Polarization as a Feynman Rule

#COMETA Polarization Workshop – Toulouse, FR

Richard Ruiz[1]

Institute of Nuclear Physics – Polish Academy of Science (IFJ PAN)

23 September 2024

[1] w/ Javurkova, Lopes de Sá, Sandesara (PLB'24) [2401.17365]

thank you for the invitation!

brief motivation

(it has been a long day)

Broad motivation for exploring helicity polarization in of high-energy scattering (req. lots of data!)

Desireable for MC tools

– loop-induced in QCD and/or EW

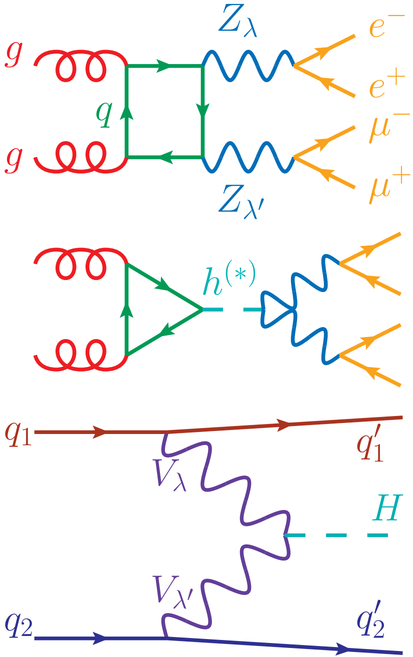
– NLO in QCD and/or EW

– off-shell/finite-width

– interference **(int.)** between resonant and non-res. diagrams

– int. between polarization configurations

– *s*- and *t*-channel configurations



state-of-the-art tools cannot do everything (yet)

motivated to consider new approaches to calculate polarized scattering rates

Popular (and successful) paradigm: decompose numerator of propagator via completeness relationship

care is need at this step!

$$-g_{\mu\nu} \ + \ q_\mu q_\nu / M_V^2 \ = \ \sum_{\lambda=\pm,0,S} \ \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda)$$

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vector boson propagator becomes sum over **truncated propagators**

similar result for massive fermions

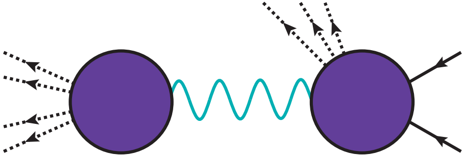
$$\Pi^V_{\mu\nu}(q) = \frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / M_V^2\right)}{q^2 - M_V^2 + iM_V\Gamma_V}$$

$$= \sum_{\lambda\in\{0,\pm1,A\}} \underbrace{\eta_\lambda}_{\pm1} \underbrace{\left(\frac{i\varepsilon_\mu(q,\lambda)\ \varepsilon^*_\nu(q,\lambda)}{q^2 - M_V^2 + iM_V\Gamma_V}\right)}_{\equiv \Pi^{V\lambda}_{\mu\nu} \text{ truncated prop.}}$$

Unpolarized matrix elements (MEs) become sum over **polarized MEs**

$$\mathcal{M} = \mathcal{M}_f^{\mu} \left(\sum_{\lambda \in \{\pm 1, 0, A\}} \eta_\lambda \times \Pi_{\mu\nu}^{V\lambda} \right) \mathcal{M}_i^{\nu}$$

$$= \sum_{\lambda \in \{\pm 1, 0, A\}} \eta_\lambda \times \underbrace{\mathcal{M}_f^{\mu} \cdot \Pi_{\mu\nu}^{V\lambda} \cdot \mathcal{M}_i^{\nu}}_{=\mathcal{M}_\lambda \text{ polarized ME}}$$

$-i\mathcal{M}_\lambda =$ 

$\mathcal{M}_f^{\mu} \cdot \quad \Pi_{\mu\nu}^{V\lambda}(q) \quad \cdot \mathcal{M}_i^{\nu}$

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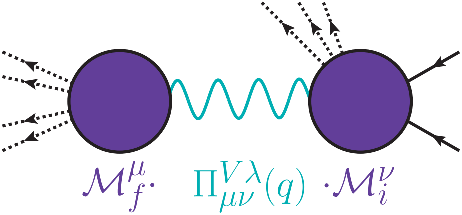

Polarized cross sections are then built from **polarized MEs**

$$d\sigma_\lambda = \frac{1}{\text{flux}} \frac{1}{\text{spin/color avg.}} \sum_{\text{dof.}} \int dPS \, |\mathcal{M}_\lambda|^2$$

etc., etc., etc.,

lots of good talks today!

one step back

Unpolarized ME becomes sum over polarized MEs

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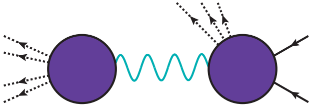
$-i\mathcal{M}_\lambda =$ 

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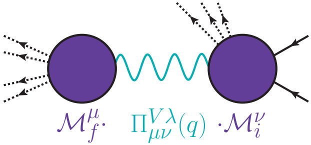
$-i\mathcal{M}_\lambda =$ 

Question: what do these two lines mean?

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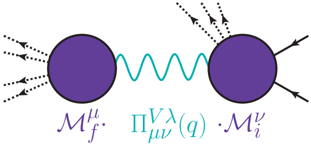
$-i\mathcal{M}_\lambda =$ 

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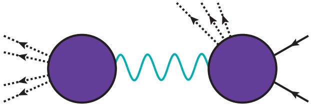
$-i\mathcal{M}_\lambda =$ 

Question: what do these two lines mean?
top: full **matrix element** is sum over **helicity polarizations**
btm: full **matrix element** is sum over ***sub*amplitudes**

the big idea

treat $\mathcal{M}_\lambda$ as a full subamplitude, not “just” as a component of a subamplitude

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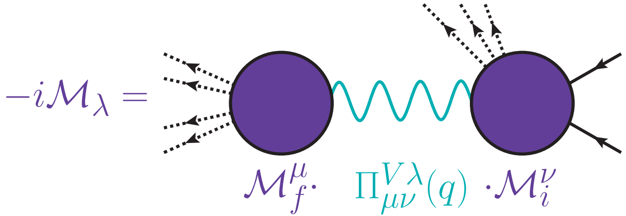

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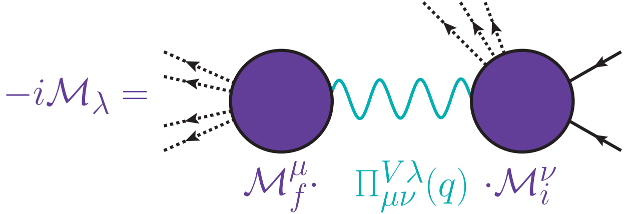
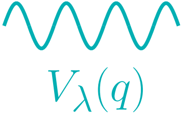
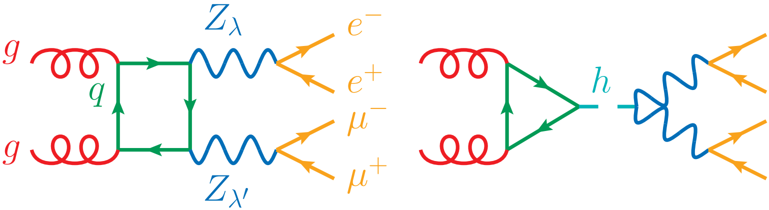


$\Longrightarrow$ **promoting truncated propagator to a Feynman rule**

$$\frac{-i\ \varepsilon_\mu(q,\lambda)\ \varepsilon^*_\nu(q,\lambda)}{q^2 - M_V^2} =$$



an example



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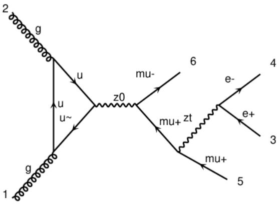
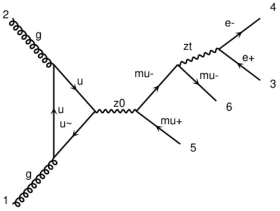


diagram 3 QCD=2, QED=4



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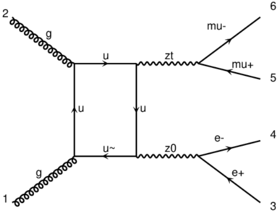

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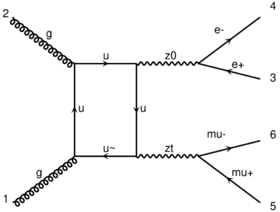

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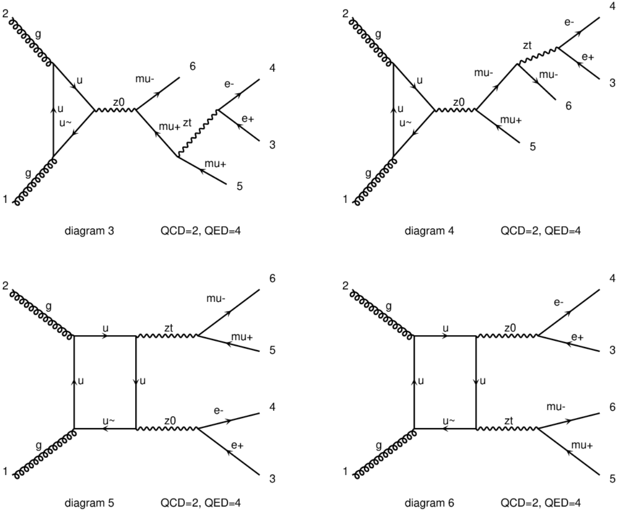
– **diagram filtering** then gives desired subset of diagrams (subamps.)



how?

by utilizing frameworks built for *new physics*

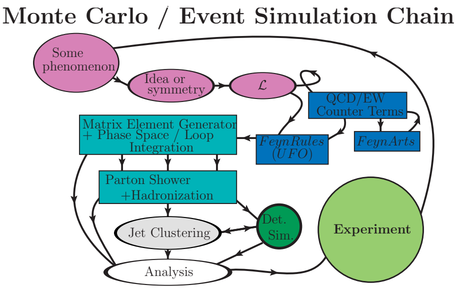
Tools for BSM@LHC

Monte Carlo Tool chains have long been adapted for new physics:

– new particles with spin 0, 1, 1/2, 2

– new vertices

– alternative propagators



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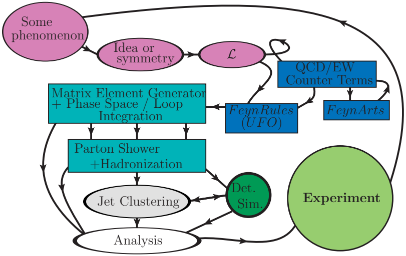
Monte Carlo / Event Simulation Chain



FeynRules

A Mathematica package to calculate Feynman rules

Universal FeynRules Object (UFO) libraries encode (.py) Feynman rules (incl. UV and R2 count terms) for MadGraph5, SHERPA, . . .

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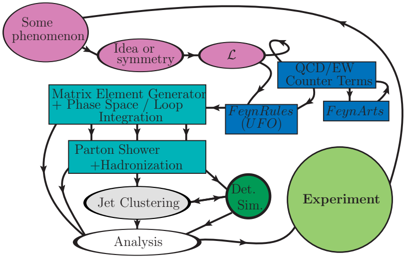


We wrote a UFO with $W^{\pm}_{\lambda}$ and Z_{λ}

`VPolar` feynrules.irmp.ucl.ac.be/wiki/VPolarization

```
...
Definitions ->
    {Z[mu_]->Z0[mu]+ZT[mu]+ZA[mu]+ZX[mu]}
...
Definitions ->
    {W[mu_]->W0[mu]+WT[mu]+WA[mu]+WX[mu]}
```

the result

Desireable approach for MC tools

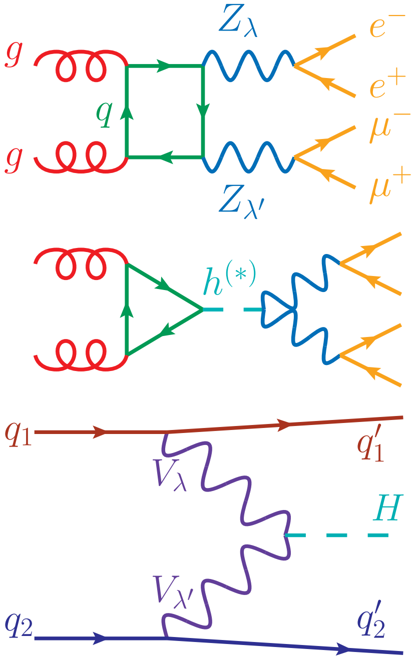
– loop-induced in QCD ✓

– off-shell/finite-width ✓

– interference **(int.)** between resonant and non-res. diagrams ✓

– int. between polarization configurations ✓

– *s*- and *t*-channel configurations ✓



some numbers

Example: full $2 \to 4$ process
$gg \to e^+e^-\mu^+\mu^-$ at $\mathcal{O}(\alpha_s^2\alpha^4)$

– `MG5aMC@NLO`
– all res. and non-res. diags.
– finite width (no NWA)
– no γ^* (for simplicity)
– phase space cuts

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$$\sigma_{ZZ} \times K^{\rm NLO} = 2.27 \text{ fb} {}^{+25\%}_{-19\%} {}^{+1\%}_{-1\%};$$
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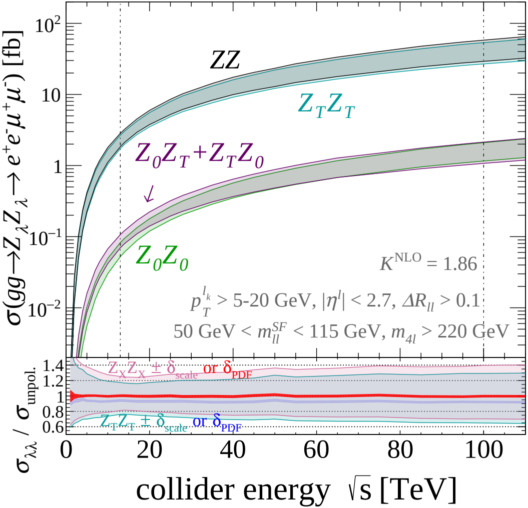


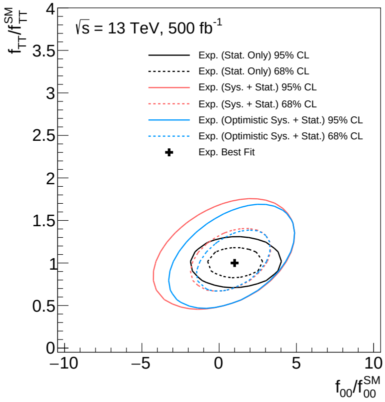
polarization fraction at $\sqrt{s} = 13$ TeV:

$f_{TT} \approx 93.3\%$ $\quad f_{T0+0T} \approx 3.8\%$

$f_{00} \approx 2.9\%$

in agreement with Denner & Pelliccioli (JHEP'21) [2107.06579]

Physics resuls with $\mathcal{L}$ =500 fb^{-1} at $\sqrt{s} = 13$ TeV



Polarization fraction sensitivity for $gg \to Z_\lambda Z_\lambda \to e^+e^-\mu^+\mu^-$ over SM "background"

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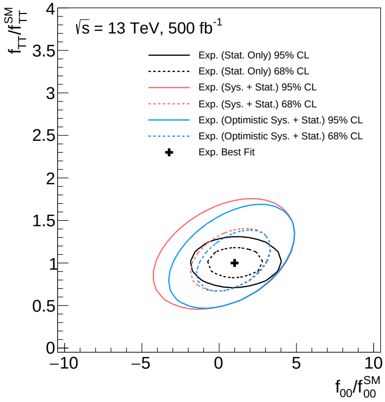

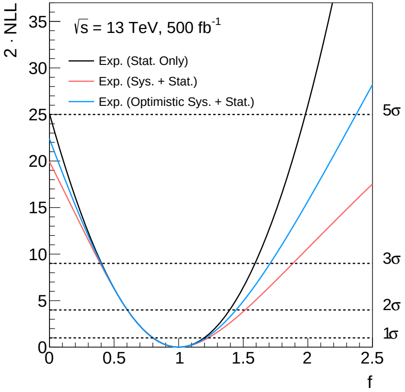

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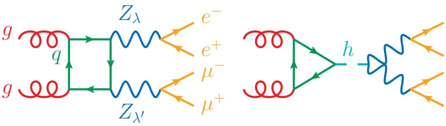
Spin correlation: falsify large, uncorrelated contribution ($f \neq 1$) by parameterizing total number of $e^+e^-\mu^+\mu^-$ events by

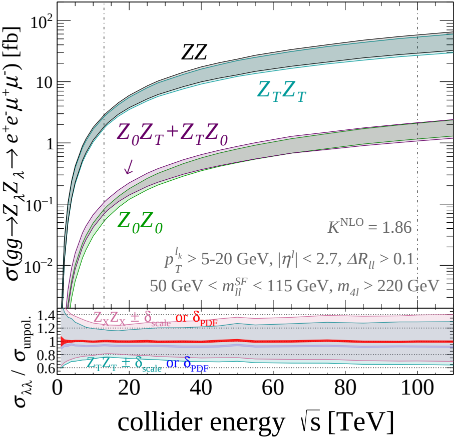
$$N = N^{q\bar{q}ZZ} + Z^{VBF} + f \cdot N^{ggF} + (1-f) \cdot N^{ggF}_{\text{uncorr.}}$$

summary

Treating **helicity polarization** as a **Feynman rule** provides a new method for computing polarized xsec

Javurkova, Ruiz, et al (PLB'24) [2401.17365]





- loop-induced processes ✓
- interference between different polarizations configurations ✓
- non-resonant diagrams ✓
- off-shell/finite-width effects ✓

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$V_\lambda(q)$

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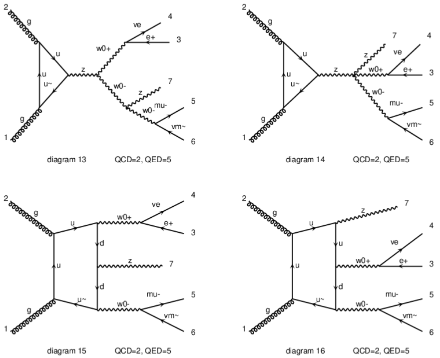
works for $gg \to VVV$



(resources limited!)

final words

senior postdoc vacancy in Krakow

3-year Adv/Senior Postdoctoral Researcher in Theoretical Particle Physics

Cracow, INP • Europe

hep-ph | hep-th | nucl-th | PostDoc

Deadline on Nov 15, 2024

Job description:
Job Title: Adv/Senior Postdoctoral Researcher

The Department of Theoretical Particle Physics (NZ42) at the **Institute of Nuclear Physics – Polish Academy of Sciences** (IFJ PAN) in Krakow, Pola
postdoctoral appointment ("adjunct" in Polish) in the group of Prof. Richard Ruiz.

inspirehep.net/jobs/2829053

thank you!

backup

Decomposing Propagators

Completeness relationships between **propagators** & **polarization vectors** in gauge theories are subtle. Example: **QED** in Feynman gauge

$\implies \xi = 1$ so $(1-\xi)q_\mu q_\nu/q^2 \to 0$:

$$-g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix} = \sum_{\lambda=\pm,0,S} \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda)$$

For $q = (q^0, 0, 0, q^3)$ and **transverse** pols $\varepsilon_\mu(\lambda = \pm) = (0, \mp 1, -i, 0)/\sqrt{2}$

$$\sum_{\lambda=\pm} \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda) = \begin{pmatrix} 0 & & & \\ & +1 & 0 & \\ & 0 & +1 & \\ & & & 0 \end{pmatrix}$$

Decomposing Propagators

For $q = (q^0, 0, 0, q^3)$ and **longitudinal** $\varepsilon_\mu(\lambda = 0) = (q^3, 0, 0, q^0)/\sqrt{q^2}$

$$\sum_{\lambda=0} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = \frac{q^2}{q^2}\begin{pmatrix} -1 & & & \\ & 0 & & \\ & & 0 & \\ & & & +1 \end{pmatrix} + \frac{q_\mu q_\nu}{q^2}$$

For "**auxiliary**" (A) or "**scalar**" (S) polarization $\varepsilon_\mu(\lambda = S) = q_\mu/\sqrt{-q^2}$

$$\sum_{\lambda=S} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = -\frac{q_\mu q_\nu}{q^2}$$

Precise form for $\lambda = 0, S$ depends on several factors:
– broken (massive) or unbroken (massless) gauge symmetry
– gauge (Feynman vs Landau vs Unitary vs Axial)
– gauge fixing ($\xi = 1$ or $n^2 = -1$)

Decomposing Propagators

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Example: for W/Z in Unitary gauge, $\varepsilon_\mu^{W/Z}(\lambda = S) = q_\mu\sqrt{\frac{1}{M_V^2} - \frac{1}{q^2}}$

$$\sum_{\lambda=S} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = -\frac{q_\mu q_\nu}{q^2} + \frac{q_\mu q_\nu}{M_V^2}$$

Decomposing Propagators

For $q = (q^0, 0, 0, q^3)$ and **longitudinal** $\varepsilon_\mu(\lambda = 0) = (q^3, 0, 0, q^0)/\sqrt{q^2}$

$$\sum_{\lambda=0} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = \frac{q^2}{q^2}\begin{pmatrix} -1 & & & \\ & 0 & & \\ & & 0 & \\ & & & +1 \end{pmatrix} + \frac{q_\mu q_\nu}{q^2}$$

For "**auxiliary**" (A) or "**scalar**" (S) polarization $\varepsilon_\mu(\lambda = S) = q_\mu/\sqrt{-q^2}$

$$\sum_{\lambda=S} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = -\frac{q_\mu q_\nu}{q^2}$$

Bonus, longitudinal polarization vectors can be written as Dawson ('85)

$$\varepsilon_\mu(\lambda = 0) = \frac{q_\mu}{\sqrt{q^2}} + \mathcal{O}\left(\frac{\sqrt{q^2}}{q^0}\right) \leftarrow \text{not an approximation}$$