The quark anomalous magnetic moment in the NJL model and how to avoid first-order phase transitions induced by regularization issues

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1 NJL-AMM SU(2) model formalism

The Lagrangian density of a two flavor Nambu–Jona-Lasinio (NJL) model, up (u) and down (d) quarks, with the effect of the anomalous magnetic moment (AMM) of the quarks included by the Schwinger *ansatz* is Ref. [1]

$$\mathcal{L} = \overline{\psi}\left(i\not{D} - \hat{m} + \frac{1}{2}\hat{a}\sigma_{\mu\nu}F^{\mu\nu}\right)\psi + G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2\right] \ , \quad (1)$$

where $\psi = (\psi_u \quad \psi_d)^T$, $\not{D} = \gamma_\mu D^\mu$, $D^\mu = (\partial^\mu + ieQA^\mu)$, $Q = \mathrm{diag}(q_u = 2/3, q_d = -1/3)$, $\hat{m} = \mathrm{diag}(m_u, m_d)$, $\hat{a} = \mathrm{diag}(a_u, a_d)$ is the quark AMM matrix, $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and G is the coupling constant.

In the mean field approximation (MFA), Eq. (1) becomes

$$\mathcal{L}^{\mathrm{MFA}} = \overline{\psi}\left(i\not{D} - M + \frac{1}{2}\hat{a}\sigma_{\mu\nu}F^{\mu\nu}\right)\psi - \frac{(M-m)^2}{4G} \ , \quad (2)$$

where the effective quark mass M is defined as

$$M = m - 2G\left\langle\overline{\psi}\psi\right\rangle \ , \quad (3)$$

we take into account for the isospin limit, i.e., $m_u = m_d \equiv m$, and $\langle\overline{\psi}\psi\rangle$ is the chiral condensate. We consider the Landau gauge, $A_\mu = \delta_{\mu 2}x_1 B$, so $\overrightarrow{B} = B\widehat{e_3}$ and $\overrightarrow{\nabla}\cdot\overrightarrow{A} = 0$. Hence, a constant magnetic field.

The thermodynamic potential at zero temperature, $T = 0$, is

$$\Omega = \frac{(M-m)^2}{4G} + \Omega^{\mathrm{mag}}(M, B) \ , \quad (4)$$

the magnetic contribution is

$$\Omega^{\mathrm{mag}}(M, B) = -N_c \sum_{f=u,d} |B_f| \sum_{n=0}^{\infty}\sum_{s=\pm 1}\int_{-\infty}^{\infty}\frac{dp_3}{4\pi^2}E^f_{n,s} \quad (5)$$

in which $B_f \equiv q_f eB$, $s = \pm 1$ is the spin index, n are Landau levels, $N_c = 3$ the number of colors and the quark-energy dispersion relation is defined as

$$E^f_{n,s} = \sqrt{p_3^2 + \left(\sqrt{|B_f|(2n + 1 - s_f s) + M^2} - s a_f B\right)^2} , \quad (6)$$

where $s_f = \mathrm{sign}(q_f)$ is the charge sign function. We can rewrite the magnetic contribution defining the function $F_f(\tau)$, Ref. [2], as

$$\Omega^{\mathrm{mag}}(M, B) = \frac{N_c}{8\pi^2}\sum_{f=u,d}\int_0^\infty \frac{d\tau}{\tau^3}e^{-\tau M^2}F_f(\tau) \ , \quad (7)$$

where the function $F_f(\tau)$ is given by

$$F_f(\tau) = \exp\left[-\tau(a_f B)^2\right] \ \tau|B_f|\left[s_f \sinh(\tau 2a_f BM) + \sum_{k=0}^{\infty}\frac{(-\tau 2a_f BM)^{2k}}{(2k)!}\times \right.$$
$$\left. \times\sum_{n=0}^{k}\binom{k}{n}(-1)^n\left(\frac{|B_f|}{M^2}\right)^n\frac{d^n}{d(\tau|B_f|)^n}\coth(|B_f|\tau)\right] . \quad (8)$$

2 Vacuum magnetic regularization (VMR) scheme

The integral of Eq. (7) diverges to $\tau \to 0$. The VMR scheme, Ref. [3], is described as

$$\Omega^{\mathrm{mag}}(M, B) = [\Omega^{\mathrm{mag}}(M, B) - \Omega^{VD}(M, B)] + \Omega^{VD}(M, B)$$
$$\to \Omega^{\mathrm{mag}}_R(M, B) + \Omega^{VM}(M, B), \quad (9)$$

where $\Omega^{\mathrm{mag}}_R = \Omega^{\mathrm{mag}}(M, B) - \Omega^{VD}(M, B)$ is the regularized magnetic part and the other contributions

$$\Omega^{\mathrm{VD}}(M, B) = \frac{N_c}{8\pi^2}\sum_{f=u,d}\int_0^\infty\frac{d\tau}{\tau^3}e^{-\tau M^2}F^0_f(\tau) \ , \quad (10)$$

$$\Omega^{\mathrm{VM}}(M, B) = \frac{N_c}{8\pi^2}\sum_{f=u,d}\int_{\frac{1}{\Lambda^2}}^\infty\frac{d\tau}{\tau^3}e^{-\tau M^2}F^0_f(\tau) \ , \quad (11)$$

the vacuum-divergent (VD) and the vacuum-magnetic contributions (VM), respectively. $F^f_0(\tau)$ is an expansion of the function $F_f(\tau)$ around $\tau = 0$ up to the order $\mathcal{O}(\tau^2)$.

3 Mass-dependent (MD) regularization

Taking the explicit expansion of Eq. (8), we have Ref. [2]

$$F^0_f(\tau) = 1 + (a_f B)^2\tau + R_f(B_f, M)\tau^2 + \mathcal{O}(\tau^3) \ , \ \tau \ll 1 \ , \quad (12)$$

where the mass-dependent coefficient is defined as

$$R_f(B_f, M) = \frac{|B_f|^2}{3} - \frac{(a_f B)^4}{6} + 2(a_f B)^2M^2 + s_f 2|B_f|(a_f B)M \ . \quad (13)$$

4 Quark AMM and numerical results

The AMM of the quarks at one-loop level are given by

$$a_f = q_f\alpha_f\mu_B \quad (14)$$

where $\alpha_f = \alpha_e q_f^2/2\pi$ is the anomalous contribution, $\alpha_e = 1/137$ the fine-structure constant. Dealing with constant AMM values, the Bohr magneton is defined as $\mu_B = e/2M_0$, where $M_0 = M(B = 0, T = 0)$. We can use a quantity defined as $\kappa_f = \alpha_f/2M_0$ and so the

$$a_f B = \kappa_f B_f = \kappa_f q_f eB \ . \quad (15)$$

Following a NJL-SU(2) model proper-time regularization parameters from Ref. [4]: $\Lambda = 886.62$ MeV, $m_0 = 7.383$ MeV and $G = 4.001/\Lambda^2$. In the left of Fig. 1 (MD), for non-zero quark AMM values, i.e. $\kappa_f \neq 0$, we see first-order phase transitions (1st-PT) as in Refs. [1, 5]. In the right of Fig. 1, assuming a mass-independent (MI) expression for F^0_f, we avoid this behavior.

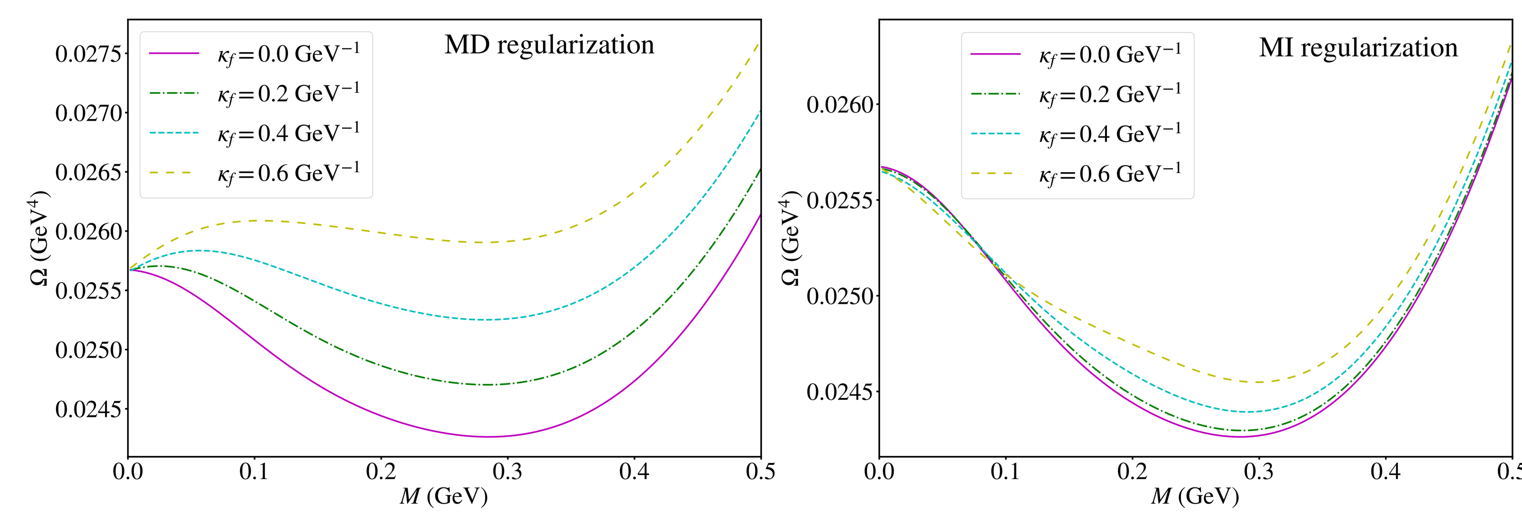

Figure 1: Left: MD regularization: Thermodynamic potential as function of the effective quark mass for different values of quark AMM with $2(a_f B)^2M^2 + s_f 2Ba_f M \neq 0$ at $eB = 0.3$ GeV^2. Right: MI regularization: Same with $2(a_f B)^2M^2 + s_f 2Ba_f M = 0$.

5 Mass-independent (MI) regularization

In Eq. (8), assuming $|B_f|/M_0^2 \ll 1$, as shown in Ref. [2] that

$$\Omega^{\mathrm{mag}}(M, B) = \frac{N_c}{8\pi^2}\sum_f\int_0^\infty\frac{d\tau}{\tau^3}e^{-\tau\mathcal{K}^2_{0,f}}\left[\tau|B_f|\frac{\cosh(c_f|B_f|\tau)}{\sinh(|B_f|\tau|)}\right] \ , \quad (16)$$

where $c_f = a_f + 1$ and $\mathcal{K}_{0,f} = \sqrt{M^2 + (a_f B)^2}$. The potential in Eq. (16) is the same as in Ref. [6] from the one-loop Schwinger–Weisskopf effective Lagrangian formalism, previously used to describe quark AMM in Ref. [7] with NJL model. The expansion of $F^0_f(\tau)$ of Eq. (16) is MI:

$$F^0_f(\tau) = 1 + \frac{(B_f\tau)^2}{6}\left(3c_f^2 - 1\right) + \mathcal{O}(\tau^3) \ , \ \tau \ll 1 \ , \quad (17)$$

thus, we can physically constrain by $|B_f|/M_0^2 \ll 1$ the absense of 1st-PT.

6 Conclusions and next steps

Mass-dependent (MD) terms uneven the thermodynamic potential and leads to artificial first-order phase transitions. The mass-independent (MI) regularization in VMR approach indicate the absence of these transitions in agreement with lattice QCD predictions Ref. [8]. This work is published at Ref. [2]. We intend to address the quark AMM using the Polyakov–NJL model to investigate the effects of magnetic catalysis and inverse magnetic catalysis.

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