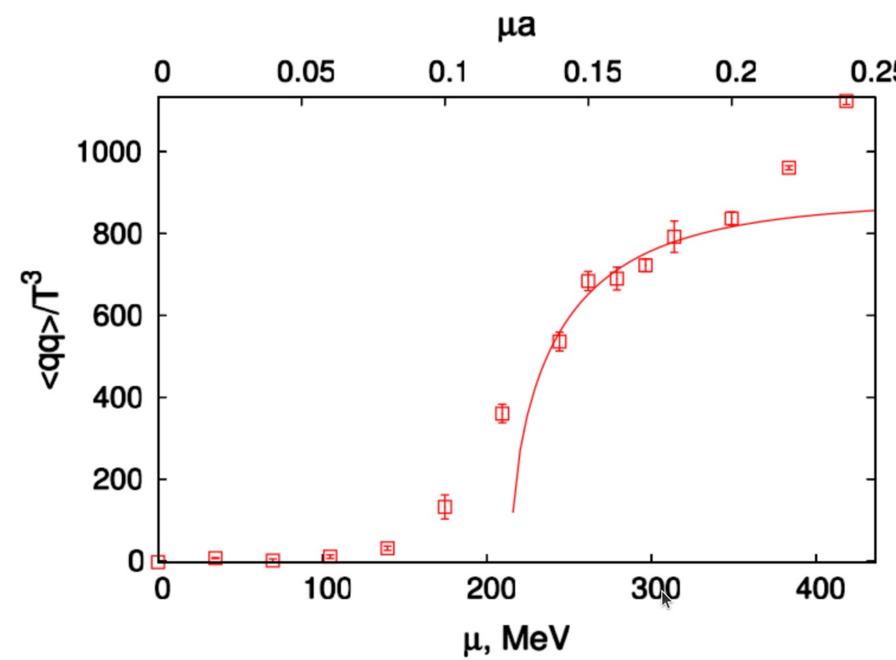
3- Qualitative agreement with ChPT (and LQCD) for the 2-color QCD

- $\Delta \longrightarrow 0$ for high values of μ_B $(= N_c\mu)$ is an artifact of the incorrect regularization also in the physical limit.



phase	$\langle\bar{\psi}\psi\rangle$	$\langle\psi\psi\rangle$	n_B
$\mu < m_\pi/2$	$\langle\bar{\psi}\psi\rangle_0$	0	0
$\mu > m_\pi/2$	$\langle\bar{\psi}\psi\rangle_0\left(\frac{m_\pi}{2\mu}\right)^2$	$\langle\bar{\psi}\psi\rangle_0\sqrt{1-\left(\frac{m_\pi}{2\mu}\right)^4}$	$8\mu N_f F^2\left(1-\left(\frac{m_\pi}{2\mu}\right)^4\right)$

Kogut et. al., NPB 582 477 (2000)



Braguta, et. al., PRD 94, 114510 (2016)

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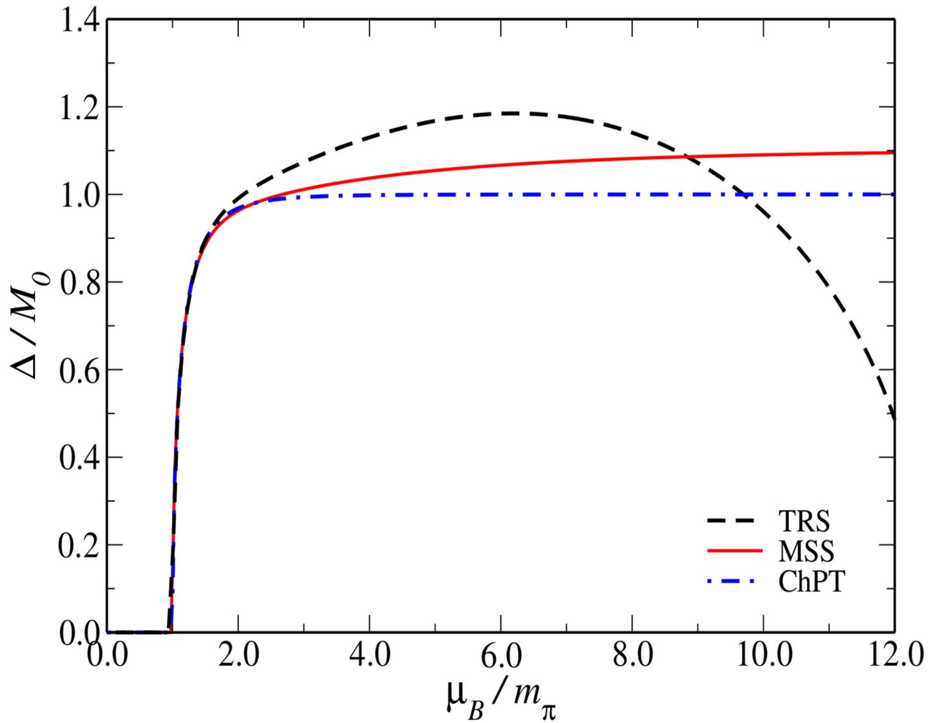
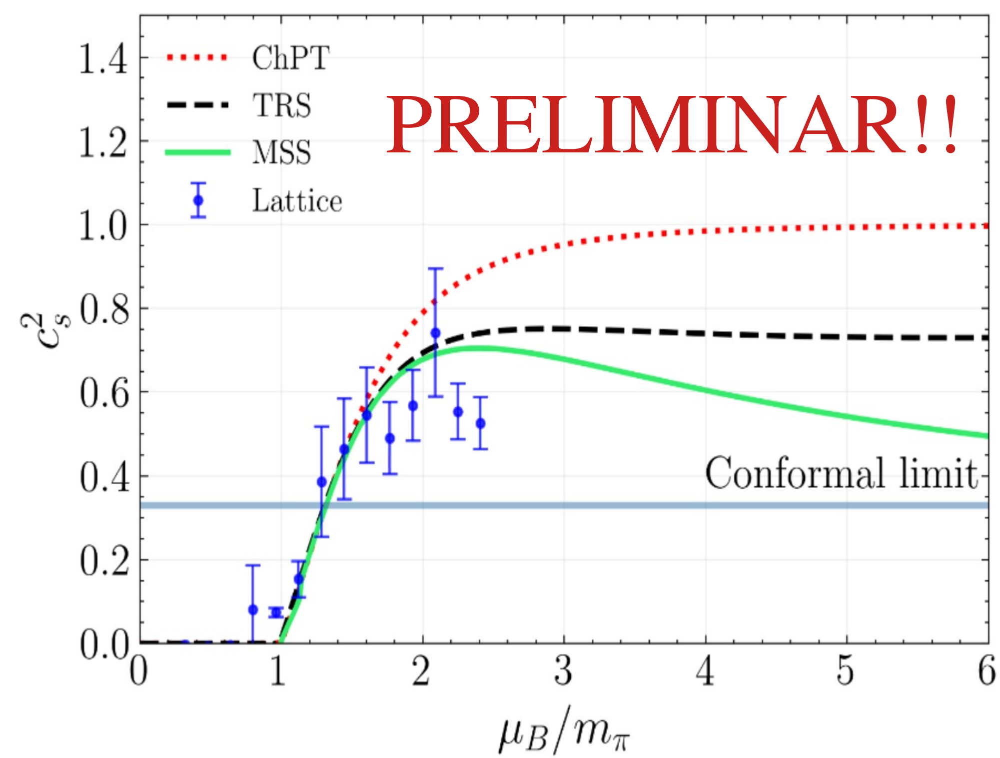
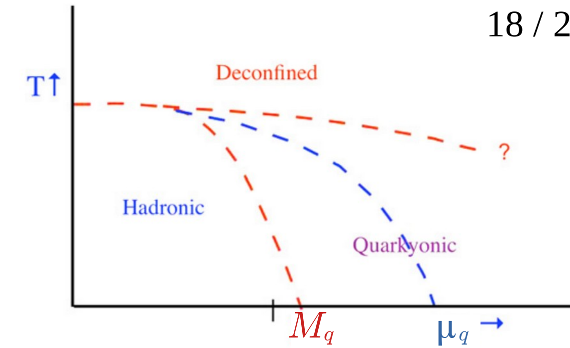




MS in collaboration with A. Pasqualotto, R. Farias

Perspectives: Quarkyonic matter from a quark model point of view



- Possible phase of QCD phase diagram, argued from large N_c that belongs completely to confined world, and takes place when the quark chemical potential exceeds its mass.

- Strong candidate to take into account the constraints of the observational data, since it naturally generates a stiff EoS in the intermediate regime;

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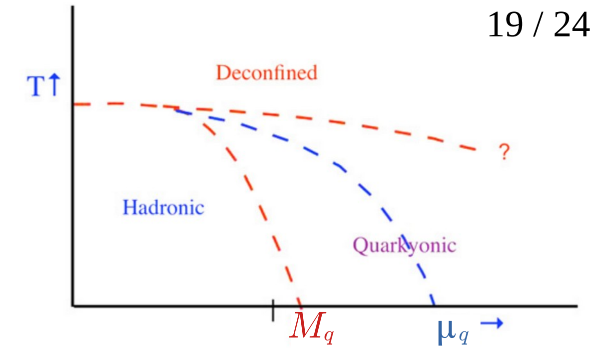
A PEDAGOGICAL DISCUSSION OF QUARKYONIC MATTER AND ITS IMPLICATION FOR NEUTRON STARS

Larry McLerran

Perspectives: Quarkyonic matter from a quark model point of view



NJL + Polyakov loop:
McLerran et al. Nuclear Physics A 824 (2009) 86.

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\mu D^\mu - m + i\mu\gamma_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\vec{\tau}\gamma_5\psi\right)^2\right] - U\left(\Phi[A], \bar{\Phi}[A]\right)$$

$$\Omega = U + \frac{(M-m)^2}{2G} - 2N_f N_c \int \frac{d^3p}{(2\pi)^3} E_p \Theta\left(\Lambda - |\vec{p}|\right) + \delta\Omega_f,$$

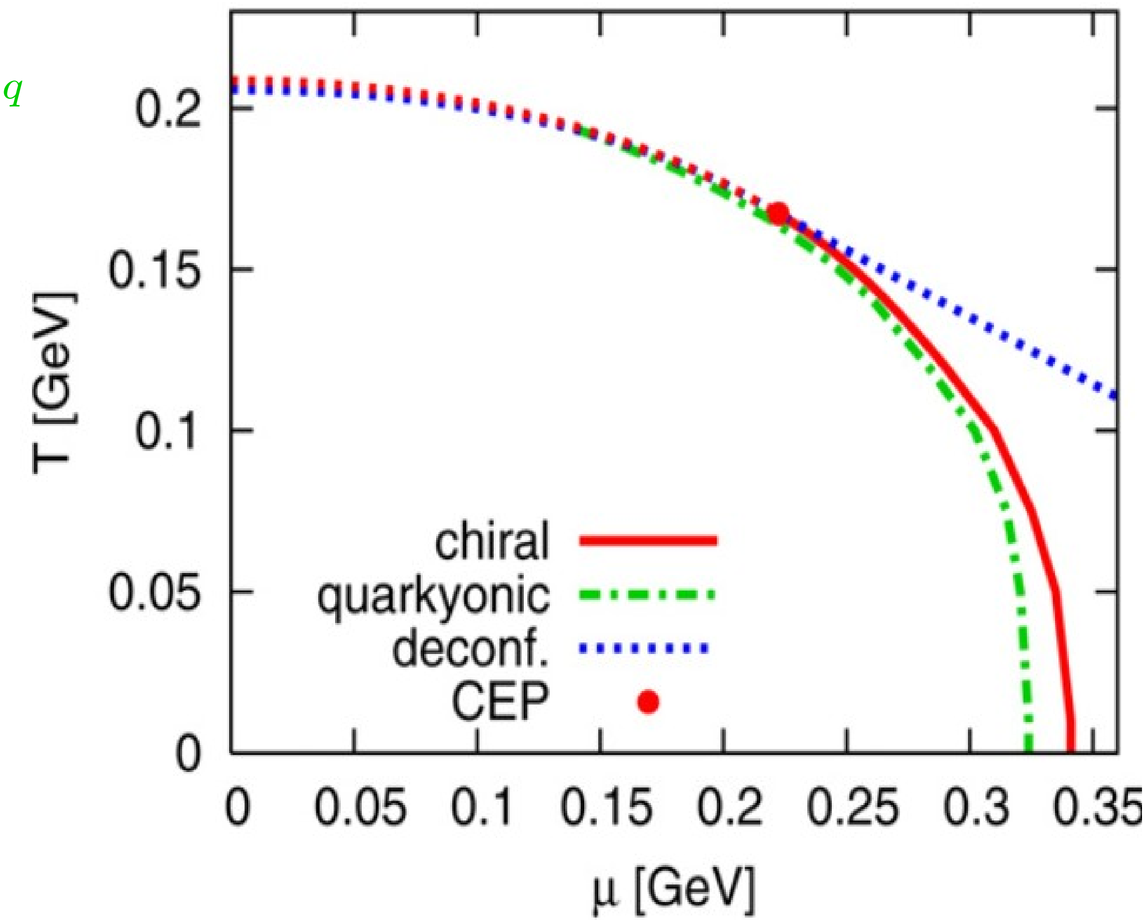
$$\delta\Omega_f = -2N_c N_f T \int \frac{d^3p}{(2\pi)^3}\left[\Theta(E_p-\mu)\Phi\left(e^{-\beta(E_p-\mu)} + e^{-\beta(E_p+\mu)}\right) + \Theta(\mu - E_p)\left\{\beta(\mu - E_p) + \Phi\left(e^{-\beta(\mu-E_p)} + e^{-\beta(\mu+E_p)}\right)\right\}\right],$$

$$\frac{\partial M}{\partial T}\;,\quad \frac{\partial \Phi}{\partial T}\;,\quad \mu_q = m_q$$

Perspectives: Quarkyonic matter from a quark model point of view

$\frac{\partial^2 M}{\partial T^2} = 0$, $\frac{\partial^2 \Phi}{\partial T^2} = 0$, $\mu_q = m_q$

"We find that at vanishing temperature and at large N_c, the quarkyonic transition occurs at densities only slightly lower than that expected for the chiral transition."



McLerran, Redlich, Sasaki, Nucl. Phys. A 824, 86 (2009).

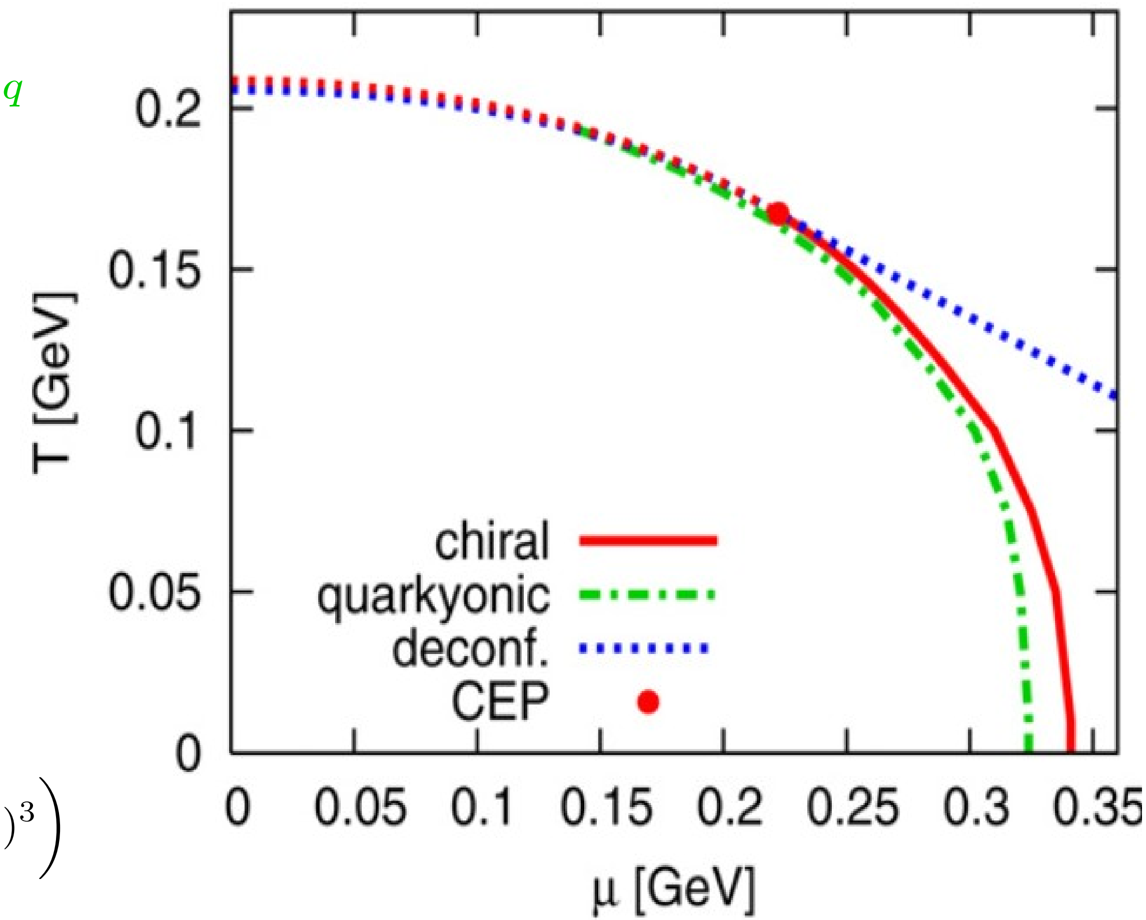
Perspectives: Quarkyonic matter from a quark model point of view

$$\frac{\partial^2 M}{\partial T^2} = 0 \ , \quad \frac{\partial^2 \Phi}{\partial T^2} = 0 \ , \quad \mu_q = m_q$$

"We find that at vanishing temperature and at large N_c, the quarkyonic transition occurs at densities only slightly lower than that expected for the chiral transition."

$$U = T^4 \left(-\frac{b_2(T)}{2}\bar{\Phi}\Phi + \frac{b_4}{4}(\bar{\Phi}\Phi)^2 + \frac{b_6}{6}(\bar{\Phi}\Phi)^3 \right)$$

Proportional to T^4: Disappear at $T \longrightarrow 0$!



McLerran, Redlich, Sasaki, Nucl. Phys. A 824, 86 (2009).

Alternative: Polyakov loop potential dependent on μ

$$U(\Phi,T)=T^4\left(-\frac{b_2(T)}{2}\bar{\Phi}\Phi+\frac{b_4}{4}(\bar{\Phi}\Phi)^2+\frac{b_6}{6}(\bar{\Phi}\Phi)^3\right)$$

$$\Downarrow$$

$$U(\Phi,T,\mu)=(a_0T^4+a_1\mu^4+a_2T^2\mu^2)\Phi^2+a_3T_0^4\log(1-6\Phi^2+8\Phi^3-4\Phi^4)$$

Use of $U(\Phi,T,\mu)$ together with

$$\begin{cases} G_s\to\mathcal{G}_s(G_s,\Phi)=G_s(1-\Phi^2), \\ G_V\to\mathcal{G}_V(G_V,\Phi)=G_V(1-\Phi^2) \\ K\to\mathcal{K}(K,\Phi)=K(1-\Phi^2). \end{cases}$$

$$\Omega_{\mathrm{PNJL0}}=G_s\sum_f\rho_{sf}^2-G_V\sum_f\rho_f^2+4K\prod_f\rho_{sf}-\frac{\gamma}{2\pi^2}\sum_f\int_0^\Lambda dk\,k^2(k^2+M_f^2)^{1/2}-\frac{\gamma}{6\pi^2}\sum_f\int_0^{k_{Ff}}\frac{dk\,k^4}{(k^2+M_f^2)^{1/2}}+\mathcal{U}(\rho_f,\rho_{sf},\Phi)$$

$$\begin{aligned}\mathcal{U}(\rho_f,\rho_{sf},\Phi)&\equiv\mathcal{U}(\rho_u,\rho_d,\rho_s,\rho_{su},\rho_{sd},\rho_{ss},\Phi)\\&=G_V\Phi^2\sum_f\rho_f^2-G_s\Phi^2\sum_f\rho_{sf}^2-4K\Phi^2\prod_f\rho_{sf}\\&\quad+\mathcal{U}_0(\Phi).\end{aligned}\tag{22}$$

Alternative: Polyakov loop potential dependent on μ

$$U(\Phi, T) = T^4\left(-\frac{b_2(T)}{2}\bar{\Phi}\Phi + \frac{b_4}{4}(\bar{\Phi}\Phi)^2 + \frac{b_6}{6}(\bar{\Phi}\Phi)^3\right)$$

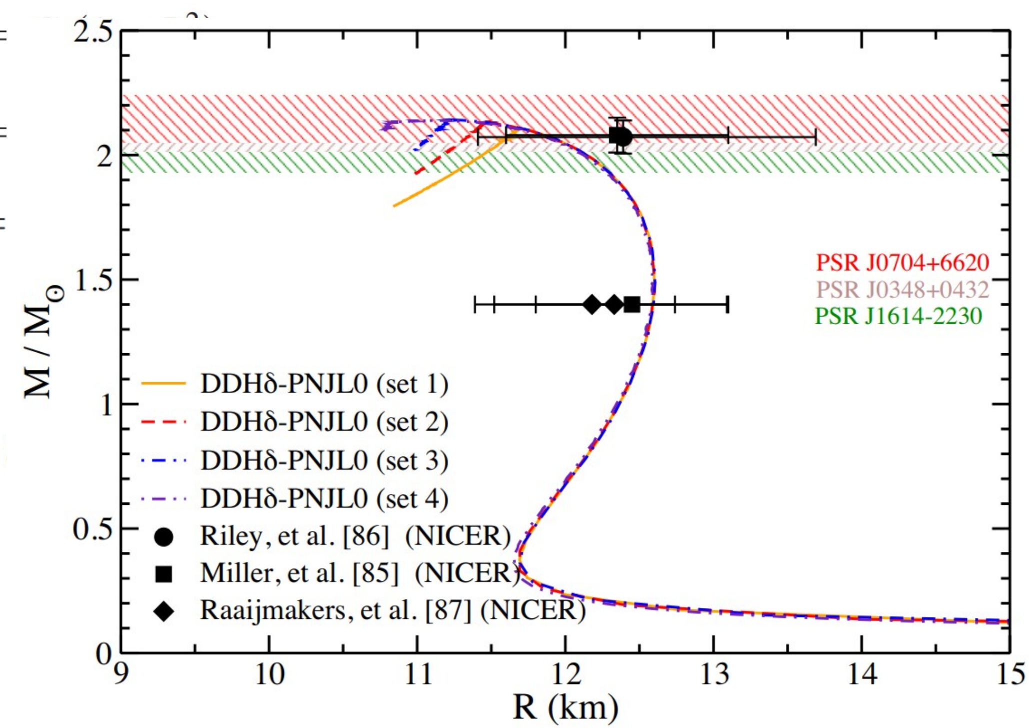
$$\Downarrow$$

$$U(\Phi, T, \mu) = (a_0 T^4 + a_1\mu^4 + a_2 T^2\mu^2)\Phi^2 + a_3 T_0^4 \log(1 - 6\Phi^2 + 8\Phi^3 - 4\Phi^4)$$

Use of $U(\Phi, T, \mu)$ together with

$$\begin{cases} G_s \to \mathcal{G}_s(G_s, \Phi) = \\ G_V \to \mathcal{G}_V(G_V, \Phi) = \\ K \to \mathcal{K}(K, \Phi) = \end{cases}$$

$$\Omega_{\mathrm{PNJL0}} = G_s\sum_f \rho_{sf}^2 - G_V\sum_f \rho_f^2 + 4K\prod_f \rho_{sf} - \frac{\gamma}{2\pi^2}\sum_f\int_0^\Lambda dk\, k^2(k^2+M_f^2)^{1/2} - \frac{\gamma}{6\pi^2}\sum_f\int_0^{k_{Ff}}\frac{dk\,k^4}{(k^2+M_f^2)^{1/2}} + \mathcal{U}(\rho_f, \rho_{sf}, \Phi)$$



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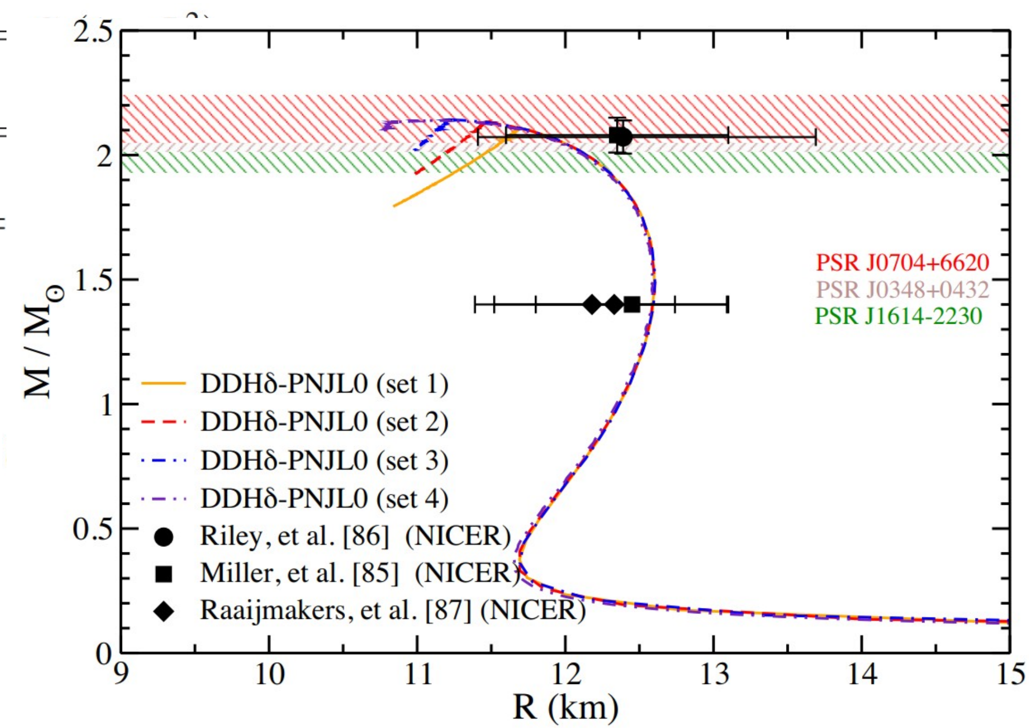
+ eB, with MSS and MFIR
In collaboration with F. Azeredo, J. Prado, R. Farias

Use of $U(\Phi,T,\mu)$ together with

$$\begin{cases} G_s \to \mathcal{G}_s(G_s,\Phi) = \\ G_V \to \mathcal{G}_V(G_V,\Phi) = \\ K \to \mathcal{K}(K,\Phi) = \end{cases}$$

$$\Omega_{\mathrm{PNJL0}} = G_s\sum_f \rho_{sf}^2 - G_V\sum_f \rho_f^2 + 4K\prod_f \rho_{sf} - \frac{\gamma}{2\pi^2}\sum_f\int_0^\Lambda dk\,k^2(k^2+M_f^2)^{1/2} - \frac{\gamma}{6\pi^2}\sum_f\int_0^{k_{Ff}}\frac{dk\,k^4}{(k^2+M_f^2)^{1/2}} + \mathcal{U}(\rho_f,\rho_{sf},\Phi)$$



Final Remarks

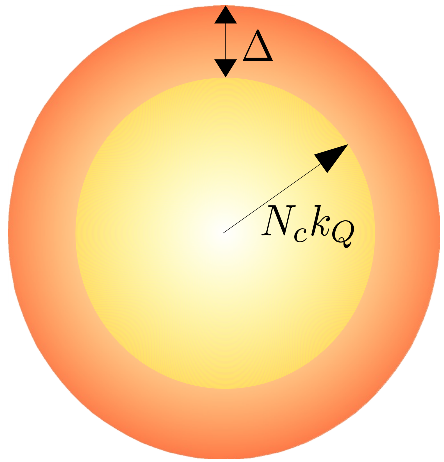
- Significant progress has been made in recent years toward describing the QCD phase diagram, but much remains to be done!

- **The careful treatment of divergences through the separation of medium-dependent terms in non-renormalizable models is essential for an accurate description of the physical quantities of interest;**

- MSS proves to be a powerful tool for this task when applied to NJL/PNJL. Extension to other models or contexts?

Thanks for your attention!

Backup:



- QCD with 3 colors: Small typical energy scales ($\sim 1/N_c$), even for nuclear interactions of $O(N_c)$ $\Longrightarrow$ matter behaves like a dilute gas.
- When density increase: Typical energies become of the same order of nuclear interactions $\Longrightarrow$ hadronic matter changes properties rapidly.
- Sound velocity increase very rapidly, exceed 1/3 at densities around (3-4) ρ_0.
$\Longrightarrow$Since it needs to approach 1/3 from below we expect that sound velocity will show a minimum at some point.

$$n_B \sim (k_F^B)^3 \sim (k_F^Q)^3 \quad \text{and} \quad k_F^B \simeq N_c k_F^Q \qquad c_s^2 = \frac{n_B}{\mu_B}\frac{d\mu_B}{dn_B}$$

If nuclear matter is composed by only nucleons (quasiparticles) their phase space increase with k_F^3, and the rapid increase in phase space available without corresponding increase in n_B suggests that nucleons are only partially filling their available phase space.