# Speed of sound, compressibility and adiabatic index in the Many-Body Forces Model for nuclear matter at finite temperature

**Rafael Bán Jacobsen[1], Ricardo L. S. Farias[1], Veronica Dexheimer[2].**

[1]Department of Physics, Federal University of Santa Maria, Santa Maria/RS, 97105-900, Brazil
[2] Department of Physics, Kent State University, Kent/Ohio, 44242, USA

## Introduction and Formalism

We present the extension for finite temperature of the *Many-Body Forces Model* (MBF Model) for the first time. The MBF Model [1] [2] describes nuclear matter in a relativistic quantum hadrodynamics formalism that takes many-body forces into account, by means of a field dependence of the nuclear interaction coupling constants. Assuming nuclear matter to be charge neutral, beta-equilibrated and populated by the baryon octet, electrons and muons, we explore the parameters of the model, different hyperonic coupling schemes (also for the first time) and temperature effects to describe some properties of nuclear matter. The most complete version of the MBF Model [3] includes baryon fields ($\psi_b$) coupled to various meson fields ($\sigma$, $\omega$, $\varrho$, $\delta$ and $\phi$) and free lepton fields ($\psi_l$); the $\delta$ meson is introduced in order to better describe the properties of asymmetric matter, while the strange meson $\phi$ has important impact on hyperon interactions:

$$\begin{aligned}\mathcal{L} = \sum_b \overline{\psi}_b \Big[\gamma_\mu\Big(i\partial^\mu - g^*_{\omega b\xi}\omega^\mu - \frac{1}{2}g^*_{\varrho b\kappa}\tau.\varrho^\mu - g^*_{\phi b\eta}\phi^\mu\Big) \\ -\Big(m_b - g^*_{\sigma b\zeta}\sigma - \frac{1}{2}g^*_{\delta b\zeta}\boldsymbol{\tau}.\boldsymbol{\delta}\Big)\Big]\psi_b + \Big(\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\Big) \\ +\Big(\frac{1}{2}\partial_\mu\boldsymbol{\delta}.\partial^\mu\boldsymbol{\delta} - m_\delta^2\boldsymbol{\delta}^2\Big) + \Big(-\frac{1}{16\pi}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{8\pi}m_\omega^2\omega_\mu\omega^\mu\Big) \\ +\Big(-\frac{1}{16\pi}\boldsymbol{\varrho}_{\mu\nu}.\boldsymbol{\varrho}^{\mu\nu} + \frac{1}{8\pi}m_\varrho^2\boldsymbol{\varrho}_\mu.\boldsymbol{\varrho}^\mu\Big) + \Big(-\frac{1}{16\pi}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{8\pi}m_\phi^2\phi_\mu\phi^\mu\Big) \\ +\sum_l \overline{\psi}_l\gamma_\mu(i\partial^\mu - m_l)\psi_l,\end{aligned} \tag{1}$$

where $\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\boldsymbol{\varrho}_{\mu\nu} = \partial_\mu\boldsymbol{\varrho}_\nu - \partial_\nu\boldsymbol{\varrho}_\mu$, $\phi_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$, and the operators $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ denote the Pauli isospin matrices. The subscripts $b$ and $l$ label, respectively, the baryon octet ($n$, $p$, $\Lambda^0$, $\Sigma^-$, $\Sigma^0$, $\Sigma^+$, $\Xi^-$, $\Xi^0$) and lepton ($e^-$, $\mu^-$) degrees of freedom. The properties of the mesons are exhibited in the table below:

| meson | particle | classification | coupling constant | mass (MeV) |
|---|---|---|---|---|
| $\sigma$ | $\sigma$ | scalar-isoscalar | $g_{\sigma_b}$ | 550 |
| $\boldsymbol{\delta}$ | $a_0$ | scalar-isovector | $g_{\delta_b}$ | 980 |
| $\omega_\mu$ | $\omega$ | vector-isoscalar | $g_{\omega_b}$ | 782 |
| $\boldsymbol{\varrho}_\mu$ | $\varrho$ | vector-isovector | $g_{\varrho_b}$ | 770 |
| $\phi_\mu$ | $\phi$ | vector-isovector | $g_{\phi_b}$ | 1020 |

The general definition of the meson-baryon couplings is:

$$\begin{gathered} g^*_{\sigma b\zeta} \equiv \Pi_{\zeta b}g_{\sigma b}, \quad g^*_{\delta b\zeta} \equiv \Pi_{\zeta b}g_{\delta b}, \\ g^*_{\omega b\xi} \equiv \Pi_{\xi b}g_{\omega b}, \quad g^*_{\varrho b\kappa} \equiv \Pi_{\kappa b}g_{\varrho b}, \quad g^*_{\phi b\eta} \equiv \Pi_{\eta b}g_{\phi b}, \end{gathered} \tag{2}$$

where the **parametric term $\Pi_{\lambda b}$** introduces nonlinear contributions:

$$\Pi_{\lambda b} \equiv \left(1 + \frac{g_{\sigma b}\sigma + \frac{1}{2}g_{\delta b}\boldsymbol{\tau}.\boldsymbol{\delta}}{\lambda m_b}\right)^{-\lambda}, \tag{3}$$

for $\lambda = \xi, \kappa, \eta, \zeta$. The expression (3) is a generalized derivative coupling form, in the spirit of the original proposal by Zimanyi and Moszkowski [4], being **$\lambda$ an adjustable parameter** that can assume different values $\xi$, $\kappa$, $\eta$ or $\zeta$ to modulate the intensity of such coupling for $\omega$, $\varrho$, $\phi$ or the scalar mesons, respectively. As a first approach, we consider the so-called **scalar version** of the model, in which the nonlinear contributions affect only the scalar mesons, i.e., **$\xi = 0, \kappa = 0, \eta = 0, \zeta \neq 0$**. Moreover, from the Lagrangian density (1), in the **mean-field approximation**, the effective mass and the effective chemical potential of the baryons can be defined as:

$$m_b^* = m_b - \Pi_{\zeta b}\left(g_{\sigma b}\sigma_0 + g_{\delta b}I^{3b}\delta_0^3\right), \tag{4}$$

$$\mu_b^* = \sqrt{k_b^2 + (m_b^*)^2} + g_{\omega b}\omega_0 + g_{\varrho b}I^{3b}\varrho_0^3 + g_{\phi b}\phi_0, \tag{5}$$

where $I^{3b}$ is the baryon isospin projection in the z-direction, $\sigma_0$, $\omega_0$, $\phi_0$, $\varrho_0^3$ and $\delta_0^3$ are the expected values of the meson fields and $k_b$ stands for the Fermi momentum of each species of baryon.

## Extension for Finite Temperature

The equation of state (EoS), i.e., the general expressions for energy density ($\varepsilon$) and pressure ($p$) can be derived from the Lagrangian density (1) extracting the diagonal components of the energy-momentum tensor (assuming an ideal fluid). In the finite temperature case ($T \neq 0$), the integrals over the Fermi momenta must extend to infinity, and both particles and antiparticles must be included in the statistical description. Therefore:

$$\varepsilon = \frac{m_\sigma^2\sigma_0^2}{2} + \frac{m_\delta^2\delta_0^{32}}{2} + \frac{m_\omega^2\omega_0^2}{2} + \frac{m_\varrho^2\varrho_0^{32}}{2} + \frac{m_\phi^2\phi_0^2}{2} + \sum_{i=b}\varepsilon_i + \sum_{i=l}\varepsilon_i, \tag{6}$$

$$p = -\frac{m_\sigma^2\sigma_0^2}{2} - \frac{m_\delta^2\delta_0^{32}}{2} + \frac{m_\omega^2\omega_0^2}{2} + \frac{m_\varrho^2\varrho_0^{32}}{2} + \frac{m_\phi^2\phi_0^2}{2} + \sum_{i=b}p_i + \sum_{i=l}p_i, \tag{7}$$

where

$$\varepsilon_i = \frac{1}{\pi^2}\int_0^\infty dk_i\, k_i^2 E_i\,(f_{i+} + f_{i-}), \quad p_i = \frac{1}{\pi^2}\int_0^\infty dk_i\, \frac{k_i^4}{E_i}\,(f_{i+} + f_{i-}), \tag{8}$$

being $E_b = \sqrt{k_b^2 + (m_b^*)^2}$ the energy for each species of baryon $b$; $E_l = \sqrt{k_l^2 + m_l^2}$ the energy for each species of lepton $l$; $f_{b+}$ e $f_{b-}$ are the Fermi-Dirac distributions for baryons and antibaryons, respectively; $f_{l+}$ e $f_{l-}$ are the Fermi-Dirac distributions for leptons and antileptons, respectively:

$$f_{b+} = \left[\frac{1}{e^{(E_b - \mu_b^*)/T} + 1}\right], \quad f_{b-} = \left[\frac{1}{e^{(E_b + \mu_b^*)/T} + 1}\right], \tag{9}$$

$$f_{l+} = \left[\frac{1}{e^{(E_l - \mu_l)/T} + 1}\right], \quad f_{l-} = \left[\frac{1}{e^{(E_l + \mu_l)/T} + 1}\right]. \tag{10}$$

In the above expressions, $m_b^*$ and $\mu_b^*$ are given by equations (4) and (5), respectively. The total baryon number density is given by:

$$n_B = \sum_{i=b} n_i = \frac{1}{\pi^2}\sum_{i=b}\int_0^\infty dk_i\, k_i^2\,(f_{i+} + f_{i-}). \tag{11}$$

## Coupling Schemes

For each parametrization ($\zeta$) of the model, the meson-baryon coupling constants can be determined in 3 steps:

**STEP 1:** At saturation, the cold isopin-symmetric nuclear matter has vanishing pressure, and is populated only by nucleons ($N$), not by leptons nor hyperons. Also, due to the isospin symmetry, the mean values of the isovector mesons $\varrho$ and $\delta$ are zero. To determine the constants $(g_{\sigma N}/m_\sigma)^2$, $(g_{\omega N}/m_\omega)^2$ and the effective mass of the nucleon $m_N^*$ at saturation, we solve the system of equations of zero pressure, experimental value of the binding energy per nucleon and the $\sigma_0$ field equation of motion self-consistently. For these calculations, we impose a saturation density of **$n_0 = 0.15\,\mathrm{fm}^{-3}$** and a binding energy per baryon of **$B/A = -15.75\,\mathrm{MeV}$**. This procedure allows us to find the compressibility modulus for symmetric nuclear matter at saturation, which is related to the curvature of the equation of state:

$$K_0 = 9n_0\left[\frac{d(\varepsilon/n_B)}{dn_B}\right]_{n_B = n_0}. \tag{12}$$

**STEP 2:** In order to determine the coupling constants of the nucleon with respect to the isovector mesons, it is necessary to consider the equation of state of cold asymmetric nuclear matter (in the absence of leptons) and solve the system of equations of the symmetry energy $a^0_{sym}$ and its slope $L_0$ to find the corresponding values of $(g_{\varrho N}/m_\varrho)^2$ and $(g_{\delta N}/m_\delta)^2$, according to:

$$a^0_{sym} = \frac{1}{2}\left[\frac{d^2(\varepsilon/n_B)}{dt^2}\right]_{t=0}, \quad L_0 = 3n_0\left[\frac{da_{sym}}{dn_B}\right]_{n_B = n_0}, \tag{13}$$

where the asymmetry between protons and neutrons is quantified by $t = (n_p - n_n)/n_B$. Considering the phenomenological values for $m_N^*$, $K_0$, $a^0_{sym}$ and $L_0$, this procedure allows us to build a map of the meson-nucleon coupling constants. Actually, the range of values for the adjustable parameter $\zeta$ is quite narrow, and the following table presents the coupling constants for the extreme values of $\zeta$ within the phenomenological range:

| $\zeta$ | $m_N^*/m_N$ | $K_0$ $(MeV)$ | $a^0_{sym}$ $(MeV)$ | $L_0$ $(MeV)$ | $(g_{\sigma N}/m_\sigma)^2$ $(fm^2)$ | $(g_{\omega N}/m_\omega)^2$ $(fm^2)$ | $(g_{\varrho N}/m_\varrho)^2$ $(fm^2)$ | $(g_{\delta N}/m_\delta)^2$ $(fm^2)$ |
|---|---|---|---|---|---|---|---|---|
| 0.040 | 0.66 | 297 | 25 | 76 | 14.51 | 8.74 | 2.56 | 0.38 |
| 0.040 | 0.66 | 297 | 25 | 90 | 14.51 | 8.74 | 6.53 | 6.50 |
| 0.040 | 0.66 | 297 | 33 | 100 | 14.51 | 8.74 | 4.74 | 0.38 |
| 0.040 | 0.66 | 297 | 33 | 115 | 14.51 | 8.74 | 6.93 | 0.66 |
| 0.129 | 0.78 | 211 | 25 | 73 | 10.78 | 5.13 | 4.10 | 3.00 |
| 0.129 | 0.78 | 211 | 25 | 90 | 10.78 | 5.13 | 9.10 | 10.60 |
| 0.129 | 0.78 | 211 | 33 | 100 | 10.78 | 5.13 | 7.60 | 4.60 |
| 0.129 | 0.78 | 211 | 33 | 115 | 10.78 | 5.13 | 11.10 | 10.50 |

**STEP 3:** Proposals for the meson-hyperon coupling constants:

**U - Universal Coupling:** The coupling constants for hyperons ($Y = \Lambda, \Sigma, \Xi$) and nucleons ($N = p, n$) are considered to be identical: $g_{\sigma Y} = g_{\sigma N}$, $g_{\omega Y} = g_{\omega N}$, $g_{\varrho Y} = g_{\varrho N}$, $g_{\delta Y} = g_{\delta N}$, $g_{\phi Y} = g_{\omega N}$. Also, $g_{\phi N} = 0$.

**M - Moszkowski Coupling:** Based on the quark content of nucleons and hyperons, Moszkowski proposed [5] that meson-hyperon coupling constants should be corrected according to: $g_{\sigma Y} = \sqrt{2/3}g_{\sigma N}$, $g_{\omega Y} = \sqrt{2/3}g_{\omega N}$, $g_{\varrho Y} = \sqrt{2/3}g_{\varrho N}$, $g_{\delta Y} = \sqrt{2/3}g_{\delta N}$, $g_{\phi Y} = \sqrt{2/3}g_{\omega N}$. Moreover, $g_{\phi N} = 0$.

**SU(6) - Spin-Flavor Symmetry Coupling:** This coupling scheme takes into account the strangeness of the particles and the isospin proportion between nucleons and hyperons [6] [7]:

$$\frac{1}{3}g_{\omega N} = \frac{1}{2}g_{\omega\Lambda} = \frac{1}{2}g_{\omega\Sigma} = g_{\omega\Xi}, \tag{14}$$

$$g_{\varrho N} = \frac{1}{2}g_{\varrho\Sigma} = g_{\varrho\Xi}, \quad g_{\varrho\Lambda} = 0, \tag{15}$$

$$-\frac{2\sqrt{2}}{3}g_{\omega N} = 2g_{\phi\Lambda} = 2g_{\phi\Sigma} = g_{\phi\Xi}, \quad g_{\phi N} = 0, \tag{16}$$

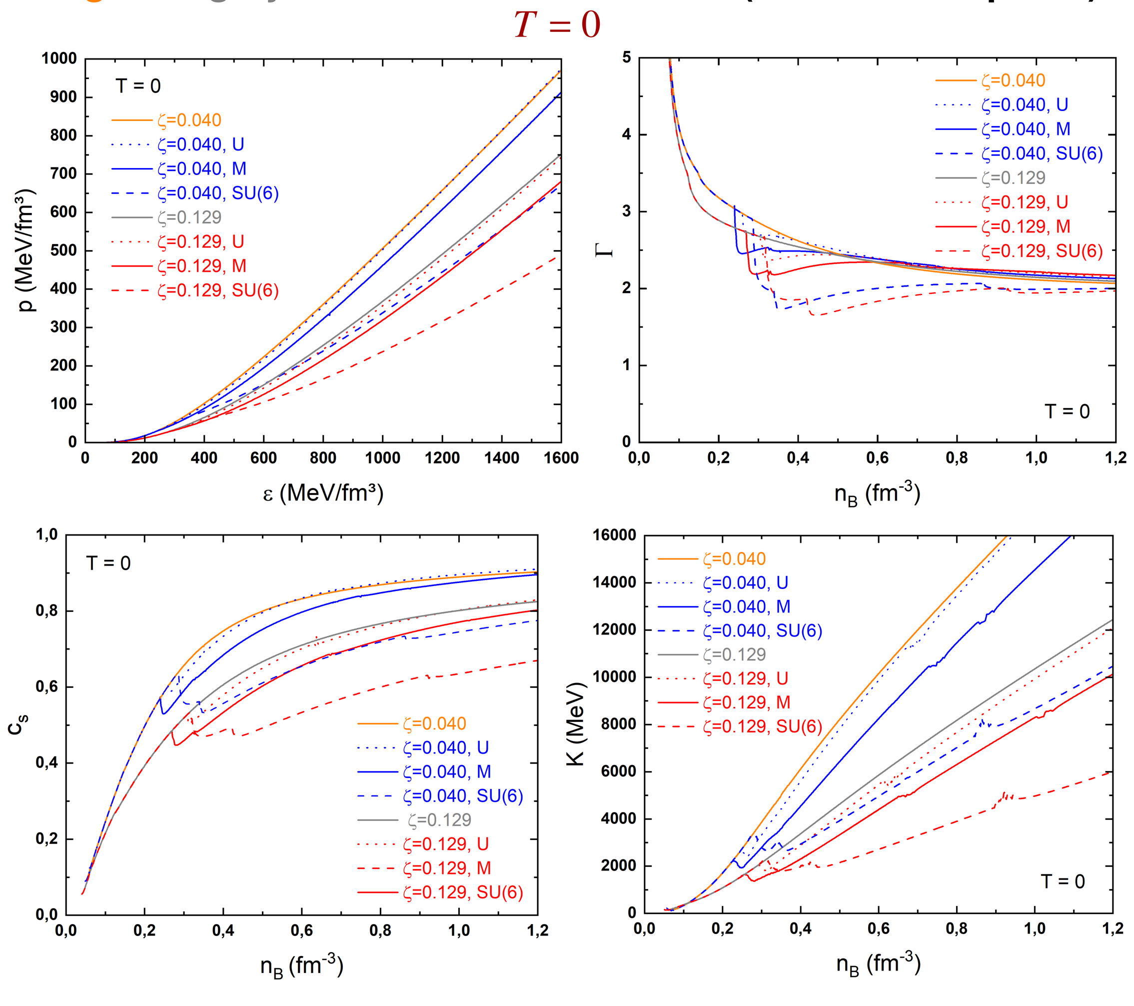
$$g_{\delta N} = \frac{1}{2}g_{\delta\Sigma} = g_{\delta\Xi}, \quad g_{\delta\Lambda} = 0. \tag{17}$$

We obtain the hyperon-sigma coupling associated to the attractive interaction between hyperons and nucleons by fitting the potential depths of the hyperons in nuclear matter [8] [9], according to the prescription: $U_Y = g_{\omega Y}\omega_0(n_0) - g_{\sigma Y}\sigma_0(n_0)$. Following the values of [10]: $U_\Lambda = -28\,\mathrm{MeV}$, $U_\Sigma = +30\,\mathrm{MeV}$ and $U_\Xi = -18\,\mathrm{MeV}$.

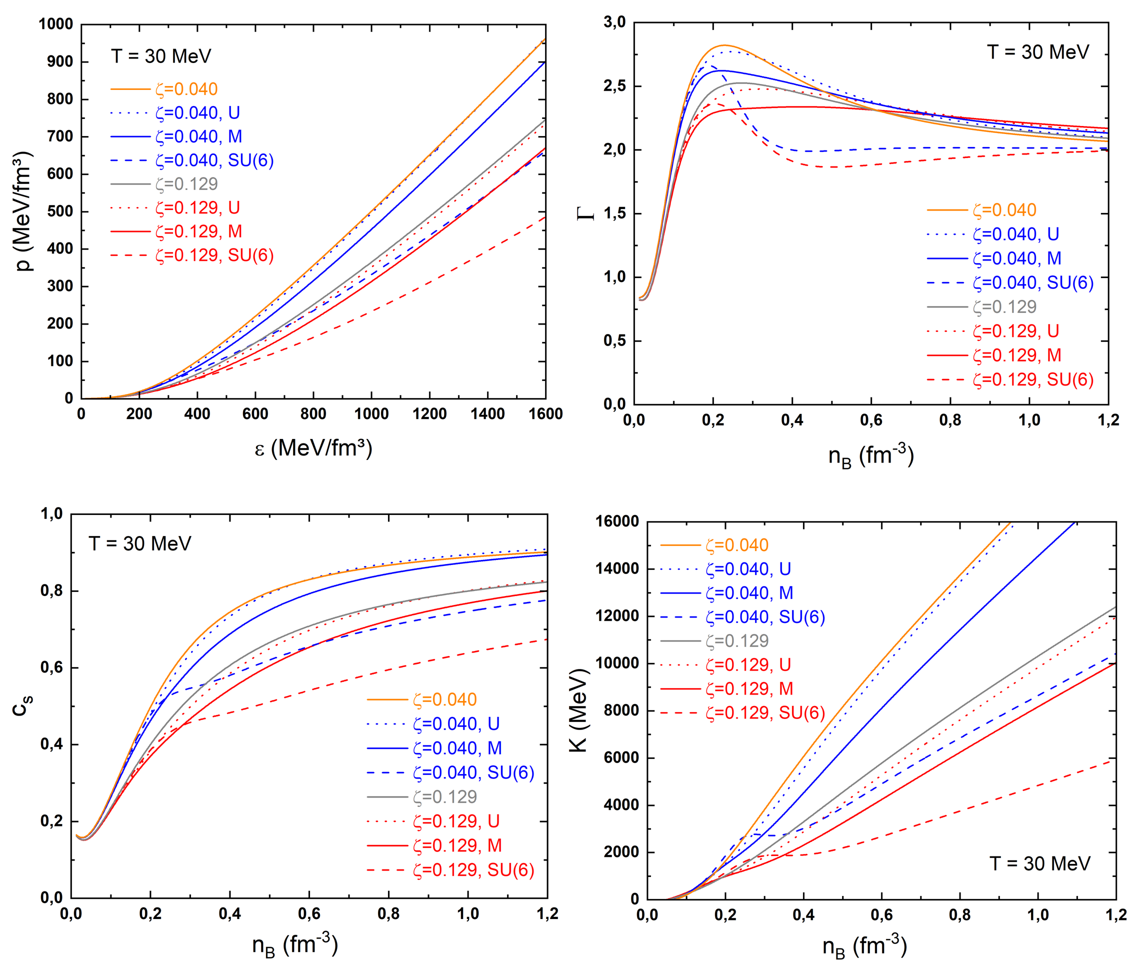
## Numerical Results

Here we present the equations of state for 3 different temperatures, followed by the results for the adiabatic index ($\Gamma$), speed of sound as a fraction of the speed of light ($c_s$) and compressibility ($K$), considering the two extreme cases for the adjustable parameter $\zeta$ within the phenomenological range: **$\zeta = 0.040$** and **$\zeta = 0.129$**, both with **$a^0_{sym} = 33\,MeV$** and **$L_0 = 100\,MeV$**. In **blue** and **red**: results for **hadronic matter (nucleons + hyperons + leptons)** in the 3 coupling schemes discussed in this approach - U, M and SU(6). In **orange** and **gray**: results for **nuclear matter (nucleons + leptons)**.

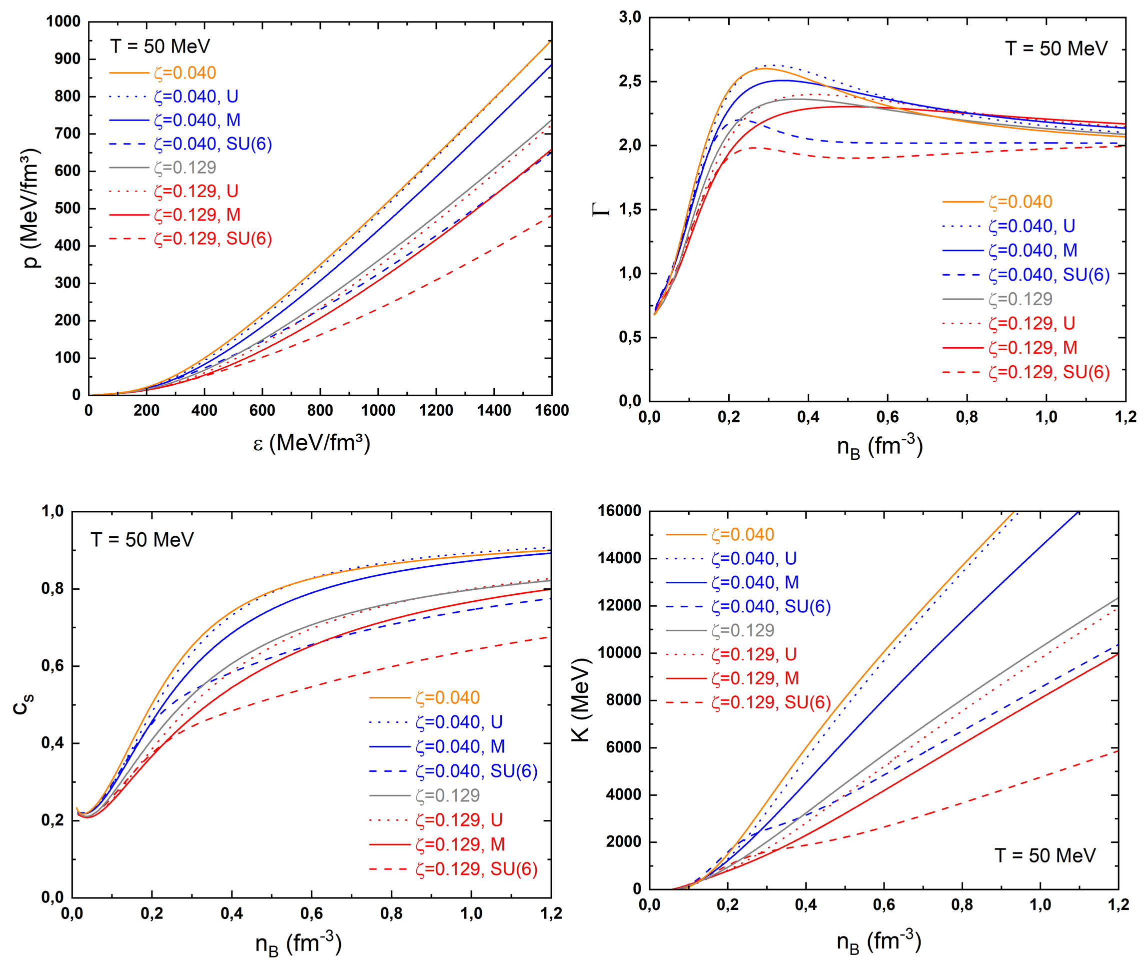
$T = 0$



$T = 30MeV$



$T = 50MeV$



## Conclusions

- The MBF Model is useful for constructing a variety of equations of state within the phenomenological range, from the stiffest ($\zeta = 0.040$) to the softest ($\zeta = 0.129$) equation of state.
- In the presence of hyperons, the coupling scheme adopted has significant impact on the stiffness of the EoS; namely, the universal coupling scheme (U) provides stiffer equations of state, while the SU(6) coupling scheme gives the softest.
- In the MBF Model, hadronic matter in the universal coupling scheme (U) and plain nuclear matter (without hyperons) present similar behavior.
- Even though the overall appearance of each EoS remains the same for different temperatures, the results for $\Gamma$, $c_s$ and $K$ reveal the expected thermal effects.
- The peaks and bumps in $\Gamma$, $c_s$ and $K$ at $T = 0$ (due to the appearance of new hyperonic degrees of freedom) are softened as temperature increases because, in this case, hyperons are already present at low densities.
- The results for $c_s$ show that the MBF Model strictly respects causality, even in the presence of strong vector interactions.
- The liquid-gas phase transition at $T = 0$ leads to a sudden increase in $\Gamma$ at low densities; however, at higher temperatures, this phase transition is not present and $\Gamma$ tends to a constant (but non-vanishing) value at low densities, as expected.
- Our results are in good agreement with literature for $T = 0$ MBF [11] and for other models at finite $T$ [12].

## References

[1] A. R. Taurines, C. A. Z. Vasconcellos, M. Malheiro, and M. Chiapparini, "Neutron stars with parametrized meson couplings," *Mod. Phys. Lett. A*, vol. 15, pp. 1789–1800, 2000.

[2] A. R. Taurines, C. A. Z. Vasconcellos, M. Malheiro, and M. Chiapparini, "Neutron stars in a class of nonlinear relativistic models," *Phys. Rev. C*, vol. 63, p. 065801, May 2001.

[3] R. O. Gomes, V. Dexheimer, S. Schramm, and C. A. Z. Vasconcellos, "Many-body forces in the equation of state of hyperonic matter," *Astrophys. J.*, vol. 808, no. 1, p. 8, 2015.

[4] J. Zimanyi and S. A. Moszkowski, "Nuclear equation of state with derivative scalar coupling," *Phys. Rev. C*, vol. 42, pp. 1416–1421, Oct 1990.

[5] S. A. Moszkowski, "Energy of neutron-star matter," *Phys. Rev. D*, vol. 9, pp. 1613–1625, 1974.

[6] C. B. Dover and A. Gal, "Hyperon nucleus potentials," *Prog. Part. Nucl. Phys.*, vol. 12, pp. 171–239, 1985.

[7] J. Schaffner, C. B. Dover, A. Gal, C. Greiner, D. J. Millener, and H. Stoecker, "Multiply strange nuclear systems," *Annals Phys.*, vol. 235, pp. 35–76, 1994.

[8] N. K. Glendenning and S. A. Moszkowski, "Reconciliation of neutron star masses and binding of the lambda in hypernuclei," *Phys. Rev. Lett.*, vol. 67, pp. 2414–2417, 1991.

[9] J. Schaffner, H. Stoecker, and C. Greiner, "Metastable exotic multihypernuclear objects," *Phys. Rev. C*, vol. 46, pp. 322–329, 1992.

[10] S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, "Hyperons and massive neutron stars: the role of hyperon potentials," *Nucl. Phys. A*, vol. 881, pp. 62–77, 2012.

[11] V. A. Dexheimer, C. A. Z. Vasconcellos, and B. E. J. Bodmann, "On the Density Dependent Nuclear Matter Compressibility," *Phys. Rev. C*, vol. 77, p. 065803, 2008.

[12] A. R. Raduta, F. Nacu, and M. Oertel, "Equations of state for hot neutron stars," *Eur. Phys. J. A*, vol. 57, no. 12, p. 329, 2021.

CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico · FAPERGS · serrapilheira · CAPES