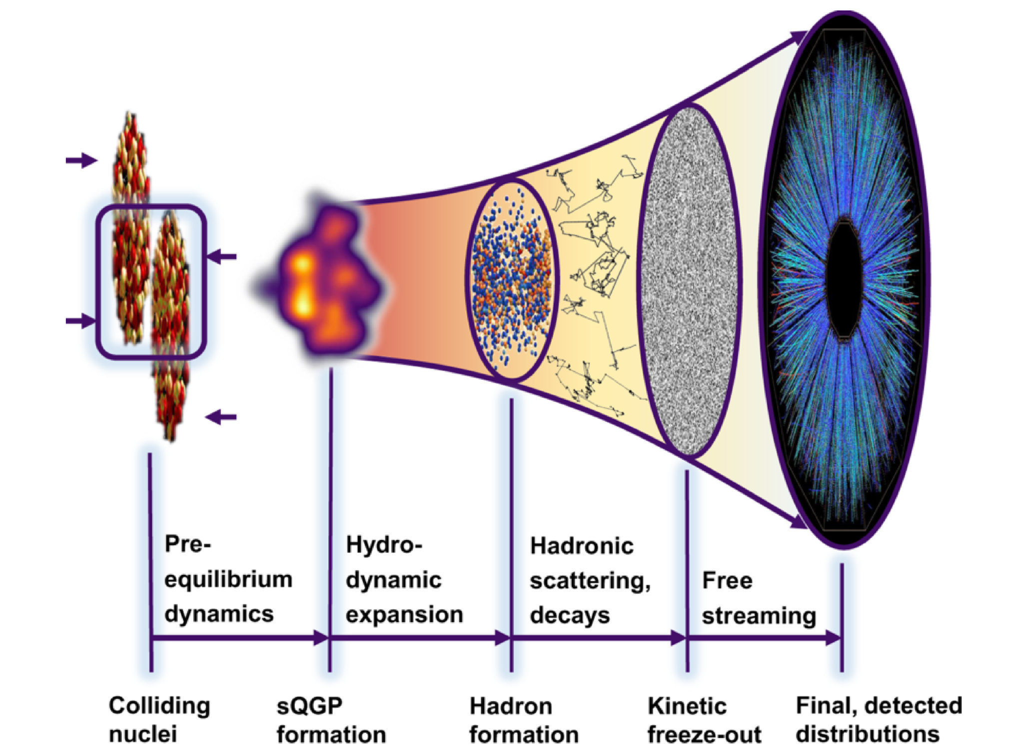
The Effect of Acausality on Bayesian Analyses

Thiago S. Domingues*, M.Luzum, R.Krupczak, T.J.Nunes da Silva, J-F Paquet, J.Noronha

Department of Mathematical-Physics, Institute of Physics, University of São Paulo.

thiago.siqueira.domingues@usp.br

Relativistic heavy-ion collisions



Heavy-ion collisions can recreate the extreme conditions necessary to form the Quark–Gluon Plasma (QGP) — a transient state in which quarks and gluons are deconfined. Multistage numerical models simulate the evolution from the initial impact through to final hadron production. We compare these simulations with experimental data to elucidate the properties of this exotic matter. **Relativistic Hydrodynamics** is a essential tool to describe the space-time evolution of the QGP. Traditionally, it is derived as an expansion around **local thermodynamic equilibrium**, yet the system produced in these collisions is initially **far-from-equilibrium.** However, recent theoretical advancements suggest that hydrodynamics may still be applicable in such regime [1]. A crucial aspect of its validity is *causality*, which can serve as a **definitive criterion.** To assess its impact, we employ *Bayesian analysis* to **quantify the extent to which causality constraints influence hydrodynamic applicability.**

Bayesian parameter estimation

Bayesian analysis provides a *model-to-data approach* for estimating **physical properties and their uncertainties.** It relies on Bayes' theorem:

$$P(\theta|D) \propto P(D|\theta)P(\theta)$$

where $P(\theta|D)$ (posterior) represents updated knowledge about the parameters θ given the data D, $P(D|\theta)$ (likelihood) quantifies how well the model describes the data, $P(\theta)$ (prior) encodes prior physical knowledge. Another important concept is the Maximum a Posteriori (MAP) point, most probable parameter set given the data, maximizing the posterior distribution.

Take-home message

Hydrodynamic simulations typically enter acausal regimes, at least sometimes. Demanding limits on acausality has **nonnegligible effects on existing Bayesian analyses.** In the era of precision heavy-ion physics, it is an issue that should be addressed!

References

[1] Thiago S. Domingues et al. *Phys. Rev. C*, 110(6):064904, 2024.

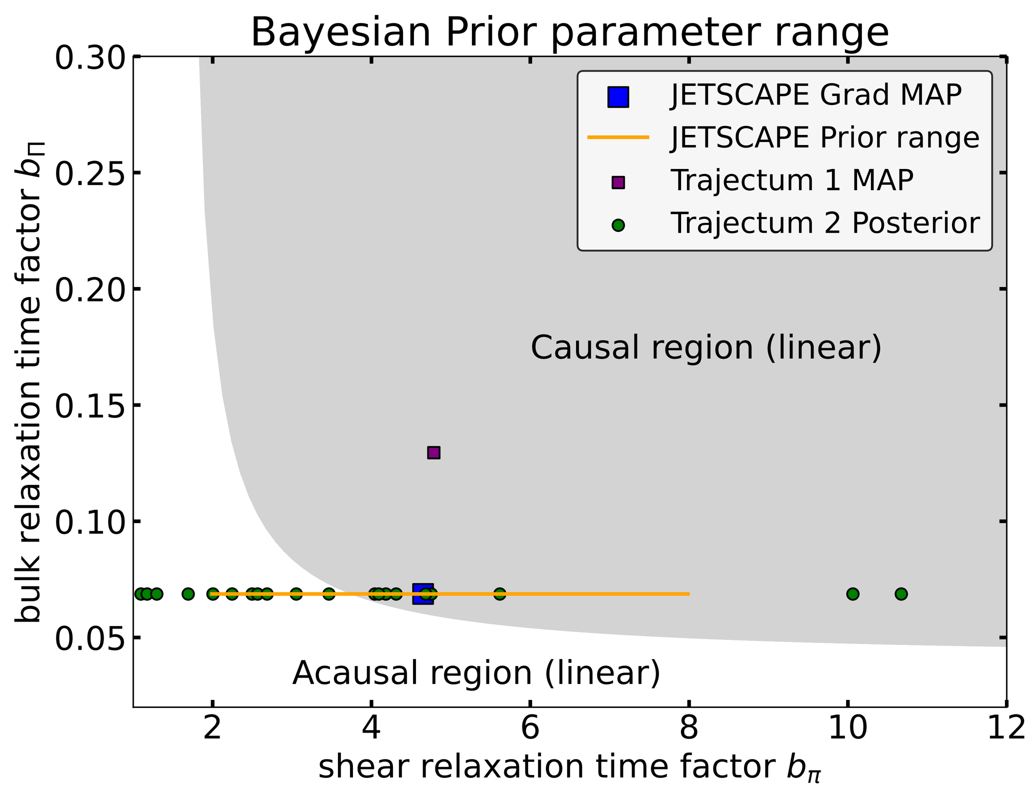
[2] Renata Krupczak et al. *Phys. Rev. C*, 109(3):034908, 2024.

Quantifying acausality in hydrodynamic simulations

Linear Causality Condition

Consider an infinitesimal perturbation about static global equilibrium. By linearizing the equations of motion, one demands that the signal propagation speed remains subluminal ($v < c$). This requirement leads to the following condition:

$$n_{\text{static}} \equiv c_s^2 + \frac{4}{3b_\pi} + \frac{\left(\frac{1}{3} - c_s^2\right)^2}{b_\Pi} \leq 1$$



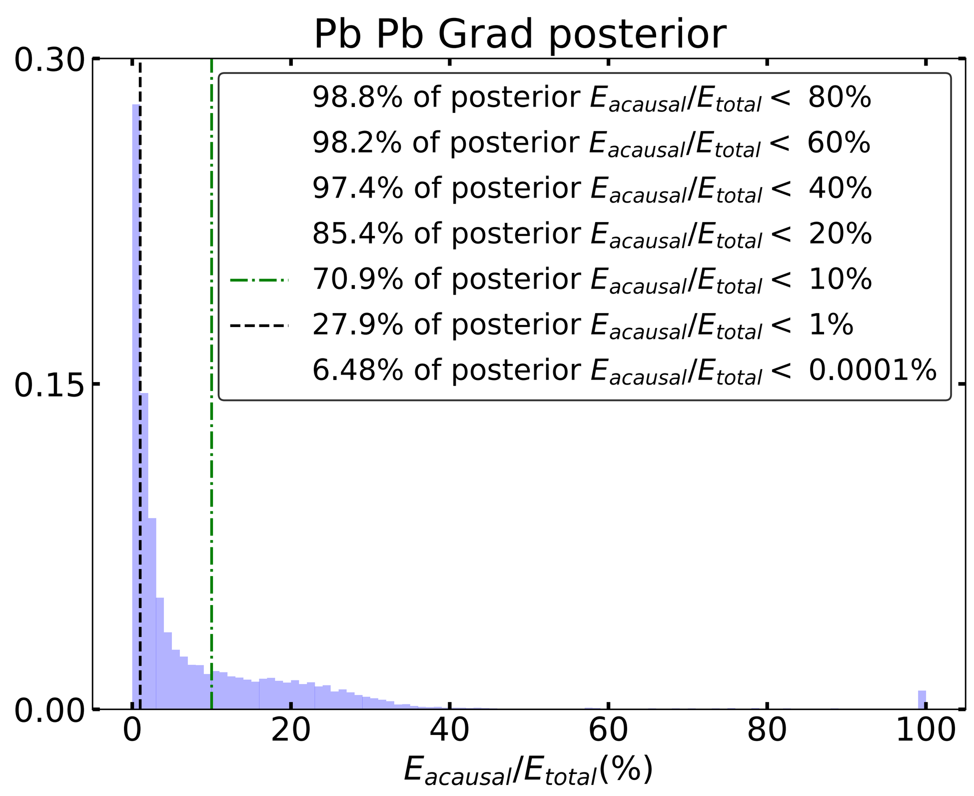
- The prior of current Bayesian analysis (and even the posterior) allow **violations of this linear causality constraint!**
- The weak dependence of observables on b_π and b_Π yields a flat posterior, thus not strongly affecting other parameters [1, 2].

Non-linear Causality Conditions

When going beyond the linear regime, more stringent conditions must be imposed.

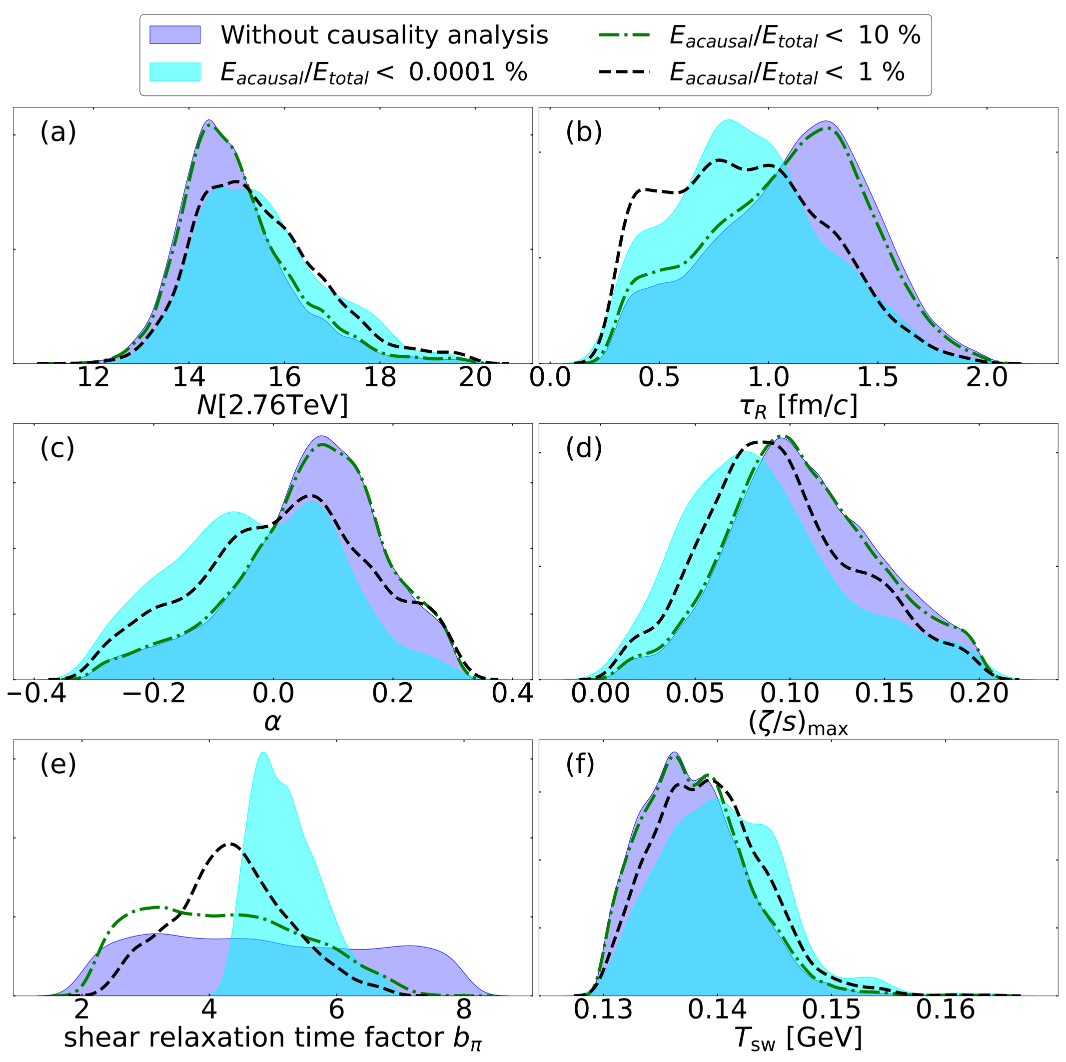
$$\begin{aligned} n_6 \equiv 1 &- \left(c_s^2 + \frac{4}{3}\frac{1}{C_\eta} + \frac{1}{C_\zeta}\right) \\ &+ \left(1 - \frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi} - c_s^2\right)\frac{\Pi}{\varepsilon + P} \\ &+ \left(1 - \frac{3\delta_{\pi\pi} + \tau_{\pi\pi}}{3\tau_\pi} - \frac{\lambda_{\Pi\pi}}{\tau_\Pi} - c_s^2\right)\frac{\Lambda_3}{\varepsilon + P} \geq 0 \end{aligned}$$

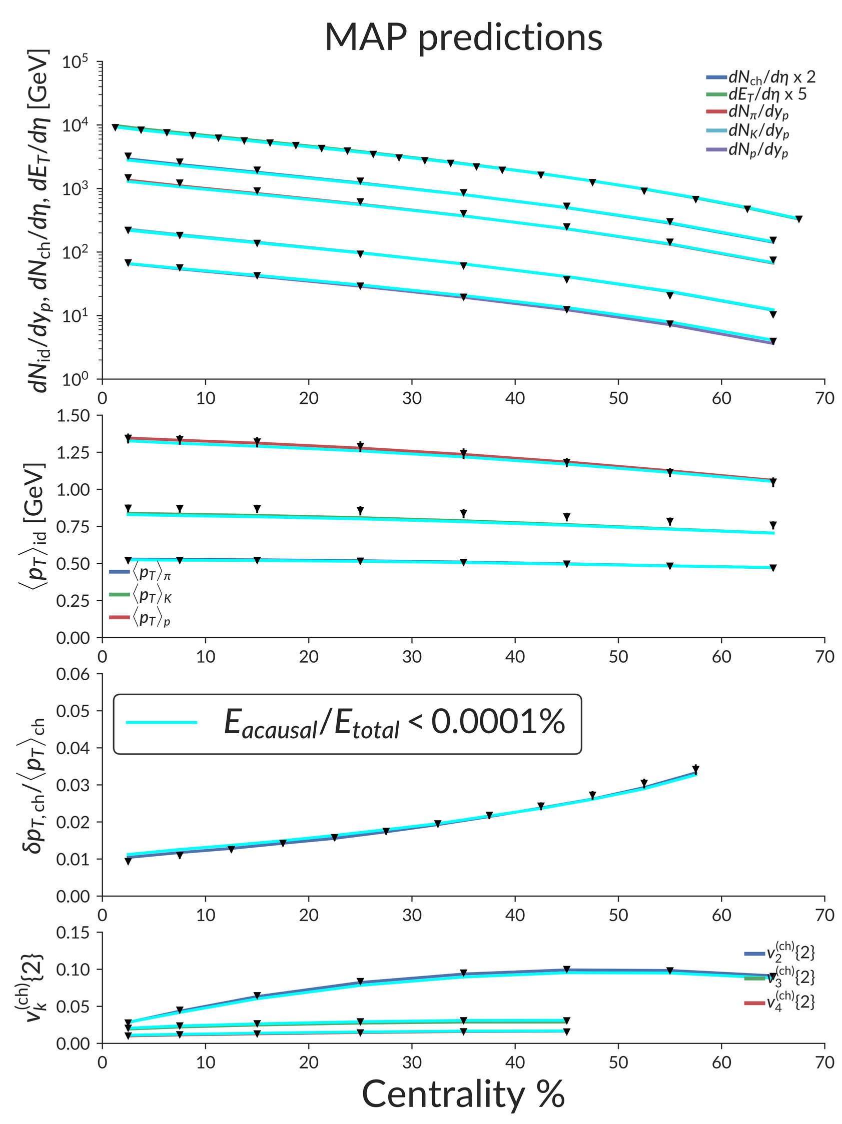
n_6 is found to be the most stringent condition to enforce causality in the non-linear regime.

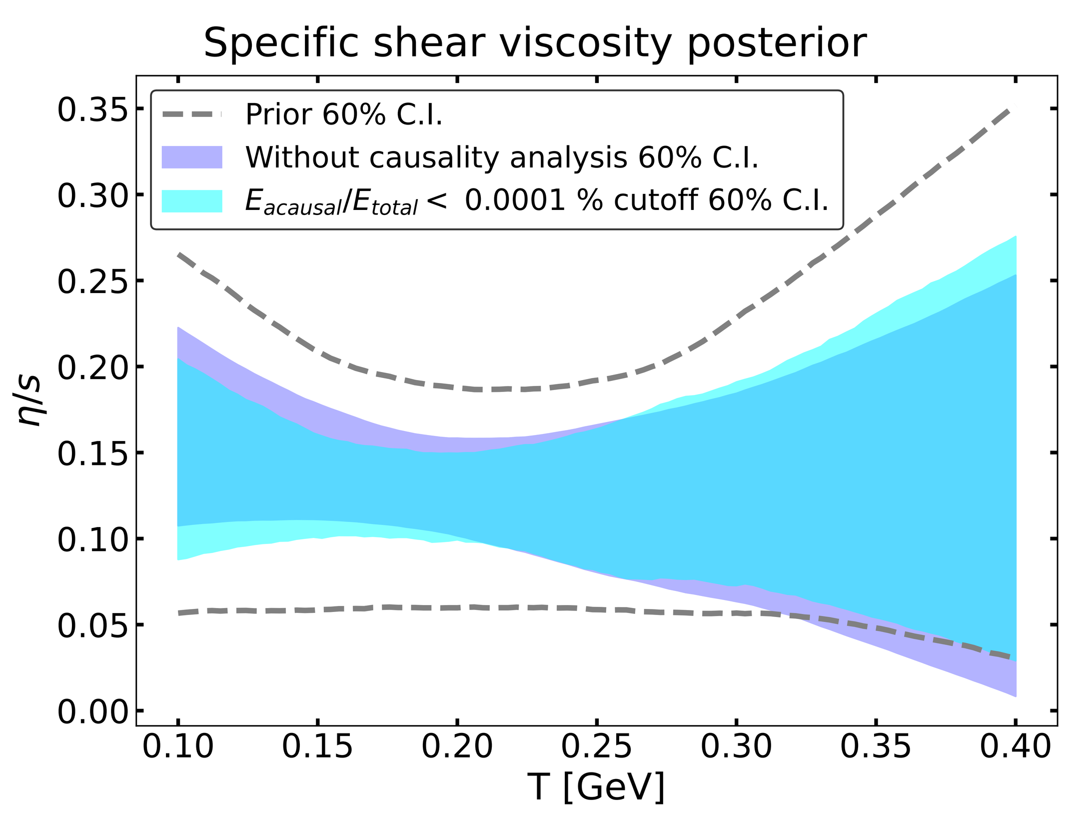


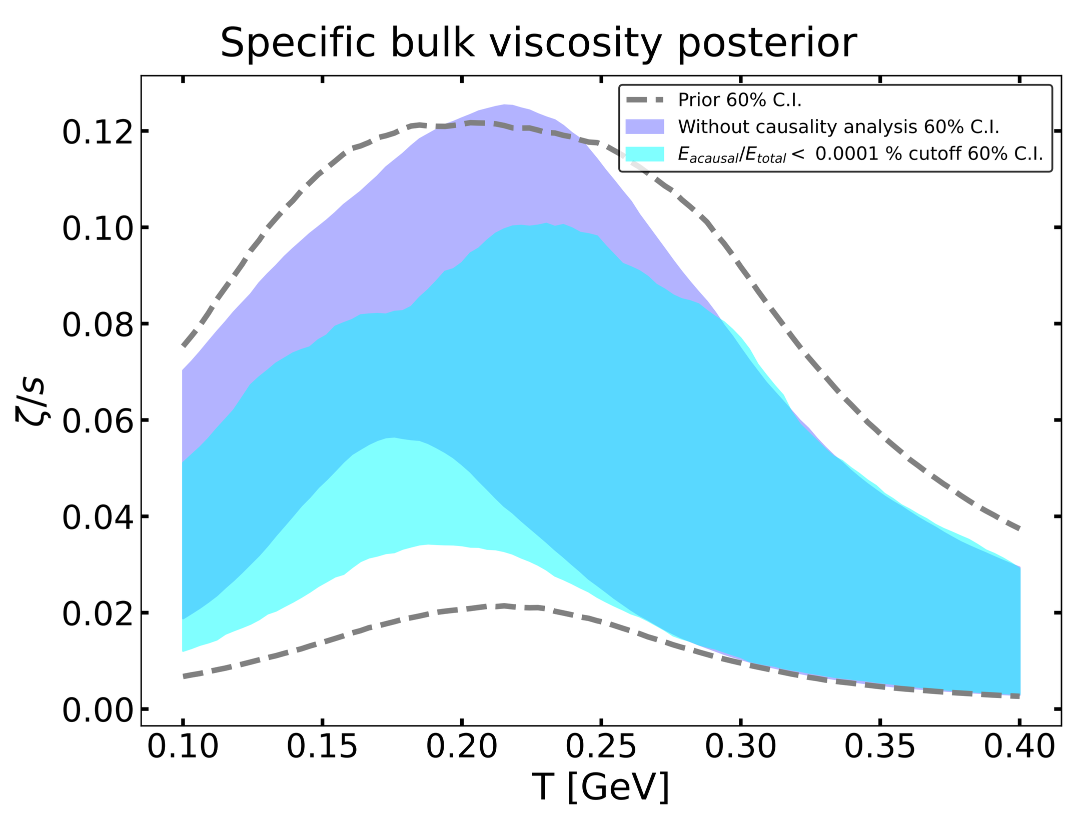
We performed $b = 0$ (central collisions) event-by-event simulations to quantify **the energy fraction of the system that becomes acausal at the onset of hydrodynamics**. **What happens if we impose cuts on the posterior?**

Effects of acausality on Bayesian parameter estimation









Enforcing causality constraints affects Bayesian posteriors!