Polarised calculations with BBMC and MoCaNLO

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Table of contents

Motivation

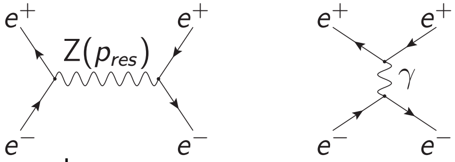
- Polarisation measurements require polarised templates to split the measured signal into the polarised contributions
- To make polarisation templates Monte Carlo Codes capable of making high accuracy predictions for polarised processes are needed
  - Need a method to define polarised cross-sections
  - Need to compute higher order corrections to the processes

Overview of BBMC and MoCaNLO

- General purpose Monte-Carlo codes
  - Can compute NLO QCD and EW accurate full off-shell and polarised processes
  - DPA is used to compute polarised processes
- Catani-Seymour dipole subtraction scheme [Catani, Seymour 9605323]
- RECOLA 1.4.4 as an amplitude provider [Actis et al. 1211.6316, 1605.01090]
- COLLIER to compute loop integrals [Denner et al. 1604.06792]

- Used for the computation of many polarised di-boson processes
  - ZZ production: [Denner, Pellicciolli 2107.06579]
  - W^+W^- production: [Denner, Pellicciolli 2006.14867; Denner, Haitz, Pellicciolli 2311.16031]
  - ZW^+ production: [Denner and Pellicciolli 2010.07149; Denner, Haitz, Pellicciolli 2211.09040]
  - W^+W^+ scattering: [Denner, Haitz, Pellicciolli 2409.03620]

Amplitude in the pole approximation



- Diagrams with and without the wanted (s-channel) resonances contribute to a given process
- Remove non-resonant diagrams in a gauge-independent way
  - By using a pole approximation
    - ★ Set resonant particles on-shell $\{p\} \Rightarrow \{\tilde{p}\}$: $\tilde{p}_{res}^2 = M_{res}^2$
    - ★ Conserve some off-shell effects by using the off-shell denominators of the propagators and applying the phase-space cuts to the off-shell momenta

$$\mathcal{M}\left(\{\tilde{p}\}, p_{res}^2\right) = \mathcal{M}_{\mu,\text{production}}\left(\{\tilde{p}\}\right) \frac{\mathcal{N}^{\mu\nu}\left(\{\tilde{p}\}\right)}{p_{res}^2 - M_{res}^2 + iM_{res}\Gamma_{res}} \mathcal{M}_{\nu,\text{decay}}\left(\{\tilde{p}\}\right)$$

On-shell projection (OSP) for two resonances

- Gauge invariance requires on-shell resonances
- Properties to ensure a physically meaningful result
  - Four-momentum conservation
  - Masses of the external particles are conserved
  - Smoothly approach the limit of on-shell resonances
- Additionally conserved in our implementation of the DPA
  - Momenta of other external particles that are not decay particles of the resonances
  - Direction of the resonant particles in the centre-of-mass frame of the two resonant particles
  - Direction of the decay particles in the rest frame of the corresponding resonance.
- When $(p_{res,1} + p_{res,2})^2 < (M_{res,1} + M_{res,2})^2$ the momenta cannot be projected on-shell and the amplitude is set to zero
- Generalisable to more resonances

LO unpolarised DPA W^+W^- pair-production

state	σ_{LO} [fb]	$\sigma_{\mathrm{NLO\,EW}}$ [fb]	δ_{EW}[%]	f_{NLO}[%]
full	259.02(2)			
unp.	249.97(2)			
LL				
LT				
TL				
TT				
int.				

- The LO unpolarised DPA cross-section can be computed
- Small non-resonant background (3.6%)

Polarised amplitude in the pole approximation

- All Numerators of the resonant propagators contain a sum over all polarisation states

$$\sum_{\text{polarisations}} \epsilon^*_\mu \epsilon_\nu = -g_{\mu\nu}$$

$$\mathcal{M}\left(\{\tilde{p}\}, p^2_{res}\right) = \sum_\lambda \mathcal{M}_{\mu,\text{production}}\left(\{\tilde{p}\}\right) \frac{\epsilon^{\mu *}_\lambda\left(\{\tilde{p}\}\right) \epsilon^\nu_\lambda\left(\{\tilde{p}\}\right)}{p^2_{res} - M^2_{res} + iM_{res}\Gamma_{res}} \mathcal{M}_{\nu,\text{decay}}\left(\{\tilde{p}\}\right)$$

$$\mathcal{M}\left(\{\tilde{p}\}, p^2_{res}\right) = \sum_\lambda \mathcal{M}_\lambda\left(\{\tilde{p}\}, p^2_{res}\right)$$

Polarised cross-section in the pole approximation

- Take the square of the matrix element to calculate the cross-section

$$\underbrace{\left|\mathcal{M}\left(\{\tilde{p}\}, p^2_{res}\right)\right|^2}_{\text{unpolarised}} = \sum_{\lambda} \underbrace{\left|\mathcal{M}_{\lambda}\left(\{\tilde{p}\}, p^2_{res}\right)\right|^2}_{\text{polarisation } \lambda}$$

$$\underbrace{+ \sum_{\lambda \neq \lambda'} \mathcal{M}^*_{\lambda}\left(\{\tilde{p}\}, p^2_{res}\right) \mathcal{M}_{\lambda'}\left(\{\tilde{p}\}, p^2_{res}\right)}_{\text{interferences}}$$

LO polarised DPA W^+W^- pair-production

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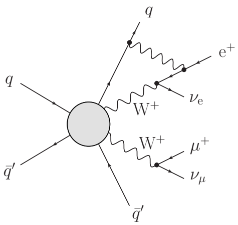
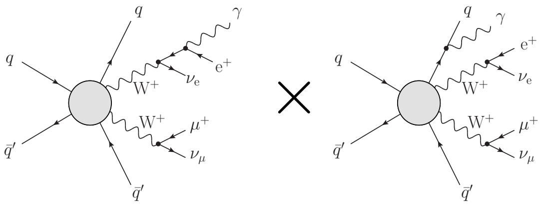
full	259.02(2)			
unp.	249.97(2)			
LL	21.007(2)			
LT	33.190(3)			
TL	34.352(5)			
TT	182.56(2)			
int.	−21.14(5)			

- The LO polarised and interference contributions can be computed
- TT dominates
- Large interference

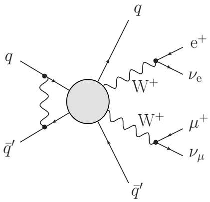
Calculation of NLO corrections in the DPA

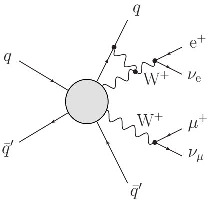
- Nonfactorisable contributions are background
  - Small when real and virtual treated in the DPA





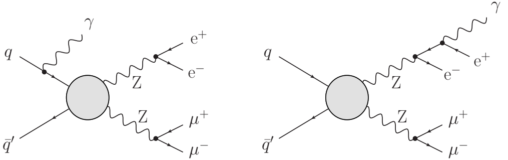
- Virtual and integrated dipole contributions are evaluated with the same methods as at LO





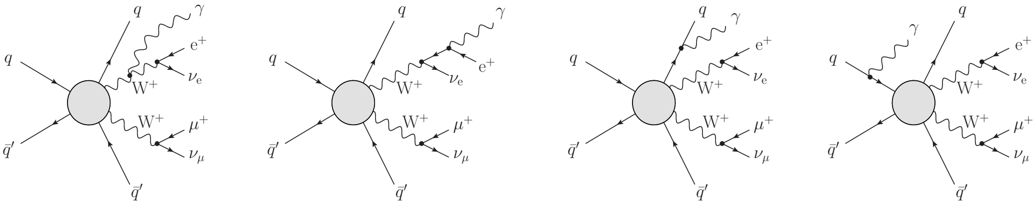
Real NLO EW corrections to neutral resonances

- Clear split between emission from production and the decay subprocess
- IR divergences can be canceled with the same type of dipole structures as in the full off-shell calculation
- For real emission from the decay the on-shell projection is done with one additional decay particle

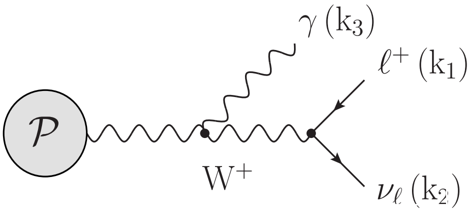


Real NLO EW corrections to charged resonances

- Diagrams with real radiation from the resonant propagators contribute
  - Split between production and decay part
- Additional divergent structures only present for on-shell resonances
  - Additional local counterterms (massive dipoles) are needed
  - Charged resonances take role of emitter and/or spectator in dipoles



Partial-fraction decomposition



$$\mathcal{A} = \mathcal{N}(k_1,k_2,k_3)\,\frac{1}{s_{123}-M_W^2+iM_W\Gamma_W}\cdot\frac{1}{s_{12}-M_W^2+iM_W\Gamma_W}$$
$$= -\frac{\mathcal{N}(k_1,k_2,k_3)}{s_{13}+s_{23}}\left(\frac{1}{s_{123}-M_W^2+iM_W\Gamma_W}-\frac{1}{s_{12}-M_W^2+iM_W\Gamma_W}\right)$$

- Use a partial fraction decomposition to split the divergence between the process where the photon is emitted from the production and from the decay amplitude

Partial-fraction decomposition

- Split resonances s_{12} and s_{123}
- Project s_{12} on-shell $\rightarrow$ divergence is only in the production amplitude

$$\tilde{\mathcal{A}}^{(2)} = \frac{1}{s_{12} - M_\mathrm{W}^2 + iM_\mathrm{W}\Gamma_\mathrm{W}} \left[\frac{\mathcal{N}(\tilde{k}_1^{(12)}, \tilde{k}_2^{(12)}, \tilde{k}_3^{(12)})}{\tilde{s}_{13}^{(12)} + \tilde{s}_{23}^{(12)}} \right]$$

- Project s_{123} on-shell $\rightarrow$ divergence is only in the decay amplitude

$$\tilde{\mathcal{A}}^{(3)} = \frac{1}{s_{123} - M_\mathrm{W}^2 + iM_\mathrm{W}\Gamma_\mathrm{W}} \left[-\frac{\mathcal{N}(\tilde{k}_1^{(123)}, \tilde{k}_2^{(123)}, \tilde{k}_3^{(123)})}{\tilde{s}_{13}^{(123)} + \tilde{s}_{23}^{(123)}} \right]$$

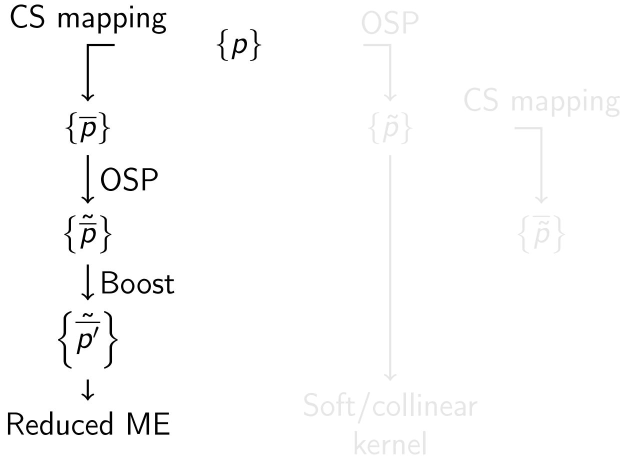
- Massive particle counterterms can be used to cancel the divergences in the production and decay amplitude

Resonance dipoles

- Decay dipoles
  - Newly derived dipole tailored to the W-boson decay reproducing its radiative decay
  - The other decay momenta (here the neutrino momentum) is used as the spectator for the subtraction mapping
- Production dipoles
  - Use the kernel structure for photon emission from a massive fermion [Catani, Dittmaier, Seymour, Trócsányi 0201036], [Dittmaier 9904440]
    - ⋆ Same singular behaviour as W-bosons

Evaluation of the local counterterms

- Off-shell real momenta $\{p\}$
- Reduced ME
  1. Catani-Seymour subtraction mapping
  2. Project on-shell (reduced)
  3. For polarised processes boost to the reference frame
- Soft/collinear kernel
  1. Project on-shell (real)
- The on-shell mapping and the subtraction mapping do not necessarily commute



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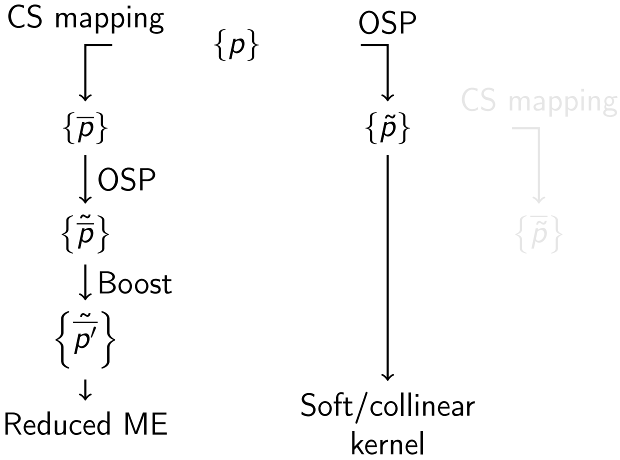

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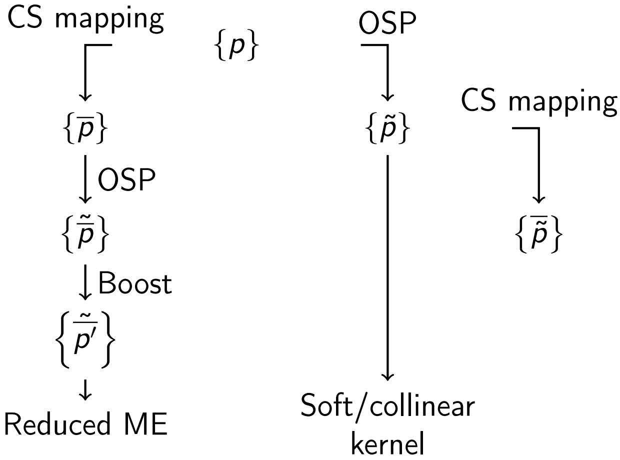


Correspondence between local and integrated counterterms

- Finite parts of the local and integrated counterterms need to cancel
- Analytic integration in d-dimensions to compute the integrated counterterms is done over the on-shell radiation phase space
- Numerical integration of the local counterterms is done over the off-shell radiation phase space
- This introduces a mismatch between the local and integrated counterterms
  - For our method this is an effect beyond the accuracy of the DPA
  - Reverse order (On-shell projection first, CS mapping second) gives potentially larger discrepancies

NLO polarised DPA W^+W^- pair-production

state	σ_{LO} [fb]	$\sigma_{\mathrm{NLO\,EW}}$ [fb]	$\delta_{\mathrm{EW}}[\%]$	$f_{\mathrm{NLO}}[\%]$
full	259.02(2)	253.95(9)	−1.96	103.4
unp.	249.97(2)	245.49(2)	−1.79	100.0
LL	21.007(2)	20.663(2)	−1.64	8.4
LT	33.190(3)	33.115(3)	−0.23	13.5
TL	34.352(5)	34.230(5)	−0.35	13.9
TT	182.56(2)	178.21(3)	−2.38	72.6
int.	−21.14(5)	−20.6(2)	−2.45	−8.4

- Now we have all parts needed to compute the polarised signals at NLO accuracy
- NLO corrections depend on the polarisation $\Rightarrow$ change polarisation fractions

Summary

- MoCaNLO and BBMC can be used to calculate polarise cross-sections at NLO accuracy
  - Showed how to treat real emission in the presence of charged resonances
- RECOLA 1.4.4 (publicly available) is used to get polarised tree-level and one-loop amplitudes
- Can be used to compute general multi-boson processes at NLO accuracy

Backup Slides

On-shell projection for two resonances (explicit form)

- Threshold for the on-shell projection

$$(M_{res,1} + M_{res,2})^2 \leq (p_{res,1} + p_{res,2})^2$$

- Begin with the construction of the on-shell momenta of the resonances
- Boost to centre-of-mass frame of the two resonances
- Set absolute value of the three momentum of the resonance

$$\left|\tilde{p}'_{1,res}\right| = \frac{\left(p_{tot}^2\right)^2 - 2p_{tot}^2\left(M_{res,1}^2 + M_{res,2}^2\right) + \left(M_{res,1}^2 - M_{res,2}^2\right)^2}{4p_{tot}^2}$$

- Set energy to full fill the on-shell condition
- Boost back to lab frame

On-shell projection for two resonances (explicit form)

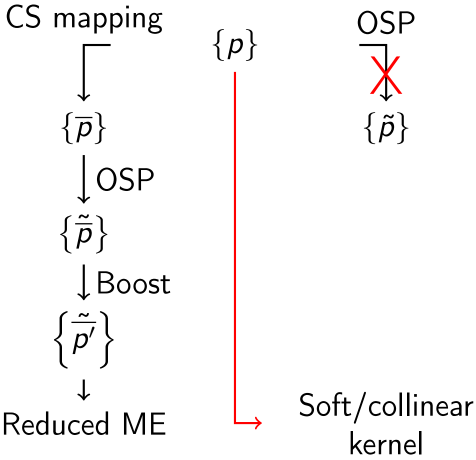
- Apply momentum rescaling to the decay momenta
  - Boost decay momentum into the rest frame of the off-shell resonance
  - Rescale decay momentum

$$\tilde{p}''_{decay} = \frac{M_{res}}{\sqrt{p^2_{res}}} p''_{decay} \qquad \text{(massless decay particles)}$$

  - Boost back from the decay frame of the on-shell resonance to the lab frame

Evaluation of the local counterterms

- The subtraction mapping is applied to the off-shell phase-space point
- There are phase-space points where
  - Reduced momenta can be projected on-shell
  - Real momenta cannot be projected on-shell
- Far from the singular regions
  - Local subtraction is not effected by the treatment of these events
- Evaluate counterterm kernels with off-shell real momenta



NLO QCD corrections

- Colour neutral resonances (Z bosons, W bosons)
  - Analogous to NLO EW with uncharged resonances
- Colour charged resonances (top quarks)
  - Similar to charged resonances in the EW case
  - Additional local counterterms are needed
  - Resonance is the emitter and/or spectator
  - Massive recoiler in the mapping of the decay counterterms
  - Need dipole for gluon entering the reduced process and the resonance as spectator